Foreign Currency Risk: Hedging Performance in Won-dollar Futures Markets and Non Deliverable Forward (NDF) Markets

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We employ hedge strategies to cover the downside risk of the Won-dollar spot exchange rate using the Won-dollar futures and Non Deliverable Forwards (NDFs). We use several hedging methodologies such as the minimum variance hedge model, the time-varying bivariate GARCH(1,1) and ECT-ARCH(1) models comparing their hedge performances among hedging methods. We also examine which instrument is more effective against the risk of Won-dollar spot. The sample period covers from January 2, 2001 to December 28, 2002 using the daily Won-dollar spot, futures and NDF data.

During both within and out-of sample period, first, in terms of hedging models there is no big difference in the hedging performances between the conventional hedge model and time-varying hedge models in the Won-dollar futures markets. However, in the NDF markets, the hedge performance of time-varying hedge models is relatively better than conventional model. Second, in terms of derivative instruments the hedge effectiveness of Won-dollar futures markets is relatively better than that of NDF markets based on conventional minimum variance model but vice versa in time-varying hedging models.

INTRODUCTION

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Hedging is to reduce the risk and offset the loss generating from the spot position like commodities, foreign exchange, and financial assets with the use of derivatives. In particular, currency futures markets different from commodities and interest rates considering complicated variables such as transaction cost, storage cost, and duration tend to work well as a hedging instrument against the risk exposure of cash. This paper provides empirical evidence on the effectiveness of hedging to reduce the Won-dollar exchange risk among the major functions of the currency futures including the price discovery and speculating.

The volatilities of Won-dollar exchange rates have been increased due to the introduction of complete floating exchange rate system, the enhancement of capital market after the 1997 Korean currency crisis. The recent strength of European, Japanese, and Korean currencies due to the weakness of dollar has given imminent alertness of exchange volatility to corporate and bank exchange managers who are in charge of foreign exchange risk management. In addition, the abolition of foreign investors' investment restriction has also contributed the increase of the volatility of the Won-dollar exchange rates. Those circumstances have increased the degree of the market integration as well as risk exposure in Korean capital market. Now, financial managers have to contemplate reducing the volatility of the exchange rate by seeking and developing various financial techniques.

Many previous papers show the empirical evidence on the effectiveness of hedging with futures and forwards to cover the downside risk of spot position in the foreign exchange markets, stock and bond markets. However, they didn't show the consistent results whether the performance of time-varying hedging models such as ARCH, GARCH, and EGARCH etc. is better than that of conventional minimum variance hedging model. This is the first paper to test the optimal hedge ratio and to compare the hedge performance of time-varying hedge model with that of traditional hedge model with constant hedge ratio as well as to compare the hedge performance between Won-dollar futures and NDF forward market.

Several previous papers, Keynes(1930), Hicks, and Working(1953) suggested the theory of conventional hedging models and Johnson(1960), Stein(1961), Ederington(1979), Figlewski(1984), Baillie & Mayers(1991), Myers(1991), Kroner & Sultan(1993), Ghosh(1993), Park and Switzer(1995), Crain & Lee(1997) presented the improvement of hedging performance by time-varying bivariate GARCH hedging model and ECM (error correction model) developing the conventional hedging models.

Cecchetti, Cumby and Figlewski(1988) estimated the hedging ratios of U.S. treasury bill and reported the hedging ratios of T-bill changed as time goes by. They employed time-varying bivariate ARCH model suggested by Engle(1982). In particular, Kroner and Sultan(1993) estimated the hedge ratios in 5 foreign currency futures, the British pound, the Canadian dollar, the German mark, the Japanese yen, and the Swiss franc by a bivariate error correction model in exchange rates of spot and futures with a GARCH error structure. They also presented the dynamic hedge model provided greater risk reduction than the conventional models in both within and out-of-sample comparisons.

In this paper, we employ the futures and NDF of the Won-dollar exchange rates to reduce the risk of the Won-dollar exchange rates. Also, we compare the hedge ratios and effectiveness of the conventional hedge model with those of the time-varying ECM-ARCH hedge model. Both within and out-of-sample tests reveal which model achieves better performance. The important findings may be summarized as follows.

During both within and out-of sample period, first, in terms of hedging models there is no big difference in the hedging performances between the conventional hedge model and time-varying hedge models in the Won-dollar futures markets. However, in the NDF markets, the hedge performance of time-varying hedge models is relatively better than conventional model. Second, in terms of derivative instruments the hedge effectiveness of Won-dollar futures markets is relatively better than that of NDF markets based on conventional minimum variance model but vice versa in time-varying hedging models.

The rest of this paper is organized as follows. Section Ⅰ explains the data and preliminary statistics, while section Ⅱ presents the methodology. We put the main result in section Ⅲ and section Ⅳ concludes the paper.

Ⅰ. Data and Preliminary Statistics

The objective of hedging is to minimize the risk of the portfolio for given level of return. Factors that influence the hedge construction and its effectiveness include basis risk, hedging horizon, and correlation between changes in the futures price and the cash price. This paper prior to empirical tests suggests the brief introduction of the data, Won-dollar futures and NDF markets, and basic statistics of each time series.

The Won-dollar exchange futures started trading according as Korea Futures Exchanges (KFE) was established on April 23, 1999. In particular, the daily trading volume of the Won-dollar futures has increased by 1,465 contracts (1999), 5,556 contracts (2000), 6,817 contracts (2001), 5,879(2002), 6073(2003) and 8,052(2004.10). Figure 4 addresses the trend of Won-dollar futures' volume and figure 5 shows the trend of Won-dollar exchange rate.

The 1997 Korean currency crisis contributed to the quick establishment of the Won-dollar exchange futures market. Many investors have tried to reduce the risk of the Won-dollar exchange spot by the Won-dollar exchange futures. Since the inception of Won-dollar futures, the trading volume of Won-dollar futures market has been increased as follows;

Trading volume of Won-dollar futures

(unit: contract)

	1999	2000	2001	2002	2003	2004
Trading volume	255,249	1,355,730	1,676,979	1,434,591	1,506,123	1,651,383
Daily trading volume	1,465	5,556	6,817	5,879	6,073	8,052
Outstanding	22,516	31,261	23,754	19,675	23,650	54,924
No. of trading days	177	244	246	244	248	205

* The data for 2004 is based on the period from January 1, 2004 to October, 30, 2004.

Trading volume of Won-dollar spot and NDFs of Non-residents

(unit: billion US$)

	2001	2002	2003	2004 1/4	2004 2/4	2004 3/4
Daily average trading volume	0.51	0.67	1.34	1.35	1.68	1.58
NDF buying			179.1	54.4	58.2	57.6
NDF selling			152.9	42.2	44.4	41.9

This study uses daily price changes of Won-dollar spot, Won-dollar futures data on the nearby contract and Non-Deliverable Forwards(NDF) from January 2, 2001 to December 28, 2002. The data are from data-stream and Bloomberg. The closing data of the Won-dollar spot futures data are from 4:00 p.m. on the basis of Seoul Standard time. The price changes of all time series are calculated as follows:

$$RST_{\tau} = ST_t - ST_{t-1} \quad (1)$$

$$RFT_{\tau} = FT_t - FT_{t-1} \quad (2)$$

The terms, RST_{τ} means the price change of Won-dollar cash price. RFT_{τ} represent the daily price changes of Won-dollar futures and NDFs. Where ST_t and ST_{t-1} are the Won-dollar spot price at time t and at time t-1 respectively. FT_t and FT_{t-1} are the closing price of the Won-dollar futures and NDFs at time t and at time t-1 respectively.

Table 1 reports the summary statistics for the daily Won-dollar spot and futures, 1 month NDF and 3 month NDF data. According to the result of Table 1 the price changes of all data are negative, suggesting the Korean currency was strong during the sample period. In terms of level variables of four time series, the 1 month NDF has the highest standard deviation among. However in terms of price changes, the 3 month NDFs are relatively more volatile than the rest 3 series. All the level variables are skewed to the left, while all price changes are skewed to the right. Measures for excess kurtosis are leptokurtic with the normal distribution. The Bera-Jacque statistics for the level and return variables of the Won-dollar cash and futures, NDFs are statistically significant, indicating the presence of serial correlation (linear dependencies). This suggests the presence of autoregressive conditional heteroskedasticity, i.e. volatility clustering, which can be properly captured by the ARCH framework of Engle (1982) and GARCH model of Bollerslev(1986).

Furthermore, all the Won-dollar exchange spot, futures and NDFs series are tested to ensure whether they are stationary. As we expected the level variables are all non-stationary which means that each has a unit root in its autoregressive representation. This indicates that each series is non-stationary, necessitating the calculation of first differences and the difference series are then checked for the presence of a unit root. We see that the ADF and the PP tests clearly reject the null hypothesis of the presence of a unit root for each series, implying that the difference series are indeed stationary, that is, I(0).

Since it is established that each series is I(1), the next step is to test the co-integration relationship between Won-dollar spot and futures as well as between Won-dollar spot and NDFs. We employ the Johansen co-integration test. According to the test results, there is a co-integration relationship between the level variables of Won-dollar cash and futures data. However there is no co-integration relationship between the level variables of Won-dollar spot and NDFs data. Therefore, when we estimated the optimal hedge ratio and hedge performance of Won-dollar futures markets we incorporate the error-correction term in our hedging model suggested by Engle and Granger(1987). The error correction term imposes the long-run restrictions into this short-run model.

Measures for skewness and excess kurtosis indicate that all foreign currency series are significantly skewed and leptokurtic with respect to the normal distribution. The Bera-Jacque statistics for the return series of the Won-dollar exchange spot, futures and NDFs are statistically significant, indicating the presence of serial correlation (linear dependencies). This suggests the presence of autoregressive conditional heteroskedasticity, i.e. volatility clustering, which can be properly modeled by the ARCH framework of Engle (1982) and Bollerslev (1986).

Table 1

Data summary statistics for daily Won-dollar spot exchange rate and Won-dollar futures exchange rate from January 2, 2001 to December 28, 2002. Return of spot exchange rate and futures exchange rate is defined as the value: $RST_\tau = ST_t - ST_{t-1}$, $RFT_\tau = FT_t - FT_{t-1}$, where ST_t and FT_t is the spot and futures exchange value at time t.

B-J is the Bera-Jarque test for normality. The statistic is

$$\text{B-J} = T\left[\frac{skewness^2}{6} + \frac{(kurtosis-3)^2}{24}\right]$$

B-J is distributed X_2^2 under the null of normality. ***, ** indicate the significance at the 0.1 and 0.5 percent level, respectively.

Panel a: Won-dollar spot and futures exchange rates

	Spot		Futures	
	Rate	Return	Rate	Return
Mean	1270.98	-0.14	1273.27	-0.15
Median	1284.00	-0.10	1286.50	-0.20
Maximum	1365.20	21.50	1367.00	23.50
Minimum	1165.60	-23.10	1167.70	-21.00
Standard deviation	44.52	5.72	44.46	5.78
Skewness	-0.54	0.19	-0.52	0.21
Kurtosis	2.09	4.37	2.07	4.13
J-B	40.53***	40.89***	39.79***	29.49***

Panel b: NDF Forwards exchange rates

	1 Month NDF		3Month NDF	
	Rate	Return	Rate	Return
Mean	1273.37	-0.17820	1278.18	-0.1684
Median	1286.70	-0.2000	1290.80	-0.1000
Maximum	1366.00	23.5000	1370.00	25.0000
Minimum	1168.40	-22.0000	1173.60	-21.5000
Standard deviation	44.5554	5.7638	44.3099	5.8661
Skewness	-0.5204	0.21031	-0.4821	0.2169
Kurtosis	2.0676	4.3436	2.0442	4.3427
J-B	39.9444***	40.5556***	37.7108***	40.7382***

Ⅱ. Methodology

A. Ederington's (1979) Risk Minimization Hedge

Ederington (1979) suggests that the minimum variance hedge model in which the spot position is considered fixed and the optimal hedge ratio (number of futures contract per spot contract) is determined from the Ordinary Least Squares (OLS) regression of spot price changes on futures price changes. The optimal hedge ratio represents the minimum risk level for the spot/futures portfolio and consists of the covariance between the spot and futures divided by the variance of the futures. The objective of the hedger is to minimize the variance of the price changes for the Won-dollar exchange spot rate/futures rate portfolio. The expected price change and variance of the hedged position are established as follows;

$$RST_t - RST_{t-1} = \alpha + \beta(RFT_t - RFT_{t-1}) + \varepsilon_t \qquad (3)$$

Where $RST_t - RST_{t-1}$ represents the price change of the Won-dollar spot exchange rate from t-1 to t, $RFT_t - RFT_{t-1}$ represents the price change of the Won-dollar futures, 1 month and 3 month NDFs exchange rate from t-1 to t. β is the optimal hedge ratio estimated by the Ordinary Least Squares (OLS) regression. The slope coefficient of equation (3) is used as the measure of optimal hedge ratio under the conventional hedge model system. We also define the optimal hedge ratio as the covariance between Won-dollar cash and futures and between Won-dollar cash and NDF.

The regression results are reported in Table 2. The hedge ratios of 0.97636 for Won-dollar futures, 0.98794 and 0.96597 for 1 month NDFs and 3 month NDFS imply that 0.97636 daily contract, 0.98794 1 month NDFs and 0.96597 3 month NDFs of the Won-dollar futures and forwards markets needs to be shorted for a long position of 1 spot exchange to minimize the variance of the hedged position value change. This hedge ratio is considerably less than one, which implies that the Won-dollar futures and forwards exchange are more volatile than the Won-dollar spot exchange rate.

Hedging effectiveness (HE) of Won-dollar futures and NDFs markets can be measured as the percent reduction in the variance of the unhedged Won-dollar spot position by the risk minimization hedge as follows;

$$\text{HE} = \frac{Var(\Delta C_{t,t+1}) - Var(\Delta Portfolio_{t,t+1})}{Var(\Delta C_{t,t+1})} \qquad (4)$$

For example, the minimum variance of the Won-dollar spot exchange and futures portfolio value change is as follows:

$$Var(\Delta Portfolio_{t,t+1}) = Var(\Delta C_{t,t+1}) - \frac{[Cov(\Delta C_{t,t+1}, \Delta F_{t,t+1})]^2}{Var(\Delta F_{t,t+1})} \tag{5}$$

The same equation is applied for the minimum variance between Won-dollar spot and NDFs portfolio value changes. Consequently, from the above equations 4 and 5, we employ the following equation(6) to figure out the hedge performance between Won-dollar futures market and NDFs market.

$$\text{HE} = \frac{[Cov(\Delta C_{t,t+1}, \Delta_{t,t+1})]^2}{Var(\Delta C_{t,t+1}) Var(\Delta F_{t,t+1}} = \rho^2 \tag{6}$$

Where ρ^2 is the population coefficient of determination between Won-dollar spot and futures exchange changes as well as Won-dollar spot and forwards rate change, and it can be estimated as R^2., the sample coefficient of determination of regression 3. Table 2 reports R^2., of 0.9736, 0.9914, 0.9817 so that a 97.36%, 99.14%, and 98.17% reduction of the daily variance of the Won-dollar spot position has been achieved by the risk minimum hedging strategy. In details, If we have a long position of one (1) Won-dollar portfolio at foreign exchange spot market theoretically we have to take a short position of 0.97636 contract at the Won-dollar futures market to hedge the downside risk of Won-dollar spot position during the period from January 2, 2001 to December 28, 2002. As a result, the variance reduction for the hedged portfolio is 97.36% compared with the unhedged spot position. In case of Won-dollar forward markets, the risk averse hedger have to sell 0.9914 and 0.9817 NDFs to cover the downside risk in Won-dollar spot position.

Table 2

The estimation results of optimal hedge ratio using conventional minimum variance hedge model with constant hedge ratio to Won-dollar futures and NDFs market

To determine the optimal hedge ratio of Won-dollar futures, 1 month NDFs and 3 month NDFs to cover the downside risk of Won-dollar spot position, the following regression is estimated using time-matched daily data for the period from January 2, 2001 to December 28, 2002.

$$(RST_{\tau} - RST_{t-1}) = \alpha + \beta(RFT_t + RFT_{t-1}) + \varepsilon_t$$

where $\varepsilon_t = \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \eta_t$, the dependent variable is the price change of Won-dollar spot exchange rate and the independent variable is the price changes of Won-dollar futures and NDFs from day t and t+1, β coefficient represents the minimum risk hedge ratio (number of futures and NDFs contracts per one(1) Won-dollar spot position), and the coefficient of determination, R^2, measures the hedging effectiveness in terms of the percent reduction of the variance of the unhedged spot position. *** indicates the significance at the 1% percent level. Asymptotic t-statistics are given in parentheses.

	Won-dollar Futures Markets	1 Month NDF Market	3 Month NDF Market
α	+0.00531 (0.04198)	-0.00764 (-0.3189)	-0.02100 (-0.6009)
Hedging Ratio (β)	0.97636*** (134.28)	+0.98794*** (+237.45)	+0.96597*** (162.01)
Hedging Effectiveness (R^2)	0.9736	0.9914	0.9817
F	18032.26***	56371.00**	26250.29***

B. Bivariate GARCH and ECT-ARCH Hedge

Ederington (1979) suggests the minimum variance hedge model assuming that the hedge ratios are constant according to constant volatility during the test period. His assumption has been questioned by many scholars, including Grmmatikos and Sauders (1983), Malliaris and Urrutia (1991a, 1991b) and Cecchetti, Cumby, and Figlewski(1988), Kroner and Sultan(1993), Crain & Lee(1997).

In particular, Cecchetti, Cumby, and Figlewski(1988) introduce Autoregressive Conditional Heteroskedasticity (ARCH) model developed by Engle(1982) to solve for time-varying volatility and find that the optimal hedge ratio varies over time. We also employ the ARCH model and we incorporate the error correction term into the estimation model due to the co-integration relationship between Won-dollar cash and futures market. According to the model specification test including maximum likelihood ratio, we find that ARCH(1) model is well fitted to test the optimal hedge ratio of Won-dollar futures market. The bivariate ECT-ARCH(1) for Won-dollar futures market is established as follows;

$$RST_\tau = \alpha_{0s} + \alpha_{1s}(RST_{t-1} + \gamma RFT_{t-1} - C) + e_{st} \quad (7)$$
$$RFT_\tau = \alpha_{0f} + \alpha_{1f}(RST_{t-1} + \gamma RFT_{t-1} - C) + e_{ft}, \quad (8)$$
$$\begin{bmatrix} e_{s,t} \\ e_{f,t} \end{bmatrix} \mid \psi_{t-1} \sim \mathrm{N}(0, \mathrm{H}_t), \quad (9)$$
$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{s,t-1} \\ \varepsilon_{s,t-1}, \varepsilon_{f,t-1} \\ \varepsilon^2_{f,t-1} \end{bmatrix} \quad (10)$$

On the other hand, when we estimate the optimal hedge ratio for 1 month and 3 month NDFs market we do not incorporate the error correction term into the Generalized Autoregressive Conditional Heteroskedastic(GARCH) model of Bollerslev(1986) because there was no co-integration relationship between the level variables of Won-dollar cash and NDF markets.

The advantage of the GARCH model is that very convenient assumption about the conditional density of Won-dollar price changes can lead to a rich model that allows for time-dependent conditional variance and leptokurtosis in the unconditional distribution of price changes. GARCH model allows for weak dependence in the form of interactions at higher moments.

GARCH models have already proved useful in explaining the distribution of foreign exchange rates, e.g. McCurdy and Morgan(1987), Milhoj(1987), Diebodl and Nerlove(1989), Baillie and Bollerslev(1989). Therefore we employ GARCH(1,1) model because this model well characterizes the dynamics in the second moments of Won-dollar spot and NDFs price. The bivariate GARCH(1,1) model to test the optimal hedge ratio of Won-dollar NDFs market against Won-dollar spot market is employed as follows;

$$RST_\tau = \alpha_{0s} + e_{st} \quad (11)$$
$$RFT_\tau = \alpha_{0f} + e_{ft} \quad (12)$$

$$\begin{bmatrix} e_{s,t} \\ e_{f,t} \end{bmatrix} \mid \psi_{t-1} \sim \mathrm{N}(0,\mathrm{H}_t) \quad (13)$$
$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} =$$

$$\mathrm{H}_t = \begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{ss,t-1} \\ h_{sf,t-1} \\ h_{ff,t-1} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{s,t-1} \\ \varepsilon_{s,t-1}, \varepsilon_{f,t-1} \\ \varepsilon^2_{f,t-1} \end{bmatrix} \quad (14)$$

where RST_τ are the price change of the Won-dollar spot and RFT_τ means Won-dollar futures contracts and NDFs, respectively. e_τ is a (2x1) vector of residuals, $\psi_{\tau-1}$ is the information set at time t-1. H_t is a (2x2) conditional variance-covariance matrix of residuals. Under the assumption of nonconstant correlation, where $h_{ss,\tau}$ and $h_{ff,\tau}$ are the variance of $e_{s,\tau}$ and $e_{f,\tau}$, respectively, and $h_{sf,\tau}$ is the covariance between $e_{s,\tau}$ and $e_{f,\tau}$. In the conditional variance equation 10 of ECT-ARCH(1) model and the conditional variance equation 14 of GARCH(1,1) model, we assume that the b and c matrices are diagonal to reduce the number of parameters from 12 to 6 and from 21 to 9 respectively. This simplifying assumption was made by most prior studies including Bollerslev, Engle, and Wooldridge (1988), due to the large number of parameters to be estimated. Accordingly, we can get the following equation.

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{s,t-1} \\ \varepsilon_{s,t-1}, \varepsilon_{f,t-1} \\ \varepsilon^2_{f,t-1} \end{bmatrix} \tag{15}$$

$$H_t = \begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{ss,t-1} \\ h_{sf,t-1} \\ h_{ff,t-1} \end{bmatrix} + \begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{s,t-1} \\ \varepsilon_{s,t-1}, \varepsilon_{f,t-1} \\ \varepsilon^2_{f,t-1} \end{bmatrix} \tag{16}$$

In the ECT-ARCH(1) model, the total coefficients that we have to estimate for the hedge ratio and hedge performance are 10; 4 from conditional mean equation 7 and 8, 6 from conditional variance equation 15. On the other hand, in the GARCH(1,1) model the total number of parameters to estimate the optimal hedge ratio and hedge performance of Won-dollar futures and forwards markets is 11; 2 from conditional mean equation 11 and 12, 9 from conditional variance equation 16. In bivariate GARCH(1,1) systems, the diagonal vech parameterization involves nine conditional variance parameters while the positive definite parameterization has 11.

These coefficients are estimated using the Berndt, Hall, Hall, and Hausman (1974) algorithm. According to Baillie and Myers(1991), the expected return to holding futures is zero, the minimum variance hedging rule leads to a hedge ratio which depends solely on the elements of the conditional variance-covariance matrix, H_t. In particular, the minimum variance hedge ratio, HR^*_{ft}, is expressed as follows;

$$HR^*_{ft} = \frac{h_{sf,t}}{h_{ff,t}} \qquad (17)$$

where $h_{sf,t}$ are the conditional covariance between the Won-dollar spot and futures markets, between Won-dollar spot and NDF markets. $h_{ff,t}$ means the conditional variance between the Won-dollar futures markets and Won-dollar forwards markets, respectively.

Table 3 and Table 4 report the dynamic optimal hedge ratios for the Won-dollar futures and forwards markets to cover the downside risk of Won-dollar spot position from the period of January 2, 2001 to December 28, 2002. The coefficients are similar to those computed form the conventional minimum variance hedging model: 0.97615 for Won-dollar futures market, 0.99537 and 0.98082 for Won-dollar 1 month NDFs and 3 month NDFs respectively. This means that 97.615%, 99.537%, and 98.082% reduction of the daily variance of the won-dollar spot exchange position has been achieved by the time varying ECT-ARCH(1) and GARCH(1,1) hedging models, respectively.

Also, similar to the risk minimization hedge, the hedge effectiveness of the bivariate ECT-ARCH(1) and GARCH(1,1) can be measured as the percent reduction in the variance of the unhedged Won-dollar spot position. Therefore the hedge performance of ECT-ARCH(1) and GARCH(1,1) models are estimated as follows;

Hedge Performance: R^2 = 1-Var(HP)/Var(UP) (18)

Where, Var(HP) is the variance of hedged portfolio, Var(UP) means the variance of unhedged portfolio.

Table 3

The estimation results of optimal hedge ratio in Won-dollar futures markets using ECT-ARCH(1) model

Estimates of the following bivariate ECT-ARCH model are established as follows;

$$RST_\tau = \alpha_{0s} + \alpha_{1s}(RST_{t-1} + \gamma RFT_{t-1} - C) + e_{st}$$
$$RFT_\tau = \alpha_{0f} + \alpha_{1f}(RST_{t-1} + \gamma RFT_{t-1} - C) + e_{ft}$$
$$\begin{bmatrix} e_{s,t} \\ e_{f,t} \end{bmatrix} \mid \psi_{t-1} \sim \mathrm{N}(0, \mathrm{H}_t)$$
$$\begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{s,t-1} \\ \varepsilon_{s,t-1}, \varepsilon_{f,t-1} \\ \varepsilon^2_{f,t-1} \end{bmatrix}$$

where RST_τ and RFT_τ are the spot rate and futures price changes, respectively, e_τ is a (2x1) vector of residuals, $\psi_{\tau-1}$ is the information set at time t-1, H_t .is a (2x2) conditional variance-covariance matrix of residuals, and the a and b matrices are assumed to be diagonal. The model is estimated using time-matched daily from January 2, 2001 to December 28, 2002. ***, **, * indicate the significance at the 0.1, 0.5, and 1 percent level, respectively. Standard deviation is given in parentheses.

	Won-dollar Futures market
a_{0s}	-1.70192** (-0.85845)
a_{0f}	-2.99985*** (-0.87939)
a_{1s}	0.44145* (0.23172)
a_{2s}	0.79816*** (0.23740)
b_{11}	26.20846*** (1.74960)
b_{22}	26.40536*** (1.79913)
b_{33}	27.10554*** (1.87635)
c_{11}	0.16816*** (0.04223)
c_{22}	0.16182*** (0.23172)
c_{33}	0.15867*** (0.04200)
Log-L	-1223.12
$\overline{HR}$	0.97615***

Table 4

The results of optimal hedge ratio using the Won-dollar NDF markets using bivariate ECT-ARCH(1) and GARCH(1,1) models

Estimates of the following bivariate GARCH(1,1) model is established as follows;

$$RST_\tau = \alpha_{0s} + e_{st},$$
$$RFT_\tau = \alpha_{0f} + e_{ft}$$

$$\begin{bmatrix} e_{s,t} \\ e_{f,t} \end{bmatrix} \mid \psi_{t-1} \sim \text{N}(0,\text{H}_t), \quad H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix}$$

$$\text{H}_t = \begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{ss,t-1} \\ h_{sf,t-1} \\ h_{ff,t-1} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{s,t-1} \\ \varepsilon_{s,t-1}, \varepsilon_{f,t-1} \\ \varepsilon^2_{f,t-1} \end{bmatrix}$$

where RST_τ and RFT_τ are the Won-dollar spot rate and NDF forwards price changes, respectively, e_τ is a (2x1) vector of residuals, $\psi_{\tau-1}$ is the information set at time t-1, H_t .is a (2x2) conditional variance-covariance matrix of residuals, and the a, b and c matrices are assumed to be diagonal. The model is estimated using time-matched daily from January 2, 2001 to December 28, 2002. ***, **, * indicate the significance at the 1, 5, and 10 percent level, respectively. Standard deviation is given in parentheses.

	1 Month NDF market	3Month NDF markets
a_{0s}	+0.011649 (+0.20868)	+0.08571 (+0.18024)
a_{0f}	+0.02309 (+0.21050)	+0.10459 (+0.17804)
a_{11}	+22.49248*** (+0.78934)	+23.21146*** (+1.71877)
a_{22}	+22.64801*** (+0.71437)	+28.29347*** (+2.96977)
a_{33}	+22.53457*** (+0.72771)	+29.45349*** (+4.90508)
b_{11}	+0.32758*** (+0.03306)	+0.42171*** (+0.04051)
b_{22}	+0.32385*** (+0.03158)	+0.30601*** (0.07292)
b_{33}	+0.32822*** (+0.03143)	+0.28779*** (+0.11809)
c_{11}	+0.23891*** (0.02730)	+0.11394*** (+0.02928)
c_{22}	+0.23833*** (+0.02609)	+0.11144*** (+0.02955)
c_{33}	+0.23778*** (0.02524)	+0.11859*** (0.03073)
Log-L	-894.23	-1181.00
$\overline{HR}$	+0.99537*** (+0.01060)	+0.98082*** (+0.01985)

Ⅲ. Comparisons of Hedging Performance among Won-dollar futures and NDF forward markets

The objective of this paper is to compare the conventional hedge model with constant hedge ratio with ARCH family model with time-varying hedge ratios. Also, we intended to address the issue of choosing between Won-dollar futures and NDF forwards to hedge the downside risk of Won-dollar spot position.

For these purposes, we divide the full sample period into two sub-period. One is within sample period from January 2, 2001 to September 28, 2002 and the other is out-of-sample period from October 1, 2002 to December 28, 2002. Figure 1, Figure 2 and Figure 3 plot the optimal hedge ratios for the conventional minimum variance hedge model and for time-varying hedging models. As we expected the hedging ratios are changing as new information comes to the foreign exchange market over time.

For within-sample hedging period, the computation of the optimal hedging ratio and the hedging effectiveness is calculated simultaneously. In other words, when we develop hedge model we assume a perfect forecasting on the future prices of Won-dollar spot, Won-dollar futures and NDFs in the future time. These kinds of assumption are obviously far from real world. Therefore we need to figure out the hedge performance during out-of-sample period in which an optimal hedge ratio from the historical data is estimated and apply it to study the hedge effectiveness.

As mentioned above, the hedging performance is measured by the percent reduction in the variance of the unhedged Won-dollar spot exchange. Table 5 provides the result on hedge performance of each hedging model during the whole sample period. According to within sample period, when we estimate the optimal hedge ratio, we assume perfect forecasting and update the optimal hedge ratio at the end of everyday for bivariate ECT-ARCH(1) and GARCH(1,1) hedge models, while we use only one constant hedge ratio (=0.97636 for Won-dollar futures, 0.98794 for 1 month NDFs forwards, 0.96597 for 3 month NDF forwards per unit Won-dollar spot exchange) during the within-sample period (1/2/2001-9/28/2002) for the conventional minimum variance hedge model.

According the test results during the within sample period, First, in terms of the comparison of hedge models there is no big difference in the hedging performances between the conventional minimum variance hedge model (+0.97626) and dynamic time-varying ECT-ARCH(1) hedging models (+0.97592) in the Won-dollar futures markets. However, in the Won-dollar NDF forward markets, the hedge performance of 1 month NDFs (+0.99013) and 3 Month NDFs (+0.97826) using bivariate GARCH(1,1) model are relatively better than conventional minimum variance hedge strategy (+0.96628 and +0.90885).

Second, in terms of the hedge performance comparison between Won-dollar futures markets and Won-dollar NDF forward markets, the hedge effectiveness of Won-dollar futures markets (+0.97626) is relatively better than that of NDF forward markets (+0.96628 and +0.90885) in the conventional minimum variance model using a constant hedge ratio. On the other hand, the hedge performance (+0.99013 and +0.97826) of Won-dollar NDF forwards is relatively better than that of Won-dollar futures markets (+0.97592) based on time-varying conditional hedging models.

Table 5

Comparisons of Hedging Effectiveness between Won-dollar futures and NDF forward markets during the within sample period

The reduction of variances in the hedged spot/futures portfolio value is reported. The within-sample results are computed based on daily hedge ratio updates for the period from January 2, 2001 to September 28, 2002. The percent reduction in variance is computed as follows;

1- (variance of the hedged position / variance of the unhedged position)

The values shown are the estimates times 10^2..

Method	Won-dollar Futures markets	1Month NDF markets	3Month NDF markets
Minimum Variance Hedge Model	0.97626	+0.96628	+0.90885
ECT-ARCH(1) or GARCH(1,1) Models	0.97592	+0.99013	+0.97826

Now, we turn to the out-of-sample hedging. The hedger tries to minimize the conditional variance of the end-of-sample period wealth by sequentially updating information and choosing the hedge ratio. The results are reported on table 5. First, in terms of the hedge performance comparison of hedge models there is no big difference in the hedging performances between the conventional minimum variance hedge model (+0.98292) and dynamic time-varying ECT-ARCH(1) hedging models(+0.98092) in the Won-dollar futures markets. However, in the Won-dollar NDF forward markets, the hedge performances (+0.99536 and +0.99595) of the time-varying bivariate GARCH(1,1) model are relatively better than conventional minimum variance hedge model (+0.92867 and +0.84481).

Second, in terms of the hedge performance comparison between Won-dollar futures markets and Won-dollar NDF forward markets, the hedge performance of Won-dollar futures markets (+0.98292) is relatively better than that of NDF forward markets (+0.92867 and

+0.84481) in the conventional minimum variance model with a constant hedge ratio. On the other hand, the hedge performance (+0.99536 and +0.99595) of Won-dollar NDF forwards markets are relatively better than that of Won-dollar futures markets (+0.98092).

Table 6

Comparisons of Hedging Effectiveness between Won-dollar futures and NDF forward markets during out-of-sample period

The reduction of variances in the hedged spot/futures portfolio value is reported. The out-of-sample results are computed based on daily hedge ratio updates for the period from October 1, 2002 to December 28, 2002. The percent reduction in variance is computed as follow;

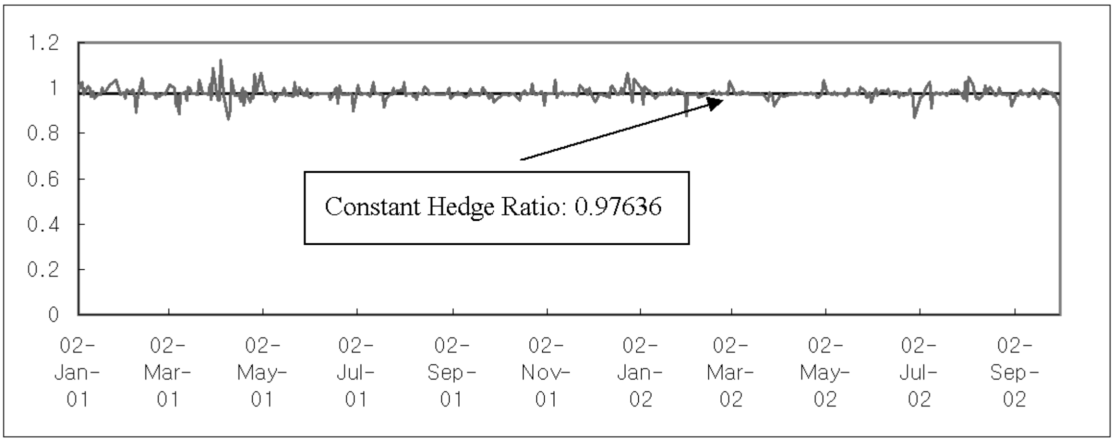
1- (variance of the hedged position / variance of the unhedged position)

The values shown are the estimates times 10^2..

Method	Won-dollar Futures markets	1Month NDF markets	3Month NDF markets
Minimum Variance Hedge Model	0.98292	+0.92867	+0.84481
ECT-ARCH(1) or GARCH(1,1) Models	0.98092	+0.99536	+0.99595

Figure 1: Optimal hedge ratio of Minimum variance hedge model and time varying GARCH(1,1) models for Won-dollar futures markets

A: Within sample period



B: Out-of sample period

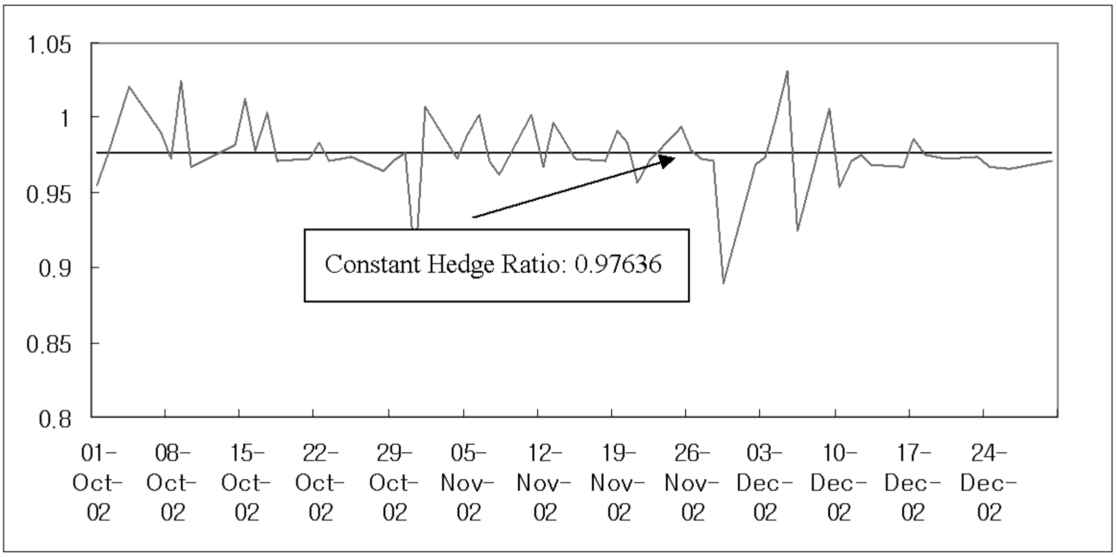
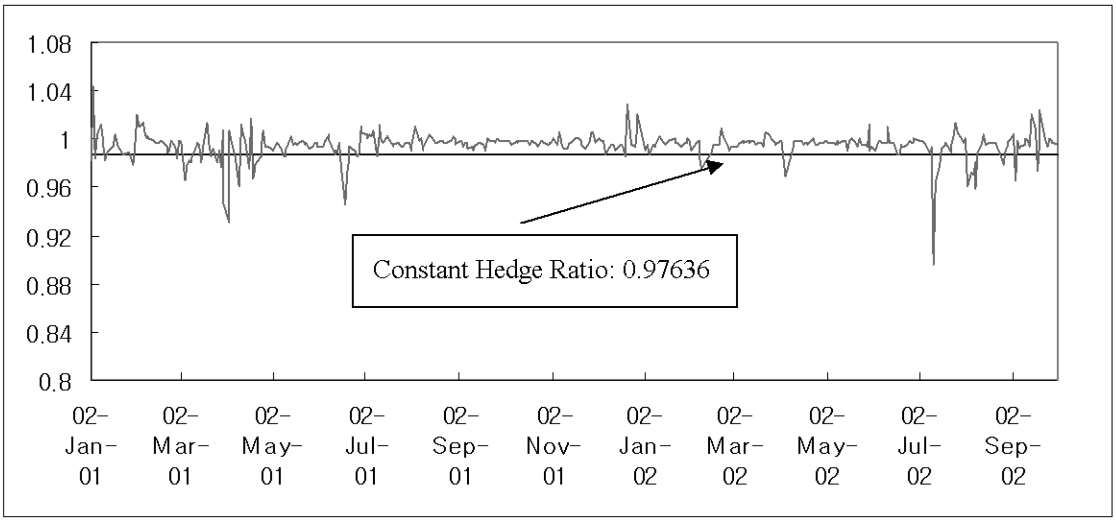


Figure 2: Optimal hedge ratio of Minimum variance hedge model and time varying GARCH(1,1) models for 1M NDF exchange rate

A: Within sample period



B: Out-of sample period

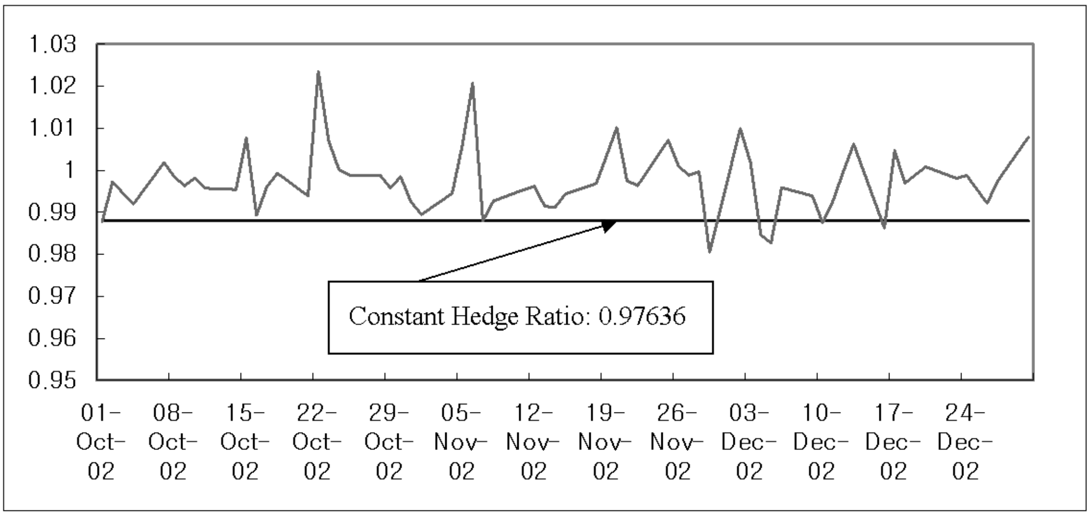
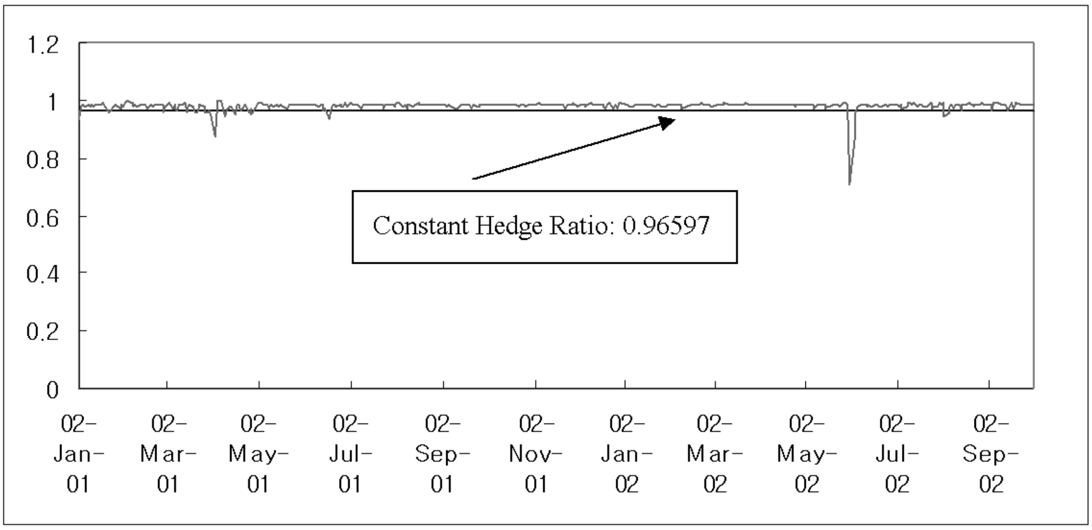
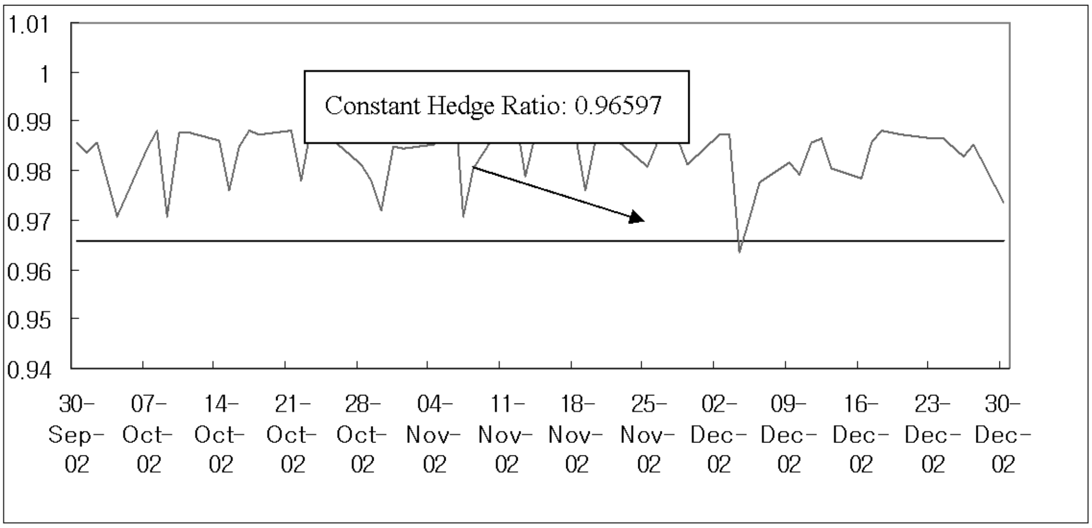


Figure 5: Optimal hedge ratio of Minimum variance hedge model and time varying GARCH(1,1) models for 3 Month NDF exchange rate

A: Within sample period



B: Out-of sample period



Ⅳ. Conclusion

This study presents alternative hedging models for calculating risk-minimizing hedge ratios in Won-dollar currency futures and Non-Deliverable Forwards contracts and compares the hedging performance of conditional hedging models with that of a conventional hedging method with constant optimal hedge ratio over time. For this purpose, we employ ECT-ARCH(1) and GARCH(1,1) models for conditional hedging models with time-varying optimal hedge ratio over time. The data we employ is the daily Won-dollar spot, Won-dollar futures, 1 month NDF and 3 Month NDF forward contracts from January 2, 2001 to December 28, 2002.

According to the estimation results on optimal hedging ratio for Won-dollar futures and forwards markets, optimal hedge ratio for Won-dollar futures markets are relatively lower than those of Won-dollar NDF forwards market both in the conventional hedge model and time-varying ECT-ARCH(1) and GARCH(1,1) model during the full sample period. The optimal hedge ratio of 1 month NDF forward is better than that of 3 month NDF forward in which these results come from the difference of the trading volume of NDFs.

According to the estimation results of hedge performance during the within sample period, the evidence presented in this paper indicate that First, in terms of the comparison of hedge models there is no big difference in the hedging performances between the conventional minimum variance hedge model and time-varying hedging models in the Won-dollar futures markets. However, in the Won-dollar forward markets, the hedge performance of time-vary bivariate GARCH(1,1) model are relatively better than conventional minimum variance hedge model. These results are consistent with Kroner and Sultan (1991), Myers (1991).

Second, in terms of the hedge performance comparison between Won-dollar futures and forward markets, the hedge effectiveness of Won-dollar futures markets is relatively better than that of Won-dollar forward markets in the conventional minimum variance model with a constant hedge ratio. However the hedge performance of Won-dollar forwards is relatively better than that of Won-dollar futures markets based on time-varying conditional hedging models.

Next, according to the estimation results of hedge performance during the out-of sample period, the evidence reports that First, in terms of the comparison of hedge models there is no big difference in the hedging performances between the conventional minimum variance hedge model and dynamic time-varying ECT-ARCH(1) hedging models in the Won-dollar futures markets. These results are consistent with Kroner and Sultan (1991), Myers (1991).

However, in the Won-dollar NDF forward markets, the hedge performance of time-varying bivariate GARCH(1,1) model are relatively better than conventional minimum variance hedge strategy. Second, in terms of the hedge performance between Won-dollar futures and forward markets, the hedge effectiveness of Won-dollar futures markets is relatively better than that of NDF forward markets in the conventional minimum variance model using a constant hedge ratio. On the other hand, the hedge performance of Won-dollar NDF forwards is relatively better than that of Won-dollar futures markets based on the time-varying ECT-ARCH(1) and GARCH(1,) models.

REFERENCES

Baillie, R. and Myers, R., Bivariate GARCH estimation of the optimal commodity futures hedge, *Journal of Applied Econometrics,* 6, (1991), 109-124.

Berndt, E. K., B. H. Hall, R. E. Hall and J. A. Hausman C.: Estimation and Inference in Nonlinear Structural Models, *Journal of Economic and Social Measurement,* 1974, 653-665.

Bollerslev, T., Modelling the Coherence in Short-Run Nominal Exchange Rates: A Mulivariate Generalized ARCH Approach, *Review of Economics and Statistics,* 72, 1990, 498-505.

Bollerslev, T., R. F. Engle, and J. M. Wooldridge, A Capital Asset Pricing Model with Time-Varying Covariances, *Journal of Political Economy,* 1998, 116-131.

Cecchetti, Stephen G., Robert E. Cumby, and Stephen Figlewski, Estimation of the Optimal Futures Hedges, *Review of Economics and Statistics*, 4, 1988, 623-630.

Chang, C. W., Chang, J. S. K. and Fang, H., Optimum futures hedges with jump risk and stochastic basis, *The Journal of Futures Markets,* 16, 1996, 441-458.

Dicky, D. A. and W. A. Fuller, Distribution of the Estimators for Autoregressive Time Series with a Unit Root, *Journal of American Statistical Association*, 74, 1979, 427～431.

Ederington, L. H., The Hedging Performance of the New Futures Markets, *The Journal of Finance*, 34, 1979, 157-170.

Engle, R. F., Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U. K. Inflation, *Econometrica*, 1982, 987-1008.

Engle, Robert F. and Granger, C., Cointegration and Error Correction Representation, Estimation, and Testing, *Econometrica,* 55, 1987, 251～1008.

Fitzgerald, M. D., *Financial Futures,* Euromoney Publication, 1983, 67.

Gagnon, L. and Lypny, G., Hedging short-term interest risk under time-varying distributions, *The Journal of Futures Markets,* 15, 1995, 767-783.

Ghosh, A., Hedging with stock index futures: Estimation and forecasting with error correction model, *The Journal of Futures Markets,* 13, 1993, 743-752.

Ghosh, Asim and Ronnie Clayton, Hedging with International Stock Index Futures: An Intertemporal Error Correction Model, *Journal of Financial Research*, 19, 1996, 477-492.

Hicks, J., *Value and Capital,* London, 1953.

Howard, C. T., and D'Antonio, L. J., Multiperiod hedging using futures: A risk minimization approach in the presence of autocorrelation, *The Journal of Futures Markets,* 11, 1991, 697-710.

Johnson, L., The Theory of Hedging and Speculation in Commodity Futures, *Review of Economic Studies,* 27, 1960, 139-151.

Jorion, P., On Jump Processes in the Foreign Exchange and Stock Markets, *Review of Financial*

Studies, 1, 1993, 427-445.

Keynes, J., *Treatise on Money*, 2, London, 1930.

Kroner, K. F. and J. Sultan, Time-Varying Distributions and Dynamic Hedging with Foreign Currency Futures, *Journal of Financial and Quantitative Analysis,* 28, 1993, 535-551.

Mackinnon, J., *Critical Value for Cointegration Tests for in R.F. Engle and C.W.J. Granger, Long-run Economic Relationships,* Oxford University Press, 1991.

Maddala, G. and I. Kim, *Unit Roots, Cointegration, and Structural Change*, Cambridge Univ. Press, Cambridge, U. K., 1998.

McNew, K. P. and Fackler, P. L., Nonconstant optimal hedge ratio estimation and nested hypothesis tests, *The Journal of Futures Markets*, 14, 1994, 619-635.

Myers, R., Estimating Time-Varying Optimal Hedge Ratios on Futures Markets, *The Journal of Futures Markets*, 11, 1991, 39-54.

Nelson, D., Conditional heteroskedasticity in asset returns: a new approach, Econometirica 59, 1991, 347-370.

Park, T. H. and Switzer, L. N., Bivariate GARCH estimation of optimal hedge ratios for stock index futures: A note, *The Journal of Futures Markets,* 15, 1995, 61-67.

Phillips, P. C. B. and P. Perron, Testing for a Unit Root in Time Series Regression, *Biometrika*, 75, 1988, 335-346.

Stein, J., The Simultaneous Determination of Spot and Futures Prices, *American Economic Review,* 51, 1961, 1012-1025.

Working, H., Futures Trading and Hedging, *American Economic Review,* 43, June 1953, 314-343.

Figure 4: Trend of Won-dollar futures' trading volume

Figure 5: Trend of Won-dollar spot exchange rate

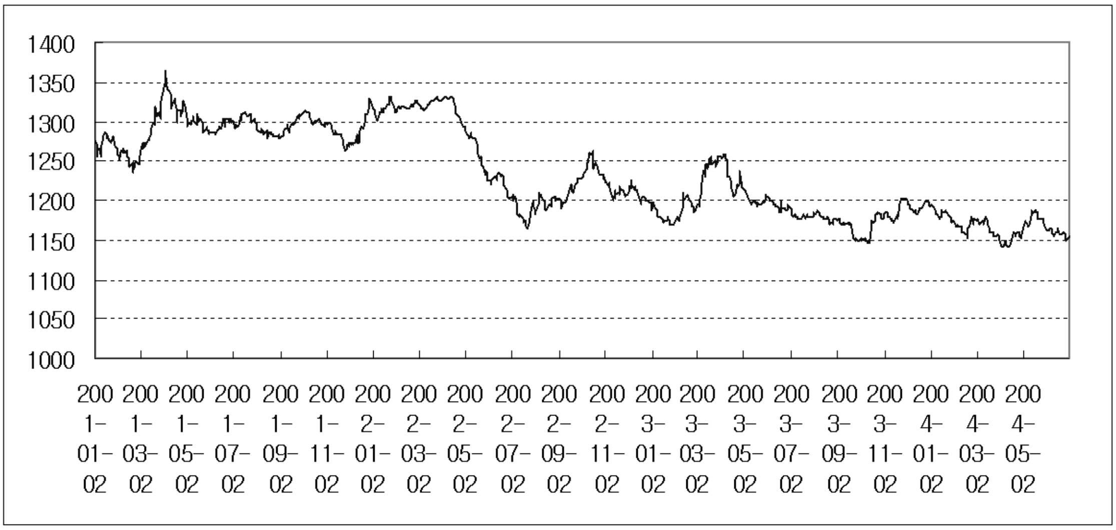


Table 7: The characteristics of US dollar futures

Underlying Asset	US Dollars
Trading Unit	US $50,000
Contract Months	The first three consecutive contract months (two serial expirations and one quarterly expiration) plus the next three months in the quarterly cycle (March, June, September, December)
Trading Hours	- 09:00 ~ 16:00 (Mon. - Fri.) - 09:00 ~ 11:30 (Last trading day)
Price Quotation	Korean Won (KRW) per US Dollar (USD)
Minimum Price Fluctuation	0.1, representing a value of KRW 5,000
Last Trading Day	Second trading day preceding the final settlement day
Final Settlement Day	Third Wednesday of the contract month
Settlement Method	Delivery settlement
Daily Price Limit	- None - However the limit on order price is imposed to prevent errors in entering orders.
Single Price Auction	Orders gathered during the pre-open session (08:30am-09:00am) will be matched at a single price auction.

Source: Korea Futures Exchange (http://www.kofex.com)