Indexing Catastrophe Securities

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Abstracts: The recent development of indexed catastrophe (CAT) securities has been a concern in insurance literature. We refer to the two existing prominent approaches as the "systematic risk approach" and the "moral hazard approach". In the systematic risk approach, the systematic risk portion is hedged by index-triggered securities, and the remaining unsystematic risk is hedged through indemnity-triggered vehicles including traditional insurance. In the moral hazard approach, indexing protects firms from the loss without incurring moral hazard problem. We argue that indexing seems to be supplementary, rather than dominant, in those approaches. We suggest two alternative rationales for indexing CAT securities. First, we argue that if firms are concerned with downside risk rather than variation, then indexing is optimal. Second, we argue that the observability of loss is another key factor for indexing, even when firms are concerned with variability. We identify the important sources of high observation costs as (i) the inherent difficulty in identifying CAT loss, (ii) the non-separability of cash flows between CAT event and other operations of the firm, and (iii) the impossibility of taking over the firm by the bondholders under a CAT event. In both cases, we show that indexing is a dominant tool, in contrast to existing approaches.

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I. Introduction

The securitization of insurance risk has been recently developed in the capital market (see Swiss Re, 1999, 2001, 2003 for more details). Since the capital market is much larger than the insurance market, securitization enables policyholders to utilize a larger pool of capital to hedge risks (Jaffee and Russell, 1997). Naturally, catastrophe risks such as earthquake and windstorm risks have received attentions for securitization. Large catastrophe losses may well wipe out the capital of the insurance industry, even though a major catastrophe is generally less than one percent of the global financial wealth.[1] A major portion of securitization transactions has involved catastrophe (CAT) bonds.[2] In a typical CAT bond contract, investors may lose all or some of the interest payment and principal if the prescribed catastrophic events occur. Another example of securitization is the insurance derivatives such as PCS options listed on the Chicago Board of Trade.

In most cases, the payoff structures of CAT securities are based on two types of

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[1] The total wealth of global financial assets is over US$60 trillion. A US$300 billion catastrophe would represent only 0.5% of the global financial wealth. (See Swiss Re, 2001) In addition, typical catastrophe losses of $50-100 billion are less than one daily standard deviation of the U.S. capital market value, while total net worth of the U.S. property-liability insurance industry is on the order of $300 billion. (See Doherty and Schlesinger, 2002)

[2] Swiss Re (2001) reports that nearly half of insurance securitization transactions involve CAT bonds.

triggers: indemnity and index. Under indemnity-triggers, payoffs to investors are based on the actual loss of the issuer. Under index-triggers, payoffs to investors are based on predetermined indexes such as industry losses or parametric values like magnitude of earthquake activities. Swiss Re (2001: p 21) reports that 32% of CAT securitization uses index-triggers, even though the majority uses indemnity-triggers.[3] Index-triggers are one of the interesting features of CAT securities. Indexing contrasts CAT bond with ordinary bond, since the forgiveness of payment in an ordinary bond is triggered by actual failure in payout, not by index.

Accordingly, the optimality of indexing has been a concern of insurance literature. We refer to the two prominent approaches as the "systematic risk approach" and the "moral hazard approach".[4] In the systematic risk approach, CAT risk is considered a mixture of systematic (or undiversifiable) risk and unsystematic (or diversifiable) risk (Doherty and Richter, 2002; Doherty and Schlesinger, 2002; Mahul, 2002). The systematic risk portion is hedged by index-triggered securities, and the remaining unsystematic risk is hedged through indemnity-triggered vehicles including traditional insurance. On the other hand, the moral hazard approach points that the index is chosen to be highly correlated to the individual firm's loss but not to be manipulated by the firm (Doherty, 1997; Doherty and Mahul, 2001; Froot 1999). Thus, indexing protects firms from the loss without incurring moral hazard problem.

Even though these approaches provide insights to indexed CAT securitization, some related questions have yet to be addressed. Let us consider the systematic risk approach. First, there seems to be no consensus on whether or not CAT risk is undiversifiable. One of the often-mentioned benefits of CAT securitization is that investors can reduce portfolio risks by purchasing CAT securities since CAT risk is virtually uncorrelated to financial risks. In other words, CAT risk is diversifiable. If CAT risk is diversifiable, indexing is redundant. For example, if a firm facing CAT risk issues indemnity-triggered CAT bonds, investors are willing to buy the bonds for a risk free return. CAT risks are fully diversified in the investors' portfolio. Firms and investors do not need index-triggered CAT bond.

Secondly, even if CAT risk can be decomposed into a diversifiable portion and an undiversifiable portion, it does not automatically guarantee the optimality of indexing. Suppose that an undiversifiable loss for a firm is x and a diversifiable loss is e. The systematic risk approach suggests that the firm will issue an indexed CAT bond (or buy an indexed CAT option) for x and buy, for example, traditional insurance for e. However, it would work equally well (and probably better if transaction costs were incurred) to issue one indemnity-triggered CAT bond for total loss x + e.[5, 6] Investors

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[3] 32% of indexing is obtained by adding 24% of "index" and 8% of "physical (or parametric) index" in Figure 12 of Swiss Re (2001). Indemnity-trigger is used 56% of the cases.

[4] We note that the two approaches are not mutually exclusive.

[5] Of course, there are other cases in which segregation of cash flows of securities is beneficial. For example, segregation will help to complete the market. However, since the focus of the systematic risk approach is on the diversifiability of CAT risk, our discussion also focuses on that.

[6] This is noticed in Doherty and Schlesinger (2002). They show that traditional

can easily diversify the unsystematic risk and keep the systematic risk with an appropriate premium. If an investor wants to hedge against the systematic risk, he will need only one index for the aggregated systematic risks, instead of many indexes for the segregated systematic risks. Therefore, it is not clear why firms have to issue indexed CAT securities even if CAT risk can be decomposed into systematic and nonsystematic risks. In sum, the systematic risk approach does not seem to provide a strong case for indexed securitization.

Now, consider the moral hazard approach. While indexing can control the moral hazard problem as pointed out by this approach, it is not clear whether or not indexing is optimal. From the standard moral hazard literature (for example, Holmström, 1979), we know the following outcomes:

(1) For an agent (or one party of the contract) to take full risk is not optimal when he is risk averse.

(2) If there is information on the efforts of the agent, then ignoring the information is not optimal.

Indexing, by itself, is not compatible with either outcome. When the actual loss of a firm can be observed, using an index and ignoring the loss is not optimal by (2). In addition, even if an index is correlated to the actual losses of a firm, there is a variation in actual losses, given the index. Indexing implies that, given an index, all remaining risk (basis risk) should be borne by the firm, which violates (1). Therefore, in order to achieve optimal results, it is necessary to synthesize contracts, one for index and one for basis risk (or actual loss) (Doherty and Mahul, 2001; Doherty and Richter, 2002). As a result, even if an index-trigger can be a part of optimal solution when an index provides additional information, it does not seem to replace an indemnity-trigger.

In summary, we think that neither approach provides a strong case for the optimality of index-triggered CAT securities. Indexing seems to be, at most, supplementary, rather than dominant, in both approaches. In this paper, we suggest two alternative rationales for indexing CAT securities. First, we argue that if firms are concerned with downside risk rather than variation, then indexing can be optimal. For firms facing large CAT risk, downside risk seems to be extremely important. Firms may go bankrupt after a CAT loss. Even if they survive, they may have to incur large costs to finance and recover the loss. Such firms are worried about the downside effect of a CAT loss, rather than about variation. We think that this situation needs to be treated differently from the usual risk aversion setting in which risk is symmetrically treated. We show that, in the case of downside risk aversion, indexing can be optimal, since indexing can remove downside risk, without incurring costs for upside risk (see Sections II and III). Unlike the moral hazard approach, indexing is optimal regardless of the presence of moral hazard.

Second, we argue that the observability of loss is another key factor for indexing, even when firms are concerned with variability (Section IV). It is rather obvious that indexing can be optimal when investors cannot observe the actual loss or can observe it only with high costs. What we do in this paper is to investigate why high costs will be incurred to observe the actual CAT loss. First, identifying the CAT

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nonparticipating insurance and a futures contract replicate (*not dominate*) variable participating contracts.

loss may well be inherently difficult. Especially, if the firm is an insurance company, then its CAT loss is the sum of the policyholders' losses. Determination of eventual policyholders' losses may well take several years. Second, observation costs are high due to the non-separability of cash flows between the CAT event and other operations of the firm. The non-separability of cash flows leads the firm to inflate the reported loss by transferring costs from other operation to the CAT loss. Third, unlike the default of ordinary bonds, a CAT event does not allow investors to take over the firm. With such weak rights to protect their claim values, bondholders will have to incur higher costs in identifying the CAT loss of the firm.

In this regard, we consider a CAT bond as a mixture of an ordinary bond and project financing. A CAT bond is project financing in that forgiveness of payout depends on the cash flows of a project (CAT event, here). A typical project financing will require the separation of cash flows of the project from those of the other operations of the firm (Brealey, Cooper, and Habib, 1999). However, due to its nature, CAT loss is not easily separable from the firm's operation. Since cash flows from the firm are mixed with CAT loss, firms have incentives to inflate (reported) CAT loss. As a result, identifying true CAT loss incurs large costs. Indexing is an efficient device to satisfy the need for project financing without incurring observation costs. Note that moral hazard can affect the use of indexing, but the reason is different from that in the moral hazard approach. In our paper, observation costs are a main focus. Moral hazard stimulates the use of indexing indirectly through its effect on the observation costs.

The remainder of the paper is composed as follows. Sections II and III will consider the optimality of indexing under downside risk aversion. We set up a simple model in Section II and show the results in Section III. We show that indexing is optimal. Section IV considers the observation costs issue. Section V discusses empirical implications and Section VI concludes.

II. A Model for Downside Risk Aversion

We consider a simple one-period model in which a firm (an issuer) issues a CAT bond to finance the potential CAT loss. In this model, the firm is downside risk averse, instead of risk (variation) averse. We model this by attaching costs to the loss that is not covered by the bond, if any. We consider the following situation. At time 0 ($t = 0$), the firm issues a CAT bond to an outside investor. The bond matures at time 1 ($t = 1$). At $t = 1$, a CAT event can occur with probability q. There is an index I for the CAT event. $I = 1$ if the CAT event occurs and $I = 0$, otherwise. The index value is public information that is observed by the issuer and the investor. When the event occurs at $t = 1$, the issuer suffers a random loss D. At $t = 0$, the issuer determines effort level E that can affect the loss size D. There are two effort levels. High effort is denoted as e and low effort is denoted as 0. We suppose that the disutility is V under high effort and 0 under low effort. If the issuer makes high effort, then, given the CAT event, $D = H$ with probability p and $D = L$ with probability 1-p. If the issuer makes low effort, then, given the CAT event, $D = H$ with probability one. If the CAT event does not occur, then $D = 0$ for both effort levels. We suppose that D is also public information, unless stated otherwise.[7]

The payout structure of the bond is designed to take into account the CAT event. The face value of the bond is denoted as F. The issuer receives the proceeds of X from the investor at t = 0. The bond is fairly priced. For simplicity, we assume a zero discount rate. At t = 1, the payout is made to the investor, depending on the information observed then.[8] When the CAT event does not occur, the investor receives the face value F. The payout under the CAT event will be denoted as F_i given observed information i. Note that if the investor observes D, information on I is redundant. Thus, we consider information i = H or L.

The wealth of the issuer at time t is denoted as W_t. We normalize the initial wealth W_0 of the issuer to be zero. Besides loss, the issuer potentially faces two sources of costs. First, financing by issuing a bond incurs costs, which will be called "financing costs" in this paper. Transaction costs for financing is one type of financing costs. The second type of financing costs is due to the restricted investment of the proceeds. Since the proceeds will be used to protect against the CAT loss, the investment will be restricted in general. For example, the issuer may forgo a risky project with positive NPV. In this case, financing costs include the lost NPV.

The second source of costs is due to downside risk aversion and/or financing after loss, which will be called "risk costs" in this paper. Suppose that the CAT event occurs and the issuer experiences loss that is not fully transferred to investors. It is commonly believed that financing after catastrophe event incurs high costs due to severe information asymmetry. Risk costs include these financing costs after the CAT event. Risk costs also include the financial distress costs following the CAT event. After the CAT event, the firm may well face severe financial distress. Such financial distress incurs costs due to, for example, loss of reputation, price reduction for urgent property sales, and incentive conflicts between stakeholders of the firm (Brealey and Myers, 2003).

We consider these financing and risk costs in the simplest forms. Financing costs are assumed to be proportional to the proceeds of the bond issue and risk costs are proportional to the loss uncovered by the bond. We denote k as the financing cost per proceeds and c as the risk cost per uncovered loss. Basic notations and assumptions are summarized in Table 1.

Table 1
Basic Notations and Assumptions

Catastrophe
Catastrophic event occurs with probability q
Index I = 1 if the catastrophic event occurs
= 0, otherwise

Effort by the issuer, E

[7] Observability of D would make it more difficult for indexing to be optimal. Thus, if indexing turns out to be optimal in our context, it will provide a strong case for indexing.
[8] We assume that observed information is also verifiable.

$E = e$, for high effort
$E = 0$, for low effort
Disutility: V under $E = e$ and 0 under $E = 0$

Loss D
Under no catastrophic event; $D = 0$
Under the catastrophic event;
(i) $E = e$;
$D = H$ with conditional probability p
$D = L$ with conditional probability 1-p
(ii) $E = 0$; $D = H$

Bond
X = proceeds from bond sale
F = face value
Payout under catastrophic event;
F_i: payout to the investor given information i at $t = 1$;
F_D: payback when information is $D = H, L$.

Financing cost per proceeds: k
Risk cost per loss uncovered: c
Wealth of the issuer at time t: W_t, $W_0 = 0$.

Suppose that no CAT bond is issued. When loss D is realized, $D + cD$ is the total costs to the firm, since no loss is covered. Therefore, the expected utility of e-effort firm is:
$EU(e\text{: no bond}) = - q[p(H + c\bullet H) + (1-p)(L + c\bullet L)] – V$
Similarly, the expected utility of 0-effort firm is $EU(0\text{: no bond}) = - q(H + c\bullet H)$
Therefore, high effort is desirable if $EU(e\text{: no bond}) – EU(0\text{: no bond}) > 0$, or $q(1+c)(1-p)(H-L) > V$. We simplify our analysis by assuming that high effort is desirable even with $c = 0$.[9] Assumption A holds throughout this paper.

Assumption A: $q(1-p)(H-L) > V$

The general framework of the issuer's problem can be described as follows: The high-effort issuer will determine an optimal payout structure to maximize his

[9] An alternative assumption is that $q(1+c)(1-p)(H-L) > V$. However, this change of assumption does not change the results substantially.

expected utility, given constraints. As constraints, we require that (i) the bond price is fairly valued and (ii) the payouts to investors are non-negative. In addition, an incentive compatibility constraint (IC) will be needed in order to prevent the low- (0-) effort issuer from mimicking the high- (e-) effort issuer. In principle, the 0-effort issuer also needs to solve a similar problem. However, we ignore the 0-effort issuer's problem, since assumption A will guarantee that high effort is desirable under fair bond price. In notation, the high-effort issuer's problem is:

$$\text{Max}_{\{F, F_H, F_L\}} \text{ EU of the e-effort issuer}$$
$$\text{s.t. Fair bond price}$$
$$\text{Incentive Compatibility}$$
$$F, F_H, F_L \geq 0$$

The gross wealth of the high-effort issuer before netting financing and risk costs at $t = 1$ is determined as follows:

X-F under no loss with probability 1-q
$X - H - F_H$: under loss = H with probability qp
$X - L - F_L$: under loss = L with probability q(1-p)

Now, consider the cost side. Disutility due to effort is V. Financing costs for proceeds X are kX. Since the uncovered loss is $\text{Max}[D - (X-F_D), 0]$, the risk costs are $c \bullet \text{Max}[D - (X-F_D), 0]$. Therefore, the (net) expected utility EU(e) can be written as:

$$EU(e) = X-(1-q)F - q[p\{H + F_H + c \bullet \text{Max}[H - (X - F_H), 0]\} + (1-p)\{L + F_L + c \bullet \text{Max}[L - (X-F_L), 0]\}] - V - kX$$

Fair pricing implies that $X = (1-q)F + q\{pF_H + (1-p)F_L\}$. Thus, $X-F_H = (1-q)(F-F_H) + q(1-p)(F_L - F_H)$, $X-F_L = (1-q)(F-F_L) - qp(F_L - F_H)$. Therefore, we can rewrite EU(e) as:

$$EU(e) = - q[p\{H + c \bullet \text{Max}[H - (1-q)(F-F_H) - q(1-p)(F_L - F_H), 0]\} + (1-p)\{L + c \bullet \text{Max}[L - (1-q)(F-F_L) + qp(F_L - F_H), 0]\}] - V - k[(1-q)F + q\{pF_H + (1-p)F_L\}]$$

For IC, the 0-effort issuer should have no incentive to mimic the e-effort issuer. If the 0-effort issuer mimics the e-effort issuer, the expected utility of the 0-effort issuer becomes;

$$EU(0\text{: mimicking}) = - q[\{H -(1-p)(F_L-F_H) + c \bullet \text{Max}[H - (1-q)(F-F_H) - q(1-p)(F_L - F_H), 0]\}] - k[(1-q)F + q\{pF_H + (1-p)F_L\}]$$

Now, IC becomes

$EU(e) – EU(0\text{: mimicking}) \geq 0$, or,

$$q(1-p)\{(H-L) - (F_L - F_H) + c \bullet \text{Max}[H - (1-q)(F-F_H) - q(1-p)(F_L - F_H), 0] - c \bullet \text{Max}[L - (1-q)(F-F_L) + qp(F_L - F_H), 0]\} - V \geq 0.$$

Therefore, the e-effort issuer will solve the following problem:

$Max_{F, FH, FL}$ EU(e) = - q[p{H + c•Max[H - (1-q)(F-F_H) - q(1-p)(F_L - F_H), 0]} + (1-p){L + c•Max[L - (1-q)(F-F_L) + qp(F_L - F_H), 0]}] - V - k[(1-q)F + q{pF_H + (1-p)F_L}]

s.t. q(1-p){(H-L) - (F_L - F_H) + c•Max[H - (1-q)(F-F_H) - q(1-p)(F_L - F_H), 0] - c•Max[L - (1-q)(F-F_L) + qp(F_L - F_H), 0]} - V ≥ 0
F, F_H, F_L ≥ 0

III. Optimality of Indexing When the Firm is Downside Risk Averse

The following observation will simplify our analysis.

Lemma 1: At an optimum:
(i). If F > 0, then F_H and F_L cannot be positive simultaneously.
(ii). We cannot have 0 ≤ F < F_i. In other words, F ≥ F_H, F_L.
(iii) If F = 0, then F_H = F_L = 0.
[proof] See the Appendix.

To understand Lemma 1, note that, except for the financing costs, the objective function is affected only by the differences of the payoffs, not by the values of payoffs. Therefore, if all payoffs are positive, the firm can increase the expected utility by reducing payoffs by the same amount that will only reduce financing costs (Lemma 1 (i)). Lemma 1 (ii) and (iii) imply that F cannot be smaller than F_i. This result is intuitive, since forgiving payout is more valuable to the firm when CAT loss is incurred.

We identify the case in which F = F_H = F_L = 0 as no bond issue. The following definition identifies "indexing" in our context.

Definition: The bond is *index-triggered* (or simply *indexed*) if F_H = F_L.

We will suppose F > 0, unless mentioned otherwise. By Lemma 1, when F > 0, the possible solutions types are:
F_H > 0, F_L = 0
F_H = 0, F_L > 0
F_H = 0, F_L = 0.

We will check whether or not each type of solution is possible at an optimum. Intuitively, we may guess that F_H > 0 and F_L = 0 will not hold at an optimum. The reason is as follows. The firm will suffer more loss with D = H than with D = L. Therefore, forgiving payout is more valuable when D = H. As a result, F_H should be lower than F_L. Lemma 2 shows that this is indeed the case.

Lemma 2: At an optimum, we cannot have F_H > 0 and F_L = 0.
[proof] See the Appendix.

Now, let us investigate the case in which F_H = 0, F_L > 0. In this case, the problem becomes:

$\text{Max}_{F, FH, FL}$ EU(e) = - q[p{H + c•Max[H - (1-q)F - q(1-p)F_L, 0]} + (1-p){L + c•Max[L - (1-q)(F-F_L) + qpF_L, 0]}] - V - k[(1-q)F + q(1-p)F_L]

s.t. q(1-p){(H-L) - F_L + c•Max[H - (1-q)F - q(1-p)F_L , 0] - c•Max[L - (1-q)(F-F_L) + qpF_L, 0]} - V ≥ 0

By solving the problem, we can show the following results.

Lemma 3: Suppose that $F_H = 0$ and $F_L > 0$ hold at an optimum. Then the following holds:

(i) H - (1-q)F - q(1-p)F_L = 0, $F_L \leq$ (H-L) - V/[q(1-p)]

(ii) EU = - q[pH + (1-p)L] - V – kH.

[proof] See the Appendix.

The intuition of this lemma is as follows. Note that $F_L > 0$ implies that the payout is positive when D = L, which means that the firm has more than needed to cover the loss. Thus, the positive payout implies that loss is covered fully, or L - (1-q)(F-F_L) + qp$F_L \leq 0$. On the other hand, $F_H = 0$ implies that the firm has (weakly) less than needed to fully cover the loss when D = H. Thus, zero payout implies that H - (1-q)F - q(1-p)$F_L \geq 0$. If the financing costs are low relative to the risk costs, then it will be desirable to maximize the loss coverage. In this case, H - (1-q)F - q(1-p)F_L = 0. If the financing costs are high relative to the risk costs, then it will be desirable to minimize the loss coverage. In this case, F and/or F_L will be lowered as much possible. However, this case will be contradictory to the requirement that F and $F_L > 0$ at an optimum. Thus, $F_H = 0$ and $F_L > 0$ hold at an optimum only if H - (1-q)F - q(1-p)F_L = 0. In this case, we obtain the expected utility as given in Lemma 3 (ii). F and F_L are not uniquely determined, since the expected utility is the same as long as H - (1-q)F - q(1-p)F_L = 0 and L - (1-q)(F-F_L) + qp$F_L \leq 0$. Finally, $F_L \leq$ (H-L) - V/[q(1-p)] is needed for IC.

Finally, let us consider the case in which $F_H = 0$ and $F_L = 0$ at an optimum. Note that the CAT bond is indexed, since $F_H = F_L$. In this case, the problem becomes:

$\text{Max}_{F, FH, FL}$ EU(e) = - q[p{H + c•Max[H - (1-q)F, 0]} + (1-p){L + c•Max[L - (1-q)F, 0]}] - V - k[(1-q)F]

s.t. q(1-p){(H-L) + c•Max[H - (1-q)F, 0] - c•Max[L - (1-q)F, 0]} - V ≥ 0

Note that H – (1-q)F > L – (1-q)F. By solving the problem, we can show the following.

Lemma 4: Suppose that $F_H = 0$ and $F_L = 0$ at an optimum.[10]

(i) If qc ≤ k, then

F = 0, EU(e) = - q(1+c)[pH + (1-p)L] - V

(ii) If qpc ≤ k < qc, then

[10] We allow F = 0 for this lemma.

$F = L/(1-q)$, $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V - kL$
(iii) If $k < qpc$, then
$F = H/(1-q)$, $EU = - q[pH + (1-p)L] - V – kH$
[proof] See the Appendix.

Note that qc is the expected unit risk costs of losses up to L. And qpc is the expected unit risk cost for loss above L. If the financing costs are too high ($qc \leq k$), then it is better not to issue the bond ($F = 0$). If the financing costs are moderate ($qpc \leq k < qc$), then it is optimal to cover only up to L. In this case, the firm needs to suffer the uncovered loss of H – L in case of D = H. If the financing costs are low ($k < qpc$), then coverage for the high loss H is desirable. The firm is over-funded in the case of D = L.

Now, in order to achieve the optimal outcomes, we compare the two possible cases: (i) $F_H = 0$ and $F_L > 0$ and (ii) $F_H = F_L = 0$. Proposition 1 shows the results.

Proposition 1: Indexing is optimal where the firm is downside risk averse.
[proof] See the Appendix.

Proposition 1 is obtained by comparing Lemmas 3 and 4. If the financing costs are high ($qc \leq k$), then it is clearly optimal to issue no CAT bond. Thus, $F = F_H = F_L = 0$ as in Lemma 4 (i). If the financing costs are moderate but too high to cover the loss above L in case of D = H ($qpc \leq k < qc$), then it is optimal to cover only up to L. Thus, in this case, $F_H = F_L = 0$, and $F = F = L/(1-q)$ as in Lemma 4 (ii). Finally, if the financing costs are low ($k < qpc$), then coverage of high loss H is desirable. In this case, $H - (1-q)F - q(1-p)F_L = 0$ and $EU = - q[pH + (1-p)L] - V – kH$. Note that this result is obtained as long as $F_L \leq (H-L) - V/[q(1-p)]$ as in Lemma 3. Therefore, there are an infinite number of solutions including $F_L = 0$. In sum, in all cases, $F_H = F_L = 0$ are an optimal solution. Thus, indexing is optimal.

Proposition 1 provides a strong case for indexing: Index-triggers dominate indemnity-triggers. This result is contrasting with the systematic risk and the moral hazard approaches, since indexing is rather supplementary in those approaches. In our context, the optimality of indexing comes from the assumption of downside risk aversion. If the firm were risk averse in the usual sense, then it would worry about the differences between wealths in different states. This would lead the payouts to depend on actual losses, not just on the index. This does not happen in our model, since the firm cares only about downside risks.

It should be emphasized that indexing does not aim to control the moral hazard problem even if indexing resolves it when it exists.[11] Indexing is optimal even if effort level can be observed.[12] This result is contrasting with the moral hazard approach. Relative sizes of financing costs and risk costs affect the financing level, not indexing

[11] Indexing resolves the moral hazard problem since indexing leaves basis risks to the firm.
[12] Technically, this result is obtained from the observation that IC is not binding with indexing.

itself. The linearity assumption of cost is also not critical for our result. Nonlinear costs may change the financing level without affecting the optimality of indexing.

In addition, the optimality of indexing still holds when the index can have multiple values. This can be easily understood from the observation that financing amount is determined by comparing the marginal financing costs with the marginal expected risk cost. For example, suppose that the index I can have values of 0, 1, or 2, with probability q_0, q_1 and q_2, respectively. With high effort, given I = 1, D = H with probability p and D = L with probability 1 – p, as above. Given I = 2, D = T with probability r and D = M with probability 1 – r, where L < H < M < T.[13] Now, loss sizes L, H, M, and T are relevant with probabilities $q_1 + q_2$, $q_1p + q_2$, q_2 and q_2r, respectively.[14] Depending on the marginal financing cost relative to the marginal (expected) risk cost, financing amount is determined. For example, if financing cost k is between q_2c and q_2rc, then the optimal proceeds are M, since financing over M incurs financing cost higher than expected risk cost. Assuming no moral hazard problem, an optimal payback design should be such that the payback amount is M for I = 2 and no risk cost incurs for I = 1. Thus, an index trigger is optimal, if it satisfies that $F_{I=2} = M$, $F_{I=1} = H$. In addition, the index trigger can also resolve the moral hazard problem, if it exists.[15] As a result, the optimality of indexing still holds in the case where index can have multiple values.

IV. Optimality of Indexing When CAT Loss Is Not Observable

We consider another case for optimal indexing. Suppose that investors cannot observe the actual CAT loss of the firm or can observe it only with high costs. In this case, payouts from the CAT bond cannot be based on the actual losses. Then, it is natural to base the payouts of bond on something observable, or index. Below, we explore why CAT loss can incur high observation costs.

It will be useful to first compare the CAT bond with the ordinary bond. Payouts from the ordinary bonds are normally backed by the firm's operation and assets. The firm that issues ordinary bonds is supposed to pay interests and principal from its overall operation and assets. The failure to pay interests or principal provides ordinary bondholders with the legal right to "take over" the firm.[16] Taking over helps bondholders to protect their claim values by preventing owners and managers from

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[13] The order is not important.

[14] For example, loss size of H is relevant when the loss size is lager than or equal to H, or D = H, M, or T. The probability that D = H, M or T is $q_1p + q_2$.

[15] Assuming no moral hazard problem, there are also other optimal solutions that are not completely indexing. For example, $\{F_{D=H} = H, F_{D=L} = L, F_{I=2} = M\}$ is also optimal since it incurs the same financing and risk costs as the indexing scheme. However, this may expose the investors to the moral hazard problem if it exists, since the firm does not bear any risk for I = 1.

[16] In reality, the default does not imply bondholder's direct control of the firm. We use the term "take over" broadly for any actions to protect bondholders' interest, including seizure of collateral and acceleration of the maturity of the bond (Smith and Warner, 1979).

deteriorating them further. In doing so, bondholders first try to observe the true value of the firm by, for example, due diligence review.

A CAT bond can be considered a mixture of an ordinary bond and project financing. As for the ordinary bond, the failure of interests and principal induces the bondholders to take over the firm. The CAT bond is also project financing in that forgiveness of payments of interests and principal is based on the CAT loss, not on the overall operations of the firm. Here, CAT event plays a role of a project. As in typical project financing, the CAT loss does not provide the CAT bondholders with the right to take over the firm. In typical project financing, the (parent) firm sets up an SPV (special purpose vehicle) to handle financing, cash flows and operation for the project. SPV is an entity legally separated from the firm. All the cash flow related to the project is accrued to the SPV, not to the firm. Project financing can thus be considered as ordinary bonds for the SPV. Since all the cash flows from the project are separated from the firm, project financing is backed only by the cash flows from the project.

In a typical CAT bond issue, like other project financing, an SPV is set up to handle the financing and cash flows. However, there is a difference between a CAT bond and typical project financing. Cash flows from the SPV are not easily separable from the cash flows from the parent firm, since the CAT loss is transferred from the parent firm to the SPV. Therefore, CAT loss is not separable from the operation of the parent firm. Non-separability of cash flows also aggravates moral hazard and adverse selection problems. Note that controlling moral hazard is one of the main purposes of separating cash flows in project financing. When cash flows are not separable and thus are under the discretion of the firm, the firm will tend to inflate the CAT loss than otherwise. The firm will have low incentives to reduce loss, ex ante and ex post. In addition, the firm may try to inflate the reported loss by transferring costs from other operation to the CAT loss. Note that taking over the SPV does not help a lot to reduce the moral hazard problem, since the CAT loss is incurred in the parent firm, not in the SPV. The adverse selection problem is also aggravated, since the firm has information about the CAT loss and cash flows from the firm, which bondholders do not have. Ex ante, the firm with higher CAT loss potentials has higher incentive to issue CAT bonds. Ex post, the firm with low CAT loss may inflate the reported CAT loss. Under the presence of moral hazard and adverse selection problems, the reported CAT loss tends to be larger than otherwise, which makes it more difficult to identify the true CAT loss. In sum, the non-separability of cash flows will aggravate moral hazard and adverse selection problems, which increases observation costs.

With the help of these observations, we conclude that (i) the inherent difficulty in identifying CAT loss, (ii) the non-separability of cash flows, and (iii) the impossibility of taking over the firm contribute to high observation costs. First, the observation costs of CAT loss are inherently large, since the true CAT loss is not easily identifiable and can often be identified only after several years of lawsuits and negotiation.[17] Difficulty in the observation of CAT loss may be severe if the firm is an insurance company. Unlike manufacturing firms, insurance firms have to settle claims

[17] This inherent difficulty in observing CAT loss seems to be well recognized in literature (see, for example, Swiss Re 2001).

from many policyholders. This settlement process normally takes lengthy time periods. Second, as shown above, the non-separability of cash flows, coupled with moral hazard and adverse selection problems, increases the costs to identify the CAT loss. Moreover, the CAT loss does not allow the bondholders to take over the firm. With weaker rights to protect their claim values, bondholders will have to incur higher costs in identifying the CAT loss of the firm.

Under high observation costs, indexing is an efficient device to finance the CAT loss without incurring observation costs. Bondholders do not need to incur observation costs to identify the actual loss under indexing. Let us summarize the results of our discussion as follows.

Proposition 2: Indexing is optimal where loss observation incurs high costs. CAT risk incurs high observation costs, due to (i) the inherent difficulty in identifying CAT loss, (ii) the non-separability of cash flows, and (iii) the impossibility of taking over the firm by the bondholders under a CAT event.

Clearly, a better index is one that is more highly correlated to the actual loss, ceteris paribus. It is also obvious that a better index is one that incurs lower observation costs (for index), ceteris paribus. Recent development of triggers sheds lights on the importance of observation costs. Two main examples of indexes are the industry loss index and the parametric index. The industry loss index is probably more highly correlated to the actual loss than the parametric index, since the former traces more closely the actual loss of the firm than the latter. On the other hand, the parametric index is observable with lower costs than the industry loss index. Therefore, if observation costs are more important, then parametric indexing will be adopted more often than industry loss indexing. Consistent with our conjecture, it seems that parametric indexing becomes more popular (Swiss Re, 1999, 2003).

Notice that our approach distinguishes itself from the moral hazard approach in two aspects. First, the index is not designed to directly control the moral hazard, unlike in the moral hazard approach. Moral hazard contributes to indexing, but only indirectly through its effect on the observation costs. In this regard, ex post moral hazard is more important than ex ante moral hazard, since ex post moral hazard directly contributes to the increase in observation costs, while ex ante moral hazard does not. This result is contrasting to Doherty (1997) in which ex ante moral hazard is treated more importantly. Second, to the extent that the industry loss index is not subject to moral hazard, the moral hazard approach seems to prefer the industry loss index to the parametric index, since the former has a higher correlation with the actual loss than the latter.

V. Empirical Implications

This paper has several empirical implications. First, firms with high observation costs will prefer indexing. Reinsurance firms and insurance firms are good examples, since identification of the CAT loss is inherently difficult and time-consuming. Another example will include firms with non-separable cash flows. We note that the non-separability may depend on the characteristics of the loss that the firm wants to cover. For example, if a firm wants to cover the loss of future operating cash flows following the CAT event, then it will be difficult to distinguish the CAT loss from loss due to other factors. In this case, the firm will prefer the index-trigger. On the other hand, if the firm's main concern is on the damage of, say, buildings and facilities, then the separation of cash flows will be relatively easy. In this case, the firm may prefer the indemnity-trigger. CAT bonds issued by Oriental Land Co. (Tokyo Disneyland) in 1999 illustrate this point. It was emphasized by the officers of Oriental Land Co. that the bonds were essentially designed to smooth out *operating cash flow volatility* (Quinn, 1999). Our results indicate that indexing is an efficient tool in such a case. Indeed, the bonds were parametric index-triggered based on the magnitude of Richter scale measured by Japanese Meteorological Agency. Note that our approach also implies that the parametric index will be more associated with higher observation costs than the industry loss index, since the former is observed more easily than the latter.

Next, our results show that indexing is preferred when downside risk is significant. Downside risk is significant when the firm faces severe losses after a CAT event. Again, reinsurance and insurance firms are examples. After a CAT event, (re) insurance firms may well face a large amount of claims. This large amount of claims may lead the (re) insurance firms to bankruptcy, if the firms have no other hedge tools. Thus, (re) insurance firms exposed to large CAT losses will prefer the index-trigger to the indemnity-trigger. The level of financing will depend on the relative sizes of financing costs and risk costs. When risk costs are relatively high (low) compared to financing costs, then the financing level will be high (low).

We note that our results do not rule out the possibility that firms simultaneously use indemnity-trigger and index-trigger. For example, as far as the non-separability of cash flows is concerned, firms may use the indemnity-trigger for losses for buildings and the index-trigger for losses of future operating cash flows. As far as downside risk is concerned, it seems plausible to suppose that, if the loss is small, firms are averse to variation, rather than to downside risk. Therefore, firms may use the indemnity-trigger for small losses and the index-trigger for large losses. The synthetic use of the indemnity-trigger and the index-trigger is similar as in the systematic risk approach and the moral hazard approach. However, the reasons are different. Different triggers are used due to the different observation costs of cash flows or the different sizes of losses in our approaches. On the other hand, the synthetic use of triggers is followed by the decomposition of the CAT risk in the systematic risk approach and by the moral hazard in the moral hazard approach.

Our results imply that reinsurance firms and primary insurance firms, to a lesser degree, are the main beneficiaries of the indexed securities, since they are exposed to high downside risks as well as high observation costs. Even if each policyholder faces small CAT losses with low observation costs, the sum of losses may well be large and unobservable. As a result, (primary) insurance firms are exposed to high downside risk with high observation costs. Reinsurance firms face much higher downside risk with higher observation costs, since their policyholders are primary insurance firms. This explains why reinsurance firms are main CAT bond issuers. On the other hand, as illustrated by the example of Oriental Land Co., manufacturing firms that are exposed to high downside risk or high observation costs will also be main issuers.

VI. Conclusion

The recent development of indexed CAT securities has been a concern in insurance literature. We refer to the two prominent approaches as the "systematic risk approach" and the "moral hazard approach". In the systematic risk approach, CAT risk is considered a mixture of systematic risk and unsystematic risk. The systematic risk portion is hedged by indexed securities, and the remaining unsystematic risk is hedged through traditional insurance vehicles. The moral hazard approach points that the index is chosen to be highly correlated to the individual firm's loss but not to be manipulated by the firm. Thus, indexing protects firms from the loss without incurring moral hazard problem.

Even though these approaches provide insights to indexed CAT securitization, indexing seems to be supplementary, rather than dominant. Based on this observation, we suggest two alternative rationales for indexing CAT securities. First, we argue that if firms are concerned with downside risk rather than variation, then indexing is optimal. Second, we argue that the observability of loss is another key factor for indexing, even when firms are concerned with variability. In both cases, we show that indexing is a dominant, not supplementary, tool, in contrast with existing approaches.

We think that downside risk aversion may be a better fit for CAT loss than usual risk aversion. In the case of CAT loss, the main concern is survival or severe financial suffering. Income smoothing does not seem to be a primary concern. Thus, it seems to be the case in which downside risk should be weighted much more heavily than upside risk. If so, our downside risk aversion model will provide new insights to the indexing behavior.

In addition, observation costs are emphasized. Even though high observation costs in the case of CAT event is generally admitted, the effects and sources of high observation costs do not seem to be investigated closely. We identify the important sources of high observation costs as (i) the inherent difficulty in identifying CAT loss, (ii) the non-separability of cash flows between CAT event and other operations of the firm, and (iii) the impossibility of taking over the firm by the bondholders under a CAT event. Indexing is an efficient device when observation costs are high.

For future research, it may be worthwhile to investigate whether or not firms use index-trigger as a dominant tool for CAT losses. More generally, it will be interesting to investigate the risk types (not just CAT risk) for which the index-trigger is efficient. In addition, the design of triggers of CAT securities seems to be worth further investigation. Recently, we have observed the development of several index types such as parametric index and modeled loss index (Swiss Re, 2003). The different roles of different indexes have not yet been fully investigated.

Appendix

For notational simplicity, we use EU for EU(e).

Lemma 1: At an optimum;

(i). If $F > 0$, then F_H and F_L cannot be positive simultaneously.

(ii). We cannot have $0 \leq F < F_i$. In other words, $F \geq F_H, F_L$.

(iii) If $F = 0$, then $F_H = F_L = 0$.

[proof]

(i) If both F_H and F_L are positive, a slight reduction of F, F_H and F_L by the same amount will satisfy all constraints and increase expected utility (EU). A contradiction.

(ii) Suppose first that $F < F_L$ and $F_H \leq F_L$. Then $L - (1-q)(F-F_L) + qp(F_L - F_H) > 0$. A slight reduction of F_L will not violate constraints and increase EU, since $dEU/dF_L = -q[p\{c\bullet[-q(1-p)]\} + (1-p)\{c\bullet[(1-q) + qp]\}] - k[1-p] = -qc[(1-p)(1-q)] - k[1-p] < 0$. A contradiction.

Now, suppose that $F < F_H$, $F_L \leq F_H$. Then $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) > 0$.

$EU(e) = -q[p\{H + c[H + (1-q)F_H - q(1-p)(F_L - F_H)]\} + (1-p)\{L + c\bullet Max[L + (1-q)F_L + qp(F_L - F_H), 0]\}] - V - kq\{pF_H + (1-p)F_L\}$

$dEU(e)/dF_H \leq -q[p\{c[(1-q) + q(1-p)]\} - (1-p)cqp] - kqp = -qpc(1-q) - kqp < 0$.

LHS of IC $= q(1-p)\{(H-L) - (F_L - F_H) + c\bullet[H + (1-q)F_H - q(1-p)(F_L - F_H)] - c\bullet Max[L + (1-q)F_L + qp(F_L - F_H), 0]\} - V$.

For $L + (1-q)F_L + qp(F_L - F_H) \geq 0$,

LHS of IC $= q(1-p)\{(H-L) - (F_L - F_H) + c\bullet[H + (1-q)F_H - q(1-p)(F_L - F_H)] - c\bullet[L + (1-q)F_L + qp(F_L - F_H)] - V$

$= q(1-p)\{(1+c)(H-L) + (1+c)(F_H - F_L)\} - V > 0$.

For $L + (1-q)F_L + qp(F_L - F_H) < 0$,

LHS of IC $= q(1-p)\{(H-L) - (F_L - F_H) + c\bullet[H + (1-q)F_H - q(1-p)(F_L - F_H)]\} - V > 0$.

Thus, a slight reduction of F_H will increase EU without violating IC. A contradiction.

(iii) This is obvious from (ii). ///

Lemma 2: At an optimum, we cannot have $F_H > 0$ and $F_L = 0$.

[proof] Suppose that $F_H > 0$ and $F_L = 0$.

In this case, we have:

$H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) = H - (1-q)F + (1-qp)F_H$

$L - (1-q)(F-F_L) + qp(F_L - F_H) = L - (1-q)F - qpF_H$

$H - (1-q)F + (1-qp)F_H > L - (1-q)F - qpF_H$

Claim 1: $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) \leq 0$.

[proof] If not, $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) > 0$. Then,

LHS of IC

$= q(1-p)\{(H-L) + F_H\} + c\bullet[H - (1-q)F + (1-qp)F_H] - c\bullet Max[L - (1-q)F - qpF_H, 0]\} - V$

$\geq q(1-p)\{(H-L) + F_H\} + c\bullet[H - (1-q)F + (1-qp)F_H] - c\bullet[L - (1-q)F - qpF_H]\} - V$

$= q(1-p)\{(1+c)(H-L + F_H)\} - V > 0$, by assumption A.

Therefore, IC is not binding. Now, EU(e) becomes

$EU(e) = - q[p\{H + c\bullet[H - (1-q)(F-F_H) - q(1-p)(F_L - F_H)]\} + (1-p)\{L + c\bullet Max[L - (1-q)(F-F_L) + qp(F_L - F_H), 0]\}] - V - k[(1-q)F + q\{pF_H + (1-p)F_L\}]$

Thus,

$dEU/dF_H = - q[p\{c\bullet[(1-q) + q(1-p)]\} + (1-p)\{c\bullet[-qp]\}] - k[qp] = -cqp(1-q) - kqp < 0$, if $L - (1-q)(F-F_L) + qp(F_L - F_H) > 0$.

$dEU/dF_H = - q[p\{c\bullet[(1-q) + q(1-p)]\}] - k[qp] = -cqp(1-qp) - kqp < 0$, if $L - (1-q)(F-F_L) + qp(F_L - F_H) < 0$.

Thus, a slight reduction of F_H increases utility in either case without violating IC. //

Claim 2: $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) = 0$.

[proof] Now, we know that $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) \leq 0$ from Claim 1. Now, suppose that $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) < 0$. Then, $EU(e) = - q[pH + (1-p)L] - V - k[(1-q)F + qpF_H]$ and IC is not binding. Thus, a slight decrease of F or F_H will increase EU(e). //

By Claims 1 and 2, we only need to consider the case of $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) = 0$. For $H - (1-q)(F-F_H) - q(1-p)(F_L - F_H) = 0$, LHS of IC $= q(1-p)(H-L + F_H) – V \geq 0$, for any F_H, satisfying IC.

Now, $EU(e) = - q[pH + (1-p)L] - V - k[(1-q)F + qpF_H]$

$= - q[pH + (1-p)L] - V - k[H + F_H]$.

$dEU/dF_H = -k < 0$.

Therefore, lower F_H is preferred under restriction of $H - (1-q)F + (1- qp)F_H = 0$. In the limit, F_H will decrease to 0 and F will increases to $H/(1-q)$. In the limit, $EU = - q[pH + (1-p)L] - V - kH$. However, note that $F_H = 0$ violates the restriction of $F_H > 0$. ///

To prove Lemma 3, we need following results.

Lemma A1: Suppose that $F_H = 0$ and $F_L > 0$ hold at an optimum. Then the following holds:

(i) $L - (1-q)(F-F_L) + qpF_L \leq 0$.

(ii) $H - (1-q)F - q(1-p)F_L \geq 0$.

[proof] (i) On the contrary, suppose that $L - (1-q)(F-F_L) + qpF_L > 0$.

$dEU/dF_L \leq - q[p\{c\bullet[- q(1-p)]\} + (1-p)\{c\bullet[(1-q + qp)]\}] - k[q(1-p)]$

$= -q(1-p)(c(1-q) + k) < 0$.

Δ LHS of IC $\leq q(1-p)\{-1 - c\bullet[1-q + qp]\} \leq 0$.

Thus, a slight reduction of F_L will not violate the IC and increase EU. A contradiction.

(ii) On the contrary, suppose that $H - (1-q)F - q(1-p)F_L < 0$. Since $L - (1-q)(F-F_L) + qpF_L \leq 0$ from (i),

$EU = - q[pH + (1-p)L] - V - k[(1-q)F + q(1-p)F_L]$

IC: $q(1-p)\{(H-L) - F_L\} – V \geq 0$.

A slight decrease of F_L will increase EU without violating IC. ///

Lemma A2: Suppose that $F_H = 0$ and $F_L > 0$ hold at an optimum. In addition, suppose that $L - (1-q)(F-F_L) + qpF_L < 0$. Then the following holds:

(i) When $qpc > k$, $H - (1-q)F - q(1-p)F_L = 0$ if IC is not binding.

(ii) When $qpc \le k$, $H - (1-q)F - q(1-p)F_L = 0$.
[proof]
Now we know that $H - (1-q)F - q(1-p)F_L \ge 0$ from Lemma A1. On the contrary, suppose that $H - (1-q)F - q(1-p)F_L > 0$.
$EU(e) = - q[p\{H + c\bullet[H - (1-q)F - q(1-p)F_L]\} + (1-p)L] - V - k[(1-q)F + q(1-p)F_L]$
$dEU(e)/dF = - q[p\{ c\bullet[- (1-q)]\}] - k(1-q) = (1-q)(qpc - k)$.
IC: $q(1-p)\{(H-L) - F_L + c[H - (1-q)F - q(1-p)F_L]\} - V$
(i) When $qpc > k$, a slight increase of F will increase utility if IC is not binding.
(ii) When $qpc \le k$, then a slight decrease of F will increase utility without violating IC.
///

Lemma 3: Suppose that $F_H = 0$ and $F_L > 0$ hold at an optimum. Then the following holds:
(i) $H - (1-q)F - q(1-p)F_L = 0$, $F_L \le (H-L) - V/[q(1-p)]$
(ii) $EU = - q[pH + (1-p)L] - V – kH$.
[proof] We prove this proposition by the following steps.
First, we find the optimality condition when $L - (1-q)(F-F_L) + qpF_L < 0$ (Claim 1).
Second, we find the optimality condition when $L - (1-q)(F-F_L) + qpF_L = 0$ (Claim 2).
Third, we compare the solutions of the two cases.

Claim 1: Suppose $L - (1-q)(F-F_L) + qpF_L < 0$.
At an optimum, the following holds.
(i) $H - (1-q)F - q(1-p)F_L = 0$, $(H-L) - V/[q(1-p)] \ge F_L$
(ii) $EU(e) = EU = - q[pH + (1-p)L] - V - kH$.
[proof] Suppose $L - (1-q)(F-F_L) + qpF_L < 0$.
Then, the problem becomes:
$Max_{F, FH, FL}\ EU(e) = - q[p\{H + c\bullet[H - (1-q)F - q(1-p)F_L]\} + (1-p)L] - V - k[(1-q)F + q(1-p)F_L]$
s.t. $q(1-p)\{(H-L) - F_L + c\bullet[H - (1-q)F - q(1-p)F_L]\} - V \ge 0$
Lagragian function becomes:
$L = - q[p\{H + c\bullet[H - (1-q)F - q(1-p)F_L]\} + (1-p)L] - V - k[(1-q)F + q(1-p)F_L] + \lambda[q(1-p)\{(H-L) - F_L + c\bullet[H - (1-q)F - q(1-p)F_L]\} - V]$, where λ is the Lagrangian multiplier related to IC.
$L_F = - q[p\{c\bullet[- (1-q)]\}] - k[1-q] + \lambda[q(1-p)\{c\bullet[- (1-q)]\}]$
$= qpc\bullet(1-q) - k[1-q] - \lambda q(1-p)c\bullet(1-q)$
$L_{FL} = - q[pc\bullet[- q(1-p)]]] - kq(1-p) + \lambda[q(1-p)\{- 1 + c\bullet[- q(1-p)]\}]$
$= qpc\bullet q(1-p) - kq(1-p) - \lambda q(1-p)\{1 + c\bullet q(1-p)\}$
$= (1-p)[q\{pcq - k\} - \lambda q\{1 + c\bullet q(1-p)\}]$
$= (1-p)q[\{pcq - k\} - \lambda\{1 + c\bullet q(1-p)\}]$
$= (1-p)q[\{pcq - k - \lambda cq(1-p)\} - \lambda]$
$= (1-p)q[L_F/(1-q) - \lambda]$
Note if $L_F \le 0$, then $L_{FL} \le 0$.
$L_\lambda = q(1-p)\{(H-L) - F_L + c\bullet[H - (1-q)F - q(1-p)F_L]\} - V$

First, consider the case of binding IC, or $\lambda > 0$.
$L_\lambda = 0 \Rightarrow q(1-p)\{(H-L) - F_L + c \bullet [H - (1-q)F - q(1-p)F_L]\} - V = 0$
$\Rightarrow F = [(1+c)H - L - (1+ cq(1-p))F_L - V/[q(1-p)]]/[c(1-q)]$

$c \bullet [H - (1-q)F - q(1-p)F_L]\} = V/[q(1-p)] - (H-L) + F_L$
$\Rightarrow - (1-q)F - q(1-p)F_L = \{V/[q(1-p)] - (H-L) + F_L\}/c - H$
$\Rightarrow (1-q)F + q(1-p)F_L = H - \{V/[q(1-p)] - (H-L) + F_L\}/c$

$L - (1-q)(F-F_L) + qpF_L = L -[(1-q)F + q(1-p)F_L] + qF_L = L - H - \{V/[q(1-p)] - (H-L) + F_L\}/c] + qF_L = L - H - \{V/[q(1-p)] - (H-L) + F_L\}/c] + qF_L$
$c[L - (1-q)(F-F_L) + qpF_L] = c(L - H) - \{V/[q(1-p)] - (H-L) + F_L\} + cqF_L$
$= (1-c)(H - L) - V/[q(1-p)] -(1-cq)F_L$
Thus,
$EU(e) = - q[p\{H + c \bullet [H - (1-q)F - q(1-p)F_L]\} + (1-p)L] - V - k[(1-q)F + q(1-p)F_L]$
$= - q[p\{H + V/[q(1-p)] - (H-L) + F_L \} + (1-p)L] - V - k [H - \{V/[q(1-p)] - (H-L) + F_L\}/c]$
$dEU(e)/dF_L = - q[p] - k [- 1/c] = -(1/c)(qpc - k)$

If $qpc \leq k$, Lemma A2 implies that $H - (1-q)F - q(1-p)F_L = 0$. Thus, the solution should satisfy $H - (1-q)F - q(1-p)F_L = 0$.
If $qpc > k$, then the optimal F_L will be the lowest possible. Under IC binding, $c \bullet [H - (1-q)F - q(1-p)F_L]\} = V/[q(1-p)] - (H-L) + F_L$ and $c[L - (1-q)(F-F_L) + qpF_L] = -(1+c)(H - L) + V/[q(1-p)] + (1+c)F_L$. For high F_L, $H - (1-q)F - q(1-p)F_L$ and $L - (1-q)(F-F_L) + qpF_L$ are positive. As F_L decreases, $L - (1-q)(F-F_L) + qpF_L = 0$ first. Therefore, optimal F_L will satisfy $H - (1-q)F - q(1-p)F_L = 0$ and $L - (1-q)(F-F_L) + qpF_L < 0$. In sum, if IC is binding, the optimal solution satisfies $H - (1-q)F - q(1-p)F_L = 0$. In this case, we have $F_L = (H-L) - V/[q(1-p)]$ for IC binding.

Secondly, consider the case of non-binding IC, or $\lambda = 0$. We know that $H - (1-q)F - q(1-p)F_L = 0$ from Lemma A2. In this case, $EU = - q[pH + (1-p)L] - V - kH$.
Note that the solution should satisfy both $L - (1-q)(F-F_L) + qpF_L < 0$ and $q(1-p)\{(H-L) - F_L\} - V > 0$ (non-binding IC). Since $L - (1-q)(F-F_L) + qpF_L < 0$ requires $F_L < H-L$, and non-binding IC requires $F_L < (H-L) - V/[q(1-p)]$, we need $F_L < (H-L) - V/[q(1-p)]$. Therefore, any (F, F_L) s.t. $H - (1-q)F - q(1-p)F_L = 0$ and $F_L < (H-L) - V/[q(1-p)]$ will be a solution.

As a result, regardless of IC binding, we have that $H - (1-q)F - q(1-p)F_L = 0$, $F_L \leq (H-L) - V/[q(1-p)]$, and $EU(e) = EU = - q[pH + (1-p)L] - V - kH$. This proves Claim 1. //

Claim 2: Suppose $L - (1-q)(F-F_L) + qpF_L = 0$:
At an optimum, the following holds.
(i) $H - L - F_L = V/[q(1-p)(1+c)]$
(ii) $EU = - q[pH + (1-p)L] - V - kH + (k-qpc)V/[q(1-p)(1+c)]$.
(iii) $qpc > k$.
[proof] When $L - (1-q)(F-F_L) + qpF_L = 0$, the problem becomes:
$Max_{F, FH, FL}\ EU(e) = - q[p\{H + c \bullet Max[H - (1-q)F - q(1-p)F_L, 0]\} + (1-p)\{L + c \bullet Max[L - (1-q)(F-F_L) + qpF_L, 0]\}] - V - k[(1-q)F + q(1-p)F_L]$

s.t. $q(1-p)\{(H-L) - F_L + c\bullet Max[H - (1-q)F - q(1-p)F_L , 0] - c\bullet Max[L - (1-q)(F-F_L) + qpF_L, 0]\} - V \geq 0$

We know that $H - (1-q)F - q(1-p)F_L = H – L – F_L \geq 0$, when $L - (1-q)(F-F_L) + qpF_L = 0$.

$EU(e) = - q[p\{H + c[H – L – F_L] + (1-p)L\}] - V - k[L + F_L]$
$dEU/dF_L= - q[p\{-c\}] – k = qpc – k$.

If $qpc > k$, higher F_L will be preferred. However, increase of F_L is restricted by $H – L – F_L \geq 0$ and IC. Since IC becomes binding before $H-L-F_L = 0$, the solution will be obtained where IC is binding. If IC is binding, $q(1-p)\{(H-L) - F_L + c\bullet[H - (1-q)F - q(1-p)F_L]\} - V = 0$
$\Rightarrow q(1-p)\{(H-L) - F_L + c\bullet[H - L - F_L]\} - V = 0$
$\Rightarrow q(1-p)(1+c)[H – L - F_L] - V = 0$
$H – L - F_L = V/[q(1-p)(1+c)]$

$EU = - q[p\{H + c\bullet [H - (1-q)F - q(1-p)F_L]\} + (1-p)L] - V - k[(1-q)F + q(1-p)F_L]$
$= - q[p\{H + c\bullet [H – L - F_L]\} + (1-p)L] - V - k[H + L + F_L -H]$
$= - q[pH + (1-p)L] - qpc\bullet [H – L - F_L] - V + k[H - L - F_L] – kH$
$= - q[pH + (1-p)L] - V – kH + (k-qpc) [H - L - F_L]$
$= - q[pH + (1-p)L] - V – kH + (k-qpc)V/[q(1-p)(1+c)]$

If $qpc \leq k$, lower F_L is preferred. In the limit, $F_L = 0$. In this case, $H - (1-q)F - q(1-p)F_L = H – L – F_L = H – L > 0$. In addition, IC is non-binding since LHS of IC = $q(1-p)(1+c)(H-L) – V > 0$ by assumption A. We have $F = L/(1-q)$ and $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V – kL$. However, $F_L = 0$ corresponds to the case in which $F_H = F_L = 0$, which will be considered later. In sum, if $F_H = 0$, $F_L > 0$ and $L - (1-q)(F-F_L) + qpF_L = 0$ at an optimum, then IC is binding and $H – L - F_L = V/[q(1-p)(1+c)]$. In this case, $EU = - q[pH + (1-p)L] - V – kH + (k-qpc)V/[q(1-p)(1+c)]$, where $qpc > k$. This proves Claim 2. //

By comparing the EUs from Claims 1 and 2, it is easy see that optimal results are obtained where $L - (1-q)(F-F_L) + qpF_L < 0$. ///

Lemma 4: Suppose that $F_H = 0$ and $F_L = 0$ at an optimum.
(i) If $qc \leq k$, then
$F = 0$, $EU(e) = - q(1+c)[pH + (1-p)L] - V$
(ii) If $qpc \leq k < qc$, then
$F = L/(1-q)$, $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V - kL$
(iii) If $k < qpc$, then
$F = H/(1-q)$, $EU = - q[pH + (1-p)L] - V – kH$
[proof] Note that $H – (1-q)F \geq 0$. For, if $H – (1-q)F < 0$, then $L – (1-q)F < 0$. In this case, a slight decrease of F will increase EU without violating IC. Now there are three possible cases:

(i) $H - (1-q)F > 0$ and $L - (1-q)F \geq 0$,
(ii) $H - (1-q)F > 0$ and $L - (1-q)F < 0$,
(iii) $H - (1-q)F = 0$ and $L - (1-q)F < 0$.
First, let us consider the case in which $H - (1-q)F > 0$ and $L - (1-q)F \geq 0$. In this case,
$EU(e) = - q[p\{H + c[H - (1-q)F]\} + (1-p)\{L + c\bullet[L - (1-q)F]\}] - V - k[(1-q)F]$
IC: $q(1-p)\{(H-L) + c\bullet[H - (1-q)F] - c\bullet[L - (1-q)F]\} - V \geq 0$

Thus, $dEU/dF = - q[pc\{-(1-q)\} + (1-p)c\{- (1-q)\}]] - k(1-q) = (1-q)[qc - k]$

If $qc > k$, then F will increase until $L - (1-q)F = 0$, or $F = L/(1-q)$. In this case, LHS of IC $= q(1-p)(1+c)(H-L) - V \geq 0$ by assumption A. $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V - kL$.
If $qc \leq k$, then F will decrease to 0. In this case,
$EU = - q[p\{H + cH\} + (1-p)\{L + cL\}] - V = - q(1+c)[pH + (1-p)L] - V$.

Secondly, if $H - (1-q)F > 0$ and $L - (1-q)F < 0$, then
$EU(e) = - q[p\{H + c[H - (1-q)F]\} + (1-p)L] - V - k[(1-q)F]$
LHS of IC $= q(1-p)\{(H-L) + c\bullet[H - (1-q)F]\} - V \geq 0$.
$dEU/dF = - q[pc\{- (1-q)\}] - k(1-q) = (1-q)(qpc - k)$
If $qpc > k$, F will increase until $H - (1-q)F = 0$, which corresponds to case (iii).
If $qpc \leq k$, F will decrease until $L - (1-q)F = 0$, which corresponds to case (i).
Therefore, there is no solution for $H - (1-q)F > 0$ and $L - (1-q)F < 0$.

Thirdly, if $H - (1-q)F = 0$ and $L - (1-q)F < 0$, then it is easy to see that $F = H/(1-q)$,
$EU(e) = - q[pH + (1-p)L] - V - kH$,
LHS of IC $= q(1-p)(H-L) - V \geq 0$.

By comparing the three cases, we obtain the following results.
For $qc \leq k$, there are two potential solutions. One is such that $F = 0$; $EU = - q(1+c)[pH + (1-p)L] - V$. The other is such that $F = H/(1-q)$; $EU(e) = - q[pH + (1-p)L] - V - kH$. The difference of EU is $qc(1-p)(H - L) > 0$. As a result, $F = 0$ is the solution.
For $qpc \leq k < qc$, $F = L/(1-q)$, $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V - kL$.
For $k < qpc$, there are also two potential solutions. One is such that $F = L/(1-q)$; $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V - kL$. The other is such that $F = H/(1-q)$; $EU = - q[pH + (1-p)L] - V - kH$. Since the difference of EU is $-(qpc - k)(H-L) < 0$, the solution is $F = H/(1-q)$. ///

Proposition 1: Indexing is optimal where the firm is downside risk averse.
[proof] Let us compare the three cases considered in Lemmas 3 and 4.
If $qc < k$;
For $F_H = 0$, $F_L = 0$, $F = 0$; $EU = - q(1+c)[pH + (1-p)L] - V$.
For $F_H = 0$, $F_L > 0$; $EU(e) = - q[pH + (1-p)L] - V - kH$
Difference of $EU = -qc[pH + (1-p)L] + kH > qc(1-p)(H-L) > 0$. Therefore, the optimal solution is: $F_H = 0$, $F_L = 0$, $F = 0$; $EU = - q(1+c)[pH + (1-p)L] - V$.

If $qpc < k < qc$;
For $F_H = 0$, $F_L = 0$; $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V - kL$
For $F_H = 0$, $F_L > 0$; $EU(e) = - q[pH + (1-p)L] - V – kH$
Difference of $EU = -qpc(H-L) + k(H-L) > 0$. Therefore, the optimal solution is: $F_H = 0$, $F_L = 0$ $F = L/(1-q)$; $EU(e) = - q[pH + (1-p)L + pc[H - L]] - V – kL$.

If $k < qpc$;
For $F_H = 0$, $F_L = 0$; $EU = - q[pH + (1-p)L] - V – kH$
For $F_H = 0$, $F_L > 0$; $EU = - q[pH + (1-p)L] - V – kH$
Since the difference of EU = 0, the optimal solution is $F_H = 0$, $F_L \leq (H-L) - V/[q(1-p)]$, $H - (1-q)F - q(1-p)F_L = 0$; $EU = - q[pH + (1-p)L] - V – kH$.

In all cases, $F_H = F_L = 0$ are optimal. ///

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