What is the real meaning of implied volatility?

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Abstract

In this study, we derive the same explicit formula for a European option as Merton's by assuming that the excess return of an underlying asset follows a lognormal process. We claim that the implied volatility by ad hoc Black-Scholes formula used by practitioners does not mean the future volatility of the returns of an underlying asset, but that of excess returns. The validity of this claim is shown by regressions among the implied volatility, realized volatility, and the historical volatility of returns and excess returns, using SPX and OMX index options. The result that implied volatility is a good predictor for future volatility is also confirmed.

Key word: implied volatility, future volatility, excess return, stochastic interest rate, S& P 500 index option

Introduction

There have been many studies on the relation between the implied volatility and realized (future) volatility of an underlying asset's returns. The volatility implied in an option price is widely believed as the option market's best forecast of future volatility over the remaining life of the relevant option. Since option price reflects market participants' expectation of future movements of the underlying asset, the volatility implied in an option price is regarded to be informationally superior to historical (past) volatility of the underlying asset's returns. If option markets are efficient and the option pricing model is correct, implied volatility should include all the information contained in other variables in explaining future volatility. There are many empirical researches on whether Black-Scholes' (1973)[BS hereafter] implied volatility of underlying asset's returns is an unbiased forecast for future volatility. However, empirical evidences on this claim are mixed.

Latane and Rendleman (1976), Jorion (1995) and Christensen and Prabhala (1998)[1], among others, confirm that implied volatility contain predictive information about future volatility. But Day and Lewis (1992) and Lamoureux and Lastrapes (1993)[2] have the opposite result that historical volatility is more informative than implied volatility. Strikingly, Canina and Figlewski (1993) report that implied volatility has virtually no correlation with future volatility, and it does not incorporate the information contained in recent observed volatility. They conclude that implied volatility is a poor forecast of subsequent realized volatility. However, as Christensen and Prabhala (1998) point out, the latter group of research has some econometric problems arising from maturity mismatch and usage of overlapping data. Christensen and Prabhala (1998) avoid these problems by using nonoverlapping data. They adopt the instrumental variables estimation to resolve the errors-in-variable problem associated with estimated implied volatility. They find that implied volatility outperforms the past volatility in forecasting future volatility and contains the information content of past volatility in their specifications.

However, most of the previous studies lack the precise understanding about the meaning of implied volatility even though they take all econometric issues into account. They use the interest rate matched with the life of the option from current interest rate term structure instead of the constant instantaneous

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[1] Latane and Rendleman (1976) focus on static cross-sectional tests using individual stock option prices. Jorion (1995) studies on foreign exchange options. And Christensen and Prabhala (1998) studies on S&P 100 index options.

[2] Day and Lewis (1992), tests on S&P 100 index options, and Lamoureux and Lastrapes on individual stock options.

interest rate to derive implied volatility through the Black-Scholes model. Even practitioners use 3 month LIBOR or 3 month T-bill rate instead of constant instantaneous short rate. This means that the model used to calculate the implied volatility is not exactly the Black-Scholes model but another option pricing model with stochastic interest rate. Naturally, the implied volatility does not forecast the future volatility of returns. In our model, we show that they are estimating implied volatility of excess returns of underlying asset.

In most of the option pricing models with stochastic interest rate including Merton(1973), Rabinovitch(1989), Amin and Jarrow(1992) etc, the volatility implied in their option formula is a mixture of volatility of the underlying asset's return and that of interest rate and covariance between them. Rabinovitch (1989) explicitly considers the covariance between the return of underlying asset and the interest rate. Using mean-reverting Ornstein-Uhlenbeck process for short interest rate, he obtains a closed form formula for the stochastic interest rate option pricing model. In his model, the correlation between the unanticipated returns of underlying asset and the changes in the short-term rate plays an important role in the option valuation. He shows that option prices calculated by his formula differ from those fitted by Black-Scholes formula when the correlation is relatively large and short interest rate is relatively volatile.

In this article we derive an option formula by assuming that excess returns, not returns of underlying assets, follow a diffusion process. This is same as using risk-free bond price as a numeraire. This gives us two advantages. One is that we do not need to specify any stochastic process for interest rate. We do not explicitly assume any stochastic process for interest rate. So we could guarantee parsimony of a model. Still it can be compatible with any stochastic interest rate model. Second, even though our model is a generalized Merton model, we can calculate the implied volatility from the formula. But it is not the implied volatility of the underlying asset returns, but of the excess returns. Based on our model, the implied volatilities used by practitioners are actually those of excess returns.

To confirm our claim, we test whether the implied volatility estimated by ad hoc Black-Scholes model or our model explains the realized volatility of excess returns better than that of the underlying asset's returns.

For empirical test, we use the SPX and OMX options data. The results show that implied volatility has a slightly higher correlation with the volatility of excess returns than with underlying -asset returns, even though the difference is statistically marginal. But this slightly higher correlation is quite robust for any change of sample.

We also find that implied volatility is less biased forecast of futures volatility than historical volatility of both underlying asset's returns and excess returns.

The rest of the article is organized as follows. Section 2 describes our option pricing model under stochastic interest rates. Section 3 contains the description of the data and methodologies. Section 4 lists empirical results, and section 5 concludes.

Ⅱ. The Model

Under the continuous time economy with the complete and frictionless market, we evaluate a call option of strike price K maturing at T. An underlying asset's price (hereafter we call stock) normalized by the bond price, $P(t)$, is defined by $S(t)/B(t)$ where $S(t)$ are a stock price and $B(t)$ a risk-free discount bond price with a payoff \$1 at T. Then the rate of return from this normalized stock is equal to the excess return. Different from Black-Scholes option pricing model, we assume that the rate of excess return follows one factor diffusion process, i.e.,

$$\frac{dP(t)}{P(t)} = \mu dt + \sigma dW(t) \tag{2.1}$$

where $\{W(t)\}$ is a Wiener process, μ the instantaneous expected rate of excess return, which is the difference between stock's log return and risk-free discount bond's log return, and σ the standard deviation is assumed to be a deterministic function of time, $\sigma = \sigma(t)$.

As for the price of an option, it seems natural to calculate the option price as units of a bond price. Let's define $V(P,\tau)$ as follows;

$$V(P,\tau) = \mathrm{C}(\mathrm{S(t)},\mathrm{B}(\tau),\tau)/B(\tau)$$

where τ is the time to maturity $T-t$, $C(S(t),B(\tau),\tau)$ price of a call option given current stock and bond price. Then $V(P,\tau)$ satisfies the following partial differential equation,

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V}{\partial P^2} = \frac{\partial V}{\partial \tau}. \tag{2.2}$$

Note that at maturity, $P = S$ and the payoff of the option will be

$$\max[P-K,0] \tag{2.3}$$

which is the terminal condition of equation (2.2). To guarantee the uniqueness of the solution we assume the following regularity condition,

$$\lim_{P\to\infty}\frac{\partial V}{\partial P}=1. \tag{2.4}$$

When P approaches infinity, the stock price S approaches infinity too since a bond price is bounded below 0. Hence the equation (2.4) means that the option will be exercised when the stock price increase infinitely. By applying Merton's result (1973) the solution of equation (2.2) with boundary conditions (2.3) and (2.4) is

$$V(P,\tau)=PN(d_1)-KN(d_2)$$

where $N(d)$ is the cumulative normal distribution function and

$$d_1=\frac{\ln(P/K)+1/2\theta^2}{\theta},\quad d_2=d_1-\theta\,,\quad \theta^2=\int_0^\tau \sigma^2(s)ds\,.$$

Replacing $P(t)$ and $V(P,\tau)$ by $S(t)/B(\tau)$ and $C(S,\tau)/B(\tau)$ respectively,

$$C(S,\tau)=S(t)N(d_1)-KB(\tau)N(d_2) \tag{2.5}$$

$$d_1=\frac{\ln(S/K)-\ln B(\tau)+1/2\theta^2}{\theta},\quad d_2=d_1-\theta\,,\quad \theta^2=\int_0^\tau \sigma^2(s)ds$$

are obtained. Note that the formula (2.5) is same as Black-Scholes formula except that $e^{-r\tau}$ is replaced by $B(\tau)$ if the standard deviation of the excess return is assumed to be a constant, $\sigma(t)=\sigma$. But it has empirically different meaning from BS model. We should note that σ is not the standard deviation of stock' return but that of stock's excess return over the remaining life of the option. When the stock continuously pays dividend with rate δ, the option formula becomes

$$C(S,\tau)=e^{-\delta\tau}S(t)N(d_1)-KB(\tau)N(d_2) \tag{2.6}$$

where δ_2 and θ remain the same but

$$d_1=\frac{\ln(S/K)-\delta\tau-\ln B(\tau)+1/2\theta^2}{\theta}.$$

At this moment, it is worthy to review Merton's option pricing formula. Merton's stochastic interest rate model is given by stock price dynamics and bond price dynamics as follows:

$$\frac{dS}{S}=\alpha dt+\sigma_1 dz$$

$$\frac{dB}{B} = \mu(\tau)dt + \sigma_2(\tau)dq(t;\tau)$$

where it is assumed that there is no serial correlation among returns while it is assumed that there is cross sectional correlation, $dq(t;\tau)dz(t) = \rho dt$.

It is obvious that Merton's formula is the same as the formula (2.5) except the integrand in θ^2. Since he assumes constant variances, it is equal to $\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$. One can realize that when the stock and bond price dynamics are given as in Merton, the excess return process i.e. $P(t) = S(t)/B(t)$ is

$$\frac{dP(t)}{r(t)} = \left(\alpha - \mu(\tau) + \sigma_2^2 - \rho\sigma_1\sigma_2\right)dt + \left(\sigma_1 dz - \sigma_2 dq(t;\tau)\right).$$

The variance of the excess return in Merton's model is $\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$ which is the variance, $\sigma(t)^2$ in our model. Note that the variance of the excess return in Merton's model is endogenously derived from the variances of the bond's return as well as the stock's return. However the variance in our model can be exogenously chosen, so ours is less restrictive than Merton's. By similar argument all other stochastic interest rate option models are special cases of our model since they are specific parameterization of Merton's.

Ⅲ. Data and Methodology

A. Data

The data sample is drawn from the set of closing bid, ask prices for all at-the-money call options on the S&P 500 index (SPX) from May 19, 2000 through February 21, 2003 provided by IVolatility.com. To implement the equation (2.6) we need data for bond prices instead of the unobservable instantaneous rates. Discount bond prices are calculated using the interest rates with the same maturity as option's life. Interest rates of various maturities are estimated by cubic spline method from rates of a week -through a year LIBOR rates. SPX option's expiration is on the Saturday following the third Friday. Since the settlement prices are based on Friday's opening stock prices, the last trading day is usually the Thursday. S&P 500 index and LIBORs are from the Thomson Financial DataStream.

As for dividends, we assume that the index pays dividends continuously. This is a reasonable approximation for large indices such as S&P500 index, since different stocks pay dividends on different days throughout the year. We treat dividends as if future payouts were known and use the actual dividends rates paid over the option's life published by S&P Company.

In order to get maximum efficiency within a limited sample period, daily observations are used. As Christensen and Prabhala (1998) point out, we face overlapping sample problem. Following Hansen-White adjustment for serial correlation, standard errors are corrected for the overlapping sample. To check the validity of this correction, nonoverlapping samples with about one-month maturity are extracted and used for the test.

During our sample period, the dollar interest rate and its volatility have been stable. If interest rate remains stable, the use of the stochastic interest rate model would not make much difference. To investigate the effects of high interest rate volatility, we also use the OMX options data. The OMX is a trading turnover weighted index of the 30 most traded stocks at the Stockholm Stock Exchange. As in Rindell (1995), we use data sample covering the interest rate turbulence period in 1992. During the period, the interest rate of Swedish bill had been very fluctuated by foreign currency crisis. Rindell (1995) use the similar data and show that stochastic interest rate option model price is different from BS option model price over that period. The OMX option is European with expirations of up to two years. The OMX options data are provided by Stockholm Stock Exchange. As the default free interest rate, we use expiration matched yields on Swedish T-bills, quoted in the Thomson Financial DataStream.

B. Methodology.

The index returns and excess returns are computed by log difference as follows,

$$\ln(S_t / S_{t-1}) \tag{3.1}$$

$$\ln(S_t / S_{t-1}) - \ln\left(B_t(T) / B_{t-1}(T)\right) \tag{3.2}$$

where S_t is index value at time t, and $B_t(T)$ the price of the risk-free zero-coupon bond expiring at T. The realized volatility, $\sigma_{t,T}^{R}$ ($\sigma_{t,T}^{ER}$), is annualized standard deviation of returns (excess returns) on date t to expiration date T, respectively. They are estimated using the data from observation date t, to option expiration date T. The historical volatility, $VOL60_t^R$ and $VOL60_t^{ER}$, is annualized standard deviation estimated with the data from trading date $t-60$ to observation date t. We choose 60 days for estimation band to compare results with previous studies. The implied volatility, $\sigma_{t,T}^{IV}$, is derived from ad hoc Black-Scholes model or our model with the date t price of the at-the money call option. Note that the implied volatility of our model is the same as that of BS model.

As the purpose of this empirical work is to compare the forecasting performances of implied volatilities between BS model and ours, we run the following pairs of regressions. We use both overlapping and non-overlapping data.

$$\sigma_{t,T}^{R} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \varepsilon_{t,T} \quad \text{vs.} \quad \sigma_{t,T}^{RR} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \varepsilon_{t,T} \tag{3.3}$$

$$\sigma_{t,T}^{R} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \beta_2 VOL60_t^{R} + \varepsilon_{t,T} \quad \text{vs.} \quad \sigma_{t,T}^{ER} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \beta_2 VOL60_t^{RR} + \varepsilon_{t,T} \tag{3.4}$$

$$\sigma_{t,T}^{R} = \beta_0 + \beta_1 VOL60_t^{R} + \varepsilon_{t,T} \quad \text{vs.} \quad \sigma_{t,T}^{ER} = \beta_0 + \beta_1 VOL60_t^{R} + \varepsilon_{t,T} \tag{3.5}$$

If implied volatility is an unbiased forecast for future volatility, $\beta_0 = 0$ and $\beta_1 = 1$ must be true. Since our primary interest is in the comparison of forecastibility, we test whether β_1 in $\sigma_{t,T}^{ER}$ regressor is greater than that in $\sigma_{t,T}^{R}$ regressor. We use t-test after correcting for the heteroscedasticity explained below.

As long as the regressors and error terms are uncorrelated with one another, the usual ordinary least squares (OLS) estimates of betas will be unbiased and consistent. However, using daily data introduces heteroscedasticity in error terms $\varepsilon_{t,T}$. Since $\sigma_{t,T}^{R}$ ($\sigma_{t,T}^{ER}$) is estimated using data from *t* to *T*, $\varepsilon_{t,T}$ is an accumulation of shocks occurring during the period. Then $COV(\varepsilon_{t,T}, \varepsilon_{d,D}) \neq 0$ if $d \le t \le D$ or $t \le d \le T$. Therefore the error terms are correlated between options whose remaining lifetimes overlap. This may cause downward bias in the OLS standard errors.

Hansen (1992) provides a correction that extends White's (1980) method to deal with heteroskedasticity and properly deals with serial correlation.[3] The corrected Hansen-White (HW) variance-covariance matrix of estimated coefficients is given by $\hat{\Omega} = (X'X)^{-1}\hat{\Psi}(X'X)^{-1}$, with

$$\hat{\Psi} = N^{-1}\sum_{n=1}^{N}(\hat{\varepsilon}_n)^2 X_n' X_n + N^{-1}\sum_{k=1}^{N}\sum_{n=k+1}^{N} Q(k,n)\hat{\varepsilon}_k\, \hat{\varepsilon}_n (X_n' X_k + X_k' X_n) \tag{3.6}$$

3 Canina and Figlewski (1993) and Jorion (1995) apply this method to volatility forecasts.

where $Q(k.n)$ is an indicator function taking value 1 if there is an overlap between observation n and k, and 0 otherwise. ε_k and ε_n are the fitted residuals for observations k and n from the OLS regression.

IV. Empirical Results

Table Ⅰ is basic descriptive statistics about return and excess return series of SPX. During the sample period, SPX shows negative return on average. Standard deviation is about 1.5% which means annualized value is about 23%. Shapiro-Wilk test is a statistic for the test of normality based on skewness and kurtosis with the null of normality. In both return and excess return series, p-values for Shapiro-Wilk test are less than 0.01 except 2002 sample. This confirms the conventional non-normality of underlying asset returns, and kurtosis greater than 3 shows fat-tail phenomenon.

<Table I>

Estimation results of equation (3.3), (3.4) and (3.5) are presented in Table II, III, and IV. Panel A shows regression of underlying asset returns' volatility and Panel B of excess returns' volatility of underlying asset. To check the possibility of differences across the expiration group, we also run regressions with three separate expiration groups. Group 1 is a sample with expiration less than one month, group 2 within two months, and group 3 within three months. The equations are estimated by OLS, and the coefficient covariance matrix is computed following the formula (3.6).

Estimation results for equation (3.3) are presented in Table II. Coefficients on implied volatility are estimated between 0.45 and 0.86. They are all statistically significant with conventional significance level. The hypothesis that $\beta_1 = 1$ cannot be rejected in all sample and group 1 and 2 regressions and the hypothesis that $\beta_0 = 0$ cannot be rejected in any case. This confirms the results that implied volatility is an unbiased forecast of future volatility. In regressions with long expiration group, this unbiasedness is rejected in both returns and excess returns regressions. As the remaining options life is long, it must be harder for market to forecast the future volatility.

Panel C shows the test result for the null hypothesis of equality of β_1 coefficients in Panel A and B.

We use t-test by calculating the standard deviation of the difference of β coefficients ($\beta_1^R - \beta_1^{ER}$) after correcting for the heteroscedasticity arising from overlapping sample. The equality cannot be rejected in all sample and group 2 regressions in any reasonable significance level. In group 1 regressions, it can be rejected if we use 10% of significance level. As for group 3 regressions, the t-statistics is not calculated since the forecastabilities of returns and excess returns volatility are so close that the variance of $\hat{\beta}_1^R - \hat{\beta}_1^{ER}$ is close to zero. Based on these results, we could cautiously conclude that the forecastibility of implied volatility increases as the remaining maturity become shorter and the implied volatility explains excess returns volatility better than returns volatility.

< Table II>

The table III shows the regression results of equation (3.4), which adds historical volatility in equation (3.3). The figures in Table III confirm the results of Table II. The intercept (β_0) is close to zero and the hypothesis that it is equal to zero is not rejected in all sample and group 1 and 2 regressions. Coefficients of implied volatility (β_1) are about 0.85 in all sample regressions. Therefore, even after we control for the information contained in historical volatility, implied volatility has over 80% of forecastability. Expiration group regressions show that the coefficient become greater as the expiration gets shorter. Implied volatilities of options with 60 to 90 days of remaining expiration have about 50% of explicability for the variation of future volatility. But the forecasting power increases dramatically to over 90% as the time to expiration becomes less than 2 months.

The comparison of forecastibilities of implied volatility for realized underlying asset returns and excess returns volatility is not formally tested in this regression setup. Since the historical volatilities added in this specification are not the same in both regressions, the coefficients of implied volatilities cannot be compared directly, and formal statistical test is not easily set up. Even though we do not execute a statistical test, we can find that the coefficients of implied volatility in excess returns regression are slightly greater in all regressions. Therefore, we could argue that the implied volatility explains excess returns volatility at least better than returns volatility of underlying asset.

Table III

Table IV reports the regression results of equation (3.5). This regression is performed to answer the question similar to that has long been debated on implied volatility: which of the implied volatility and historical volatility of excess returns has more predictive power for future excess returns volatility.

As for the underlying asset returns volatility, Latane and Rendleman(1976), Jorion(1995), and Christensen and Prabhala(1998) find that implied volatility contains predictive information for future volatility while Day and Lewis(1992) , Lamoureux and Lastrapes(1993), and Canina and Figlewski(1993) report that implied volatility has less predictive power than historical volatility does. In our regression, if we compare Panel A of Table II and Table IV, we find that implied volatility outperforms historical volatility-in forecasting future volatility. The coefficient of implied volatilities are about 0.7~0.8, while that of historical volatilities are around 0.3. Even though the data used in regressions for Tables II, III, IV are overlapped, our results are supportive of the first group like Christensen and Prabhala (1998) than the other group.

Following the setup of our model, implied volatility has better predictive power for future excess returns volatility than historical volatility does. In all sample regression, the coefficient of implied volatility is 0.7242 while that of historical volatility is 0.3041. Expiration group regressions show similar extent of differences in estimated coefficients.

Table IV

Table V are estimated figures for equation (3.3), (3.4) and (3.5) with non-overlapping data. Christensen and Prabhala(1998) point out that overlapping data gives less precise and potentially inconsistent estimates. They suggest that non-overlapping data with exactly one observation in each option's remaining lifetime yields more reliable regression estimates. With our SPX options data, we could extract 33 non-overlapping samples.

When we use non-overlapping sample, the coefficient of implied volatility (β_1) increases up to 0.97 in regression with both returns volatility and excess returns volatility. If we control for the lag of realized volatility, the coefficient becomes bigger than 1. When we include implied volatility in realized volatility regressions, the intercept(β_0) is always close to zero. As in overlapping sample regressions, the implied volatility is a good forecaster of future volatility.

When we compare the figures in top rows of Panel A and B, we find that the coefficient of implied volatility in excess returns volatility regression is a bit greater than in returns volatility regression. Test result for the equality of β_1 coefficients when $\beta_2 = 0$ is in Panel C. P-value for the null hypothesis is 0.0755, and it is not rejected at 5% significance level, but it can be rejected at 10% level. Therefore we get to the similar conclusion as in overlapping sample regressions that implied volatility is slightly more correlated with excess returns volatility than with returns volatility.

With non-overlapping sample, again we have the opposite result to Canina and Figlewski(1993). Historical volatility explains about 30% of the variation of realized volatility while implied volatility accounts for around 97% of the volatility.

Table V

The empirical results with OMX options data are presented in appendix because the results are not much different from those with SPX options data. The coefficients of implied volatility in excess returns regression are greater than those in returns volatility regression. The absolute magnitudes of the difference in coefficients are larger than those in Table II or III. The statistical significance of this difference is not guaranteed, but the pattern happens consistently in most of the regressions.

The striking results of Canina and Figlewski(1993) do not show up in regressions with OMX options data either.

IV. Conclusions

We propose an option pricing model similar to Merton's (1973). Instead of assuming the underlying asset return process, we assume that the excess return of underlying asset follows a lognormal process. This model is compatible with an option pricing model with any stochastic interest rate. Without assuming any specific stochastic process for interest rate, we are able to derive an option formula similar to Black-Scholes. With this model we argue that the implied volatilities used by practitioners and some academic researchers are actually the forecast of excess returns' volatility, not of returns' volatility.

This argument is supported by some regressions with SPX and OMX option data. The results show that implied volatility has a slightly higher correlation with the volatility of excess returns than with underlying asset returns, even though the difference is statistically marginal. But this slightly higher correlation is quite robust for any change of sample.

In addition, different from Canina and Figlewski(1993), we find that implied volatility contains good predictive power for future volatility with overlapping and non-overlapping sample. This predictive power increases as option's remaining lifetime becomes shorter.

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Table Ⅰ. Descriptive Statistics for daily returns and excess returns of SPX

Statistics	Returns	Excess Returns
2000/1/3~2000/12/29		
N	251	251
Mean	-0.00039	-0.00063
Median	-0.00034	-0.00064
Std. Dev.	0.01402	0.01400
Skewness	-0.00607	0.00828
Kurtosis	4.43742	4.43577
Shapiro-Wilk Test	0.98490	0.98483
(*p*-value)	(0.0093)	(0.0091)
2001/1/2~2001/12/28		
N	246	246
Mean	-0.00041	-0.00064
Median	-0.00043	-0.00060
Std. Dev.	0.01350	0.01353
Skewness	0.02429	0.04493
Kurtosis	4.56791	4.55399
Shapiro-Wilk Test	0.98221	0.98215
(*p*-value)	(0.0036)	(0.0035)
2002/1/2~2002/12/27		
N	249	249
Mean	-0.00111	-0.00121
Median	-0.00189	-0.00121
Std. Dev.	0.01644	0.01647
Skewness	0.43576	0.43153
Kurtosis	3.66915	3.64815
Shapiro-Wilk Test	0.98658	0.98695
(*p*-value)	(0.0196)	(0.0230)

* Shapiro-Wilk test is for normality of a variable. The null hypothesis is that the data are drawn from normal distribution.

Table II. Regression of Realized volatility of returns and of excess returns on Implied volatility with SPX options data

Panel A: $\sigma^{R}_{ht,T} = \beta_0 + \beta_1 \sigma^{IV}_{t,T} + \varepsilon_{t,T}$

Maturity Group	β_0	β_1	Adj. R^2
All	0.05118	0.72371**	0.2709
	(0.03867)	(0.17188)	
1	0.01198	0.86028**	0.4084
	(0.03464)	(0.14865)	
2	0.04042	0.76456**	0.2886
	(0.04129)	(0.19034)	
3	0.12343**	0.45262*	0.1053
	(0.04659)	(0.20249)	

Panel B: $\sigma^{RR}_{t,T} = \beta_0 + \beta_1 \sigma^{IV}_{t,T} + \varepsilon_{t,T}$

Maturity Group	β_0	β_1	Adj. R^2
All	0.05113	0.72416**	0.2708
	(0.03869)	(0.17193)	
1	0.01178	0.86127**	0.4083
	(0.03465)	(0.14867)	
2	0.04039	0.76481**	0.2883
	(0.04134)	(0.19050)	
3	0.12359**	0.45244*	0.1052
	(0.04657)	(0.20253)	

Panel C: Test for the equality of β_1 coefficients

Maturity Group	t	p-value
All	0.09630	0.46165
1	1.52608	0.06381
2	0.05154	0.47945

The coefficients are fitted by OLS with standard errors in parentheses corrected for heteroskedasticity using the Hansen and White procedure. Expiration Group labeled 1, 2, and 3 means samples with expirations less than one, two and three months.

* p-value < 0.05, ** p-value < 0.01

Table III. Regression of Realized volatility of returns and of excess returns on Implied and Historical volatility with SPX options data

Panel A: $\sigma_{t,T}^{R} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \beta_2 VOL60_t^{R} + \varepsilon_{t,T}$

Maturity Group	β_0	β_1	β_2	Adj. R^2
All	0.05930	0.85408**	-0.16920	0.2822
	(0.04234)	(0.22499)	(0.17888)	
1	0.02481	0.96666**	-0.16920	0.4179
	(0.04296)	(0.14925)	(0.16880)	
2	0.04886	0.91806**	-0.19380	0.3025
	(0.04297)	(0.29594)	(0.22993)	
3	0.12436	0.53479*	-0.08769	0.1071
	(0.04878)	(0.30995)	(0.22240)	

Panel B: $\sigma_{t,T}^{ER} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \beta_2 VOL60_t^{RR} + \varepsilon_{t,T}$

Maturity Group	β_0	β_1	β_2	Adj. R^2
All	0.05946	0.85781**	-0.17334	0.2826
	(0.04244)	(0.22374)	(0.17756)	
1	0.02471	0.96874**	-0.16872	0.4179
	(0.04298)	(0.14926)	(0.16897)	
2	0.04898	0.92176**	-0.19784	0.3027
	(0.04308)	(0.29546)	(0.22925)	
3	0.12465	0.54253*	-0.09619	0.1077
	(0.04898)	(0.30752)	(0.21941)	

The coefficients are fitted by OLS with standard errors in parentheses corrected for heteroskedasticity using the Hansen and White procedure. Expiration Group labeled 1, 2, and 3 means samples with expirations less than one, two and three months.

* p-value < 0.05

** p-value < 0.01

Table IV. Regression of Realized volatility of returns and of excess returns on Historical volatility with SPX options data

Panel A: $\sigma^{R}_{t,T} = \beta_0 + \beta_1 VOL60^{R}_{t} + \varepsilon_{t,T}$

Maturity Group	β_0	β_1	Adj. R^2
All	0.15064**	0.30517	0.0664
	(0.04498)	(0.20551)	
1	0.12369*	0.39520	0.0927
	(0.05540)	(0.24513)	
2	0.14653**	0.31433	0.0686
	(0.04938)	(0.22980)	
3	0.18417**	0.19797	0.0341
	(0.04326)	(0.17820)	

Panel B: $\sigma^{ER}_{t,T} = \beta_0 + \beta_1 VOL60^{R}_{t} + \varepsilon_{t,T}$

Maturity Group	β_0	β_1	Adj. R^2
All	0.15089**	0.30412	0.0657
	(0.04508)	(0.20596)	
1	0.12360*	0.39563	0.0926
	(0.05548)	(0.24529)	
2	0.14668**	0.31366	0.0681
	(0.04944)	(0.23013)	
3	0.18507**	0.19432	0.0327
	(0.04336)	(0.17907)	

The coefficients are fitted by OLS with standard errors in parentheses corrected for heteroskedasticity using the Hansen and White procedure. Expiration Group labeled 1, 2, and 3 means samples with expirations less than one, two and three months.

* p-value < 0.05

** p-value < 0.01

Table V. Regression of Realized volatility of returns and of excess returns on Implied and Historical volatility with non-overlapping SPX options data

Panel A: $\sigma_{t,T}^{R} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \beta_2 VOL60_t^{R} + \varepsilon_{t,T}$

β_0	β_1	β_2	Adj. R^2	DW
-0.00967	0.97046**		0.5166	2.057
(0.03883)	(0.16356)			
0.14802**		0.31210	0.0673	2.128
(0.03930)		(0.17343)		
-0.00592	1.19567**	-0.26205	0.5429	1.494
(0.03864)	(0.21069)	(0.15805)		

Panel B: $\sigma_{t,T}^{ER} = \beta_0 + \beta_1 \sigma_{t,T}^{IV} + \beta_2 VOL60_t^{ER} + \varepsilon_{t,T}$

β_0	β_1	β_2	Adj. R^2	DW
-0.01001	0.97195**		0.5171	2.061
(0.03886)	(0.16366)			
0.14819**		0.31134	0.0669	2.128
(0.03932)		(0.17347)		
-0.00625	1.19836**	-0.26335	0.5437	1.497
(0.03864)	(0.21069)	(0.15787)		

Panel C: Test for the equality of β_1 coefficients when $\beta_2 = 0$.

t	p-value
1.47475	0.07553

Numbers in parentheses are standard errors.

* p-value < 0.05, ** p-value < 0.01

Appendix: Estimation results with OMX call options

Table A-I. Regression of Realized volatility of returns and of excess returns on Implied volatility

Panel A: $\sigma^{R}_{ht,T} = \beta_0 + \beta_1 \sigma^{IV}_{t,T} + \varepsilon_{t,T}$

N	β_0	β_1	Adj. R^2
851	0.12389* (0.05565)	0.60012** (0.16736)	0.2366

Panel B: $\sigma^{ER}_{ht,T} = \beta_0 + \beta_1 \sigma^{IV}_{t,T} + \varepsilon_{t,T}$

N	β_0	β_1	Adj. R^2
851	0.11629* (0.05201)	0.61897** (0.16217)	0.2570

† p-value < 0.10, * p-value < 0.05, ** p-value < 0.01

Table A-II. Regression of Realized volatility of returns and of excess returns on **historical volatility**

Panel A: $\sigma^{R}_{t,T} = \beta_0 + \beta_1 VOL60^{R}_{t} + \varepsilon_{t,T}$

N	β_0	β_1	Adj. R^2
851	0.18117* (0.08080)	0.28106 (0.24632)	0.0758

Panel B: $\sigma^{ER}_{t,T} = \beta_0 + \beta_1 VOL60^{ER}_{t} + \varepsilon_{t,T}$

N	β_0	β_1	Adj. R^2
851	0.16986* (0.07932)	0.32901 (0.26515)	0.0851

† p-value < 0.10, * p-value < 0.05, ** p-value < 0.01

Table A-III. Regression of Realized volatility of returns and of excess returns on Implied and

Historical volatility

Panel A: $\sigma^{R}_{t,T} = \beta_0 + \beta_1 \sigma^{IV}_{t,T} + \beta_2 VOL60^{R}_{t} + \varepsilon_{t,T}$

N	β_0	β_1	β_2	Adj. R^2
851	0.13041* (0.06872)	0.69773** (0.17993)	-0.11615 (0.26367)	0.2425

Panel B: $\sigma^{ER}_{t,T} = \beta_0 + \beta_1 \sigma^{IV}_{t,T} + \beta_2 VOL60^{ER}_{t} + \varepsilon_{t,T}$

N	β_0	β_1	β_2	Adj. R^2
851	0.12541* (0.06669)	0.73740** (0.19339)	-0.15315 (0.29025)	0.2654

† p-value < 0.10, * p-value < 0.05, ** p-value < 0.01