Stochastic Simulation Method for the Term Structure Models with Jump

Kisoeb Park[1], Moonseong Kim[2], and Seki Kim[1]

[1] Department of Mathematics, Sungkyunkwan University
440-746, Suwon, Korea +82-31-290-7030, 7034
{kisoeb, skim}@skku.edu
[2] School of Information and Communication Engineering
Sungkyunkwan University
440-746, Suwon, Korea +82-31-290-7226
moonseong@ece.skku.ac.kr

Stochastic Simulation Method for the Term Structure Models with Jump *

Kisoeb Park[1], Moonseong Kim[2], and Seki Kim[1]

[1] Department of Mathematics, Sungkyunkwan University
440-746, Suwon, Korea +82-31-290-7030, 7034
{kisoeb, skim}@skku.edu
[2] School of Information and Communication Engineering
Sungkyunkwan University
440-746, Suwon, Korea +82-31-290-7226
moonseong@ece.skku.ac.kr

Abstract. Monte Carlo Method as a stochastic simulation method is used to evaluate many financial derivatives by financial engineers. Monte Carlo simulation is harder and more difficult to implement and analyse in many fields than other numerical methods. In this paper, we derive term structure models with jump and perform Monte Carlo simulations for them. We also make a comparison between the term structure models of interest rates with jump and HJM models based on jump. Bond pricing with Monte Carlo simulation is investigated for the term structure models with jump.

1 Introduction

Before mentioning the procedure in derivation of bond pricing models with jumps, we discuss general models of the term structure of interest rates. Approaches to modeling the term structure of interest rates in continuous time may be broadly described in terms of either the equilibrium approach or the no-arbitrage approach even though some early models include concepts from both approaches.

We introduce one-state variable model of Vasicek (1977)[16], Cox, Ingersoll, and Ross (CIR)[5], the extended model of the Hull and White[10], and the development of the model is the jump-diffusion model of the Ahn and Thompson[1] and the Baz and Das[2]. Conventionally, financial variables such as stock prices, foreign exchange rates, and interest rates are assumed to follow a diffusion processes with continuous paths when pricing financial assets. Also, Heath, Jarrow and Morton(HJM)[6] is widely accepted as the most general methodology for term structure of interest rate models.

In pricing and hedging with financial derivatives, jump-diffusion models are particularly important, since ignoring jumps in financial prices will cause pricing and hedging rates. Term structure model solutions under jump-diffusions

* corresponding author.

are justified because movements in interest rates display both continuous and discontinuous behavior. These jumps are caused by several market phenomena money market interventions by the Fed, news surprise, and shocks in the foreign exchange markets, and so on.

We study a solution of the bond pricing for the term structure models with jump. The term structure models with jump which allows the short term interest rate, the forward rate, the follow a random walk. We compare between the term structure model of interest rate with jump and the HJM model based on jump. We introduce the Monte Carlo simulation. One of the many uses of Monte Carlo simulation by financial engineers is to place a value on financial derivatives. Interest in use of Monte Carlo simulation for bond pricing is increasing because of the flexibility of the methods in handling complex financial instruments. One measure of the sharpness of the point estimate of the mean is Mean Standard Error(MSE). For the term structure models with jump, we study bond prices by the Monte Carlo simulation. Numerical methods that are known as Monte Carlo methods can be loosely described as statistical simulation methods, where statistical simulation is defined in quite general terms to be any method that utilizes sequences of random numbers to perform the simulation.

The structure of the remainder of this paper is as follows. In Section 2, the basic of bond prices with jump are introduced. In Section 3, the term structure models in jump are presented. In Section 4, we calculate numerical solutions using Monte Carlo simulation for the term structure models with jump. In Section 5, we investigate bond prices given for the eight models using the Vasicek and CIR models. Conclusions are in Section 6.

2 Preliminaries for the Bond Prices

2.1 Stochastic Differential Equation with Jump

We will first recall some notations. All our models will be set up in a given complete probability space (Ω, F_t, P) and an argumented filtration $(F_t)_{t\geq 0}$ generated by a Winear process $W(t)$ in $\Re$. We will ignore taxes and transaction costs. We denote by $V(r, r, T)$ the price at time t of a **discount bond**. It follows immediately that $V(r, T, T) = 1$. Now consider a quite different type of random environment. Suppose $\pi(t)$ represents the total number of extreme shocks that occur in a financial market until time t. The time dependence can arise from the cyclical nature of the economy, expectations concerning the future impact of monetary policies, and expected trends in other macroeconomic variables.

In the same way that a model for the asset price is proposed as a lognormal random walk, let us suppose that the interest rate r and the forward rate is governed by a **Stochastic differential equation(SDE)** of the form

$$dr = u(r,t)dt + \omega(r,t)dW^Q + Jd\pi \tag{1}$$

and

$$df(t,T) = \mu_f(t,T)dt + \sigma_f(t,T)dW^Q(t) + Jd\pi. \tag{2}$$

, where $\omega(r,t)$ is the instantaneous volatility, $u(r,t)$ is the instantaneous drift, $\mu_f(t,T)$ represents drift function, ${\sigma^2}_{f_i}(t,T)$ is volatility coefficients, and jump size J is normal variable with mean μ and standard deviation γ.

2.2 The Zero-Coupon Bond Pricing Equation

When interest rates follow the **SDE**(1), a bond has a price of the form $V(r,t)$; the dependence on T will only be made explicit when necessary. Pricing a bond is technically harder than pricing an option, since there is no underlying assert with which be hedge. We set up a portfolio containing two bonds with different maturities T_1, T_2. The bond with maturity T_1 has price V_1, and the bond with maturity T_2 has price V_2. Thus, the riskless portfolio is

$$\varPi = V_1 - \varDelta V_2. \tag{3}$$

And then we applied he jump-diffusion version of Ito's lemma. Hence we derive the the partial differential bond pricing equation.

Theorem 1. *If r satisfy Stochastic differential equation $dr = u(r,t)dt + \omega(r,t)dW^Q + Jd\pi$ then the zero-coupon bond pricing equation in jumps is*

$$\frac{\partial V}{\partial t} + \frac{1}{2}\omega^2 \frac{\partial^2 V}{\partial r^2} + (u - \lambda\omega)\frac{\partial V}{\partial r} - rV + hE[V(r+J,t) - V(r,t)] = 0 \tag{4}$$

, where $\lambda(r,t)$ is the market price of risk. The final condition corresponds to the payoff on maturity and so $V(r,T,T) = 1$. Boundary conditions depend on the form of $u(r,t)$ and $\omega(r,t)$.

3 Bond Pricing Models with Jump

The dependence of the yield curve on the time to maturity, $T - t$, is called the **term structure of interest rates**. It is common experience from market data that yield curve typically come in three distinct shapes, each associated with different economic conditions. A wide variety of yield curves can be predicted by the model, including **Increasing**, **decreasing**, and **humped**. Now we consider the term structure models with jump.

3.1 Jump-Diffusion Version of Extended Vasicek's Model

The time dependence can arise from the cyclical nature of the economy, expectations concerning the future impact of monetary policies, and expected trends in other macroeconomic variables. In this study, we extend the jump-diffusion version of equilibrium single factor model to reflect this time dependence. We proposed the mean reverting process for interest rate r is given by

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)dW^Q(t) + Jd\pi(t) \tag{5}$$

We will assume that the market price of interest rate diffusion risk is a function of time, $\lambda(t)$. Let us assume that jump risk is diversifiable. From equation (4) with the drift coefficient $u(r,t) = \theta(t) - a(t)r(t)$ and the volatility coefficient $\omega(r,t) = \sigma(t)$, we get the partial differential difference bond pricing equation:

$$[\theta(t) - a(t)r(t) - \lambda(t)\sigma(t)]V_r + V_t + \frac{1}{2}\sigma(t)^2 V_{rr} - rV + hV[-\mu A(t,T) + \frac{1}{2}(\gamma^2 + \mu^2)A(t,T)^2] = 0. \tag{6}$$

Then the yield on zero-coupon bond price expiring $T - t$ periods hence is given by:

$$Y(r,t,T) = -\frac{\ln V(r,t,T)}{T-t} \tag{7}$$

is defined the entries **term structure of interest rates**. The price of a discount bond that pays off \$ 1 at time T is the solution to (6) that satisfies the boundary condition $V(r,T,T) = 1$. A solution of the form:

$$V(r,t,T) = \exp[-A(t,T)r + B(t,T)] \tag{8}$$

can be guessed. Bond price derivatives can be calculated from (7). We omit the details, but the substitution of this derivatives into (6) and equating powers of r yields the following equations for A and B.

Theorem 2.

$$-\frac{\partial A}{\partial t} + a(t)A - 1 = 0 \tag{9}$$

and

$$\frac{\partial B}{\partial t} - \phi(t)A + \frac{1}{2}\sigma(t)^2 A^2 + h[-\mu A + \frac{1}{2}(\gamma^2 + \mu^2)A^2] = 0, \tag{10}$$

where, $\phi(t) = \theta(t) - \lambda(t)\sigma(t)$ and all coefficients is constants. In order to satisfy the final data that $V(r,T,T) = 1$ we must have $A(T,T) = 0$ and $B(T,T) = 0$.

3.2 Jump-Diffusion Version of Extended CIR Model

We proposed the mean reverting process for interest rate r is given by

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)\sqrt{r(t)}dW^Q(t) + Jd\pi(t) \tag{11}$$

We will assume that the market price of interest rate diffusion risk is a function of time, $\lambda(t)\sqrt{r(t)}$. Let us assume that jump risk is diversifiable.

In jump-diffusion version of extended Vasicek's model the short-term interest rate, r, to be negative. If Jump-diffusion version of extended CIR model is proposed, then rates are always non-negative. This has the same mean-reverting drift as jump-diffusion version of extended Vasicek's model, but the standard deviation is proportional to $\sqrt{r(t)}$. This means that its standard deviation increases when the short-term interest rate increases. From equation (4) with the drift coefficient $u(r,t) = \theta(t) - a(t)r(t)$ and the volatility coefficient $\omega(r,t) = \sigma(t)\sqrt{r(t)}$, we get the partial differential bond pricing equation:

$$[\theta(t) - a(t)r(t) - \lambda(t)\sigma(t)r(t)]V_r + V_t + \frac{1}{2}\sigma(t)^2 r(t)V_{rr} - rV \\ + hV[-\mu A(t,T) + \frac{1}{2}(\gamma^2 + \mu^2)A(t,T)^2] = 0. \tag{12}$$

Bond price partial derivatives can be calculated from (12). We omit the details, but the substitution of this derivatives into (6) and equating powers of r yields the following equations for A and B.

Theorem 3.

$$-\frac{\partial A}{\partial t} + \psi(t)A + \frac{1}{2}\sigma(t)^2 A^2 - 1 = 0 \tag{13}$$

and

$$\frac{\partial B}{\partial t} - (\theta(t) + h\mu)A + \frac{1}{2}h[(\gamma^2 + \mu^2)A^2] = 0, \tag{14}$$

where, $\psi(t) = a(t) + \lambda(t)\sigma(t)$ and all coefficients is constants. In order to satisfy the final data that $V(r,T,T) = 1$ we must have $A(T,T) = 0$ and $B(T,T) = 0$.

Proof) In equations (12) and (13), by using the solution of this Ricatti's equation formula we have

$$A(t,T) = \frac{2(e^{\omega(t)(T-t)} - 1)}{(\omega(t) + \psi(t))(e^{\omega(t)(T-t)} - 1) + 2\omega(t)} \tag{15}$$

with $\omega(t) = \sqrt{\psi(t)^2 + 2\sigma(t)}$. Similarly way, we have

$$B(t,T) = \int_t^T \left\{ -(\theta(t) + h\mu)A + \frac{1}{2}h(\gamma^2 + \mu^2)A^2 \right\} dt'. \tag{16}$$

These equation yields the exact bond prices in the problem at hand. Equation (16) can be solved numerically for B. Since (15) gives the value for A, bond prices immediately follow from equation (6).

3.3 Heath-Jarrow-Merton(HJM) Model with Jump

The HJM consider forward rates rather than bond prices as their basic building blocks. Although their model is not explicitly derived in an equilibrium model, the HJM model is a model that explains the whole term structure dynamics in a no-arbitrage model in the spirit of Harrison and Kreps[**?**], and it is fully compatible with an equilibrium model. If there is one jump during the period $[t, t+dt]$ then $d\pi(t) = 1$, and $d\pi(t) = 0$ represents no jump during that period.

We know that the **zero coupon bond prices** are contained in the forward rate informations, as bond prices can be written down by integrating over the forward rate between t and T in terms of the risk-neutral process

$$V(t,T) = \exp\left(-\int_t^T f(t,s)ds\right). \tag{17}$$

As we mentioned already, a given model in the HJM model with jump will result in a particular behavior for the short term interest rate. We introduce relation between the short rate process and the forward rate process as follows. In this study, we jump-diffusion version of Hull and White model to reflect this restriction condition. We know the following model for the interest rate r;

$$dr(t) = a(t)[\theta(t)/a(t) - r(t)]dt + \sigma_r(t)r(t)^\beta dW^Q(t) + Jd\pi(t), \tag{18}$$

where, $\theta(t)$ is a time-dependent drift; $\sigma_r(t)$ is the volatility factor;$a(t)$ is the reversion rate; $dW(t)$ is standard Wiener process; $d\pi(t)$ represents the Poisson process.

Theorem 4. *Let be the jump-diffusion process in short rate $r(t)$ is the equation (18). Let be the volatility form is*

$$\sigma_f(t,T) = \sigma_r(t)(\sqrt{r(t)})^\beta \eta(t,T) \tag{19}$$

with $\eta(t,T) = \exp\left(-\int_t^T a(s)ds\right)$ is deterministic functions. We know the jump-diffusion process in short rate model and the "corresponding" compatible HJM model with jump

$$df(t,T) = \mu_f(t,T)dt + \sigma_f(t,T)dW^Q(t) + Jd\pi(t) \tag{20}$$

, where $\mu_f(t,T) = \sigma_f(t,T)\int_t^T \sigma_f(t,s)ds$. Then we obtain the equivalent model is

$$\begin{aligned} f(0,T) = r(0)\eta(0,T) &+ \int_0^T \theta(t)\eta(s,T)ds \\ &- \int_0^T \sigma_r^2(s)(r(s)^2)^\beta \eta(s,T)\int_s^T (\eta(s,u)du)ds \end{aligned} \tag{21}$$

that is, all forward rates are normally distributed. Note that we know that $\beta = 0$ case is an extension of Vasicek's jump diffusion model; the $\beta = 0.5$ case is an extension of CIR jump diffusion model.

Note that the forward rates are normally distributed, which means that the bond prices are log-normally distributed. Both the short term rate and the forward rates can become negative. As above, we obtain the bond price from the theorem 1. By the theorem 2, we drive the relation between the short rate and forward rate.

Corollary 1. *Let be the HJM model with jump of the term structure of interest rate is the stochastic differential equation for forward rate $f(t,T)$ is given by*

$$df(t,T) = \sigma_f(t,T) \int_t^T \sigma_f(t,s)dsdt + \sigma_f(t,T)dW^Q(t) + Jd\pi(t) \tag{22}$$

where, dW_i^Q is the Wiener process generated by an equivalent martingale measure Q and $\sigma_f(t,T) = \sigma_r(t)(\sqrt{r(t)})^\beta \exp\left(-\int_t^T a(s)ds\right)$.
Then the discount bond price $V(t,T)$ for the forward rate is given by the formula

$$V(t,T) = \frac{V(0,T)}{V(0,t)} \exp\{-\frac{1}{2}\left(\frac{\int_t^T \sigma_f(t,s)ds}{\sigma_f(t,T)}\right)^2 \int_0^t \sigma_f^2(s,t)ds - \frac{\int_t^T \sigma_f(t,s)ds}{\sigma_f(t,T)}[f(0,t) - r(t)]\}$$

with the equation (21).

Note that we know that $\beta = 0$ case is an extension of Vasicek's jump diffusion model; the $\beta = 0.5$ case is an extension of CIR jump diffusion model.

4 Monte Carlo Simulation of The Term Structure Models with Jump

By and application of Girsanov's theorem the dependence on the market price of risk can be absorbed into an equivalent martingale measure. Let $W(t), 0 \le t \le T$, be a Wienear process on a probability space (Ω, F, P). Let $\lambda(t), 0 \le t \le T$, be a process adapted to this filtration. The Wiener processes $dW^Q(t)$ under the equivalent martingale measure Q are given by $W^Q(t) = W(t) + \int_0^t \lambda(s)ds$ so that

$$dW_i^Q(t) = dW_i(t) + \lambda_i(t)ds.$$

A **risk-neutral measure**Q is any probability measure, equivalent to the market measure P, which makes all discounted bond prices martingales.

We now move on to discuss Monte Carlo simulation. A Monte Carlo simulation of a stochastic process is a procedure for sampling random outcomes for the process. This uses the risk-neutral valuation result. The bond price can be expressed as:

$$V(r_t,t,T) = E_t^Q\left[e^{-\int_t^T r_s ds}|r(t)\right] \quad \text{or} \quad V(f_t,t,T) = E_t^Q\left[e^{-\int_t^T f(t,s)ds}\right] \tag{23}$$

, where E_t^Q is the expectations operator with respect to the equivalent risk-neutral measure. Under the equivalent risk-neutral measure, the local expectation hypothesis holds(that is, $E_t^Q\left[\frac{dV}{V}\right]$).To execute the Monte Carlo simulation, we discretize the equations (5) and (12). we divide the time interval $[t,T]$ into

m equal time steps of length Δt each. For small time steps, we are entitled to use the discretized version of the risk-adjusted stochastic differential equations (5), (11), and (22):

$$\begin{aligned} r_j = r_{j-1} &+ [(\theta \cdot t) - (a \cdot t) r_{j-1} \cdot t - (\lambda \cdot t)(\sigma \cdot t)] \Delta t \\ &+ (\sigma \cdot t) \varepsilon_j \sqrt{\Delta t} + J_j N_{\Delta t}, \end{aligned} \tag{24}$$

$$\begin{aligned} r_j = r_{j-1} &+ [(\theta \cdot t) - (a \cdot t) r_{j-1} - (\lambda \cdot t)(\sigma \cdot t) \sqrt{r_{j-1} \cdot t}] \Delta t \\ &+ (\sigma \cdot t) \sqrt{r_{j-1} \cdot t}\ \varepsilon_j \sqrt{\Delta t} + J_j N_{\Delta t} \end{aligned} \tag{25}$$

and

$$f_j = f_{j-1} + \left[\sigma_f(t,T) \int_t^T \sigma_f(t,s) ds dt \right] \Delta t + \sigma_f(t,T) \varepsilon_j \sqrt{\Delta t} + J_j N_{\Delta t} \tag{26}$$

, where $\sigma_f(t,T) = \sigma_r(t)(\sqrt{r(t)})^\beta \exp\left(-\int_t^T a(s)ds\right)$, $j = 1, 2, \cdots, m$, ε_j is standard normal variable with $\varepsilon_j \sim N(0,1)$, and $N_{\Delta t}$ is a Poisson random variable with parameter $h\Delta t$. We know that $\beta = 0$ case is an extension of Vasicek's jump diffusion model; the $\beta = 0.5$ case is an extension of CIR jump diffusion model. We can investigate the value of the bond by sampling n spot rate paths under the discrete process approximation of the risk-adjusted processes of the equations (24), (25), and (26). The bond price estimate is given by:

$$V(r_t, t, T) = \frac{1}{n} \sum_{i=1}^{n} \exp\left(-\sum_{j=0}^{m-1} r_{ij} \Delta t \right) \text{ or } V(f_t, t, T) = \frac{1}{n} \sum_{i=1}^{n} \exp\left(-\sum_{j=0}^{m-1} f_{ij} \Delta t \right)$$

, where r_{ij} is the value of the short rate and f_{ij} is the value of the forward rate under the discrete risk-adjusted process within sample path i at time $t + \Delta t$. Numerical methods that are known as Monte Carlo methods can be loosely described as statistical simulation methods, where statistical simulation is defined in quite general terms to be any method that utilizes sequences of random numbers to perform the simulation. The Monte Carlo simulation is clearly less efficient computationally than the numerical method. One measure of the sharpness of the point estimate of the mean is MSE, defined as

$$MSE = \nu / \sqrt{n} \tag{27}$$

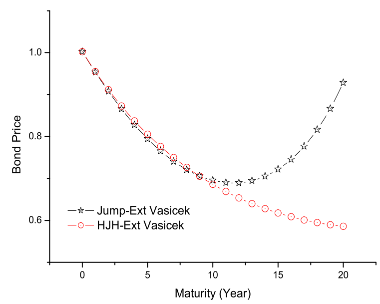
where, ν^2 is the estimate of the variance of bond prices as obtained from n sample paths of the short rate:

$$\nu^2 = \frac{\sum_{i=1}^{n} \left[\exp\left(-\sum_{j=0}^{m-1} f_{ij} \Delta t \right) - \nu \right]}{n-1}. \tag{28}$$

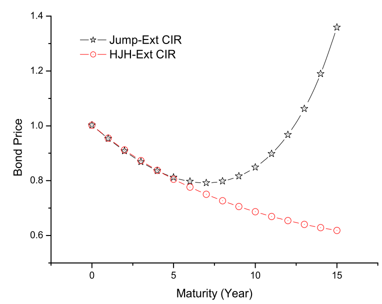
This reduces the MSE by increasing the value of n. However, highly precise estimates with the brute force method can take a long time to achieve. For the purpose of simulation, we conduct three runs of 1,000 trials each and divide the year into 365 time steps.

5 Experiments

In this section, we investigate the jump-diffusion version of extended Vasicek and CIR model and HJM model with jump. Experiments are consist of the numerical method and Monte Carlo simulation. Experiment 1, 2 plot estimated term structure using the various models. In experiment 1, 2, the parameter values are assumed to be $r = 0.05$, $a = 0.5$, $b = 0.05$, $\theta = 0.025$, $\sigma = 0.08$, $\sigma_r = 0.08$, $\lambda = -0.5$, $\gamma = 0.01$, $\mu = 0$, $h = 10$, $t = 0.05$, and $T = 20$. Experiment 3, 4 examine bond prices by the Monte Carlo simulation. In experiment 3, 4, the parameter values are assumed to be $r = 0.05$, $f[0,t] = 0.049875878$, $\sigma_r = 0.08$, $a = 0.5$, $b = 0.05$, $\theta = 0.025$, $\sigma = 0.08$, $\lambda = -0.5$, $\Delta t = (T-t)/m$, $m = 365$, $n = 1000$, $\gamma = 0.01$, $\mu = 0$, $h = 10$, $t = 0.05$, and $T = 20$.



(a) Bond prices based on extended Vasicek model



(b) Bond prices based on extended CIR model

Fig. 1: The various bond prices for the term structure models with jump

	J-Vasicek	J-E_Vasicek	HJM-E_Vasicek	J-HJM-E_Vasicek
CFS	0.93596	0.953704	0.954902	0.954902
MCS	0.933911	0.95031	0.951451	0.951722
Diff(CFS-MCS)	0.00150833	0.0002868	5.03495E-06	0.000319
Variance	0.00122814	0.00053554	7.09574E-05	0.00178619
MSE	0.000933	0.00286	0.00205	0.003394

Experiment 3. Bond prices estimated by the Monte Carlo simulation for the jump-diffusion and HJM model with jump based on Vasicek model.

	J-CIR	J-E_CIR	HJM-E_CIR	J-HJM-E_CIR
CFS	0.942005	0.953478	0.95491	0.95491
MCS	0.947482	0.951688	0.951456	0.950456
Diff(CFS-MCS)	0.0002863	0.00030634	1.2766E-06	0.0002897
Variance	0.000535	0.0005535	0.000113	0.001702
MSE	-0.005478	0.00179042	0.00345374	0.00444414

Experiment 4. Bond prices estimated by the Monte Carlo simulation for the jump-diffusion and HJM model with jump based on CIR model.

6 Conclusion

Even though Monte Carlo simulation is both harder and conceptually more difficult to implement than the other numerical methods, interest in use of Monte Carlo simulation for bond pricing is getting stronger because of its flexibility in evaluating and handling complicated financial instruments. However, it takes a long time to achieve highly precise estimates with the brute force method. In this paper we investigate bond pricing models and their Monte Carlo simulations with several scenarios. The bond price is humped in the jump versions of the extended Vasicek and CIR models while the bond prices are decreasing functions of the maturity in HJM models with jump.

References

1. C. Ahn and H. Thompson, "Jump-Diffusion Processes and the Term Structure of Interest Rates," Journal of Finance, vol. 43, pp. 155-174, 1998.
2. J. Baz and S.R.Das, "Analytical Approximations of the Term Structure for Jump-Diffusion Processes : A Numerical Analysis," Journal of Fixed Income, vol. 6(1), pp.78-86, 1996
3. D. Beaglehole and M. Tenney, "Corrections and Additions to 'A Nonlinear Equilibrium Model of the Term Structure of Interest Rates'," Journal of Financial Economics, vol. 32, pp.345-353, 1992
4. E. Briys, "Options, Futures and Exotic Derivatives," John Wiley, 1985.
5. J.C. Cox, J. Ingersoll, and S. Ross, "A Theory of the Term Structure of Interest Rate," Econometrica, vol. 53, pp.385-407, 1985
6. David Health., Robert Jarrow, and Andrew Morton, "Bond Pricing and the Term Structure of Interest Rates," Econometrica, vol. 60. NO.1, pp.77-105, 1992
7. T. S. Ho and S. LEE, " Term Structure Movements and Pricing Interest Rate Contingent Claims," Journal of Finance, vol. 41, pp. 1011-1028, 1986.
8. F. Jamshidian, "An Exact Bond Option Formula," Journal of Finance, vol 44, 1989.
9. J. Frank, and CFA. Fabozzi, "Bond markets Analysis and strategies," Fourth Edition, 2000.
10. J. Hull and A. White, "Pricing Interest Rate Derivative Securities," Review of Financial Studies, vol. 3, pp.573-92, 1990
11. J. Hull and A. White, "Options, Futures, and Derivatives," Fourth Edition, 2000
12. F. A. Longstaff and S. Schwartz, "Interest Rate Volatility and the Term structure: A Two-Factor General Equilibrium Model," Journal of Finance, vol. 47, pp.1259-1282, 1992
13. Michael J. Brennan and Eduardo S. Schwartz, "A Continuous Time Approach to the Pricing of Bonds," Journal of Banking and Finance, vol 3, pp.133-155, 1979
14. J. Strikwerda, "Finite Difference Schmes and Partial Differential Equations," Wadsworth and Brooks/Cole Advanced Books and Software, 1989
15. J.W. Drosen, "Pure jump shock models in reliability," Adv. Appl. Probal. vol. 18, pp.423-440, 1986
16. O. A. Vasicek, "An Equilibrium Characterization of the Term Structure," Journal of Financial Economics, vol. 5, pp.177-88, 1977