Long Run Probability of Default and BASEL II Capital Allocation

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Abstract

Basel II regulatory capital formula could imply substantial gaps between the long run PD and the short run historical average. Hence, banks might need to raise their short run historical average of internal PD substantially. Under through-the-cycle rating system, they might have to increase it even more when the economy is in booming period. With more realistic assumption of credit migration, however, we find that gaps are much smaller in many cases. We show, through simulation and a credit card portfolio, that rating pooling can generate substantial variation in BASEL II regulatory capital.

Key Words: BASEL II, probability of default, regulatory capital, pooling, business cycle, rating system

1 Introduction

There has been growing attention on capital adequacy under BASEL II. Recently Kupiec (2004) suggested, using a one factor Black-Scholes-Merton model, that advanced internal rating based (A-IRB) approach could underestimate the true capital by a large margin. The regulatory capital under BASEL II is directly related to how the long-run probability of default (PD) is computed and how the risk rating is pooled. However, little attention has been paid to the relationship between the PD of the historical average and the long-run PD and to the impact of pooling of obligors on regulatory capital.

The BASEL II document (2004) states that the long run probability of default (PD) for borrowers in each grade has to be an average of one-year default rates using at least five years of historical data. Since five years is not sufficient to cover the business cycle and the worst experience over five years could fall short of the true long run PD, banks and regulators have been working to fill the gap between the long run PD and short run average of historically available PD data.

From the one-factor model that the capital formula of BAESEL II is based on, we can derive a distribution of one-year PD for a given long-run probability of default. We can derive the same distribution from the models of Gordy (2003) and Heitfield (2004) and Perli and Nayda (2002). Using this relationship[1] we can study whether a short-term historical average, typically five year average, of one-year default rates based on bank's internal data could be a conservative estimate for a long run average of one-year default rates if such data is available. We find substantial gaps between the long-run one-year PD and the short run historical average of one-year PD. It implies that banks need to increase their five or more years of historical average of one-year PD substantially to approximate the long-run PD used in the capital formula.

We extend the analysis to Through-the-Cycle (TTC) rating systems and include restriction on lower bound. The analysis generally applies only for rating systems under Point-in-Time (PIT) philosophy. Since TTC rating systems utilize economic state variables, we use macro economic variables to estimate the stress level and study the relationship between the short-run PD and the long-run PD. Most borrowers are also subject to positive default probability. Thus it is reasonable to restrict a low bound on default probability instead of using the unrestricted distribution from the one-factor model. Moreover, banks use internal or external rating systems to divide borrowers into homogeneous pools. Banks may assign the borrowers into another pool when the credit quality changes. Hence, it seems to be reasonable to set low bounds on one-year PD depending on the level of PD of each rating instead of assuming that all the borrowers have same low bound of zero default probability.

In this paper we show how the gaps between the long-run PD and the short-run PD can change with assumptions on the underlying economic states and business cycles. Under

[1] OSFI (2004) did a preliminary analysis related to these issues.

TTC rating system the short run PD is higher when the economy is in recession compared to expansion periods. We also show that the gap is much smaller with credit migration and more realistic economic states. Finally, we show that risk bucketing or pooling is more important in terms of regulatory capital.

2 PD Estimation

We start from the one factor model used by Gordy (2003) and Perli and Nayda (2002). Assuming that assets are driven by a single common factor and an idiosyncratic noise component E_i:

$$V_i = \sqrt{r}X + \sqrt{1-r}E_i \tag{1}$$

The Default state is defined by an indicator function:

$$I_i = \begin{cases} 1 & if \quad V_i < D_i \\ 0 & otherwise \end{cases} \tag{2}$$

The conditional default indicator value is computed as follows:

$$E(I_i \mid X = x) = p(x) = \Pr(V_i < D_i \mid X = x) = \Phi\left(\frac{D_i - \sqrt{r}x}{\sqrt{1-r}}\right) \tag{3}$$

Since the proportion of portfolio that defaults, denoted as Y, converges almost surely to the probability of default defined in equation (3), the probability distribution is simplified as follows:

$$\begin{aligned} F(y) = \Pr(Y \le y) &= \int_{-\infty}^{\infty} \Pr(Y = p(x) \le y \mid X = x)\phi(x)dx \\ &= \Phi(x^*) \end{aligned} \tag{1}$$

where $p(-x^*) = y$. Hence, the fraction of losses can be derived as follows[2]:

$$F(y) = \Phi\left(\frac{\sqrt{1-r}}{\sqrt{r}}\Phi^{-1}(y) - \frac{1}{\sqrt{r}}\Phi^{-1}(pd)\right) \tag{4}$$

where $pd = \Phi(D_i)$. The fraction of defaults such that the probability of y_α or less defaults happening will be exactly α is given by $F(y_\alpha) = \alpha$ i.e.,

[2] Refer to Perli and Nayda (2002).

$$y_{\alpha} = \Phi\left(\frac{\sqrt{r}}{\sqrt{1-r}} \Phi^{-1}(\alpha) + \frac{1}{\sqrt{1-r}} \Phi^{-1}(pd) \right) \quad (5)$$

Similarly, from the new Basel II capital formula, with some substitutions, we can derive the following equation[3]:

$$PD_{q_y} = \Phi\left[\Phi^{-1}(PD^{LR}) \frac{1}{\sqrt{1-r}} + \Phi^{-1}(q_y) \frac{\sqrt{r}}{\sqrt{1-r}} \right] \quad (6)$$

The correlation r varies by the type of exposure. For example, for corporate exposures, the correlation r is defined as follows:

$$r = 0.12 * \frac{1-e^{(-50*PD^{LR})}}{1-e^{(-50)}} + 0.24 * \left(1 - \frac{1-e^{-(50*PD^{LR})}}{1-e^{(-50)}} \right)$$

The state of the economy is represented by y, which follows standard normal distribution. Hence, the percentile parameter q_y is defined as $q_y = N(y)$, which follows uniform distribution, $U(0,1)$. For a given long-run PD^{LR}, we get the distribution of one-year PD_{q_y}, by varying economic state y or percentile q_y. The shape of the distribution is sensitive to the correlation r and the shape of the inverse cumulative normal distribution. To find out this, suppose the following:

$$x = \Phi^{-1}(PD^{LR})$$
$$z = \Phi^{-1}(q_y)$$

Then the level of q_y that makes the long run PD^{LR} to be equal to the one-year PD_{q_y} is determined by:

$$z = x\beta \equiv x\left(\frac{\sqrt{1-r} - 1}{\sqrt{r}} \right)$$

where $\beta = -1$ when $r = 1$, and β increases to 0 as r decreases to 0. When the long run PD^{LR} is less than *0.5*, i.e., $x < 0$, it implies that $0 < z < -x$. Hence, the percentile $q_y(PD^{LR} = PD_{q_y})$ is greater than 0.5. The higher the correlation r is, the higher the

[3] The capital formula for banks with internal rating-based approach is from the document [2] of Basel Committee on Banking Supervision. We set LGD=EAD=1 to derive the equation (6) and additionally set the maturity adjustment equal to 1 for the corporate case.

percentile $q_y(PD^{LR} = PD_{q_y})$ will be. Because of the shape of the inverse cumulative normal distribution, as the long run PD^{LR} increases approaching to *0.5*, the percentile $q_y(PD^{LR} = PD_{q_y})$ will get smaller approaching to *0.5*.

Figure 1 shows the level of q_y that makes the long run PD^{LR} to be equal to the one-year PD_{q_y}. It shows that the shape of the distribution for one-year PD is sensitive to the correlation, r. It also shows that the lower the long run PD, the bigger the discrepancy between the long run PD and one-year PD in percentage wise (the more skewed the distribution to the right).

Based on the distribution of one-year PD drawn from the long run PD of *10* basis point, *50* basis points, and *100* basis points, we randomly draws, *1* million times, one-year PD for five years and calculated the average of five-year samples. Although less skewed than the distribution of the one-year PD, the distribution of the average of five-year samples is still skewed and long-tailed as shown in second graph of Figure 2.

Without the information about underlying economy, the mean of five-year time series would most likely fall at the mode of the above distribution of five-year samples. Hence, we need to compute the mode of the distribution. Since observations of the distribution are irregularly spaced and the mode can be sensitive to the length of intervals, we use the following kernel density estimation to compute the mode:

$$\pi(z_i) = \frac{1}{Nh}\sum_{t=1}^{N} K\left(\frac{(z_i - x_t)}{h}\right) \tag{7}$$

where $\{x_t\}_{t=1}^{N}$ is a set of mean of five-year sample, *K* is a Gaussian kernel, and $\{z_i\}_{i=1}^{M}$ is a set of *M* points defining an equally spaced partition of a subset of the support of the stationary density. The bandwidth h is computed using the formula[4], $4\hat{\sigma}N^{-1/5}$, where $\hat{\sigma}$ is the standard deviation of sample $\{x_t\}_{t=1}^{N}$. Figure 3 shows that the mode of one-year PD is difficult to pick when we use1 basis point interval. However, the density becomes very smooth when we use 33 basis points interval based on the automatic bandwidth.

Table 2 shows the mode of the five-year average of one-year PD for each long run PD. From these results we could determine the long run average corresponding to a given mode. For example, the five-year average default rate that a bank had observed over 5 years was *0.62%*. Assuming this to be the mode most likely, the bank found that this value is the mode of the distribution of five-year sample estimates when the long-term average is *1%*. Hence the bank needs to increase their PD estimate by *61%*. Figure 4

[4] Stanton (1997) used the same bandwidth in his nonparametric estimation of interest rate. For various bandwidth selections, refer to Silverman (1986).

shows the approaching speed to the true long run PD. As shown above, the higher the long-run PD, the closer the sample average to the long-run PD because of the decreasing correlation and the shape of cumulative normal distribution.

3 PD Estimation with Rating Migration

Since most exposures are considered to be risky, there should be a lower bound of default that one-year PD_{q_y} cannot go below. Either by internal or by external, banks have rating or pooling category. In other words, banks have PD boundary for each rating and they assign obligors to each rating by their credit behavior score. When the credit behavior score for an obligor changes, it may migrate to another rating. Hence, it implies that we need to restrict at least the lower boundaries of PD for each rating consistent with rating classification.

3.1 Fixed Lower Bound of PD

From the equation (6), for a given long run PD^{LR} and one-year PD_{q_y}, the percentile q_y is determined as follows:

$$q_y = \Phi\left[\Phi^{-1}(PD_{q_y}) \frac{\sqrt{1-r}}{\sqrt{r}} - \Phi^{-1}(PD^{LR}) \frac{1}{\sqrt{r}} \right] \tag{8}$$

We set the lower boundary of q_y as 10% of the long-run PD. Table 3 shows the level of q_y for lower boundaries of one-year PD_{q_y}. Table 4 shows the mode of the five-year average of one-year PD for each long run PD under the above lower boundary. Now the *5*-year short-run average is much closer to the long-run PD. For example, under the long-run PD of *10* basis points, the mode of *5*-year short-run average is only *4.2* basis points without any restriction on low bound. However, the mode of *5*-year short-run average becomes *8.1* basis points with the low bound of only 1 basis point. With *10*-year data, it almost reaches the long run PD at *9.9* basis points while the unrestricted mode is still only half of the long run PD at *5.4* basis points.

3.2 Variable Lower Bound with Rating Migration

Since banks have rating category, we set the low bound one the one-year PD for each rating proportional to the higher rating PD. Table 5 shows the mode of the five-year average of one-year PD for each long run PD under variable lower bound on the one-year PD. It shows that the mode of the *5*-year average is higher than the long run PD when the higher rating PD is used as a lower bound on the one-year PD. The results can vary as we change the rating category.

Depending on the assumption of the low bound on the one-year PD, we get substantially different adjustment factor for the long-run PD. In the next section, we assume that each low bound on the one-year PD represents the true economic state and generate historical average of one-year PD as above. We compare the resulting regulatory capital under BASEL II.

4 PD and Rating Systems with Business Cycles

We extend the model in Section 2 following Heitfield (2004). We assume that asset values, hence defaults, are also affected by an exogenous risk factor $Z_{i,t}$ which describes economic state or business cycle. Thus asset values follow the process:

$$V_{i,t+1} = \beta_z Z_t + U_{i,t+1} \quad (9),$$

$$U_{i,t+1} = \sqrt{r} X_{t+1} + \sqrt{1-r} E_{i,t+1} \quad (10).$$

If β_z is equal to zero, then the model is as same as the one in Section 2. The exogenous variable Z_t is a risk factor that affects the credit quality of all obligors in a bank's portfolio. The standard normal random variable $U_{i,t+1}$ reflects information that affects an obligor's default status at date *t+1* that cannot be observed by a bank at date *t*. It is a weighted average of systematic risk factor X_{t+1} shared by all obligors and an idiosyncratic risk factor $E_{i,t+1}$ that is unique to an obligor i.

The default frequency for Point-In-Time (PIT) rating is computed as follows:

$$\begin{aligned} DF_{i,t}^{PIT} &= E\left[D_{i,t+1} \mid i \in \Gamma_t^{PIT}, Z_t = z_t, X_{t+1} = x_{t+1}\right] \\ &= \Phi\left(\frac{\gamma_{PIT} - \sqrt{r}x_{t+1}}{\sqrt{1-r}}\right) \\ &= \Phi\left(\frac{\Phi^{-1}(LRDF^{PIT}) - \sqrt{r}x_{t+1}}{\sqrt{1-r}}\right) \\ &= \Phi\left(\frac{1}{\sqrt{1-r}}\Phi^{-1}(LRDF^{PIT}) + \frac{\sqrt{r}}{\sqrt{1-r}}\Phi^{-1}(q)\right) \end{aligned} \quad (11)$$

where $q = \Phi(-x_{t+1})$ and $LRDF^{PIT}$ denotes long-run probability of default under PIT. This is equivalent to the formula (6) derived from the Basel II framework. However, the default frequency for Through-The-Cycle (TTC) rating is

$$
\begin{aligned}
DF_{i,t}^{TTC} &= E\left[D_{i,t+1} \mid i \in \Gamma_t^{TTC}, Z_t = z_t, X_{t+1} = x_{t+1}\right] \\
&= \Phi\left(\gamma_{TTC} - \frac{\beta_Y z_t - \psi + \sqrt{r}x_{t+1}}{\sqrt{1-r}}\right) \\
&= \Phi\left(\frac{1}{\sqrt{1-r}}\left[\Phi^{-1}(LRDF^{TTC})\sqrt{1+\beta_Z^2} - \beta_Y z_t\right] + \frac{\sqrt{r}}{\sqrt{1-r}}\Phi^{-1}(q)\right)
\end{aligned} \qquad (12).
$$

For TTC rating system, short-run PD depends not only on the long-run PD but also on the macroeconomic environment and business cycle. It implies that the short-run PD will be lower when the economy is booming and will be higher when the economy is recession.

We estimate the coefficient β_Z using the Moody's probability of default for six credit ratings, AA, A, BBB, BB, B, CCC and U.S. federal fund rate from 1995 through 2004. We use the normalized federal fund rate as a proxy for economic state. First we find time series of long run PD for each rating such that the mode of short-run PD matches the historical PD of each rating. Using the time series of long run PD, we can find the time series of stress level q and β_Z that maximizes the occurrence of observed time series of historical PD.

$$
(\beta_Z^*, q^*) = \underset{\beta_z, q}{Arg}\, Min\left\{\sum_{t=1}^{T}\sum_{i=1}^{6}\left[DF_{i,t}^{TTC} - DF_{i,t}\right]^2\right\} \qquad (13).
$$

Figure 5 shows estimated stress level, q, and the fitted PD. The estimated coefficient β_Z is *0.09*. The fitted long run PD is about 10% higher than the historical average for all credit ratings, AA, A, BBB.

Under the normal economic period, $z = 0$, the following relationship holds between the long-run PD under TTC rating system and that under PIT rating system:

$$
LRDF^{TTC} = \Phi\left(\frac{\Phi^{-1}(LRDF^{PIT})}{\sqrt{1+\beta_Z^2}}\right) \qquad (14).
$$

Using the estimated coefficient β_Z, we compute a long-run PD under TTC rating for a given long-run PD under PIT rating. Table 6 shows the mode of the five-year average of one-year PD for each long run PD under the TTC rating system when the economy is at recession: β_Z is *0.09 and* z is *–1.5*.

Figure 6 shows the corresponding long run PD under PIT and TTC rating systems when the economy is booming and in recession. When the economy is booming, banks using TTC rating system should increase their PD more than the banks using PIT rating system.

5 Credit Portfolio Example

5.1 Simulation

To illustrate the impact of low bound of one-year PD and pooling method on regulatory capital at portfolio level, we set up two portfolios. We report the case under PIT rating system since the adjustment under TTC rating system is obvious as was shown in section 3. Portfolio 1 has three pools of long-run PD of 0*.1%*, *1%*, and *10%*, respectively. Portfolio 2 has seven pools of long-run PD of *0.1%, 0.3%, 0.5%, 1%, 3%, 5%,* and *10%.*

We draw one-year PD from the equation (6) using three low bounds – unrestricted low bound (ULB, Table 2), 10% fixed low bound (FLB, Table 3), and variable low bound (VLB, Table 4). For the portfolio 1, we randomly select 1000 obligors from each pool and we adjust individual PD so that the mean of each 1000 obligor match the mode five-year average of one-year PD from each pool as shown in Table 2, Table 4, and Table 5[5]. Hence, we maintain the hypothesis that the historical average is the mode of the average of one-year PD from equation (6). For the portfolio 2, we randomly select 400 obligors from each pool and proceed as in the portfolio 1. We further assume that there are two pooling systems for portfolios –one with two ratings and the other with five ratings.

Pooling method changes the number of obligors in each rating and thus the average of PD in each pool. In other words, pooling triggers variations in transition probability. Table 7 displays 10 different rating boundaries for the two pooling systems. We report three regulatory capital numbers in Figure 6 and Figure 7. The BASEL II regulatory capital formula for a corporate exposure under the advanced IRB approach is as follows:

$$K(Capital\ \mathrm{Re}\,quirement) = \left\{ LGD * N\left[\frac{1}{(1-R)} * N^{-1}(PD) + \left(\frac{R}{1-R} \right)^{0.5} * N^{-1}(0.999) \right] - PD * LGD \right\} * \frac{(1+(M-2.5)*b)}{(1-1.5*b)}$$

where

$$R = 0.12 * \frac{(1-e^{-50*PD})}{1-e^{-50}} + 0.24 * \left[1 - \frac{(1-e^{-50*PD})}{1-e^{-50}} \right],$$

and

$$b = (0.11852 - 0.05478 * \ln(PD))^2$$

[5] In the case of VLB for portfolio 1, we use the mode from Table 5 instead of Table 6 since lower boundaries of long-run PD match with those of Table 5.

Table 8 and Table 9 show more detailed capital number – capital based on the true long-run PD, capital based on observed average PD of each pool, and capital based on mode-adjustment from the previous section i.e., Table 2, Table 4, and Table 5[6].

The results are very sensitive to how the rating boundary was chosen. While the low bound restriction on the one-year PD can make a difference in regulatory capital amount, it is very small compared to what the rating boundary i.e., pooling method can cause in regulatory capital amount.

5.2 Credit Card Example

We collected the probability of default from CIBC visa card based on the number of accounts from 1998 through 2002. Default includes accounts which are past due 180 days. We transform the default rate not to disclose the true numbers of CIBC. As a comparison we collected the delinquency rate of Canadian credit card loans that are past due 90 days or more from the Canadian Bankers Association (CBA) over the period from 1977 through 2003 (yearly data). The Canadian delinquency rate is based on the outstanding balance not on the number of accounts.

Figure 9 displays the yearly delinquency rate of Canadian credit card loans that are past due 90 days or more from the Canadian Bankers Association over the period 1977 through 2003 and compare them with the default rate of CIBC credit card. It displays up and downs for CBA delinquency rate since it includes business cycles.

Since the CIBC default rate is based on the delinquency rate of 180 days or more, we expect the CBA delinquency rate of 90 days or more to be more volatile than the CIBC default rate. To control the volatility of the transformed delinquency rate, we report the volatility ratio of the transformed delinquency rate. Volatility ratio is computed by dividing the volatility of the transformed delinquency rate over 27 years by the volatility of CIBC PD over 5 years.

Without the volatility restriction the transformed delinquency rate has volatility nine times higher than that of the CIBC default rate. Hence, we add two more cases: the volatility of the delinquency rate is twice higher than the volatility of the default rate and the volatility of the delinquency rate is as same as the volatility of the default rate. The first graph of Figure 10 displays the adjusted yearly delinquency rate of Canadian credit card loans under three volatility ratios. The second graph shows the estimated stress level.

Table 9 shows the results based on Canadian delinquency rate that was computed using the outstanding balance and not the number of accounts. It shows that the 5-year average substantially underestimated the long run PD by 118 basis points (3.30% vs. 4.48%) without the volatility restriction. When the volatility of the delinquency rate is as same as

[6] We linearly interpolate for the numbers not covered in the tables.

the volatility of the CIBC default rate, the 5-year average underestimated the long run PD by 32 basis points (3.30% vs. 3.62%; 11% increase).

Figure 11 shows the regulatory capital computed under three different pooling methods for the CIBC visa portfolios. We do not report the actual amount not to disclose the CIBC information directly. It shows that the regulatory capital is sensitive to the pooling method.

6 Conclusion

Basel II regulatory capital formula could imply substantial gaps between the long run PD and the short run historical average. Hence, banks might need to raise their short run historical average of internal PD substantially. Under through-the-cycle rating system, they might have to increase it even more when the economy is in expansion period. With more realistic assumption of credit migration, however, we find that gaps are much smaller in many cases. We show through simulation and a credit card portfolio that rating category or pooling boundary can generate substantial variation in BASEL II regulatory capital.

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Table 1. The Percentage of One-Year PD Less Than the Long Run PD

Long run PD is in basis point. Numbers in column 2 through column 5 indicate the percentage that one-year PD drawn from equation (6) is less than the long run PD. For example, when the long run PD is 1 basis point, 83% of the one-year PD is less than 1 basis point for a corporate exposure.

Long run PD	Coporate	Retail_Mortgage	Retail_Revolver	Retail_Other
1	83%	77%	65%	78%
3	81%	76%	64%	76%
5	80%	75%	63%	75%
10	79%	73%	62%	74%
20	77%	72%	61%	72%
30	75%	71%	61%	71%
50	74%	70%	60%	69%
70	72%	69%	60%	68%
100	70%	68%	59%	66%
150	68%	67%	59%	64%
200	67%	66%	58%	63%
300	65%	65%	58%	60%
500	62%	63%	57%	58%
700	61%	62%	56%	56%
1000	59%	60%	55%	55%
1500	57%	58%	54%	54%
2000	56%	57%	53%	53%
3000	54%	54%	52%	52%
4000	52%	52%	51%	51%
5000	50%	50%	50%	50%
6000	48%	48%	49%	49%
7000	46%	46%	48%	48%
8000	44%	43%	47%	47%
9000	41%	40%	45%	46%
9500	38%	37%	43%	44%
9900	34%	32%	41%	42%
9990	29%	27%	38%	39%
9999	25%	23%	35%	37%

Table 2. The Mode of Historical Average of One-Year PD

PD numbers are in basis point. For each long run PD at column 1, we randomly draw one-year PD from equation (6) for the number of years at the second raw of below tables and compute the average. We repeat the process 1 million times and report the mode of each distribution at column 2 through 7. For example, when the long run PD is 10 basis points, the mode of 5-year historical average is 4.2 basis points for a corporate exposure.

Corporate Exposure

	Number of Years Used to Compute Average					
Long run PD	1 year	3 years	5 years	10 years	20 years	30 years
3	0.70	1.00	**1.20**	1.40	1.80	2.00
5	1.20	1.70	**2.00**	2.50	3.10	3.50
10	2.40	3.50	**4.20**	5.40	6.70	7.40
30	8	12	**15**	19	23	25
50	14	21	**26**	34	40	43
100	31	50	**62**	76	87	91
300	125	207	**239**	268	284	288
500	247	389	**431**	465	483	488
1000	616	866	**918**	960	981	986
2000	1533	1867	**1919**	1963	1981	1987
3000	2613	2903	**2939**	2973	2986	2991

Retail Mortgage Exposure

	Number of Years Used to Compute Average					
Long run PD	1 year	3 years	5 years	10 years	20 years	30 years
3	0.90	1.30	**1.50**	1.90	2.30	2.40
5	1.50	2.20	**2.60**	3.30	3.90	4.20
10	3.10	4.60	**5.50**	6.90	8.10	8.60
30	10	15	**18**	23	26	27
50	17	27	**32**	40	44	46
100	36	58	**70**	83	91	93
300	123	205	**238**	267	283	288
500	224	371	**419**	459	480	485
1000	536	830	**897**	950	976	983
2000	1396	1832	**1899**	1954	1977	1984
3000	2485	2879	**2925**	2966	2982	2989

Table 3. The Percentile Corresponding to the Low Bound on the One-year PD

Both long run PD and lower bound are in basis point. Long run PD is in basis point. Numbers in column 3 through column 6 indicate the percentage that the one-year PD drawn from equation (6) is less than the low bound at column 2 of each long run PD at column 1. For example, when the long run PD is 1 basis point, 50% of one-year PD is less than one-tenth of a basis point for a corporate exposure.

Long-run PD	Low Bound	Coporate	Retail_Mortgage	Retail_Revolver	Retail_Other
1	0.1	0.499598	0.291156	0.010766	0.316371
3	0.3	0.442346	0.244465	0.006199	0.265803
5	0.5	0.412370	0.222029	0.004563	0.240376
10	1	0.367031	0.191036	0.002818	0.203245
20	2	0.314408	0.159697	0.001579	0.162134
30	3	0.279189	0.141381	0.001061	0.135834
50	5	0.229282	0.118552	0.000595	0.100375
70	7	0.193075	0.103805	0.000384	0.076180
100	10	0.152666	0.088569	0.000226	0.051106
150	15	0.106820	0.071942	0.000113	0.026146
200	20	0.076980	0.060734	6.37E-05	0.012956
300	30	0.043102	0.046010	2.48E-05	0.002798
500	50	0.017719	0.029735	5.55E-06	8.49E-05
700	70	0.009508	0.020726	1.59E-06	2.30E-06
1000	100	0.004818	0.012911	3.04E-07	2.43E-08
1500	150	0.001965	0.006391	2.53E-08	1.69E-10
2000	200	0.000868	0.003305	2.41E-09	5.06E-12
3000	300	0.000172	0.000895	2.17E-11	8.83E-15
4000	400	3.04E-05	0.000221	1.34E-13	1.01E-17
5000	500	4.21E-06	4.51E-05	3.87E-16	4.26E-21
6000	600	3.87E-07	6.65E-06	3.24E-19	3.40E-25
7000	700	1.79E-08	5.66E-07	3.36E-23	1.67E-30
8000	800	2.27E-10	1.72E-08	6.90E-29	4.37E-38
9000	900	1.15E-13	4.00E-11	8.36E-39	2.60E-51
9500	950	5.32E-17	8.74E-14	7.27E-49	9.82E-65
9900	990	9.76E-25	5.90E-20	2.98E-72	6.02E-96
9990	999	1.43E-35	1.35E-28	5.10E-105	1.10E-139
9999	999.9	4.18E-46	5.36E-37	7.20E-137	3.50E-182

Table 4. The Mode of Historical Average of One-Year PD with Low Bound

PD numbers are in basis point. For each long run PD at column 1, we randomly draw one-year PD from equation (6), with the restriction that the one-year PD is higher than the low bound at column 2, for the number of years at the second raw of below tables and compute the average. We repeat the process 1 million times and report the mode of each distribution at column 2 through 7. For example, when the long run PD is 10 basis points and the low bound is 1 basis point, the mode of 5-year historical average is 8.1 basis points for a corporate exposure.

Corporate Exposure

Long run PD	Lower Bound	Upper Bound	Number of Years Used to Compute Average					
			1 year	3 years	5 years	10 years	20 years	30 years
3	0.3	10000	1.70	2.30	**2.60**	3.10	3.20	3.20
5	0.5	10000	2.70	3.70	**4.20**	5.10	5.40	5.40
10	1	10000	5.10	7.00	**8.10**	9.90	10.90	10.90
30	3	10000	14	20	**23**	29	33	33
50	5	10000	22	32	**38**	47	54	54
100	10	10000	43	64	**77**	93	104	108
300	30	10000	136	218	**251**	280	296	301
500	50	10000	252	397	**439**	473	491	496
1000	100	10000	616	869	**922**	965	985	991
2000	200	10000	1533	1868	**1920**	1964	1983	1989
3000	300	10000	2613	2904	**2939**	2973	2986	2991

Retail Mortgage Exposure

Long run PD	Lower Bound	Upper Bound	Number of Years Used to Compute Average					
			1 year	3 years	5 years	10 years	20 years	30 years
3	0.3	10000	1.40	1.90	**2.20**	2.70	3.10	3.30
5	0.5	10000	2.30	3.20	**3.70**	4.50	5.20	5.50
10	1	10000	4.50	6.30	**7.40**	8.90	10.20	10.80
30	3	10000	13	19	**23**	27	31	31
50	5	10000	22	32	**38**	45	50	52
100	10	10000	44	67	**79**	91	100	102
300	30	10000	135	217	**251**	280	297	302
500	50	10000	236	384	**433**	473	494	500
1000	100	10000	540	840	**909**	962	988	995
2000	200	10000	1395	1836	**1905**	1959	1983	1990
3000	300	10000	2485	2880	**2927**	2969	2985	2991

Table 5. The Mode of Historical Average of One-Year PD with Low Bound

PD numbers are in basis point. For each long run PD at column 1, we randomly draw one-year PD from equation (6), with the restriction that the one-year PD is higher than the low bound at column 2, for the number of years at the second raw of below tables and compute the average. We repeat the process 1 million times and report the mode of each distribution at column 2 through 7. For example, when the long run PD is 10 basis points and the low bound is 5 basis points, the mode of 5-year historical average is 11 basis points for a corporate exposure.

Corporate Exposure

Long run PD	Lower Bound	Upper Bound	Number of Years Used to Compute Average					
			1 year	3 years	5 years	10 years	20 years	30 years
3	0.3	10000	1.70	2.30	**2.60**	3.10	3.20	3.20
5	3	10000	5.40	5.40	**5.40**	5.40	5.40	5.40
10	5	10000	10.90	10.90	**11.00**	10.90	10.90	10.90
30	10	10000	25	33	**33**	33	33	33
50	30	10000	55	55	**54**	54	54	54
100	50	10000	93	110	**109**	109	109	110
300	100	10000	196	280	**315**	329	329	330
500	300	10000	440	550	**550**	550	550	550
1000	500	10000	768	1047	**1100**	1100	1100	1100
2000	1000	10000	1577	2063	**2128**	2179	2200	2200
3000	2000	10000	2676	3282	**3300**	3299	3299	3299

Retail Mortgage Exposure

Long run PD	Lower Bound	Upper Bound	Number of Years Used to Compute Average					
			1 year	3 years	5 years	10 years	20 years	30 years
3	0.3	10000	1.40	1.90	**2.20**	2.70	3.10	3.30
5	3	10000	5.40	5.50	**5.40**	5.40	5.50	5.40
10	5	10000	9.90	10.90	**10.90**	10.90	10.90	11.00
30	10	10000	22	29	**33**	33	33	33
50	30	10000	51	55	**55**	55	55	54
100	50	10000	87	110	**110**	110	109	109
300	100	10000	196	281	**316**	329	329	329
500	300	10000	449	550	**550**	550	550	550
1000	500	10000	776	1076	**1100**	1100	1100	1100
2000	1000	10000	1547	2115	**2196**	2200	2200	2200
3000	2000	10000	2666	3299	**3300**	3299	3299	3300

Table 6. The Mode of Historical Average of One-Year PD under TTC with Recession

PD numbers are in basis point. For each long run PD at column 1, we randomly draw one-year PD from equation (11), β_Y is *0.1 and* y is *–1.5*, for the number of years at the second raw of below tables and compute the average. We repeat the process 1 million times and report the mode of each distribution at column 2 through 7. For example, when the long run PD is 10.53 basis points, the mode of 5-year historical average is 7 basis points for a corporate exposure.

Corporate Exposure

	Number of Years Used to Compute Average					
Long run PD	1 year	3 years	5 years	10 years	20 years	30 years
3.19	1.2	1.7	**2.1**	2.6	3.2	3.5
5.3	2	2.9	**3.5**	4.4	5.5	5.8
10.53	4	6	**7**	9	11	12
31.27	13	19	**24**	31	34	34
51.88	22	34	**43**	54	57	57
103.12	48	78	**97**	113	113	113
306.41	184	304	**337**	337	337	337
508.48	358	549	**559**	559	559	560
1011.21	858	1112	**1112**	1112	1112	1112
2011.71	2001	2212	**2212**	2212	2212	2212
3009.05	3241	3309	**3309**	3309	3309	3309

Retail Mortgage Exposure

	Number of Years Used to Compute Average					
Long run PD	1 year	3 years	5 years	10 years	20 years	30 years
3.19	1.5	2.3	**2.7**	3.4	3.5	3.5
5.3	2.6	3.8	**4.6**	5.8	5.8	5.8
10.53	5	8	**10**	11	11	11
31.27	16	25	**30**	34	34	34
51.88	27	43	**52**	57	57	57
103.12	55	90	**108**	113	113	113
306.41	181	301	**337**	337	337	337
508.48	324	527	**559**	559	559	559
1011.21	756	1112	**1112**	1112	1112	1112
2011.71	1861	2212	**2212**	2212	2212	2212
3009.05	3138	3309	**3309**	3309	3309	3309

Table 7. The Rating Boundaries for the Pooling System

The following tables display ten different rating boundaries for two pooling systems – one with two ratings and the other with 5 ratings.

Two-Rating System

	Rating A	Rating BB
Pooling 1	PD <= 0.01	PD > 0.01
Pooling 2	PD <= 0.02	PD > 0.02
Pooling 3	PD <= 0.03	PD > 0.03
Pooling 4	PD <= 0.04	PD > 0.04
Pooling 5	PD <= 0.05	PD > 0.05
Pooling 6	PD <= 0.06	PD > 0.06
Pooling 7	PD <= 0.07	PD > 0.07
Pooling 8	PD <= 0.08	PD > 0.08
Pooling 9	PD <= 0.09	PD > 0.09
Pooling 10	PD <= 0.10	PD > 0.10

Five-Rating System

	Rating AA	Rating A	Rating BBB	Rating BB	Rating B
Pooling 1	PD<=0.001	0.001<PD<= 0.003	0.003<PD<= 0.005	0.005<PD<= 0.01	PD> 0.01
Pooling 2	PD<=0.002	0.002<PD<= 0.005	0.005<PD<= 0.007	0.007<PD<= 0.02	PD> 0.02
Pooling 3	PD<=0.003	0.003<PD<= 0.007	0.007<PD<= 0.01	0.01<PD<= 0.03	PD> 0.03
Pooling 4	PD<=0.004	0.004<PD<= 0.009	0.009<PD<= 0.02	0.02<PD<= 0.04	PD> 0.04
Pooling 5	PD<=0.005	0.005<PD<= 0.01	0.01<PD<= 0.03	0.03<PD<= 0.05	PD> 0.05
Pooling 6	PD<=0.006	0.006<PD<= 0.02	0.02<PD<= 0.04	0.04<PD<= 0.06	PD> 0.06
Pooling 7	PD<=0.007	0.007<PD<= 0.03	0.03<PD<= 0.05	0.05<PD<= 0.07	PD> 0.07
Pooling 8	PD<=0.008	0.008<PD<= 0.04	0.04<PD<= 0.06	0.06<PD<= 0.09	PD> 0.09
Pooling 9	PD<=0.009	0.009<PD<= 0.05	0.05<PD<= 0.07	0.07<PD<= 0.10	PD> 0.10
Pooling 10	PD<=0.010	0.010<PD<= 0.06	0.06<PD<= 0.08	0.08<PD<= 0.12	PD> 0.12

Table 8. Regulatory Capital for Portfolio 1 (PD = *0.1%, 1%, and 10%*)

The following tables display Basel II regulatory capital for each pooling and rating. The regulatory capital is computed using the formula for corporate exposure under the advanced IRB approach.

Two-Rating System (Corporate Exposure)

		ULB			FLB		
	Long-run	Mean	Mode (ULB)	Mode (FLB)	Mean	Mode (ULB)	Mode (FLB)
Pooling 1	3.569%	2.614%	3.089%	2.838%	2.797%	3.304%	3.036%
Pooling 2	3.569%	2.622%	3.103%	2.852%	2.811%	3.326%	3.057%
Pooling 3	3.569%	2.657%	3.152%	2.893%	2.837%	3.366%	3.089%
Pooling 4	3.569%	2.879%	3.416%	3.150%	3.016%	3.580%	3.300%
Pooling 5	3.569%	3.267%	3.773%	3.568%	3.354%	3.873%	3.663%
Pooling 6	3.569%	3.577%	3.986%	3.853%	3.634%	4.050%	3.914%
Pooling 7	3.569%	3.753%	4.115%	4.012%	3.796%	4.161%	4.057%
Pooling 8	3.569%	3.846%	4.187%	4.098%	3.882%	4.226%	4.136%
Pooling 9	3.569%	3.893%	4.226%	4.143%	3.926%	4.262%	4.178%
Pooling 10	3.569%	3.917%	4.247%	4.167%	3.948%	4.281%	4.200%

Five-Rating System (Corporate Exposure)

		ULB			FLB		
	Long-run	Mean	Mode (ULB)	Mode (FLB)	Mean	Mode (ULB)	Mode (FLB)
Pooling 1	3.569%	2.494%	2.870%	2.682%	2.669%	3.071%	2.870%
Pooling 2	3.569%	2.511%	2.911%	2.703%	2.672%	3.098%	2.876%
Pooling 3	3.569%	2.534%	2.946%	2.729%	2.691%	3.129%	2.898%
Pooling 4	3.569%	2.551%	2.981%	2.753%	2.709%	3.165%	2.923%
Pooling 5	3.569%	2.566%	3.010%	2.774%	2.726%	3.198%	2.947%
Pooling 6	3.569%	2.577%	3.031%	2.789%	2.741%	3.223%	2.967%
Pooling 7	3.569%	2.588%	3.049%	2.804%	2.756%	3.247%	2.986%
Pooling 8	3.569%	2.597%	3.063%	2.815%	2.774%	3.272%	3.006%
Pooling 9	3.569%	2.601%	3.071%	2.821%	2.781%	3.284%	3.016%
Pooling 10	3.569%	2.604%	3.078%	2.827%	2.786%	3.292%	3.024%

Table 9. Regulatory Capital for Portfolio 2 (PD = *0.1%, 0.3%, 0.5%, 1%, 3%, 5%, and 10%*)

The following tables display Basel II regulatory capital for each pooling and rating. The regulatory capital is computed using the formula for corporate exposure under the advanced IRB approach.

Two-Ratings System (Corporate Exposure)

		ULB			FLB		
	Long-run	Mean	Mode (ULB)	Mode (FLB)	Mean	Mode (ULB)	Mode (FLB)
Pooling 1	3.431%	2.489%	2.976%	2.718%	2.653%	3.173%	2.898%
Pooling 2	3.431%	2.679%	3.224%	2.961%	2.809%	3.380%	3.104%
Pooling 3	3.431%	2.852%	3.394%	3.152%	2.964%	3.528%	3.277%
Pooling 4	3.431%	2.966%	3.479%	3.272%	3.067%	3.598%	3.384%
Pooling 5	3.431%	3.077%	3.554%	3.372%	3.167%	3.658%	3.470%
Pooling 6	3.431%	3.197%	3.638%	3.479%	3.275%	3.726%	3.564%
Pooling 7	3.431%	3.298%	3.709%	3.570%	3.366%	3.786%	3.643%
Pooling 8	3.431%	3.366%	3.760%	3.632%	3.428%	3.828%	3.699%
Pooling 9	3.431%	3.407%	3.790%	3.669%	3.465%	3.854%	3.732%
Pooling 10	3.431%	3.428%	3.806%	3.689%	3.484%	3.869%	3.749%

Five-Ratings System (Corporate Exposure)

		ULB			FLB		
	Long-run	Mean	Mode (ULB)	Mode (FLB)	Mean	Mode (ULB)	Mode (FLB)
Pooling 1	3.431%	2.361%	2.757%	2.560%	2.545%	2.972%	2.759%
Pooling 2	3.431%	2.337%	2.748%	2.537%	2.516%	2.959%	2.732%
Pooling 3	3.431%	2.342%	2.762%	2.546%	2.518%	2.970%	2.737%
Pooling 4	3.431%	2.347%	2.780%	2.555%	2.522%	2.987%	2.746%
Pooling 5	3.431%	2.362%	2.801%	2.573%	2.537%	3.009%	2.764%
Pooling 6	3.431%	2.374%	2.821%	2.588%	2.549%	3.030%	2.780%
Pooling 7	3.431%	2.395%	2.851%	2.611%	2.569%	3.058%	2.801%
Pooling 8	3.431%	2.416%	2.882%	2.636%	2.587%	3.087%	2.823%
Pooling 9	3.431%	2.439%	2.918%	2.665%	2.607%	3.119%	2.848%
Pooling 10	3.431%	2.464%	2.955%	2.695%	2.627%	3.151%	2.873%

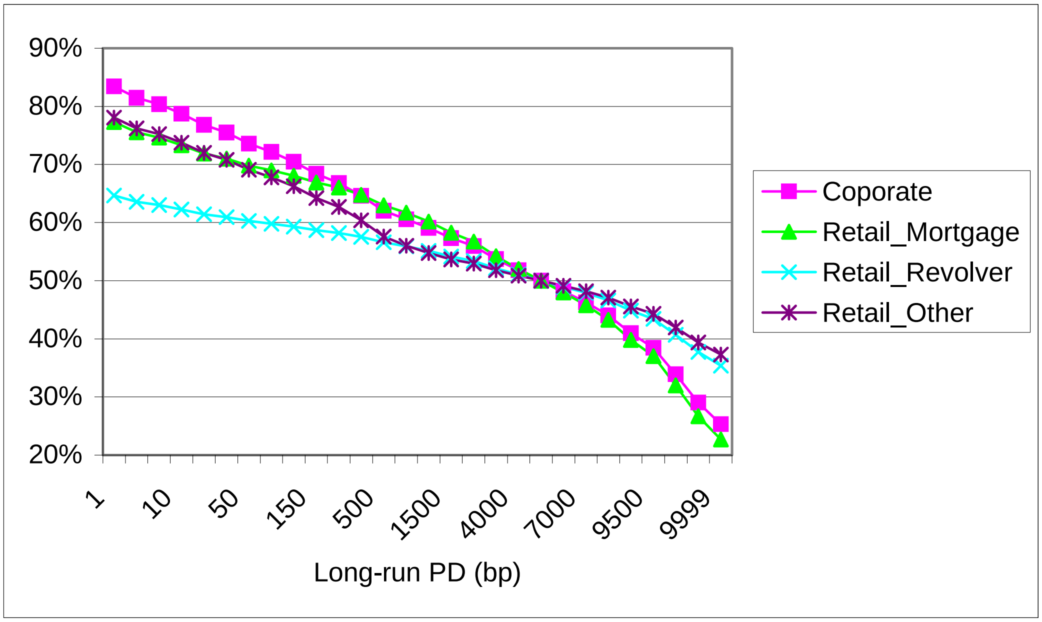
Table 10. Long Run PD Estimated using the Stress Levels Inferred from a 27-year History of Candian Delinquency Rate.

Volatility ratio is computed by dividing the volatility of the transformed delinquency rate over 27 years by the volatility of CIBC PD over 5 years.

Volatility Ratio	Long-term PD fitted for 27 years data (1977 –2003)	Long-term PD fitted for 5 years of data (Aug. 1998 – Dec. 2002)	Average PD over 27 years (1977 – 2003)	Average PD over 5 years (Aug. 1998 – Dec. 2002)
9:1	**4.48%**	4.48%	3.96%	**3.30%**
2:1	**4.01%**	4.01%	3.46%	**3.30%**
1:1	**3.62%**	3.62%	3.39%	**3.30%**

Figure 1. Percentage of One-year PD less than the Long-run PD and the Correlation (Long-run PD is in basis point)

Perecentage of One-year PD less than the Long-run PD



Correlation

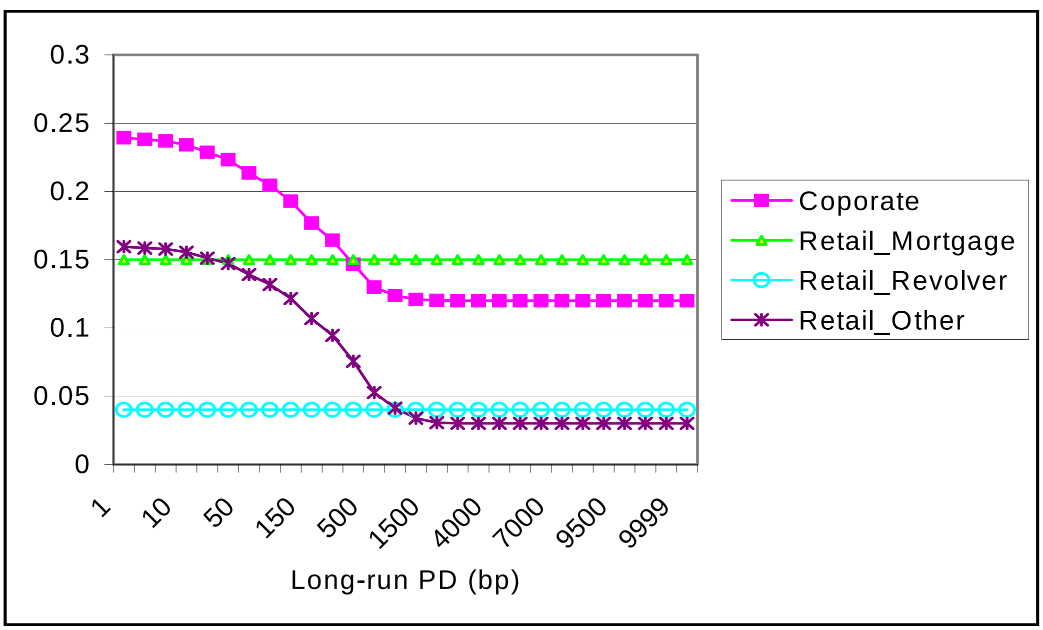
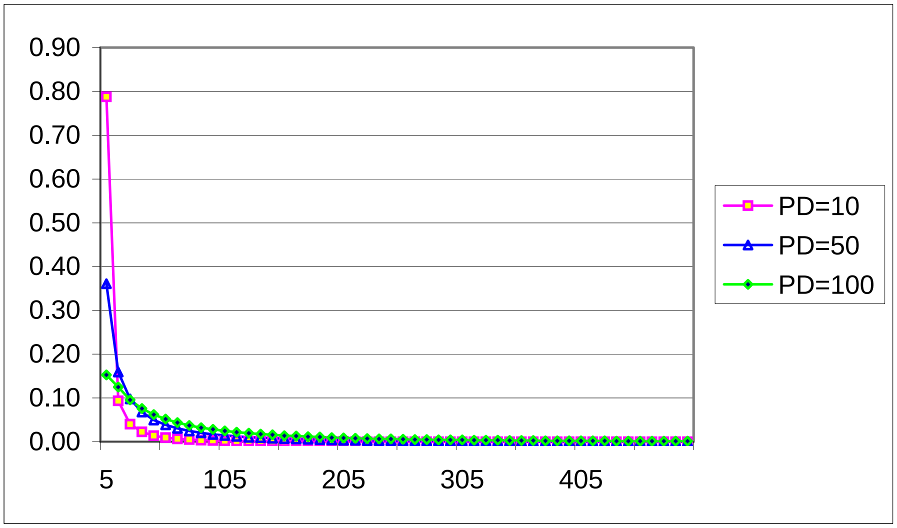


Figure 2. Distribution of One-year PD Corresponding to the Long-run PD
(PD Numbers are in basis point)

(Corporate Exposure)

1-Year Sample



5-Year Sample Average

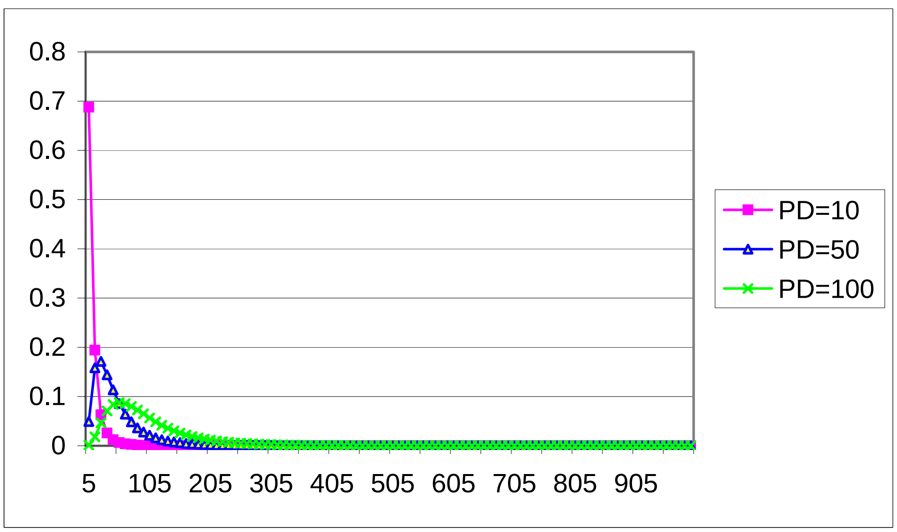
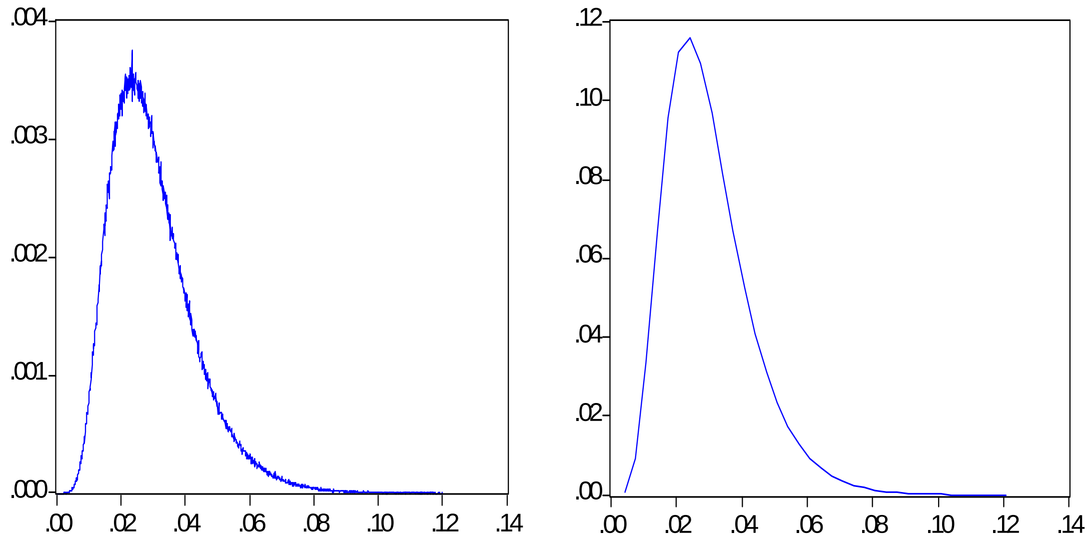


Figure 3. Distribution of the Average of 5 years Sample of the One-year PD

(Corporate Exposure; Long-run PD of 300 basis points)

Histogram

Interval = 1 bp (Mode = 234.5 bps) Interval = 33.2 bps (Mode PD = 241.14 bps)



Kernel Density

Interval = 1 bp (Mode PD = 234 bps) Interval = 33.2 bps (Mode PD = 239 bps)

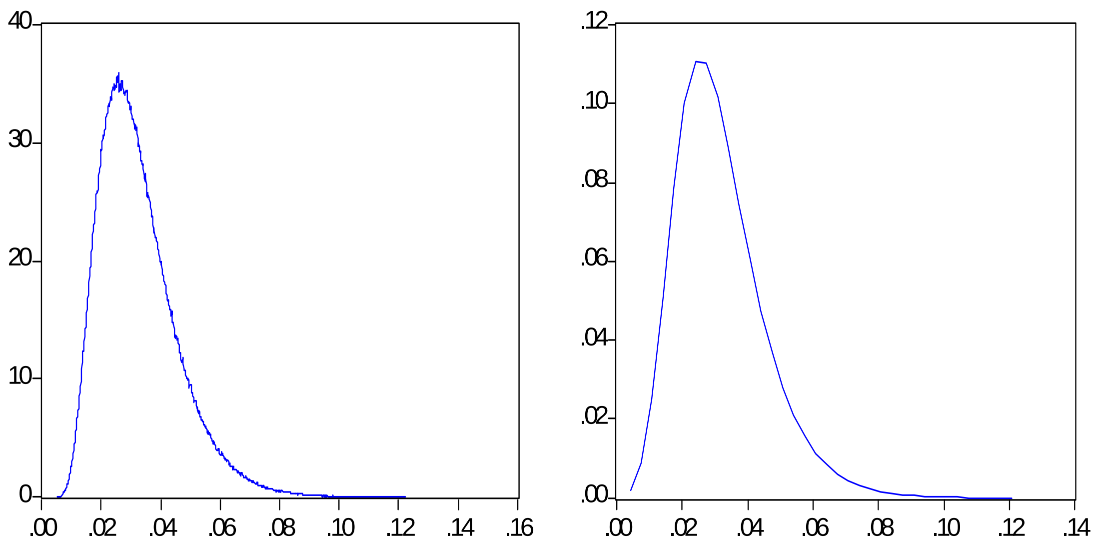
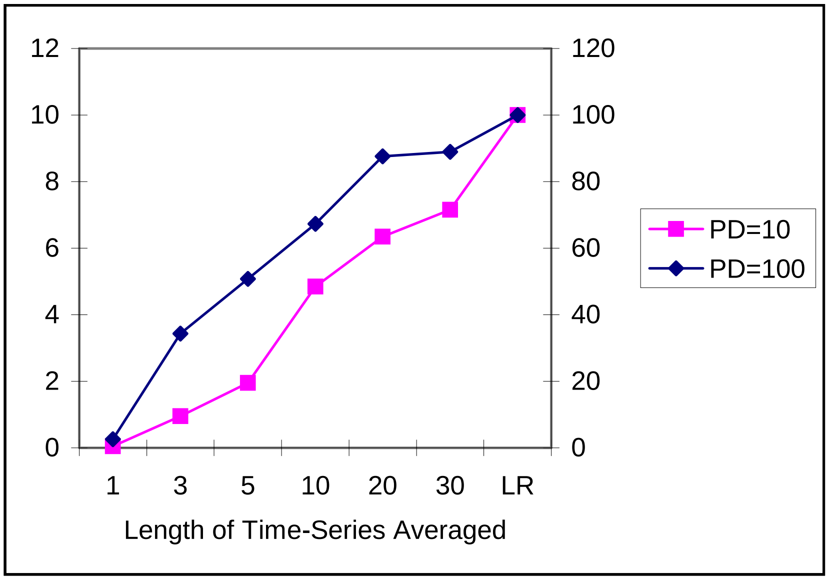


Figure 4. Estimates of One-Year PD Corresponding to the Long Run PD
(Corporate Exposure: Numbers are in basis point)



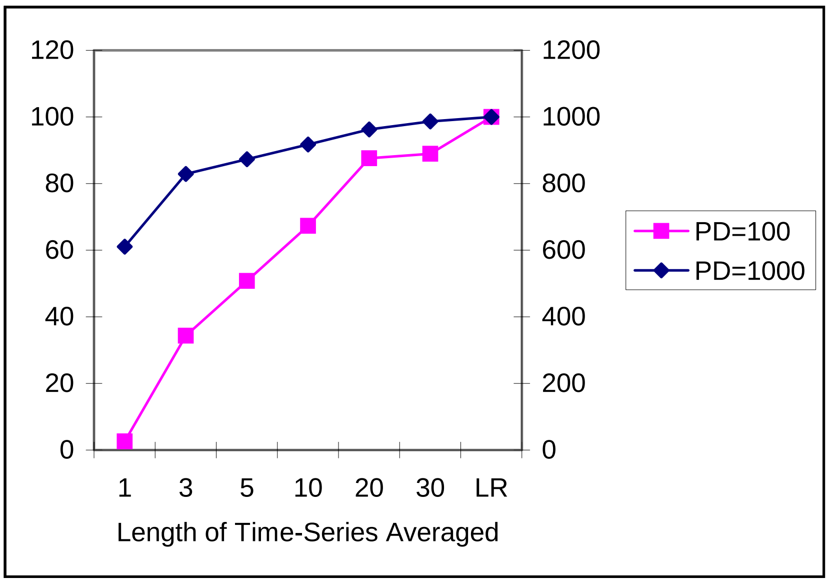
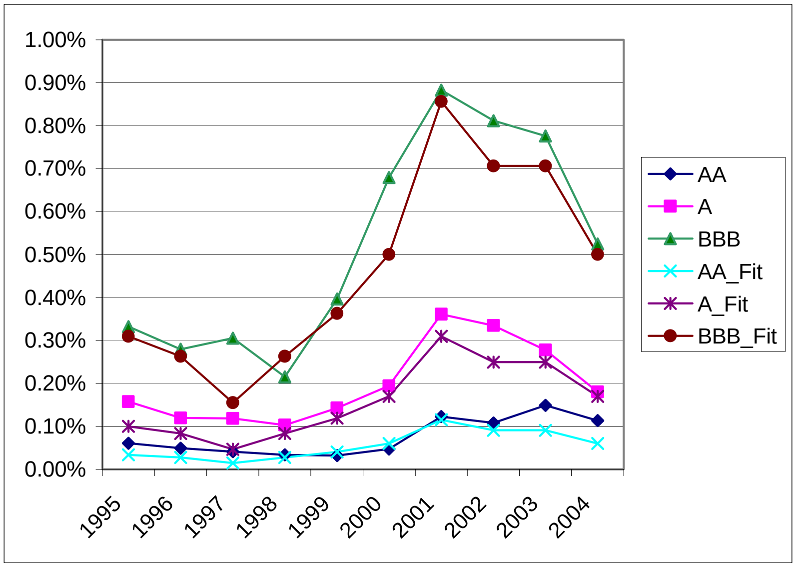


Figure 5. Estimation of Stress Level and Default Probability

(PD numbers are in percentage)

Default Probability



Stress Level

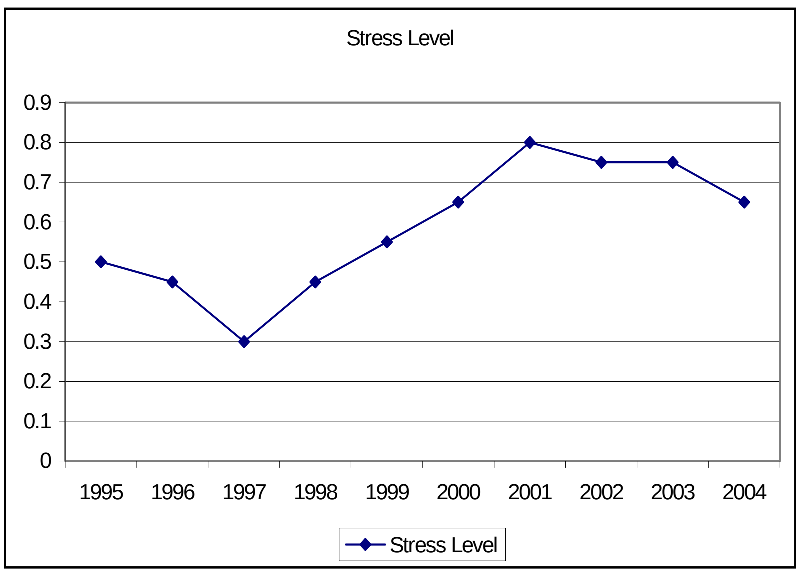
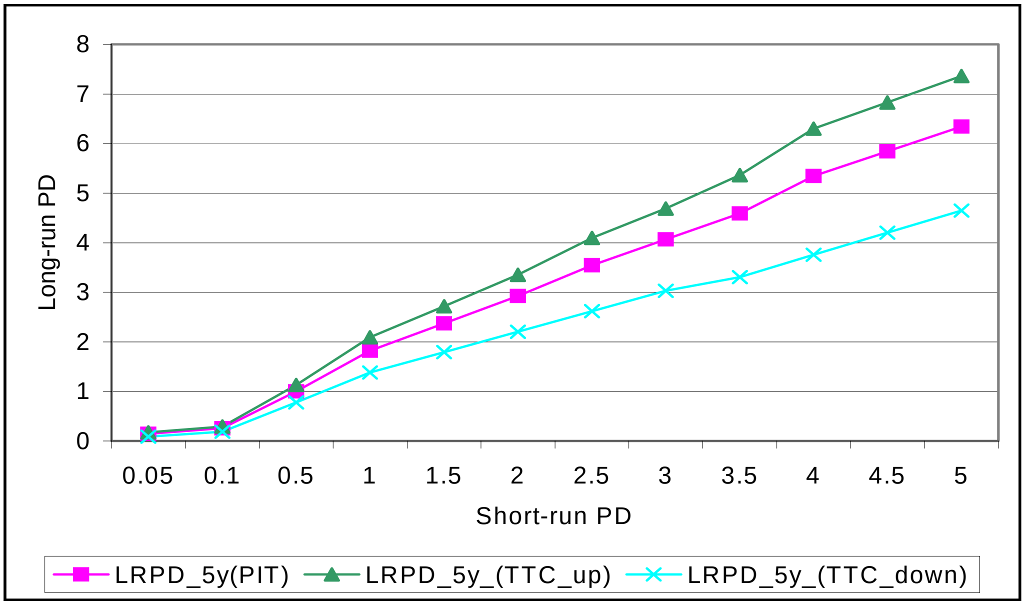


Figure 6. Short-run PD and Long-run PD under PIT and TTC
(PD numbers are in percentage)

5-year Average Mapping



30-year Average Mapping

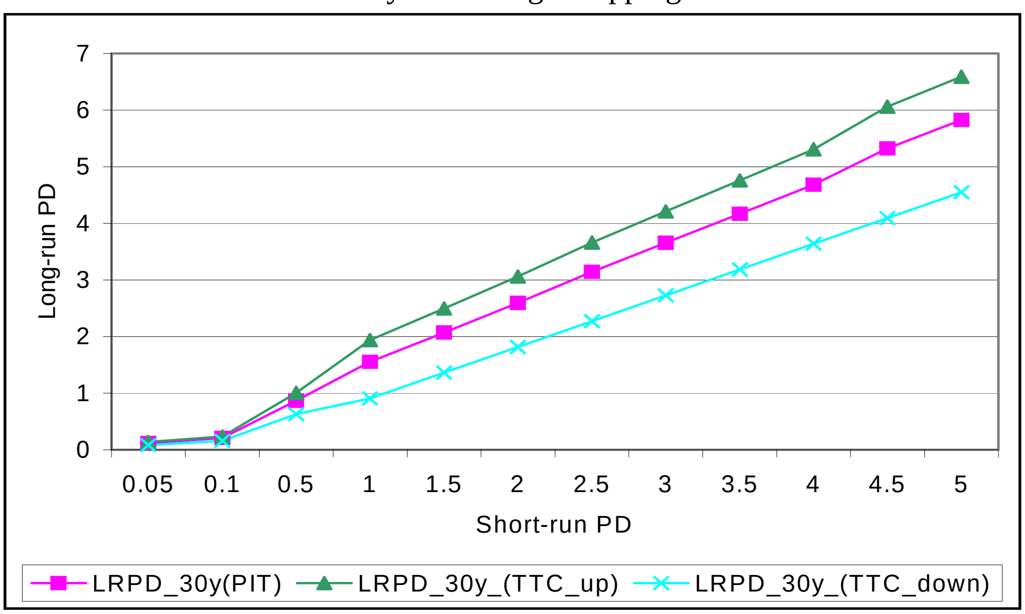
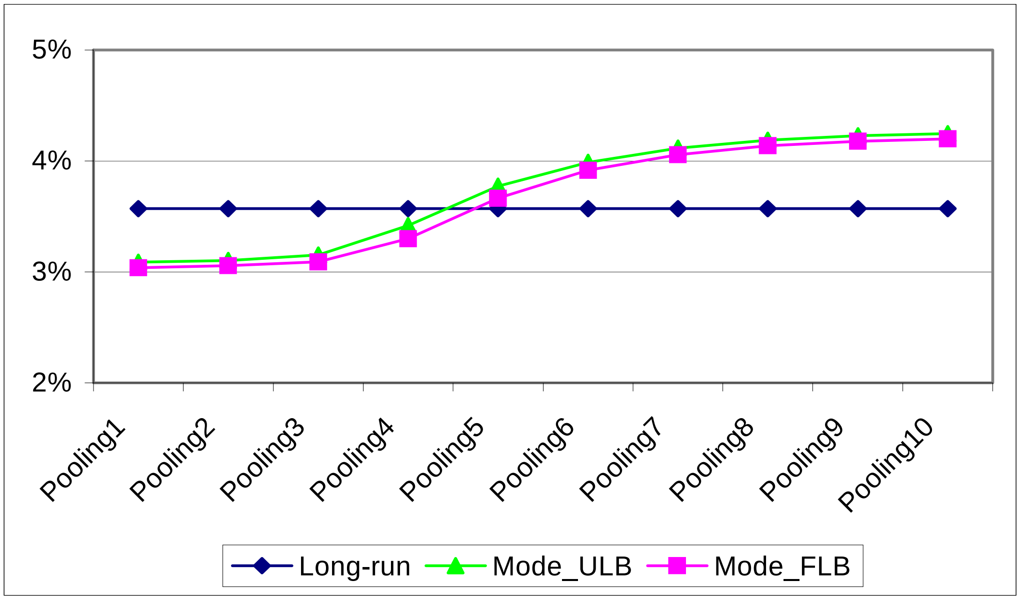


Figure 7. Regulatory Capital for Portfolio 1 (PD = *0.001, 0.01, 0.1*)
Two-Rating System (Corporate Exposure)



Five-Rating System (Corporate Exposure)

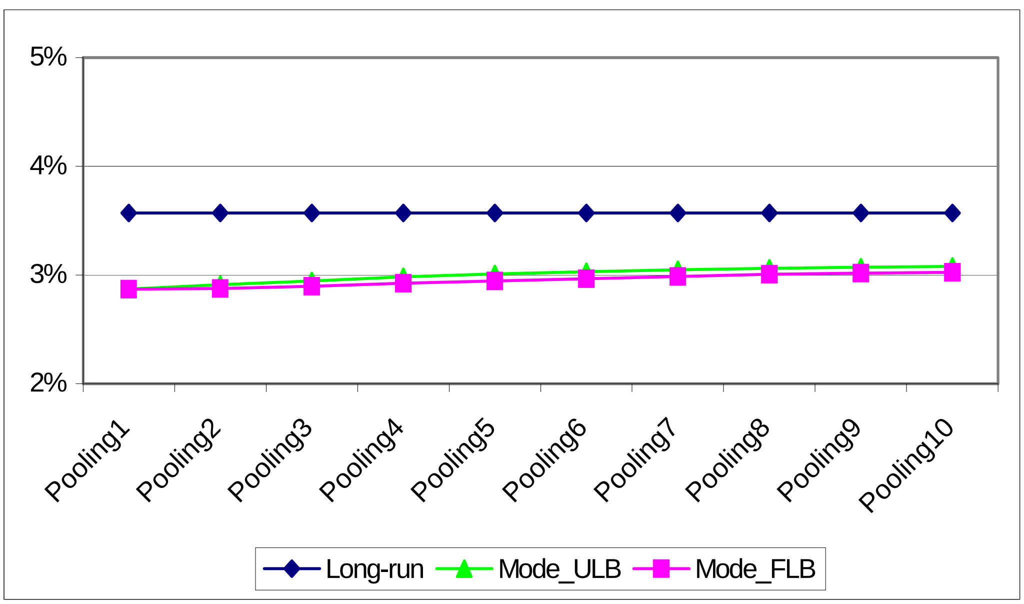
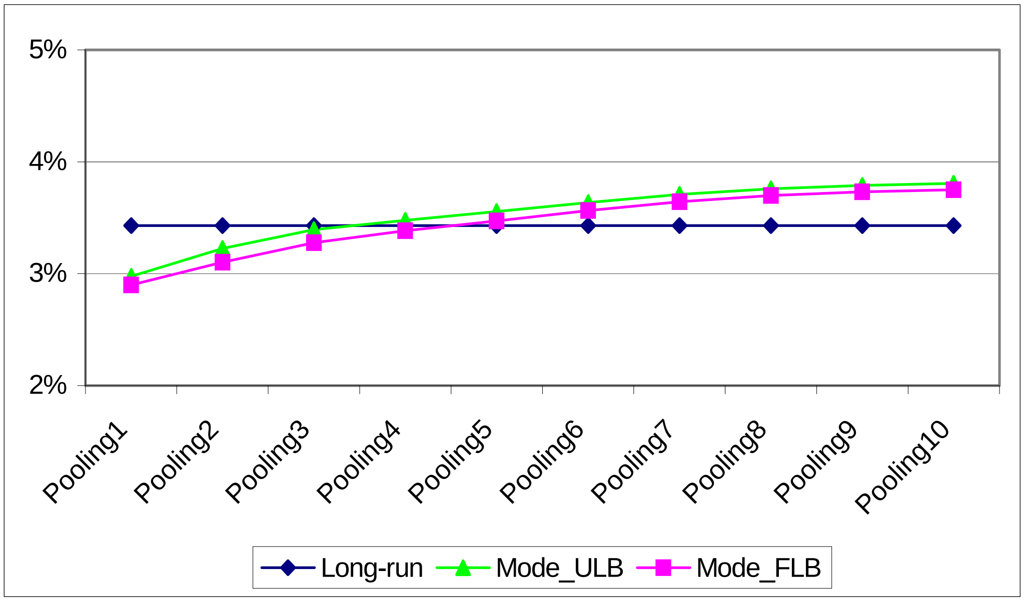


Figure 8. Regulatory Capital for Portfolio 2 (PD = *0.001*, *0.003*, *0.005*, *0.01*, *0.03*, *0.05*, *0.1*)

Two-Rating System (Corporate Exposure)



Five-Rating System (Corporate Exposure)

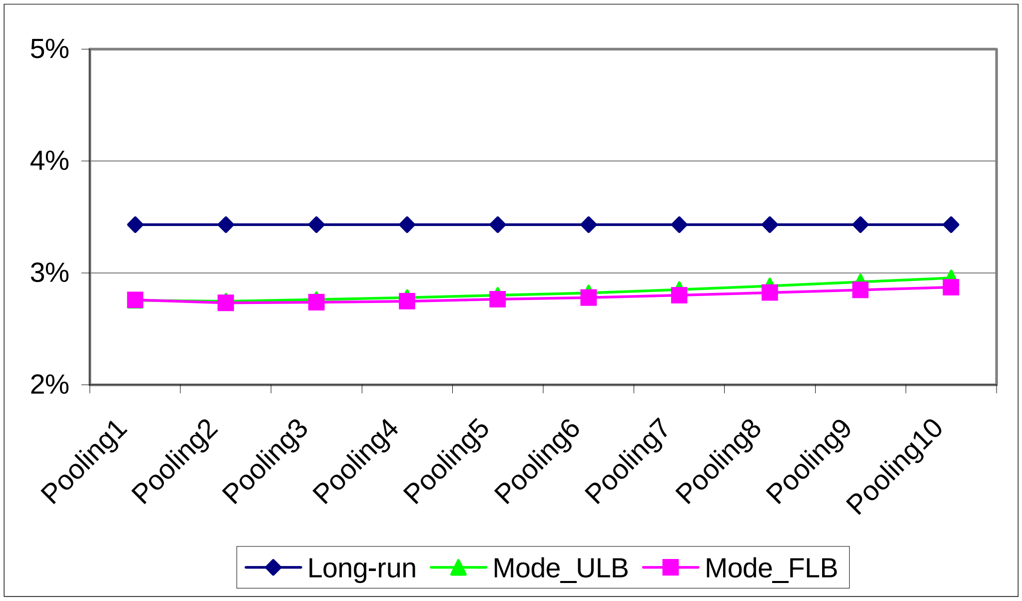


Figure 9. Probability of Default and Delinquency Rate

(Probability of Default of CIBC has been modified)

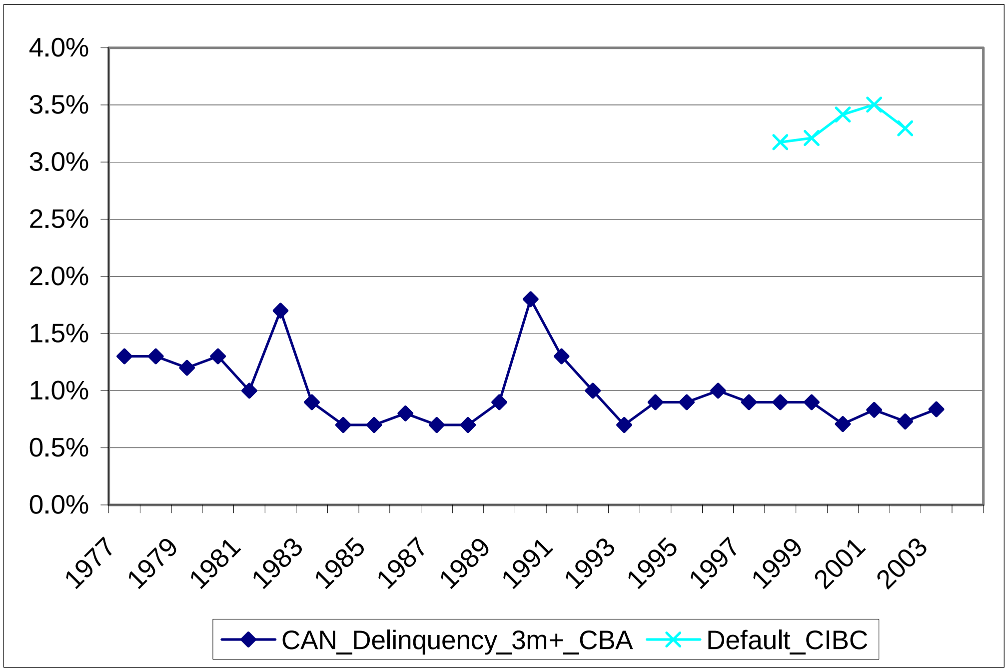
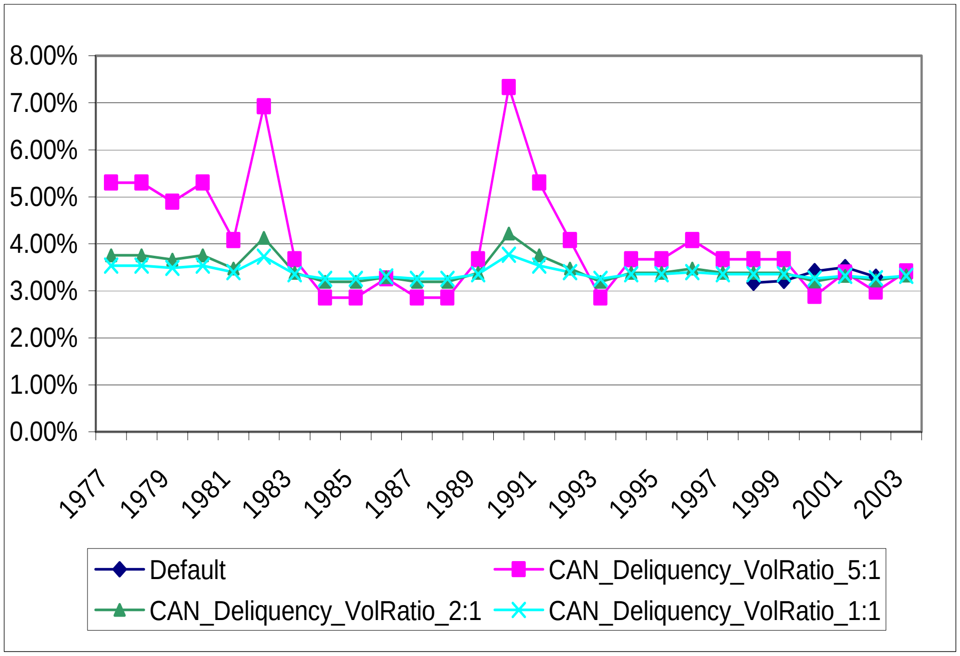


Figure 10. Delinquency Rate under Various Volatility Ratios

Delinquency Rate under Various Volatility Ratios



Estimated Stress Level

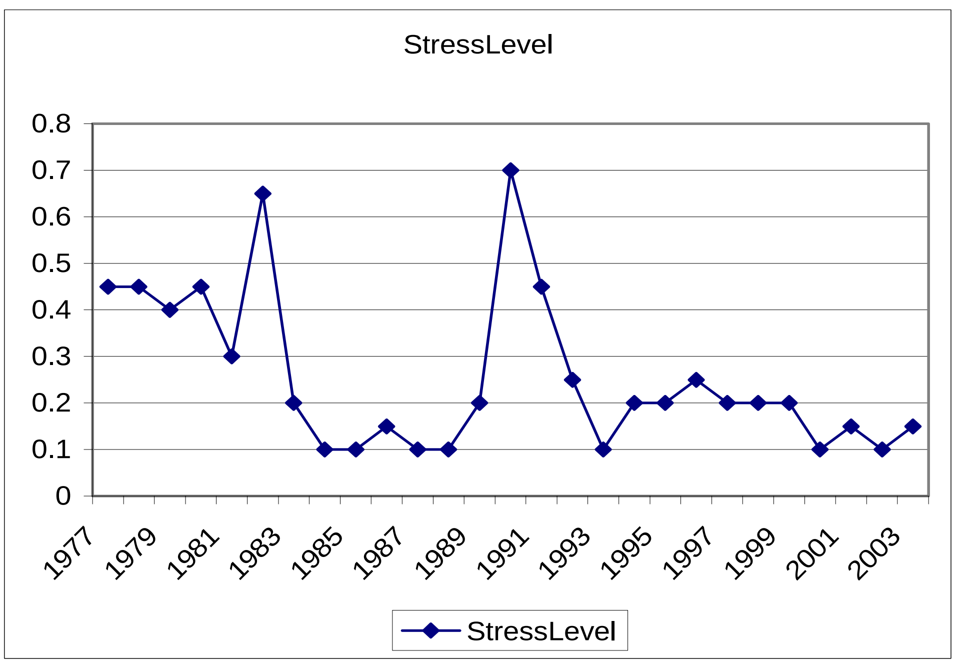


Figure 11. Regulatory Capitals under Different Poolings