SHORT-SALE CONSTRAINT, ASSYMETRIC LEAD-LAG RELATION AND IMPLIED VOLATILITY DISCREPANCY

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Abstracts

We investigate why implied volatility of call option on KOSPI 200 index is significantly different from that of put option. From May 2002 to May 2005, the sample average of implied volatility discrepancy (implied volatility of call option minus implied volatility of put option) is negative. We show that short-sale constraint in the spot market causes negative implied volatility discrepancy. We also show that, due to the short-sale constraint, the lead-lag relation is stronger under bullish market than under bearish market.

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I. INTRODUCTION

The implied volatility in an option's price reflects the market's assessment of the average volatility of the underlying asset over the remaining life of the option. As such, the implied volatility has been widely used as an ex ante estimate of future price movements, and its informational content has been documented in numerous previous studies. However, it is commonplace to observe that implied volatility of call option is different from that of put option with the same underlying asset, strike price and expiration. We define this phenomenon as the implied volatility discrepancy, or shortly the volatility discrepancy. Market practitioners and researchers usually ignore the volatility discrepancy, and choose one of the two for their convenience or make an arithmetic average of the two implied volatilities.

The implied volatility discrepancy means put-call parity relation does not hold. Put-call parity is a strong arbitrage relation between European call option and put option with the same underlying asset, strike price and expiration,

$$S^I - Xe^{-r(T-t)} = C^O - P^O$$

where S is underlying asset, P is price of put option, C is price of call option, X is strike price of the options, r is riskless interest rate, t is current time, and T is maturity of the options. If the relation holds, volatility of call option should be equal to that of put option. If the relation violates, they cannot be the same, resulting in the implied volatility discrepancy.

According to previous literature there may be many reasons why put-call parity violates. Although Kamara and Miller (1995) reported that if transaction cost involves, there are many small violations that cannot be arbitraged away, and they do produce the implied volatility discrepancy. Another possibility is that the underlying stock market and the option market are not synchronized, i.e. either the underlying stock market or the option market is inefficient, or both. This is known as the lead-lag relation.

The lead-lag relation refers to how fast one market reacts to new information relative to the other, and how tightly the two markets are linked. In a perfect economy, price movements of the two markets are cointegrated, i.e. contemporaneously correlated and not cross-correlated. In a real economy futures and option markets are reported to lead the underlying spot market. For example, Kawaller, Koch, and Koch (1987) report that the lead from S&P 500 futures to S&P index price extends between 20 and 45 minutes, while the lead from the index to futures prices rarely beyond one minute. Stoll and Whaley (1990) find that S&P 500 and Major Market Index (MMI) futures returns lead stock index returns by about five minutes on average, and occasionally by as long as 10 minutes or more, but the feedback from the cash market into the futures market is much shorter than that. Manaster and Rendleman (1982), Bhattacharya (1987), Anthony (1988) find evidence that option market leads the spot market.

The lead-lag relation between the linked markets may be attributed to several factors. First, the futures or option market performs superior price discovery to the spot markets. Investors with valid private information generally want to take advantage of high leverage in the futures or option market. Thus, we might expect the futures or option market leads the spot market if the information trading is effective in the market.

Second, the lead-lag relation may be attributed to infrequent trading in the spot market. For example, since some component stocks consisting of index portfolio may not be traded frequently, observed index value, which is the average of the last transaction prices of component stocks, cannot reflect the actual developments in the stock market and lag behind true index levels. If futures and option trades actively and can update information at once, they will lead the index level. Harris (1989) derives an estimator of the underlying index values using a simple one-factor representation of the value-generating process to estimate the nonsynchronous trading adjustment. Stoll and Whaley (1990) use an ARMA process to purge the effects of infrequent trading, and extract the return innovations to proxy for true index returns. Lo and MacKinlay (1990) examine the effects of nonsynchronous trading on the cross-autocorrelation pattern. They show that the relative nontrading probabilities of securities i and j are given by the ratio of the covariance of past returns of j to current returns of i, and the covariance of past returns of i to current returns of j. In other words, if security i has a higher nontrading probability than security j, security i lags security j more than it leads. This relation suggest that if the lad is only due to nonsynchronous trading, futures returns will be dominant in leading returns of only those component stocks that have higher nontrading probabilities than futures.

Third, the lead-lag relation might be due to transaction cost which is higher in the spot market than in the futures or option market. Transaction cost consists of the spread between bid and offer price, brokerage fee, and tax. In Korean market, tax in trading individual stocks costs 0.3% of dollar amount while trading futures involves no tax. Brokerage fee and average bid and offer spread are usually smaller in the futures or option market than in the spot market. If economy-wide information arrives the trader will prefer trading futures or option to the stocks, and future and option will be dominant in revealing the information

The last reason for the lead-lag relation is the short-sale constraint in the spot market, the inability to borrow and short. Diamond and Verrecchia (1987) show that prohibiting traders from shorting slows the adjustment of prices to private information, especially with respect to private bad news. Since there is no short-sale constraint in the futures and option market, their prices are symmetric in revealing private good news and bad news. Therefore, the lead-lag relation would not be the same under bullish and bearish markets, and future and option prices should lead the cash index to a greater degree under bad news. However, if the marginal arbitrageur has long-positions in the stocks, and he is not constrained by short-sale restrictions, the index prices should not lag futures and option to a greater degree under bad news.

The short-sale constraint in the spot market influences the implied volatility discrepancy differently from the other reasons for the lead-lag relation. If the lead-lag relation is due to one of price discovery, infrequent trading of the stock market, or transaction cost, the implied volatility discrepancy should be symmetric, i.e. the sample average of the volatility discrepancy is zero. However, if short-sale restriction exists in the spot market, the spot price lags the futures or option price to a greater degree under bad news, and the greater volatility discrepancy will be observed. Thus, we expect that the sample average of the implied volatility discrepancy should be significantly different from zero.

If the spot market lags the option market and this produce the implied volatility discrepancy, it deserves investigation to see if there are any alternative methods which does not produce the volatility discrepancy. Using futures price rather than the spot price as the underlying price of the options can be a natural alternative because futures price is also reported to lead the spot price. Like option market, futures market is actively traded and less costly, and provides higher leverage for the informed traders than the spot market. There is usually no short-sale constraint in the future market. Therefore it is reasonable to exploit futures price rather than the spot price.

In this paper we show that the implied volatilities of call option and put option are significantly different because the lead-lag relation exists between the two markets. We also show that the lead-lag relation is asymmetric due to the short-sale constraint in the spot market, and this makes the implied volatility discrepancy significantly different from zero. Then we suggest alternative methods that may not produce the implied volatility discrepancy.

In the next section we develop the model to investigate the implied volatility discrepancy. In section III, we suggest several alternative methods. Section IV describes the data used in this paper and presents empirical results, and section V is a conclusion.

II. IMPLIED VOLATILITY DISCREPANCY AND LEAD-LAG RELATION

Let us define the implied volatility discrepancy as the difference between the implied volatility of call option and put option:

$$IVD^{S} = \sigma_{call}^{S} - \sigma_{put}^{S}$$

where IVD^S is the implied volatility discrepancy, σ_{call}^{S} and σ_{put}^{S} are implied volatilities of call option, and put option respectively. The superscript, S, means that the implied volatilities are calculated using the spot price as the underlying asset price. Later we use futures price or option implied spot price as the underlying price to calculate implied volatilities. We also define the spot price discrepancy as the difference between the observed spot price and the option implied spot price.

$$SPD = S^O - S^I$$

where SPD is spot price discrepancy, S^O is observed spot price, and S^I is option implied spot price. The option implied spot price is calculated from the put-call parity relation.

$$S^I = C - P + Xe^{-r(T-t)}$$

where C and P are market prices of call and put option with the same strike price, X, and expiration, T, r is riskless interest rate, and t is current time.

Suppose option market is more efficient than the spot market, i.e. option market leads the spot market. If negative information arrives at both markets, option price will be immediately adjusted while the spot does not. Thus, at least for a while, the observed spot price will be higher than the option implied spot price, so the spot price discrepancy will be observed positive:

$$SPD = S^O - S^I > 0 \qquad \text{(When negative information arrives)}$$

Now we back out implied volatilities by using the observed call and put option prices and spot price.

$$\sigma\left(C^O, S^O\right) = f_{call}^{-1}\left(S^O, X, r, T-t, C^O\right) = f_{call}^{-1}\left(S^I + SPD, X, r, T-t, C^O\right)$$

$$\sigma\left(P^O, S^O\right) = f_{put}^{-1}\left(S^O, X, r, T-t, P^O\right) = f_{put}^{-1}\left(S^I + SPD, X, r, T-t, P^O\right)$$

where $\sigma(C^O,S^O)$ and $\sigma(P^O,S^O)$ are implied volatilities calculated from the observed spot price and, call option and put option price respectively, f_{call} and f_{put} are pricing functions of call option and put option, and f_{call}^{-1} and f_{put}^{-1} are their inverse functions with respect to the volatility. f_{call} is an increasing function of both underlying price and volatility while f_{put} is increasing with respect to underlying price but decreasing with respect to volatility. When SPD is positive, $\sigma(C^O,S^O)$ will be lower than when SPD is zero, while $\sigma(P^O,S^O)$ will be higher. That is to say, negative information creates negative implied volatility discrepancy when option market leads the spot market. With the same argument it is obvious that positive information creates positive implied volatility discrepancy when a lean-lag relation exists between the two markets.

$$IVD^{S} = \sigma_{call}^{S} - \sigma_{put}^{S} < 0 \text{ (When negative information arrives)}$$

$$IVD^{S} = \sigma_{call}^{S} - \sigma_{put}^{S} > 0 \text{ (When positive information arrives)}$$

If the reason why the spot market lags option market is due to one of its inferior price discovery to option market, higher transaction cost, or infrequent trading, spot price discrepancy and implied volatility discrepancy should be zero[1] on average under the assumption that the effect of information is distributed symmetrically around zero. However if it the spot market lags option market due to short-sale constraint, i.e. investors in the spot market are prohibited to borrow and sell, the spot price movement cannot be symmetric although the information effect is symmetric. When positive information arrives at the market, investors rush immediately to buy the spot resulting in raising its price. However, when negative information arrives at the market, the only marginal investors that have already had positions in the spot can sell it immediately. Consequently the observed spot price will show an upward bias on average.

While investors are restricted to short sell in the spot market, they can simply make a short position in option market, and the option price movements are symmetric. Thus, the observed spot price will be higher than the option implied spot price, i.e. positive spot price discrepancy. As we have seen before, positive spot price discrepancy leads to negative implied volatility discrepancy. To summarize, if short-sale restriction exists not in option market, but in the spot market, the spot price discrepancy will be

[1] Although the effect of information is distributed symmetrically around zero, the implied volatility discrepancy may not be zero because option pricing function is convex. However, the effect of information is small, the discrepancy is locally zero.

positive on average, and implied volatility discrepancy will be negative.

III. ALTERNATIVE METOHDS

One natural alternative method which circumvents the implied volatility discrepancy is using futures price rather the spot price. Like option market, futures market is also reported to lead the stock market and is actively traded and less costly. Furthermore investors can make a short position on futures, so there exists no short-sale constraint in the futures market. Therefore it is reasonable to make use of futures price rather than the spot price for estimating implied volatility, which is not expected to represent the implied volatility discrepancy.

The second alternative method is to back out implied index and volatility simultaneously from the observed call option and put option prices. If option market is efficient, so both call and put options are fairly priced, we can find out the fair level of the underlying spot price that equates the implied volatilities of call option and put option.

$$C^O = f_{call}\left(S^I, X, r, T-t, \sigma^I\right)$$

$$P^O = f_{put}\left(S^I, X, r, T-t, \sigma^I\right)$$

where the superscript 'O' means observed price, and 'I' means implied price. The unique solution pair, (S^I, σ^I) of the above simultaneous equations can always be found numerically if there exists no arbitrage opportunity.

We can find out the solution pair , (S^I, σ^I) more easily by using put-call parity relation. Because the unique solution always satisfies the put-call parity,

$$S^I = C^O - P^O + Xe^{-r(T-t)}$$

find out option implied spot price first by using put-call parity. Then, calculate implied volatilities of call option and put option by using option implied spot price as the underlying price.

IV. EMPIRICAL RESULTS

A. Data

We perform an empirical investigation using KOSPI 200 index option which is European-style. We choose Korean market because it is one of the most actively traded option markets, and the underlying stock market of KOSPI 200 index option restricts short-selling. Sample period of KOSPI 200 index option is 1st May 2002 through 31st May 2005, 37 months in aggregate. This is chosen to take advantage of the most recent trend of Korean option market which has grown up to be one of the most actively traded option markets. The data set are provided by The Korean Exchange.

We include options only with at-the-money strike price due to the liquidity consideration. When an option is in-the-money, its liquidity is too low to rely upon its price. The at-the-money options are defined as the options with the strike price at which the difference of call option price and put option price is smallest. Options with time to maturity less than five days are excluded from the sample. These options may have market microstructure concerns. Also we include only the nearest term to maturity options. KOSPI 200 index options which do not expire at the nearest term to maturity are rarely traded and show large bid-ask spreads. Following Bakshi, Cao, and Chen (2000) we use the arithmetic average of the best bid and offer quotes to avoid the bid-ask bounce problem. We calculate daily implied volatility of KOSPI 200 index option by inverting Black-Scholes option pricing formula. The yield of certificate of deposit maturing after 91 days after is used as the proxy of risk free interest rate for calculating implied volatility of KOSPI 200 index option. Dividend yield on KOSPI 200 index is also provide by Korea Exchange. It is the sum of all the dividend of the component stocks divided by the market capital of the component stocks, and based on previous dividend history of each component stock with adjustment if necessary.

B. Descriptive statistics

To calculate implied volatility of index option we use Black-Scholes (1973) option pricing formula.

$$C(S,X,r,q,\sigma,T-t) = Se^{-q(T-t)}N(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$P(S,X,r,q,\sigma,T-t) = Xe^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{Se^{-q(T-t)}}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

where C and P are European call and put option prices on the spot price, t is current time, S is the spot index at time t, X is exercise price of the options, r is (constant) riskless interest rate, q is (constant) continuous dividend yield, σ is volatility of the spot index, and T is expiration date of option. To calculate implied volatility by using futures price as underlying price, Black (1976) model is used.

$$C(F,X,r,\sigma,T-t) = e^{-r(T-t)}\left[FN(d_1) - XN(d_2)\right]$$

$$P(S,X,r,\sigma,T-t) = e^{-r(T-t)}\left[XN(-d_2) - FN(-d_1)\right]$$

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

where F is futures prices on the same underlying asset as the options. To use the above formula delivery date of futures contract should be the same as expiry of the options. In Korean market futures contracts on KOSPI 200 index matures only on the second Thursday of March, June, September, and December while options on KOSPI 200 index expires on the second Thursday every month. Thus we separate the sample into two groups. The first group consists of options with their expiration dates the same as futures delivery, and the second consists of options with expiration shorter than futures. Futures prices in the second group are calculated using cost of carry relationship.

$$F_1 = S^{1-\frac{T_1}{T_2}} F_2^{\frac{T_1}{T_2}}$$

where T_1 and T_2 are the expiration date of option and delivery date of futures respectively, F_2 is the observed futures prices maturing at time T_2, and F_1 is the theoretical future price implied by spot price and observed future price. Now as we define implied volatility discrepancy when the underlying price is the spot price, we define implied volatility discrepancy when futures price is the underlying of options.

$$IVD^F = \sigma^F_{call} - \sigma^F_{put}$$

where IVD^F, σ^F_{call}, and σ^F_{put} are the implied volatility discrepancy and implied volatilities of call option, and put option respectively. The subscript, F means that the underlying asset is futures.

Figure 1 represents implied volatility of call option and put option during the sample period. When the spot price is the underlying price in panel A, we can see that implied volatilities of put options are higher than those of call options. Sometimes the implied volatility discrepancy amounts to more than 10%. The descriptive statistics is shown at table I. Mean and standard deviation of implied volatility of call option are 27.11% and 7.55%, and those of put option are 29.26% and 8.81%. On average implied volatility of put option is 2.15% higher than that of call option, i.e. implied volatility discrepancy is negative. Maximum and minimum discrepancy amounts to 13.19% and -19.99%.

C. Hypothesis test on the implied volatility discrepancy

Here we test the null hypothesis that the implied volatility discrepancy is zero by typical t-test and regression test. We test the hypothesis on the volatility discrepancy when the underlying price is the spot index as well as when the underlying price is the futures price.

$$H_0 : IVD^S = \sigma^S_{call} - \sigma^S_{put} = 0 \qquad \text{(When underlying price is the spot index)}$$

$$H_0 : IVD^F = \sigma^F_{call} - \sigma^F_{put} = 0 \qquad \text{(When underlying price is the futures price)}$$

Before testing the hypothesis we divide the sample into two sub-samples. The options whose expiration dates are equal to futures delivery dates are included in the first sub-sample, and the others are included in the second sub-sample. Therefore futures and stock index in the first sub-sample do not show basis risk whereas it exists in the second sub-sample. As we mentioned before, the implied volatility of futures option cannot be estimated correctly if there is a basis risk.

Table II shows the empirical results of t-test. Panel A shows the results of the aggregate sample while panel B and panel C contain the results of the first and second sub-sample respectively. The symbol, *, represnts the significance at the level of 1%. In every panel, sample mean of the implied volatility discrepancy of index option, IVD^S, has a negative value and is significantly different from zero. For example, in panel A, sample mean and t-statistics of the discrepancy are -2.15% and -15.25. This indicates that short-sale constraint in the stock market causes the positive spot index discrepancy. On the other hand, we cannot reject the null hypothesis that the implied volatility discrepancy when underlying price is futures price, IVD^F, is zero in every panel. The sample mean and t-statistics are 0.07%, 0.9190 in panel A, 0.03%, 0.5347 in panel B, and 0.1%, 0.8105 in panel C. We can conclude that the IVD^F is zero on average, futures put-call parity is satisfies, and there exists no lead-lag relation between futures and option markets. One more thing that is deserved a comment is that in panel B sample mean and sample standard deviation of the discrepancy of index futures option, 0.03% and 0.41%, are much lower in panel B, 0.1% and 2.64%. This is because the second sub-sample, i.e., when option expiry is not equal to futures delivery, contains a basis risk.

Typical t-test is based on the sample mean. If the sample mean of the implied volatility discrepancy is near zero, the hypothesis may not be rejected. Regression test, however, tells us if the implied volatility discrepancy is zero for every observation. Following (Literature) we set up the simple linear equation for regression test.

$$\sigma_{call}^{S} = \alpha^{S} + \beta^{S}\sigma_{put}^{S} + \varepsilon^{S} \qquad \text{(When underlying price is the spot index)}$$

$$\sigma_{call}^{F} = \alpha^{F} + \beta^{S}\sigma_{put}^{S} + \varepsilon^{F} \qquad \text{(When underlying price is the futures price)}$$

where α^i and β^i (i = S or F) are regression coefficients, and ε^i is the white noise. We expect that α^i is zero and β^i is one if the implied volatility discrepancy is zero, so the null hypothesis follows:

$$H_0 : \alpha^{S} = 0 \ \text{ and } \ \beta^{S} = 1 \qquad \text{(When underlying price is the spot index)}$$

$$H_0 : \alpha^F = 0 \text{ and } \beta^F = 1 \qquad \text{(When underlying price is the spot index)}$$

Regression results are presented in Table III. Again the results of the aggregate sample, the equal maturity sample, and the different maturity sample are exhibited in panel A, B, and C respectively, and the symbol, *, represents the significance at the level of 1%. The first column shows regression results based on implied volatilities of stock index. In panel A, the intercept, α, and the slope coefficient, β, are 0.0456 and 0.7704, both of which are significant at the 1% level. Similar results are shown in panel B and C although the estimated values and their significances are more or less changed. The F-statistics that tests the null hypothesis are 300.35, 131.89, and 169.25 in panel A, B, and C. The intercept is too higher than zero and the slope coefficient is too lower than one not to reject the null hypothesis. This suggests the implied volatility discrepancy of stock index is significantly different from zero, which is the same conclusion as results of t-test in Table II.

We observe quite different results in the second column, which presents regression results based on implied volatilities when underlying price is futures price. In panel C, the intercept and the slope coefficient are 0.0165, 0.9438, and F-statistics is 7.4282. The intercept is lower than in the first column, and the slope coefficient is higher suggesting that the implied volatility discrepancies are diminished. But the null hypothesis is still rejected at the 1% level. This contradicts the results of t-test. In panel B, the intercept and the slope coefficient, 0.0008 and 0.9982, are even closer to zero and one, respectively. In this time, F-statistics shows that the null hypothesis cannot be rejected, which agrees to the results of t-test. In panel C, F-statistics is 7.4282, which reject s the null hypothesis. That is to say, if maturity date of option and delivery date of futures are the same, futures price does not lead the spot index, and no volatility discrepancy is shown. The aggregate result in panel A shows that the implied volatility discrepancy is not zero with F-statistics, 7.4839.

D. Negative implied volatility discrepancy and asymmetric lead-lag relation

So far we have seen that implied volatility discrepancy calculated from the spot index and option price contains a negative bias, and that can be eliminated if we use futures price that is to be delivered at maturity date of option. Here we relate implied volatility discrepancy to the lead-lag relation. Before going further we represent the lead-lag relation between the spot and option market. To estimate the relation we follow the error correction model of Engle and Granger (1987).

$$\Delta S_t^O = \alpha^O + \beta^O Z_{t-1} + \gamma_1^O \Delta S_{t-1}^O + \gamma_2^O \Delta S_{t-2}^O + \Lambda + \gamma_p^O \Delta S_{t-p}^O$$
$$+ \delta_1^O \Delta S_{t-1}^I + \delta_1^O \Delta S_{t-2}^I + \Lambda + \delta_p^O \Delta S_{t-p}^I + \varepsilon^O$$

$$\Delta S_t^I = \alpha^I + \beta^I Z_{t-1} + \gamma_1^I \Delta S_{t-1}^O + \gamma_2^I \Delta S_{t-2}^O + \Lambda + \gamma_p^I \Delta S_{t-p}^O$$
$$+ \delta_1^I \Delta S_{t-1}^I + \delta_2^I \Delta S_{t-2}^I + \Lambda + \delta_p^I \Delta S_{t-p}^I + \varepsilon^I$$

where $Z_{t-1} = S_{t-1}^O - S_{t-1}^I$, α^O and α^I are constant terms, and p is the number of lags. As before the superscript, 'O' means the observed price, 'I' means the option implied price. The error correction term, Z_{t-1}, maintains the long-run equilibrium relationship, $S_t^O = S_t^I$.

Table IV shows the lead-lag relation between KOSPI 200 index and option implied index. We restrict the sample period to 2004, and to avoid inaccurate dividend estimation problem use the sample in which maturity date of option is the same as delivery date of futures. The symbol, *, represents significance of the 1% level. When dependent variable is ΔS_t^O, the coefficient of error correction term, Z_{t-1}, 0.2548, is significantly positive with its t-statistics, 17.52. From the 1st lagged variable, ΔS^{I}_{t-1}, to the 28th lagged variable, ΔS^{I}_{t-28}, their coefficients are all significant at the 1% level. This means that the option implied index leads the observed index, i.e. the option market leads the spot market by 28 minutes. However, when dependent variable is ΔS^{I}_{t}, the result is quite different. The coefficient of Z_{t-1}, 0.4660, is still significant, but the coefficients of lagged variables are insignificant except ΔS^{O}_{t-1}. This means that the observed index hardly leads the option implied index, i.e. the spot market does not leads the option market. To summarize the results of table IV coincides with those of the previous studies.

Although the fact that option market leads the spot market is still valid, the size of lead-lag can be asymmetric, if short-sale constraint exists in the spot market. Under bad news the spot index cannot move down, due to short-sale constraint, as fast as option market reacts and option implied index decreases. The coefficients of short-term lagged variables are expected to be insignificant while those of long-term lagged variables are expected to be significant.

Table V represents the lead-lag relation when the market is bullish. To obtain bullish market sample we calculate the daily increment of option implied index, which is defined as the close price minus the open price. 80% or above of all the daily increment distribution are days when the market is bullish. Likewise, 20% or less are days when the market is bearish. When dependent variable is ΔS^{O}_{t}, the overall results are similar to the results of table IV. The 1st to 27th lagged variables of ΔS^{I}_{t} are significant at the 1% level except three lagged variables(19th, 20th, and 21st). When dependent variable is ΔS_t^I, constant, error correction term, and the 1st lagged variable ΔS_{t-1}^O is only significant. Option market leads the spot market to a similar degree of table IV.

However, table VI represents that option market leads the spot market less strongly when the market is bearish. The regression results of ΔS_t^O against shows that the number of coefficients of the lagged variables of ΔS_t^I are much less. Only the 1st to 16th lagged variables are significant except 20th and 28th lagged ones. This implies that when the market is bearish the lead-lag relation is less strong than in the bullish market.

V. CONCLUSION

We investigate why implied volatility of call option on KOSPI 200 index is significantly different from that of put option. According to our empirical analysis of 37 months, the sample average of implied volatility discrepancy is significantly different from zero. This suggests that put-call parity violates and furthermore a lead-lag relation exists between the spot market and option market. We show that short-sale constraint in the spot market causes negative implied volatility discrepancy. We also show that due to the short-sale constraint option market leads the spot market to greater degree under bullish market than under bearish market.

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Figure 1

Implied volatility of call option and put option

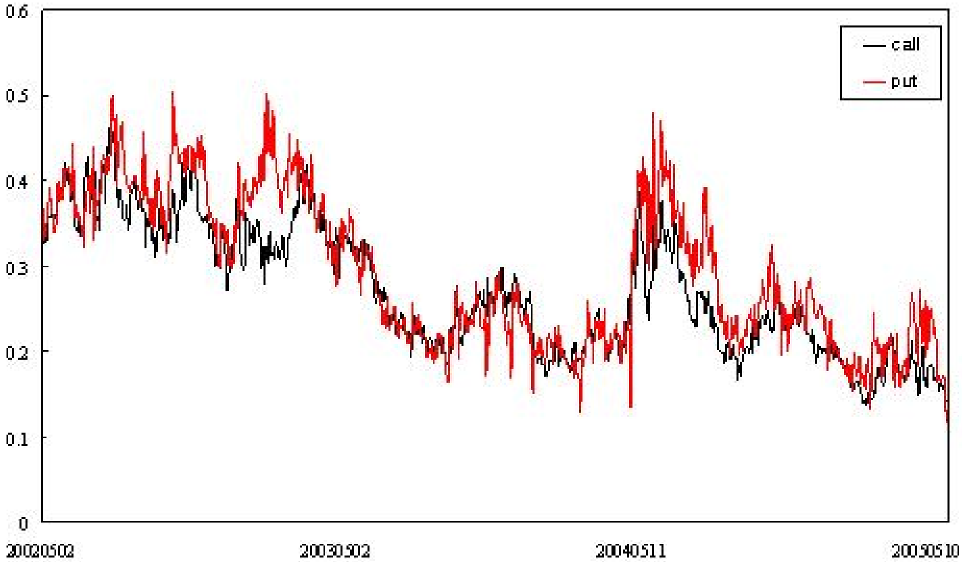

Table I

Descriptive statistics of the implied volatility

	KOSPI 200 index option		
	Call	Put	Discrepancy
Obs	760	760	760
Mean	0.2711	0.2926	-0.0215
Median	0.2586	0.2673	-0.0143
Maximum	0.4610	0.5046	0.1319
Minimum	0.13699	0.1162	-0.1999
Std Dev	0.0755	0.0881	0.0388
Skewness	0.2968	0.3163	-0.7797
Kurtosis	1.9620	1.8662	4.9621
Jarque-Bera	45.2807	53.3783	198.9239
Probability	0.0000	0.0000	0.0000

Table II

Student t-tests on the implied volatility discrepancy

	KOSPI 200	
	IVD^S	IVD^F
Panel A: Total sample		
N	760	760
Mean	-0.0215	0.0007
Stdev	0.0388	0.0215
t-statistics	-15.2524	0.9190
Probability	0.0000	0.3584
Panel B: When option expiry is equal to futures delivery		
N	261	261
Mean	-0.0246	0.0003
Stdev	0.0402	0.0041
t-statistics	-9.8612	0.5347
Probability	0.0000	0.5933
Panel C: When option expiry is NOT equal to futures delivery		
N	499	499
Mean	-0.0199	0.0010
Stdev	0.0380	0.0264
t-statistics	-11.6780	0.8105
Probability	0.0000	0.4180

* represnts the significance at the level of 1%.

Table III

Regression test on the implied volatility discrepancy

	KOSPI 200	
	Stock index	Index futures
Panel A: Aggregate sample		
α	0.0456*	0.0109*
(s.e.)	0.0042	0.0028
β	0.7704*	0.9633*
(s.e.)	0.0137	0.0098
F-statistics	300.35	7.4839
Probability	0.0000	0.0006
Panel B: When option expiry is equal to futures delivery		
α	0.0488*	0.0008
(s.e.)	0.0070	0.0009
β	0.7541*	0.9982*
(s.e.)	0.0223	0.0031
F-statistics	131.89	0.6896
Probability	0.0000	0.5027
Panel C: When option expiry is NOT equal to futures delivery		
α	0.0434*	0.0165
(s.e.)	0.0052	0.0043
β	0.7815*	0.9438
(s.e.)	0.0173	0.0149
F-statistics	169.25	7.4282
Probability	0.0000	0.0007

* represnts the significance at the level of 1%.

Table IV

Lead-lag relation

		Dependent variable							
		ΔS^{O}				ΔS^{I}			
CONSTANT		-0.0254*	(-17.14)			-0.0366*	(-20.22)		
z(t-1)		0.2548*	(17.52)			0.3627*	(20.39)		
ΔS^{O}(t-1)	ΔS^{I}(-1)	-0.3476*	(-46.16)	0.5192*	(84.63)	0.0734*	(7.96)	-0.0138	(-1.83)
ΔS^{O}(t-2)	ΔS^{I}(-2)	-0.2578*	(-32.04)	0.3378*	(46.33)	0.0229	(2.32)	-0.0329*	(-3.69)
ΔS^{O}(t-3)	ΔS^{I}(-3)	-0.2248*	(-27.09)	0.2611*	(33.67)	0.0070	(0.69)	-0.0186	(-1.96)
ΔS^{O}(t-4)	ΔS^{I}(-4)	-0.1899*	(-22.44)	0.2161*	(26.95)	-0.0022	(-0.22)	0.0115	(1.17)
ΔS^{O}(t-5)	ΔS^{I}(-5)	-0.1632*	(-19.03)	0.1661*	(20.30)	0.0206	(1.96)	0.0012	(0.12)
ΔS^{O}(t-6)	ΔS^{I}(-6)	-0.1540*	(-17.77)	0.1347*	(16.28)	-0.0057	(-0.54)	-0.0189	(-1.87)
$\Delta\backslash S^{O}$(t-7)	ΔS^{I}(-7)	-0.1493*	(-17.10)	0.1238*	(14.83)	0.0018	(0.17)	-0.0226	(-2.21)
ΔS^{O}(t-8)	ΔS^{I}(-8)	-0.1189*	(-13.53)	0.1035*	(12.33)	0.0090	(0.83)	-0.0195	(-1.90)
ΔS^{O}(t-9)	ΔS^{I}(-9)	-0.1225*	(-13.88)	0.0994*	(11.79)	0.0036	(0.34)	-0.0062	(-0.60)
ΔS^{O}(t-10)	ΔS^{I}(-10)	-0.0998*	(-11.27)	0.1035*	(12.26)	0.0118	(1.09)	0.0060	(0.58)
ΔS^{O}(t-11)	ΔS^{I}(-11)	-0.0951*	(-10.73)	0.0848*	(10.03)	0.0130	(1.20)	-0.0144	(-1.39)
ΔS^{O}(t-12)	ΔS^{I}(-12)	-0.1057*	(-11.91)	0.0857*	(10.11)	-0.0206	(-1.89)	-0.0134	(-1.29)
ΔS^{O}(t-13)	ΔS^{I}(-13)	-0.0911*	(-10.26)	0.0805*	(9.48)	-0.0093	(-0.86)	0.0086	(0.82)
ΔS^{O}(t-14)	ΔS^{I}(-14)	-0.0795*	(-8.95)	0.0666*	(7.86)	0.0046	(0.42)	-0.0055	(-0.53)
ΔS^{O}(t-15)	ΔS^{I}(-15)	-0.0740*	(-8.33)	0.0714*	(8.42)	0.0002	(0.02)	0.0204	(1.96)
ΔS^{O}(t-16)	ΔS^{I}(-16)	-0.0692*	(-7.80)	0.0693*	(8.17)	0.0026	(0.24)	0.0166	(1.60)
ΔS^{O}(t-17)	ΔS^{I}(-17)	-0.0502*	(-5.67)	0.0481*	(5.68)	0.0150	(1.38)	0.0017	(0.17)
ΔS^{O}(t-18)	ΔS^{I}(-18)	-0.0614*	(-6.95)	0.0579*	(6.84)	0.0008	(0.08)	0.0104	(1.00)
ΔS^{O}(t-19)	ΔS^{I}(-19)	-0.0696*	(-7.92)	0.0400*	(4.75)	0.0089	(0.83)	-0.0066	(-0.64)
ΔS^{O}(t-20)	ΔS^{I}(-20)	-0.0613*	(-7.00)	0.0528*	(6.28)	0.0237	(2.21)	-0.0168	(-1.64)
ΔS^{O}(t-21)	ΔS^{I}(-21)	-0.0476*	(-5.45)	0.0463*	(5.52)	0.0015	(0.14)	-0.0125	(-1.22)
ΔS^{O}(t-22)	ΔS^{I}(-22)	-0.0413*	(-4.76)	0.0362*	(4.34)	0.0051	(0.48)	0.0003	(0.03)
ΔS^{O}(t-23)	ΔS^{I}(-23)	-0.0389*	(-4.52)	0.0367*	(4.43)	0.0088	(0.84)	0.0076	(0.75)
ΔS^{O}(t-24)	ΔS^{I}(-24)	-0.0379*	(-4.46)	0.0331*	(4.03)	-0.0006	(-0.05)	-0.0100	(-1.00)
ΔS^{O}(t-25)	ΔS^{I}(-25)	-0.0278*	(-3.31)	0.0349*	(4.29)	0.0025	(0.25)	0.0152	(1.52)
ΔS^{O}(t-26)	ΔS^{I}(-26)	-0.0447*	(-5.41)	0.0327*	(4.07)	-0.0098	(-0.97)	0.0108	(1.10)
ΔS^{O}(t-27)	ΔS^{I}(-27)	-0.0324*	(-4.01)	0.0377*	(4.78)	-0.0061	(-0.61)	0.0027	(0.28)
ΔS^{O}(t-28)	ΔS^{I}(-28)	-0.0288*	(-3.68)	0.0347*	(4.50)	-0.0148	(-1.55)	0.0152	(1.62)
ΔS^{O}(t-29)	ΔS^{I}(-29)	-0.0171	(-2.32)	0.0120	(1.63)	-0.0113	(-1.25)	0.0117	(1.29)
ΔS^{O}(t-30)	ΔS^{I}(-30)	0.0049	(0.80)	0.0081	(1.19)	0.0140	(1.86)	0.0110	(1.32)

* represnts the significance at the level of 1%.

Table IV

Lead-lag relation on bullish markets, 2004

		Dependent variable							
		ΔS^{O}				ΔS^{I}			
CONSTANT		-0.0209*	(-6.45)			-0.0282*	(-6.78)		
z(t-1)		0.2487*	(7.82)			0.3313*	(8.11)		
ΔS^{O}(t-1)	ΔS^{I}(-1)	-0.3403*	(-19.97)	0.5171*	(39.11)	0.1041*	(4.75)	-0.0267	(-1.57)
ΔS^{O}(t-2)	ΔS^{I}(-2)	-0.2514*	(-13.76)	0.3635*	(22.71)	0.0358	(1.52)	-0.0440	(-2.14)
ΔS^{O}(t-3)	ΔS^{I}(-3)	-0.2356*	(-12.52)	0.2551*	(14.80)	-0.0145	(-0.60)	0.0054	(0.25)
ΔS^{O}(t-4)	ΔS^{I}(-4)	-0.2048*	(-10.67)	0.2254*	(12.73)	-0.0153	(-0.62)	0.0254	(1.11)
ΔS^{O}(t-5)	ΔS^{I}(-5)	-0.1336*	(-6.86)	0.1458*	(8.07)	0.0371	(1.48)	-0.0022	(-0.10)
ΔS^{O}(t-6)	ΔS^{I}(-6)	-0.1466*	(-7.48)	0.1036*	(5.68)	0.0231	(0.92)	-0.0546	(-2.33)
$\Delta\backslash S^{O}$(t-7)	ΔS^{I}(-7)	-0.1229*	(-6.22)	0.0883*	(4.81)	0.0327	(1.29)	-0.0724	(-3.07)
ΔS^{O}(t-8)	ΔS^{I}(-8)	-0.1216*	(-6.12)	0.0882*	(4.78)	0.0063	(0.25)	-0.0268	(-1.13)
ΔS^{O}(t-9)	ΔS^{I}(-9)	-0.0949*	(-4.75)	0.0938*	(5.07)	0.0110	(0.43)	0.0072	(0.30)
ΔS^{O}(t-10)	ΔS^{I}(-10)	-0.1165*	(-5.83)	0.0636*	(3.43)	0.0170	(0.66)	-0.0396	(-1.66)
ΔS^{O}(t-11)	ΔS^{I}(-11)	-0.0792*	(-3.95)	0.0803*	(4.34)	0.0284	(1.10)	-0.0333	(-1.40)
ΔS^{O}(t-12)	ΔS^{I}(-12)	-0.0890*	(-4.43)	0.0545*	(2.93)	0.0052	(0.20)	-0.0569	(-2.38)
ΔS^{O}(t-13)	ΔS^{I}(-13)	-0.0854*	(-4.25)	0.0784*	(4.21)	0.0215	(0.83)	-0.0391	(-1.64)
ΔS^{O}(t-14)	ΔS^{I}(-14)	-0.0787*	(-3.90)	0.0637*	(3.41)	0.0319	(1.23)	-0.0362	(-1.51)
ΔS^{O}(t-15)	ΔS^{I}(-15)	-0.0699*	(-3.47)	0.0558*	(2.99)	0.0055	(0.21)	-0.0082	(-0.34)
ΔS^{O}(t-16)	ΔS^{I}(-16)	-0.0569*	(-2.83)	0.0687*	(3.67)	0.0371	(1.43)	-0.0047	(-0.20)
ΔS^{O}(t-17)	ΔS^{I}(-17)	-0.0466	(-2.32)	0.0478	(2.56)	0.0065	(0.25)	0.0076	(0.32)
ΔS^{O}(t-18)	ΔS^{I}(-18)	-0.0537*	(-2.68)	0.0488*	(2.63)	0.0168	(0.65)	-0.0139	(-0.58)
ΔS^{O}(t-19)	ΔS^{I}(-19)	-0.0416	(-2.09)	0.0191	(1.03)	0.0190	(0.74)	-0.0026	(-0.11)
ΔS^{O}(t-20)	ΔS^{I}(-20)	-0.0348	(-1.76)	0.0075	(0.41)	0.0695*	(2.72)	-0.0776	(-3.28)
ΔS^{O}(t-21)	ΔS^{I}(-21)	-0.0599*	(-3.03)	0.0333	(1.82)	-0.0014	(-0.05)	-0.0203	(-0.86)
ΔS^{O}(t-22)	ΔS^{I}(-22)	-0.0724*	(-3.69)	0.0566*	(3.10)	-0.0189	(-0.75)	0.0091	(0.39)
ΔS^{O}(t-23)	ΔS^{I}(-23)	-0.0631*	(-3.25)	0.0581*	(3.20)	-0.0254	(-1.02)	0.0174	(0.74)
ΔS^{O}(t-24)	ΔS^{I}(-24)	-0.0920*	(-4.79)	0.0578*	(3.22)	-0.0481	(-1.95)	0.0260	(1.12)
ΔS^{O}(t-25)	ΔS^{I}(-25)	-0.0674*	(-3.57)	0.0646*	(3.65)	-0.0218	(-0.90)	0.0323	(1.42)
ΔS^{O}(t-26)	ΔS^{I}(-26)	-0.0539*	(-2.89)	0.0507*	(2.90)	0.0364	(1.52)	0.0051	(0.22)
ΔS^{O}(t-27)	ΔS^{I}(-27)	-0.0177	(-0.97)	0.0480*	(2.80)	0.0294	(1.26)	-0.0063	(-0.29)
ΔS^{O}(t-28)	ΔS^{I}(-28)	-0.0055	(-0.31)	0.0157	(0.94)	0.0506	(2.23)	-0.0554	(-2.59)
ΔS^{O}(t-29)	ΔS^{I}(-29)	-0.0278	(-1.69)	0.0271	(1.71)	-0.0130	(-0.61)	0.0049	(0.24)
ΔS^{O}(t-30)	ΔS^{I}(-30)	0.0130	(0.96)	-0.0230	(-1.59)	0.0210	(1.21)	-0.0268	(-1.44)

* represnts the significance at the level of 1%.

Table V

Lead-lag relation on bearish markets, 2004

		Dependent variable							
		ΔS^{O} (t)				ΔS^{I} (t)			
CONSTANT		-0.0347*	(-10.35)			-0.0527*	(-11.84)		
z(t-1)		0.3062*	(9.39)			0.4660*	(10.78)		
ΔS^{O} (t-1)	ΔS^{I} (-1)	-0.3516*	(-20.40)	0.5310*	(40.93)	0.0535	(2.34)	-0.0316	(-1.84)
ΔS^{O} (t-2)	ΔS^{I} (-2)	-0.2361*	(-12.86)	0.3370*	(21.06)	0.0534	(2.19)	-0.0191	(-0.90)
ΔS^{O} (t-3)	ΔS^{I} (-3)	-0.2441*	(-12.91)	0.2775*	(16.29)	0.0044	(0.18)	-0.0281	(-1.24)
ΔS^{O} (t-4)	ΔS^{I} (-4)	-0.1851*	(-9.58)	0.2251*	(12.70)	-0.0327	(-1.28)	0.0336	(1.43)
ΔS^{O} (t-5)	ΔS^{I} (-5)	-0.1738*	(-8.88)	0.1629*	(8.98)	0.0251	(0.97)	0.0115	(0.48)
ΔS^{O} (t-6)	ΔS^{I} (-6)	-0.1402*	(-7.07)	0.1448*	(7.90)	-0.0056	(-0.21)	-0.0063	(-0.26)
$\Delta\backslash S^{O}$ (t-7)	ΔS^{I} (-7)	-0.1188*	(-5.95)	0.0960*	(5.19)	0.0468	(1.77)	-0.0423	(-1.72)
ΔS^{O} (t-8)	ΔS^{I} (-8)	-0.0987*	(-4.91)	0.0793*	(4.26)	0.0386	(1.45)	-0.0609	(-2.47)
ΔS^{O} (t-9)	ΔS^{I} (-9)	-0.1431*	(-7.09)	0.0758*	(4.05)	0.0073	(0.27)	-0.0556	(-2.24)
ΔS^{O} (t-10)	ΔS^{I} (-10)	-0.0844*	(-4.16)	0.1125*	(5.99)	0.0261	(0.97)	0.0011	(0.04)
ΔS^{O} (t-11)	ΔS^{I} (-11)	-0.0702*	(-3.46)	0.0729*	(3.87)	0.0189	(0.70)	-0.0188	(-0.75)
ΔS^{O} (t-12)	ΔS^{I} (-12)	-0.0990*	(-4.88)	0.0756*	(4.01)	-0.0433	(-1.61)	-0.0030	(-0.12)
ΔS^{O} (t-13)	ΔS^{I} (-13)	-0.0753*	(-3.72)	0.0547*	(2.90)	-0.0451	(-1.68)	0.0372	(1.49)
ΔS^{O} (t-14)	ΔS^{I} (-14)	-0.0532*	(-2.63)	0.0404	(2.15)	0.0108	(0.40)	-0.0216	(-0.87)
ΔS^{O} (t-15)	ΔS^{I} (-15)	-0.0786*	(-3.89)	0.0558*	(2.97)	-0.0244	(-0.91)	0.0365	(1.47)
ΔS^{O} (t-16)	ΔS^{I} (-16)	-0.0575*	(-2.85)	0.0579*	(3.09)	-0.0174	(-0.65)	0.0580	(2.34)
ΔS^{O} (t-17)	ΔS^{I} (-17)	-0.0200	(-0.99)	0.0171	(0.91)	0.0083	(0.31)	0.0055	(0.22)
ΔS^{O} (t-18)	ΔS^{I} (-18)	-0.0735*	(-3.66)	0.0472	(2.53)	-0.0269	(-1.01)	0.0271	(1.09)
ΔS^{O} (t-19)	ΔS^{I} (-19)	-0.0687*	(-3.43)	0.0409	(2.19)	0.0293	(1.10)	-0.0103	(-0.42)
ΔS^{O} (t-20)	ΔS^{I} (-20)	-0.0778*	(-3.90)	0.0606*	(3.25)	0.0259	(0.98)	-0.0226	(-0.92)
ΔS^{O} (t-21)	ΔS^{I} (-21)	-0.0337	(-1.70)	0.0458	(2.46)	0.0086	(0.33)	-0.0424	(-1.72)
ΔS^{O} (t-22)	ΔS^{I} (-22)	0.0143	(0.72)	-0.0035	(-0.19)	0.0251	(0.96)	-0.0341	(-1.39)
ΔS^{O} (t-23)	ΔS^{I} (-23)	-0.0296	(-1.50)	0.0029	(0.16)	0.0196	(0.75)	0.0002	(0.01)
ΔS^{O} (t-24)	ΔS^{I} (-24)	-0.0041	(-0.21)	0.0033	(0.18)	0.0336	(1.30)	-0.0338	(-1.40)
ΔS^{O} (t-25)	ΔS^{I} (-25)	-0.0126	(-0.66)	0.0169	(0.94)	0.0187	(0.73)	0.0188	(0.78)
ΔS^{O} (t-26)	ΔS^{I} (-26)	-0.0351	(-1.86)	0.0115	(0.65)	-0.0234	(-0.93)	0.0151	(0.64)
ΔS^{O} (t-27)	ΔS^{I} (-27)	-0.0582*	(-3.14)	0.0407	(2.33)	-0.0383	(-1.56)	-0.0090	(-0.39)
ΔS^{O} (t-28)	ΔS^{I} (-28)	-0.0429*	(-2.38)	0.0564*	(3.31)	-0.0411	(-1.72)	0.0471	(2.08)
ΔS^{O} (t-29)	ΔS^{I} (-29)	0.0005	(0.03)	-0.0020	(-0.12)	-0.0011	(-0.05)	0.0143	(0.66)
ΔS^{O} (t-30)	ΔS^{I} (-30)	-0.0086	(-0.62)	0.0125	(0.85)	0.0045	(0.25)	0.0073	(0.37)

* represnts the significance at the level of 1%.