# What makes the two phase behavior in financial markets?

Gyuchang Lim[1], SooYong Kim[1], and Kyungsik Kim[2,**]
[1] *Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea*
[2] *Department of Physics, Pukyong National University, Pusan 608-737, Korea*

The two phase behavior of Korean treasury bond (KTB) futures is investigated in Korean exchange market. We examine the real data such as KTB futures and the correlated Brownian process. By adding those correlations inherent in KTB futures series to Brownian walks, we study whether the volatility clustering plays a key role in the equilibrium and nonequilibrium states of financial markets. It is shown that the two phase behavior is basically resulted from the heavy-tailed behavior of the distribution of price increments.

Several stylized facts of financial markets, such as the power-law distribution of price increments, traded volumes, and volatilities, have recently been reported $[1-5]$. Critical phenomena in the physical systems have been the most interesting fields, and in these fields the twin concepts of scaling and universality have been proved to be important in a number of scientific fields [6]. Following these arguments, several works have been performed to financial systems and they have also reported some reasonable results $[7-9]$. In those works, the authors argued that the analogy with thermodynamic systems at critical point can be led to the explanation of the two-phase behavior of financial markets.

In the minority games, the volatility clustering is detected. To investigate the possible relationship between volatility clustering and the two phase phenomena, Zheng et al [10] have analyzed the two-phase phenomena with the German DAX and compare it with those in minority games and herding models. They concluded that the volatility clustering and the two-phase phenomena are independent characteristics of financial dynamics. Matia and Yamasaki [11] showed that the scaling of trading volumes was to be a key factor in the emergence of two-phase behavior of volume imbalance conditional to the local intensity.

The purpose of this paper is to examine the two-phase phenomena of the futures market and more importantly to focus on how the volatility clustering and the distribution of price increments affect the emergence of two-phase phenomena. As done in [10], we analyzed the two-phase phenomena with the volatility instead of the volume imbalance [9]. In order to investigate the dependence of two-phase phenomena on the volatility clustering and the distribution of price increments, respectively, we generated the geometric Brownian walks and added to them a higher-order correlation inherent in the real futures quotations.

The logarithmic increment is represented in units of ticks as

$$G(t) = \log y(t+\Delta) - \log y(t) \tag{1}$$

where $y(t)$ denotes the futures quotation at time t, and $\Delta$ is the size of window, over which the logarithmic increment and the volatility are defined. The volatility of logarithmic increments is described as

$$r(t) = \langle | \log y(t''+1)/y(t'') - \langle \log y(t''+1)/y(t'')\rangle | \rangle \tag{2}$$

Here the bracket denotes the average over the interval $[t, t+\Delta]$. The window size $\Delta$ is set to be 10 min. The window size has no effect on the two phase behavior of the time series. To guarantee a large number of statistics under analysis, the statistics can be calculated using the overlapping window.

Next we calculate the autocorrelation function and the detrended fluctuation analysis on the KTB412, shuffled KTB412, geometric Brownian walk, and correlated Brownian walk in order to check the existence of higher-order correlation. This analysis is performed on the volatility with $\Delta = 1$ min. The detrended fluctuation analysis helps us to detect an inherent correlation in time series, and the detrended fluctuation is defined as

$$F(s) = [\frac{1}{N_s}\sum_{\nu=1}^{N_s} F^2(\nu, s)]^{1/2} \sim s^{\alpha}, \tag{3}$$

*Corresponding author. Tel.:+82-51-620-6354; fax:+82-51-611-6357.
E-mail address: kskim@pknu.ac.kr.

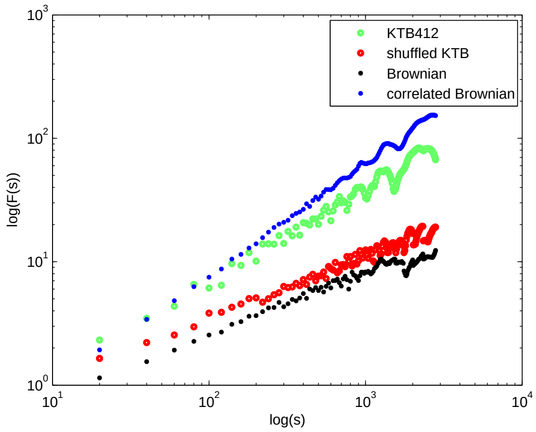


FIG. 1: Plot of the detrended fluctuation analysis of four time series; the KTB412, shuffled KTB412, geometric Brownian walk, and correlated geometric Brownian walk for $\Delta = 1$ min.

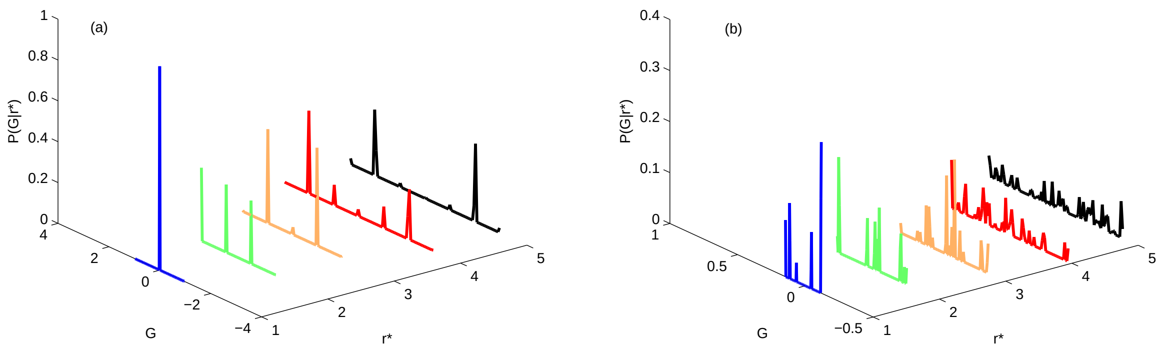


FIG. 2: $P(G|r^*)$ of (a) the KTB412 and (b) the correlated geometric Brownian walk. These are conditional to the volatility. $r^*$ is used as a scale of real volatility and the scale is determined by dividing the range of volatilities into 100 intervals. A large $r$ is excluded since the corresponding data size is too small to get a reliable result.

where the root mean square fluctuation is determined by

$$F^2(\nu, s) = \frac{1}{s}\sum_{k=1}^{s}[Y\{(\nu-1)s+k\} - Z_\nu(k)]^2 \tag{4}$$

for each segment $\nu$, where $\nu = 1, \cdots, N_s$. Here $Y(i)$ is the integrated time series or profile and $Z_\nu(k)$ is the fitting polynomial in segment $\nu$.

We analyze the two-phase behavior of the derivative security from the data set of tick-by-tick recorded KTB412 futures quotations which was traded for six months from 1st July of 2004. This tick data consists of about 14,000 quotations. In general, since the trade of contracts becomes active as getting close to the maturity, the quotations are concentrated in the period of 3 months close to the maturity of contracts.

For a cumulative distribution $P(\geq G) \sim G^{-\mu}$, we obtain $\mu \simeq 2.3$ and $\mu \simeq 2.1$ for logarithmic increments and volatility($\Delta = 10$) of the KTB412, respectively. Fig. 1 shows that $\alpha \simeq 0.50$ and $\alpha \simeq 0.50$ for shuffled KTB412 and the Brownian walk, respectively, which means that there is no correlation. However, for the KTB412 and the correlated Brownian walk, we also obtain $\alpha \simeq 0.75$ and $\alpha \simeq 0.91$, respectively. Next we calculate the distribution of the logarithmic increments conditional to the volatility to examine the two phase behavior of KTB412 following the procedure taken in Ref. [10]. Figs. 2(a) and 2(b) show the distribution $P(G|r^*)$ for the KTB412 and correlated Brownian walk. For convenience, we use the rescaled volatility, $r^* = 1, 2, \cdots$, for which a volatility range is defined as $[r_{min} + (r^*-1) \times (r_{max} - r_{min})/100, r_{min} + r^* \times (r_{max} - r_{min})/100]$, where $r_{min} = 0$ and $r_{max} = 44.46$. For the

KTB412, it appears two phase phenomena at $r^* = 5$, comparable to the German DAX [10]. The other data set of the correlated Brownian walk shows no clear distinction between distributions conditional to varying volatilities. It means that the two phase phenomena is not consistent with the volatility clustering.

In conclusion, we have examined the two phase phenomena for the KTB412 and the correlated Brownian walk. We have confirmed that KTB412 with heavy-tailed distribution has the two phase behavior, while the Brownian walk has no clue of it, which means that the two phase behavior is independent of the clustering volatility. For a small volatility the price changes accumulate near zero, which implies that the market is efficient. As the volatility increases, it goes to the two phase. According to this result, we can easily determine the state of the market if we find a reliable and novel method to estimate a volatility from small number of quotations.

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