February 25, 2007

Theory-Based Illiquidity and Asset Pricing

Tarun Chordia*

Sahn-Wook Huh**

Avanidhar Subrahmanyam***

*Goizueta Business School, Emory University, Atlanta, GA 30327.

**Faculty of Business, Brock University, St. Catharines, Ontario, Canada L2S 3A1.

***The Anderson School, 110 Westwood Plaza, University of California at Los Angeles, Los Angeles, CA 90095-1481.

We thank Antonio Bernardo, Michael Brennan, Adam Kolasinski, Jiro Kondo, Tony Tang, seminar participants at Brock University and the 2006 Annual Conference on Market Structure and Market Integrity for valuable feedback. Research assistance was ably provided by Mi-Ae Kim. Huh gratefully acknowledges the generous financial support from the Social Sciences and Humanities Research Council of Canada (SSHRC) and the Faculty of Business at Brock University. All errors are solely ours.

Abstract

Theory-Based Illiquidity and Asset Pricing

Many proxies of illiquidity have been used in the literature that relates illiquidity to asset prices. These proxies have been motivated from an empirical standpoint. In this study, we approach liquidity estimation from a theoretical perspective. Our method explicitly recognizes the analytic dependence of illiquidity on more primitive drivers such as trading activity and information asymmetry. More specifically, we estimate illiquidity using structural formulae for Kyle's (1985) lambda for a comprehensive sample of NYSE/AMEX and NASDAQ stocks. The empirical results provide convincing evidence that theory-based estimates of illiquidity are priced in the cross-section of expected stock returns, even after accounting for risk factors, firm characteristics known to influence returns, and other illiquidity proxies prevalent in the literature.

The question of whether investors demand higher returns from less liquid securities is an enduring one in financial economics. In a seminal paper on this issue, Amihud and Mendelson (1986) find evidence that asset returns include a significant premium for the quoted bid-ask spread. Since that study, Brennan and Subrahmanyam (1996), Brennan, Chordia and Subrahmanyam (1998), Jacoby, Fowler, and Gottesman (2000), Jones (2002), and Amihud (2002) all elaborate upon the role of liquidity as a determinant of expected returns. Further, Pástor and Stambaugh (2003) and Acharya and Pedersen (2005) relate liquidity risk to expected stock returns.[1]

An important issue in studies that relate illiquidity to asset prices is the measurement of illiquidity. Other than direct empirical measurements of illiquidity by the bid-ask spread, the approach taken in the literature has been to employ empirical arguments and econometric techniques to measure illiquidity. For example, Amihud (2002) proposes the ratio of absolute return to volume as a measure of illiquidity. Brennan and Subrahmanyam (1996), based on the analysis of Glosten and Harris (1988), suggest measuring illiquidity by the relation between price changes and order flows. Pástor and Stambaugh (2003) measure illiquidity by the extent to which returns reverse upon high volume, an approach based on the notion that such a reversal captures inventory-based price pressures. Hasbrouck (2005) provides a comprehensive set of estimates of these and other measures, and we will not duplicate his efforts by describing these measures in detail.

These empirical proxies have added considerably to our understanding of illiquidity. However, there are some issues related to this literature that are matters of concern. First, because these measures have yielded mixed results, the significance of the findings is difficult to interpret. Thus, for example, Brennan and Subrahmanyam (1996) find a negative relation between bid-ask spread and expected returns which is at odds with the

[1]Two recent theoretical papers attempt to endogenize liquidity in asset-pricing settings. Eisfeldt (2004) relates liquidity to the real sector and finds that productivity, by affecting income, feeds into liquidity. Johnson (2005) models liquidity as arising from the price discounts demanded by risk-averse agents to change their optimal portfolio holdings. He shows that such a measure may dynamically vary with market returns and, hence, help provide a rationale for liquidity dynamics documented in the literature.

liquidity premium argument. Spiegel and Wang (2005) do not find a significant relation between expected returns and a variety of empirically-motivated illiquidity proxies, which also appears to muddy the conclusions on whether illiquidity is related to asset returns. Second, the empirical arguments proposed to justify such proxies often do not mesh well with theory. For example, although many microstructure theories have been developed, extant economic models are unable to map precisely on to the Amihud (2002) construct of the ratio of absolute return to volume. Third, illiquidity is endogenous and depends on many variables that are related to asset prices via other models. For instance, illiquidity depends on volatility, but volatility is related to expected returns via traditional risk-return arguments.

In this paper, we propose a new approach to measure illiquidity and relate our measures to expected asset returns, thus providing stronger theoretical underpinnings to the empirical illiquidity-return relation relative to the existing literature. Specifically, we turn to theory in order to consider illiquidity estimates that can be estimated by way of closed-form expressions. The basis for our work emanates from Brennan and Subrahmanyam (1995), who test a structural representation of a theoretically derived estimate of illiquidity and relate it to analyst following. Their estimates derive from the price impact measure, lambda, which is, in turn, based on the Kyle (1985) model and its adaptation by Admati and Pfleiderer (1988) to explain intraday patterns. The advantage of estimating the equilibrium versions of Kyle lambdas is that the expressions are in terms of quantities that are relatively easy to comprehend and for which plausible empirical proxies can be devised at low cost.

Our analysis considers illiquidity to be endogenous insofar as it arises as an outcome of trading patterns in financial markets. Unlike a stock's return beta, which depends on influences extraneous to financial markets, such as a firm's line of business and the cyclicality of a firm's revenue stream, the endogeneity of illiquidity makes it difficult to interpret results from asset pricing regressions. For example, illiquidity depends on total volatility, including systematic risk.[2] Lack of adequate controls for systematic risk (e.g., only through the CAPM, as in Amihud and Mendelson, 1986; Brennan and Wang, 2005) could create the appearance that illiquidity is priced because illiquidity depends on true systematic risk. Liquidity also depends on volume. If volume captures investor sentiment (viz., Baker and Stein, 2004), then illiquidity may again appear to be priced, even though what the researcher may be capturing is the impact of volume through its impact on illiquidity. As such, a complete approach to understanding the pricing of illiquidity would model illiquidity's dependence on primitive economic forces and separately control for systematic risk and trading volume, which is what our study attempts to accomplish. Furthermore, in contrast to the *ad hoc* measures of illiquidity in the literature, the functional form of illiquidity that we use is obtained from an equilibrium setting.

We estimate two variants of closed-form expressions for Kyle lambdas, one of which assumes perfectly correlated information signals, while the other postulates diverse signals. Many of the empirical proxies for the inputs to the Kyle (1985) model are similar to those used by Brennan and Subrahmanyam (1995), and some are new. In examining the time-series behavior of such lambdas, we find a decline in the measures over time, which mirrors the behavior of other illiquidity proxies, such as bid-ask spreads (Jones, 2002). We then examine whether these lambdas are priced in the cross-section of stock returns, using a comprehensive set of NYSE/AMEX and NASDAQ stocks over the last three decades. After controlling for known characteristics such as book-to-market equity and momentum as well as for known sources of risk such as the Fama and French (1993) factors, we find convincing evidence that both our versions of Kyle lambdas are priced in the cross-section of stock returns. We check the robustness of our findings by using midpoint returns and conducting quarterly regressions with quarterly compounded returns. In addition, we run a "horse race" with other commonly used (il)liquidity measures, demonstrating that the theory-based illiquidity is a priced factor even after accounting

[2] This holds as long as agents have private information about firm-specific as well as systematic components of firm value, as in Brennan, Jegadeesh, and Swaminathan (1993), Subrahmanyam (1991), or Kumar and Seppi (1993).

for the effects of other competing (il)liquidity measures.

The remainder of this paper is organized as follows. In Section 1, we present the theoretical background and estimation of the two theory-based illiquidity measures in the context of Kyle lambdas. Section 2 describes the methodology. Section 3 outlines the data, definitions, and descriptive statistics. Section 4 discusses the empirical results and robustness checks. In Section 5, we compare the effects of the theory-based measures with those of other alternative (il)liquidity measures. Section 6 concludes.

1 Estimates of Kyle Lambdas

In this section, we provide the theoretical background for our lambda estimation. We estimate two versions of Kyle lambdas, with and without signal noise. We begin by linking illiquidity and asset pricing in the context of Kyle lambdas.

1.1 The Link between Kyle Lambdas and Illiquidity Pricing

The Kyle (1985) model does not provide a direct link to illiquidity pricing. However, assuming that a liquidity trader is the marginal agent allows the incorporation of a link. Thus, suppose that an asset is traded over two dates. At date 2, it pays off $\widetilde{W} = \overline{W} + \delta$, where $\overline{W}$ is nonstochastic and the payoff innovation (δ) is a normally distributed variable with a mean of zero. Informed traders obtain a (possibly noisy) signal about δ. Trading of the asset occurs at date 1. As usual the price P is set to be of the form $P = \overline{W} + \lambda Q$, where λ is the impact of order flows on prices and Q is the total order flow. Prior to the date 1 trading (at date 0), a "discretionary" uninformed (liquidity) trader contemplates investing in the asset. This trader's demand is denoted by D, which is normally distributed with mean zero. There also is a set of non-discretionary liquidity traders, whose total demand is also normally distributed with a zero mean, and equals U, with D and U being independent of each other. We define $z \equiv U + D$ to be the total demand from the liquidity traders. The discretionary and non-discretionary liquidity demands are independent of δ. For a given λ, the expected cost to the discretionary liquidity trader is then given by $E[(P - \widetilde{W})D] = \lambda D^2$.

Assuming that the risk-free discount rate is zero across dates 0 and 2, we normalize the asset's supply to one share.[3] The date 0 price is the shadow price at which the discretionary trader is indifferent between holding the stock and not doing so. At date 0, the risk-neutral discretionary liquidity trader will be willing to pay an amount

$$\overline{W} - \lambda D^2.$$

Thus, the expected price change across dates 0 and 2 is given by[4] λD^2, and is thus proportional to λ (ignoring cross-sectional variation in D for convenience). It follows that expected future returns are linearly related to λ divided by the initial price of the stock. As we describe below, for our empirical work we estimate λ's each month for each stock and proxy the initial price by the market price of the stock (P) as of the end of the month previous to the one in which λ is measured.

Our study uses structural estimates of two versions of Kyle lambdas: one with, and the other without, signal noise. In the following two subsections, we present the theory-based illiquidity measures and discuss how to estimate them using proxy variables. We first present our main results using a base set of inputs and then examine in section 4.3 the robustness of our results to using alternative input variables, mid-point returns, and different frequency in regressions.

[3]The supply of shares does not play any role at date 1 because prices are set by risk neutral market makers who are willing to absorb any quantity of excess shares at an unbiased price.

[4]Note that the expected price change across dates 1 and 2 is zero (the date 1 price is semi-strong efficient), and the expected price change across dates 0 and 1 is equal to the expected price change across dates 0 and 2. The expected price change across dates 0 and 2, therefore, is the only unique, non-zero expected price change in our model.

1.2 An Illiquidity Measure Without Noise in the Information Signals

When the informed traders observe, without any noise, a signal that is informative about the payoff on a risky asset, the Appendix shows in detail that the illiquidity (or price impact) measure, lambda, in a standard Kyle (1985) market is given by

$$\lambda = \frac{\sqrt{N v_\delta}}{(N+1)\sqrt{v_z}}, \tag{1}$$

where N is the number of informed traders, v_δ is the variance of the payoff, and v_z is the variance of uninformed trades. Dividing both sides of Eq.(1) by the price P in order to get a price-scaled illiquidity measure, we have[5]

$$\begin{aligned} \frac{\lambda}{P} &= P^{-1}\frac{\sqrt{N v_\delta}}{(N+1)\sqrt{v_z}} \\ &= \frac{\sqrt{N}\sqrt{Var(R)}}{(N+1)\sqrt{v_z}} \\ &= \frac{N^{0.5} std(R)}{(N+1) std(z)}, \end{aligned} \tag{2}$$

where R is the asset return, and $std(z)$ is the standard deviation of uninformed trades. Eq.(2) is our first measure of illiquidity used in this study, and we call it $ILLIQ\_1$.

To estimate $ILLIQ\_1$ each month for each stock, we employ proxy variables as inputs for each of the original variables in Eq. (2). Our approach in this subsection is not to condition on any specific source of information (such as earnings) but to assume that private information is about value innovations as reflected in the series of stock price movements. We use analyst following to proxy for informed agents. This approach toward estimating lambdas is very similar to that of Brennan and Subrahmanyam (1995). Our specific inputs are as follows:

[5]For Eq.(2), note that at time t the conditional variance of returns is $Var(R) = Var\left(\frac{P_t}{P_{t-1}} - 1 \mid I_{t-1}\right) = \frac{Var(P_t)}{P_{t-1}^2}$.

N: One plus the number of analysts following a firm in each month, notated as ANA.[6]

$std(R)$: This is proxied by the standard deviation of daily returns within the previous month (month $t-1$), notated as STD(RET). To obtain this variable for each month, we use firms that have at least 10 daily returns in the previous month from the CRSP daily file.

$std(z)$: The average of daily dollar volume (in \$million) within the previous month, notated as AVG(DVOL).[7] To obtain this variable for each month, we use firms that have at least 10 daily trading records in the previous month from the CRSP daily file.

We now discuss how to measure lambdas when information signals are assumed to be noisy and diverse.

1.3 An Illiquidity Measure with Noisy Information Signals

When the informed traders observe diverse signals, so that each trader observes the asset's payoff plus an error term that is independent and identically distributed across agents, the Appendix shows that Kyle's (1985) measure, λ, is in this case given by

$$\lambda = \frac{v_\delta}{(N+1)v_\delta + 2v_\varepsilon}\sqrt{\frac{N(v_\delta + v_\varepsilon)}{v_z}}, \tag{3}$$

where N is the number of informed traders, v_δ is the variance of the asset payoff, v_z is the variance of uninformed trades, and v_ε is the variance of signal innovations. Dividing both sides of Eq.(3) by P_{t-1}, we have

$$\frac{\lambda}{P} = P^{-1}\frac{v_\delta}{(N+1)v_\delta + 2v_\varepsilon}\sqrt{\frac{N(v_\delta + v_\varepsilon)}{v_z}}. \tag{4}$$

Eq.(4) is our second measure of illiquidity used in this study, and we call it $ILLIQ\_2$. Note that this measure requires a proxy for signal noise variance as well as the signal

[6] If N is zero, then the illiquidity measure in equation (2) will also be zero, which is not reasonable. To get around this, we use a variable ANA, which is one plus the number of analysts. In this way, we avoid a sample bias because firms not covered by analysts are included in our sample (our approach mimicks that of Brennan and Subrahmanyam, 1995).

[7] Note that since uninformed trades (z) follow the normal distribution, i.e., $z \sim N(0, v_z)$, $E[|z|] = \sqrt{\frac{2}{\pi}}std(z)$. Thus, $\sqrt{v_z} = std(z) = \sqrt{\frac{\pi}{2}}E[|z|]$, which in turn can be proxied by average trading volume.

itself. It is difficult to obtain such a proxy from the return series alone. Therefore, in an approach that is different from that in the previous section, we condition on a specific informational event, namely, earnings announcements, in the calculation of the lambda with signal noise. In this case, the noise in the signal can readily be calculated in this context by considering the discrepancy between actual earnings and analysts' earnings forecasts. Thus, to estimate $ILLIQ\_2$ for each stock in each month, our proxy variables for each of the original variables in Eq.(4) are as follows (as mentioned earlier, we consider alternative proxies for the inputs in Subsection 4.3):

P: the stock price at the previous month's end.

v_δ: This variable is proxied by EVOLA-sqr, which is the squared value of earnings volatility (EVOLA), where EVOLA is the standard deviation of earnings per share (EPSs) from the most recent eight quarters.

v_ε: This variable is proxied by ESURP-sqr, which is the squared value of earnings surprise (ESURP), where ESURP is the absolute value of the current earnings per share (EPS) minus the EPS forecast four quarters ago.

v_z: We proxy this variable by AVG(DVOL)-sqr, which is the squared value of the average of daily dollar volume (in $million) within the previous month. To obtain this variable for each month, we use firms that have at least 10 daily trading records in the previous month from the CRSP daily file.

1.4 Estimation of the Illiquidity Measures

To estimate our illiquidity measures ($ILLIQ\_1$ and $ILLIQ\_2$) according to Eq.(2) and Eq.(4), the input variables related to the number of analysts (ANA), earnings surprise (ESURP, ESURP-sqr), and earnings volatility (EVOLA, EVOLA-sqr) are extracted from the I/B/E/S database. If a firm has one or more missing value(s) in the number of analysts, the missing months are filled with the previous month's value up to two quarters. We use the CRSP daily and monthly files to obtain other input variables: STD(RET), AVG(DVOL), AVG(DVOL)-sqr, and P. The average numbers of component stocks used each month to estimate $ILLIQ\_1$ and $ILLIQ\_2$ for NYSE/AMEX stocks are 1,845.1 and 1,683.1, respectively. Those for NASDAQ stocks are 2,663.8 and 1,967.0, respectively.

Table I contains the descriptive statistics for the input variables of the first illiquidity measure, $ILLIQ\_1$. As one would expect, the mean of ANA for NASDAQ (interchangeably, the "OTC market") stocks (3.15) is much lower than that for NYSE/AMEX (interchangeably, the "exchange market") stocks (5.52). Given that the NASDAQ market is often comprised of smaller firms, it is intuitive that average daily dollar volume, AVG(DVOL), in this market (\$3.61 million) is lower than that in the exchange market (\$5.26 million). Because NASDAQ stocks are more high-tech oriented as well as smaller, it also is not surprising that daily return volatility, STD(RET), in the OTC market (4.4%) is far higher than that in the exchange market (2.7%).

To check the descriptive statistics of the input variables for the second illiquidity measure, $ILLIQ\_2$, we again see that ANA and AVG(DVOL)-sqr are qualitatively similar to the corresponding input variables for $ILLIQ\_1$. In the same context, the price level in the exchange market (\$31.31) is much higher than in the OTC market (\$14.16). While earnings surprise variables (ESURP and ESURP-sqr) are higher in the exchange market than in NASDAQ, earnings volatility variables (EVOLA and EVOLA-sqr) are considerably higher in the NASDAQ market. Also note that ESURP-sqr is much more variable across firms in the NASDAQ market than in the exchange market.

After having described our estimation procedure and summary statistics, we discuss and reiterate some advantages of our approach relative to estimation of illiquidity using intradaily data (e.g., as in Brennan and Subrahmanyam, 1996 or Sadka, 2006). In addition to using closed-form expressions obtained from equilibrium economic theory, our estimation method also avoids noise that might be induced when obtaining order flows through the Lee and Ready (1991) algorithm. Furthermore, the Lee and Ready (1991) algorithm only signs market orders, which may induce additional errors. We also do not have to worry about microstructural issues that may complicate the estimation process, such as price discreteness and inventory concerns, or about the appropriate aggregation interval for order flows. Last, our method enables us to use a far broader cross-section and longer time-series of data because we do not need to process the Institute for the Study of Securities Markets (ISSM) and TAQ databases, which are not available prior to 1983. We realize that in spite of all these advantages, the key challenge is to show if our measures actually are priced after accounting for other popular illiquidity proxies also estimable over long time-periods; and we will show below that this is indeed the case.

2 Methodology

Assume that returns are generated by an L-factor approximate factor model:

$$\tilde{R}_{jt} = E(\tilde{R}_{jt}) + \sum_{k=1}^{L} \beta_{jk}\tilde{f}_{kt} + \tilde{e}_{jt}, \tag{5}$$

where $\tilde{R}_{jt}$ is the return on security j at time t, and $\tilde{f}_{kt}$ is the return on the k-th factor $(k = 1, 2, ..., L)$ at time t. The exact or equilibrium version of the arbitrage pricing theory (APT) in which the market portfolio is well diversified with respect to the factors (Connor, 1984; Shanken, 1985, 1987) implies that the expected excess returns may be written as

$$E(\tilde{R}_{jt}) - R_{Ft} = \sum_{k=1}^{L} \theta_{kt}\beta_{jk}, \tag{6}$$

where R_{Ft} is the return on the risk-free asset and θ_{kt} is the risk premium on the factor portfolio k. Plugging Eq.(6) into Eq.(5), the APT implies that realized returns are given by

$$\tilde{R}_{jt} - R_{Ft} = \sum_{k=1}^{L} \beta_{jk}\tilde{F}_{kt} + \tilde{e}_{jt}, \tag{7}$$

where $\tilde{F}_{kt} \equiv \theta_{kt} + \tilde{f}_{kt}$ is the sum of the risk premium and return on the factor k.

Our goal is to test whether the two illiquidity measures derived in Section 1, based on the strategic microstructure model, have incremental explanatory power for returns relative to the Fama and French (FF, 1993) 3-factor benchmark, after controlling for other security characteristics. For this purpose, a standard application of the Fama-MacBeth (1973) procedure would involve estimation of the following equation:

$$\tilde{R}_{jt+1} - R_{Ft+1} = c_0 + \phi ILLIQ\_i_{jt} + \sum_{k=1}^{L} \theta_k \beta_{jkt} + \sum_{m=1}^{M} c_m Z_{mjt} + \tilde{e}_{jt+1}, \tag{8}$$

where $ILLIQ\_i_{jt}$ ($i = 1$ or 2) is one of our illiquidity measures ($ILLIQ\_1$ or $ILLIQ\_2$) for security j in month t estimated in Section 1, and a vector of control variables, Z_{mjt}, is firm characteristic m ($m = 1$, ..., M) for security j in month t. Note that the right-hand side variables in Eq.(8) are all lagged one-period in order to ensure that we capture pure predictive relations. Under the null hypothesis that expected excess returns depend only on the risk characteristics of the returns notated by β_{jk}, then ϕ and c_m ($m = 1$, ..., M) will be zero. This hypothesis can be tested in principle by first estimating the factor loadings each month using the past data, conducting a cross-sectional regression for each month in which the independent variables are an illiquidity measure, factor loadings, and other non-risk characteristics, and then averaging the monthly coefficients over time and computing their standard errors. This basic Fama-MacBeth approach, however, will present a problem if the factor loadings are measured with errors.

In order to address the above issue arising from error-prone loadings estimates, we adopt the Brennan, Chordia, and Subrahmanyam (1998) approach. Specifically, we perform risk adjustments in returns using the Fama-French (1993) factors (MKT_t, SMB_t, and HML_t)[8] in two different ways. In the first method, we compute risk-adjusted returns, $\tilde{R}^{*1}_{jt}$, for each month as the sum of the intercept and the residual, i.e.,

$$\begin{aligned} \tilde{R}^{*1}_{jt} &= (\tilde{R}_{jt} - R_{Ft}) - (\widehat{\beta}^{*}_{j1} MKT_t + \widehat{\beta}^{*}_{j2} SMB_t + \widehat{\beta}^{*}_{j3} HML_t) \\ &= \widehat{\alpha}^{*} + \widehat{\tilde{e}}^{*}_{jt}, \end{aligned} \tag{9}$$

after conducting regressions in Eq.(7) (but with a constant term α) using the *entire* sample range (from January 1972 to December 2002 for NYSE/AMEX stocks and from

[8] MKT is the excess return on the market portfolio, SMB is the return on a zero net investment portfolio which is long in small firms and short in large firms, and HML is the return on a zero net investment portfolio which is long in high book-to-market firms and short in low book-to-market firms.

January 1983 to December 2002 for NASDAQ stocks) of the data.[9] We call this risk-adjusted return ($\tilde{R}^{*1}_{jt}$) *FF3-adj EXSRET1.* We also use another version of risk adjustment for robustness. Thus, in the second method, we obtain *rolling* estimates of the factor loadings, β_{jk}, for each month over the sample period for all securities using the time series of the past 60 months (at least 24 months) with Eq.(7). Given the current month's data ($\tilde{R}_{jt}$-R_{Ft}, MKT_t, SMB_t, and HML_t) and the factor loadings ($\widehat{\beta}^{**}_{jk}$) estimated each month for all stocks, we can compute the risk-adjusted return on each of the securities, $\tilde{R}^{*2}_{jt}$, for each month t as follows:

$$\tilde{R}^{*2}_{jt} = (\tilde{R}_{jt} - R_{Ft}) - (\widehat{\beta}^{**}_{j1} MKT_t + \widehat{\beta}^{**}_{j2} SMB_t + \widehat{\beta}^{**}_{j3} HML_t). \tag{10}$$

We call this risk-adjusted return ($\tilde{R}^{*2}_{jt}$) *FF3-adj EXSRET2.*

The risk-adjusted returns from Eq.(9) and Eq.(10) constitute the raw material for the estimates that we present in the following Fama-Macbeth (1973) cross-sectional regressions:

$$\tilde{R}^{*h}_{jt+1} = c_{0t} + \phi_t ILLIQ\_i_{jt} + \sum_{m=1}^{M} c_{mt} Z_{mjt} + \tilde{e}'_{jt+1}, \text{ h=1 or 2.} \tag{11}$$

Note that the error term in Eq.(11) is different from that in Eq.(8) because the error in Eq.(11) also contains terms arising from the measurement error associated with the factor loadings.

To check whether illiquidity is priced, we report three types of statistics based on regressions in Eq.(11): the statistics based on regressions with the dependent variable in Eq.(11) being 1) risk-*unadjusted* excess returns (we call this unadjusted return as EXSRET); 2) risk-adjusted excess returns using the first method, FF3-adj EXSRET1; and 3) risk-adjusted excess returns using the second method, FF3-adj EXSRET2. For our purposes, we estimate the vector of coefficients $\mathbf{c}_t = [c_{0t}\ \phi_t\ c_{1t}\ c_{2t} ... c_{Mt}]'$ from Eq.(11) each month with a simple OLS regression as

$$\widehat{\mathbf{c}}_t = (\mathbf{Z}'_t \mathbf{Z}_t)^{-1} \mathbf{Z}'_t \tilde{\mathbf{R}}^{*h}_{t+1},$$

[9]In the first method, therefore, for each stock we have only *one* set of the factor loadings ($\widehat{\beta}^{*}_{jk}$) estimated using the whole time-series of the data.

where $h = 1$ or 2, $\mathbf{Z}_t = [ILLIQ\_i \;\; Z_1 \; Z_2 ... Z_M]'$, and $\tilde{\mathbf{R}}^{*h}_{t+1}$ is the vector of risk-adjusted excess returns based on Eq.(9) or Eq.(10). The standard Fama-MacBeth (1973) estimator is the time-series average of the monthly coefficients, and the standard error of this estimator is taken from the time series of monthly coefficient estimates, $\widehat{\mathbf{c}}_t$. Note that although factor loadings are estimated with error in Eq.(7), this error affects only the dependent variable, $\tilde{R}^{*h}_{t+1}$, as we see in Eq.(9), Eq.(10), and Eq.(11). While the factor loadings will be correlated with vector $\mathbf{Z}_t = [ILLIQ\_i \;\; Z_1 \; Z_2 ... Z_M]'$, there is no *a priori* reason to believe that the errors in the estimated loadings will be correlated with the vector $\mathbf{Z}_t$. This implies that the coefficient vector $\widehat{\mathbf{c}}_t$ estimated in Eq.(11) is unbiased.[10]

3 Data, Definitions, and Descriptive Statistics

For this study, we use data at a daily and/or monthly frequency over the 372 months (31 years: 197201-200212) for NYSE/AMEX stocks and the 240 months (20 years: 198301-200212) for NASDAQ stocks. For those cases in which accounting variables and other data are available only on a yearly (or quarterly) basis, we keep the relevant values constant for 12 months (or 3 months) in the regressions.[11]

The three dependent variables (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2) defined in Section 2 for the Fama and MacBeth (1973) regressions are obtained or estimated using the CRSP monthly file and the FF 3 factors are available from Ken-

[10] If the errors in the estimated factor loadings are correlated with the explanatory variables $\mathbf{Z}_t = [ILLIQ\_i \;\; Z_1 \; Z_2 ... Z_M]'$, the monthly estimates of the coefficients, $\widehat{\mathbf{c}}_t$, will be correlated with the factor realizations, and thus the mean of these estimates (which is the Fama-MacBeth estimator) will be biased by an amount that depends on the factor realizations. Therefore, as a check on the robustness of our results, we also obtained a "purged" estimator for each of the explanatory variables in the regressions of FF3-adj EXSRET1 and FF3-adj EXSRET2: i.e., the constant term (and its t-value) from the regression of the monthly coefficients ($\widehat{\mathbf{c}}_t$) estimated in Eq.(11) on the time series of FF 3 factor realizations. This estimator, which was developed by Black, Jensen, and Scholes (1972), purges the monthly estimates of the factor-dependent component so that it is unbiased even when the errors in the factor loading estimates are correlated with vector $\mathbf{Z}_t$.

[11] The data series available only on a yearly basis are the variables related to the book-to-market ratio (BM_Raw, BM_Trim, and BTM) and the effective cost measure (*Roll_Gibbs*: to be explained later). Those available only on a quarterly basis are accounting performance-related variables, ESURP-sqr and EVOLA-sqr.

neth French's web site. In addition to the variables mentioned above, we use six firm characteristics in the regressions as control variables: SIZE, BTM, MOM1-MOM4. The definitions of the control and related variables are as follows:

MV: The market value defined as the month-end stock price times the number of shares outstanding (in $million).

SIZE: The natural logarithm of MV.

BM_Raw: The untrimmed book-to-market ratio defined as BV/MV, where the book value (BV) is common equity plus deferred taxes (in $million).

BM_Trim: The trimmed book-to-market ratio, where BM_Raw values greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively.

BTM: The natural logarithm of BM_Trim. Following Fama and French (1992), we fill monthly BM_Raw (hence BM_Trim and BTM) values for July of year t to June of year $t+1$ with the value computed using the accounting data at the end of year $t-1$, assuming a lag of six months before the annual accounting numbers are known to investors.

MOM1: The compounded holding period return of a stock over the most recent 3 months (from month $t-1$ to month $t-3$).

MOM2: The compounded holding period return over the next recent 3 months (from month $t-4$ to month $t-6$).

MOM3: The compounded holding period return over the 3 months from month $t-7$ to month $t-9$.

MOM4: The compounded holding period return over the 3 months from month $t-9$ to month $t-12$. For each of the above four momentum variables to exist, a stock should have all three consecutive monthly returns over the corresponding three-month period.

Later, in Section 5, we run a horse race to compare the effects of our two illiquidity measures with those of four other alternative (il)liquidity measures commonly used in the literature. The alternative measures to be analyzed in our study are notated and defined as follows:

Amihud: The illiquidity measure of Amihud (2002). We estimate this measure each month as the average of $|r|/DVOL$, where r is the daily stock return, and $DVOL$ is the daily dollar volume in $1000.

Roll_Gibbs: The market risk-adjusted effective bid-ask spread of Roll (1984), estimated at an annual frequency using the Gibbs sampler. This measure was obtained from Joel Hasbrouck's website.

PS: The illiquidity measure (gamma) of Pastor and Stambaugh (2003). We estimate this measure by running monthly regressions, using the CRSP daily data whose transaction records are kept for at least fifteen days within a month (see Section 5 for details).

$TURN$: The average of daily share turnover values within each month for each stock.

The variables related to the book-to-market ratio are constructed using the CRSP and CRSP/Compustat Merged (CCM) files. Other firm characteristic and related variables (MV, SIZE, and MOM1-MOM4) are also extracted from the CRSP monthly file. The three (il)liquidity measures (*Amihud*, PS, and $TURN$) are estimated using the CRSP daily or monthly file. The average number of component stocks used each month in the Fama-MacBeth (1973) cross-sectional regressions for NYSE/AMEX stocks is 1,845.1, while that for NASDAQ stocks is 2,667.5.

Table II reports the time-series average values of monthly means, medians, standard deviations (STD), and other descriptive statistics for our key variables. The values of each statistic are first computed cross-sectionally and then averaged in the time-series over the sample period. Insofar as the average means of the two illiquidity measures ($ILLIQ\_1$ and $ILLIQ\_2$) in the NASDAQ market are higher by a factor of almost five than in the exchange market, the NASDAQ market appears to be less liquid in general. Considering that the NASDAQ market is characterized by young, small, and high-tech firms (with large values of information asymmetry) on average, this result seems reasonable. Moreover, the variations of the measures across stocks in the NASDAQ market are also far higher. The differences in the levels of the two illiquidity measures across the two different markets point out that proxying (il)liquidity by share turnover only may be misleading. Chordia, Huh, and Subrahmanyam (2005) document that turnover in the NASDAQ market has been much higher than that in the exchange market. However, this finding may partly result from the double counting in the NASDAQ market (Atkins and Dyl, 1997), indicating that despite the higher turnover in the NASDAQ market, we cannot say that the NASDAQ market is more liquid than the exchange market.

Another noteworthy aspect is that $ILLIQ\_1$ and $ILLIQ\_2$ are highly leptokurtic as well as significantly skewed to the left in both markets. The large kurtoses of $ILLIQ\_1$ and $ILLIQ\_2$ also imply that sample distributions of the two measures exhibit many extreme observations. To alleviate this problem, Hasbrouck (1999, 2005, 2006) advocates employing the square-root transform of liquidity measures.[12] For this reason, we include in our analyses the empirical results based on the square-root transform of our theory-based measures, in addition to those based on the raw measures themselves. To obtain the equivalent transformation of Pastor and Stambaugh's (2003) measure, whose sign often varies, we multiply the sign of the measure by the square root of the absolute value of the measure as $Sign(PS)[|PS|]^{1/2}$. As we see in Table II, the skewnesses and kurtoses of the corresponding measures ($[ILLIQ\_1]^{1/2}$, $[ILLIQ\_2]^{1/2}$) are substantially reduced by the square-root transformation.

To examine the time-series behavior of our two illiquidity measures, we plot the value-weighted series of the transformed measures in Figures 1 and 2 for the two markets over the sample period. As we see in Figure 1(a), value-weighted $[ILLIQ\_1]^{1/2}$ of NYSE/AMEX stocks demonstrates a huge run-up after the oil crisis in November 1973. However, it generally exhibits a decreasing time trend after 1974, suggesting that market

[12] Hasbrouck (2006) eschews the logarithmic transformation because it is theoretically possible for illiquidity to be zero. Given that our two illiquidity measures often have values near zero, we opt to use the square-root instead of the log transform.

liquidity has improved since the mid-1970s in the exchange market. As mentioned above, the average level of monthly illiquidity in the OTC market [Figure 1(b)] is consistently higher, but the measure is also decreasing from the late 1980s. Some large spikes in the four graphs in common also present that the market liquidity decreased during the months after the stock market crash in October 1987, the Iraqi invasion of Kuwait in August 1990, and the Russian default as well as the Long-Term Capital Management (LTCM) debacle in June to October 1998. Reflecting the recent economic recession and the 9/11 events, the transformed $ILLIQ\_1$ in the OTC market shows a more salient increase in 2001-2002 than that in the exchange market. As can be seen in Figure 2, the trend of $[ILLIQ\_2]^{1/2}$ is qualitatively similar to that of $[ILLIQ\_1]^{1/2}$, with its volatility and 2001-2002 level increase being more pronounced in the NASDAQ market. For brevity, we do not report the graphs based on the series of the equal-weighted illiquidity measures. But their trends are qualitatively similar, with the absolute levels generally being higher than those of the value-weighted series.

Other discernible facts in Table II are as follows. NYSE/AMEX stocks are larger by a substantial margin, and NASDAQ stocks are more likely to be growth stocks (see BM_Raw). The four momentum variables (MOM1-MOM4) are consistently negative in the NASDAQ market, while they are positive in the exchange market. In addition, the firm size (MV) and the book-to-market ratio (BM_Raw) tend to be left-skewed.

Next, we examine the average correlation coefficients between our explanatory variables in Table III.[13] The lower and upper triangles in the table present the correlations for NYSE/AMEX and NASDAQ stocks, respectively. Our two illiquidity measures are highly correlated in both markets: 64% between the two untransformed measures and 79% between the two transformed measures in the exchange market; and 46% between the first ones and 64% between the second ones in the NASDAQ market. The two measures are negatively correlated with the four momentum variables, suggesting that

[13]To save space, we do not report the correlation coefficients between the three types of excess returns to be used as a left-hand side variable in the regressions. They are highly correlated (coefficients greater than 94%) in both markets.

good past price performance of a stock tends to contribute to improving the liquidity of that stock. It also is not surprising to observe that correlation of SIZE with the two illiquidity measures is negative and statistically significant at any conventional level in both markets because we would expect larger firms with greater breadth of ownership to be more liquid than smaller ones. In both markets, the correlation coefficients between the book-to-market ratio and the two illiquidity measures are positive and statistically significant. This indicates that value stocks are likely to be more illiquid.

4 Empirical Results

4.1 Features of the Portfolios Formed on Illiquidity and Size

Before moving on to regression analyses, we report the average values of monthly return, firm size, and illiquidity for the 25 portfolios formed by sorting on illiquidity and firm size. For this purpose, each month we first sort sample stocks by $ILLIQ\_1$ in ascending order and split them into five portfolios with the equal number of stocks. Then, each of the five portfolios is again sorted by firm size (MV) and split into five portfolios, resulting in the 25 portfolios. Next, the average values of return, size, and illiquidity are computed each month for each of the 25 portfolios, and the time-series averages of the three variables over the sample period are reported in each panel of Table IV.

Panel A in Table IV shows that for a given size group (especially size groups 1-3) the average return tends to increase with illiquidity in both markets, while for a given illiquidity group the average return tends to decrease with firm size. The latter confirms the small-firm effect documented by many researchers. The t-values (italicized in Panel A) also demonstrate that monthly portfolio returns are mostly significantly different from zero. In particular, note that the average return of the portfolio with smallest size and highest illiquidity (2.91%) is three times higher than that of the other extreme portfolio (with largest size and lowest illiquidity, 0.93%) in the NYSE/AMEX market, and eight times higher in the NASDAQ market. Within a given size group, the return difference between the two extreme portfolios (the highest and lowest illiquidity groups) is always positive and is significant at the 5% level for the first two size groups, and at the 10% level for the third size group. This finding supports the notion that illiquidity pricing may be especially pronounced for the smaller firms. Of course, the portfolio analysis is preliminary in the sense that it does not account for other characteristics that may affect stock returns; moreover, averaging returns across stock groups may obscure illiquidity pricing phenomena that may be particularly prominent at the individual stock level. We address these issues by way of regression analysis in the next subsection.

As can be seen in Panels B and C of Table IV, average firm size (within a given size group) is related negatively to illiquidity, and average illiquidity (within a given $ILLIQ\_1$ group) is mostly negatively related to firm size. To save space, we do not report the analog table with $ILLIQ\_2$, but the results are qualitatively similar to those using $ILLIQ\_1$.

4.2 Cross-Sectional Regressions

We have observed in Table IV that within a given size group the average portfolio return is likely to increase with illiquidity, suggesting that theory-based illiquidity is a priced factor. In this section, we formally test whether our two illiquidity measures predict returns. As mentioned above, our test involves the following cross-sectional regression estimated at the monthly frequency:

$$\tilde{R}^*_{jt+1} = c_{0t} + \phi_t ILLIQ\_i_{jt} + \sum_{m=1}^{M} c_{mt} Z_{mjt} + \tilde{e}'_{jt+1}, \text{ i=1 or 2,} \qquad (12)$$

where $\tilde{R}^*_{jt+1}$ represents either the risk-unadjusted excess return (EXSRET) or the two risk-adjusted excess returns (FF3-adj EXSRET1 and FF3-adj EXSRET2) defined and estimated in Section 2, $ILLIQ\_i_{jt}$ is either of our two theory-based illiquidity measures ($ILLIQ\_1$, $ILLIQ\_2$, and their transforms) derived and estimated in Section 1, and Z_{mjt} denotes firm characteristic m for stock j in month t.[14]

[14]Note that unlike the contemporaneous regressions in Fama and MacBeth (1973), our explanatory variables are all lagged one period because we are interested in capturing pure predictive relations.

We report the standard Fama-MacBeth statistics (the time-series average of the estimated coefficients from the equation above and its t-statistic) in Tables V and VI. Along with the average coefficients and t-statistics, we provide two other types of statistics in the tables: the average of the adjusted R^2 values from the individual regressions (Avg R-sqr), and the average number of companies used in the regression each month over the sample period (Avg Obs).

The regression results based on Eq.(12) with $ILLIQ\_1$ or its transform are presented in Table V, while those with $ILLIQ\_2$ appear in Table VI. As we see in Panel A of Table V, the average number of component stocks used in the monthly regressions for NYSE/AMEX stocks ranges from 1,797.7 to 1,845.1, and that for NASDAQ stocks ranges from 2,406.3 to 2,663.8, depending on data availability of the variables. *Avg R-sqr* is in the 3-5% range in the exchange market, and that for NASDAQ stocks is slightly lower. The explanatory power of the regressions tends to be higher with the unadjusted excess returns (EXSRET) than with the risk-adjusted returns (FF3-adj EXSRET1, FF3-adj EXSRET2) in both markets. Given that $ILLIQ\_2$ requires more input variables, Table VI shows that the average number of component stocks used in the regressions with this illiquidity measure decreases by 7-8% to 1667.6-1683.1 for NYSE/AMEX stocks, and by 21-26% to 1907.3-1967.0 for NASDAQ stocks (see Panel A). Other aspects of the explanatory power are similar to those with $ILLIQ\_1$.

We first discuss the results from the Fama-MacBeth regressions of EXSRET on $ILLIQ\_1$ as well as other firm characteristics that are best known to be associated with expected returns, namely, SIZE, BTM, and the four momentum variables (MOM1-MOM4). Panel A of Table V shows that the average coefficients of $ILLIQ\_1$ are positive and statistically significant at any conventional level in both markets after controlling for other firm characteristics, confirming the hypothesis that stocks with higher illiquidity are expected to have higher (excess) returns. The coefficients of SIZE and BTM are respectively negative and positive, and they are statistically significant at the 5% level in both markets. These size and book-to-market effects are consistent with previous studies, such as Fama and French (1992). We also find that the sensitivity of returns to these variables is much greater in the NASDAQ market than in the exchange market. The four momentum variables are all strongly positively related to returns in the exchange market, whereas they become weaker in the OTC market.

We now consider whether the relations observed above are maintained when the dependent variable is risk-adjusted using the FF factors. The estimates of illiquidity and characteristic rewards ($\widehat{\phi}$, $\widehat{c}_m$) for returns adjusted by the first method in Section 2 (FF3-adj EXSRET1) are presented in the next column of Panel A. By risk-adjusting, the coefficients of the right-hand side variables tend to attenuate slightly, but the relations are essentially unchanged, with the levels of statistical significance becoming even reinforced in may cases. $ILLIQ\_1$ continues to be strongly positively related to risk-adjusted returns, firm size is negatively related to returns, and a higher book-to-market ratio predicts higher returns in both markets. Overall, SIZE and BTM play more important roles in the NASDAQ market than in the NYSE/AMEX market in predicting stock returns. The momentum variables again demonstrate that better price performance in the past is expected to provide higher returns in the current month, especially in the NYSE/AMEX market. This finding confirms the continuation of short-term returns documented by Jegadeesh and Titman (1993) and Fama and French (1996).

In the last column, we report the estimates of illiquidity and characteristic rewards ($\widehat{\phi}$, $\widehat{c}_m$) for excess returns (FF3-adj EXSRET2), which are now risk-adjusted by the second method in Section 2. First, the impact of $ILLIQ\_1$ on risk-adjusted returns is virtually the same as the result with FF3-adj EXSRET1: positive and statistically significant in the two markets. SIZE and BTM continue to have a strong impact on excess returns in both markets, though their statistical significance is slightly lower in the exchange market. Another discernible feature is that the momentum variables now show a contrast between the two different markets. Using FF3-adj EXSRET2, these variables are strongly positively related to returns in the exchange market. In the NASDAQ market, however, the coefficients of the variables become insignificant without exception, an interesting aside, which deserves investigation in future research.

Now we examine the results with $[ILLIQ\_1]^{1/2}$ in Panel B. Overall, the features with the transformed measure are very similar to those in Panel A. However, two aspects are worth mentioning. First, the sensitivity of returns to illiquidity and the statistical significance of the relevant coefficients are reinforced, particularly for NASDAQ stocks. To gauge the effect of illiquidity on the stock return in the EXSRET specification, we find that an increase in illiquidity ($[ILLIQ\_1]^{1/2}$) by one standard deviation requires higher monthly (excess) returns of 0.35% in the exchange market and 0.72% in the OTC market. The magnitude of these additionally required monthly returns is economically significant, given that Chordia, Huh, and Subrahmanyam (2005) document that average monthly return is 1.19% for 1,647.2 NYSE/AMEX stocks over the past 39.5 years and 1.46% for 1,722.1 NASDAQ stocks over the past twenty years. The other noteworthy point is that with the transformed measure, the size effect is attenuated in the exchange market.

Next, we investigate in Table VI how the effects of illiquidity and other firm characteristics on returns change when we employ the second measure, $ILLIQ\_2$, or its transform in the regressions. We see in Panel A of Table VI that the coefficients of $ILLIQ\_2$ are also statistically different from zero at the 5% level in the exchange market after accounting for the effects of other characteristics, while the significance phases out in the NASDAQ market. It is possible that analyst forecasts for tech-oriented NASDAQ stocks are more prone to error due to greater cash flows uncertainty for such stocks. This may induce noise in the estimates of $ILLIQ\_2$, which relies on analyst-related data to estimate signal noise. However, if we use the transformed measure, which subdues the influence of extreme observations, our second theory-based measure is strongly related to excess returns as we see in Panel B. As observed in Table V about the roles of SIZE and BTM, the size of the coefficients and the level of their statistical significance are again larger in the NASDAQ market than in the exchange market. The momentum effects for NYSE/AMEX stocks are similar to those in Table V, but the effects become more salient for NASDAQ stocks than in Table V. For example, the coefficient estimates of more recent past returns (MOM1-MOM2) for NASDAQ stocks are now statistically significant at the 5% level in both panels for the FF3-adj EXSRET2 specification.

4.3 Robustness Checks

4.3.1 Different Combinations of Input Variables in Estimating the Illiquidity Measures

As checks on the robustness of our results, we have used three different types of excess returns (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2).[15] We have also considered the effects of our choices of input variables used to estimate the two illiquidity measures. As pointed out in Section 1, the two important input variables in estimating $ILLIQ\_1$ and $ILLIQ\_2$ are, among others, the standard deviation of daily returns in month $t-1$ and the average of daily dollar volume in month $t-1$. For robustness, we have obtained standard deviations of returns computed each month with daily returns in month $t-2$, daily returns in the past 36 months, and monthly returns in the past 60 months. Moreover, as a proxy for $std(R)$, we also have used an idiosyncratic risk measure against the FF 3 factors using data from the past 60 months in line with Spiegel and Wang (2005). For the average volume as a proxy for $std(z)$, we tried many candidates, including average daily share volume, average daily dollar volume, and daily turnover in month $t-1$, $t-2$, and in the past 36 months. We estimated the two illiquidity measures using a number of combinations of these as inputs. Cross-sectional regressions using illiquidity measures estimated with different combinations of input variables do not significantly change our results, especially the effects of illiquidity, firm size, and book-to-market equity.

[15]As mentioned earlier, we additionally obtained a "purged" estimator of Black et al. (1972) for each of the explanatory variables in the regressions of FF3-adj EXSRET1 and FF3-adj EXSRET2. The results were very similar to those of the "raw" estimator, and are not reported. The results imply that the estimation errors in factor loadings are not correlated with the vector of explanatory variables. The results are all available from the authors upon request.

4.3.2 Using Quote Mid-point Returns

A recent study by Bessembinder and Kalcheva (2006) argues that empirical pricing tests using observed returns calculated using the reported closing prices might induce microstructure biases because of the bid-ask bounce, suggesting that asset-pricing tests with quote mid-point returns can reduce this problem. To address this issue, we obtain mid-point returns for both markets. For NYSE/AMEX stocks, monthly quote mid-point returns are calculated based on the first (open) quote mid-point and the last (close) quote mid-point (open-to-close mid-points) within each month over the 180 months from 1988 to 2002 (fifteen years: 198801-200212). For NASDAQ stocks, we compute returns based on the mid-points of the monthly closing quotes (close-to-close mid-points) using the NASDAQ National Market System (NMS) data in the CRSP file over the 240 months (twenty years: 198301-200212) from 1983 to 2002.[16]

The cross-sectional regression results using mid-point returns are reported in Table VII. As shown in Panel A, the coefficients of $[ILLIQ\_1]^{1/2}$ are statistically significant at 5% in the exchange market and at 1% in the OTC market, with the magnitude of them being much larger compared to the result in Panel B of Table V. The slightly lower t-values of the illiquidity measure in the exchange market might result from the shorter sample period used to compute mid-quote returns. Panel B shows that with the usage of mid-point returns the magnitude and statistical significance of our second illiquidity measure ($[ILLIQ\_2]^{1/2}$) are actually reinforced in the NASDAQ market, while the corresponding coefficient in the exchange market is comparable to that obtained using transaction returns in Panel B of Table VI.[17]

[16] Month-end bid and ask quotes for the NASDAQ National Market System (NMS) stocks are obtained using the CRSP file, in which closing bid and ask NASDAQ market quotes are available from November 1982 onwards.

[17] We do not report the tables with non-transformed measures (ILLIQ_1 and ILLIQ_2), but the results are qualitatively similar to those with the transformed measures.

4.3.3 Quarterly Cross-sectional Regressions

Brennan and Wang (2006) argue that in asset-pricing tests with monthly returns, variables correlated with short-horizon mispricing, such as illiquidity measures, may be spuriously related to future returns. They indicate that the mispricing bias is likely to be attenuated in longer-horizon quarterly returns. Thus, we now present the results from quarterly cross-sectional regressions. We compute quarterly returns (compounded) for the dependent variable and three-month average values for the explanatory variables. The sample range is 124 quarters (31 years: 197201-200204) for NYSE/AMEX stocks and 80 quarters (20 years: 198301-200204) for NASDAQ stocks.

The results are contained in Table VIII. Panel A exhibits that the statistical significance level of the loadings on $[ILLIQ\_1]^{1/2}$ is very similar to the monthly regression results reported in Panel B of Table V. Panel B shows that the magnitude and statistical significance of the coefficient for $[ILLIQ\_2]^{1/2}$ becomes stronger in quarterly regressions, especially for NASDAQ stocks, than those in the monthly regressions of Table VI.

So far, we have demonstrated that the two theory-based illiquidity measures continue to be priced in the cross-section of stock returns, regardless of using: 1) different input variables in estimating the illiquidity measures; 2) mid-point returns; or 3) quarterly returns. However, two more questions still remain to be answered: (i) Do the theory-based measures perform better than the other commonly used (il)liquidity measures in the finance literature? (ii) Do the two measures continue to be priced after accounting for the effects of other competing (il)liquidity measures? To further test the robustness of our findings, we compare the results of the theory-based measures to those of alternative illiquidity measures in the next section.

5 A Horse Race with Alternative Measures

5.1 Selection of Alternative Measures and their Relations to the Theory-based Illiquidity Measures

There are a number of (il)liquidity measures that have been used in the asset-pricing or microstructure literature. Some measures have been obtained or estimated from the TAQ intradaily file, while others come from the CRSP daily file. As Hasbrouck (2005) admits, however, estimating the measures using high-frequency trade and quote data, such as the TAQ database, limits the availability to the relatively small and recent data samples. Moreover, Merton (1980) suggests that the accuracy in estimating first moments hinges upon the length of the data sample and not the sampling frequency. It also is relevant to recognize the computational economy of and hence the importance of liquidity measures that can be constructed from data of daily or lower frequency. Of course, our two theory-based measures can be constructed from the CRSP daily file and the lower frequency I/B/E/S database. As such, given the issues described above in selecting alternative measures for comparison purposes, we limit our choices to the measures that can be estimated using the CRSP daily file, and thus over the same horizons as our measures.

First, we consider Amihud's (2002) illiquidity measure, which is defined as $|r|/DVOL$, where r is the daily stock return and $DVOL$ is the daily dollar volume (in \$1,000). For monthly regressions, we compute each month the average of the daily estimates of illiquidity within a month. Roughly speaking, this measure (notated as *Amihud* in our analysis) is similar to Kyle's (1985) λ, which is the basis of our two theory-based illiquidity measures. However, the Amihud measure is distinct from Kyle's λ in the sense that *Amihud* captures the absolute return impact of unsigned volume, while λ is the price impact of signed order flows. From an operational standpoint, our closed-form expressions for lambdas include the impact of analyst following, over and above the volatility and volume measures, since it is an economic link between illiquidity and information flows. Given the fact that the Amihud measure has been used widely in recent literature, however, we include *Amihud* as one of the competing illiquidity measures.

Attempting to answer the question of how well high-frequency measures can be proxied using daily data, Hasbrouck (2005) suggests that the market risk-adjusted effective cost of Roll (1984), estimated using the Gibbs sampler, is one of appropriate CRSP-based proxies for a TAQ-based effective cost. We thus consider this measure (notated as *Roll_Gibbs*) in our study.[18] Next, if a stock is not perfectly liquid, order flows or signed volume may induce corrections in stock prices that initially overshoot and subsequently revert to the true values. Therefore, we estimate a reversal measure of illiquidity each month for each stock using the CRSP daily file as in Pastor and Stambaugh (2003) who estimate γ from the regression equation,

$$r^e_{j,d+1,t} = a + br_{j,d,t} + \gamma sign(r^e_{j,d,t}) DVOL_{j,d,t} + \varsigma_{j,d+1,t},$$

where $r_{j,d,t}$ is the raw return and $r^e_{j,d,t}$ is the excess return (over the CRSP value-weighted index return) of stock j at day d within month t (we require at least fifteen days of data per month in the CRSP daily file to estimate γ). We call this measure PS. Lastly, share turnover has been used as an (il)liquidity proxy by many researchers such as Brennan, Chordia, and Subrahmanyam (1998). For this reason, we consider monthly share turnover (notated as $TURN$) as one of the liquidity proxies in the analysis. We estimate each month the three illiquidity measures (*Amihud*, PS, and $TURN$), while we obtain *Roll_Gibbs* from the web site of Joel Hasbrouck. The *Roll_Gibbs* measure is at an annual frequency.

Table IX presents the correlation coefficients between the (il)liquidity measures. The lower triangle in Panel A shows that in the NYSE/AMEX market, both $ILLIQ\_1$ and $ILLIQ\_2$ are most highly correlated with *Amihud*, followed by *Roll_Gibbs*. Both PS and $TURN$ are negatively correlated with the two theory-based measures, but the size of the coefficients is small. For NASDAQ stocks (in the upper triangle), the two theory-based

[18]We initially used c^{Gibbs}, estimated in Hasbrouck (2005), and later replaced it with c_BMA, estimated in Hasbrouck (2006), for he updates the paper and data. The results are virtually the same.

measures are most highly correlated with *Roll_Gibbs*. But PS is very weakly related to the two theory-based measures in both markets. Another discernible fact is that while PS is weakly correlated in general with any other (il)liquidity measures, *Amihud* is highly correlated with *Roll_Gibbs* in both markets. Panel B is the analog to Panel A with all the transformed measures. With the square-root transforms, the absolute correlations substantially increase, but the patterns are qualitatively the same as in Panel A.

5.2 Cross-Sectional Regressions with Alternative Illiquidity Measures

In this subsection, we conduct a horse race between one of our two theory-based illiquidity measures and one (or all) of the four competing measures considered in the previous subsection. Our goal is to test whether the effects of our theory-based measures on expected returns are comparable to those of other competing measures and, going one step further, to check whether each of the theory-based measures still has an incremental impact on returns after accounting for the effects of the four alternative measures.

First, we run the regression with each of the competing measures or their transforms by replacing $ILLIQ\_i_{jt}$ with one of the four measures (*Amihud*, *Roll_Gibbs*, PS, and $TURN$) in Eq.(12). Because *Roll_Gibbs* is of annual frequency, we keep the annual values of this measure constant over the twelve months within each year for the monthly regressions. For brevity, we report in Table X the results with only the transformed measures.[19]

As the correlation coefficients in Table IX suggest, Panel A of Table X exhibits that the impact of the transformed *Amihud* on returns is consistently strong in both markets, which is comparable to that of the transformed $ILLIQ\_1$ or $ILLIQ\_2$ in Panel B of Tables V-VI. However, the impact of $[Roll\_Gibbs]^{1/2}$ in Panel B has the wrong sign (negative), although it is statistically significant. This is surprising, given that this measure is highly correlated with $[Amihud]^{1/2}$ as well as with the two transformed theory-based measures.

[19]However, the results with raw measures are similar to those with transformed measures.

Hasbrouck (2006) as well as Spiegel and Wang (2005) also conduct similar analyses with the same measure, documenting that *Roll_Gibbs* is positively related to returns but mostly not significant.[20] The reason(s) that this measure exhibits such equivocal effects may be one or more of the following: 1) The estimation is not sufficiently accurate; 2) Estimates at the annual frequency are not suitable for monthly regressions; or 3) This is not a priced characteristic. As Hasbrouck (2006) indicates, the limitation of *Roll_Gibbs* stems from the fact that it does not explicitly incorporate the price impact effects of trading volume or order flow, which may be endogenous with price dynamics. As we see in Panels C-D, the impact of transformed PS and $TURN$ on returns is negligible after controlling for other firm characteristics. The weak role of PS in our study is consistent with Hasbrouck (2005).[21]

Next, we run a horse race between one of our two theory-based illiquidity measures and all the four competing measures. For this purpose, we augment Eq.(12) by including four more variables as in the equation,

$$\tilde{R}^*_{jt+1} = c_{0t} + \phi_t ILLIQ\_i_{jt} + \sum_{n=1}^{4} \varphi_{nt} ALT_{njt} + \sum_{m=1}^{M} c_{mt} Z_{mjt} + \tilde{e}'_{jt+1}, \text{ i=1 or 2,} \qquad (13)$$

where ALT_{njt} (n = 1,..., 4) denotes one of the four alternative (il)liquidity measures (*Amihud*, *Roll_Gibbs*, PS, and $TURN$). We again report the results with only the transformed measures.

As shown in Panel A of Table XI, by including additional four (il)liquidity measures in the regressions, the number of average component stocks decreases, while the adjusted R^2 increases relative to the results in Panel B of Table V. We also observe in Panel A that the transformed $ILLIQ\_1$ continues to be priced even after controlling for the alternative measures. The effects of $[Amihud]^{1/2}$ and $[Roll\_Gibbs]^{1/2}$ are similar to those of Panels A and B in Table X, respectively, although there are some differences in the size or statistical significance of their coefficients. It is interesting to see that by including the competing measures, the coefficients of the transformed PS and $TURN$ tend to be

[20]See Table 6 and Table 8 of Hasbrouck (2006).
[21]See Table 5 of Hasbrouck (2005).

significant in the NASDAQ market. However, their effects in the exchange market are insignificant or of the wrong sign. With the additional (il)liquidity variables, the size effect still exists in both markets, but the book-to-market effect disappears for NASDAQ stocks. Panel B of Table XI reports the analog of Panel A with $[ILLIQ\_2]^{1/2}$. The impact of $[ILLIQ\_2]^{1/2}$ continues to be positive and statistically significant at the 5% level in the exchange market, but it phases out in the OTC market after accounting for the effects of competing measures. PS plays no role. The other variables show patterns similar to those in Panel A.

To conserve space, we do not report the horse-racing results analogous to Table XI with mid-point returns or quarterly returns. But a couple of features from these results deserve mentioning. With mid-point returns, the loadings on $[ILLIQ\_1]^{1/2}$ are statistically significant at 1-10% in both markets, but the loadings on the four alternative measures are no longer significant. Another noticeable aspect with mid-point returns is that the impact of $[ILLIQ\_2]^{1/2}$ is positive and statistically significant at 1% in the NASDAQ market. In quarterly regressions, the coefficients of both $[ILLIQ\_1]^{1/2}$ and $[ILLIQ\_2]^{1/2}$ are significant at 1-10% in both markets, with the loadings on $[ILLIQ\_2]^{1/2}$ being so at the 1% level in the NASDAQ market.

Overall, the empirical tests show that while our second illiquidity measure is priced in the exchange market in any case, it exhibits relatively weaker behavior in the NASDAQ market in some situations, indicating that information about the earnings of highly volatile firms is harder to come by, which would imply that our measure is not a perfect measure of information asymmetry. On balance, however, our analyses provide strong evidence that theory-based illiquidity is a priced attribute, even after controlling for the four alternative measures.

6 Conclusion

Empirical proxies for illiquidity have been subject to controversy because they have achieved mixed results in answering the question of whether illiquidity is related to asset returns. Further, these proxies typically do not emanate from an equilibrium model, raising the question of whether the informal reasoning that justifies the measures is the cause of conflicting conclusions about the illiquidity-return relation.

We use an alternative approach to measuring illiquidity. Specifically, we explicitly model the functional relation between illiquidity and its primitive drivers and thus provide stronger economic underpinnings for the estimation of illiquidity relative to those in the extant literature. We estimate Kyle lambdas, using analytic formulae that are derived from an equilibrium framework. We use plausible empirical proxies for inputs to the theoretical expressions, along the lines of Brennan and Subrahmanyam (1995). Our lambdas are estimated for a comprehensive sample of NYSE/AMEX and NASDAQ stocks, spanning more than 30 years. In asset pricing regressions, although our measure of illiquidity does not completely subsume other measures of illiquidity, it generally remains significant even after accounting for the effects of these empirically-motivated measures. Overall, the results provide convincing evidence that our theory-based lambdas are priced in the cross-section of expected stock returns.

Future research would focus on illiquidity-risk pricing and also why and how such theory-based illiquidity measures vary over time and across firms. For example, are such lambdas relatively high when the firm is young and information asymmetry plays a predominant role? Do they decline as the clientele for holding stocks changes from ostensibly informed institutions to uninformed individual investors? Can these lambdas be estimated for other markets, such as fixed income, and do these vary with credit risk (presumably because the potential for asymmetric information is greater in bonds with high default probability)? Such issues are left for future research.

References

Anderson, T., 1984, *An Introduction to Multivariate Statistical Analysis* (2nd ed.), John Wiley & Sons, New York.

Acharya, V., and L. Pedersen, 2005, Asset pricing with liquidity risk, *Journal of Financial Economics* 77, 385-410.

Admati, A. and P. Pfleiderer, 1988, A theory of intraday patterns: Volume and price variability, *Review of Financial Studies* 1, 3-40.

Amihud, Y., 2002, Illiquidity and stock returns: Cross-section and time-series effects, *Journal of Financial Markets* 5, 31-56.

Amihud, Y., and H. Mendelson, 1986, Asset pricing and the bid-ask spread, *Journal of Financial Economics* 17, 223-249.

Amihud, Y., H. Mendelson, and B. Lauterbach, 1997, Market microstructure and securities values: Evidence from the Tel Aviv Exchange, *Journal of Financial Economics* 45, 365-390.

Atkins, A., and E. Dyl, 1997, Market structure and reported trading volume: NASDAQ versus the NYSE, *Journal of Financial Research* 20, 291-304.

Baker, M., and J. Stein, 2004, Market liquidity as a sentiment indicator, *Journal of Financial Markets* 7, 271-299.

Bessembinder, H., and I. Kalcheva, 2006, Liquidity biases in asset pricing tests, working paper, University of Utah.

Black, F., Jensen, M., and M. Scholes, 1972, The capital asset pricing model: Some empirical tests, In: Jensen, M. (Ed.), *Studies in the Theory of Capital Markets*, Praeger Publishers, New York.

Brennan, M., Chordia, T., and A. Subrahmanyam, 1996, Cross-sectional determinants of expected returns, In: Lehmann, M. (Ed.), *In honor of Fischer Black*, Oxford University Press.

Brennan, M., Chordia, T., and A. Subrahmanyam, 1998, Alternative factor specifications, security characteristics, and the cross-section of expected stock returns, cross-sectional determinants of expected returns, *Journal of Financial Economics* 49, 345–373.

Brennan, M., N. Jegadeesh, and B. Swaminathan, 1993, Investment analysis and the adjustment of stock prices to common information, *Review of Financial Studies* 6, 799-824.

Brennan, M., and A. Subrahmanyam, 1995, Investment analysis and price formation in securities markets, *Journal of Financial Economics* 38, 361-381.

Brennan, M., and A. Subrahmanyam, 1996, Market microstructure and asset pricing: On the compensation for illiquidity in stock returns, *Journal of Financial Economics* 41, 441-464.

Brennan, M., and A. Wang, 2006, Asset pricing and mispricing, working paper, University of California at Irvine.

Chordia, T., S. Huh, and A. Subrahmanyam, 2005, The cross-section of expected trading activity, forthcoming, *Review of Financial Studies.*

Chordia, T., R. Roll, and A. Subrahmanyam, 2002, Order imbalance, liquidity, and market returns, *Journal of Financial Economics* 65, 111-130.

Chordia, T., A. Sarkar, and A. Subrahmanyam, 2005, An empirical analysis of stock and bond market liquidity, *Review of Financial Studies* 18, 85-129.

Chordia, T., A. Subrahmanyam, and V. Anshuman, 2001, Trading activity and expected stock returns, *Journal of Financial Economics* 59, 3-32.

Connor, G., 1984, A unified beta pricing theory, *Journal of Economic Theory* 34, 13-31.

Cooper, S, J. Groth, and W. Avera, 1985, Liquidity, exchange listing, and common stock performance, *Journal of Economics and Business* 37, 19-33.

Eisfeldt, A., 2004, Endogenous liquidity in asset markets, *Journal of Finance* 59, 1-30.

Emmons, W., and F. Schmid, 2000, The Asian crisis and the exposure of large U.S. firms, Federal Reserve Bank of St. Louis Review, January 2000, 15-34.

Fama, E., and K. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-466.

Fama, E., and K. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, E., and K. French, 1996, Multifactor explanations for asset pricing anomalies, *Journal of Finance 51*, 55-84.

Fama, E., and J. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607-636.

Glosten, L., 1989, Insider trading, liquidity, and the role of the monopolist specialist, *Journal of Business* 62, 211-235.

Glosten, L., and L. Harris, 1988, Estimating the components of the bid-ask spread, *Journal of Financial Economics* 21, 123-142.

Harris, L., 1995, Minimum price variations, discrete bid-ask spreads, and quotation sizes, *Review of Financial Studies* 7, 149-178.

Hasbrouck, J., 1991, Measuring the information content of stock trades, *Journal of Finance* 56, 179-207.

Hasbrouck, J., 1999, The dynamics of discrete bid and ask quotes, *Journal of Finance* 54, 2109-2142.

Hasbrouck, J., 2005, Trading costs and returns for US equities: The evidence from daily data, working paper, New York University.

Hasbrouck, J., 2006, Trading costs and returns for US equities: Estimating effective costs from daily data, working paper, New York University.

Jacoby, G., D. Fowler, and A. Gottesman, 2000, The capital asset pricing model and the liquidity effect: A theoretical approach, *Journal of Financial Markets* 3, 69-81.

Jegadeesh, N., and S. Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48,65-92.

Johnson, T., 2005, Dynamic liquidity in endowment economies, *Journal of Financial Economics*, 80, 531-562.

Jones, C., 2002, A century of stock market liquidity and trading costs, working paper, Columbia University.

Kyle, A., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315-1335.

Kumar, P., and D. Seppi, 1994, Information and index arbitrage, *Journal of Business* 67, 481-509.

Lee, C., and M. Ready, 1991, Inferring trade direction from intraday data, *Journal of Finance* 46, 733-747.

Lehmann, B., 1990, Residual risk revisited, *Journal of Econometrics* 45, 71-97.

Lehmann, B., 1990, Fads, martingales, and market efficiency, *Quarterly Journal of Economics* 105, 1-28.

Litzenberger, R., and K. Ramaswamy, 1979, Dividends, short-selling restrictions, tax-induced investor clienteles and market equilibrium, *Journal of Financial Economics* 7, 163-196.

Lo, A., and J. Wang, 2000, Trading volume: Definitions, data analysis, and implications of portfolio theory, *Review of Financial Studies* 13, 257-300.

Merton, R., 1980, On estimating the expected rate of return on the market, *Journal of Financial Economics* 8, 323-362.

O'Hara, M., and G. Oldfield, 1986, The microeconomics of market making, *Journal of Financial and Quantitative Analysis* 21, 361-376.

Pástor, L., and R. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 113, 642-685.

Roll, R., 1984, A simple implicit measure of the effective bid-ask spread in an efficient market, *Journal of Finance* 39, 1127-1139.

Sadka, R., 2006, Momentum and post-earnings-announcement drift anomalies: the role of liquidity risk, *Journal of Financial Economics* 80, 309-349.

Shanken, J., 1985, Multibeta CAPM or equilibrium APT? A reply, *Journal of Finance* 40, 1189-1196.

Shanken, J., 1987, Multivariate proxies and arbitrage pricing relations: living with the Roll critique, *Journal of Financial Economics* 18, 91-110.

Spiegel, M., and X. Wang, 2005, Cross-sectional variation in stock returns: liquidity and idiosyncratic risk, working paper, Yale University.

Stoll, H., 1978, The pricing of security dealer services: An empirical study of NASDAQ stocks, *Journal of Finance* 33, 1153-1172.

Stoll, H., 2000, Presidential address: Friction, *Journal of Finance* 55, 1479-1514.

Subrahmanyam, A., 1991, A theory of trading in stock index futures, *Review of Financial Studies 4*, 17-51.

Appendix: Derivation of the Two Theory-based Illiquidity Measures

In this appendix, we derive the illiquidity measure (λ) used in our analysis, assuming that there are many informed traders in Kyle's (1985) setting. We begin our analysis by stating a few standard assumptions that are made in much of the literature on Kyle (1985)-type frameworks. Consider an asset that pays off $\tilde{W} = \overline{W} + \tilde{\delta}$, where $\tilde{W}$ is the liquidation value of the asset (or the common value that all traders assign to it), $\overline{W}$ is the expected value of the asset, and $\tilde{\delta}$ is the innovation in the asset payoff that is normally distributed (with its mean being zero), i.e., $\tilde{\delta} \sim N(0, v_\delta)$. There are N informed traders who observes a signal that is informative about $\tilde{\delta}$. For now, we assume that informed trader i observes a signal with an error, $\tilde{\delta} + \tilde{\varepsilon_i}$ (i=1, 2, 3, ..., N), where $\tilde{\varepsilon_i}'$s are *iid* and normally distributed, i.e., $\tilde{\varepsilon_i} \sim iid\ N(0, v_\varepsilon)$. Informed traders maximize expected profits. There are also uninformed traders who trade randomly, and their total trades, z, are normally distributed, i.e., $z \sim N(0, v_z)$. It is assumed that δ, ε, and z are all independent. Risk-neutral market makers set the prices of assets equal to the expected values of the liquidation values, conditional on information about the quantities traded by other participants. They are competitive and efficient, earning zero expected profits and ensuring that markets clear.

At each auction, trading of an asset occurs in two steps. In the first step, the informed and uninformed traders submit orders simultaneously to a market maker. In the second step, the market maker quotes a price contingent on the *combined* trades (order flows) of both types of traders. The market maker does not observe the *individual* quantities traded by the informed or the uninformed. He does not have any other information than the combined total trades by the two types of traders. Therefore, price fluctuations of an asset are purely a result of order flow innovations.

Suppose that informed trader i conjectures that other informed traders use trading strategies of a form $\gamma(\tilde{\delta} + \tilde{\varepsilon}_j)$., i.e., a trade of informed trader j is given by

$$x_j = \gamma(\tilde{\delta} + \tilde{\varepsilon}_j), \tag{14}$$

and also that for informed trader i is

$$x_i = \gamma(\tilde{\delta} + \tilde{\varepsilon}_i). \tag{15}$$

From the above equations, the combined total trades (order flows), ω, are expressed as a sum of informed and uninformed trades, i.e.,

$$\begin{aligned} \omega &= \{x_i + (N-1)x_j\} + z \\ &= \{x_i + (N-1)\gamma\tilde{\delta} + \gamma\sum_{j\neq i}\tilde{\varepsilon}_j\} + z \qquad (16) \\ &= N\gamma\tilde{\delta} + \gamma\sum_i \tilde{\varepsilon}_i + z. \qquad (17) \end{aligned}$$

The asset price, P, is set by the market maker after he observes ω so that[22]

$$P = E\left[\overline{W} + \tilde{\delta} \mid \omega = \mathrm{N}\gamma\tilde{\delta} + \gamma\sum_{\mathrm{i}}\tilde{\varepsilon}_{\mathrm{i}} + \mathrm{z}\right] \tag{18}$$

$$= \overline{W} + \frac{Cov\left[\tilde{\delta}, \mathrm{N}\gamma\tilde{\delta} + \gamma\sum_{\mathrm{i}}\tilde{\varepsilon}_{\mathrm{i}} + \mathrm{z}\right]}{Var\left[N\gamma\tilde{\delta} + \gamma\sum_i\tilde{\varepsilon}_i + z\right]}\left(N\gamma\tilde{\delta} + \gamma\sum_i\tilde{\varepsilon}_i + z\right). \tag{19}$$

In addition, Kyle (1985) suggests that P should also be a linear function of order flows in a form,

$$\begin{aligned} P &= \overline{W} + \lambda\omega \qquad (20) \\ &= \overline{W} + \lambda\left(N\gamma\tilde{\delta} + \gamma\sum_i\tilde{\varepsilon}_i + z\right), \qquad (21) \end{aligned}$$

where λ is the sensitivity of the asset price to order flows. Also, the profit of informed trader i is expressed as

$$\pi_i = (\tilde{W} - P)x_i. \tag{22}$$

[22]Equation (19) comes from the property of the multivariate normal distribution. Let two random variables, X_1 and X_2, be jointly normally distributed so that $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right]$. Then, we can show that $E[X_1|X_2] = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(X_2 - \mu_2)$, and $Var[X_1|X_2] = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$. For details, see Anderson (1984).

In this setting, we can solve for the equilibrium that satisfies the two conditions: profit maximization by the informed and market efficiency.

In Eq.(20) and Eq.(21), λ is an illiquidity measure and its inverse, $\frac{1}{\lambda}$, is sometimes called the "depth" of the market. If a market is very liquid, one would expect that combined trades, ω, may not affect the asset price very much, and hence the level of λ is low. Our goal is to solve λ as a measure of illiquidity from the equilibrium conditions.

First, informed trader i's problem is:

$$\begin{aligned}
Max \quad &: \quad \mathrm{E}\left[\pi_{\mathrm{i}} \mid \tilde{\delta}+\tilde{\varepsilon}_{\mathrm{i}}\right] \\
&= \quad \mathrm{E}\left[\left\{\overline{\mathrm{W}}+\tilde{\delta}-\overline{\mathrm{W}}-\lambda\left(\mathrm{x_i}+(\mathrm{N}-1)\gamma\tilde{\delta}+\gamma\sum_{\mathrm{j}\neq\mathrm{i}}\tilde{\varepsilon}_{\mathrm{j}}\}+\mathrm{z}\right)\right\}\mathrm{x_i} \ \mid \ \tilde{\delta}+\tilde{\varepsilon}_{\mathrm{i}}\right] \\
&= \quad \left\{E\left[\tilde{\delta}\mid\tilde{\delta}+\tilde{\varepsilon}_i\right]-\lambda x_i-\lambda(N-1)\gamma E\left[\tilde{\delta}\mid\tilde{\delta}+\tilde{\varepsilon}_i\right]\right\}\mathrm{x_i} \\
&= \quad -\lambda x_i^2+x_i\left\{1-\lambda(N-1)\gamma\right\}E\left[\tilde{\delta}\mid\tilde{\delta}+\tilde{\varepsilon}_i\right]. \qquad (23)
\end{aligned}$$

The first order condition of Eq.(23) gives $-2\lambda x_i+\{1-\lambda(N-1)\gamma\}E\left[\tilde{\delta}\mid\tilde{\delta}+\tilde{\varepsilon}_i\right]=0$. Thus,

$$\begin{aligned}
x_i &= \frac{1}{2\lambda}\{1-\lambda(N-1)\gamma\}E\left[\tilde{\delta}\mid\tilde{\delta}+\tilde{\varepsilon}_i\right] \\
&= \frac{1}{2\lambda}\{1-\lambda(N-1)\gamma\}\frac{v_\delta}{v_\delta+v_\varepsilon}(\tilde{\delta}+\tilde{\varepsilon}_i). \qquad (24)
\end{aligned}$$

Therefore, from Eq.(15) and Eq.(24), we have $\gamma=\frac{1}{2\lambda}\{1-\lambda(N-1)\gamma\}\frac{v_\delta}{v_\delta+v_\varepsilon}$, which in turn leads to

$$\gamma=\frac{\frac{v_\delta}{v_\delta+v_\varepsilon}}{\lambda\left\{2+\frac{v_\delta}{v_\delta+v_\varepsilon}(N-1)\right\}}. \qquad (25)$$

Next, from Eq.(19) and Eq.(21), the market efficiency condition is equivalent to

$$\begin{aligned}
\lambda &= \frac{Cov\left[\tilde{\delta},\mathrm{N}\gamma\tilde{\delta}+\gamma\sum_{\mathrm{i}}\tilde{\varepsilon}_{\mathrm{i}}+\mathrm{z}\right]}{Var\left[N\gamma\tilde{\delta}+\gamma\sum_i\tilde{\varepsilon}_i+z\right]} \\
&= \frac{N\gamma v_\delta}{N^2\gamma^2 v_\delta+\gamma^2 N v_\varepsilon+v_z}
\end{aligned}$$

$$= \frac{N v_\delta}{\gamma(N^2 v_\delta + N v_\varepsilon) + \frac{1}{\gamma} v_z}. \tag{26}$$

Plugging Eq.(25) into Eq.(26) gives

$$\lambda = \frac{v_\delta}{(N+1)v_\delta + 2v_\varepsilon} \sqrt{\frac{N(v_\delta + v_\varepsilon)}{v_z}}. \tag{27}$$

Note that initially we assumed informed traders observe a signal with noise, $\tilde{\varepsilon_i}$ (i=1, 2, 3, ..., N). Now suppose there is no noise in the signal so that $v_\varepsilon = 0$. Then, Eq.(27) is reduced to

$$\lambda = \frac{\sqrt{N v_\delta}}{(N+1)\sqrt{v_z}}. \tag{28}$$

In this study, Eq.(27) and Eq.(28) are used as the primary basis of our two illiquidity measures.

Table I

Descriptive Statistics for the Input Variables of the Two Illiquidity Measures

This table reports descriptive statistics for the input variables of our two theoretically derived illiquidity measures, *ILLIQ_1* = $\frac{N^{0.5}std(R)}{(N+1)std(z)}$, and *ILLIQ_2* = $P^{-1}\frac{v_\delta}{(N+1)v_\delta+2v_\varepsilon}\sqrt{\frac{N(v_\delta+v_\varepsilon)}{v_z}}$, where each input variable is defined as follows: *N*: the number of informed traders; *std(R)*: standard deviation of returns; *std(z)*: standard deviation of noise trades; *P*: asset price; v_δ: variance of payoff innovations; v_ε: variance of signal innovations; and v_z: variance of noise trades. The above original input variables are in turn proxied by the variables shown in the second column of the table below. Each proxy variable is defined as follows: *ANA*: one plus the number of analysts following a firm; *STD(RET)*: standard deviation of daily returns in the previous month; *AVG(DVOL)*: average of daily dollar volume (in $million) in the previous month; *P*: month-end stock price of the previous month; *EVOLA-sqr*: squared value of earnings volatility (EVOLA), which is defined as standard deviation of EPSs from the most recent eight quarters; *ESURP-sqr*: squared value of earnings surprise (ESURP), which is defined as the absolute value of the current earnings per share (EPS) minus the EPS from four quarters ago; and *AVG(DVOL)-sqr*: squared value of AVG(DVOL). The sample periods are the past 372 months (31 years: 197201-200212) for NYSE/AMEX stocks and the 240 months (20 years: 198301-200212) for NASDAQ stocks. The values of each statistic are first calculated cross-sectionally each month and then the time-series averages of those values are reported here. The average numbers of component stocks used each month for *ILLIQ_1* and *ILLIQ_2* in Panel A (NYSE/AMEX stocks) are 1,845.1 and 1,683.1, respectively. Those in Panel B (NASDAQ stocks) are 2,667.5 and 1,967.0, respectively.

Panel A: NYSE/AMEX

Original Input Variables	Proxy Variables Employed	For ILLIQ_1			For ILLIQ_2		
		Mean	Median	STD	Mean	Median	STD
N	ANA	5.52	2.46	6.34	5.73	2.67	6.47
std(R)	STD(RET)	0.027	0.022	0.025	-	-	-
std(z)	AVG(DVOL)	5.26	0.53	16.88	-	-	-
P	P	-	-	-	31.31	18.06	379.92
v_δ	EVOLA-sqr	-	-	-	173.80	0.06	6298.46
v_ε	ESURP-sqr	-	-	-	69.68	0.02	2920.24
v_z	AVG(DVOL)-sqr	-	-	-	933.97	0.75	11061.09

Panel B: NASDAQ

Original Input Variables	Proxy Variables Employed	For ILLIQ_1			For ILLIQ_2		
		Mean	Median	STD	Mean	Median	STD
N	ANA	3.15	1.37	4.01	3.70	1.79	4.56
std(R)	STD(RET)	0.044	0.036	0.039	-	-	-
std(z)	AVG(DVOL)	3.61	0.12	27.86	-	-	-
P	P	-	-	-	14.16	9.76	14.80
v_δ	EVOLA-sqr	-	-	-	1781.81	0.03	96989.02
v_ε	ESURP-sqr	-	-	-	3.22	0.01	73.02
v_z	AVG(DVOL)-sqr	-	-	-	2639.84	0.04	53269.12

Table II
Descriptive Statistics for Key Variables

This table reports descriptive statistics (Mean, Median, Standard Deviation (STD), Coefficient of Variation (CV), Skewness, and Kurtosis) for the key variables to be used on the right-hand side in the Fama-MacBeth (1973) cross-sectional regressions. Each variable is defined as follows: *ILLIQ_1*: the first illiquidity measure defined as in Table I; *ILLIQ_2*: the second illiquidity measure defined as in Table I; *MV*: market value defined as the month-end stock price times the number of shares outstanding (in $million); *SIZE*: natural logarithm of MV; *BM_Raw*: the untrimmed book-to-market ratio defined as BV/MV, where the book value (BV) is common equity plus deferred taxes (in $million); *BM_Trim*: the trimmed book-to-market ratio, where BM_Raw values greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *BTM*: natural logarithm of BM_Trim; *MOM1*: compounded holding period return of a stock over the most recent 3 months (from month *t-1* to month *t-3*); *MOM2*: compounded holding period return over the next recent 3 months (from month *t-4* to month *t-6*); *MOM3*: compounded holding period return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period return over the 3 months from month *t-9* to month *t-12*. The sample periods are the past 372 months (31 years: 197201-200212) for NYSE/AMEX stocks and the 240 months (20 years: 198301-200212) for NASDAQ stocks. The values of each statistic are first calculated cross-sectionally each month and then the time-series averages of those values are reported here. The average number of component stocks used in a month to compute the statistics for each variable in Panel A (NYSE/AMEX stocks) is 1,845.1 (except that it is 1,683.1 for ILLIQ_2), while that in Panel B (NASDAQ stocks) is 2,667.5 (except that it is 1,967.0 for ILLIQ_2).

Panel A: NYSE/AMEX

Variables	Mean	Median	STD	CV	Skewness	Kurtosis
ILLIQ_1	2.14	0.11	14.92	686.22	17.29	441.55
$[\text{ILLIQ\_1}]^{1/2}$	0.60	0.24	1.12	199.87	6.12	74.05
ILLIQ_2	9.34	0.04	161.08	1229.57	24.13	730.84
$[\text{ILLIQ\_2}]^{1/2}$	0.73	0.16	2.38	329.14	10.90	200.60
MV	1722.92	245.01	6140.44	368.03	12.05	215.85
SIZE	5.15	5.13	1.98	39.10	0.09	-0.42
BM_Raw	1.02	0.87	0.78	78.08	4.92	80.07
BM_Trim	1.01	0.87	0.69	69.69	2.10	8.40
BTM	-0.27	-0.20	0.71	-387.46	-0.81	2.66
MOM1	0.011	0.016	0.191	-872.70	-0.27	5.55
MOM2	0.012	0.016	0.187	-1599.66	-0.18	4.94
MOM3	0.013	0.016	0.184	257.10	-0.10	4.41
MOM4	0.014	0.017	0.183	38.04	-0.06	4.14

Panel B: NASDAQ

Variables	Mean	Median	STD	CV	Skewness	Kurtosis
ILLIQ_1	10.25	0.18	271.22	1154.10	29.12	1130.67
$[\text{ILLIQ\_1}]^{1/2}$	0.96	0.41	2.20	211.36	10.49	243.27
ILLIQ_2	44.63	0.06	1272.14	1503.41	27.50	937.70
$[\text{ILLIQ\_2}]^{1/2}$	1.02	0.24	4.28	353.70	13.62	315.04
MV	323.01	47.56	2770.90	570.96	20.12	624.91
SIZE	3.84	3.76	1.65	43.49	0.24	-0.05
BM_Raw	0.75	0.58	0.71	93.08	4.23	49.95
BM_Trim	0.74	0.58	0.65	86.47	2.40	9.96
BTM	-0.66	-0.55	0.92	-147.06	-0.85	2.14
MOM1	-0.013	-0.001	0.289	-1496.29	-0.27	4.71
MOM2	-0.008	0.001	0.284	-11200.06	-0.16	4.05
MOM3	-0.004	0.004	0.280	64.58	-0.09	3.83
MOM4	-0.003	0.004	0.278	745.88	-0.06	3.78

Table III
Correlations between Explanatory Variables

The lower triangle shows the average correlations between the key variables for NYSE/AMEX stocks over the 372 months (31 years: 197201-200212), and the upper triangle shows those for NASDAQ stocks over the 240 months (20 years: 198301-200212) The cross-sectional correlation coefficients are first calculated each month and then the time-series averages of those values over the sample periods are reported here. The definitions of the variables are as follows: *ILLIQ_1*: the first illiquidity measure defined as in Table I; *ILLIQ_2*: the second illiquidity measure defined as in Table I; *SIZE*: natural logarithm of MV, which is defined as the month-end stock price times the number of shares outstanding (in $million); *BTM*: natural logarithm of BM_Trim, which is the trimmed book-to-market ratio, where book-to-market ratios greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *MOM1*: compounded holding period return of a stock over the most recent 3 months (from month *t-1* to month *t-3*); *MOM2*: compounded holding period return over the next recent 3 months (from month *t-4* to month *t-6*); *MOM3*: compounded holding period return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period return over the 3 months from month *t-9* to month *t-12*. The average number of component stocks used in a month for NYSE/AMEX stocks is 1,845.1 (except that it is 1,683.1 for ILLIQ_2), while that for NASDAQ stocks is 2,667.5 (except that it is 1,967.0 for ILLIQ_2).

Average Correlations: NYSE/AMEX (Lower Triangle) and NASDAQ (Upper Triangle)

	ILLIQ_1	$[ILLIQ\_1]^{1/2}$	ILLIQ_2	$[ILLIQ\_2]^{1/2}$	SIZE	BTM	MOM1	MOM2	MOM3	MOM4
ILLIQ_1	1	0.813	0.464	0.538	-0.182	0.072	-0.055	-0.049	-0.040	-0.036
$[ILLIQ\_1]^{1/2}$	0.846	1	0.412	0.642	-0.501	0.174	-0.127	-0.114	-0.103	-0.099
ILLIQ_2	0.639	0.533	1	0.860	-0.154	0.065	-0.066	-0.048	-0.046	-0.036
$[ILLIQ\_2]^{1/2}$	0.726	0.794	0.853	1	-0.393	0.163	-0.134	-0.105	-0.103	-0.092
SIZE	-0.302	-0.617	-0.184	-0.445	1	-0.247	0.176	0.173	0.172	0.171
BTM	0.091	0.184	0.064	0.167	-0.290	1	0.089	0.081	0.045	-0.009
MOM1	-0.088	-0.139	-0.074	-0.137	0.122	0.043	1	0.019	0.045	0.037
MOM2	-0.079	-0.128	-0.059	-0.115	0.120	0.039	0.015	1	0.014	0.040
MOM3	-0.072	-0.124	-0.056	-0.113	0.121	0.009	0.047	0.012	1	0.007
MON4	-0.073	-0.124	-0.054	-0.112	0.122	-0.034	0.052	0.044	0.006	1

Table IV

Average Values of Monthly Return, Size, and Illiquidity for the 25 Portfolios Formed on Illiquidity and Size

This table reports the average values of monthly stock return (Panel A), firm size (Panel B), and illiquidity (Panel C) for the 25 portfolios formed on illiquidity and size. ILLIQ_1 is the illiquidity measure defined as in Table I and size is the market value (MV) of a firm (in $million). The component stocks are first split into 5 portfolios (with the equal number of stocks) after being sorted in an ascending order by ILLIQ_1 and then each of the 5 portfolios is again split into another 5 portfolios after being sorted by size, resulting in 25 portfolios each month. The average values of return, size, and illiquidity are computed each month for each of the 25 portfolios, and then the time-series averages of the 3 variables over the sample period are reported in each panel of the table. Panel A also contains t-statistics (italicized), in addition to the average returns. The row High-Low contains the return differential between the portfolios with the highest and lowest values of the illiquidity measure (with t-statistics for the null hypothesis that difference equals zero in the following row). The sample periods are the past 372 months (31 years: 197201-200212) for NYSE/AMEX stocks and the 240 months (20 years: 198301-200212) for NASDAQ stocks. The average number of component stocks in each portfolio in a month for NYSE/AMEX stocks is 73.4, while that for NASDAQ stocks is 106.1.

Panel A: Average Return

	Size Group (NYSE/AMEX)					Size Group (NASDAQ)				
ILLIQ_1 Group	1 small	2	3	4	5 big	1 small	2	3	4	5 big
1 low	0.0118	0.0116	0.0104	0.0096	0.0093	0.0110	0.0100	0.0089	0.0083	0.0081
	3.94	*4.29*	*4.07*	*3.95*	*3.91*	*2.85*	*2.27*	*2.28*	*2.19*	*2.36*
2	0.0120	0.0124	0.0118	0.0103	0.0103	0.0110	0.0101	0.0076	0.0080	0.0079
	3.45	*4.45*	*4.09*	*3.72*	*4.21*	*1.90*	*2.17*	*2.02*	*2.29*	*3.22*
3	0.0120	0.0125	0.0119	0.0110	0.0103	0.0131	0.0101	0.0078	0.0089	0.0088
	3.35	*3.72*	*3.31*	*4.03*	*3.80*	*2.28*	*2.10*	*1.61*	*2.16*	*2.61*
4	0.0136	0.0127	0.0124	0.0112	0.0104	0.0207	0.0107	0.0078	0.0091	0.0095
	3.54	*3.54*	*3.79*	*3.56*	*3.66*	*3.31*	*2.27*	*1.80*	*1.79*	*2.37*
5 high	0.0291	0.0127	0.0125	0.0114	0.0101	0.0640	0.0244	0.0141	0.0090	0.0089
	5.27	*2.80*	*2.55*	*2.40*	*2.47*	*8.70*	*4.58*	*3.03*	*2.02*	*1.27*
High-Low	0.0173	0.0011	0.0021	0.0018	0.0008	0.0530	0.0144	0.0052	0.0007	0.0008
	3.58	*1.97*	*1.66*	*0.92*	*0.61*	*8.25*	*2.70*	*1.88*	*0.92*	*1.35*

Panel B: Average Size ($million)

	Size Group (NYSE/AMEX)					Size Group (NASDAQ)				
ILLIQ_1 Group	1 small	2	3	4	5 big	1 small	2	3	4	5 big
1 low	1194.29	2179.59	3397.65	5898.20	23237.69	126.85	270.57	437.02	769.11	5160.62
2	317.96	528.95	739.70	1034.03	2008.78	46.09	79.50	113.26	164.76	335.73
3	104.28	172.21	240.05	338.44	656.28	18.69	32.40	46.13	66.94	150.88
4	30.00	52.66	77.31	112.35	238.31	7.93	14.21	20.77	30.49	69.87
5 high	4.31	9.46	15.65	24.84	61.47	2.36	4.80	7.56	11.97	29.73

Panel C: Average ILLIQ_1

	Size Group (NYSE/AMEX)					Size Group (NASDAQ)				
ILLIQ_1 Group	1 small	2	3	4	5 big	1 small	2	3	4	5 big
1 low	0.008	0.007	0.005	0.003	0.001	0.006	0.008	0.006	0.004	0.002
2	0.037	0.033	0.029	0.024	0.019	0.056	0.047	0.041	0.037	0.033
3	0.145	0.126	0.114	0.102	0.091	0.245	0.216	0.196	0.181	0.166
4	0.683	0.574	0.485	0.418	0.357	1.106	0.965	0.891	0.826	0.752
5 high	31.766	8.288	4.703	3.175	2.436	173.772	32.932	15.874	20.877	7.853

Table V

Results of Monthly Cross-sectional Regressions: with ILLIQ_1 and [ILLIQ_1]$^{1/2}$ for NYSE/AMEX and NASDAQ Stocks

This table reports the monthly Fama-MacBeth (1973)-type cross-sectional regressions using ILLIQ_1 (in Panel A) and its square-root values (in Panel B) for NYSE/AMEX stocks over the 372 months (31 years: 197201-200212) and for NASDAQ stocks over the 240 months (20 years: 198301-200212). The dependent variables (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2) are all one-month leading values (no contemporaneous regressors are used). The definitions of the variables are as follows: *EXSRET*: the monthly risk-unadjusted excess return, i.e., the monthly return less the risk-free rate proxied by the one-month T-bill rate; *FF3-adj EXSRET1*: the risk-adjusted excess return using the Fama-French (FF) 3 factors, i.e., the constant term plus the residual from the time-series regression of the excess return on the FF 3 factors using the *entire* sample range of the data; *FF3-adj EXSRET2*: the risk-adjusted excess return using the Fama-French (FF) 3 factors with factor loadings being estimated from the 5-year rolling regressions, i.e., R_i^* computed each month with the current month data from the equation, $R_i^* = (R_i - R_f) - [\hat{\beta}_1 MKT + \hat{\beta}_2 SMB + \hat{\beta}_3 HML]$, after the factor loadings ($\alpha, \beta_1, \beta_2, \beta_3$) are first estimated for *each month* using the time-series data of the past 60 months in the monthly regression, $R_i - R_f = \alpha + \beta_1 MKT + \beta_2 SMB + \beta_3 HML + \varepsilon$, where R_i, R_f, and R_m are the individual stock return, the risk-free rate, and the market index return, respectively, while *MKT*, *SMB*, and HML are FF 3 factors; *ILLIQ_1*: the first illiquidity measure defined as $\frac{N^{0.5} std(R)}{(N+1) std(z)}$, where *N* is the number of informed traders, *std(R)* is the standard deviation of returns, and *std(z)* is the standard deviation of noise trades (the original input variables are proxied by the variables as shown in Table I); *SIZE*: natural logarithm of MV, which is defined as the month-end stock price times the number of shares outstanding (in $million); *BTM*: natural logarithm of BM_Trim, which is the trimmed book-to-market ratio, where book-to-market ratios greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *MOM1*: compounded holding period return of a stock over the most recent 3 months (from month *t-1* to month *t-3*); *MOM2*: compounded holding period return over the next recent 3 months (from month *t-4* to month *t-6*); *MOM3*: compounded holding period return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period return over the 3 months from month *t-9* to month *t-12*. The average number of component stocks used in the monthly regressions for NYSE/AMEX stocks is 1,845.1, while that for NASDAQ stocks is 2,667.5. The values in the first row for each explanatory variable are the time-series averages of coefficients obtained from the month-by-month cross-sectional regressions, and the values italicized in the second row of each variable are t-statistics computed based on Fama-MacBeth (1973). The coefficients are all multiplied by 100. *Avg R-sqr* is the average of adjusted R-squared. *Avg Obs* is the monthly average number of companies used in the cross-sectional regressions. Coefficients significantly different from zero at the significance levels of 1% and 5% are indicated by ** and *, respectively.

(Table V continued: Panel A)

Panel A: with ILLIQ_1												
Explanatory	NYSE/AMEX						NASDAQ					
Variables	EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2		EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2	
Intercept	0.994	**	0.187		0.239		2.337	**	1.694	**	1.759	**
	2.65		*1.19*		*1.40*		*4.75*		*4.58*		*4.50*	
ILLIQ_1	0.044	**	0.039	**	0.041	**	0.035	**	0.033	**	0.033	**
	3.55		*3.18*		*3.12*		*3.94*		*3.97*		*3.87*	
SIZE	-0.096	*	-0.053	*	-0.053	*	-0.351	**	-0.325	**	-0.325	**
	-2.15		*-2.19*		*-2.08*		*-4.25*		*-4.43*		*-4.17*	
BTM	0.223	**	0.161	**	0.102	*	0.323	**	0.341	**	0.308	**
	3.17		*3.40*		*2.27*		*2.87*		*4.34*		*3.37*	
MOM1	0.863	**	0.938	**	0.907	**	0.324		0.431		0.333	
	2.71		*3.63*		*2.94*		*0.93*		*1.66*		*0.87*	
MOM2	1.177	**	1.144	**	1.043	**	0.677	*	0.495	*	0.492	
	4.31		*5.12*		*3.91*		*2.35*		*2.18*		*1.61*	
MOM3	1.435	**	1.373	**	1.112	**	0.613	*	0.473	*	0.294	
	5.47		*6.04*		*4.10*		*2.54*		*2.20*		*0.95*	
MOM4	1.006	**	0.961	**	0.720	**	0.394		0.343		0.209	
	4.76		*4.97*		*2.77*		*1.83*		*1.90*		*0.75*	
Avg R-sqr	0.049		0.029		0.032		0.034		0.023		0.028	
Avg Obs	1845.1		1843.4		1797.7		2663.8		2658.9		2406.3	

(Table V continued: Panel B)

Panel B: with Square Root of ILLIQ_1

Explanatory Variables	NYSE/AMEX EXSRET		NYSE/AMEX FF3-adj EXSRET1		NYSE/AMEX FF3-adj EXSRET2		NASDAQ EXSRET		NASDAQ FF3-adj EXSRET1		NASDAQ FF3-adj EXSRET2	
Intercept	0.635		-0.121		-0.054		1.817	**	1.189	**	1.219	**
	1.72		*-0.91*		*-0.36*		*3.50*		*3.20*		*3.06*	
$[ILLIQ\_1]^{1/2}$	0.315	**	0.274	**	0.274	**	0.326	**	0.310	**	0.325	**
	3.78		*3.41*		*3.20*		*5.02*		*5.54*		*5.38*	
SIZE	-0.051		-0.014		-0.017		-0.268	**	-0.245	**	-0.239	**
	-1.23		*-0.76*		*-0.83*		*-3.21*		*-3.33*		*-3.04*	
BTM	0.221	**	0.158	**	0.100	*	0.305	**	0.325	**	0.291	**
	3.16		*3.36*		*2.22*		*2.73*		*4.14*		*3.18*	
MOM1	0.906	**	0.975	**	0.943	**	0.376		0.476		0.381	
	2.86		*3.78*		*3.07*		*1.08*		*1.83*		*1.00*	
MOM2	1.225	**	1.190	**	1.085	**	0.705	*	0.524	*	0.528	
	4.51		*5.35*		*4.07*		*2.45*		*2.30*		*1.72*	
MOM3	1.474	**	1.411	**	1.151	**	0.630	**	0.487	*	0.310	
	5.65		*6.25*		*4.25*		*2.61*		*2.26*		*1.01*	
MOM4	1.034	**	0.983	**	0.739	**	0.411		0.353	*	0.222	
	4.91		*5.09*		*2.85*		*1.92*		*1.96*		*0.80*	
Avg R-sqr	0.050		0.029		0.033		0.034		0.022		0.028	
Avg Obs	1845.1		1843.4		1797.7		2663.8		2658.9		2406.3	

Table VI

Results of Monthly Cross-sectional Regressions: with ILLIQ_2 and [ILLIQ_2]$^{1/2}$ for NYSE/AMEX and NASDAQ Stocks

This table reports the monthly Fama-MacBeth (1973)-type cross-sectional regressions using ILLIQ_2 (in Panel A) and its square-root values (in Panel B) for NYSE/AMEX stocks over the 372 months (31 years: 197201-200212) and for NASDAQ stocks over the 240 months (20 years: 198301-200212). The dependent variables (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2) are all one-month leading values (no contemporaneous regressors are used). The definitions of the variables are as follows: *EXSRET*: the monthly risk-unadjusted excess return, i.e., the monthly return less the risk-free rate proxied by the one-month T-bill rate; *FF3-adj EXSRET1*: the risk-adjusted excess return using the Fama-French (FF) 3 factors, i.e., the constant term plus the residual from the time-series regression of the excess return on the FF 3 factors using the *entire* sample range of the data; *FF3-adj EXSRET2*: the risk-adjusted excess return using the Fama-French (FF) 3 factors with factor loadings being estimated from the 5-year rolling regressions, i.e., R_i^* computed each month with the current month data from the equation, $R_i^* = (R_i - R_f) - [\hat{\beta}_1 MKT + \hat{\beta}_2 SMB + \hat{\beta}_3 HML]$, after the factor loadings ($\alpha, \beta_1, \beta_2, \beta_3$) are first estimated for *each month* using the time-series data of the past 60 months in the monthly regression, $R_i - R_f = \alpha + \beta_1 MKT + \beta_2 SMB + \beta_3 HML + \varepsilon$, where R_i, R_f, and R_m are the individual stock return, the risk-free rate, and the market index return, respectively, while *MKT*, *SMB*, and HML are FF 3 factors; *ILLIQ_2*: the second illiquidity measure defined as $P^{-1} \dfrac{v_\delta}{(N+1)v_\delta + 2v_\varepsilon} \sqrt{\dfrac{N(v_\delta + v_\varepsilon)}{v_z}}$, where *P* is the asset price, *N* is the number of informed traders, v_δ is the variance of payoff innovations, v_ε is the variance of signal innovations, and v_z is the variance of noise trades (the original input variables are proxied by the variables as shown in Table I); *SIZE*: natural logarithm of MV, which is defined as the month-end stock price times the number of shares outstanding (in $million); *BTM*: natural logarithm of BM_Trim, which is the trimmed book-to-market ratio, where book-to-market ratios greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *MOM1*: compounded holding period return of a stock over the most recent 3 months (from month *t-1* to month *t-3*); *MOM2*: compounded holding period return over the next recent 3 months (from month *t-4* to month *t-6*); *MOM3*: compounded holding period return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period return over the 3 months from month *t-9* to month *t-12*. The average number of component stocks used in the monthly regressions for NYSE/AMEX stocks is 1,683.1, while that for NASDAQ stocks is 1,967.0. The values in the first row for each explanatory variable are the time-series averages of coefficients obtained from the month-by-month cross-sectional regressions, and the values italicized in the second row of each variable are t-statistics computed based on Fama-MacBeth (1973). The coefficients are all multiplied by 100. *Avg R-sqr* is the average of adjusted R-squared. *Avg Obs* is the monthly average number of companies used in the cross-sectional regressions. Coefficients significantly different from zero at the significance levels of 1% and 5% are indicated by ** and *, respectively.

(Table VI continued: Panel A)

Panel A: with ILLIQ_2						
Explanatory	NYSE/AMEX			NASDAQ		
Variables	EXSRET	FF3-adj EXSRET1	FF3-adj EXSRET2	EXSRET	FF3-adj EXSRET1	FF3-adj EXSRET2
Intercept	1.120 **	0.295	0.356 *	1.937 **	1.257 **	1.318 **
	2.91	*1.74*	*1.97*	*3.97*	*3.54*	*3.47*
ILLIQ_2	0.010 **	0.008 *	0.009 *	0.008	0.004	0.004
	2.84	*2.37*	*2.45*	*1.04*	*0.55*	*0.54*
SIZE	-0.112 *	-0.064 *	-0.068 *	-0.242 **	-0.220 **	-0.223 **
	-2.42	*-2.44*	*-2.46*	*-3.09*	*-3.20*	*-2.98*
BTM	0.184 **	0.134 **	0.073	0.366 **	0.361 **	0.349 **
	2.59	*2.74*	*1.58*	*2.98*	*4.15*	*3.60*
MOM1	0.806 *	0.847 **	0.874 **	0.812 *	0.876 **	0.817 *
	2.51	*3.26*	*2.87*	*2.08*	*2.91*	*2.05*
MOM2	1.068 **	1.054 **	1.004 **	0.960 **	0.795 **	0.707 *
	3.90	*4.70*	*3.82*	*3.05*	*2.99*	*2.14*
MOM3	1.362 **	1.301 **	1.071 **	0.699 **	0.612 **	0.323
	5.15	*5.66*	*3.91*	*2.73*	*2.58*	*1.04*
MOM4	0.954 **	0.930 **	0.652 *	0.568 *	0.575 **	0.440
	4.33	*4.57*	*2.43*	*2.20*	*2.59*	*1.47*
Avg R-sqr	0.050	0.029	0.032	0.040	0.028	0.032
Avg Obs	1683.1	1682.0	1667.6	1967.0	1961.6	1907.3

(Table VI continued: Panel B)

Panel B: with Square Root of ILLIQ_2						
Explanatory	NYSE/AMEX			NASDAQ		
Variables	EXSRET	FF3-adj EXSRET1	FF3-adj EXSRET2	EXSRET	FF3-adj EXSRET1	FF3-adj EXSRET2
Intercept	0.903 *	0.111	0.191	1.670 **	1.064 **	1.096 **
	2.41	*0.76*	*1.21*	*3.46*	*3.11*	*3.01*
$[ILLIQ\_2]^{1/2}$	0.145 **	0.124 **	0.121 **	0.150 **	0.100 *	0.117 *
	3.74	*3.30*	*3.06*	*2.61*	*1.97*	*2.10*
SIZE	-0.084	-0.041	-0.047 *	-0.200 **	-0.190 **	-0.189 **
	-1.93	*-1.82*	*-1.98*	*-2.65*	*-2.86*	*-2.62*
BTM	0.176 *	0.125 **	0.066	0.352 **	0.351 **	0.338 **
	2.49	*2.60*	*1.44*	*2.88*	*4.01*	*3.46*
MOM1	0.866 **	0.899 **	0.921 **	0.863 *	0.914 **	0.857 *
	2.72	*3.47*	*3.03*	*2.21*	*3.05*	*2.17*
MOM2	1.136 **	1.117 **	1.057 **	0.998 **	0.831 **	0.743 *
	4.18	*5.01*	*4.03*	*3.14*	*3.07*	*2.22*
MOM3	1.429 **	1.365 **	1.134 **	0.709 **	0.615 **	0.328
	5.47	*6.00*	*4.16*	*2.76*	*2.58*	*1.05*
MOM4	0.976 **	0.947 **	0.664 *	0.582 *	0.584 **	0.453
	4.48	*4.71*	*2.49*	*2.26*	*2.65*	*1.52*
Avg R-sqr	0.050	0.030	0.033	0.040	0.027	0.032
Avg Obs	1683.1	1682.0	1667.6	1967.0	1961.6	1907.3

Table VII

Results of Monthly Cross-sectional Regressions Using Quote Mid-point Returns: with $[ILLIQ\_1]^{1/2}$ and $[ILLIQ\_2]^{1/2}$ for NYSE/AMEX and NASDAQ Stocks

This table reports the monthly Fama-MacBeth (1973)-type cross-sectional regressions using quote *mid-point returns* with $[ILLIQ\_1]^{1/2}$ (in Panel A) and $[ILLIQ\_2]^{1/2}$ (in Panel B) for NYSE/AMEX stocks over the 180 months (15 years: 198801-200212) and for NASDAQ stocks over the 240 months (20 years: 198301-200212). Monthly quote mid-point returns for NYSE/AMEX stocks are calculated based on the first (open) quote mid-point and the last (close) quote mid-point (open-to-close mid-points) within each month from 1988 to 2002, while those for NASDAQ stocks are calculated based on the mid-points of the monthly closing quotes (close-to-close mid-points) from 1983 to 2002. The dependent variables (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2) are all one-month leading values (no contemporaneous regressors are used). The definitions of the variables are as follows: *EXSRET*: the monthly risk-unadjusted excess return, i.e., the monthly return less the risk-free rate proxied by the one-month T-bill rate; *FF3-adj EXSRET1*: the risk-adjusted excess return using the Fama-French (FF) 3 factors, i.e., the constant term plus the residual from the time-series regression of the excess return on the FF 3 factors using the *entire* sample range of the data; *FF3-adj EXSRET2*: the risk-adjusted excess return using the Fama-French (FF) 3 factors with factor loadings being estimated from the 5-year rolling regressions, i.e., R_i^* computed each month with the current month data from the equation, $R_i^* = (R_i - R_f) - [\hat{\beta}_1 MKT + \hat{\beta}_2 SMB + \hat{\beta}_3 HML]$, after the factor loadings ($\alpha, \beta_1, \beta_2, \beta_3$) are first estimated for *each month* using the time-series data of the past 60 months in the monthly regression, $R_i - R_f = \alpha + \beta_1 MKT + \beta_2 SMB + \beta_3 HML + \varepsilon$, where R_i, R_f, and R_m are the individual stock return, the risk-free rate, and the market index return, respectively, while *MKT*, *SMB*, and HML are FF 3 factors; *ILLIQ_1* and *ILLIQ_2*: the two illiquidity measures defined in Table I; *SIZE*: natural logarithm of MV, which is defined as the month-end stock price times the number of shares outstanding (in $million); *BTM*: natural logarithm of BM_Trim, which is the trimmed book-to-market ratio, where book-to-market ratios greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *MOM1*: compounded holding period mid-point return of a stock over the most recent 3 months (from month *t-1* to month *t-3*); *MOM2*: compounded holding period mid-point return over the next recent 3 months (from month *t-4* to month *t-6*); *MOM3*: compounded holding period mid-point return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period mid-point return over the 3 months from month *t-9* to month *t-12*. The average numbers of component stocks used in the monthly regressions for NYSE/AMEX stocks are 1680.7-1748.8, while those for NASDAQ stocks are 2656.9-2693.5 [note that the model specification for FF3-adj EXSRET2 in the table loses the first 2 year observations in the sample because of rolling estimation for the FF3 factor loadings (using the past 60 months or at least 24 months)]. The values in the first row for each explanatory variable are the time-series averages of coefficients obtained from the month-by-month cross-sectional regressions, and the values italicized in the second row of each variable are t-statistics computed based on Fama-MacBeth (1973). The coefficients are all multiplied by 100. *Avg R-sqr* is the average of adjusted R-squared. *Avg Obs* is the monthly average number of companies used in the cross-sectional regressions. Coefficients significantly different from zero at the significance levels of 1% and 5% are indicated by ** and *, respectively.

(Table VII continued: Panel A)

Panel A: with Square Root of ILLIQ_1 for Quote Mid-point Returns

Explanatory Variables	NYSE/AMEX EXSRET		NYSE/AMEX FF3-adj EXSRET1		NYSE/AMEX FF3-adj EXSRET2		NASDAQ EXSRET		NASDAQ FF3-adj EXSRET1		NASDAQ FF3-adj EXSRET2	
Intercept	-8.096		-12.668		-24.803		2.603	**	2.177	**	3.073	**
	-1.15		*-1.44*		*-1.72*		*4.39*		*4.42*		*4.81*	
$[ILLIQ\_1]^{1/2}$	0.975	**	1.421	*	2.213	*	0.510	**	0.488	**	0.534	**
	4.55		*1.96*		*2.42*		*4.16*		*3.98*		*3.41*	
SIZE	2.314		3.278		4.464		-0.502	**	-0.514	**	-0.682	**
	1.13		*1.58*		*1.59*		*-4.81*		*-5.29*		*-5.39*	
BTM	5.633		6.553		6.345		-0.038		-0.036		-0.265	
	1.06		*1.25*		*0.94*		*-0.26*		*-0.30*		*-1.55*	
MOM1	-1.260		16.416		14.314		-1.370	**	-1.128	**	-1.486	*
	-0.26		*1.40*		*0.87*		*-3.61*		*-3.61*		*-2.51*	
MOM2	-4.457		-8.637		-10.147		-0.833	*	-0.708		-0.654	
	-0.67		*-1.12*		*-0.91*		*-2.10*		*-1.95*		*-1.14*	
MOM3	-0.236		-22.224		-54.627		-0.637		-0.962	*	-1.811	**
	-0.16		*-1.51*		*-1.52*		*-1.69*		*-2.54*		*-2.84*	
MOM4	10.753		4.748		12.236		-0.304		-0.499		-0.707	
	1.03		*0.34*		*0.50*		*-1.03*		*-1.81*		*-1.47*	
Avg R-sqr	0.034		0.024		0.053		0.023		0.015		0.019	
Avg Obs	1680.7		1686.7		1748.8		2693.5		2690.0		2656.9	

(Table VII continued: Panel B)

Panel B: with Square Root of ILLIQ_2 for Quote Mid-point Returns

Explanatory Variables	NYSE/AMEX EXSRET		NYSE/AMEX FF3-adj EXSRET1		NYSE/AMEX FF3-adj EXSRET2		NASDAQ EXSRET		NASDAQ FF3-adj EXSRET1		NASDAQ FF3-adj EXSRET2	
Intercept	-7.919		-11.627		-23.765		2.163	**	1.729	**	2.314	**
	-1.06		*-1.33*		*-1.71*		*4.28*		*4.34*		*4.44*	
$[ILLIQ\_2]^{1/2}$	0.346	**	0.332	*	0.507	*	0.338	**	0.291	**	0.373	**
	3.64		*2.37*		*2.36*		*5.30*		*4.87*		*4.87*	
SIZE	2.371		3.011		4.439		-0.378	**	-0.396	**	-0.488	**
	1.08		*1.38*		*1.53*		*-4.37*		*-4.96*		*-4.67*	
BTM	6.043		6.423		6.828		0.164		0.130		0.041	
	1.05		*1.14*		*0.94*		*1.17*		*1.14*		*0.26*	
MOM1	-2.559		5.015		12.695		-0.803		-0.671		-0.839	
	-0.43		*0.57*		*0.73*		*-1.85*		*-1.80*		*-1.42*	
MOM2	-4.890		2.045		-11.869		-0.121		-0.123		0.062	
	-0.70		*0.18*		*-1.06*		*-0.33*		*-0.35*		*0.12*	
MOM3	-0.032		-24.588		-56.674		0.071		-0.301		-0.610	
	-0.02		*-1.60*		*-1.52*		*0.18*		*-0.76*		*-1.02*	
MOM4	11.636		5.132		12.847		0.082		0.042		-0.115	
	1.02		*0.34*		*0.50*		*0.25*		*0.13*		*-0.23*	
Avg R-sqr	0.034		0.037		0.055		0.027		0.017		0.021	
Avg Obs	1547.7		1553.1		1640.8		2064.6		2062.6		2219.8	

Table VIII

Results of Quarterly Cross-sectional Regressions: with $[ILLIQ\_1]^{1/2}$ and $[ILLIQ\_2]^{1/2}$ for NYSE/AMEX and NASDAQ Stocks

This table reports the *quarterly* Fama-MacBeth (1973)-type cross-sectional regressions with $[ILLIQ\_1]^{1/2}$ (in Panel A) and $[ILLIQ\_2]^{1/2}$ (in Panel B) for NYSE/AMEX stocks over the 124 quarters (31 years: 197201-200204) and for NASDAQ stocks over the 80 quarters (20 years: 198301-200204). The dependent variables (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2) are all one-quarter leading values (no contemporaneous regressors are used). The definitions of the variables are as follows: *EXSRET*: the quarterly return (compounded over 3 months in each quarter) using the monthly risk-unadjusted excess return defined in Table V; *FF3-adj EXSRET1*: the quarterly return (compounded over 3 months in each quarter) using the monthly risk-adjusted excess return (by the first method) defined in Table V; *FF3-adj EXSRET2*: the quarterly return (compounded over 3 months in each quarter) using the monthly risk-adjusted excess return (by the second method) defined in Table V; *ILLIQ_1* and *ILLIQ_2*: the averages (over 3 months in each quarter) of the two illiquidity measures defined in Table I; *SIZE*: the average (over 3 months in each quarter) of the natural logarithm of MV, which is defined as the month-end stock price times the number of shares outstanding (in $million); *BTM*: the average (over 3 months in each quarter) of the natural logarithm of BM_Trim, which is the trimmed book-to-market ratio, where book-to-market ratios greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *MOM1*: compounded holding period return of a stock over the most recent 3 months; *MOM2*: compounded holding period return over the next recent 3 months; *MOM3*: compounded holding period return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period return over the 3 months from month *t-9* to month *t-12*. The average numbers of component stocks used in the quarterly regressions for NYSE/AMEX stocks are 1724.0-1758.8, while those for NASDAQ stocks are 2454.5-2612.4. The values in the first row for each explanatory variable are the time-series averages of coefficients obtained from the quarter-by-quarter cross-sectional regressions, and the values italicized in the second row of each variable are t-statistics computed based on Fama-MacBeth (1973). The coefficients are all multiplied by 100. *Avg R-sqr* is the average of adjusted R-squared. *Avg Obs* is the average number of companies used in the quarterly cross-sectional regressions. Coefficients significantly different from zero at the significance levels of 1% and 5% are indicated by ** and *, respectively.

(Table VIII continued: Panel A)

Panel A: Quarterly Cross-sectional Regressions with Square Root of ILLIQ_1

Explanatory Variables	NYSE/AMEX EXSRET		NYSE/AMEX FF3-adj EXSRET1		NYSE/AMEX FF3-adj EXSRET2		NASDAQ EXSRET		NASDAQ FF3-adj EXSRET1		NASDAQ FF3-adj EXSRET2	
Intercept	-1.076		-2.355	**	-2.653	**	2.801		1.431		1.256	
	-0.62		*-3.19*		*-3.47*		*1.40*		*1.05*		*0.89*	
$[ILLIQ\_1]^{1/2}$	2.135	**	2.043	**	2.037	**	1.183	**	1.155	**	1.242	**
	4.09		*4.11*		*3.92*		*4.48*		*5.16*		*5.33*	
SIZE	0.358	*	0.328	**	0.350	**	-0.262		-0.277		-0.220	
	2.09		*3.39*		*3.61*		*-0.87*		*-1.03*		*-0.77*	
BTM	0.673		0.662	**	0.572	*	0.988	*	1.031	**	0.988	**
	1.91		*2.65*		*2.18*		*2.16*		*3.17*		*2.81*	
MOM1	-1.611		-1.850		-2.558		-2.601	**	-2.723	**	-3.717	**
	-1.26		*-1.68*		*-1.92*		*-2.57*		*-3.07*		*-3.14*	
MOM2	2.936	*	3.276	**	4.390	**	1.411		1.893	**	2.628	**
	2.08		*2.91*		*3.35*		*1.94*		*2.91*		*2.63*	
MOM3	3.915	**	3.020	**	1.794		1.686	*	1.013		0.089	
	3.23		*3.30*		*1.45*		*2.43*		*1.80*		*0.09*	
MOM4	2.501	**	2.447	**	1.759		1.175		0.870		0.212	
	2.76		*3.33*		*1.65*		*1.61*		*1.69*		*0.29*	
Avg R-sqr	0.063		0.041		0.049		0.046		0.036		0.045	
Avg Obs	1758.8		1758.5		1724.0		2612.4		2611.1		2454.5	

(Table VIII continued: Panel B)

Panel B: Quarterly Cross-sectional Regressions with Square Root of ILLIQ_2

Explanatory	NYSE/AMEX			NASDAQ		
Variables	EXSRET	FF3-adj EXSRET1	FF3-adj EXSRET2	EXSRET	FF3-adj EXSRET1	FF3-adj EXSRET2
Intercept	0.525	-0.802	-1.025	2.959	1.491	1.470
	0.30	*-0.96*	*-1.24*	*1.55*	*1.14*	*1.10*
$[ILLIQ\_2]^{1/2}$	0.777 **	0.717 **	0.679 **	0.671 **	0.645 **	0.720 **
	3.12	*3.02*	*2.71*	*3.91*	*4.03*	*4.03*
SIZE	0.144	0.123	0.136	-0.194	-0.207	-0.176
	0.75	*1.01*	*1.19*	*-0.67*	*-0.81*	*-0.67*
BTM	0.537	0.548 *	0.461	1.052 *	1.084 **	1.088 **
	1.49	*2.15*	*1.72*	*2.27*	*3.28*	*3.04*
MOM1	-1.227	-1.603	-2.328	-1.623	-1.794	-2.785 *
	-0.99	*-1.50*	*-1.79*	*-1.54*	*-1.94*	*-2.34*
MOM2	2.959 *	3.201 **	4.308 **	2.634 **	3.078 **	3.591 **
	2.11	*2.83*	*3.23*	*3.24*	*4.35*	*3.60*
MOM3	3.759 **	2.808 **	1.589	2.213 **	1.412 *	0.544
	2.97	*3.00*	*1.27*	*2.98*	*2.27*	*0.56*
MOM4	2.558 **	2.523 **	1.809	1.455	1.356 *	0.499
	2.76	*3.45*	*1.75*	*1.87*	*2.43*	*0.66*
Avg R-sqr	0.064	0.042	0.049	0.047	0.036	0.043
Avg Obs	1630.7	1630.6	1621.5	2021.1	2020.6	1992.8

Table IX

Relations of the Theory-based Illiquidity Measures to Alternative Measures

In Panel A, the lower triangle shows the monthly average correlations between the key (il)liquidity measures for NYSE/AMEX stocks over the past 372 months (31 years: 197201-200212), and the upper triangle shows those for NASDAQ stocks over the 240 months (20 years: 198301-200212). The cross-sectional correlation coefficients are first calculated each month and then the time-series averages of those values over the sample periods are reported here. The definitions of the measures are as follows: *ILLIQ_1*: the first illiquidity measure defined as in Table I; *ILLIQ_2*: the second illiquidity measure defined as in Table I; *Amihud*: the illiquidity measure of Amihud (2002) estimated each month as the average of $|r|/DVOL$, where r is the daily stock return and *DVOL* is the daily dollar volume in $1000; *Roll_Gibbs*: the market risk-adjusted effective bid-ask spread of Roll (1984) estimated using the Gibbs sampler, which is of annual frequency obtained from the web site of Joel Hasbrouck; *PS*: the illiquidity measure (gamma) of Pastor and Stambaugh (2003) estimated each month using the CRSP daily file from the regression equation,

$$r^{e}_{j,d+1,t} = a + br_{j,d,t} + \gamma sign(r^{e}_{j,d,t})DVOL_{j,d,t} + \varsigma_{j,d+1,t},$$

where $r_{j,d,t}$ is the raw return and $r^{e}_{j,d,t}$ is the excess return (over the CRSP value-weighted market return) of stock j at day d within month t (the number of days should be equal to or greater than 15 within each month); *TURN*: the average of daily share turnover within each month for each stock. Panel B reports the average correlations between the square-root values of the six corresponding (il)liquidity measures. The average number of component stocks used in a year for NYSE/AMEX stocks is 2,236.4, while that for NASDAQ stocks is 2,799.4.

Panel A: Average Correlations between (Il)liquidity Measures: NYSE/AMEX (Lower Triangle) and NASDAQ (Upper Triangle)

	ILLIQ_1	ILLIQ_2	Amihud	Roll_Gibbs	PS	TURN
ILLIQ_1	1	0.467	0.249	0.388	-0.120	-0.057
ILLIQ_2	0.602	1	0.248	0.292	-0.032	-0.042
Amihud	0.657	0.466	1	0.485	-0.156	-0.072
Roll_Gibbs	0.503	0.322	0.564	1	-0.101	-0.235
PS	-0.045	-0.022	-0.058	-0.024	1	0.012
TURN	-0.090	-0.048	-0.097	-0.109	0.005	1

Panel B: Average Correlations between Square-Root Values of Measures: NYSE/AMEX (Lower Triangle) and NASDAQ (Upper Triangle)

	$[ILLIQ\_1]^{1/2}$	$[ILLIQ\_2]^{1/2}$	$[Amihud]^{1/2}$	$[Roll\_Gibbs]^{1/2}$	$Sign(PS)[\|PS\|]^{1/2}$	$[TURN]^{1/2}$
$[ILLIQ\_1]^{1/2}$	1	0.638	0.559	0.657	-0.114	-0.280
$[ILLIQ\_2]^{1/2}$	0.776	1	0.462	0.503	-0.063	-0.187
$[Amihud]^{1/2}$	0.867	0.701	1	0.704	-0.120	-0.307
$[Roll\_Gibbs]^{1/2}$	0.698	0.538	0.720	1	-0.070	-0.361
$Sign(PS)[\|PS\|]^{1/2}$	-0.022	-0.013	-0.022	-0.008	1	0.037
$[TURN]^{1/2}$	-0.280	-0.182	-0.297	-0.163	0.008	1

Table X

A Horse Race with Each of the Four Alternative (Il)liquidity Measures

This table runs a horse race in the monthly Fama-MacBeth (1973)-type cross-sectional regressions for comparison purposes using 4 alternative (il)liquidity measures for NYSE/AMEX stocks over the past 372 months (31 years: 197201-200212) and for NASDAQ stocks over the past 240 months (20 years: 198301-200212). Each of Panels A-D reports the regression results comparable to those in Tables V-VI using each of the square-root values of the 4 alternative measures. The dependent variables (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2) are all one-month leading values (no contemporaneous regressors are used). The definitions of the variables are as follows: *EXSRET*: the monthly risk-unadjusted excess return, i.e., the monthly return less the risk-free rate proxied by the one-month T-bill rate; *FF3-adj EXSRET1*: the risk-adjusted excess return using the Fama-French (FF) 3 factors, i.e., the constant term plus the residual from the time-series regression of the excess return on the FF 3 factors using the *entire* sample range of the data; *FF3-adj EXSRET2*: the risk-adjusted excess return using the Fama-French (FF) 3 factors with factor loadings being estimated from the 5-year rolling regressions, i.e., R_i^* computed each month with the current month data from the equation, $R_i^* = (R_i - R_f) - [\hat{\beta}_1 MKT + \hat{\beta}_2 SMB + \hat{\beta}_3 HML]$, after the factor loadings $(\alpha, \beta_1, \beta_2, \beta_3)$ are first estimated for *each month* using the time-series data of the past 60 months in the monthly regression, $R_i - R_f = \alpha + \beta_1 MKT + \beta_2 SMB + \beta_3 HML + \varepsilon$, where R_i, R_f, and R_m are the individual stock return, the risk-free rate, and the market index return, respectively, while *MKT*, *SMB*, and HML are FF 3 factors; *Amihud*: the illiquidity measure of Amihud (2002) estimated each month as the average of |*r*|/*DVOL*, where *r* is the daily stock return and *DVOL* is the daily dollar volume in $1000; *Roll_Gibbs*: the market risk-adjusted effective bid-ask spread of Roll (1984) estimated using the Gibbs sampler, which is of annual frequency obtained from the web site of Joel Hasbrouck; *PS*: the illiquidity measure (gamma) of Pastor and Stambaugh (2003) estimated each month using the CRSP daily file from the regression equation,

$$r^e_{j,d+1,t} = a + br_{j,d,t} + \gamma sign(r^e_{j,d,t}) DVOL_{j,d,t} + \varsigma_{j,d+1,t},$$

where $r_{j,d,t}$ is the raw return and $r^e_{j,d,t}$ is the excess return (over the CRSP value-weighted market return) of stock *j* at day *d* within month *t* (the number of days should be equal to or greater than 15 within each month); *TURN*: the average of daily share turnover within each month for each stock; *SIZE*: natural logarithm of MV, which is defined as the month-end stock price times the number of shares outstanding (in $million); *BTM*: natural logarithm of BM_Trim, which is the trimmed book-to-market ratio, where book-to-market ratios greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *MOM1*: compounded holding period return of a stock over the most recent 3 months (from month *t-1* to month *t-3*); *MOM2*: compounded holding period return over the next recent 3 months (from month *t-4* to month *t-6*); *MOM3*: compounded holding period return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period return over the 3 months from month *t-9* to month *t-12*. For monthly regressions, we keep the annual values of Roll_Gibbs constant over the 12 months within each year. The average number of component stocks used in the regressions for NYSE/AMEX stocks is 1,845.1, while that for NASDAQ stocks is 2,667.5. The values in the first row for each explanatory variable are the time-series averages of coefficients obtained from the month-by-month cross-sectional regressions, and the values italicized in the second row of each variable are t-statistics computed based on Fama-MacBeth (1973). The coefficients are all multiplied by 100. *Avg R-sqr* is the average of adjusted R-squared. *Avg Obs* is the monthly average number of companies used in the cross-sectional regressions. Coefficients significantly different from zero at the significance levels of 1% and 5% are indicated by ** and *, respectively.

(Table X continued: Panel A)

Panel A: with Square Root of Amihud												
Explanatory	NYSE/AMEX						NASDAQ					
Variables	EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2		EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2	
Intercept	0.655		-0.099		-0.044		1.810	**	1.209	**	1.184	**
	1.77		*-0.74*		*-0.30*		*3.52*		*3.30*		*3.09*	
$[\text{Amihud}]^{1/2}$	6.049	**	5.304	**	5.549	**	5.013	**	4.534	**	5.114	**
	4.13		*3.76*		*3.69*		*4.34*		*4.47*		*4.89*	
SIZE	-0.053		-0.017		-0.019		-0.271	**	-0.251	**	-0.237	**
	-1.28		*-0.90*		*-0.90*		*-3.27*		*-3.48*		*-3.14*	
BTM	0.220	**	0.158	**	0.098	*	0.295	**	0.317	**	0.281	**
	3.14		*3.35*		*2.19*		*2.63*		*4.03*		*3.06*	
MOM1	0.912	**	0.980	**	0.950	**	0.426		0.521	*	0.439	
	2.87		*3.79*		*3.08*		*1.22*		*2.00*		*1.15*	
MOM2	1.216	**	1.180	**	1.071	**	0.726	*	0.541	*	0.551	
	4.47		*5.29*		*4.01*		*2.53*		*2.39*		*1.81*	
MOM3	1.463	**	1.405	**	1.140	**	0.671	**	0.528	*	0.355	
	5.57		*6.21*		*4.19*		*2.78*		*2.46*		*1.16*	
MOM4	1.063	**	1.009	**	0.767	**	0.407		0.346		0.211	
	5.01		*5.22*		*2.97*		*1.90*		*1.90*		*0.76*	
Avg R-sqr	0.049		0.029		0.032		0.035		0.023		0.029	
Avg Obs	1844.8		1843.1		1797.4		2664.7		2659.9		2407.2	

(Table X continued: Panel B)

Panel B: with Square Root of Roll_Gibbs

Explanatory Variables	NYSE/AMEX EXSRET	NYSE/AMEX FF3-adj EXSRET1	NYSE/AMEX FF3-adj EXSRET2	NASDAQ EXSRET	NASDAQ FF3-adj EXSRET1	NASDAQ FF3-adj EXSRET2
Intercept	3.392 **	2.802 **	3.000 **	12.099 **	11.166 **	11.228 **
	10.07	*12.58*	*12.59*	*15.61*	*18.84*	*18.10*
$[\text{Roll\_Gibbs}]^{1/2}$	-16.306 **	-17.919 **	-18.728 **	-41.655 **	-40.456 **	-40.480 **
	-7.46	*-9.03*	*-8.92*	*-16.41*	*-17.84*	*-17.32*
SIZE	-0.323 **	-0.297 **	-0.313 **	-1.451 **	-1.395 **	-1.389 **
	-8.78	*-14.65*	*-14.25*	*-15.23*	*-15.97*	*-14.94*
BTM	0.219 **	0.158 **	0.100 *	0.307 **	0.312 **	0.274 **
	3.18	*3.42*	*2.25*	*2.69*	*3.94*	*2.97*
MOM1	0.125	0.189	0.110	-0.029	0.066	-0.117
	0.40	*0.74*	*0.36*	*-0.08*	*0.24*	*-0.30*
MOM2	0.412	0.379	0.230	-0.086	-0.206	-0.285
	1.53	*1.72*	*0.86*	*-0.29*	*-0.83*	*-0.87*
MOM3	0.633 *	0.555 *	0.242	0.126	0.016	-0.270
	2.48	*2.52*	*0.91*	*0.49*	*0.07*	*-0.84*
MOM4	0.410 *	0.378 *	0.113	0.087	0.010	-0.093
	1.97	*2.05*	*0.45*	*0.35*	*0.05*	*-0.31*
Avg R-sqr	0.053	0.032	0.036	0.047	0.033	0.037
Avg Obs	1745.0	1744.0	1700.2	2190.4	2186.0	1981.3

(Table X continued: Panel C)

Panel C: with Sign(PS) x (Square Root of |PS|)												
Explanatory	**NYSE/AMEX**						**NASDAQ**					
Variables	EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2		EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2	
Intercept	1.184	**	0.353	*	0.415	*	2.542	**	1.889	**	1.957	**
	3.05		*1.98*		*2.16*		*5.21*		*5.11*		*5.02*	
Sign(PS)[|PS|]$^{1/2}$	0.037		0.024		-0.008		0.052		0.042		0.045	
	0.55		*0.37*		*-0.13*		*0.92*		*0.78*		*0.78*	
SIZE	-0.125	**	-0.078	**	-0.080	**	-0.389	**	-0.361	**	-0.362	**
	-2.62		*-2.80*		*-2.70*		*-4.71*		*-4.93*		*-4.65*	
BTM	0.218	**	0.156	**	0.097	*	0.336	**	0.353	**	0.319	**
	3.10		*3.30*		*2.15*		*2.98*		*4.51*		*3.49*	
MOM1	0.798	*	0.881	**	0.856	**	0.278		0.388		0.291	
	2.49		*3.40*		*2.77*		*0.79*		*1.49*		*0.76*	
MOM2	1.137	**	1.112	**	1.000	**	0.646	*	0.462	*	0.460	
	4.12		*4.93*		*3.71*		*2.25*		*2.04*		*1.50*	
MOM3	1.364	**	1.307	**	1.045	**	0.592	*	0.456	*	0.274	
	5.13		*5.67*		*3.81*		*2.45*		*2.12*		*0.89*	
MOM4	1.037	**	0.989	**	0.745	**	0.369		0.320		0.180	
	4.82		*5.05*		*2.86*		*1.72*		*1.78*		*0.65*	
Avg R-sqr	0.046		0.026		0.029		0.033		0.022		0.028	
Avg Obs	1845.0		1843.2		1797.5		2667.5		2662.7		2409.7	

(Table X continued: Panel D)

Panel D: with Square Root of TURN												
Explanatory	NYSE/AMEX						NASDAQ					
Variables	EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2		EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2	
Intercept	1.255	**	0.486	*	0.568	**	2.456	**	1.879	**	2.004	**
	3.43		*2.50*		*2.75*		*5.57*		*5.26*		*5.36*	
$[TURN]^{1/2}$	-0.317		-2.897		-3.213		4.972		2.803		0.937	
	-0.11		*-1.29*		*-1.36*		*1.39*		*1.23*		*0.38*	
SIZE	-0.135	**	-0.079	**	-0.082	**	-0.430	**	-0.392	**	-0.384	**
	-2.75		*-2.83*		*-2.75*		*-5.12*		*-5.22*		*-4.72*	
BTM	0.221	**	0.151	**	0.091	*	0.370	**	0.366	**	0.313	**
	3.25		*3.26*		*2.05*		*3.95*		*5.10*		*3.63*	
MOM1	0.974	**	1.029	**	1.010	**	0.387		0.462		0.380	
	3.22		*4.07*		*3.31*		*1.25*		*1.81*		*1.01*	
MOM2	1.221	**	1.193	**	1.087	**	0.659	*	0.494	*	0.477	
	4.56		*5.34*		*4.05*		*2.44*		*2.21*		*1.58*	
MOM3	1.332	**	1.320	**	1.069	**	0.502	*	0.413	*	0.233	
	5.12		*5.71*		*3.86*		*2.20*		*1.95*		*0.76*	
MOM4	1.058	**	0.988	**	0.756	**	0.376		0.316		0.200	
	5.12		*5.11*		*2.93*		*1.91*		*1.77*		*0.72*	
Avg R-sqr	0.050		0.027		0.030		0.038		0.023		0.029	
Avg Obs	1845.0		1843.2		1797.5		2667.5		2662.7		2409.7	

Table XI
A Horse Race with All the (Il)liquidity Measures

This table runs a horse race in the monthly Fama-MacBeth (1973)-type cross-sectional regressions using one of our illiquidity measures together with the 4 alternative (il)liquidity measures for NYSE/AMEX stocks over the past 372 months (31 years: 197201-200212) and for NASDAQ stocks over the past 240 months (20 years: 198301-200212). Panel A contains the results for ILLIQ_1 with the 4 alternative measures, while Panel B does the same for ILLIQ_2. The dependent variables (EXSRET, FF3-adj EXSRET1, and FF3-adj EXSRET2) are all one-month leading values (no contemporaneous regressors are used). The definitions of the variables are as follows: *EXSRET*: the monthly risk-unadjusted excess return, i.e., the monthly return less the risk-free rate proxied by the one-month T-bill rate; *FF3-adj EXSRET1*: the risk-adjusted excess return using the Fama-French (FF) 3 factors, i.e., the constant term plus the residual from the time-series regression of the excess return on the FF 3 factors using the *entire* sample range of the data; *FF3-adj EXSRET2*: the risk-adjusted excess return using the Fama-French (FF) 3 factors with factor loadings being estimated from the 5-year rolling regressions, i.e., R_i^* computed each month with the current month data from the equation, $R_i^* = (R_i - R_f) - [\hat{\beta}_1 MKT + \hat{\beta}_2 SMB + \hat{\beta}_3 HML]$, after the factor loadings $(\alpha, \beta_1, \beta_2, \beta_3)$ are first estimated for *each month* using the time-series data of the past 60 months in the monthly regression, $R_i - R_f = \alpha + \beta_1 MKT + \beta_2 SMB + \beta_3 HML + \varepsilon$, where R_i, R_f, and R_m are the individual stock return, the risk-free rate, and the market index return, respectively, while *MKT*, *SMB*, and HML are FF 3 factors; *ILLIQ_1*: the first illiquidity measure defined as $\frac{N^{0.5} std(R)}{(N+1) std(z)}$, where *N* is the number of informed traders, *std(R)* is the standard deviation of returns, and *std(z)* is the standard deviation of noise trades (the original input variables are proxied by the variables as shown in Table I); *ILLIQ_2*: the second illiquidity measure defined as $P^{-1} \frac{v_\delta}{(N+1)v_\delta + 2v_\varepsilon} \sqrt{\frac{N(v_\delta + v_\varepsilon)}{v_z}}$, where *P* is the asset price, *N* is the number of informed traders, v_δ is the variance of payoff innovations, v_ε is the variance of signal innovations, and v_z is the variance of noise trades (the original input variables are proxied by the variables as shown in Table I); *Amihud*: the illiquidity measure of Amihud (2002) estimated each month as the average of |*r*|/*DVOL*, where *r* is the daily stock return and *DVOL* is the daily dollar volume in $1000; *Roll_Gibbs*: the market risk-adjusted effective bid-ask spread of Roll (1984) estimated using the Gibbs sampler, which is of annual frequency obtained from the web site of Joel Hasbrouck; *PS*: the illiquidity measure (gamma) of Pastor and Stambaugh (2003) estimated each month using the CRSP daily file from the regression equation, $r^e_{j,d+1,t} = a + br_{j,d,t} + \gamma sign(r^e_{j,d,t}) DVOL_{j,d,t} + \varsigma_{j,d+1,t}$, where $r_{j,d,t}$ is the raw return and $r^e_{j,d,t}$ is the excess return (over the CRSP value-weighted market return) of stock *j* at day *d* within month *t* (the number of days should be equal to or greater than 15 within each month); *TURN*: the average of daily share turnover within each month for each stock; *SIZE*: natural logarithm of MV, which is defined as the month-end stock price times the number of shares outstanding (in $million); *BTM*: natural logarithm of BM_Trim, which is the trimmed book-to-market ratio, where book-to-market ratios greater than the 99.5 percentile value or less than the 0.5 percentile value in a month are set equal to the 99.5 and 0.5 percentile values, respectively; *MOM1*: compounded holding period return of a stock over the most recent 3 months (from month *t-1* to month *t-3*); *MOM2*: compounded holding period return over the next recent 3 months (from month *t-4* to month *t-6*); *MOM3*: compounded holding period return over the 3 months from month *t-7* to month *t-9*; *MOM4*: compounded holding period return over the 3 months from month *t-9* to month *t-12*. For monthly regressions, we keep the annual values of Roll_Gibbs constant over the 12 months within each year. The average number of component stocks used in the regressions for NYSE/AMEX stocks is 1,845.1, while that for NASDAQ stocks is 2,667.5. The values in the first row for each explanatory variable are the time-series averages of coefficients obtained from the month-by-month cross-sectional regressions, and the values italicized in the second row of each variable are t-statistics computed based on Fama-MacBeth (1973). The coefficients are all multiplied by 100. *Avg R-sqr* is the average of adjusted R-squared. *Avg Obs* is the monthly average number of companies used in the cross-sectional regressions. Coefficients significantly different from zero at the significance levels of 1%, 5%, and 10% are indicated by ***, **, and *, respectively.

(Table XI continued: Panel A)

Panel A: with Square Root of ILLIQ_1

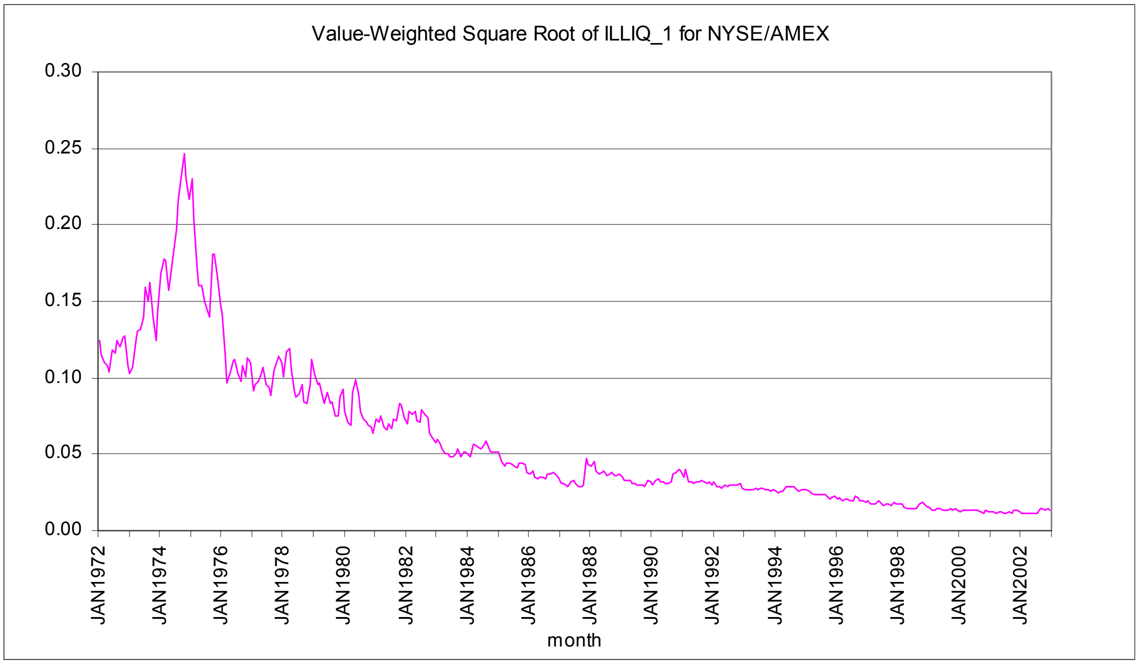
Explanatory Variables	NYSE/AMEX EXSRET	NYSE/AMEX FF3-adj EXSRET1	NYSE/AMEX FF3-adj EXSRET2	NASDAQ EXSRET	NASDAQ FF3-adj EXSRET1	NASDAQ FF3-adj EXSRET2
Intercept	3.168 ***	2.676 ***	2.888 ***	14.580 ***	13.811 ***	14.097 ***
	9.81	*11.58*	*11.73*	*19.10*	*20.60*	*20.09*
[ILLIQ_1]$^{1/2}$	0.550 ***	0.529 ***	0.519 ***	1.466 ***	1.347 ***	1.522 ***
	4.89	*4.99*	*4.44*	*7.43*	*6.98*	*5.76*
[Amihud]$^{1/2}$	11.892 ***	11.520 ***	11.989 ***	10.515 ***	9.476 **	9.933 *
	6.80	*6.88*	*6.51*	*2.76*	*2.50*	*1.86*
[Roll_Gibbs]$^{1/2}$	-30.059 ***	-31.452 ***	-32.519 ***	-71.463 ***	-69.394 ***	-71.368 ***
	-14.32	*-16.54*	*-16.14*	*-22.75*	*-22.82*	*-22.77*
Sign(PS)[|PS|]$^{1/2}$	0.047	0.035	-0.003	0.176 **	0.164 *	0.095
	0.68	*0.53*	*-0.05*	*1.96*	*1.87*	*0.87*
[TURN]$^{1/2}$	4.933 *	1.922	1.682	-6.666 *	-8.761 ***	-10.338 ***
	1.68	*0.83*	*0.70*	*-1.83*	*-3.44*	*-3.66*
SIZE	-0.217 ***	-0.188 ***	-0.200 ***	-1.431 ***	-1.390 ***	-1.386 ***
	-5.75	*-9.73*	*-9.51*	*-14.89*	*-15.65*	*-14.45*
BTM	0.210 ***	0.141 ***	0.083 *	0.104	0.097	0.035
	3.21	*3.16*	*1.92*	*1.09*	*1.34*	*0.40*
MOM1	0.454	0.487 *	0.421	0.519	0.556 **	0.446
	1.52	*1.93*	*1.39*	*1.61*	*2.06*	*1.15*
MOM2	0.560 **	0.523 **	0.384	-0.016	-0.146	-0.196
	2.17	*2.41*	*1.45*	*-0.06*	*-0.61*	*-0.62*
MOM3	0.632 ***	0.591 ***	0.294	-0.052	-0.104	-0.373
	2.59	*2.72*	*1.12*	*-0.22*	*-0.47*	*-1.18*
MOM4	0.438 **	0.372 **	0.116	0.039	-0.040	-0.160
	2.23	*2.06*	*0.47*	*0.17*	*-0.20*	*-0.54*
Avg R-sqr	0.070	0.045	0.049	0.067	0.049	0.054
Avg Obs	1744.9	1743.9	1700.1	2189.4	2185.1	1980.4

(Table XI continued: Panel B)

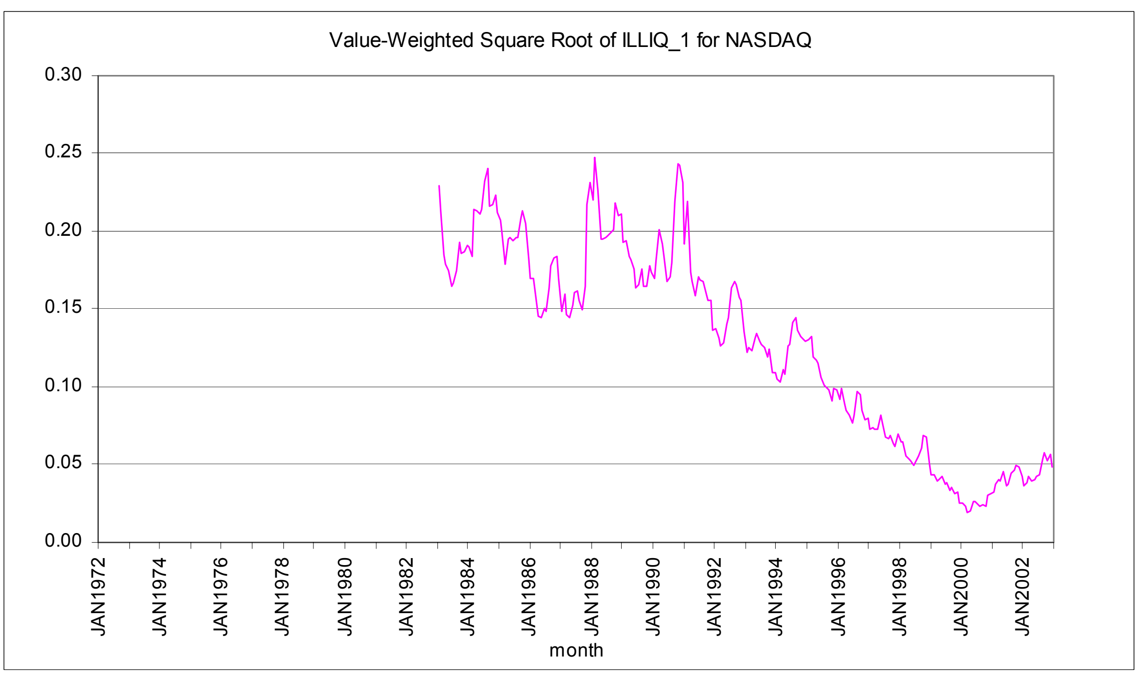
Panel B: with Square Root of ILLIQ_2												
Explanatory	NYSE/AMEX						NASDAQ					
Variables	EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2		EXSRET		FF3-adj EXSRET1		FF3-adj EXSRET2	
Intercept	3.166	***	2.678	***	2.855	***	13.614	***	12.875	***	13.051	***
	9.62		*11.28*		*11.43*		*16.75*		*17.58*		*17.16*	
$[\text{ILLIQ\_2}]^{1/2}$	0.141	***	0.120	***	0.116	**	0.328		0.276		0.267	
	2.97		*2.60*		*2.39*		*1.40*		*1.25*		*1.04*	
$[\text{Amihud}]^{1/2}$	15.533	***	15.269	***	16.035	***	29.828	***	25.624	***	30.463	***
	8.45		*8.50*		*8.38*		*3.35*		*3.08*		*3.35*	
$[\text{Roll\_Gibbs}]^{1/2}$	-27.715	***	-29.116	***	-30.114	***	-65.958	***	-64.194	***	-65.759	***
	-12.98		*-14.99*		*-14.71*		*-19.49*		*-19.86*		*-19.96*	
$\text{Sign(PS)}[\lvert\text{PS}\rvert]^{1/2}$	0.071		0.057		0.017		-0.047		-0.062		-0.043	
	1.00		*0.82*		*0.24*		*-0.27*		*-0.35*		*-0.22*	
$[\text{TURN}]^{1/2}$	3.781		0.622		1.093		-4.248		-6.010	**	-7.152	**
	1.29		*0.27*		*0.46*		*-1.08*		*-2.09*		*-2.34*	
SIZE	-0.223	***	-0.191	***	-0.205	***	-1.334	***	-1.302	***	-1.293	***
	-5.70		*-9.39*		*-9.50*		*-13.40*		*-14.03*		*-12.97*	
BTM	0.179	***	0.121	***	0.065		0.108		0.091		0.080	
	2.71		*2.65*		*1.46*		*0.93*		*0.94*		*0.73*	
MOM1	0.443		0.439	*	0.450		0.423		0.419		0.435	
	1.47		*1.73*		*1.50*		*1.15*		*1.24*		*1.02*	
MOM2	0.577	**	0.546	**	0.465	*	0.300		0.165		0.105	
	2.23		*2.50*		*1.78*		*0.88*		*0.54*		*0.28*	
MOM3	0.661	***	0.625	***	0.359		0.052		0.002		-0.212	
	2.71		*2.86*		*1.36*		*0.20*		*0.01*		*-0.66*	
MOM4	0.388	*	0.337	*	0.052		0.199		0.205		0.080	
	1.91		*1.79*		*0.20*		*0.77*		*0.86*		*0.26*	
Avg R-sqr	0.071		0.047		0.050		0.072		0.054		0.057	
Avg Obs	1602.0		1601.2		1587.0		1743.2		1738.2		1690.2	

Figure 1. Trends of the Value-weighted Illiquidity Measure: for $[ILLIQ\_1]^{1/2}$

The following graphs show the trends of the market value-weighted series for the square-root values of ILLIQ_1 over the past 372 months (31 years: 197201-200212). The value-weighted illiquidity series are the monthly cross-sectional (market-value weighted) averages of the square root of ILLQ_1 over the sample period. ILLIQ_1 is defined as in Table I. Figure 2(a) is for the stocks on the NYSE/AMEX, and Figure 2(b) for those on the NASDAQ (available from January 1983 to December 2002 only). The average numbers of component stocks used each month are 1,845.1 for NYSE/AMEX (197201-200212) stocks and 2,667.5 for NASDAQ (198301-200212) stocks.



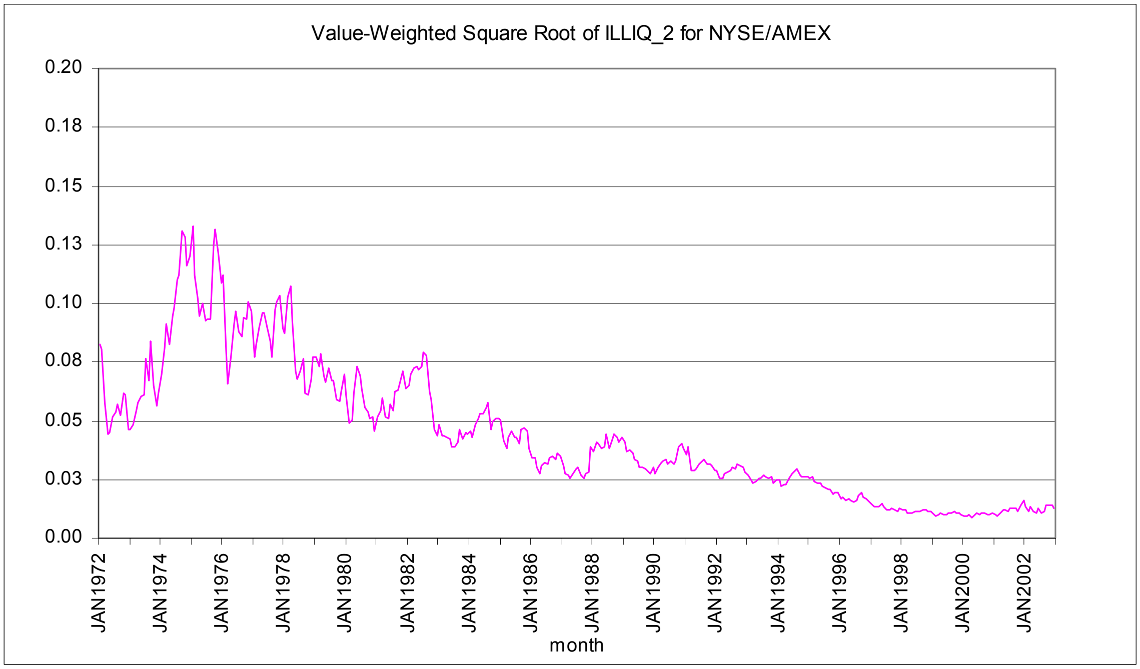
(a)



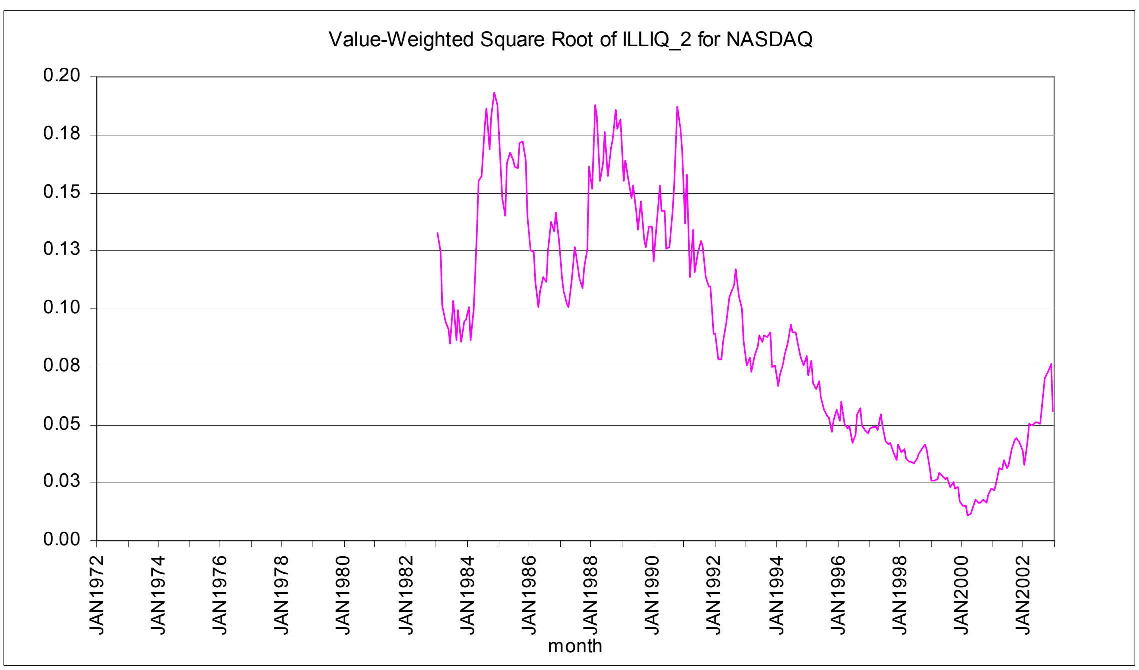
(b)

Figure 2. Trends of the Value-weighted Illiquidity Measure: for $[ILLIQ\_2]^{1/2}$

The following graphs show the trends of the market value-weighted series for the square-root values of ILLIQ_2 over the past 372 months (31 years: 197201-200212). The value-weighted illiquidity series are the monthly cross-sectional (market-value weighted) averages of the square root of ILLQ_2 over the sample period. ILLIQ_2 is defined as in Table I. Figure 2(a) is for the stocks on the NYSE/AMEX, and Figure 2(b) for those on the NASDAQ (available from January 1983 to December 2002 only). The average numbers of component stocks used each month are 1,683.1 for NYSE/AMEX (197201-200212) stocks and 1,967.0 for NASDAQ (198301-200212) stocks.



(a)



(b)