

# IS STOCHASTIC VOLATILITY PRICED ON KOSPI 200 INDEX OPTIONS ?

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# IS STOCHASTIC VOLATILITY PRICED ON KOSPI 200 INDEX OPTIONS ?

## Abstract

This study investigates whether stochastic volatility is priced on KOSPI 200 index options by using the delta-hedged gains on a portfolio of a long position in a call, hedged by a short position in the underlying asset, following Bakshi and Kapadia (2003). Contrary to other financial markets such as the S&P index options market, volatility risk is not systematically and consistently compensated on the KOSPI options. Rather jump fear influences most in determining KOSPI 200 option prices. Our results are consistent with extant literatures which have shown that Korean derivatives market is dominated by directional traders, and so there might be no hedging demands on option trades. In our research, we do not impose any specification on the stochastic processes of the underlying asset, volatility, and jumps, consequently setting our results free from misspecification errors.

*JEL classification:* G11; G12; J22

*Keywords:* KOSPI 200 index; KOSPI 200 index option; volatility risk premium; stochastic volatility/jump diffusion; risk-neutral skewness; risk-neutral kurtosis

# 1 Introduction

Since the seminal works of Black and Sholes (1976) and Merton (1976), a large number of option pricing models have been proposed to explain the dynamics of derivatives and the regularities such as the volatility smile/smirk, which could not be interpreted by Black-Sholes model (hereafter BS model). These endeavors can be classified into three groups: the deterministic volatility model [Cox and Ross (1976), Rubinstein (1994), Dupire (1994), and Derman and Kani (1994)], the stochastic volatility model [Hull and White (1987), Stein and Stein (1991), Heston (1993), and Heston and Nanci (2000)], and jump/diffusion model [Merton (1976), Bates (1991), and Pan (2002)].

Deterministic volatility models have some virtues that derivatives securities can be priced by no-arbitrage without any assumption to a pricing kernel, because markets are dynamically complete. In addition, they can easily fit the volatility smile (surface) by calibrating with respect to moneyness and maturity. Stochastic volatility models or jump/diffusion models, in contrast, enable us to capture additional risk factors such as volatility risk and/or jump risk which might affect option prices. Adding these risk factors makes the options pricing models more flexible.<sup>1</sup> Bakshi et al. (1997), Dumas et al. (1998), Bates (2000), and Jackwerth and Rubinstein (1998) have compared the empirical performances of these alternative option pricing models. Even though BS model is dominated by deterministic models as well as stochastic models, both pricing and hedging errors are too large to judge the superiority of each model.<sup>2</sup>

Recently, Bakshi et al. (2000), and Buraschi and Jackwerth (2001) found the evidence for an additional risk factor through a statistical method, which implies the rejection of deterministic volatility models. S&P 500 index options are not redundant assets spanned by the underlying asset and the risk-free investment, implying the existence of additional risk factors. Given these facts, there are two associated questions. The first is which factors

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<sup>1</sup>Another kind of explanation for pricing errors by the BS formula is related to the market imperfections such as discrete trades and transaction cost. Kim, Kim, and Ziskind (1994) and Hentschel (2003) have shown that noises by such a market imperfection can cause pricing errors reflected on the nonconstant volatility, and they are likely to make ITM and OTM options to be overpriced.

<sup>2</sup>Furthermore, the alternative models do not seem to explain fully both the underlying market and the options market, as shown in Ait-Sahalia, Wang, and Yared (2001), Anderson, Benzoni, Lund (2002), and Chernov and Ghysels (2000).

actually exist in the options market, and the second is how the additional risk factors are compensated.

One approach is to choose the model specification and to estimate the model both under the physical and under the risk-neutral measure. The difference between the parameter under the two measure infers the risk premia implied on option prices. Chernov and Ghysels (2000), Anderson et al. (2002), and Benzoni (2002) estimate a variety of the models with stochastic volatility and support the negative volatility risk premia. On the other hand, Pan (2002) estimates the stochastic volatility/jump diffusion model, and concludes that the volatility risk premia are not significantly different from zero but rather jump size risk requires a considerable premium. Like these, the different choice of models induces the different results for the risk premia, and the mis-specification of models can thereby lead us to incorrect conclusions.

Another approach is based on the hedging performance, which reflects the compensation for bearing the additional risk besides stock price risk. This kind of studies usually do not depend on the choice of models, and free from the mis-specification errors. For instance, by examining the sign and size of delta-hedged gains, Bakshi and Kapadia (2003) have found out the additional risk factor, i.e. the volatility risk, in S&P index options market without an exact specification. Delta-hedged gains on the portfolio of a long position in a call, hedged by a short position in the underlying asset, are systemically and consistently negative, which supports a negative risk premium. In other words, if the volatility risk is priced, its risk premium increases (decreases) the risk-neutral drift of a volatility process and thus increases (decreases) the option prices.

The negative risk premium corresponds to a hedging effect against significant market declines. The volatilities become higher as market moves down [French, Schwert, and Stambaugh (1987) and Glosten, Jagannathan, and Runkle (1993)]. Since option values are proportional to the volatility of the underlying asset, option buyers are ready to pay a premium for protecting market depreciation. Such an argument is consistent with the empirical evidence that the BS implied volatilities are higher than the historical volatilities of the underlying asset over the world derivatives market as well as the S&P index options markets [Jackwerth and Rubinstein (1996)].

Compared with other index options markets, however, KOSPI (Korean Composite Stock Price Index) 200 index options market of Korea Stock Exchange (KSE) differs from the

established markets in several respects. Following differences invoke the question of whether the volatility risk premia influence the option values.

- BS implied volatilities of ATM (at-the-money) options on KOSPI 200 index are not apparently higher than the realized volatilities of the underlying asset.<sup>3</sup> In the sample from 1999:01 to 2006:07, mean (standard deviation) implied volatilities and historical volatilities of at-the-money options are 30.57% (12.74%) and 30.36% (10.73%), respectively. That is, ATM KOSPI 200 index options are consistently and systematically not overpriced.
- The excess skewness and kurtosis of the KOSPI 200 index distribution to the normal distribution are relatively small, compared to those of developed countries' indices. As shown in Table 1, physical skewness and kurtosis for daily returns on KOSPI 200 index from 1999:01 to 2006:07 are about -0.3 and 5.6, respectively.<sup>4</sup> In contrast, distributions of S&P indices are severely left-skewed and fat-tailed [Anderson, Benzoni, and Lund (2002)]. The relatively low level of higher moments of KOSPI index returns is likely to reduce the ability to hedge against the underlying asset's downward movement by holding related options.
- As shown in Table 2, over 50 percent of the trades in KOSPI 200 index options market is composed of trades by individual investors, far higher than those in other markets.<sup>5</sup> Individuals tend to prefer contracts involving smaller cash outlays, and usually do not have large and well-diversified portfolios to reduce the idiosyncratic risk, hence having the low level of open interests. Under this circumstance, it is hard to expect the hedging effect, i.e., negative volatility risk premium, on option prices. A considerable proportion of extremely large trading volume seems the results from the speculation

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<sup>3</sup>Mentioned in Jackwerth and Rubinstein (1996), and Bakshi and Kapadia (2003), BS implied volatilities of ATM S&P 500 index options is consistently and systematically higher than realized volatilities.

<sup>4</sup>This phenomenon might be due to the existence of price limit in KSE, which tends to restrict the extreme price changes. Thus it is likely to decrease the excess skewness and kurtosis of the return distribution.

<sup>5</sup>In Japan, individual investors account for 12% of customer trading (ie excluding inter-dealer transactions) in Nikkei 225 futures and 8% in options, while their share in the more heavily traded TOPIX contracts is essentially zero. Comparable data for the United States and Europe do not exist, but all the available evidence suggests that individual investors account for only a small proportion of derivatives trades. (BIS Quarterly Review (2005))

demands by directional traders, not from hedgers [BIS quarterly report (2003), and Kang and Park (2007)].

Given these curiosities invoked by the facts above, the present goal of this paper is to investigate empirically whether volatility risk requires the premium in KOSPI index options market. Our empirical methods are supported by Bakshi and Kapadia (2003), in which the discrete delta-hedged gains are used to test the existence of the volatility risk premia. This approach do not impose any specification on the pricing kernel and the volatility process, thereby making the results free from the misspecification errors. The setup is a portfolio of a long position in a call option, daily-hedged by a short position in the stock, which is not hedged for other risk factors than the market risk. We call the gain on such a portfolio as the “delta-hedged gains.” If volatility is deterministic or if volatility is stochastic but not compensated, the delta-hedged gains will be zero. Otherwise, the gains are skewed subject to the sign and magnitude of a risk premium. The dataset used is the KOSPI 200 index and its related options. The KOSPI 200 is a market-capitalization-weighted-index composed of 200 major stocks in the Korea Stock Exchange (KSE), and reflects about eighty percent of the total market capitalization.

Our empirical results provide evidences supporting the following arguments.

- First, the delta-hedged gain on the portfolio of buying the ATM call, hedged by the underlying stock (KOSPI 200 index), is not far from zero. Second, there seems to be no relation between the delta-hedged gains and option vega. These mean that, in KOSPI 200 index options market, the volatility risk is not priced. Or even if the volatility risk is priced, its magnitude is very small.
- Rather, a jump risk is the most important factor in the KOSPI 200 index options market.<sup>6</sup>

The KSE, which we explore, is the most important Asian emerging market. As illustrated in Table 3, with successfully overcoming the 1997 Asian financial crisis, KSE has

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<sup>6</sup>This seems to be contrary with the results of extant literatures [Kim and Kim (2004)]. Kim and Kim (2004) compare the empirical performance of alternative option pricing models in terms of hedging and forecasting under the risk-neutral measure, and conclude that in KOSPI 200 index options market, only assuming the stochastic volatility is better than assuming both the stochastic volatility and the jump component on the risk-neutral distribution implied in options prices. However, their results is not linked to the physical process, and hence they cannot say about the volatility risk premium.

an incredible growth in the size, globalization, and capitalization. According to the WFE (World Federation of Exchanges), the KSE is the 14th largest stock exchange market in the world in 2005. In terms of turnover ratio, the KSE is as highly liquid as the most developed equity markets. As a result, a substantial proportion of Korean equities are held by foreign investors. Foreign investors from 91 countries hold about 40 percent of the KSE market capitalization at the end of 2005. More importantly, the KOSPI 200 index options in KSE are obviously the most actively traded derivatives in the world, although the size of the underlying market is smaller than those of the developed equity markets. Table 4 shows the top 5 exchange-traded derivatives in terms of the trading volume from 2000 to 2005. The trading volume of KOSPI index options is over 6 times higher than that of secondly liquid derivatives.

Besides, Korean derivatives market is somewhat different from more developed markets [BIS quarterly review (2003)]. First, trading is heavily geared toward options, which accounted for 93% of trading volume in the third quarter of 2005. Secondly, the high trading volume in Korea does not result in a large open position. The open interest in Korean derivatives contract at the end of 2005 amounts to a mere \$ 64 billion, whereas that in US contracts stands over 50 times higher at \$3.3 trillion. Both the predominance of options and the low level of open interest are related to the third characteristic, i.e., the composition of individual investors in derivatives trades. KOSPI 200 index options market is driven mainly by individuals inclining toward directional traders [Kang and Park (2007)].

In addition, these days electrical trading system disperses over the world. This trend will change the composition of market participants. Foremost, individual investors are likely to possess more opportunities to contact directly to the trade-exchange, and then their trading composition will increase. For instance, in Korea where e-trading systems are mostly well-facilitated and individuals can complete most trading by e-trading system such as Home Trading System (HTS), the trading frequency of individual investors is extremely high. Like this, the dispersion of e-trading system will raise the individual investors' trading volume, and hence the studies about Korean derivatives market, mainly driven by individuals, can give good implications on the possible changes in more developed markets.

The remainder of the paper is organized as follows. Section 2 represents the properties of the delta-hedged gains, and shows how to retrieve the risk-neutral skewness and kurtosis from option prices. In Section 3, we describes the KOSPI 200 index options market and the

criteria screening the used dataset, and includes the way of how to calculate the volatility. Section 4 documents the empirical results associated with the delta-hedged gains. We test whether delta-hedged gains are far from zero statistically, and whether it can be interpreted by the relationship with option vega, moneyness, and maturities. Section 5 explores the jump effect on option prices. Lastly, Section 6 concludes and discusses future research issues.

## 2 Theoretical Background

In this section, we simply describe the theoretical backgrounds of this paper, and the related testable implications. Based on the results of Bertsimas, Kogan, and Lo (2000), Bakshi and Kapadia (2003) develop the properties of the “delta-hedged gains,” when the volatility is stochastic. Bakshi, Kapadia, and Madan (2003) formalize the way of retrieving the risk-neutral skewness and kurtosis from out-of-the-money options, as a proxy of the jump fears. These theoretical foundations are the cornerstones of our empirical analysis.

### 2.1 Delta-Hedged Gains and Risk Premium

Denote the stock price and its volatility by  $S_t$  and  $\sigma_t$ , respectively, whose processes are as follows.

$$\frac{dS_t}{S_t} = \mu_t(S, \sigma)dt + \sigma_t dZ_t^1, \quad (1)$$

$$d\sigma_t = \theta_t(\sigma)dt + \eta(\sigma)dZ_t^2, \quad (2)$$

where the correlation between two Brownian motions is  $\rho$ .

Let  $C(t, \tau; K)$  represent the price of a European call maturing in  $\tau$  periods from time  $t$  with exercise price  $K$ ;  $\Delta(t, \tau; K)$  indicates the corresponding option delta. The delta hedged gains,  $\Pi_{t,t+\tau}$ , on the hedged option portfolio is given by

$$\Pi_{t,\tau} = C_{t+\tau} - C_t - \int_t^{t+\tau} \Delta_u dS_u - \int_t^{t+\tau} r(C_u - \Delta_u S_u) du, \quad (3)$$

where  $S_t$  is the underlying stock price at the time  $t$  and  $r$  is the riskfree rate at time  $t$ .

Under the BS economy with the constant volatility, the gains on the portfolio which is continuously hedged will be zero over every horizon. Even if it is discretely hedged, the



distribution of the gains converges to zero asymptotically [Bertsimas, Kogan, and Lo (2000)]. Consider an option portfolio, hedged discretely  $N$  times over the life of options. Its delta-hedged gains are represented as

$$\pi_{t,t+\tau} = C_{t+\tau} - C_t - \sum_{n=0}^{N-1} \Delta_{t_n} (S_{t_{n+1}} - S_{t_n}) - \sum_{n=0}^{N-1} r_n (C_t - \Delta_{t_n} S_{t_n}) \frac{\tau}{N}, \quad (4)$$

where  $t_0 = t$ ,  $t_N = t + \tau$ .<sup>7</sup> Bertsimas, Kogan, and Lo (2000) derived the asymptotic distribution of the discrete delta-hedged gains, and showed that the delta-hedged gains are distributed around zero regardless of the rebalancing frequency.

Similarly, consider the economy wherein the volatility is stochastic. According to Bakshi and Kapadia (2003), the delta-hedged gains would be satisfied the following relationships, given the volatility risk premia,  $\lambda$ .<sup>8</sup>

- The delta hedged-gains,  $\Pi_{t,t+\tau}$ , is given by

$$E_t [\Pi_{t,t+\tau}] = \int_t^{t+\tau} E_t \left( \lambda_u \frac{\partial C_u}{\partial \sigma_u} \right) du, \quad (5)$$

where  $\lambda_t = -\text{cov} \left( \frac{dm_t}{m_t}, d\sigma_t \right)$ , and  $m_t$  is the pricing kernel.

- If the volatility risk does not require the premia, the discrete delta-hedged gains,  $\pi_{t,t+\tau}$ , are, on average, zero with the order of  $O(1/N)$ .

$$E_t (\pi_{t,t+\tau}) = O(1/N) \quad (6)$$

In consequence, if the volatility is stochastic and its risk is compensated, the delta-hedged gains are affected by the volatility risk premia,  $\lambda_t$ , and the option vega,  $\partial C_t / \partial \sigma_t$ . We apply such a relationship into KOSPI 200 index options to find the evidence of the volatility risk premia.

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<sup>7</sup>This portfolio gain (4) does not satisfy the self-financing strategy, since  $\int r_u C_u du$  is approximated into  $\sum_{n=1}^N r_n C_t \frac{\tau}{N}$ . Thus we redo with self-financing portfolio, but the results are not changed. The reported results, hereafter, are based on the self-financing portfolio gains.

<sup>8</sup>The rigorous proof of above relationships is reported in Bakshi and Kapadia (2003). For saving the space, we do not rewrite it.

## 2.2 Risk-Neutral Skewness and Kurtosis as a Proxy of Jump Component

Jump risk is another factor which can induce the underperformance of the delta-hedged gains [Bakshi and Kapadia (2003)]. In this section, we simply describe the theoretical mechanism of how jump risk influences the delta-hedged gains and how we measure the extent of jump risk.

The linkage between jump fears and delta-hedged gains is a little complicate. To comprehend the impact of jump fears on delta-hedged gains, consider the simple jump-diffusion model for a stock process [Bate (1991, 1996, 2000), and Pan (2002)].

$$\frac{dS_t}{S_t} = (\mu_t - \Lambda\mu_J) dt + \sigma_t dZ_1 + J_t dq_t, \quad (7)$$

$$d\sigma_t = -\kappa\sigma_t dt + \nu dZ_2, \quad (8)$$

$$\text{Cov}(dZ_1, dZ_2) = \rho dt, \quad (9)$$

$$\text{Prob}(dq_t = 1) = \Lambda dt, \quad (10)$$

$$\ln(1 + J_t) \sim N(\ln(1 + \mu_J) - \frac{1}{2}\delta^2, \delta^2), \quad (11)$$

where  $\sigma_t$  represents the instantaneous return volatility at time  $t$ ,  $q$  is a Poisson process with intensity  $\Lambda$ , and  $J_t$  is the random percentage jump conditional on a jump occurring at time  $t$ .

Under the consideration of the jump risk, the delta-hedged gains are given by

$$\begin{aligned} E_t[\Pi_{t,t+\tau}] &= \int_t^{t+\tau} E_t \left( \lambda_u \frac{\partial C_u}{\partial \sigma_u} \right) du + \mu_J^* \Lambda^* \int_t^{t+\tau} E_t \left[ \frac{\partial C_u}{\partial S_u} S_u \right] du \\ &\quad - \Lambda^* \int_t^{t+\tau} \int_{-\infty}^{\infty} (C_u(S_u(J+1)) - C_u(S_u)) \text{prob}^*(J) dJ du \\ &\quad + \Lambda \int_t^{t+\tau} \int_{-\infty}^{\infty} (C_u(S_u(J+1)) - C_u(S_u)) \text{prob}(J) dJ du, \end{aligned} \quad (12)$$

where  $\text{prob}(J)$  is the physical density represented by (11), and  $\text{prob}^*(J)$  is the risk-neutral density. The jump risk premia  $\Lambda/\Lambda^*$  and  $\mu_J - \mu_J^*$  reflect the compensation required for bearing the systematic jump risk. Assume that jumps occur only in the stock market and the representative agent has constant relative risk aversion (CRRA) utility. When average jumps are negative, the risk-neutral jump frequency and the risk-neutral average drop size are likely to exaggerate the downside jump risk:  $\Lambda^* > \Lambda$ ,  $\mu_J^* < \mu_J$  [Bate (1991, 2000)]. The

first term in (12) reflects the effect of a volatility risk premium, and the other terms reflect the effect of the jump risk.

When  $\lambda_u = 0$ , the sign of delta-hedged gains is indefinite.<sup>9</sup> The delta-hedged gains depend on all of  $\Lambda, \Lambda^*, \mu_J, \mu_J^*, \partial C_u / \partial S_u$ , and  $\Delta C(\cdot)$ . In particular, the risk-neutral jump density  $\Lambda^*$  and  $\mu_J^*$  are related to the marginal utility of nominal wealth for the representative investor, which is difficult to be defined on empirical studies. Only with assuming that the third term and the fourth term are comparable, the delta-hedged gains are determined mainly by the second term, thereby being negative.

We adopt the risk-neutral skewness and kurtosis as a proxy for jump fears [Bakshi, Kapadia, and Maddan (2003)].<sup>10</sup> This step relies on the possibility of using only OTM calls and puts. We do not impose any specific structure on the jump process, leading our results free from misspecification errors.

Specifically, the cubic contract, which is a specific position simultaneously involving a long position in OTM calls and a short position in OTM puts, can quantify the return asymmetry. For example, when the risk-neutral distribution is left-skewed, the cost of holding puts is larger than that of holding calls, thereby reflecting the degree of the skewness. Similarly, the price of quartic contract can be transformed to the kurtosis.

According to Bakshi et al. (2003), the  $\tau$ -period risk-neutral return skewness is given by

$$\begin{aligned} \text{SKEW}(t, \tau) &\equiv \frac{E_t^* \left[ (R_{t,t+\tau} - E_t^*[R_{t,t+\tau}])^3 \right]}{\left\{ E_t^* (R_{t,t+\tau} - E_t^*[R_{t,t+\tau}])^2 \right\}^{3/2}} \\ &= \frac{e^{r\tau} W(t, \tau) - 3\mu(t, \tau)e^{r\tau} V(t, \tau) + 2\mu(t, \tau)^3}{[e^{r\tau} V(t, \tau) - \mu(t, \tau)^2]^{3/2}}. \end{aligned} \quad (13)$$

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<sup>9</sup>Bakshi and Kapadia (2003) assert that delta-hedged gains are negative, if the mean jump size is negative and if only a jump size is priced. This is supported by following argument:  $\int_{-\infty}^{\infty} C_u(S_u(J+1)) \text{prob}^*(J) dJ - \int_{-\infty}^{\infty} C_u(S_u(J)) \text{prob}(J) dJ$  is positive. Under  $\Lambda^* > \Lambda$ , and  $\mu_J^* < \mu_J$ , however, the integral term is likely to be negative rather than positive to call options. Thus the total delta-hedged gains, represented on equation (24) in the page 538 of Bakshi and Kapadia (2003), has a possibility to be positive albeit very small. Therefore it is hard to say that negative mean jumps result in the negative delta-hedged gains *absolutely*.

<sup>10</sup>Alternatively, we can also employ Bates (1991, 2000)'s skewness premium measure as a proxy. The skewness premium is also free from any specification error.

The risk-neutral kurtosis is given by

$$\begin{aligned} \text{KURT}(t, \tau) &\equiv \frac{E_t^* [(R_{t,t+\tau} - E_t^*[R_{t,t+\tau}])^4]}{\left\{E_t^* (R_{t,t+\tau} - E_t^*[R_{t,t+\tau}])^2\right\}^2} \\ &= \frac{e^{r\tau} X(t, \tau) - 4\mu(t, \tau)e^{r\tau} W(t, \tau) + 6e^{r\tau} \mu(t, \tau)^2 V(t, \tau) - 3\mu(t, \tau)^4}{[e^{r\tau} V(t, \tau) - \mu(t, \tau)^2]^2}, \end{aligned} \quad (14)$$

where  $V(t, \tau)$ ,  $W(t, \tau)$ , and  $X(t, \tau)$  are the prices of the volatility contract, the cubic contract, and the quartic contract, respectively. The expected rate of returns from  $t$  to  $t + \tau$ ,  $\mu(t, \tau)$ , are obtained by the Taylor expansion. To compute above equations from finite option data, Riemann integral should be approximated discretely, like Bakshi and Madan (2006).

$$V(t, \tau) = \int_{S_t}^{\infty} \frac{2(1 - \ln(K/S_t))}{K^2} C(t, \tau; K) dK + \int_0^{S_t} \frac{2(1 + \ln(S_t/K))}{K^2} P(t, \tau; K) dK, \quad (15)$$

$$\begin{aligned} W(t, \tau) &= \int_{S_t}^{\infty} \frac{6 \ln(K/S_t) - 3(\ln(K/S_t))^2}{K^2} C(t, \tau; K) dK \\ &\quad - \int_0^{S_t} \frac{6 \ln(S_t/K) + 3(\ln(S_t/K))^2}{K^2} P(t, \tau; K) dK, \end{aligned} \quad (16)$$

$$\begin{aligned} X(t, \tau) &= \int_{S_t}^{\infty} \frac{12(\ln(K/S_t))^2 - 4(\ln(K/S_t))^3}{K^2} C(t, \tau; K) dK \\ &\quad + \int_0^{S_t} \frac{12(\ln(S_t/K))^2 + 4(\ln(S_t/K))^3}{K^2} P(t, \tau; K) dK, \end{aligned} \quad (17)$$

$$\mu(t, \tau) \simeq e^{r\tau} - 1 - \frac{e^{r\tau}}{2} V(t, \tau) - \frac{e^{r\tau}}{6} W(t, \tau) - \frac{e^{r\tau}}{24} X(t, \tau). \quad (18)$$

In summary, the risk-neutral higher moments are used as a proxy for jump fears. If jump fears are implied on options prices, it tends to drive the delta-hedged gains to on average negative, and the risk-neutral skewness and kurtosis can explain the variation of those gains.

## 2.3 Testable Implications

This section includes three testable implications related to the delta-hedged gains.

- First, if the volatility risk premium is nonzero, the delta-hedged gains for ATM options would be also nonzero.

From the equation (5), the delta-hedged gains reflect the sign and magnitude of the volatility risk premium, given a fixed vega. To fix the option's vega, we employ only a fixed maturity of ATM options. The gains for ATM options are most sensitive to a risk premium. Moreover,

the ATM options are known to be most irrelevant with market frictions. Thus the ATM option prices are little contaminated, and the ATM-based-results are reliable.

- Second, if the volatility risk requires the premia, the magnitude of absolute delta-hedged gains (loss) coincides to the level of option vega with respect to moneyness.

To conform the first hypothesis that the volatility risk is compensated in KOSPI 200 index options, we investigate the relationship between the delta-hedged gains and option vega. Since the delta-hedged gains are determined by the risk premium,  $\lambda_t$ , and the option vega,  $\frac{\partial C}{\partial \sigma}$ , the difference of option vega induces the difference of delta-hedged gains.

According to Ito-Taylor expansion of delta-hedged gains shown in Bakshi and Kapadia (2003), the delta-hedged gains,  $E_t(\Pi_{t,t+\tau})$ , are related to the current underlying asset, the level of volatility, and the parameters governing the option vega, i.e., maturity and moneyness. For a broad class of option pricing models, option prices are homogenous of degree one in the stock price,  $S_t$ , and exercise price,  $K$  [Merton (1973)]. Therefore the option vega,  $\partial C_t / \partial S_t$ , is also linearly correlated with the stock price, given a fixed moneyness and maturity. By *Lemma 1* of Bakshi and Kapadia (2003), the delta-hedged gains can be rewritten as

$$E_t [\Pi_{t,t+\tau}] = S_t \times g_t(\sigma_t, \tau, y; \lambda_t), \quad (19)$$

where  $g(\cdot)$  is the model specific function of volatility,  $\sigma_t$ , time to maturity,  $\tau$ , and moneyness,  $y$ , given the risk premium  $\lambda_t$ .  $E_t [\Pi_{t,t+\tau}] / S_t$  varies with the physical volatility in the time series, and with the option moneyness in the cross section.

Hence such a relation can be applied to the cross-sectional analysis. Once we fix  $\sigma_t$ , the delta-hedged gains change according to the option vega, which varies with moneyness.<sup>11</sup> The option vega is maximized for at-the-money, and so are the delta-hedged losses (gains) for at-the-money. On the other hand, the vega and the losses (gains) are minimized for the away-from-the-money. If we find these relations between delta-hedged gains and moneyness, we can reject the hypothesis that the volatility risk premium is zero.

- Third, if jump fears are implied on option prices, the coefficients of risk-neutral skewness and/or kurtosis in the regression are significant.

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<sup>11</sup>It is important to fix  $\sigma$ , because the option price is nonlinear in  $\sigma_t$  for away-from-the-money strikes.

Jump fears can induce the under-or-over performance of delta-hedged gains. A jump fear can dichotomize the risk-neutral distribution from the physical distribution, thereby change the delta-hedged gains even without a volatility risk premium [Bates (2000), and Pan (2002)]. Usually, the risk-neutral distribution is more volatile and left-skewed, and leptokurtic than the physical distribution [Rubinstein (1994), Jackwerth (2000), and Bakshi, Kapadia, and Madan (2003)]. The mean jump size governs the risk-neutral skewness, while the jump intensity determines the risk-neutral kurtosis. Thus the higher-order risk-neutral moments, retrieved using a position in out-of-money calls and puts, are applied as a proxy of jump fears.

## **3 Data Description**

### **3.1 KOSPI 200 Index Options Market**

KOSPI 200 index options are written on the KOSPI 200 index, consisting of 200 blue-chip stocks in KSE. Introduced in July 7 1997, KOSPI 200 options market has become the most active options market in the world in terms of the trading volume, despite its short history. Recently, trading volume reaches to 2,535 million contracts, which is tremendously high compared with those in other derivatives markets.

Three consecutive near-term delivery months and one additional month from the quarterly cycle (March, June, September and December) make up four contract months. The expiration day is the second Thursday of each contract month. Option contract per each month has at least five strike prices. The number of strike prices may, however, increase according to the price movement. Trade in the KOSPI 200 index options is European and thus contracts can be exercised only on the expiration date. The KOSPI 200 index options market opens at 9:00 and trades until 15:05 continuously. Prior to 9:00 and from 15:05 to 15:15, options are traded on a single price, and the trade closes at 15:15. On the other hand, the underlying market closes at 15:00.

### **3.2 Screening Criteria**

We employ daily observations on KOSPI 200 index options. To avoid the synchronizing problem occurring in selecting the index and corresponding options, both prices are captured

at 14:50 on every trading day.<sup>12</sup>

The options used are screened by the following criteria. First, only the option data from January 1 1999 to July 31 2006 are used. Until 1990s, Korean interest rate was very high, but so were not the average stock returns. This causes a negative excess stock return for a certain period. More importantly, since January 1 1999, Korea Stock Exchange (KSE), additionally, has excluded Saturday as well as Sunday in the trading days, thereby having 250 trading days a year.<sup>13</sup> To avoid the negative excess return and the change of trading days, the data prior to 1999 are eliminated. Second, we delete the options that violate following arbitrage bounds.

$$S_t e^{-d\tau} \geq C_{t,\tau} \geq S_t e^{-d\tau} - e^{-r\tau} K, \quad (20)$$

$$e^{-r\tau} K - S_t e^{-d\tau} \geq P_{t,\tau} \quad (21)$$

Third, the options with maturities less than 10 (trading days) and longer than 40 (trading days, two months) are excluded due to a very low trading volume. Fourth, we also eliminate the option whose implied volatilities arrange less than 5 % or more 95% to reduce the impact induced by mispriced data. Fifth, we delete the deep-in-the-money options (with price lower than 0.03p or moneyness ( $S/K$ ) higher than 1.1) and deep-out-of-the-money options (with price higher than 15p or moneyness ( $S/K$ ) lower than 0.9). The number of option samples satisfying above criteria are 19,987 calls and 20,088 puts.

We get the index dividend yields from the website of KSE.<sup>14</sup> In addition, we use the call rate<sup>15</sup> as the risk-free rate.

### 3.3 The Historical Volatility

For robustness, we adopt two volatility estimates for KOSPI 200 index returns: the GARCH(1,1) and the sample standard deviation [Bakshi and Kapadia (2003)].

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<sup>12</sup>We have adopted various sampling times (2:00, 2:30, and others), but the results are similar across all sampling times.

<sup>13</sup>For a reference, the total trading days before 1999 are about 290.

<sup>14</sup>[www.kse.or.kr](http://www.kse.or.kr) The dividend yields of the KOSPI 200 index are calculated as the total dividend from the KOSPI 200 index constituents over the total market value of the KOSPI 200 index constituents. KSE has updated this dividend yield monthly.

<sup>15</sup>The short-term interbank interest rate offered by the Bank of Korea

- GARCH (1,1)

$$R_{t-1,t} = \bar{R} + \epsilon_t, \quad (22)$$

$$\sigma_t = a_0 + a_1 \epsilon_{t-1}^2 + a_2 \sigma_{t-1}^2 \quad (23)$$

$$\epsilon_t \sim \text{i.i.d } N(0, \sigma_t) \quad (24)$$

$$\text{VOL}_t = \sqrt{\frac{250}{\tau} \sum_{n=t-\tau}^t \hat{\sigma}_n^2}, \quad (25)$$

where the  $\tau$ -period return is defined as  $R_{t,t+\tau} \equiv \log(S_{t+\tau}/S_t)$ ;  $\sigma_t$  is the conditional volatility;  $\hat{\sigma}_n$  is the evaluated value by GARCH estimation.

- Sample standard deviation

$$\text{VOL}_t = \sqrt{\frac{250}{\tau} \sum_{n=t-\tau}^t (R_{n-1,n} - \bar{R})^2} \quad (26)$$

where  $\bar{R}$  is the average daily return for a nonoverlapping period.

## 4 Empirical Results

In Section 4.1 we documents the empirical and statistical properties of delta-hedged gains on a KOSPI 200 call option portfolio. In Section 4.2, by using the relationship between delta-hedged gains and option vega, we investigate the existence of a volatility risk premium cross-sectionally.

### 4.1 Delta-Hedged Gains and Risk Premium

As shown in equation (4), delta-hedged gains for each call option are calculated by a long position in each call at date  $t$ , daily-hedged by a short position in the underlying asset as much as the option delta,  $\partial C/\partial S$ , until the maturity date  $t + \tau$ .

For tractability, a delta-hedge ratio,  $\Delta_{t_n}$ , is implemented as the BS hedge ratio,  $N(d_1)$ , where  $N(d_1)$  is the cumulative normal distribution and

$$d_1 = \frac{\log(S_t/K_t) + \left(r_n + \frac{1}{2}\sigma_{t,t+\tau}^2\right) \tau_n}{\sigma_{t,t+\tau} \sqrt{\tau_n}}. \quad (27)$$



Under the allowance of time-varying volatility, BS delta will bias the delta-hedged gains if the volatility is correlated with the stock return. As proven through the simulation by Bakshi and Kapadia (2003), however, the bias by the usage of the BS delta is negligible.<sup>16</sup> They show for 30-day options, the mean  $\pi/S$  is -0.0018% with the stochastic volatility hedge ratio versus 0.0022% with BS hedge ratio. Thus the usage of BS hedge ratio as the delta-hedge ratio is reasonable.

The volatilities needed to compute the BS delta are calculated by two alternative methods, i.e., the GARCH (1,1) and the sample standard deviation, reported in Section 3.3. Since both estimates move together very closely, we report only the results based on the GARCH (1,1) volatilities.<sup>17</sup>

Table 5 presents the descriptive statistics of delta-hedged gains on the KOSPI 200 index call option portfolio, classified by moneyness and maturity.<sup>18</sup> The reported numbers are the average delta-hedged gains  $\pi_{t,\tau}$ , the delta-hedged gains scaled by the index level  $\pi_{t,t+\tau}/S_t$ , and delta-hedged gains scaled by the call price  $\pi_{t,t+\tau}/C_t$ . Panel A shows the gains over the total sample period 1999:01-2006:07. Over the entire ranges of moneyness and maturities, the delta-hedged strategy loses the money about 0.38 at maximum, which can be translated to 0.4 percent of the index level. Roughly speaking, the delta-hedged losses for ATM options are not higher, and it is hard to find any pattern with regard to maturities and moneyness consistent to a volatility risk premium. This is contrary to the results for S&P 500 index

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<sup>16</sup>Branger and Schlag (2004) have investigated the impact of the discrete trading and the use of BS delta through the simulation. They have shown that the discrete trading and the model mis-specification may cause the standard test to yield unreliable results under the stochastic volatility model. For the hedging interval of one day, however, discretization errors can be negligible, so that all the error of the test procedure is due to the choice of the mis-specified delta. Even for mis-specification errors, the errors by the BS delta is very small over the range of time-to-maturity (shorter than 2 month), volatility level (around 0.3), and excess return (around 0.1) consistent with our option sample, as shown in Branger and Schlag (2004). Moreover, during the sample period, the correlation between the stock return dynamics and the volatility dynamics is much low compared to the parameter value (-0.65) set in Branger and Schlag (2004). This relatively low correlation increases the validity of the BS delta. Therefore our results based on delta-hedged gains do not lose a reliability.

<sup>17</sup>Over the total sample period, the mean and standard deviation of the volatilities estimated by the sample standard deviation method are 32.1% and 10.79% respectively, while those of the volatilities estimated by GARCH (1,1) are 30.59% and 12.71%.

<sup>18</sup>The results for put options are similar with those for call options. To save a space, we do not report them.

options shown in Bakshi and Kapadia (2003). Under the existence of volatility risk premia, the absolute delta-hedged gains for at-the-money and large maturities options should be higher due to the relationship with option vega. To guarantee that such results are not caused by the extreme values, we additionally include the last column,  $1_{\pi < 0}$  statistic, which measures the frequency of negative delta-hedged gains. The frequency of negative delta-hedged gains is comparable to the trend of the average losses, and thus such results are not because of extremes.

Panel B presents, for robustness, the mean delta-hedged gains for subsamples: Set 1 and Set 2 over 1999:01-2002:12 and the 2003:01-2006:07 sample periods, respectively. As shown in Panel B, we cannot also find any pattern with respect to moneyness or maturities consistent to the volatility risk premium. Only particular thing is the apparent difference of mean delta-hedged gains between subsample periods. The losses on Set 1 are twice or three times as high as those on Set 2. Without the volatility risk premium, one possible explanation for this phenomenon is jump fears proportional to the return volatility. When the intensity of jump occurrence increases in the level of volatility, the delta-hedged gains are magnified, because the second, third, and fourth terms in the equation (12) are proportional to volatilities.<sup>19</sup>

In reality, going through the Asian financial crisis and IMF (International Monetary Fund) relief loan at late 1990s, Korean financial market had been destabilized until the early 2000s. There had been plenty of bad news as well as good news. Korean financial market was very volatile and seemed to be risky. Furthermore, immediately after overcoming such disasters, the collapse of KOSDAQ market had induced the critical depression of financial markets, albeit trivial compared to the market crash at October '87. As such, jump fears are likely to become an important risk factor in Korean financial markets, as other financial markets have gone through [Jackwerth and Rubinstein (1996), Bate (2000), Chernov and Ghysels (2000), and Anderson et al. (2002)]. The more detail analysis for jump fears on KOSPI 200 index options follows in the later Section.

Next, we examine further specifically whether the delta-hedged gains, sign of which corresponds to that of volatility risk premium, are far from zero statistically. Unless the volatility

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<sup>19</sup>To support this argument, we find out that the delta-hedged losses for ATM options increase in the level of volatility regime, albeit not reporting. Furthermore, the volatility on Set 1 is approximately twice as high as that on Set 2, according to Table 1.

risk requires a risk premium, the delta-hedged gain for at least ATM options should have the same sign with the volatility risk premium. ATM options are not only most sensitive to the volatility risk premium, but also known to be unaffected by market imperfections, such as transaction cost or asymmetric pricing errors. The results based on ATM thereby lead our results to be uncontaminated by other pricing errors.<sup>20</sup>

Table 6 presents the mean delta-hedged gain and statistics for ATM calls with a fixed maturity of 20, 30, and 40 trading days. For 20, 30, and 40 day calls, the mean gains are  $\pi = -0.11, -0.19,$  and  $-0.0079,$  which correspond to t-statistic of  $-1.56, -2.16,$  and  $-0.07.$  Only for options with a maturity of 30 days, the delta-hedged gains are statistically significant at 95% significance level. Even the delta-hedged gains for 30 day calls are insignificant, once the data during the former subsample period (especially 1999 and 2000) are excepted.

For robustness, we additionally use the alternative method using the standard deviation of discrete delta-hedged gains. Under the BS economy wherein the expected return and volatility is given, the standard deviation of delta-hedged gains is known in Bertsimas, Kogan, and Lo (2000). As such, first, we derive  $\tilde{\pi}_{t,t+\tau}$  by standardizing each  $\pi_{t,t+\tau}$  by the corresponding standard deviation. Second, we compute the t-statistic as  $\sum \tilde{\pi}_{t,t+\tau} / \sqrt{N},$  where  $N$  is the number of observations. With the historical value of  $\mu = 0.128$  and  $\sigma = 0.34,$  the t-statistics are  $-0.17, -0.30,$  and  $-0.05.$  For other mean and volatility values, we cannot reject the hypothesis that the delta-hedged gains are zero, as shown in Table 6-B.<sup>21</sup> Notice that this method starts from the assumption for BS economy, and these results might be inconsistent with the true process with time-varying volatilities. When daily-updated GARCH volatilities are implemented, however, the bias will decrease and results will be reliable.

To sum up, the hypothesis proposing that the delta-hedged gains are zero cannot be rejected through two alternative test-statistics based on ATM options with a fixed maturity. It means that, in KOSPI 200 index options market, the volatility risk is likely to be unpriced. Or even if priced, its magnitude is very small.

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<sup>20</sup>Kim, et al. (1994) and Hentchel (2003) show that ITM and OTM options suffers from asymmetric pricing errors, which can generate the volatility smile/smirk.

<sup>21</sup>For a reference, this method is sensitive to the volatility of the underlying asset. For the volatile markets including Korea Stock Exchange, this standardized method has a low testing power.

## 4.2 Delta-Hedged Gains and Option Vega in the Cross Section

In former Section, we could not reject the hypothesis proposing that delta-hedged gains are zero statistically. It weakly means the absence of a volatility risk premium. In this Section, to ensure the above hypothesis, we explore the cross-sectional relationship between delta-hedged gains and option vega. As is shown in Section 2.3, the delta-hedged gains have a relation with option vega given a fixed  $\sigma_t$ , and hence the level of absolute delta-hedged gains should coincide to the level of the option vega. As such, we can test the existence of the volatility risk premia implicitly. The specification adopted is as follows.<sup>22</sup>

$$\text{GAIN}_t^i = \Psi_0 + \Psi_1 \text{VEGA}_t^i + \epsilon_t^i, \quad i = 0, \dots, I, \quad (28)$$

where  $\text{GAIN}_t^i \equiv \pi_{t,t+\tau}/S_t$  and  $\text{VEGA}_t^i$  is the option vega.

To regress the gains on the option vega as in (28), a proxy for VEGA has to be specified. Following Bakshi and Kapadia (2003), for robustness of the estimation, we adopt two option vegas:

$$\text{VEGA} = \begin{cases} \exp(-d_1^2/2) & \text{BS vega,} \\ |y - 1| & \text{Absolute moneyness,} \end{cases} \quad (29)$$

where  $d_1$  is as presented in equation (27). Since the BS vega reaches a maximum at nearest-the-money, a negative volatility risk premium corresponds to  $\Psi_1 < 0$ , and the magnitude of  $\Psi_0 + \Psi_1$  approximates the mean delta-hedged gains for ATM options. On the other hand, the absolute moneyness as a proxy of option vega reaches a minimum at nearest-the-money, and hence a negative volatility risk premium corresponds to  $\Psi_0 < 0$  and  $\Psi_1 > 0$ , and  $\Psi_0$  approximates the mean delta-hedged gains for ATM options.

For each estimation of Equation (28), it is necessary to fix the volatility. To do this, we divide the total sample into several volatility regimes with intervals of 5%. Each sample includes the data observed at dates where volatility is within one of these intervals. The reported results are based on the options with the maturity of 20 and 30 trading days. With two vega proxies, 36 distinct panels are used in our tests.

When we implement the regressions (28), merging the data observed at several dates to one panel makes it non-trivial. It is possible that the day-specific components exist in the

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<sup>22</sup>For easily comparing with the results for S&P index options market by Bakshi and Kapadia (2003), we try to use as same notations to those in Bakshi and Kapadia (2003) as possible.

delta-hedged gains, and day-specific components can mislead us to wrong conclusions. This economic issue can be resolved by the fixed effect model or the random effect model [Greene (1997)]. The fixed effect model adopts dummy variables as many as the number of included days, while the random effect model classifies the disturbance term into the date-specific component and the white noise. The random effect model will be biased if the day-specific component is correlated with regressors, but it is more efficient than the fixed effect model otherwise. We prove that there is no correlation among them through the Hausman test for the fixed effect model versus the random effect model. The following results are, therefore, based on the random effect model where the coefficients are estimated by Feasible Generalized Least Square panel regression (FGLS).

Table 7 presents the coefficient values estimated by a random effect panel regression, which do not support the existence of a risk premium, as conjectured in Section 4.1. For 20-day options, only two coefficients  $\Psi_1$  among eight volatility regimes are significant at the significant level of 95% with respect to the BS vega. More seriously their signs are opposite to those expected by a negative volatility risk premium. For 30-day options, even though three coefficients are significant, their signs are also not apparent: two are negative, whereas the other is positive. In addition, the hypothesis that the mean delta-hedged gain is zero,  $\Psi_0 + \Psi_1 = 0$ , could not be rejected by the Wald test-statistic in most panels. These are opposite to those on S&P 500 options as in in Bakshi and Kapdai (2003), in which the sign of  $\Psi_1$  is significantly negative, and thus supports the negative volatility risk premium. The use of the absolute moneyness as a proxy for option vega also gives similar implications. For each panel with a maturity of 20-day and 30-day, just two coefficients,  $\Psi_1$ , are significant. Moreover only three coefficients  $\Psi_0$ , which indicate the mean delta-hedged gains, are significant among 16 panels. Therefore, our results do not support the existence of the volatility risk premium with respect to both proxies of option vega.

The absence of a risk premium on the volatility risk is comprehended as no hedging demands for market declines. As we mentioned in Section 1, the percent of individual investors in KOSPI 200 index options market is considerable, far higher than in other markets. Individuals tend to prefer contracts which involve smaller cash outlays and usually do not have large and well-diversified portfolio. Under this circumstance, it is hard to expect the hedging demand on option trades. That is, a considerable proportion of extremely large trading volume is likely to be the results from the directional traders, not from hedgers.

This is supported by Kang and Park (2007), in which they prove that KOSPI 200 index options market is driven by directional traders rather than volatility traders (hedgers) by using the information contents of net-buying pressure.

## 5 The Effects by Jump Fears

Although we have not found any evidence for the volatility risk premium on the delta-hedged gains, the delta-hedged gains tend to be negative in the majority of moneyness and maturities as shown in Table 5. Especially, it is manifest for out-of-money options. So we trace the reason of the negative delta-hedged gains. One possible reason is the fears for the market crash as occurred in '87 October. The delta-hedged gains reflect not only the volatility premium but also the effect for tail event [Jackwerth and Rubinstein (1996)]. For instance, the difference of the delta-hedged gains between subsample 1 and subsample 2 in Table 5 and 6 supports the jump fears implied on option prices. According to Bates (2000), Pan (2002), and Eraker, Johannes, and Polson (2003), if a jump process is dependent on the level of volatility, the delta-hedged gains are dependent, too. The rate of returns for subsample 1 are more volatile than those for the subsample 2, and the losses are over twice as much as those for the subsample 2.<sup>23</sup>

### 5.1 A Proxy of Jump Fears: Risk-Neutral Skewness and Kurtosis [Bakshi, Kapadia, and Madan (2003)]

Jump fears can dichotomize the risk-neutral distribution from the physical distribution. Usually, the risk-neutral distribution is more volatile, more left-skewed, and more leptokurtic than the physical distribution. Under the absence of the volatility risk premium, hence, the risk-neutral skewness and kurtosis can approximate the jump fears embedded on option prices [Jackwerth and Rubinstein (1996), Bates (2000), Bakshi and Kapadia (2003), and Bakshi, Kapadia, and Madan (2003)]. Generally, the frequency of asymmetric jumps is captured

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<sup>23</sup>Even though we conjecture that subsample 1 is exposed to jump risk more, the difference of physical kurtosis among subsamples is small. But notice that the movement by jumps are usually captured firstly by the volatility, and secondly by kurtosis. Hence that is not weird.

by the (risk-neutral) skewness, and the severity of jumps is captured by the (risk-neutral) kurtosis.

The risk-neutral skewness and kurtosis are based on the model-free approach of Bakshi et al. (2003), in which the risk-neutral higher moments are retrieved using out-of-the-money calls and puts. Using such proxies for jump fears, we examine the effect of jump fears on option prices. We adopt the following specification.<sup>24</sup>

$$\text{Gain}_t = \Omega_0^* + \Omega_1^* \text{Gain}_{t-1} + \Omega_2^* \text{SKEW}_t^* + \Omega_3^* \text{KURT}_t^* + \epsilon_t, \quad (30)$$

where  $\text{Gain}_t \equiv \pi_{t,t+\tau}/S_t$  is the option closest to at-the-money,  $\text{SKEW}^*$  is the risk-neutral skewness, and  $\text{KURT}^*$  is the risk-neutral kurtosis. To correct the serial correlation of the residuals, a lagged variable is included in the regression specification. The coefficients are estimated by OLS.

Before conducting the regression (30), we compare the mean risk-neutral higher moments with the physical higher moments. Table 8 presents the mean risk-neutral skewness and kurtosis implied on OTM options with a maturity of 20, 30, and 40 trading days. The option implied distribution is more left-skewed and more leptokurtic than the physical distribution. This difference is likely to be the indirect evidence of jump fears in the KOSPI 200 options market.

Table 9 presents the coefficients estimated from the specification (30) for at-the-money calls and puts with a maturity of 20, 30, and 40 trading days: Panel A is for calls and Panel B is for puts. For calls, the coefficients of the risk-neutral skewness are statistically significant in seven of nine estimations, and range between  $-0.0057$  and  $-0.0234$  for significant values. Even the panels with insignificant coefficients hold the significance except subsample 2. On the other hand, the risk-neutral kurtosis is likely to be insignificant for calls. Only three panel have a significance.

One interesting thing is the sign of the coefficient  $\Omega_2^*$  estimated from (30) for calls. As shown in Table 9-A, the sign is consistently negative in all estimates, which implies that the

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<sup>24</sup>The specification adopted is a little different from that adopted in Bakshi and Kapadia (2003). Bakshi and Kapadia (2003) examine whether a volatility risk premium (assumed to be proportional to the volatility) loses its significance under the consideration of jump fears, while we examine whether jump fears can solely explain the delta-hedged gains (option prices) without the volatility risk premium. This difference generates the change of specifications.

more negative skewed the risk-neutral distribution is, the higher the delta-hedged losses are. This is opposite to the results on S&P index options [Bakshi and Kapadia (2003)].<sup>25</sup>

To understand it, we consider the delta-hedged gains (12), which are derived under assuming jump fears. The sign of the coefficient depends on which term dominates in equation (12). If the delta-hedged gains are dominated by the second term  $\mu_J^* \Lambda^* \int_t^{t+\tau} E_t \left[ \frac{\partial C_u}{\partial S_u} S_u \right] du$  which is usually negative, the coefficient of the risk-neutral skewness will be positive. On the other hand, if they are dominated by the third term  $-\Lambda^* \int_t^{t+\tau} \int_{-\infty}^{\infty} (C_u(S_u(J+1)) - C_u(S_u)) \text{prob}^*(J) dJ du$  which is usually positive, the coefficient will be negative. In consequence, the negative coefficients estimated from KOSPI 200 options are consistent with the dominance of third term.

This might be due to the effect of directional traders, not hedgers. The hedging traders, who hold a call and hedge it by a short position in the underlying asset, are relatively less sensitive, because the direction of gains in both positions are opposite. On the other hand, the directional traders not hedging by the underlying asset will be touchy to jump fears. This overanxiety for jumps enlarges the price change of options  $C_u(S_u(J+1)) - C_u(S_u)$  under *the risk-neutral measure*, and hence makes the third term in (12) to be prominent.

Table 9-B presents the coefficients for puts. The coefficients of risk-neutral higher moments for put are more significant than those for call. Similar to calls, the skewness coefficients are significant for 7 in 9 panels, and their signs are all positive (opposite to calls) as expected. This implies that the negative skewness reduces the delta-hedged gains. The sign of kurtosis coefficients are consistently positive, and thus the increase in kurtosis raises the delta-hedged gains. One interesting thing is that the coefficients of the risk-neutral kurtosis for put are significant in all panels except one, contrary to the results for calls. This discrepancy between calls and puts reflects the importance of left-tail events. The left-tail event influences puts more directly than it does calls, since it increases the probability for puts to be in-the-money. This is consistent the fact that investors are severely averse to the large downward-losses.

To sum up, we show that a jump fear rather than the volatility premium plays an important role in explaining the underperformance of the delta-hedging strategy. As shown

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<sup>25</sup>Furthermore, the risk-neutral skewness loses the significance once volatility and kurtosis are omitted on the S&P 500 index options. On the other hand, in KOSPI 200 index options market skewness is solely significant without any other variable in all estimations.



in Table 5, the delta-hedged losses by the jump risk are about 15,000 won. This implies that the call prices with a consideration of jump fears are expensive as much as about 15,000 won, compared to those without a consideration of jump fears. Given the extremely large trading volume of KOSPI 200 call options about 2500 million per a year, the total amount impacts by the overvaluation of calls are as high as 37.5 trillion won, which amount to \$37.5 billion in U.S dollars.

## 6 Conclusion

This paper has investigated the existence of the volatility risk premium and jump fears on KOSPI 200 index options through the delta-hedged gains.

The main results of our paper can be summarized as follows, First, although the delta-hedged gains are slightly negative over the entire range of moneyness and maturities, we cannot reject that the delta-hedged gains for at-the-money options are zero statistically. Second, the delta-hedged gains are irrelevant to option vega. These means that the volatility risk is not priced on KOSPI 200 options or even if, its risk premium is very small. The absence of the volatility risk premium is the unique features in KOSPI 200 index options market compared to other derivatives market. Third, jump fears influence KOSPI 200 option prices apparently, which is proven by using the risk-neutral higher moments as the proxies of jump fears.

These results may be connected to regularities such as the volatility smile/smirk observed in KOSPI 200 options market. Although we do not mention the linkage with volatility smile or the slope of implied volatility directly, jump fears can have a critical power to explain them.

Moreover, another interesting feature of KOSPI 200 index options market is the amazingly higher portion of individual investors in derivatives trades. Such a high composition is likely to be due to the home-trading-system, i.e. well-established e-trading system, in Korean households. It gives individuals more chances to directly contact the trade-exchange, increasing individuals' trading portion. Now, the e-trading system is dispersing over the world, hence the studies about Korean derivative market may give implications on possible changes in the other financial markets.

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Table 1: Summary statistics for daily KOSPI 200 index returns

| Panel A: Full range of Sample (1999:01:01-2006:07:31) |           |          |          |           |        |       |               |
|---|-----------|----------|----------|-----------|--------|-------|---------------|
| Annual. Mean  | Std. Dev. | Skewness | Kurtosis | Autocorr. | Min    | Max   | JB-statistics |
| 12.80%  | 34.00%    | -0.2996  | 5.6041   | 0.0322    | -3184% | 2104% | 550.09        |
| Panel B: Subsample 1 (1999:01:01-2002:12:31)          |           |          |          |           |        |       |               |
| Annual. Mean  | Std. Dev. | Skewness | Kurtosis | Autocorr. | Min    | Max   | JB-statistics |
| 5.28%   | 40.66%    | -0.2455  | 4.5574   | 0.0267    | -3184% | 2104% | 108.88        |
| Panel C: Subsample 2 (2003:01:01-2006:07:31)          |           |          |          |           |        |       |               |
| Annual. Mean  | Std. Dev. | Skewness | Kurtosis | Autocorr. | Min    | Max   | JB-statistics |
| 21.3%   | 24.4%     | -0.3053  | 4.1355   | 0.006     | -1517% | 1259% | 60.17         |

Table 1 presents summary statistics for daily return of KOSPI 200 index from 1999:01:01 to 2006:07:31. Data set is also classified into two subsample panels: Panel B includes the observations from 1999:01:01 to 2002:12:31 ,while Panel C includes the observation from 2003:01:01 to 2006:07:31. Annualized return is calculated as  $\log(P_t/P_{t-1}) \times 250$ , where  $P_t$  is the KOSPI 200 index price. JB-statistics refer to Jarque-Berra test statistics.

Table 2: The compositions of KOSPI 200 index options market by the types of investors

| Year  | Call                 |                      |                      |                    | Put                  |                      |                      |                    |
|-------|----------------------|----------------------|----------------------|--------------------|----------------------|----------------------|----------------------|--------------------|
|       | individual           | institution          | foreigner            | else               | individual           | institution          | foreigner            | else               |
| 1999  | 51.82m<br>(69.03%)   | 20.29m<br>(27.03%)   | 1.56m<br>(2.09%)     | 1.39m<br>(1.85%)   | 61.40m<br>(72.41%)   | 20.44m<br>(24.11%)   | 2.09m<br>(2.47%)     | 0.86m<br>(1.02%)   |
| 2000  | 158.01m<br>(70.46%)  | 51.11m<br>(22.80%)   | 10.99m<br>(4.90%)    | 4.13m<br>(1.84%)   | 116.60m<br>(71.36%)  | 34.52m<br>(21.13%)   | 9.45m<br>(5.79%)     | 2.81m<br>(1.73%)   |
| 2001  | 652.80m<br>(70.84%)  | 196.69m<br>(21.35%)  | 56.34m<br>(6.11%)    | 15.62m<br>(1.70%)  | 536.76m<br>(74.03%)  | 127.68m<br>(17.61%)  | 46.91m<br>(6.47%)    | 13.74m<br>(1.90%)  |
| 2002  | 1360.55m<br>(64.54%) | 575.57m<br>(27.30%)  | 151.96<br>(7.21%)    | 19.90m<br>(0.94%)  | 1127.47m<br>(67.45%) | 399.69m<br>(23.91%)  | 127.85m<br>(7.65%)   | 16.62m<br>(0.99%)  |
| 2003  | 1612.80m<br>(54.41%) | 991.31m<br>(33.40%)  | 323.10m<br>(10.90%)  | 37.12m<br>(1.25%)  | 1496.72m<br>(55.21%) | 875.67m<br>(32.30%)  | 304.36m<br>(11.23%)  | 34.33m<br>(1.27%)  |
| 2004  | 1321.66m<br>(50.25%) | 991.23m<br>(37.68%)  | 294.91m<br>(11.21%)  | 22.58m<br>(0.86%)  | 1196.38m<br>(49.59%) | 891.01m<br>(36.93%)  | 306.59m<br>(12.71%)  | 18.72m<br>(0.78%)  |
| 2005  | 1182.95m<br>(45.14%) | 1065.15m<br>(40.64%) | 350.32m<br>(13.37%)  | 22.41m<br>(0.86%)  | 989.48m<br>(40.39%)  | 1064.88m<br>(43.47%) | 379.31m<br>(15.49%)  | 15.86m<br>(0.65%)  |
| 2006  | 921.54m<br>(38.14%)  | 1127.13m<br>(46.65%) | 353.86m<br>(14.64%)  | 13.80m<br>(0.57%)  | 885.07m<br>(36.69%)  | 1102.01m<br>(45.68%) | 410.39m<br>(17.01%)  | 15.00m<br>(0.62%)  |
| Total | 7262.15m<br>(50.62%) | 5018.52m<br>(35.95%) | 1543.07m<br>(11.05%) | 136.99m<br>(0.98%) | 6409.92m<br>(50.75%) | 4515.94m<br>(35.75%) | 1586.98m<br>(12.56%) | 117.97m<br>(0.93%) |

Table 2 shows the composition of trading volume of the KOSPI 200 index options by the types of investors. The trading volume is the number of contracts traded, including both sales contracts and purchase contracts. ‘m’ means the unit of million. The number in parenthesis indicates the percentage share of each group among total contracts. All data presented in Table are collected from Korea Stock Exchange, [www.krx.co.kr](http://www.krx.co.kr).

Table 3: The size, its growth rate, and the turnover ratio of global equity markets

|               | 2001   |             | 2003           |        | 2005        |                |        |       |       |
|---------------|--------|-------------|----------------|--------|-------------|----------------|--------|-------|-------|
|               | Size   | Growth rate | Turnover ratio | Size   | Growth rate | Turnover ratio |        |       |       |
| U.S. (NYSE)   | 11,027 | -3.6        | 86.9           | 11,329 | 2.7         | 89.5           | 13,311 | 17.5  | 99.1  |
| U.S. (NASDAQ) | 2,740  | -47.4       | 359.2          | 2,844  | 3.8         | 280.7          | 3,604  | 26.7  | 250.4 |
| England       | 2,165  | -24.2       | 83.8           | 2,460  | 13.6        | 106.6          | 3,058  | 24.3  | 110.1 |
| Germany       | 1,072  | -25.1       | 118.3          | 1,079  | 0.7         | 148.1          | 1,221  | 13.2  | 149.4 |
| Italy         | 527    | -27.6       | 113.4          | 615    | 16.7        | 134.8          | 798    | 29.8  | 160.0 |
| Japan         | 2,265  | -50.3       | 60             | 2,953  | 30.4        | 82.6           | 4,573  | 54.9  | 115.3 |
| Korea         | 195    | -36.3       | 218.7          | 298    | 52.8        | 193.1          | 646    | 116.8 | 152.3 |
| Taiwan        | 293    | -21.9       | 206.8          | 379    | 29.4        | 190.7          | 476    | 25.6  | 131.4 |
| Brazil        | 186    | -18.4       | 33.9           | 226    | 21.5        | 40.2           | 475    | 110.2 | 42.8  |
| Mexico        | 126    | -18.2       | 37.8           | 123    | -2.4        | 22.4           | 239    | 94.3  | 27.2  |
| Israel        | 58     | -7.9        | 27.3           | 69     | 19.0        | 33.0           | 123    | 78.3  | 45.1  |
| Turkey        | 47     | -58.4       | 178.8          | 68     | 44.7        | 211.9          | 162    | 138.2 | 169.9 |

Table 3 presents the size, its growth rate and the average monthly turnover ratio of global stock exchanges such as U.S., England, Japan and others from 2001 to 2005. The size is the market capitalization (in billions of US dollar), which is calculated as the product of the total number of issued shares of domestic companies and their respective closing prices at the end of each year. The growth rate means the growth rate of market globalization (in percentage form). Data on market capitalization do not include investment funds, warrants, ETFs, convertibles, foreign companies and include common and preferred shares. Monthly turnover ratio is the ratio of trading value during each month to market capitalization at the end of each month and is expressed in percentages on an annual basis. The stock exchanges for each country are as follows: U.S. (NYSE, NASDAQ), England (London Stock Exchange), Germany (Deutsche Börse), Italy (Borsa Italiana), Japan (Tokyo Stock Exchange), Korea (Korea Stock Exchange), Taiwan (Taiwan Stock Exchange Co.), Brazil (Sao Paulo Stock Exchange), Mexico (Mexican Exchange), Israel (Tel Aviv Stock Exchange), Turkey (Istanbul Stock Exchange). All data presented in Table are collected from WFE (World Federation of Exchange, [www.world-exchange.org](http://www.world-exchange.org)) and Korea Stock Exchange.

Table 4: The top 5 exchange-traded derivatives by the trading volume

|      | 2000                    |     | 2001                    |     | 2002                         |       |
|------|-------------------------|-----|-------------------------|-----|------------------------------|-------|
| Rank |                         |     |                         |     |                              |       |
| 1    | KOSPI 200 Index Options | 194 | KOSPI 200 Index Options | 823 | KOSPI 200 Index Options      | 1,890 |
| 2    | Euro-Bond Futures       | 151 | Eurodollar Futures      | 184 | Eurodollar Futures           | 202   |
| 3    | Eurodollar Futures      | 108 | Euro-Bond Futures       | 178 | Euro-Bond Futures            | 191   |
| 4    | CAC 40 Index Options    | 84  | CAC 40 Index Options    | 107 | E-mini S&P 500 Index Futures | 116   |
| 5    | T-Bond Futures          | 63  | Euro-Bobl Futures       | 100 | Euro-Bobl Futures            | 115   |

  

|      | 2003                               |       | 2004                               |       | 2005                         |       |
|------|------------------------------------|-------|------------------------------------|-------|------------------------------|-------|
| Rank |                                    |       |                                    |       |                              |       |
| 1    | KOSPI 200 Index Options            | 2,837 | KOSPI 200 Index Options            | 2,521 | KOSPI 200 Index Options      | 2,535 |
| 2    | Euro-Bond Futures                  | 244   | Eurodollar Futures                 | 298   | Eurodollar Futures           | 410   |
| 3    | Eurodollar Futures                 | 209   | Euro-Bond Futures                  | 240   | Euro-Bond Futures            | 299   |
| 4    | TIIE 28-Day Interbank Rate Futures | 162   | TIIE 28-Day Interbank Rate Futures | 206   | 10-Year T-Note Futures       | 215   |
| 5    | E-mini S&P 500 index Futures       | 161   | 10-Year T-Note Futures             | 196   | E-mini S&P 500 index Futures | 207   |

Table 4 illustrates the top 5 exchange-traded derivatives in terms of trading volume from 2000 to 2005. In each year, the first column is the name of derivatives product and the second column is the trading volume in million of contracts. Data is available from the annual volume survey of FIA (Futures Industry Association, [www.futuresindustry.org](http://www.futuresindustry.org)).



Table 5: Delta-hedged gains for calls written on KOSPI 200 index

Panel A: Full sample period, 1999:01 - 2006:07

| Moneyness<br>y-1 | N    | $\pi$ (in 100,000 Won) |                     |                     | $\pi/S$ (in percent %) |                 |                 | $\pi/C$ (in percent %) |                  |                  | $1_{\pi < 0}$<br>% |
|------------------|------|------------------------|---------------------|---------------------|------------------------|-----------------|-----------------|------------------------|------------------|------------------|--------------------|
|                  |      | 10-20                  | 20-40               | All                 | 10-20                  | 20-40           | All             | 10-20                  | 20-40            | All              |                    |
| -10% to -7.5%    | 2537 | -0.2609<br>(0.026)     | -0.3816<br>(0.0267) | -0.3369<br>(0.0194) | -0.29<br>(0.03)        | -0.40<br>(0.03) | -0.36<br>(0.02) | -56.18<br>(6.05)       | -30.40<br>(3.44) | -39.94<br>(3.12) | 66.69              |
| -7.5% to -5%     | 2956 | -0.3506<br>(0.0256)    | -0.3782<br>(0.0255) | -0.3679<br>(0.0186) | -0.35<br>(0.03)        | -0.38<br>(0.03) | -0.37<br>(0.02) | -61.25<br>(4.91)       | -19.93<br>(1.97) | -35.35<br>(2.24) | 65.79              |
| -5% to -2.5%     | 2999 | -0.3553<br>(0.0272)    | -0.2631<br>(0.0271) | -0.2976<br>(0.0198) | -0.35<br>(0.03)        | -0.27<br>(0.03) | -0.30<br>(0.02) | -29.37<br>(2.55)       | -6.34<br>(1.15)  | -14.97<br>(1.21) | 63.48              |
| -2.5% to 0%      | 2815 | -0.2536<br>(0.027)     | -0.1942<br>(0.0298) | -0.2163<br>(0.0212) | -0.27<br>(0.03)        | -0.23<br>(0.03) | -0.24<br>(0.02) | -9.33<br>(1.1)         | -1.41<br>(0.74)  | -4.36<br>(0.62)  | 61.10              |
| 0% to 2.5%       | 2692 | -0.1207<br>(0.0275)    | 0.003<br>(0.0308)   | -0.0432<br>(0.0219) | -0.15<br>(0.03)        | -0.05<br>(0.03) | -0.09<br>(0.02) | -1.89<br>(0.67)        | 2.63<br>(0.62)   | 0.94<br>(0.46)   | 55.49              |
| 2.5% to 5%       | 2437 | -0.0571<br>(0.0295)    | 0.0687<br>(0.0349)  | 0.0196<br>(0.0242)  | -0.09<br>(0.03)        | 0.02<br>(0.04)  | -0.03<br>(0.03) | -0.25<br>(0.56)        | 2.98<br>(0.55)   | 1.72<br>(0.40)   | 52.31              |
| 5% to 7.5%       | 2044 | -0.025<br>(0.0345)     | 0.0855<br>(0.0376)  | 0.0415<br>(0.0265)  | -0.05<br>(0.04)        | 0.05<br>(0.04)  | 0.01<br>(0.03)  | 0.34<br>(0.48)         | 2.39<br>(0.48)   | 1.58<br>(0.35)   | 50.83              |
| 7.5% to 10%      | 1507 | -0.0519<br>(0.0414)    | 0.1167<br>(0.0438)  | 0.0497<br>(0.0311)  | -0.06<br>(0.04)        | 0.09<br>(0.05)  | 0.03<br>(0.03)  | 0.00<br>(0.50)         | 2.17<br>(0.48)   | 1.31<br>(0.35)   | 49.83              |

Panel B: Delta-hedged gains across the 1999:01-2002:12 and 2003:01-2006:07 subsamples

| Moneyness<br>y-1 | Sample | N    | $\pi$ (in 100,000 Won) |         |         | $\pi/S$ (in percent %) |       |       | $\pi/C$ (in percent %) |        |        | $1_{\pi < 0}$<br>% |
|------------------|--------|------|------------------------|---------|---------|------------------------|-------|-------|------------------------|--------|--------|--------------------|
|                  |        |      | 10-20                  | 20-40   | All     | 10-20                  | 20-40 | All   | 10-20                  | 20-40  | All    |                    |
| -10% to -7.5%    | Set 1  | 1302 | -0.3978                | -0.4834 | -0.451  | -0.46                  | -0.54 | -0.51 | -28.05                 | -8.56  | -15.94 | 71.81              |
|                  | Set 2  | 1235 | -0.1096                | -0.2772 | -0.2167 | -0.09                  | -0.25 | -0.19 | -87.27                 | -52.81 | -65.25 | 61.29              |
| -7.5% to -5%     | Set 1  | 1322 | -0.4673                | -0.5148 | -0.4969 | -0.54                  | -0.57 | -0.57 | -23.32                 | -8.89  | -14.35 | 70.49              |
|                  | Set 2  | 1634 | -0.2538                | -0.2692 | -0.2636 | -0.20                  | -0.23 | -0.22 | -92.70                 | -28.73 | -52.34 | 61.99              |
| -5% to -2.5%     | Set 1  | 1334 | -0.4377                | -0.3761 | -0.3995 | -0.49                  | -0.41 | -0.44 | 14.89                  | -2.42  | -7.17  | 65.36              |
|                  | Set 2  | 1665 | -0.2873                | -0.1741 | -0.216  | -0.23                  | -0.17 | -0.19 | -41.31                 | -9.44  | -21.23 | 61.98              |
| -2.5% to 0%      | Set 1  | 1233 | -0.4184                | -0.4085 | -0.4122 | -0.46                  | -0.44 | -0.44 | -10.10                 | -2.55  | -5.37  | 65.28              |
|                  | Set 2  | 1582 | -0.124                 | -0.028  | -0.0636 | -0.12                  | -0.07 | -0.09 | -8.73                  | -0.53  | -3.57  | 57.83              |
| 0% to 2.5%       | Set 1  | 1169 | -0.3155                | -0.223  | -0.2581 | -0.34                  | -0.23 | -0.27 | -5.13                  | 1.11   | -1.26  | 61.84              |
|                  | Set 2  | 1523 | 0.0334                 | 0.1732  | 0.1217  | 0.00                   | 0.09  | 0.05  | 0.68                   | 3.77   | 2.63   | 50.62              |
| 2.5% to 5%       | Set 1  | 1108 | -0.2648                | -0.1638 | -0.2038 | -0.28                  | -0.16 | -0.21 | -2.73                  | 1.96   | 0.10   | 58.574             |
|                  | Set 2  | 1329 | 0.121                  | 0.259   | 0.2058  | 0.07                   | 0.16  | 0.12  | 1.88                   | 3.82   | 3.08   | 47.10              |
| 5% to 7.5%       | Set 1  | 982  | -0.2336                | -0.0746 | -0.1397 | -0.24                  | -0.09 | -0.15 | -1.73                  | 1.84   | 0.38   | 57.33              |
|                  | Set 2  | 1062 | 0.1784                 | 0.2283  | 0.2089  | 0.13                   | 0.17  | 0.16  | 2.37                   | 2.89   | 2.69   | 44.82              |
| 7.5% to 10%      | Set 1  | 761  | -0.2093                | -0.0574 | -0.1207 | -0.22                  | -0.07 | -0.13 | -1.32                  | 1.23   | 0.16   | 55.06              |
|                  | Set 2  | 746  | 0.1249                 | 0.2833  | 0.2234  | 0.11                   | 0.24  | 0.19  | 1.49                   | 3.07   | 2.47   | 44.50              |

Table 5 presents the delta-hedged gain on a portfolio of a long position in a call, hedged by a short position in the underlying stock, which satisfies a self-financing strategy. The option delta is computed as the Black-Scholes hedge ratio based on the GARCH volatility. The rebalancing frequency is set to 1 day. We report (i) the delta-hedged gains ( $\pi_{t,t+\tau}$ ), (ii) the delta-hedged gains normalized by the index ( $\pi_{t,t+\tau}/S_t$ ), and (iii) the delta-hedged gains normalized by the option price ( $\pi_{t,t+\tau}/C_t$ ). The moneyness of options are defined as  $y = S/K$ . The standard error, shown in parentheses, is computed as the sample standard deviation divided by the square root of the number of observation.  $1_{\pi < 0}$  is the proportion of delta-hedged gains with  $\pi < 0$ , and  $N$  is the number of computed options. Subsample: set 1 corresponds to 1999:01-2002:12; Subsample: set 2 corresponds to 2003:01-2006:07.

Table 6: Mean delta-hedged gains and statistics for ATM calls with a fixed maturity of 20, 30, and 40 trading days

A. t-statistics

|                        | ATM: [-2,5%, 2.5%] |         |         |
|------------------------|--------------------|---------|---------|
|                        | 20 days            | 30days  | 40 days |
| N                      | 192                | 213     | 173     |
| Mean delta-hedge gains | -0.1116            | -0.1973 | -0.0079 |
| Standard errors        | 0.0715             | 0.0913  | 0.1089  |
| t-statistics           | -1.5614            | -2.162  | -0.0726 |

B. Bertsimas, Kogan, and Lo (2000)'s t-statistics

|  | ATM: [-2,5%, 2.5%] |        |         |
|--|--------------------|--------|---------|
|  | 20 days            | 30days | 40 days |
| N  | 192                | 213    | 173     |
| (i) $\mu = 12.8\%$ , $\sigma = 34\%$ (historical values) |                    |        |         |
| t-statistics   | -0.17              | -0.30  | -0.05   |
| (ii) $\mu = 12.8\%$ , $\sigma = 23\%$                    |                    |        |         |
| t-statistics   | -0.29              | -0.53  | -0.07   |
| (iii) $\mu = 12.8\%$ , $\sigma = 12\%$                   |                    |        |         |
| t-statistics   | -0.97              | -1.75  | 0.31    |

Table 6 presents the statistics for at-the-money calls with a fixed maturity of 20, 30, and 40 trading days. Panel A indicates the simple t-statistic, while Panel B indicates the t-statistic of the gains, normalized by the standard deviation derived by Bertimas, Kogan, and Lo (2000). The delta-hedged gains are based on the volatility estimated by GARCH.

Table 7: Cross-sectional results with respect to option vega on Delta-hedged gains for KOSPI 200 calls by volatility regimes

| VOL <sup>h</sup> | 20 trading day options |                       |                      |                      |                       |     | 30 trading day options |                       |                       |                       |          |          |
|------------------|------------------------|-----------------------|----------------------|----------------------|-----------------------|-----|------------------------|-----------------------|-----------------------|-----------------------|----------|----------|
|                  | Vega: $\exp(-d_t^2/2)$ |                       |                      | Vega: $ y-1 $        |                       |     | Vega: $\exp(-d_t^2/2)$ |                       |                       | Vega: $ y-1 $         |          |          |
|                  | N                      | $\Psi_0$              | $\Psi_1$             | $\Psi_0$             | $\Psi_1$              | N   | $\Psi_0$               | $\Psi_1$              | $\Psi_0$              | $\Psi_1$              | $\Psi_0$ | $\Psi_1$ |
| < 15             | 77                     | -0.000432<br>[-0.327] | 0.001918*<br>[2.19]  | 0.001549<br>[1.29]   | -0.022017*<br>[-2.00] | 41  | 0.000437<br>[0.20]     | 0.005931*<br>[2.62]   | 0.007852*<br>[5.68]   | -0.085613*<br>[-3.22] |          |          |
| 15-20            | 96                     | 0.000342<br>[0.25]    | 0.000517<br>[0.41]   | 0.001<br>[0.83]      | -0.007474<br>[-0.57]  | 149 | 0.002518<br>[1.54]     | -0.001116<br>[-0.69]  | 0.001608<br>[1.26]    | 0.002100<br>[0.16]    |          |          |
| 20-25            | 147                    | 0.002492<br>[0.80]    | -0.003359<br>[-1.53] | -0.000915<br>[-0.32] | 0.020531<br>[1.18]    | 167 | 0.005126<br>[1.25]     | -0.007455<br>[-1.79]  | -0.001970<br>[-0.75]  | 0.023555<br>[0.94]    |          |          |
| 25-30            | 83                     | 0.00354<br>[0.94]     | -0.005349<br>[-1.43] | -0.001717<br>[-0.68] | 0.020895<br>[0.91]    | 84  | 0.001741<br>[0.30]     | 0.001339<br>[0.25]    | 0.004078<br>[1.09]    | -0.024655<br>[-1.07]  |          |          |
| 30-35            | 91                     | -0.001980<br>[-0.41]  | 0.001795<br>[0.39]   | 0.000523<br>[0.16]   | -0.020403<br>[-0.95]  | 105 | 0.013403<br>[1.49]     | -0.022847*<br>[-2.43] | -0.010435*<br>[-2.95] | 0.071512*<br>[2.24]   |          |          |
| 35-40            | 63                     | -0.014421<br>[-1.75]  | 0.019530*<br>[2.26]  | 0.006432<br>[1.84]   | -0.077389*<br>[-2.41] | 62  | -0.020379<br>[-0.91]   | 0.016583<br>[0.70]    | -0.006457<br>[-1.13]  | 0.032779<br>[0.56]    |          |          |
| 40-45            | 52                     | 0.01105<br>[0.75]     | -0.022303<br>[-1.47] | -0.011296<br>[-1.88] | 0.045955<br>[0.96]    | 56  | 0.028828<br>[1.51]     | -0.036919<br>[-1.92]  | -0.009467<br>[-1.29]  | 0.077470<br>[1.78]    |          |          |
| >45              | 86                     | 0.031587*<br>[2.23]   | -0.04533*<br>[-3.09] | -0.013337<br>[-3.33] | 0.056346<br>[1.62]    | 81  | 0.040935<br>[1.66]     | -0.049848*<br>[-2.01] | -0.006025<br>[-0.88]  | -0.013645<br>[-0.34]  |          |          |

Table 7 presents the estimates of a FGLS random effect panel regression of delta-hedged gains on the Vega:

$$\begin{aligned} \text{GAIN}_t^i &= \Psi_0 + \Psi_1 \text{Vega}_t^i + \epsilon_t^i \\ \epsilon_t^i &= u_t + \omega_t^i \end{aligned}$$

where  $\text{GAIN}_t^i = \pi_{t,t+\tau}^i/S_t$ . Two proxies for the option vega are used. One is  $\exp(-d_t^2)$ , where  $d_1 = \frac{\log(S/K) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$ , the other is the absolute level of moneyness,  $|y-1|$ . The data consists of monthly call options with maturity of 20 and 30 trading day over 1999:01-2006:07. Since the estimation method is not least squares, the coefficient of determination is omitted. N is the number of observation. Numbers in square brackets show the z-statistics [Greene (1997)]. An asterisk \* is attached when the coefficient is significant at 5% significant level.

Table 8: The risk-neutral skewness and kurtosis from OTM calls and puts

| Maturity<br>( $\tau$ ) | Sample | mean risk-neutral<br>skewness* | mean risk-neutral<br>kurtosis* |
|------------------------|--------|--------------------------------|--------------------------------|
| 20<br>days             | Full   | -0.54                          | 4.76                           |
|                        | set 1  | -0.31                          | 3.28                           |
|                        | set 2  | -0.81                          | 6.42                           |
| 30<br>days             | Full   | -0.57                          | 4.30                           |
|                        | set 1  | -0.36                          | 3.12                           |
|                        | set 2  | -0.81                          | 5.59                           |
| 40<br>days             | Full   | -0.59                          | 3.64                           |
|                        | set 1  | -0.44                          | 2.65                           |
|                        | set 2  | -0.75                          | 4.68                           |

Table 8 presents the risk-neutral skewness and kurtosis, implied in out-of-the-money calls and puts with a maturity of 20, 30, 40 trading days. Set 1 corresponds to 1999:01-2002:12; Set 2 corresponds to 2003:01-2006:07.

Table 9: Effect of jumps on delta-hedged gains for ATM calls and puts

| Panel A: ATM Calls     |        |                                      |              |                                      |                                      |       |
|------------------------|--------|--------------------------------------|--------------|--------------------------------------|--------------------------------------|-------|
| Maturity<br>( $\tau$ ) | Sample | $\Omega_0^*$<br>( $\times 10^{-2}$ ) | $\Omega_1^*$ | $\Omega_2^*$<br>( $\times 10^{-2}$ ) | $\Omega_3^*$<br>( $\times 10^{-2}$ ) | $R^2$ |
| 20<br>days             | Full   | -0.68                                | -0.0069      | -0.37                                | 0.09                                 | 7.08  |
|                        |        | [-1.73]                              | [-0.07]      | [-1.22]                              | [0.94]                               |       |
|                        | Set 1  | -2.8*                                | -0.10        | -1.35*                               | 0.74*                                | 30.97 |
|                        |        | [-3.61]                              | [-0.80]      | [-2.70]                              | [3.23]                               |       |
|                        | Set 2  | -0.14                                | 0.10         | 0.02                                 | 0.02                                 | 0.88  |
|                        |        | [-0.25]                              | [0.56]       | [0.06]                               | [0.25]                               |       |
| 30<br>days             | Full   | -0.61                                | 0.14         | -1.43*                               | -0.04                                | 29.57 |
|                        |        | [-1.94]                              | [1.79]       | [-5.40]                              | [-0.49]                              |       |
|                        | Set 1  | -1.10*                               | 0.10         | -2.34*                               | 0.15                                 | 36.19 |
|                        |        | [-2.30]                              | [1.02]       | [-4.88]                              | [1.10]                               |       |
|                        | Set 2  | -0.91*                               | 0.29*        | -0.68*                               | 0.08                                 | 33.13 |
|                        |        | [-2.43]                              | [2.34]       | [-2.61]                              | [0.98]                               |       |
| 40<br>days             | Full   | -0.85*                               | 0.0363       | -0.70*                               | 0.12*                                | 10.27 |
|                        |        | [-3.07]                              | [0.35]       | [-2.75]                              | [3.08]                               |       |
|                        | Set 1  | -0.88*                               | 0.01         | -0.90*                               | 0.15                                 | 8.32  |
|                        |        | [-2.12]                              | [0.09]       | [-2.00]                              | [1.91]                               |       |
|                        | Set 2  | -1.20*                               | 0.26         | -0.57*                               | 0.20*                                | 26.51 |
|                        |        | [-2.69]                              | [1.83]       | [-2.24]                              | [2.06]                               |       |

Panel B: ATM Puts

| Maturity<br>( $\tau$ ) | Sample | $\Omega_0^*$<br>( $\times 10^{-2}$ ) | $\Omega_1^*$ | $\Omega_2^*$<br>( $\times 10^{-2}$ ) | $\Omega_3^*$<br>( $\times 10^{-2}$ ) | $R^2$ |
|------------------------|--------|--------------------------------------|--------------|--------------------------------------|--------------------------------------|-------|
| 20<br>days             | Full   | -0.69*                               | -0.05        | 1.16*                                | 0.18*                                | 18.45 |
|                        |        | [-3.13]                              | [-0.56]      | [3.95]                               | [3.75]                               |       |
|                        | Set 1  | -1.55*                               | -0.03        | 0.85                                 | 0.43*                                | 15.71 |
|                        |        | [-2.50]                              | [-0.22]      | [1.47]                               | [2.40]                               |       |
|                        | Set 2  | -0.61*                               | 0.02         | 1.20*                                | 0.17*                                | 44.93 |
|                        |        | [-2.47]                              | [0.10]       | [4.78]                               | [4.76]                               |       |
| 30<br>days             | Full   | -0.55*                               | 0.13         | 1.38*                                | 0.18*                                | 30.17 |
|                        |        | [-2.34]                              | [1.62]       | [5.88]                               | [4.05]                               |       |
|                        | Set 1  | -0.41                                | 0.10         | 1.55*                                | 0.14*                                | 30.83 |
|                        |        | [-1.14]                              | [0.86]       | [4.34]                               | [1.72]                               |       |
|                        | Set 2  | -0.67*                               | 0.32*        | 0.93*                                | 0.17*                                | 39.13 |
|                        |        | [-2.93]                              | [2.77]       | [3.13]                               | [3.92]                               |       |
| 40<br>days             | Full   | -1.09*                               | 0.30*        | 1.66*                                | 0.42*                                | 35.25 |
|                        |        | [-3.70]                              | [3.30]       | [3.96]                               | [4.52]                               |       |
|                        | Set 1  | -1.25*                               | 0.31*        | 2.09*                                | 0.59*                                | 36.12 |
|                        |        | [-2.27]                              | [2.38]       | [3.23]                               | [2.43]                               |       |
|                        | Set 2  | -1.19*                               | 0.24         | 0.86*                                | 0.30*                                | 29.84 |
|                        |        | [-3.30]                              | [1.75]       | [2.04]                               | [3.19]                               |       |

We apply the risk-neutral skewness and kurtosis as proxies for jump fear. Table show the regression results based on the following specification between delta-hedged gains and the higher-order moments of the risk-neutral return distribution.

$$\text{Gain}_t = \Omega_0^* + \Omega_1^* \text{Gain}_{t-1} + \Omega_2^* \text{SKEW}_t^* + \Omega_3^* \text{KURT}_t^* + \epsilon_t,$$

where  $\text{Gain}_t \equiv \pi_{t,t+\tau}/S_t$  of puts closest to at-the-money. To correct the serial correlation of the residuals, a lagged variable is included in the regression specification. Table includes the estimates, test-statistics (t-statistic) in square brackets, the  $R^2$ . An asterisk \* is attached when the coefficient is significant at 5% significant level. Panel A corresponds to ATM call options, while Panel B corresponds to ATM put options. Full sample is from total sample periods: 1999:01 to 2006:07. Set 1 refers to the subsample of 1999:01 to 2002:12, while Set 2 refers to the subsample of 2003:01 to 2006:07. All results are tested for options with a fixed maturity of 20 days, 30 days, and 40 days (trading days).