NONLINEAR DRIFT MODEL IN THE SHORT-TERM INTEREST RATE

BY YOUNGSOO CHOI AND YOONDONG LEE

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Abstract. This paper propose a new short-term interest rate model having a different nonlinear drift function and the same diffusion coefficient with Chan, Karolyi, Longstaff, and Sanders (1992) model. The fractional polynomial power of the drift function in our model is linked to the local volatility elasticity of the diffusion coefficient. While the nonlinear drift function estimated by Aït-Sahalia (1996a) and others has a feature that higher interest rates tend to revert downward and low rates upward, the drift function estimated by our nonlinear model shows that higher interest rate mean-reverts strongly, but, medium rates has almost zero drift and low rates has a very small drift. This characteristic coincides the empirical result based on the nonparametric methodology by Stanton (1997) and the implication by the scatter plot of the short rate data. Furthermore, if our model is transformed to make the diffusion process have a constant term, the drift term in our model is very similar to that in Aït-Sahalia model. In the viewpoint of data, while his model is applied to the original interest rate data, our model is applied to the transformed data.

JEL classification: G10; G13

Keywords: Short-term interest rate model; Box-Cox transformation

Choi is with Department of Mathematics, Hankuk University of Foreign Studies, Yongin-Shi, Kyongki-Do, KOREA 449-791. Email address: choiys@hufs.ac.kr. Lee is with Department of Statistics at Konkuk University, Seoul, KOREA. .

1. Introduction

Risk-free short-term interest rate (or short rate) is used in financial economics to determine term structure of interest rates and prices of bonds of various maturities at any given time. Short rates also serve as an important element in the development of tools for risk management of valuing and hedging the huge institutional holdings of fixed income securities and in empirical work on term premiums and yield curves, where short rates are treated as reference rates for other interest rates.

The dynamics of the short rate, r_t, are usually described in the form of continuous-time diffusion models having linear drifts (local expected changes) and often linear diffusion coefficients (variances of local changes). However, these linear term structure models are developed, since it is convenient for driving and estimating explicit forms of the term structure of interest rates. Empirical observations by Chan et al. (1992; hereafter CKLS) on the U.S. Treasury Bill rate show that "the dynamics of the short rate should be those that allow the volatility of interest rate changes to be highly sensitive to the level of the short rate." Volatilities may be estimated relatively accurately using high-frequency observations of the short rate with the Generalized Method of Moments.

To find the genuine data property of the short rate, Aït-Sahalia (1996a) and Stanton (1997) propose nonparametric methods for estimating the drift and diffusion of the short rate. Both papers show that, while the estimated diffusion term is similar to that estimated by CKLS, the substantial nonlinearity in the drift term is observed and this feature is the main source of rejection of the linear drift models of CKLS and others. Aït-Sahalia finds that the short rate behaves like a random walk around its mean, reverting toward the mean when it is far away from the mean. Stanton shows that the drift term is close to zero for low and medium rates, but revert toward the mean at higher interest rates. Similar results are reported by Conley et al. (1997; hereafter CHLS), who show that "when interest rates are high, local mean reversion is small and the mechanism for inducing stationarity is the increased volatility of the diffusion process."

Figure 1 plots the drift functions – the local expected change in the short rate per year as a function of the level of the rate – estimated by Aït-Sahalia and our generalized nonlinear drift, constant elasticity of variance (GNCEV) models using one-month U.S. Treasury bill rates like CKLS and one-month Eurodollar deposits rates. This figure is similar to Figure 1 of Jones (2003); in the case of having the same drift functions of Aït-Sahalia, higher interest rates tend to revert downward and low rates upward. However, the drift function estimated by our nonlinear model shows that, while higher interest rate mean-reverts strongly, medium rates has almost zero drift and low rates has a very small drift depending on the data. This characteristic coincides the empirical result based on the nonparametric methodology by Stanton (1997) and the implication of scatter plot of the short rate data.

Our model is proposed on the base of empirical observations; the QQ-plot and Jarque-Bera test for the short rate show that it does not satisfy the normality assumption. As an improvement tool, we use the common Box-Cox (1964) transformation to produce a Gaussian time series for the short rate, but transformed series has serial correlations. Hence, it is natural to assume that transformed series follow the Ornstein-Uhlenbeck (O-U) process. Applying Ito's lemma to the transformation and extending the output, the continuous-time diffusion model having nonlinear drift and diffusion term is obtained. The fractional polynomial power of the drift function in our model is linked to the local volatility elasticity of the diffusion coefficient.

If our model is transformed to make the diffusion process have a constant term, the drift term in our model is very similar to that in Aït-Sahalia and CHLS models. That is, while their models are applied to the original interest rate data, our model is applied to the transformed data.

The remainder of this paper is organized as follows: In Section 1, we derive a new short rate model and compare it with the existing models. In Section 2, we illustrate the empirical analysis with the Generalized method of moments (GMM) and conditional multiple regression methods, and in Section 4, we draw conclusions.

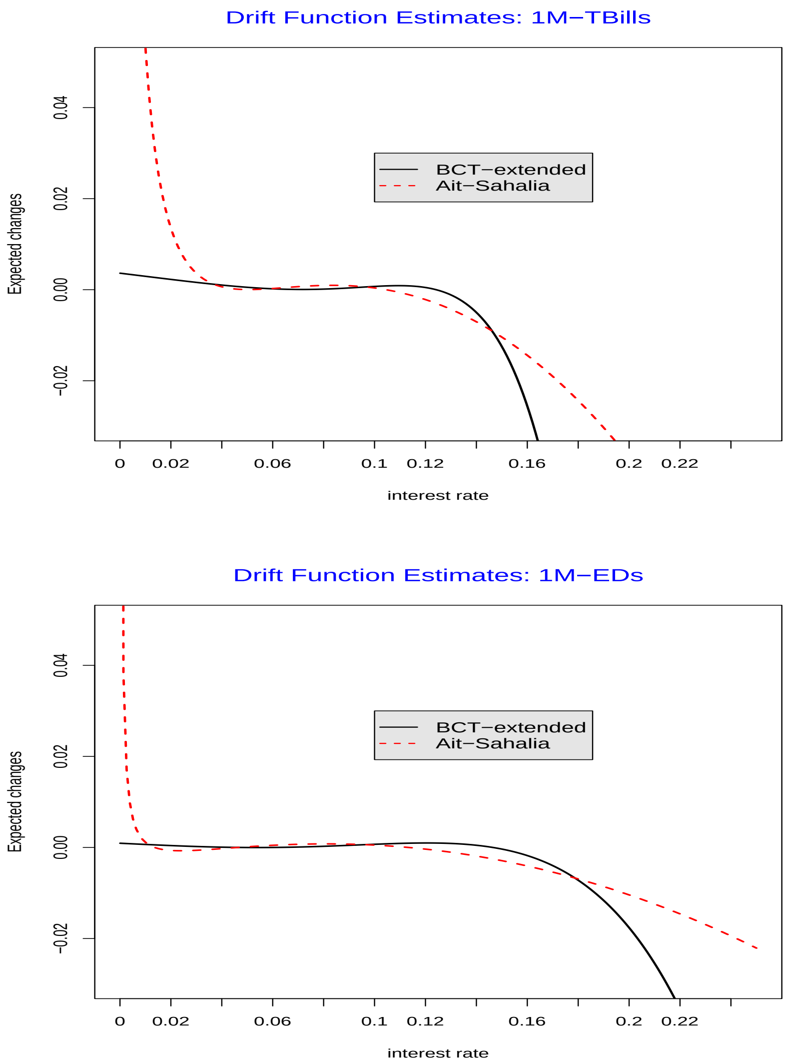

FIGURE 1. **Drift function estimates**: The first (second) figure contains the drift function estimated by Aït-Sahalia and our nonlinear drift models using monthly one-month U.S. Treasury bill (Eurodollar deposit) rates.

2. ONE FACTOR INTEREST RATE MODELS

In this section, we construct a new model for the short-term interest rate using the Box-Cox transformation model and introduce several existing models as a comparison.

Modelling process and other models

Empirical observations on the TBills show that the short term interest rate, r_t, does not satisfy the normality assumption through the QQ-plot and Jarque-Bera test. We use the Box-Cox transform to produce a Gaussian time series for the short term interest rate as follows:

$$r_t^{(\lambda)} = \begin{cases} \frac{r_t^\lambda - 1}{\lambda} & \text{for } \lambda \neq 0, \\ \ln r_t & \text{for } \lambda = 0. \end{cases} \tag{1}$$

Thus, it is natural to assume that a new state variable $r^{(\lambda)}$ follows the OU-process:

$$dr_t^{(\lambda)} = (\alpha + \beta r_t^{(\lambda)})dt + \sigma dW_t. \tag{2}$$

Applying Ito's lemma to the function $r_t = (1 + \lambda r_t^{(\lambda)})^{1/\lambda}$, we have

$$\begin{aligned} dr_t &= (1 + \lambda r_t^{(\lambda)})^{\frac{1-\lambda}{\lambda}} \Big[(\alpha + \beta r_t^{(\lambda)})dt + \sigma dW_t \Big] + \tfrac{1}{2}(1-\lambda)(1 + \lambda r_t^{(\lambda)})^{\frac{1-2\lambda}{\lambda}} \sigma^2 dt \\ &= \Big[\big(\alpha - \tfrac{\beta}{\lambda}\big) r_t^{1-\lambda} + \tfrac{\beta}{\lambda} r_t + \tfrac{1}{2}(1-\lambda)\sigma^2 r_t^{1-2\lambda} \Big] dt + \sigma r_t^{1-\lambda} dW_t. \end{aligned} \tag{3}$$

Thus, replacing $1-\lambda$ by γ, that is, $1-\lambda=\gamma$, (3) can be written as

$$dr_t = \Big[\big(\alpha - \tfrac{\beta}{1-\gamma}\big) r_t^{\gamma} + \tfrac{\beta}{1-\gamma} r_t + \tfrac{1}{2}\gamma\sigma^2 r_t^{2\gamma-1} \Big] dt + \sigma r_t^{\gamma} dW_t. \tag{4}$$

Note that (4) nests the Vasicek process in the case of $\gamma = 0$:

$$dr_t = \big(\alpha - \beta + \beta r_t\big)dt + \sigma dW_t$$

and the Black-Karasinski model as γ approaches 1, since

$$\lim_{\gamma \to 1} r_t^{(\lambda)} = \lim_{\lambda \to 0} \frac{r_t^\lambda - 1}{\lambda} = \ln r_t.$$

Thus we consider the generalized nonlinear drift, constant elasticity of variance (CEV) process as follows

$$dr_t = \big(a + b r_t + c r_t^{\gamma} + d r_t^{2\gamma-1}\big)dt + \sigma r_t^{\gamma} dW_t, \tag{5}$$

which nests the CKLS model with the linear drift term $a + br_t$.

Several existing one-factor short rate models having nonlinear drifts are introduced to compare them with our model. Aït-Sahalia (1996b) proposes the following model:

$$dr_t = \left(a + br_t + cr_t^2 + d/r_t\right)dt + \sqrt{b_0 + b_1 r_t + b_2 r_t^{b_3}}\, dW_t. \tag{6}$$

CHLS (1997) adopt the same drift parameterization as Aït-Sahalia but keep the CEV diffusion used by Chan et. al. (1992):

$$dr_t = \left(a + br_t + cr_t^2 + d/r_t\right)dt + \sigma r_t^{\gamma} dW_t. \tag{7}$$

Eom (1998) considers the nonlinear drift CEV (NCEV) process with the CEV diffusion used by Chan et. al. (1992), but selects some drift parts of equation (5):

$$dr_t = \left(br_t + dr_t^{2\gamma-1}\right)dt + \sigma r_t^{\gamma} dW_t. \tag{8}$$

As a comparison, we also consider the nonlinear drift CEV process with the CEV diffusion used by Chan et. al. (1992), but selects another drift parts of equation (5):

$$dr_t = \left(br_t + cr_t^{\gamma}\right)dt + \sigma r_t^{\gamma} dW_t. \tag{9}$$

The properties of our model

While the coefficient of $r_t^{2\gamma-1}$ in the equation (4) is restricted to be $\gamma\sigma^2/2$, that of equation (5) is free. It is related to the normality assumption of the conditional probability density function and the existence of a weak solution for the continuous-time parametric diffusion process (5). Let $p_R(\Delta, r \,|\, r_0; \boldsymbol{\theta})$ denote the conditional density of $r_{t+\Delta} = r$ given $r_t = r_0$ induced by the model (5), also called the transition function. If sampling of the process is discrete, the transition function p_R is not only unavailable in closed-form expression, but also cannot be approximated for fixed Δ around a Normal density.

However, following the methodology of Aït-Sahalia (2002), we can obtain an expansion that converges as more correction terms are added while Δ remains fixed. Since the empirical estimate by CKLS (1992) guarantees $\gamma > 1$[1], we transform r_t into a new variable as follows:

$$\int^{r_t} \frac{1}{u^\gamma}\,du = \frac{r_t^{1-\gamma}}{1-\gamma},$$

Consider the transformed process $y_t = r_t^{1-\gamma}$. Then y_t maps from the domain of $D_R = (0,\infty)$ onto $D_Y = (0,\infty)$ reversely. Applying Ito's lemma, y_t satisfies a diffusion equation with constant diffusion term:

$$dy_t = -(\gamma-1)\big(ay_t^{\overline{\gamma-1}^{\gamma}} + by_t + c + (d-\gamma\sigma^2/2)\frac{1}{y_t}\big)dt + (1-\gamma)\sigma dW_t. \tag{10}$$

Note that the above diffusion process has a similar nonlinear drift term like models (6) and (7) with just a different term $y_t^{\gamma/(\gamma-1)}$ instead of y_t^2. That is, our model is very similar to the Aït-Sahalia and CHLS models for the drift term with a different diffusion term. While their models are applied to the original interest rate data, our model is applied to the transformed data to make the diffusion process have a constant term.

While the $1/y_t$ term in the drift function is the dominant growth factor for the left boundary of D_Y, $y_t^{\gamma/(\gamma-1)}$ and y_t terms works as a leading growth factor for the right boundary of D_Y. If the coefficient $(1-\gamma)(d-\gamma\sigma^2/2)$ of $1/y_t$ is greater than 1, it satisfies the left boundary assumption of Assumption 3 in Aït-Sahalia (2002). Also, if the coefficient $(1-\gamma)a$ is less than 0 and $(1-\gamma)b$ is greater than 0, it satisfies the right boundary assumption of Assumption 3. If the above conditions are satisfied, there exists a weak solution $\{y_t \,|\, t \geq 0\}$, unique in probability law, for every distribution of its initial value y_0.

Since the diffusion term for the transformed process is constant, the random term in $y_{t+\Delta}$ given y_t for Euler and Milstein schemes can be represented by a Normal distribution and that for order 1.5 (2.0) strong Taylor scheme is written as the weighted sum of two independent Normal distributions. Thus, the conditional density of $y_{t+\Delta} \,|\, y_t$, denoted by $p_Y(\Delta, y \,|\, y_0; \boldsymbol{\theta})$, is approximated for fixed Δ around a Normal density.

[1]This restriction is unessential. It is OK as far as $\gamma > 0$.

3. Empirical analysis

In this section, we introduce the data used in our empirical work, find the appropriate parameters of several generalized nonlinear drift, CEV process and Aït-Sahalia model by using the Generalized Method of Moments (GMM) or the multiple regression analysis, and finally describe the model feature on the basis of the drift estimate function.

Data sources

The time series data used to model the short-term interest rate are the same monthly one-month Treasury bill rates used by Chan et. al (1992) and monthly one-month Eurodollar deposit rates. The Treasury bills cover the period from June 1964 to November 1989, providing 306 observations. The one-month Eurodollar yields are based on bid rates for Eurodollar deposits collected around 9:30 a.m. Eastern time and annualized using a 360-day year or bank interest. They cover the period from January 1971 to January 2007, providing 433 observations. Their time series data are plotted in Figure 2.

Parameter estimates using GMM

The Euler scheme of the generalized nonlinear drift, CEV process for r_t in equations (5) and (8)-(9) is needed for parameters estimation and can be written over time steps of length h:

$$\Delta r = r_{t+h} - r_t = \mu(r_t)h + \sigma r_t^{\gamma} z_{t+h}, \qquad z_{t+h} \sim N(0, h),$$

where $\mu(r_t) = a + br_t + cr_t^{\gamma} + dr_t^{2\gamma-1}$ ($\mu(r_t) = br_t + dr_t^{2\gamma-1}$, $\mu(r_t) = br_t + cr_t^{\gamma}$) is for equation (5) ((8), (9)). The error term $u_{t+h} = \sigma r_t^{\gamma} z_{t+h} = \Delta r - \mu(r_t)h$ satisfies the

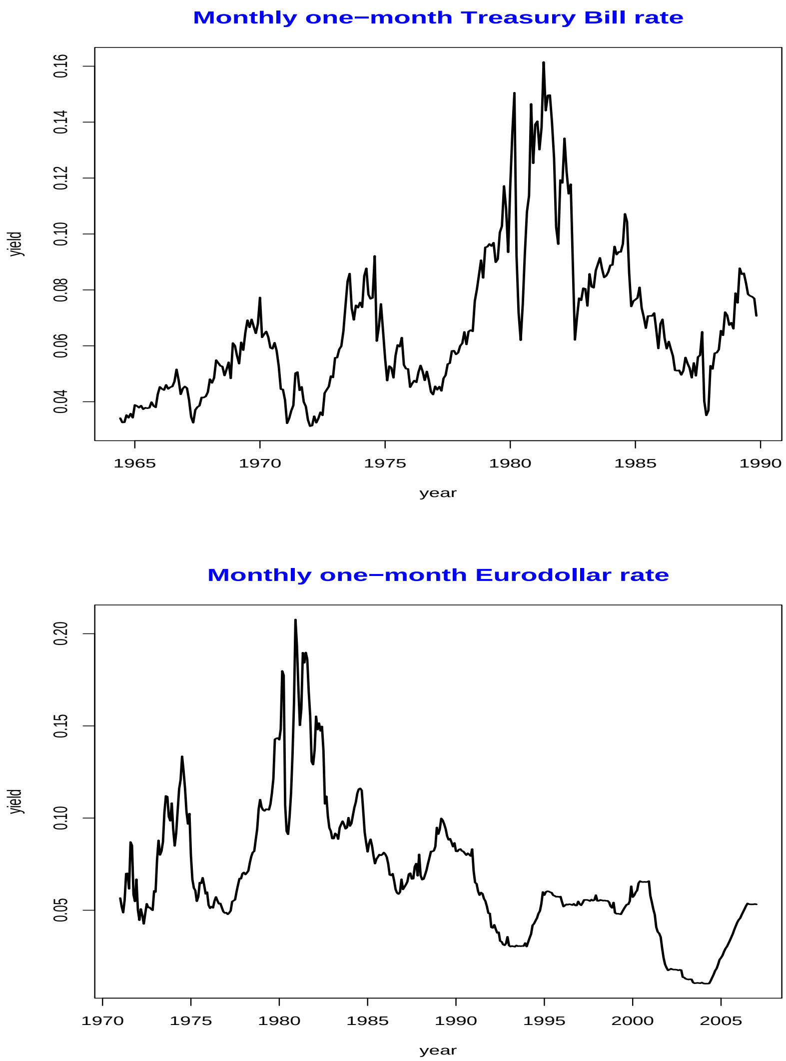

FIGURE 2. **Monthly one-month rate**: The first (second) figure contains the monthly one-month Treasury bill (Eurodollar) rate from June 1964 to November 1989 (from January 1971 to January 2007).

following conditional expectations:

$$\mathrm{E}_t[\, u_{t+h}] = 0$$

$$\mathrm{E}_t[\, u_{t+h}^2] = \sigma^2 r_t^{2\gamma} h.$$

Using instruments $Z_t = \begin{pmatrix} 1 & r_t \end{pmatrix}$, we have unconditional moment conditions

$$\begin{aligned} \mathrm{E}[\, u_{t+h} \,] &= 0 \\ \mathrm{E}[\, r_t u_{t+h} \,] &= 0 \\ \mathrm{E}[\, u_{t+h}^2 - \sigma^2 r_t^{2\gamma} h \,] &= 0 \\ \mathrm{E}[\, r_t (u_{t+h}^2 - \sigma^2 r_t^{2\gamma} h) \,] &= 0 \end{aligned}$$

The number of parameters is restricted to be less than or equal to four for these above moment conditions. Thus, the process for equation (5) is restricted to be like equation (3) as follows:

$$\mu(r_t) = b r_t + c r_t^{\gamma} + \frac{1}{2}\sigma^2 \gamma r_t^{2\gamma-1}. \tag{11}$$

Table 1 reports the parameter estimates, associated standard deviation errors and p-values, and R_j^2 information about how well the corresponding models are able to forecast the future rate change and volatility. The R_j^2 statistics are computed as the proportion of the total variation of the actual yield changes, $y_t \equiv r_t - r_{t-h}$, for $j = 1$ and their volatility (squared yield changes) for $j = 2$ explained by the predictive values of corresponding models, $\hat{y}_t$, as follows:

$$R_1^2 = 1 - \frac{\hat{\mathrm{Var}}(y_t - \hat{y}_t)}{\hat{\mathrm{Var}}(y_t)} \tag{12}$$

$$R_2^2 = 1 - \frac{\hat{\mathrm{Var}}((y_t - \hat{y}_t)^2)}{\hat{\mathrm{Var}}(y_t^2)} \tag{13}$$

Parameter estimates using multiple regression

If the number of parameters in the model is greater than four, a simple extension of instruments like $Z_t = \begin{pmatrix} 1 & r_t & r_t^2 \end{pmatrix}$ does not guarantee the convergence of the sample moment conditions for the GMM estimation. Thus, we employ the two-stage procedure: First, given γ in the appropriate range, run the multiple regression for the dependent

TABLE 1. **Estimates of alternative models for the short rate**: The estimation horizon for r_t, the annualized one-month U.S. Treasury bill yield, is from June, 1964 to December, 1989 (306 observations). The parameters are estimated by the GMM with standard deviations in parentheses and p-values in [ ]. The R_j^2 statistics are computed as the proportion of the total variation of the actual rate changes ($j = 1$) and their volatility (squared rate changes) ($j = 2$) explained by the predictive values of corresponding models.

Model	Equation (11)	Equation (9)	Equation (8)	CKLS	Vasicek
a	0	0	0	0.042	0.030
				(0.016)	(0.014)
				[0.0009]	[0.033]
b	1.268	1.170	0.593	-0.608	-0.449
	(0.631)	(0.647)	(0.317)	(0.270)	(0.247)
	[0.045]	[0.071]	[0.061]	[0.024]	[0.069]
c or d	-4.825	-4.108	-6.801	0	0
	(2.511)	1.913	(7.311)		
	[0.055]	[0.032]	[0.352]		
σ^2	1.625	1.614	1.552	1.779	0.0004
	(2.572)	(2.532)	(2.366)	(2.906)	(0.0001)
	[0.527]	[0.524]	[0.512]	[0.540]	[0.0009]
γ	1.491	1.490	1.482	1.508	0
	(0.304)	0.301	(0.293)	(0.313)	
	[0.000]	[0.000]	[0.000]	[0.000]	
$R_1^2(\Delta r)$	0.037	0.038	0.041	0.027	0.025
$R_2^2(\Delta r)$	0.209	0.213	0.230	0.163	0.130

variable, $y_t = r_t - r_{t-1}$, as follows

$$y_t = f(r_t, \boldsymbol{\beta}) + \epsilon_t, \qquad \epsilon_t \overset{i.i.d}{\sim} \mathcal{D}(0, \sigma^2),$$

where the independent variables with $\boldsymbol{\beta} = \big(a,\ b,\ c,\ d\big)$ are given by

$$f(r_t, \boldsymbol{\beta}) = \begin{cases} a + br_t & \text{CKLS model,} \\ a + br_t + cr_t^{\gamma} & \text{Equation (9),} \\ a + br_t + dr_t^{2\gamma-1} & \text{Equation (8),} \\ a + br_t + cr_t^{\gamma} + dr_t^{2\gamma-1} & \text{Equation (5),} \\ a + br_t + cr_t^2 + d\frac{1}{r_t} & \text{Aït-Sahalia.} \end{cases} \tag{14}$$

Figure 3 plots the multiple R^2-values as a function of γ: The first (second) figure contains the multiple R^2 values measured by equation (12) as a function of γ using the one-month Treasury bill (Eurodollar deposit) rates. For example, the legend, BCT-γ (CEV, BCT-ext'ed), means that the R^2-values are obtained from the multiple regression with independent variables, Equation (9) ((8), (5)). The R^2-value of the CKLS model is independent of the value of γ and is similar to the value shown in Table 1. The R^2-value of the Aït-Sahalia model, 0.058, is also independent of the value of γ and is twice larger than that of the CKLS model, 0.027. However, Figure 3 implies that the best predictive model is the GNCEV process by taking the appropriate value for γ.

Second, select the optimal γ having the largest R^2-value for the multiple regression model in (14) with equation (5) and then run the multiple regression for the equation (5) and Aït-Sahalia models. Table 2 reports the parameter estimates, F-statistics, and multiple R^2-values. While just two independent variables in the GNCEV process are significant, four all independent variables including the intercept term in the Aït-Sahalia model are significant. However, regression results such as R^2-values and F-statistics for both data sets imply that the GNCEV process is better than the Aït-sahalia model.

The drift function estimates using the parameter estimates in Table2 is plotted in Figure 1 and is similar to Figure 1 of Jones (2003); in the case of having the same drift functions of Aït-Sahalia, higher interest rates tend to revert downward and low rates upward. However, the drift function estimated by our GNCEV model shows that,

TABLE 2. **Estimates of multiple regression**: The annualized one-month U.S. Treasury bill yield covers from June 1964 to December 1989 (306 observations) and the annualized one-month Eurodollar rate covers from January 1971 to January 2007 (433 observations). The parameters are estimated by the multiple regression analysis with standard deviations in parentheses and p-values in [ ]. The F-statistics are reported with p-values in parentheses and associated degrees of freedom (d.f.). The R_j^2 statistics are computed as the proportion of the total variation of the actual rate changes ($j = 1$) and their volatility (squared rate changes) ($j = 2$) explained by the predictive values of corresponding models.

Data	Treasury Bill		Eurodollar rate	
Models	Aït-Sahalia	Eq. (5)	Aït-Sahalia	Eq. (5)
a	-0.045	3.64e-03	-6.22e-03	9.34e-04
	(0.021)	(2.59e-03)	(3.30e-03)	(1.32e-03)
	[.036]	[.162]	[.060]	[.479]
b	0.707	-6.88e-02	1.46e-01	-2.70e-02
	(0.295)	(5.18e-02)	(5.88e-02)	(3.01e-02)
	[.017]	[.185]	[.013]	[.370]
c	-3.448	7.23e+01	-8.40e-01	4.28
	(1.232)	(4.15e+01)	(2.67e-01)	(2.50)
	[.005]	[0.082]	[.0018]	[.087]
d	0.0009	-2.95e+04	6.00e-05	-1.67e+02
	(0.0005)	(1.11e+04)	(4.08e-05)	(6.00e+1)
	[.051]	[.008]	[.143]	[.0058]
γ		4.08		3.06
σ	0.008	.008	.008	.007
F-stat.	6.281	8.637	5.551	8.669
	(.0004)	(1.6e-05)	(9.59e-04)	(1.35e-05)
d.f.	(3,301)	(3,301)	(3,428)	(3, 428)
$R_1^2(\Delta r)$	0.059	0.079	0.037	.057
$R_2^2(\Delta r)$	0.343	0.422	0.163	.202

while higher interest rate mean-reverts strongly, medium rates has almost zero drift and low rates has a very small drift depending on the data. This characteristic comes from the negative reaction between independent variables r_t^γ and $r_t^{2\gamma-1}$. Furthermore, this feature coincides the empirical result based on the nonparametric methodology by Stanton (1997) and the implication of scatter plot of the short rate data in Figure 3. Figure 3 shows that our model drift function is close to that obtained by a LOWESS nonparametric regression at low and medium rates, but our model has a very strong mean-reversion trend at high rates: this phenomenon coincides the scattering plot of changes in the short rate.

4. Conclusion

In this article, we employ the Box-Cox transformation to fix the normality assumption for the short rate and propose a new short-term interest rate model having a different nonlinear drift function and the same diffusion coefficient with Chan, Karolyi, Longstaff, and Sanders (1992) model. The drift function in our model has two fractional polynomial powers, linked to the local volatility elasticity of the diffusion coefficient. While the nonlinear drift function estimated by Aït-Sahalia (1996a) and others has a feature that higher interest rates tend to revert downward and low rates upward, the drift function estimated by our nonlinear model shows that higher interest rate mean-reverts strongly, but, medium rates has almost zero drift and low rates has a very small drift. This characteristic comes from the negative reaction between two fractional polynomials as an independent variables. Furthermore, this feature coincides the empirical result based on the nonparametric methodology by Stanton (1997) and the implication of scatter plot of the short rate data.

In future research, we will estimate the parameters in several nonlinear CEV and Aït-Sahalia models using the maximum likelihood estimation and Bayesian method to make sure that nonlinear drift is a feature of daily, weekly, and monthly data, which is contrary to the result obtained by Jones (2003).

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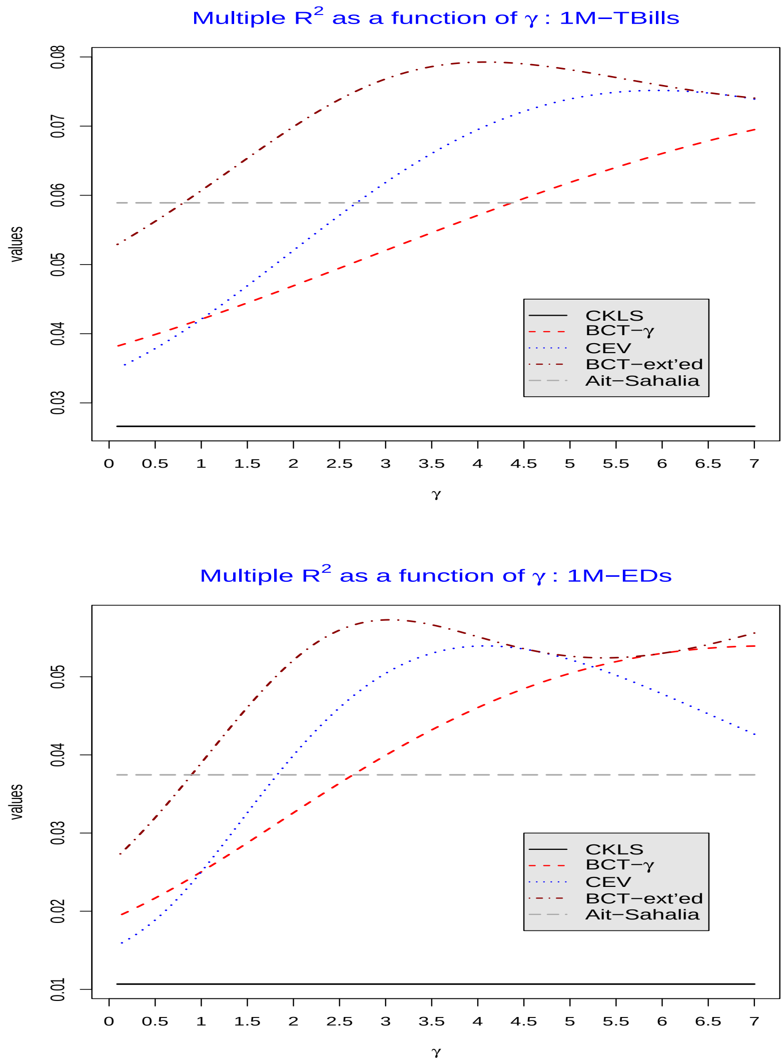

FIGURE 3. **Multiple R^2 as a function of γ**: The first (second) figure contains the multiple R^2 values measured by equation (12) as a function of γ using the one-month Treasury bill (Eurodollar deposit) rates.

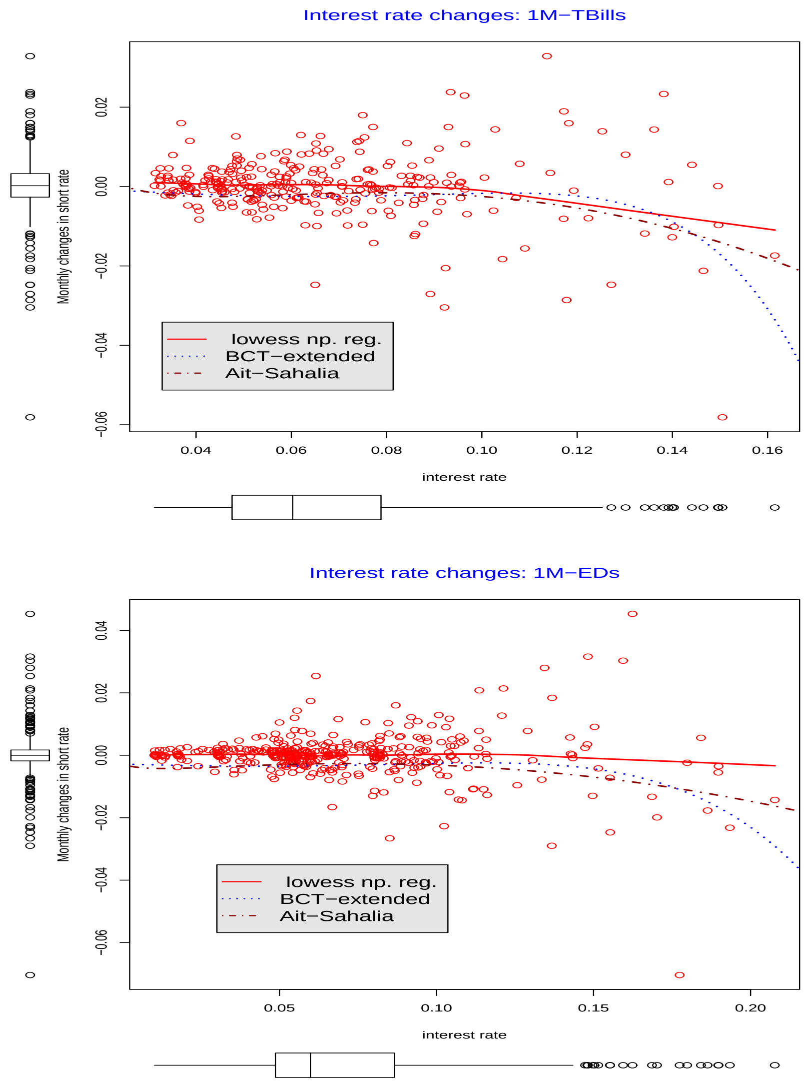

FIGURE 4. **Monthly interest rate changes**: The first (second) figure contains the monthly changes in the one-month Treasury bill (Eurodollar deposit) rates plotted against rates on preceding month. It also contains three regression lines drawn by a LOWESS nonparametric regression, and our nonlinear drift and Aït-Sahalia models.