Horizon Effects on Dependence Structure between Hedge Funds and the Equity Market[◊]

Francis In[a], Byoung Uk Kang[*b], Gunky Kim[c], Tong Suk Kim[b]

[a] *Department of Accounting and Finance, Monash University, Clayton, Victoria, 3168, Australia*

[b] *Graduate School of Management, Korea Advanced Institute of Science and Technology, 207-43, Cheongyangri 2-dong, Dongdaemoon-gu, Seoul, 130-722, Republic of Korea*

[c] *Department of Econometrics and Business Statistics, Monash University, Caulfield East, Victoria, 3145, Australia*

ABSTRACT

This paper investigates the investment horizon effects on dependence structure between hedge fund returns and market returns. The key question is whether the nonlinearity (i.e., asymmetry) in the dependence structure, observed by several previous studies using *monthly* frequency data, continues to exist between longer horizon returns. In the context of hedge funds, this question is particularly important because investing in a hedge fund often involves liquidity restrictions such as lock-up periods and redemption notice periods. Two main results emerge. First, the asymmetry in their dependence relationship is more short-term in nature. Second, the lower tail dependence decreases along with the increasing investment time horizon, which suggest the possibility that the tail risk of hedge funds can be diversified through time. Our finding of diminishing asymmetry in dependence structure along with lengthening investment horizon implies that the value of knowing such asymmetries may not be as substantial for long-term investors as the previous studies suggest.

[◊] This draft is very preliminary. Please do not quote or cite.

[*] Corresponding author. Tel.: +82-2-958-3689; fax: +82-2-958-3618. wayfarer@kgsm.kaist.ac.kr

1. Introduction

Returns on many hedge funds exhibit a nonlinear relationship with the return on market portfolio. Its implications are important for asset allocation and risk management involving hedge funds. For example, when a hedge fund has a payoff structure resembling a short position in a put option on the market, which indeed is the case according to Agrawal and Naik (2004) among others, the benefit of diversification is overstated by those who do not consider the stronger downside (than upside) co-movements between the fund and the market.

Despite the importance of its implications, our understanding of the nonlinear dependence between hedge fund returns and market returns is obtained largely by analyzing on *monthly* frequency data. Thus, the question that arises is: Does the asymmetry in the dependence structure continue to exist between longer horizon returns? In the context of hedge funds, this question is particularly important because investing in a hedge fund often involves liquidity restrictions such as lock-up periods and redemption notice periods. Hence, we simply have to look beyond one month horizon.

In this paper, we investigate the dependence structure between hedge fund returns and market returns, using short as well as long horizon returns. Thus, this paper can be viewed as an attempt to address the suggestions of Poon et al. (2004), who propose studying the effect of investment time horizons on tail dependence.

Two main results emerge. First, the asymmetry (i.e., nonlinearity) in the dependence relationship between a hedge fund and the market return is more short-term in nature. Specifically, the dependence structure between short horizon returns on the hedge fund and the market is found to be asymmetric, where their co-movements are much greater for downside moves than for upside moves. However, as investment horizon increases, the asymmetry in the dependence structure is gradually resolved.

Second, the lower tail dependence (i.e., the probability of an extremely low return on a hedge fund conditional on an extremely low return on the market) decreases along with the increasing investment time horizon. This indicates that the tail risk of hedge funds can be diversified through time. Interestingly, according to Brown and Spitzer (2006), this type of risk is not diversifiable by forming a portfolio of hedge funds.

The remainder of this paper is organized as follows. Section 2 discusses our research design. Section 3 presents the evidence for horizon effects on dependence structure between hedge funds and market returns. Section 4 concludes.

2. Research design

To investigate the horizon effects on dependence structure between a hedge fund and the market returns, we create horizon-by-horizon joint distributions of the hedge fund and the market returns. Specifically, we simulate quarterly, half-yearly, yearly, three-yearly and five-yearly return distributions of the fund and the market. To this end, we use both parametric and non-parametric approaches, which we discuss below. We first simulate, parametrically and non-parametrically, a path of 60 monthly returns on a given fund and the market, and take the first *k* realizations to compute a *k*-month holding period return:

$$R_{h,0\to k} = \prod_{j=1}^{k} R_{h,j} = \prod_{j=1}^{k} \frac{NAV_j}{NAV_{j-1}} \qquad k = 1, 3, 6, 12, 36, 60$$

where $R_{h,0\to k}$ is the holding period return over *k* months beginning from time 0, and $R_{h,j}$ is the monthly return on the hedge fund at the end of the *j*-th month (i.e., the *j*-th realization of monthly return on the hedge fund). A similar computations can be done for the market return, $R_{m,j}$. From each simulated path, we obtain a *single* realization of one-quarter, six-month, one-year, three-year, and five-year holding period returns (i.e., $R_{i,0\to 3}$, $R_{i,0\to 6}$, $R_{i,0\to 12}$, $R_{i,0\to 36}$, and $R_{i,0\to 60}$ for *i*=*h, m*) on the fund and the market, respectively. We repeat this procedure *B* (→∞) times to construct the simulated joint distribution of the fund and the market returns for each holding period.

2.1. Non-parametric simulation

The bootstrap has been traditionally used for estimating the distribution of an estimator or test statistics by re-sampling historical data. In this paper, we employ a block bootstrap technique as a non-parametric way to obtain empirical distribution of *k*-month holding period return. The main advantage of this approach is that the univariate properties such as serial correlation and heteroskedasticity in the financial time series are preserved in block-bootstrapped return series, without making any assumption on the underlying process. Specifically, we use the stationary bootstrap of Politis and Romano (1994), and the optimal average block size is determined similar to Patton (2006): We first apply the algorithm of Politis and White (2004) to returns on a given hedge fund and the market, squared returns on the hedge fund and the market, and the product of the two returns, then choose the largest of these lengths as the block length for both assets. Unlike Patton (2006), however, we re-sample the vector of the fund and the market returns, rather than re-sample the fund and the market returns separately. By doing so, we ensure that along with the statistical artefacts in univariate series, *monthly* cross-sectional dependence structure between the two assets is also maintained in each block-bootstrapped return series. In application to our dataset, we use 5,000 bootstrap replications (i.e., B=5,000) with the size of each bootstrap sample equal to 60.[1]

2.2. Parametric simulation

Unlike the non-parametric method, the parametric approach involves model specification and parameter calibration before used to generate return paths. For this reason, Monte Carlo simulation is subject to some degree of model misspecification error. Nevertheless, this approach can be very useful in investigating the distributional properties of asset returns derived from one's *intended* model. Related to this paper, Monte Carlo method enables us to examine whether or not lower tail dependency is still evident in longer investment horizons when underlying process driving *monthly* hedge fund and market returns is explicitly modelled to generate the lower tail co-movement. Of course, no such experiment is possible without a parametric model that produces the desired property. Note that the standard families of multivariate distributions such as the multivariate normal or the elliptic distributions are not flexible enough to generate such tail behaviours. For this reason, we adopt copulas.

[1] This may be interpreted as a class of '*m out of n'* bootstrap since our original sample data includes 150 monthly observations, which is larger than each bootstrap sample size of 60. It is well known that the '*m out of n'* bootstrap technique performs successfully in many situations where the conventional bootstrap fails.

2.2.1 Copula-based model specification for simulation

Typically, the marginal distributions of hedge funds and the market returns are skewed and fat-tailed, while their means and variances are time-varying. For hedge fund returns, in particular, existence of autocorrelation is well documented in recent literature. (see, e.g., Asness et al., 2001; Getmansky et al., 2004). We incorporate these univariate statistical properties of the hedge funds and the market returns, along with their dependence relationship, for the simulation. For exposition, let us assume that the marginal distributions for a hedge fund (i=h) and the market (i=m) returns follow AR(p_i)-GARCH(q_i,r_i) processes:

$$R_{i,t} = \mu_i + \sum_{l=1}^{p_i} \phi_{i,l} R_{i,t-l} + u_{i,t}$$

$$h_{i,t} = \omega_i + \sum_{l=1}^{q_i} \alpha_i u_{i,t-1}^2 + \sum_{l=1}^{r_i} \beta_i h_{i,t-l} \qquad \textit{for } i = m, h \quad (1)$$

$$u_{i,t} = \sqrt{h_{i,t}}\,\upsilon_{i,t}$$

where the distribution of the standardized residual (i.e., $\upsilon_{i,t}$, i =m, h) is either the standard normal, the standardized Student's *t* or the skewed Student's *t* distribution of Hansen (1994), and that their dependence structure is determined by a copula function C:

$$F_{m,h}\left(\frac{u_{m,t}}{\sqrt{h_{m,t}}},\frac{u_{h,t}}{\sqrt{h_{h,t}}}\right)=C\left(F_m\left(\frac{u_{m,t}}{\sqrt{h_{m,t}}}\right),F_h\left(\frac{u_{h,t}}{\sqrt{h_{h,t}}}\right);\theta\right) \tag{2}$$

where $F_{m,h}$ is the joint *cdf* of $u_{m,t}\big/\sqrt{h_{m,t}}$ and $u_{h,t}\big/\sqrt{h_{h,t}}$, and F_i is the marginal *cdf* of $u_{i,t}\big/\sqrt{h_{i,t}}$, which is the inverse function of $\upsilon_{i,t}$. Based on this specification, the simulation proceeds as follow: First, we simulate two series of 60 uniformly distributed random variables *x* and *y* from the copula function *C* with parameter θ calibrated to the data. Second, we compute two series of standardized innovations $u_{m,t}\big/\sqrt{h_{m,t}}$ and $u_{h,t}\big/\sqrt{h_{h,t}}$ by taking their respective inverse *cdf*s on *x* and *y*. Based on the simulated time series of standardized innovations and other calibrated parameters, the rest are the same as simulating univariate AR(p_i)-GARCH(q_i,r_i) models. We repeat this procedure 20,000 times (i.e., *B*=20,000) to create horizon-by-horizon joint distributions of the hedge fund and the market returns as illustrated above. While we expect that the dependence structure between one-month holding period returns (i.e., $R_{i,0\to 1}$, $i=m, h$) is most directly influenced by our choice of copula function, how this dependency changes with the varying investment horizon remains to be seen.

3. Empirical results

Section 3.1 describes our data. Sections 3.2 and 3.3, the heart of this paper, examine the effect of investment time horizons on the dependence structure between a given hedge fund and the market portfolio. Our findings are discussed mainly in Section 3.2 based on results from non-parametric simulation. Using Monte Carlo simulation, Section 3.3 serves as a robust check of the results reported in Section 3.2.

3.1. Data

We use monthly net-of-fee returns of Live and Dead (or Graveyard) hedge funds in the TASS database. Specifically, we focus on those that categorize themselves as "Long/Short Equity" (LSE hereafter) or "Event-Driven" (ED hereafter) among other style designations in the database. There are at least two motivations behind this: First, LSE and ED funds are *by far* the most popular and rapidly growing style categories within the hedge fund industry. For example, according to the Tremont Asset Flows Report (Fourth Quarter, 2006), the total amount of hedge fund capital devoted to LSE and ED funds are in excess of \$326.5 billion and \$224.2 billion, respectively. These amounts respectively account for more than 31.0% and 21.3% of sum of the all hedge funds under management. Second, and more importantly, LSE and ED funds are found by several previous studies to exhibit a strong nonlinear relation with the return on market portfolio. In particular, Brown and Spitzer (2006) observe that the nonlinear dependency, where hedge funds are more sensitive to market risk in bear markets than in bull markets, is most pronounced for LSE and ED funds among other style designations. Thus, LSE and ED funds provide us with a good laboratory to test whether or not the nonlinear dependence structure established between short-horizon (i.e., monthly) returns still remains between longer-horizon returns.

Our hedge fund data run from January 1994 through June 2006 (i.e., 150 monthly observations). For analysis, we construct an equal-weighted return on all eligible Live and Dead funds under each category. To be included in the sample, we require a fund to report U.S. dollar denominated returns, net of management and incentive fees, on a monthly basis. We also require a fund to have a minium of \$5 million of assets under management and a minimum one-year track record. As a result, we are left with 1,296 LSE funds (696 in the Live and 600 in the Dead databases) and 420 ED funds (241 in the Live and 179 in the Dead databases). As the market index, we employ the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) throughout this paper.

3.2. Analysis on non-parametrically simulated data

As a preliminary look at the dependence structure between a given hedge fund and the market portfolio, Figures 1 and 2 plot the empirical *cdf* of $R_{m,0\to k}$ ($\widetilde{F}_{R_{m,0\to k}}$

hereafter) against that of $R_{h,0\to k}$ ($\widetilde{F}_{R_{h,0\to k}}$ hereafter) for the LSE funds and the ED funds, respectively. The advantage of doing this, compared to plotting $R_{m,0\to k}$ against $R_{h,0\to k}$ is that we focus solely on the dependence structure by filtering out the influence of marginal distributions. Clearly, both Figures 1 and 2 present evidence for the impact of the investment horizon on the dependence structure: For the short-term investment horizon (i.e., quarterly), the points tend to concentrate near (0, 0), whereas for the long-term investment horizon (i.e., five-yearly), the points near the two vertices (0, 0) and (1, 1) in the unit square are of about equal densities.

Following Longing and Solnik (2000), and Ang and Chen (2002), we also consider the exceedance correlation based on the empirical *cdf*s of $R_{m,0\to k}$ and $R_{h,0\to k}$:

$$\widetilde{\rho}(q) \equiv \begin{cases} corr\left(\widetilde{F}_{R_{m,0\to k}}, \widetilde{F}_{R_{h,0\to k}} \,\middle|\, \widetilde{F}_{R_{m,0\to k}} \le Q_m(q), \widetilde{F}_{R_{h,0\to k}} \le Q_h(q)\right), & for\ q \le 0.5 \\ corr\left(\widetilde{F}_{R_{m,0\to k}}, \widetilde{F}_{R_{h,0\to k}} \,\middle|\, \widetilde{F}_{R_{m,0\to k}} \ge Q_m(q), \widetilde{F}_{R_{h,0\to k}} \ge Q_h(q)\right), & for\ q \ge 0.5 \end{cases} \tag{3}$$

where $Q_i(q)$ is the q-th quantile of $\widetilde{F}_{R_{i,0\to k}}$, $i = m, h$. Based on $\widetilde{F}_{R_{m,0\to k}}$ and $\widetilde{F}_{R_{h,0\to k}}$, the exceedance correlation in Eq. (3) enables the isolation of the dependence structure from the influence of marginal distributions. Thus, any subsequent finding of asymmetry will be entirely due to the asymmetry in dependence structure.

Figures 3 and 4 show the empirical exceedance correlations based on $\widetilde{F}_{R_{m,0\to k}}$ and $\widetilde{F}_{R_{h,0\to k}}$ for the LSE funds and the ED funds, respectively. Similar in spirit to Longing and Solnik (2000), and Ang and Chen (2002), who compare the empirical exceedance correlations with the exceedance correlations derived under the assumption of the multivariate normality, we provide a plot of what would be obtained from the normal copula in the figures. Analogous to the multivariate normality which some authors have used as the benchmark in determining asymmetry in correlation structure, we adopt the normal copula as the benchmark for asymmetry in dependence structure. Also presented in the figures is a plot of exceedance correlations implied by the Clayton copula, the inclusion of which is due to the qualitative similarity between our empirical exceedance correlations and the type of dependence suggested by the Clayton copula.

By inspection of Figures 3 and 4, the investment time horizon again appears to have an impact on the dependence structure between the hedge funds and the market. Relative to those implied by the normal and Clayton copula models, the empirical exceedance correlations based on $\widetilde{F}_{R_{m,0\to k}}$ and $\widetilde{F}_{R_{h,0\to k}}$ almost always lie somewhere in between the two. Interestingly, it is clearly demonstrated the tendency towards the normal copula (i.e., symmetry in dependence structure) as with the increasing investment horizon. Note that, in particular, the shape of its exceedance correlations for five-year holding period return on the ED funds is almost indistinguishable from what would be obtained if the funds and the market had the normal copula. In short, consistent with Figures 1 and 2, the asymmetry in the dependence relation, which indicates the stronger downside co-movement for the short-term investment horizon, is resolved for the long-term investment horizon.

Next, we examine the investment time horizon effect on the *lower* tail part of dependence structure (i.e., tail dependence). Our goal is to check whether the tail risk of hedge funds, which Brown and Spitzer (2006) among others find to be not diversified by forming a portfolio of hedge funds, is *time*-diversifiable. To this end, we compute the following measure of tail dependency, and compare the results across all holding periods:

$$\lambda_L = \lim_{u \to 0} \frac{C(u,u)}{u} \tag{4}$$

where C denotes the copula function that determines the dependent structure between a given hedge fund and the market portfolio. Joe (1997) gives the reasoning behind the Eq. (4): When $(U_1, U_2) \sim C$, then

$$\lambda_L = \lim_{u \to 0} \Pr(U_1 < u \mid U_2 < u) = \lim_{u \to 0} \Pr(U_2 < u \mid U_1 < u)$$

In words, λ_L in Eq. (4) is the probability of an extremely low return on the hedge fund, conditional on the fact that an extremely low return on the market portfolio is observed (and vice versa).

For the copula function that will be used in Eq. (4), we propose, following Hong et al. (2006) among others, a mixture copula that linearly combines the normal copula with the Clayton copula:

$$C_{mix}(u_1, u_2; \kappa, \rho, \tau) = \kappa C_{norm}(u_1, u_2; \rho) + (1-\kappa) C_{clay}(u_1, u_2; \tau) \tag{5}$$

where κ ($0 \leq \kappa \leq 1$) is the mixture parameter, C_{norm} is the normal copula function, and C_{clay} is the Clayton copula function. Our choice of the mixture copula is motivated by the previous observations in Figures 3 and 4, where the shape of empirical exceedance correlation plot appears to be a *mixture* of those implied by the normal and the Clayton copula functions.[2] In addition, by adjusting the mixture parameter κ, the mixture copula captures the gradual change in dependence structure (i.e., from near the Clayton towards the normal) with increasing investment horizons. The specification in Eq. (4) clearly shows that both the normal and the Clayton copula functions are nested within the mixture models as a special case. From Eq. (4) and (5), it can be shown that the lower tail dependence for the mixture copula is expressed in terms of the model parameters:

$$\lambda_L^{mix} = \kappa \lambda_L^{norm} + (1-\kappa)\lambda_L^{clay} = 2^{-1/\tau}(1-\kappa) \tag{6}$$

[2] Also, the mixture copula model is found to best fit the simulated sample data across all holding periods (except for the quarterly holding period return on the LSE funds) among other commonly used copula functions in the literature: the normal, the Clayton, the Gumbel, and the Frank copula functions. The comparison is based on Junker and May (2002)'s χ^2 goodness-of-fit test.

In what follows, the λ_L^{mix} for each holding period will be calculated based on the estimates of κ and τ.

Panel A of Table 1 reports the estimation results of the mixture model for the LSE funds. The model parameters are estimated by maximizing the *pseudo loglikelihood*, the likelihood of a parametric copula evaluated at nonparametric marginal distributions (i.e., the empirical marginal *cdf*s).[3] Throughout, the estimates are very precise due to the size of our simulated data. Consistent with what we visually observe in Figure 3, Panel A reveals that the mixture parameter $\tilde{\kappa}$, which determines the weight of the normal copula in the mixture model, increases monotonically along with the increasing investment horizon. In contrast, the normal parameter $\tilde{\rho}$ remains almost unchanged, indicating no horizon effects on the overall linear dependency. Although it alone implies little change in tail dependence, the Clayton parameter $\tilde{\tau}$, when inputted into Eq. (4) together with $\tilde{\kappa}$, yields monotonically decreasing lower tail dependence $\tilde{\lambda}_L^{mix}$ along with the lengthening investment horizon, mostly due to the effect of the mixture parameter $\tilde{\kappa}$. Panel B tells the same story for the ED funds, though virtual tail

[3] The extensive simulation study done in Kim et al. (2006) suggests that the proposed method performs better than the ML (maximum likelihood) and the IFM (inference function for margins) methods, overall. Their results show that the ML and the IFM methods are not robust against misspecification of the marginal distributions. Thus, care should be taken when applying theses methods when the marginal distributions are unknown, which is almost always the case in practice.

independence (or 'tail neutrality', termed by Patton (2006)) for more than one-year holding period return is one notable difference.

In summary, the tail dependence between the hedge funds and the market decreases as the investment time horizon increases. Indeed, the tail risk of the hedge fund can be, to varying extent, time-diversified.

3.3. Analysis on parametrically simulated data

Monte Carlo simulation involves model specifications and parameter calibrations. For marginal distributions of the return on the LSE funds and the market return, we choose AR(1), skewed *t*-GARCH(1,1) specifications, whereas for marginal distribution of the return on the ED funds, we adopt AR(1), skewed *t*-GARCH(0,0), where there is no time-varying volatility while the residuals are skewed and fat-tailed. Though not shown, the distributions of estimated standardized residuals justify the proposed specifications. We estimate the parameters in the univariate models, using the ML method, and the estimated standardized residuals are stored for the purpose of the copula parameter estimation. As explained above, the copula parameter estimations are done by maximizing the pseudo likelihood.[4] Regarding the choice of copula model, we choose the Clayton copula, which imposes the non-zero lower tail dependence. Although the true dependence structure may not exactly conform to this one parameter model, we find no evidence against the Clayton copula based on Junker and May (2002)'s χ^2 goodness-of-fit test (not shown). Moreover, though the tail dependence generated by the Clayton model may exaggerate the true tail dependence, this should provide no problem: Our goal in the end is to check the extent of tail dependence (or the asymmetry in dependence structure) between the *longer* horizon returns when the dependence structure between *short* horizon returns on the hedge fund and the market is characterized by those properties.

The results based on Monte Carlo simulations are presented in Figures 5 to 8 and in Table 2. The figures and table provide qualitatively similar results to those presented in the previous sub-section.

4. Conclusions

[4] This class of semiparametric copula-based model is first introduced in Chen and Fan (2006). They specify the conditional mean and the conditional variance of a multivariate time series parametrically, but specify the multivariate distribution of the standardized innovation semiparametrically as a parametric copula evaluated at nonparametric marginal distributions. They show that the limiting distribution of the estimator of the pseudo true value of the copula parameter is not affected by the estimation of the parameters involved in conditional mean and conditional variance.

We have two main results concerning the investment horizon effects on dependence structure between hedge fund returns and market returns. First, the nonlinearity in their dependence relationship is more short-term in nature. Specifically, the short horizon returns on hedge funds and market exhibit the asymmetric dependence structure, where their co-movements are much greater for downside moves than for upside moves. However, the asymmetry gradually diminishes as investment horizon increases. Second, the lower tail dependence decreases along with the increasing investment time horizon, which suggest the possibility that the tail risk of hedge funds can be diversified through time. Interestingly, according to Brown and Spitzer (2006), this type of risk is not diversifiable by forming a portfolio of hedge funds (i.e., fund of funds).

Future research could study the long-term asset allocation implications. Some recent papers have evaluated the economic significance of incorporating asymmetric dependence into investment decisions (see, e.g., Ang and Chen, 2002; Patton, 2004; Hong et al., 2006). Our finding of diminishing asymmetry in dependence structure along with lengthening investment horizon implies that the value of knowing such asymmetries may not be as substantial for long-term investors as the previous studies suggest.

References

Agarwal, V., Naik, N., 2004. Risks and portfolio decisions involving hedge funds. Review of Financial Studies 17, 63-98.

Ang, A., Chen, J., 2002. Asymmetric correlations of equity portfolios. Journal of Financial Econometrics 63, 443-494.

Asness, C., Krail, R., Liew, J., 2001. Do hedge funds hedge? Journal of Portfolio Management 28, 6-19.

Brown, S., Spitzer, J., 2006. Caught by the tail: tail risk neutrality and hedge fund returns. Unpublished working paper.

Chen, X., Fan, Y., 2006. Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification. Journal of Econometrics 135, 125-154.

Getmansky, M., Lo, A., Makarov, I., 2004. An econometric model of series correlation and illiquidity in hedge fund returns. Journal of Financial Economics 74, 529-610.

Hansen, B., 1994. Autoregressive conditional density estimation. International Economic Review 35, 705-730.

Hong, Y., Tu, J., Zhou, G., 2006. Asymmetries in stock returns: statistical tests and economic evaluation. Review of Financial Studies, forthcoming.

Joe, H., 1997. Multivariate models and dependence concepts. Chapman & Hall/CRC, London.

Junker, M., May, A., 2002. Measurement of aggregate risk with copulas. Econometrics Journal 8, 428-454.

Kim, G., Silvapulle, M., Silvapulle, P., 2007. Comparison of semiparametric and parametric methods for estimating copulas. Computational Statistics and Data Analysis 51, 2836-2850.

Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. Journal of Finance 56, 649-676.

Patton, A., 2004. On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. Journal of Financial Econometrics 2, 130-168.

Patton, A., 2006. Are “market neutral” hedge funds really market neutral? Unpublished working paper, London School of Economics, London.

Politis, D., Romano, J., 1994. The stationary bootstrap, Journal of the American Statistical Association 89, 1303-1313.

Politis, D., White, H., 2004. Automatic block-length selection for the dependent bootstrap, Econometric Reviews 23, 53-70.

Poon, S., Rockinger, M., Tawn, 2004. Extreme value dependence in financial markets: diagnostics, models, and financial implications. Review of Financial Studies 17, 581-610.

Figure 1 Scatter Diagram for the Long/Short Equity Funds based on Non-parametrically Simulated Samples

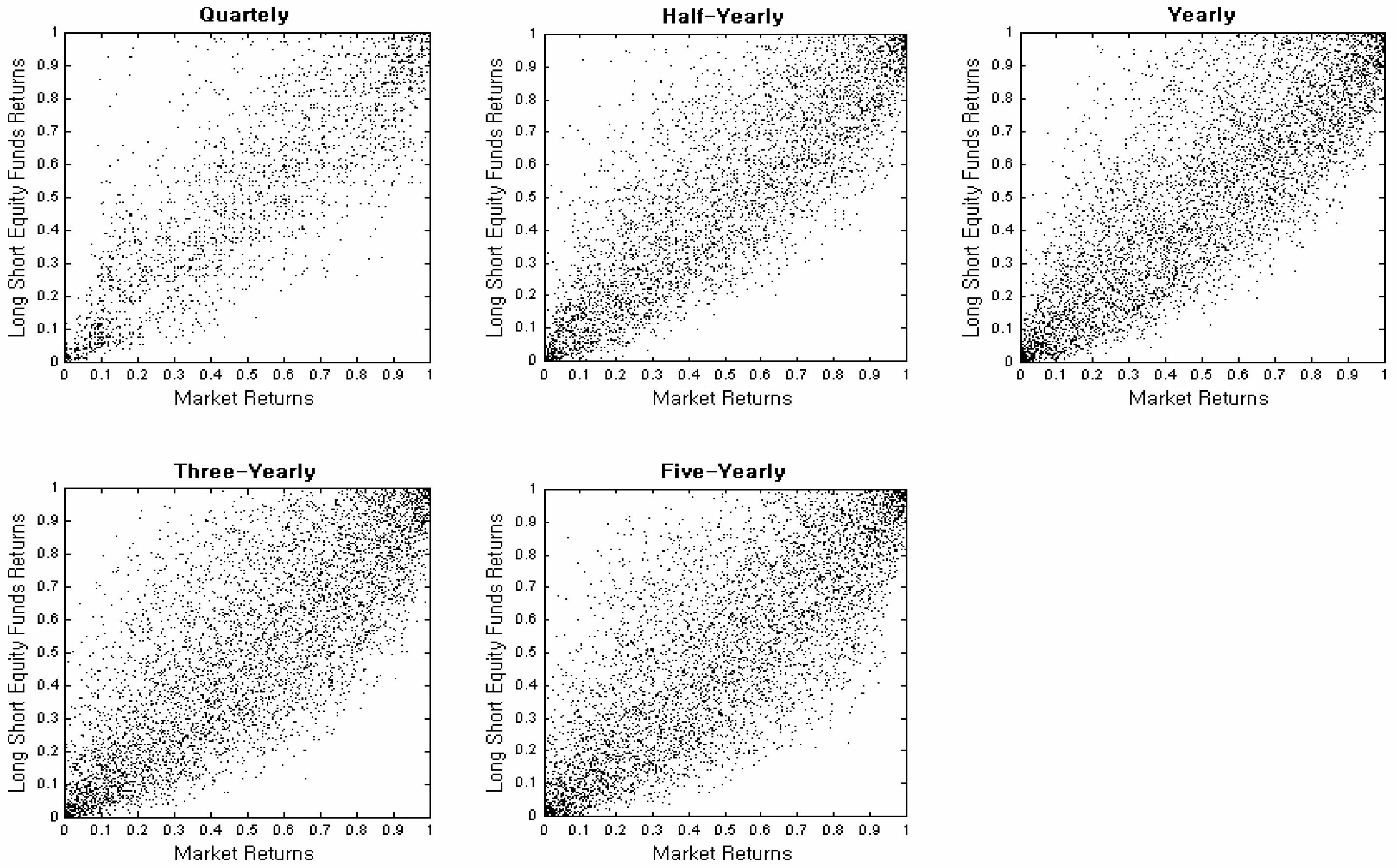

Figure 2 Scatter Diagram for the Event-Driven Funds based on Non-parametrically Simulated Samples

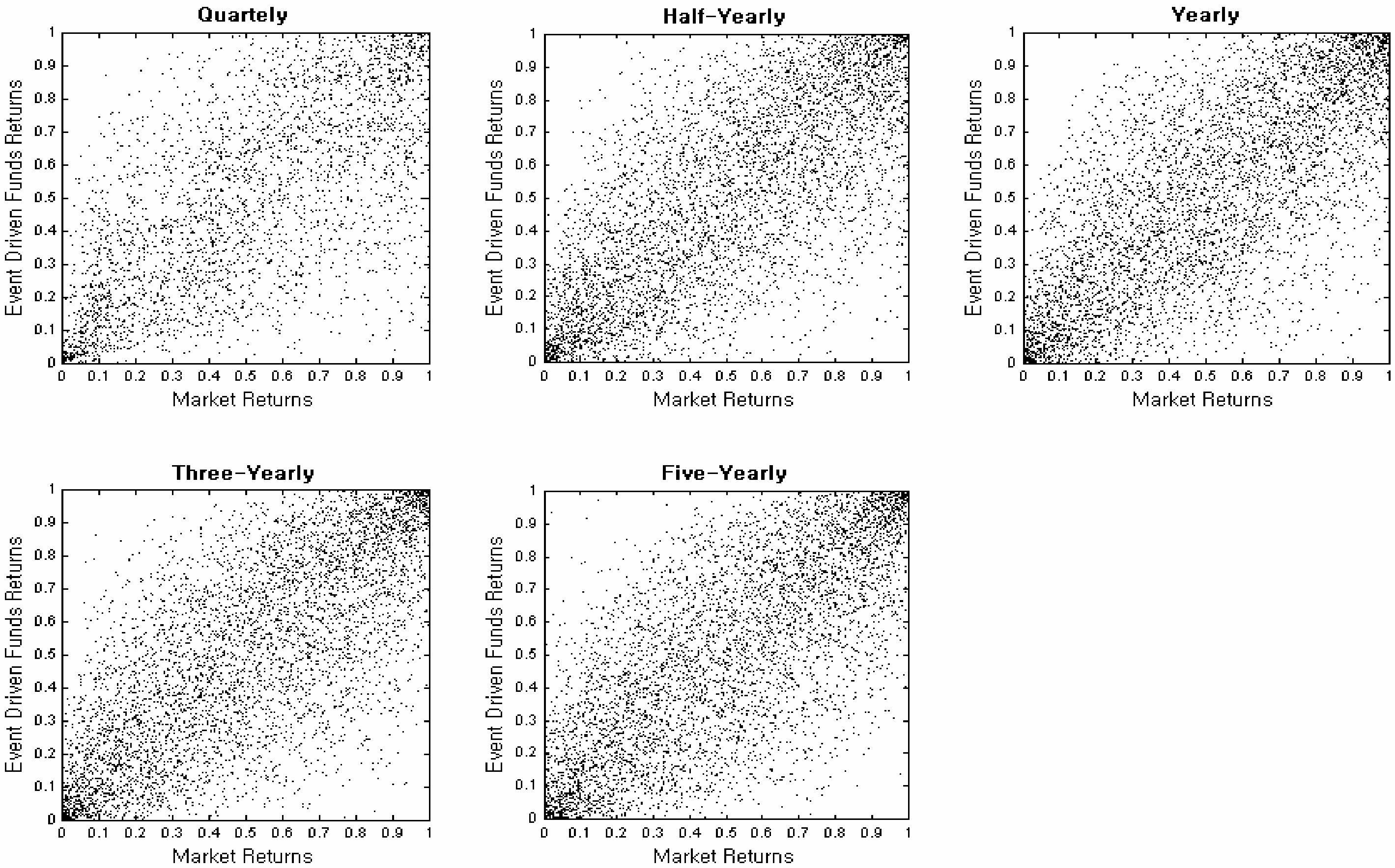

Figure 3 Exceedance Correlation Plot for the Long/Short Equity Funds based on Non-parametrically Simulated Samples

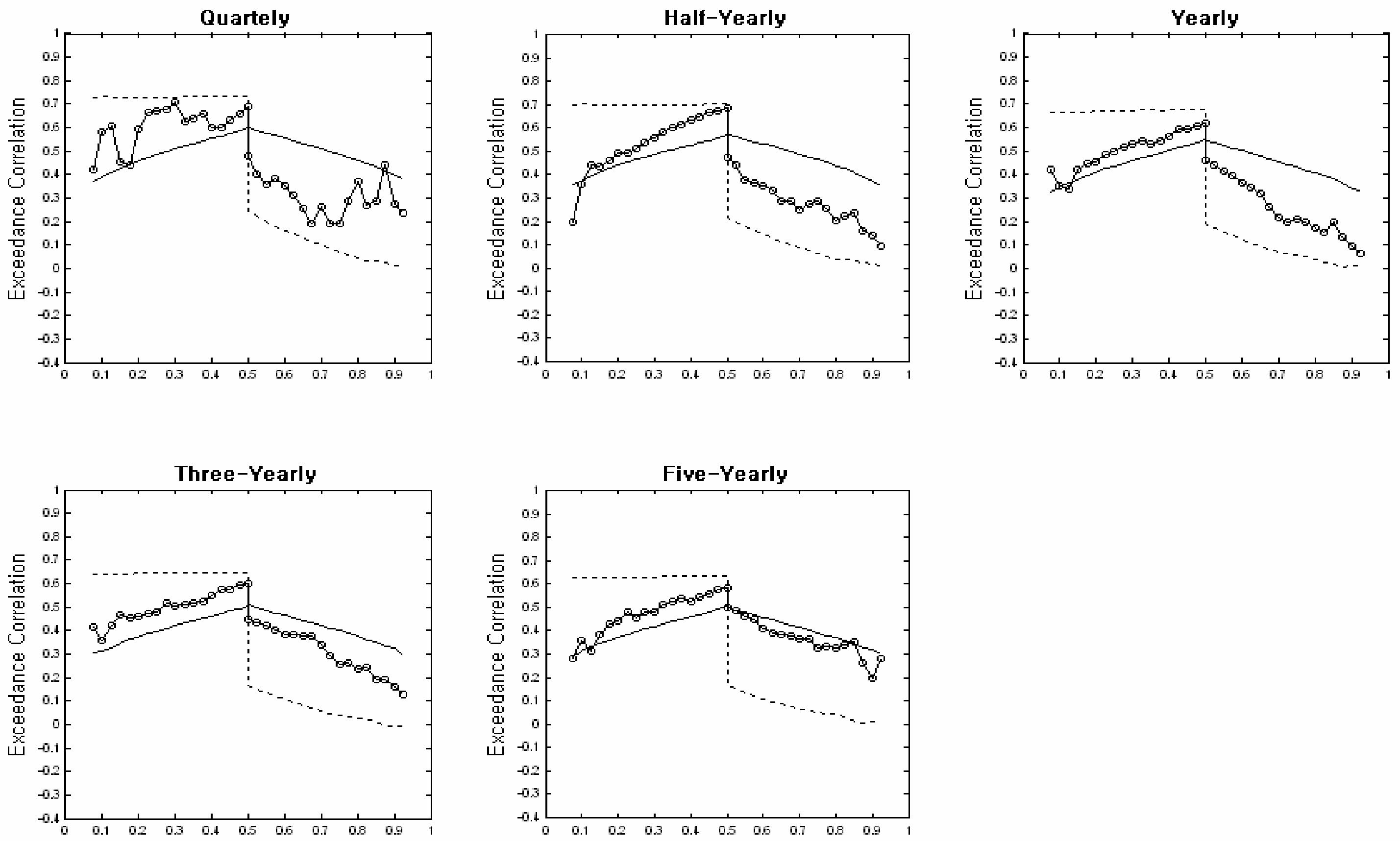

Figure 4 Exceedance Correlation Plot for the Event-Driven Fund based on Non-parametrically Simulated Samples

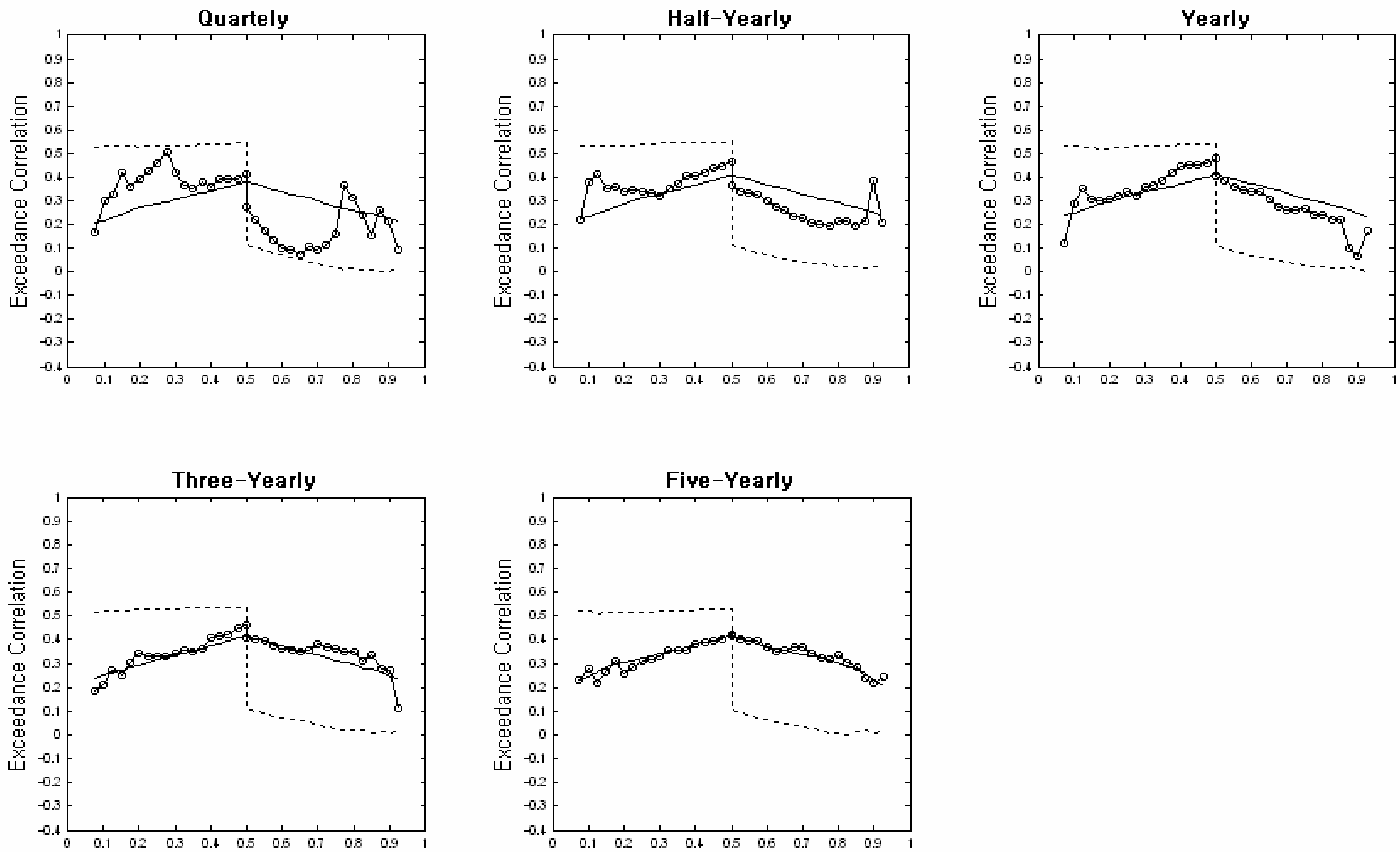

Figure 5 Scatter Diagram for the Long/Short Equity Funds based on Parametrically Simulated Samples

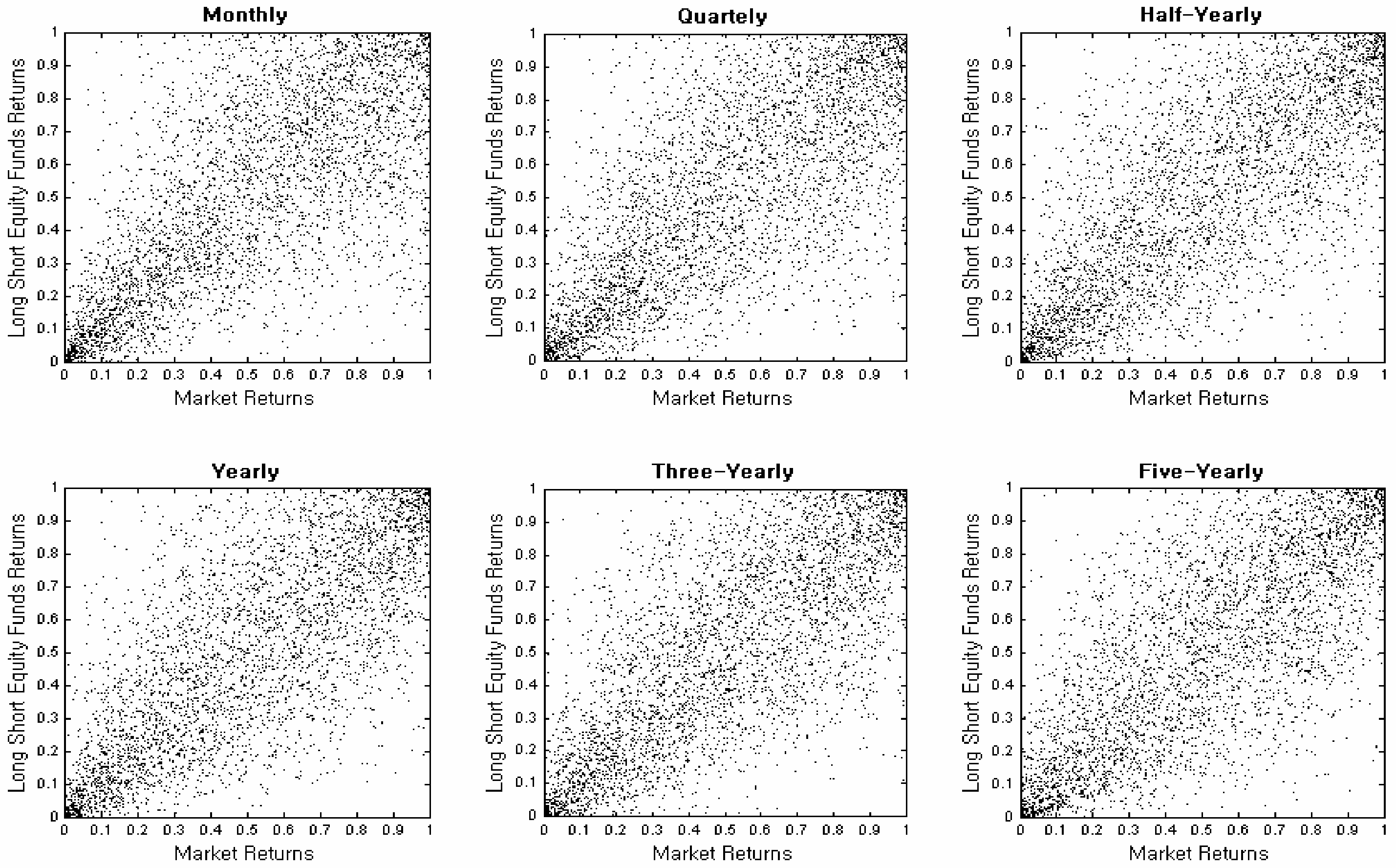

Figure 6 Scatter Diagram for the Event-Driven Funds based on Parametrically Simulated Samples

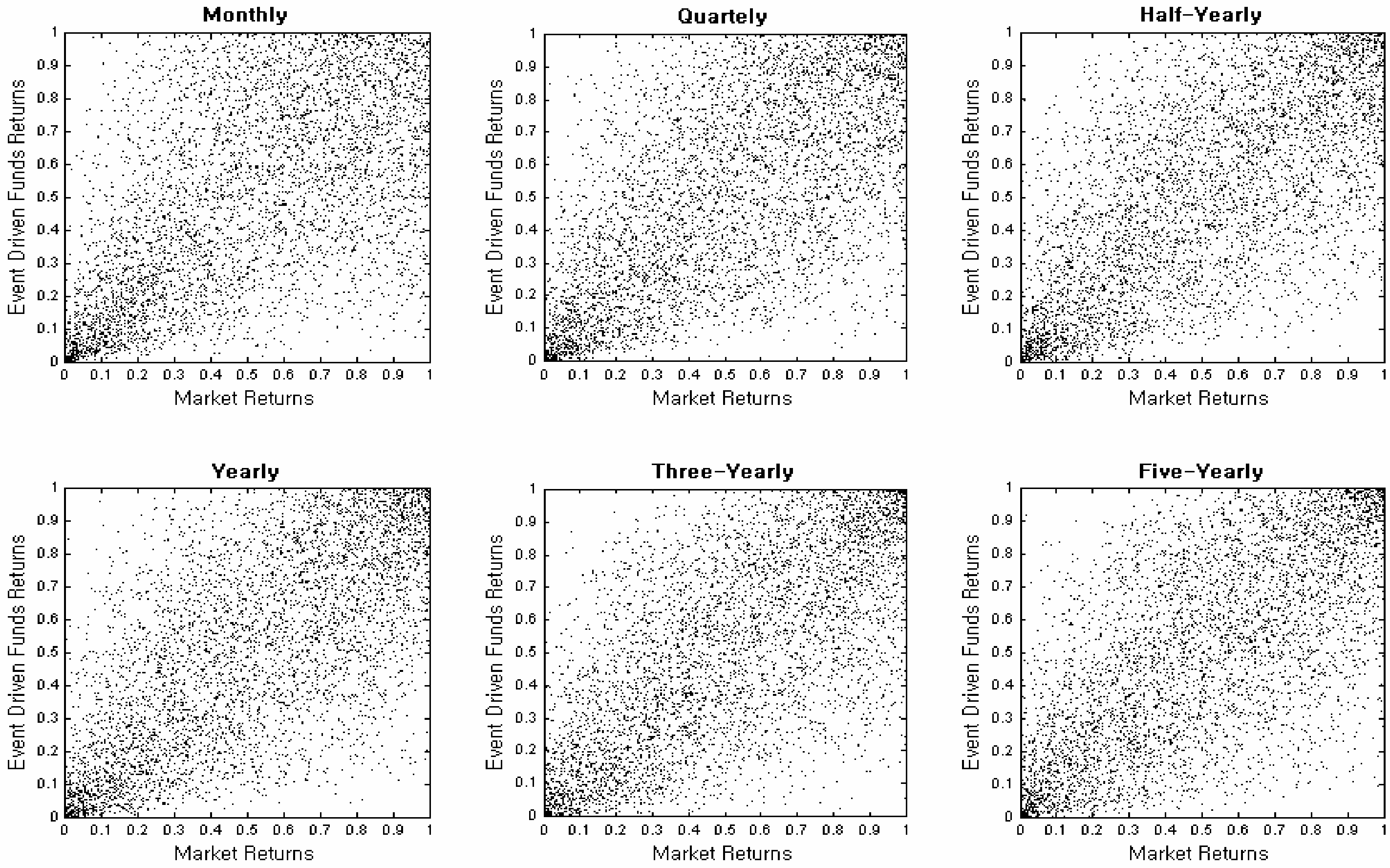

Figure 7 Exceedance Correlation Plot for the Long/Short Equity Funds based on Parametrically Simulated Samples

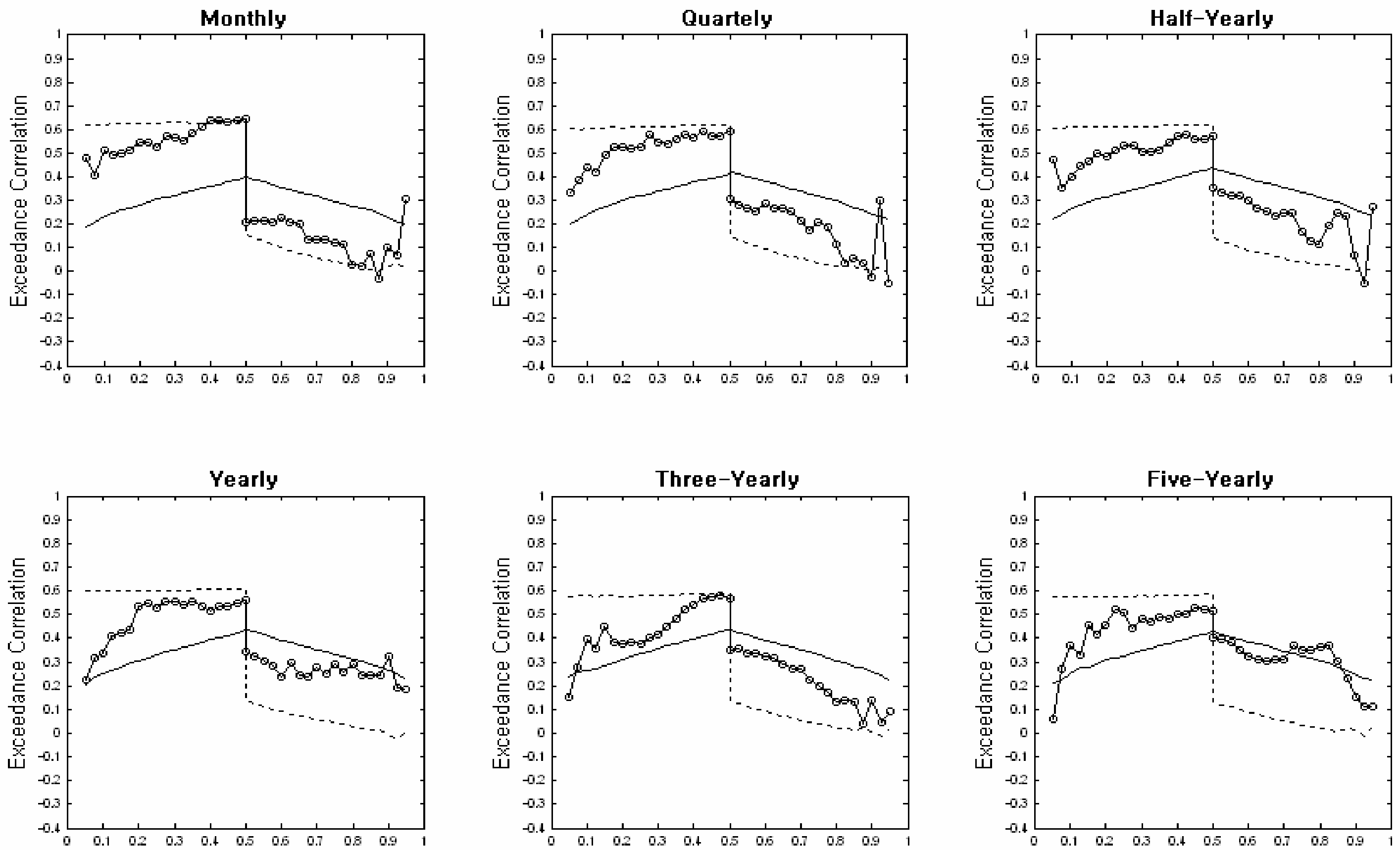

Figure 8 Exceedance Correlation Plot for the Event-Driven Funds based on Parametrically Simulated Samples

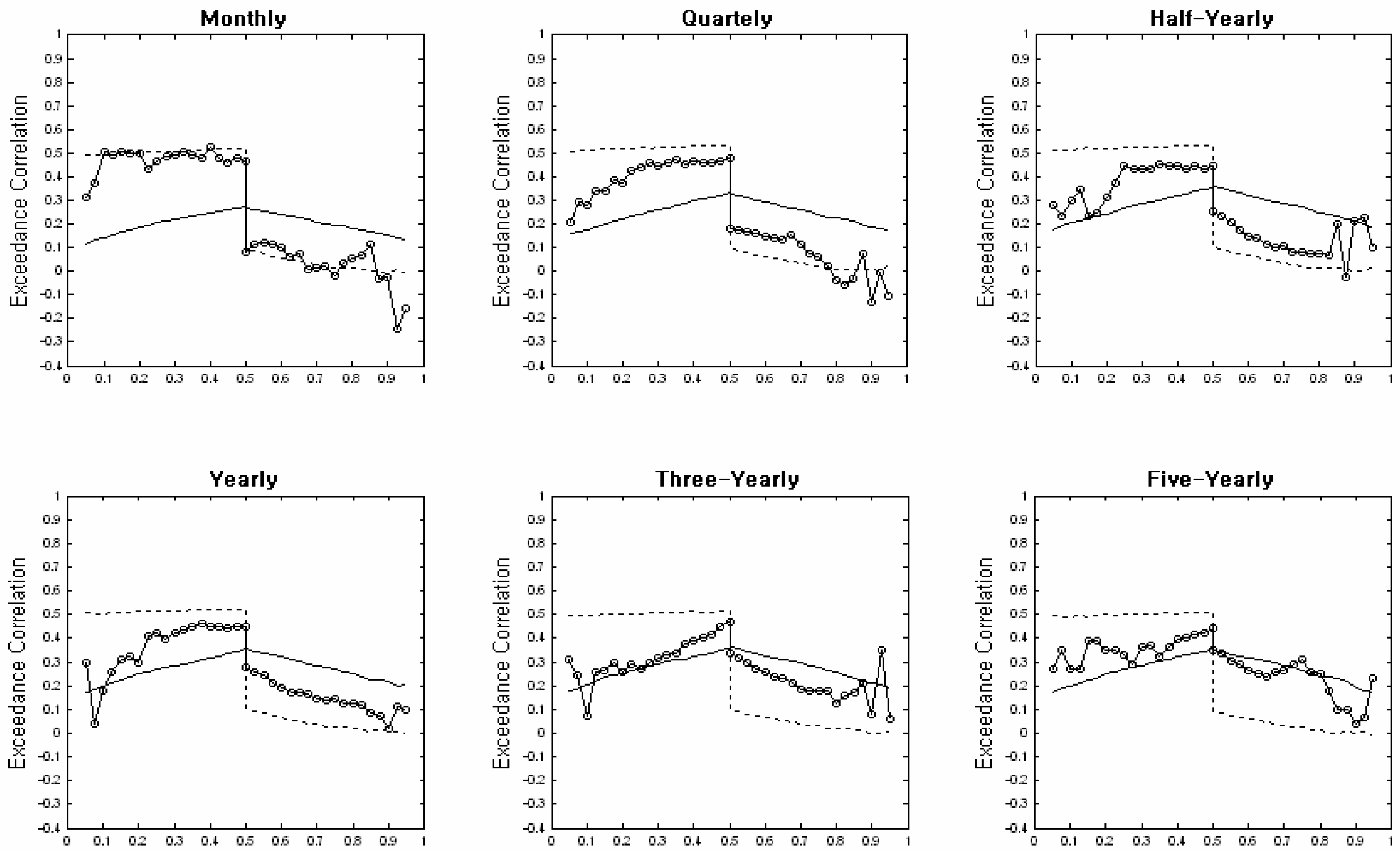

TABLE 1
Estimation of the Mixture Copula based on Non-parametrically Simulated Samples

Panel A: Long/Short Equity

	Q	HY	Y	Y3	Y5
κ	0.599	0.666	0.675	0.792	0.868
	(0.033)	(0.033)	(0.033)	(0.034)	(0.032)
ρ	0.876	0.861	0.842	0.808	0.811
	(0.006)	(0.006)	(0.007)	(0.006)	(0.006)
τ	3.348	2.900	2.754	2.920	2.634
	(0.199)	(0.173)	(0.179)	(0.294)	(0.365)
λ_L^{mix}	0.326	0.263	0.253	0.164	0.102

Panel B: Event-Driven

	Q	HY	Y	Y3	Y5
κ	0.595	0.736	0.865	0.953	0.990
	(0.036)	(0.034)	(0.030)	(0.017)	(0.005)
ρ	0.752	0.737	0.721	0.749	0.742
	(0.012)	(0.009)	(0.007)	(0.007)	(0.007)
τ	1.453	1.923	2.846	0.748	0.013
	(0.092)	(0.163)	(0.434)	(0.221)	(0.471)
λ_L^{mix}	0.251	0.184	0.106	0.018	0.000

Table 1 reports the pseudo maximum likelihood estimates of the parameters in the mixture copula model. The estimations are based on parametrically simulated data. The mixture copula is one that linearly combines the normal copula with the Clayton copula:

$$C_{mix}(u_1, u_2; \kappa, \rho, \tau) = \kappa C_{norm}(u_1, u_2; \rho) + (1-\kappa) C_{clay}(u_1, u_2; \tau)$$

where κ ($0 \leq \kappa \leq 1$) is the mixture parameter, C_{norm} is the normal copula function, and C_{clay} is the Clayton copula function. The last row in each column represents the lower tail dependence between hedge fund returns and market returns, which can be expressed in terms of the model parameters:

$$\lambda_L^{mix} = \kappa \lambda_L^{norm} + (1-\kappa)\lambda_L^{clay} = 2^{-1/\tau}(1-\kappa)$$

The reported λ_L^{mix}s for each holding period are calculated based on the corresponding estimates of κ and τ.

TABLE 2

Estimation of the Mixture Copula based on Parametrically Simulated Samples

Panel A: Long/Short Equity

	Q	HY	Y	Y3	Y5
κ	0.425	0.554	0.594	0.654	0.712
	(0.019)	(0.019)	(0.018)	(0.019)	(0.017)
ρ	0.766	0.767	0.782	0.752	0.776
	(0.006)	(0.005)	(0.005)	(0.004)	(0.004)
τ	2.161	2.278	2.047	2.349	1.768
	(0.041)	(0.059)	(0.060)	(0.069)	(0.058)
λ_L^{mix}	0.417	0.329	0.290	0.258	0.195

Panel B: Event-Driven

	Q	HY	Y	Y3	Y5
κ	0.370	0.521	0.659	0.719	0.761
	(0.022)	(0.021)	(0.021)	(0.020)	(0.019)
ρ	0.722	0.699	0.705	0.699	0.694
	(0.010)	(0.007)	(0.006)	(0.005)	(0.005)
τ	1.414	1.627	1.506	1.593	1.620
	(0.035)	(0.052)	(0.060)	(0.073)	(0.080)
λ_L^{mix}	0.386	0.313	0.215	0.182	0.156

Table 2 reports the pseudo maximum likelihood estimates of the parameters in the mixture copula model. The estimations are based on non-parametrically simulated data. The mixture copula is one that linearly combines the normal copula with the Clayton copula:

$$C_{mix}(u_1,u_2;\kappa,\rho,\tau) = \kappa C_{norm}(u_1,u_2;\rho) + (1-\kappa)C_{clay}(u_1,u_2;\tau)$$

where κ ($0 \le \kappa \le 1$) is the mixture parameter, C_{norm} is the normal copula function, and C_{clay} is the Clayton copula function. The last row in each column represents the lower tail dependence between hedge fund returns and market returns, which can be expressed in terms of the model parameters:

$$\lambda_L^{mix} = \kappa\lambda_L^{norm} + (1-\kappa)\lambda_L^{clay} = 2^{-1/\tau}(1-\kappa)$$

The reported λ_L^{mix}s for each holding period are calculated based on the corresponding estimates of κ and τ.