A Theory of Screening and Debt Financing Choices: Bank Loan versus Finance Loan

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Abstract

This paper extends a screening model in financial intermediation and provides a model for the choice of financing sources between bank loans and finance loans. This paper incorporates the positive marginal rents or mark-up of a seller in a monopolistically competitive products (capital goods) market into the equilibrium loan contracts in a perfectly competitive loan market and examines borrower's choice of different financing sources. When the sale of differentiated products is tied to financing and the additional sale of product extracts positive marginal rents, a captive finance company offers a pooling loan contract with higher loan rate and approval rate. A pooling finance loan contract needs to be subsidized by the additional sale of products to be sustainable. Banks offer separating low- and high-risk loan contracts. A low-risk borrower is indifferent between a separating bank low-risk loan contract and a pooling finance loan contract while a high-risk borrower strictly prefers a pooling finance loan contract to a separating bank high-risk loan contract. Hence, the model successfully explains the prevailing wisdom in lending practices and previous empirical findings that on average finance companies service a riskier pool of borrowers, offering more lenient loan approval rates and higher loan rates, than banks do. In addition, this paper shows the interaction between the product and debt markets in that the effective number of buyers in the monopolistically competitive products market is determined by the number of borrowers approved for loans in the debt market.

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1. Introduction

Financing is usually required for the purchase of costly capital goods. Banks and finance companies are the two main providers of the financing for purchase of capital goods. Especially, major captive finance companies have large market shares in the debt market of the US. Banks includes independent deposit-taking lending institutions such as commercial banks, credit unions, and other depository institutions while captive finance companies are the subsidiaries that finance the sales of products of their parent firms.[1] This paper provides a theoretical model explaining how these two different types of lending institutions service borrowers of different risk types in the loan market, how borrowers choose the loan types and lender types, and how the products and debt markets interact. In answering these questions, this paper examines the credit rationing induced by the adverse selection problem while banks and finance companies co-exist in the debt market.

Several papers have examined the issues related to finance companies from an empirical perspective. Remolona *et al*. (1992) explain the differential performances of banks and finance companies in the loan markets. They found that finance companies lost market share to banks while much of the finance company's growth took place in niches, market segments of relatively risky loan where command of specialized information was critical to lending institutions.

Carey *et al.* (1998) examine the existence of specialization in the private corporate loan market, applying the distinction between public versus private debts. Comparing corporate loans made by commercial banks and finance companies, they find that the two types of lending institutions are equally likely to finance information-problematic firms. However, finance

[1] Examples are captive finance companies of the automobile companies such as General Motors Acceptance Corporations, Ford Credit, and Toyota Financial Services in the USA. Most of the domestic and foreign automobile companies in the USA have their own captive finance companies to facilitate the financing for purchase of their products.

companies tend to service observably high-risk borrowers. They found that both regulatory and reputation-based explanations are significant for this specialization.

The existing literature examines the risk segmentation in the loan markets in empirical perspective without any relevant theory of why finance companies emerge in the loan markets and how lending policy of finance companies is different from that of banks. In order to present a theory for the choice of debt financing sources in the debt market, this paper follows a screening model as in Besanko and Thakor (1987) and Calem and Stutzer (1995). Both studies provide a useful analytical tool for the analysis of adverse selection in the loan market. They model the loan rate and loan approval rate as the choice variable of a borrower and the screening devices of a lending institution to form a separating equilibrium. Following Besanko and Thakor (1987) and Calem and Stutzer (1995), this paper also uses the loan rate and loan approval rate as screening devices to model the choice of financing sources between bank and finance loans in the debt market.

Another contribution of this paper to the existing literature on financial intermediation is that this paper incorporates the search behavior in the purchase of differentiated products for which financing is facilitated in conjunction with the adverse selection problem and the resulting credit rationing in the loan market. This aspect of the model in this paper provides a theory of how debt and products markets are linked to each other. For modeling the search behavior in the capital goods market, this paper follows Perloff and Salop (1985) providing a prototype model for search behavior in a monopolistically competitive products market. With this sort of market imperfectness in the monopolistically competitive products market, note importantly that, in capital goods market, an additional sale of a product provides extra marginal rents or mark-up.

These extra marginal rents in turn give a seller the incentive to offer financing to facilitate the increased sale of its products.

This paper shows that, given that the capital goods can be sold only on financing, a captive finance company offers a pooling loan contract with a higher approval rate and the associated higher loan rate to extract more marginal rents from the additional sale of products, while banks offer separating low- and high-risk loan contracts. Low-risk borrowers are indifferent between separating bank low-risk loan contract and pooling finance loan contract while high-risk borrowers strictly prefer pooling finance loan contract to separating bank high-risk loan contract. The losses from offering pooling finance loans need to be subsidized by the additional sale of products.

The remainder of this paper is organized as follows. Section 2 presents a screening model in financial intermediation mainly based on Calem and Stutzer (1995) where only banks operate in a perfectly competitive loan market. The resulting outcome is a separating equilibrium. With the equilibrium in the loan market characterized only with banks, the interaction between debt and products markets and the associated equilibrium in a monopolistically competitive capital goods market is discussed. Section 3 presents a model for the emergence of captive finance companies and the equilibrium in the loan market with the co-existence of banks and captive finance companies. The outcome shows that the captive finance companies offer a pooling loan contract with a higher loan rate and approval rate so as to extract more marginal rents from the additional sale of capital goods, while banks offer separating loan contracts. As a consequence, the captive finance companies service both low- and high-risk borrowers while banks service only low-risk borrowers, leading to the theoretical prediction that finance loans are less likely to be repaid than banks loans. The characterization of equilibrium in monopolistically competitive capital goods market is discussed when the banks and captive finance companies co-exist in the debt market. Section 4 summarizes and concludes.

2. Equilibrium Only with Banks

This section first characterizes the equilibrium loan contracts offered by an independent lending institution, a *bank*, in a perfectly competitive debt, and then discusses the characterization of the equilibrium in a monopolistically competitive capital goods market with the link to the equilibrium in the debt market.

This paper assumes that there is a fixed number, M , of purchasers all of whom require financing for the purchase of one unit of capital good in a monopolistically competitive market. Each buyer (simultaneously a borrower) can purchase one unit of a capital good only when he is approved for a loan at a lending institution. If the buyer is denied a loan, he cannot purchase the capital good. We assume that each buyer does not apply for a loan at other lending institutions.[2]

This paper assumes that the revenue from collection is zero when a borrower defaults. This is justified when collection costs imposed by legal restrictions are huge and the salvage values of collateral are relatively small and may be ignored.

The borrower type is private information of the borrower. Borrowers are classified into two types – *low-risk* (denoted by subscript l) and *high-risk* (denoted by subscript h) types. This assumption allows the effects of asymmetric information to be examined with simplicity, but

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[2] This assumption is strong, and is adopted to simplify the analysis. We could instead assume that a borrower who is denied a loan at a lending institution of a particular type can apply, at some cost, for a loan at a second lending institution. The result would be a type of "winner's curse" in lending. In particular, if credit evaluation is imperfectly correlated across lending institutions and each lending institution is unaware of whether a borrower has been rejected by other lending institution(s), then the pool of borrowers will worsen (Shaffer, 1998). If lending institutions do know whether a borrower has been rejected at other lending institution(s), lending institutions may not be willing to lend to a borrower who has been previously rejected, and the result would be similar to our assumption that a borrower not approved for a loan cannot buy the product.

without loss of generality. The probability that a high-risk borrower will repay a loan is lower than that of a low-risk borrower. Let $\gamma \in (0,1)$ and $(1-\gamma) \in (0,1)$ denote the exogenous and known probability that a borrower is low- and high-risk, respectively. Note that γ and $(1-\gamma)$ can also be interpreted as the known proportions of low- and high-risk borrowers, respectively.

Each borrower is uncertain about whether he will be approved or denied in his loan application. However, he does know the loan contract terms, i.e., loan rate, r, and loan approval rate, a, offered by a lending institution of a particular type.

Assume that there are two periods. In the first period, each prospective borrower does not have wealth for the purchase of one unit of capital good. The price of the capital goods is p. Each buyer may apply for a loan of size p to finance the purchase one unit of the capital good.[3] A borrower who is approved contracts to repay principle plus interest, $p(1+r)$, in the second period and can purchase one unit of capital good in the first period.

Following Perloff and Salop (1985), we assume that there are N firms selling differentiated products in a monopolistically competitive capital goods market, and that these firms incur identical marginal production cost κ, and fixed cost, K. To model the search behavior in the capital goods market, this paper assumes that each buyer (simultaneously a borrower), $j = 1, 2, ..., M$, attaches relative values to the N differentiated capital goods according to his valuation vector $\boldsymbol{v} = (v_{1j}, v_{2j}, ...v_{ij}, ..., v_{Nj})$. Given prices $\boldsymbol{p} = (p_1, p_2, ..., p_i, ..., p_N)$ for the N available differentiated products, each buyer searches for the differentiated capital

[3] We assume that the loan amount equals the price of capital goods, *p*, for which a buyer/borrower obtains financing. Later, the characterization of equilibrium price of the products, *p*, in monopolistically competitive market will be discussed with a link to the equilibrium in the debt market. No down payment is assumed in the model although inclusion of a common down payment would not change the conclusions of the model.

goods and chooses the product for which his surplus is maximized – his best buy. A particular buyer *j*'s net surplus from purchasing firm i's product is given by:

$$b_{ij} = v_{ij} - p_i \,. \tag{1}$$

where b_{ij} is j's net surplus from purchasing firm i's product, v_{ij} is j's valuation of firm i's product, and p_i is the price of firm i's product. If $b_{ij} \geq b_{kj}$ for a given buyer, then $v_{kj} \leq p_k - p_i + v_{ij}$, and the buyer will choose to purchase the product from firm *i* over firm *k*.

Buyer j's valuation of the N differentiated capital goods, $\boldsymbol{v} = \left(v_{1j}, v_{2j}, \ldots v_{ij}, \ldots, v_{Nj}\right)$, is symmetric and drawn from the identical and independent distribution, i.e., $g\left(v\right) = g\left(v_{ij}\right) = g\left(v_{kj}\right)$, $i \neq k$, $\forall i, k = 1, 2, \ldots, N$, with mean μ.

We assume that each borrower receives a utility from the consumption of capital good in the second period. That is, if a borrower can repay the loan to keep the capital good, it will be worth v in the second period.

In the second period, each borrower receives stochastic income. Assume that a borrower's second-period income is either $Y = y > p\left(1+r\right)$, in which case the borrower has the ability to repay the loan and keep the capital good purchased, or $Y = 0$, in which case the borrower defaults. A high-risk borrower receives $Y = 0$ with probability δ_h while a low-risk borrower receives $Y = 0$ with probability δ_l. By definition, $0 < \delta_l < \delta_h < 1$. A lending institution does not know whether a particular borrower is low- or high-risk, but does know δ_l and δ_h.

This paper first characterizes the Nash equilibrium in the perfectly competitive debt market only with independent lending institutions or banks. The Nash equilibrium is defined by a set of loan contracts and assignments of those contracts to corresponding types of borrowers such that *(i)* each bank earns zero profit, and *(ii)* no loan contract other than the given set of contracts attracts borrowers generating non-negative profits.

A profit-maximizing bank offers a menu of loan contracts to screen a borrower under the asymmetric information on a borrower's ability of loan repayment. That is, each bank offers the loan rates and loan approval rates for a low-risk borrower, $\langle r_L, a_L \rangle$, and for a high-risk borrower, $\langle r_H, a_H \rangle$, to maximize the expected profits per loan approved, taking as given loan terms offered by other lending institutions. Banks announce loan contracts and compete *ex ante* on the terms of these loan contracts. Perfect competition among banks drives their profits to zero.

We assume that a lending institution does not skim the cream from the other lending institutions. That is, a lending institution does not offer loan contracts to attract low-risk borrowers of other lending institutions when those other lending institutions could, in response, withdraw their offers to all parties so that the cream-skimmer will unavoidably get stuck unprofitably servicing high-risk borrowers as well.

Given a price of a capital good, p, the zero-profit condition of a lender offering the loan contract $\langle r, a \rangle$ per approved borrower is given by:

$$\pi\left(r_j\right) = p\left[\left(1-\delta_i\right)\left(1+r_j\right)-\left(1+\rho\right)\right] = 0 . \tag{2}$$

where $i = h, l$, $j = H, L$ and ρ denotes the cost of funds.[4] This zero-profit condition implies that antitrust regulation and free entry in banking industry are effective in eliminating excess profits.

The Nash equilibrium in the debt market may take either of two possible forms: pooling or separating. In a pooling equilibrium, all borrowers of different risk types are offered only one

[4] We assume $r_i > \rho$. Recent statistics in the USA shows that the cost of funds appears to have been very similar for banks and finance companies. Finance companies raise funds largely by issuing *CP*s and corporate bonds while banks raise funds by issuing large *CD*s.

type of loan contract. Only one combination of loan rate, r, and loan approval rate, a, will be offered in such a pooling equilibrium by all lenders. In a separating equilibrium, to screen low- and high-risk borrowers under asymmetric information on their ability of loan repayment, a bank offers a menu of loan contracts.

Under asymmetric information and with identical banks in the perfectly competitive debt market, the Nash equilibrium is not a pooling one. This is because there always exist other loan contracts that attract only low-risk borrowers and yields positive expected profits. Thus, separating loan contracts for low- and high-risk borrowers, $\langle r_L, a_L \rangle$ and $\langle r_H, a_H \rangle$ respectively, will be offered by each bank in equilibrium.

The separating loan contracts of a bank are incentive compatible. Each borrower will apply for the loan contract, $\langle r_j, a_j \rangle$, $j = H, L$, that maximizes his expected utility subject to the zero-profit conditions of a bank.

When a borrower is denied a loan, he can receive the utility from his realized income and alternative consumption, z_i, $i = h, l$. Following Calem and Stutzer (1995), this paper assumes that a high-risk borrower obtains relatively less utility if he is denied a loan. For example, this is because it is more likely that a high-risk borrower maintains worse status of current capital good and so he has a more urgent need to replace it. In addition, a high-risk borrower is more likely to default. All these factors make a high-risk borrower relatively more willing to accept a loan with higher loan rate in return for obtaining a higher *ex ante* chance of being approved for a loan.

Given a price of the capital good, p, which is also the loan amount in the model, the expected utility of a borrower of type i purchasing a capital good while applying for a loan of a type j is given by:

$$U_{ij} = \int \left[a_j \left(1-\delta_i\right) \left[v + y - p\left(1+r_j\right) \right] + \left(1-a_j\right) \left[\left(1-\delta_i\right) y + z_i \right] \right] g\left(v\right) dv$$
$$= a_j \left(1-\delta_i\right) \left[\mu + y - p\left(1+r_j\right) \right] + \left(1-a_j\right) \left[\left(1-\delta_i\right) y + z_i \right] \quad (3)$$

where $i = h,l$ and $j = H,L$.

The expected utility decreases in the loan rate, r_j, $j = H,L$. This paper assumes that μ is sufficiently larger than z_i, $i = h,l$, so that the expected utility increases in the loan approval rate, a_j, and the indifference curves of low- and high-risk borrowers are upward sloping on the $\langle r,a \rangle$ plane. In particular, $\mu - p\left(1+r_j\right) > z_i / \left(1-\delta_i\right)$ so that $\partial U_{ij} / \partial a_j > 0$ and $da_j / dr_j > 0$, where $i = h,l$ and $j = H,L$.

The incentive compatibility (self-selection) constraints ensure that both low- and high-risk borrowers voluntarily accept the loan contracts for their own types. The incentive compatibility constraints for low- and high-risk borrowers are given by:

$$\begin{aligned} U_{lL} &= a_L \left(1-\delta_l\right) \left[\mu + y - p\left(1+r_L\right) \right] + \left(1-a_L\right) \left[\left(1-\delta_l\right) y + z_l \right] \geq \\ U_{lH} &= a_H \left(1-\delta_l\right) \left[\mu + y - p\left(1+r_H\right) \right] + \left(1-a_H\right) \left[\left(1-\delta_l\right) y + z_l \right] ; \\ U_{hH} &= a_H \left(1-\delta_h\right) \left[\mu + y - p\left(1+r_H\right) \right] + \left(1-a_H\right) \left[\left(1-\delta_h\right) y + z_h \right] \geq \\ U_{hL} &= a_L \left(1-\delta_h\right) \left[\mu + y - p\left(1+r_L\right) \right] + \left(1-a_L\right) \left[\left(1-\delta_h\right) y + z_h \right] \end{aligned} \quad (4)$$

The slope of an indifference curve of a borrower of type i borrowing a loan of type j on the $\langle r,a \rangle$ plane is given by:[5]

[5] Note that the indifference curve in this paper has a different interpretation from typical indifference curves. Indifference curves typically represent the bundles of the two goods which leave an individual consumer indifferent, with all points along the curve giving the same level of utility. The indifference curve depicted in this paper represents the combinations of screening devices (loan rate, r, and loan approval rate, a) which give a consumer the same level of utility between the two loan terms, with the utility level increasing as the indifference curve shifts northwest.

$$\left.\frac{da_j}{dr_j}\right|_i = -\frac{\partial U_{ij} / \partial r_j}{\partial U_{ij} / \partial a_j} = \frac{a_j p}{\left[\mu - p\left(1+r_j\right)\right] - \left(\frac{z_i}{1-\delta_i}\right)} . \tag{5}$$

To incorporate the circumstances where a high-risk borrower maintains worse status of current capital good in the utility function and needs to replace it more urgently, the following parametric specification is assumed:

$$\frac{z_h}{\left(1-\delta_h\right)} < \frac{z_l}{\left(1-\delta_l\right)} . \tag{6}$$

Therefore, on a loan contract $\left\langle r_j, a_j \right\rangle$, $j = H, L$, the slope of a high-risk borrower's indifference curve, $da_j / dr_j\big|_h$, is less than the slope of a low-risk borrower's indifference curve, $da_j / dr_j\big|_l$. That is, the single-crossing condition holds as follows:

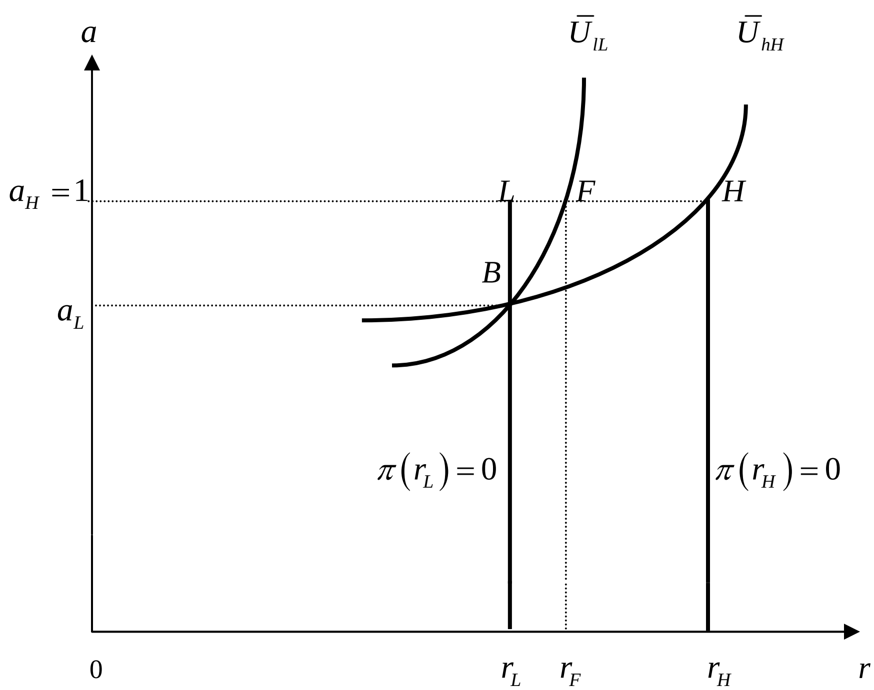
$$\left.\frac{da_j}{dr_j}\right|_h < \left.\frac{da_j}{dr_j}\right|_l \tag{7}$$

where $j = H, L$.

Evaluating the slopes of indifference curves of low- and high-risk borrowers at a loan contract $\left\langle r_j, a_j \right\rangle$, $j = H, L$, shows that (6) is sufficient to ensure (7). The inequality (6) requires z_l to be sufficiently larger than z_h. Because z_i, $i = h, l$, represents a borrower's utility obtained from alternative consumption if he is denied a loan, the parametric specification (6) will be satisfied when the consequences of being denied a loan harm high-risk borrowers substantially more. That is, the inequality, $z_h < z_l$, incorporates into a borrower's utility function the circumstance where the existing capital good of a high-risk borrower is a lot worse than that of a low-risk one.

In a separating equilibrium, the zero-profit condition of a bank determines the equilibrium loan rates for a low-risk borrower, r_L, and for a high-risk borrower, r_H. The Nash equilibrium in the perfectly competitive loan market is depicted in Figure 1. In Figure 1, given the identical costs of funds ρ, $0 < \delta_l < \delta_h < 1$ provides the two different vertical zero-profit lines, $\pi(r_L) = 0$ and $\pi(r_H) = 0$, through the two zero-profit equilibrium loan rates, r_L and r_H, respectively. On $\langle r, a \rangle$ plane, the zero-profit lines are vertical because zero-profit loan rates are independent of the loan approval rates.

Figure 1: Equilibrium Loan Contracts in Perfectly Competitive Loan Market



The separating equilibrium loan contracts are $\langle r_L, a_L \rangle$ for a low-risk borrower and $\langle r_H, 1 \rangle$ for a high-risk borrower as shown in Figure 1 when the incentive compatibility constraints and single-crossing condition hold. Low- and high-risk borrowers voluntarily apply for the loans for their own types. $\bar{U}_{lL}$ and $\bar{U}_{hH}$ denote the indifference curve of a low- and high-risk borrower, respectively. The expected utility increases to the northwest.

These separating equilibrium loan contracts show that, to screen low- and high-risk borrowers, each bank offers a lower loan approval rate for a low-rate loan as these loan terms provide the mechanism for borrower separation, i.e., the incentive to induce low- and high-risk borrowers to self-select loan contracts for their types.

In the sense of Stiglitz and Weiss (1981), this separating equilibrium depicted in Figure 1 creates an adverse selection as the low-risk borrowers are credit-rationed while all the high-risk borrowers are approved in their loan applications. That is, the low-risk borrowers would demand a loan contract with lower loan rate and higher approval rate, which is commensurate with their low-risk in loan repayment, but such a loan contract is not offered because it would then attract high-risk borrowers, with resulting losses to a bank. The asymmetric information on a borrower's ability to repay the loan distorts the choice of loan contracts available in the debt market, preventing the low-risk borrowers from being approved at approval rate 1.

With these equilibrium loan contracts, in particular with $a_L < 1$ and $a_H = 1$, offered by banks in the perfectly competitive loan market, the effective number of buyers who are approved for loans and so can purchase the capital goods is given by:

$$\bar{M} = \left[\gamma a_L + (1-\gamma)\right] M \tag{8}$$

With this effective number of buyers, $\bar{M}$, we can characterize the equilibrium in the monopolistically competitive capital goods market.

Since buyer's valuation of the product of each firm, v, is independently and identically drawn from the common distribution function $G(v)$ with density function $g(v)$, the probability of $b_{ij} \geq b_{kj}$ is $\Pr\left(b_{ij} \geq b_{kj}\right) = G\left(p_k - p_i + v\right)$.[6] Then, the proportion of buyers who can purchase firm i's capital good is given by:

$$\Pr\left(b_{ij} \geq \max_{k \neq i} b_{kj}\right) = \int \prod_{k \neq i} \left[G\left(p_k - p_i + v\right)\right] g\left(v\right) dv \,. \tag{9}$$

We examine a special case where each buyer purchases one unit of his best buy. It follows that the expected demand for capital goods sold by firm i, $D_i\left(p_1, p_2, ..., p_i, ..., p_N\right)$, equals the proportion of buyers who can buy the product given by equation (9) times the effective number of buyers $\bar{M}$ as follows:

$$D_i\left(p_1, p_2, ..., p_i, ..., p_N\right) = \bar{M} \Pr\left(b_{ij} \geq \max_{k \neq i} b_{kj}\right) = \bar{M} \int \prod_{k \neq i} \left[G\left(p_k - p_i + v\right)\right] g\left(v\right) dv \tag{10}$$

Under the assumption of symmetry that each firm has the identical, constant marginal cost, κ, and fixed costs, K, the expected profits of firm i, $\forall i = 1, 2, ..., N$, are given by:

$$\Pi_i\left(p_1, p_2, ..., p_i, ..., p_N\right) = \left(p_i - \kappa\right) D_i\left(p_1, p_2, ..., p_i, ..., p_N\right) - K \,. \tag{11}$$

Following Perloff and Salop (1985), we consider the case where a unique symmetric equilibrium price exists such that $p_i = p$, $\forall i = 1, 2, ..., N$.[7] This implies an expected demand of firm i given by:

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[6] If v is assumed to be uniformly distributed, i.e., $f\left(v\right) = 1/q$ over the finite support, [0,q], and 0 otherwise, then $\int_0^q \left[F\left(v\right)\right]^{N-2} \left[f\left(v\right)\right]^2 dv = \int_0^q \left(v/q\right)^{N-2} \left(1/q\right)^2 dv = \left(1/q\right)[1/(N-1)]$. Hence, $p = \kappa + q/N$. Note that q and $1/N$ indicate the degree of product differentiation and degree of market concentration, respectively.

[7] Perloff and Salop (1985) show that, given identical marginal and fixed costs of production across firms, the equilibrium in the monopolistically competitive products market is a unique zero-profit single-price equilibrium. See Perloff and Salop (1985) for the details regarding the form of the demand function and further discussion characterizing the market equilibrium. Since the products market reaches a unique zero-profit single-price monopolistically-competitive equilibrium and the loan market reaches a separating equilibrium with perfectly competitive loan contracts for low-risk borrower, $\langle r_L, a_L \rangle$, and for high-risk borrower, $\langle r_H, 1 \rangle$, in the model, this study rules out the case where pricing/financing combinations are used as a scheme for

$$D_i\left(p_1, p_2, ..., p_i, ..., p_N\right) = \bar{M}\int\left[G\left(p - p_i + v\right)\right]^{N-1} g\left(v\right)dv \qquad (12)$$

Under the Bertrand-Nash assumption that firms choose price to maximize expected profits, taking other firms' prices as given, firm i's first-order condition with respect to p_i is given by:

$$p_i = \frac{D_i\left(p_1, p_2, ..., p_i, ..., p_N\right)}{\left[\frac{\partial D_i\left(p_1, p_2, ..., p_i, ..., p_N\right)}{\partial p_i}\right]} \qquad (13)$$

Let p denote the unique symmetric zero-profit equilibrium price of capital goods. When only banks operate in the perfectly competitive debt market, we denote the equilibrium number of sellers as $\bar{N}$.

Given the form of expected demand (10), we obtain the following characterization for the optimal price of product of firm i , $\forall i = 1, 2, ..., N$:

$$p = \kappa + \frac{\bar{M}\int\left[G\left(v\right)\right]^{\bar{N}-1} g\left(v\right)dv}{\left(\bar{N}-1\right)\bar{M}\int\left[G\left(v\right)\right]^{\bar{N}-2}\left[g\left(v\right)\right]^2 dv} = \kappa + \frac{1}{\bar{N}\left(\bar{N}-1\right)\int\left[G\left(v\right)\right]^{\bar{N}-2}\left[g\left(v\right)\right]^2 dv} \qquad (14)$$

Equation (14) characterizes the symmetric optimal price for capital goods which lies strictly above the competitive price, i.e., $\left(p_i - \kappa\right) > 0$, $\forall i = 1, 2, ..., N$. Note that the optimal price is independent of the number of buyers.

Since all firms are assumed to be identical in their marginal and fixed costs of producing differentiated products, the equal expected demand of each firm is given by $\bar{M} / \bar{N}$ and the zero-profit condition is given by:

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differentiating *ex ante* different types of borrowers. However, modeling the pricing/financing discrimination scheme in imperfect product and loan markets might explain more real aspects of those markets.

$$\left(\frac{\bar{M}}{\bar{N}}\right)(p-\kappa)-K=0. \tag{15}$$

Equation (15) represents the Chamberlinian tangency condition - the zero-profit condition characterized by the usual tangency of demand curve with average cost curve in a monopolistically competitive market.[8]

Given the zero-profit loan rates for a low-risk borrower, r_L, and for a high-risk borrower, r_H, the expected utility of a low-risk borrower at $\langle r_L, a_L \rangle$ equal to the expected utility of a high-risk borrower at $\langle r_H, 1 \rangle$, the optimal price condition (14), and the zero-profit condition (15), characterize the separating bank low-risk loan approval rate a_L, the unique symmetric zero-profit equilibrium price p, and number of sellers $\bar{N}$, in the monopolistically competitive capital goods market when only banks operate to finance the purchases of capital goods.

3. Equilibrium with Banks and Captive Finance Companies

This section provides the model for the equilibrium loan contracts in the perfectly competitive debt market where two different types of lending institutions - independent lending institutions (banks) versus captive finance companies - operate. This section examines why captive finance companies emerge and operate in the debt market and how the equilibrium loan contract of a captive finance company is different from the separating loan contracts of a bank. In doing so, this paper shows how borrowers select the types of lending institutions and their specialized loan types and as a consequence how the debt market is segmented by banks and captive finance companies based on borrowers' risk types. Then, this paper discusses the

[8] In monopolistically competitive market, each capital good firm takes a unique symmetric zero-profit equilibrium price, p, of capital goods as given since there are substantially many firms in the industry. Given this equilibrium price, p, free entry and exit of firms in the monopolistically competitive industry will drive each firm to make zero profit and the associated zero-profit equilibrium number of firms will be determined.

characterization of the equilibrium in the monopolistically competitive capital goods market with the link to the equilibrium in the debt market when the banks and captive finance companies co-exist.

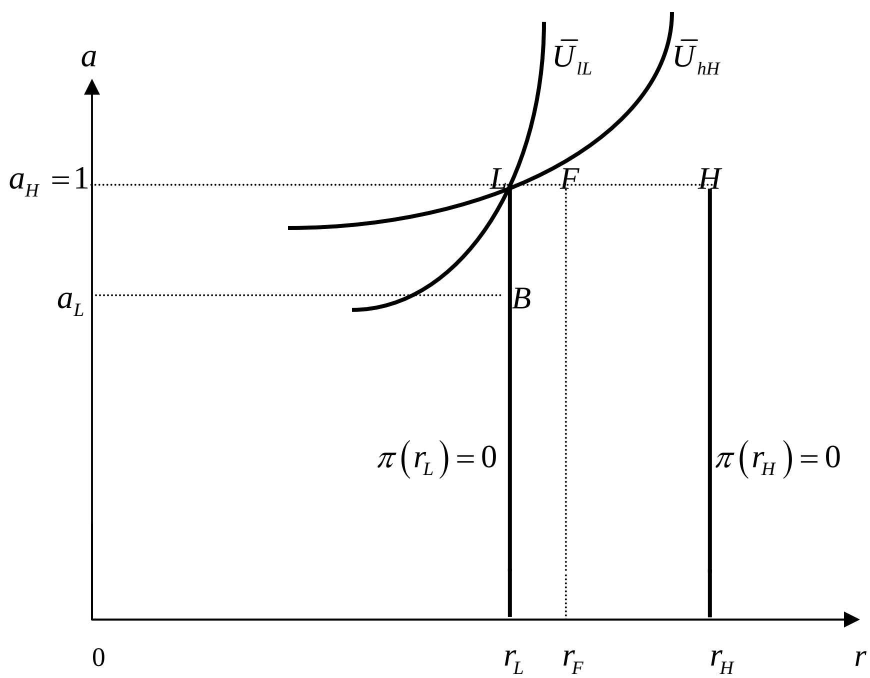
Banks and captive finance companies are not symmetric in their behavior in the debt market because the optimal lending activity of a captive finance company depends not only on the expected profits from selling products, but also on expected profits from lending. Hence, we cannot apply the Nash equilibrium concept with identical players to modeling of the co-existence of banks and finance companies in the perfectly competitive debt market.

Given the equilibrium separating bank loan contracts, $\langle r_L, a_L \rangle$ and $\langle r_H, 1 \rangle$, as shown in Figure 1, a captive finance company offers optimal loan rate and loan approval rate taking into account the additional marginal rents from the sale of capital goods of its parental seller, $(p-\kappa) > 0$, extracted by offering a loan.

As shown in Figure 2, a captive finance company would initially offer the loan contracts $\langle r_L, 1 \rangle$ to a low-risk borrower and $\langle r_H, 1 \rangle$ to a high-risk borrower in order to increase the expected marginal rents from selling more capital goods. The increase in the loan approval rate for a low-risk borrower from $a_L < 1$ to 1 will increase the expected marginal rents per borrower or $\gamma(1-a_L)(p-\kappa) > 0$.

When only low-risk borrowers apply for $\langle r_L, 1 \rangle$, the increased marginal profits from selling product and lending is positive or $\gamma(1-a_L)(p-\kappa)+\pi(r_L) > 0$. However, $\langle r_L, 1 \rangle$ cannot be an equilibrium loan contract. As shown in Figure 2, since a low-risk borrower prefers $\langle r_L, 1 \rangle$ to $\langle r_L, a_L \rangle$ and a high-risk borrower prefers $\langle r_L, 1 \rangle$ to $\langle r_H, 1 \rangle$, both low- and high-risk borrowers will choose contract $\langle r_L, 1 \rangle$, resulting in the captive finance company's expected losses from lending.

Figure 2: Emergence of Captive Finance Companies in Loan Market



In general, a capital-good firm will establish its captive finance company and offer a pooling loan contract if and only if the expected combined marginal profits per borrower with pooling finance loan contract, $(p-\kappa)+\pi(r)$, are strictly larger than the expected profits per borrower when a borrower can obtain a loan only from a bank, $a_L\gamma(p-\kappa)+(1-\gamma)(p-\kappa)$. That is, a captive finance company will emerge and grant a loan to a borrower if and only if $\gamma(1-a_L)(p-\kappa)+\pi(r)>0$ while offering loan approval rate 1.

When a captive finance company offers the loan approval rate 1 to extract more marginal rents from the sale of capital goods, it has to set an associated optimal loan rate. This paper examines all possible ranges of pooling finance loan rate. In Figure 2, we first exclude the possible pooling finance loan rates on (r_F, r_H) range because each low-risk borrower prefers the separating bank low-risk loan contract, $\langle r_L, a_L \rangle$, to any pooling finance loan contract on this range.[9] In this case, a capital-good seller cannot extract additional marginal rents. We also exclude the possible pooling loan rates on $(0, r_L)$ range since a captive finance company can find willing low-risk borrowers by charging the same low-risk loan rate, r_L, as a bank does.

The only range of interest is $[r_L, r_F]$ over which each low-risk borrower prefers a possible pooling contract to separating bank low-risk loan contract, $\langle r_L, a_L \rangle$, or at least is indifferent between separating bank low-risk loan contract, $\langle r_L, a_L \rangle$, and the possible pooling finance loan contract. r_F is the only optimal loan rate of the captive finance company since each low-risk borrower is indifferent between $\langle r_F, 1 \rangle$ and $\langle r_L, a_L \rangle$ while the captive finance company receives the highest possible loan rate on $[r_L, r_F]$ range.

Note that, given the separating equilibrium obtained by the menu of loan contracts - $\langle r_L, a_L \rangle$ for a low-risk borrower and $\langle r_H, 1 \rangle$ for a high-risk borrower - offered by each bank, the expected profits per borrower from pooling finance loan $\langle r_F, 1 \rangle$ is negative. If the expected profit from lending were non-negative on $\langle r_F, 1 \rangle$, a bank would have offered the pooling finance loan contract $\langle r_F, 1 \rangle$ in Figure 2, which attracts high-risk borrowers keeping low-risk borrowers

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[9] r_F is the loan rate when the indifference curve of a low-risk borrower, $\bar{U}_{lL}$, intersects the $a_H = 1$ line in Figure 1.

indifferent to $\langle r_L, a_L \rangle$. It is obvious that non-negative expected profits from pooling loan contact $\langle r_F, 1 \rangle$ contradict the definition of separating equilibrium obtained in the previous section.

In sum, since a capital-good seller and its captive finance company can earn additional marginal rents by offering a pooling loan contract $\langle r_F, 1 \rangle$ to low-risk borrowers, and the captive finance company incurs expected losses from its lending, i.e., $\pi(r) < 0$, and $\partial \pi(r) / \partial r > 0$, $\forall r \in [r_L, r_F]$, they would optimally set the pooling finance loan rate at the highest level possible subject to the constraint that they need to keep each low-risk borrower at least indifferent to separating bank low-risk loan contract, $\langle r_L, a_L \rangle$.

The pooling finance loan contract, $\langle r_F, 1 \rangle$, can be granted and sustainable if and only if the expected additional marginal rents per borrower, $\gamma(1-a_L)(p-\kappa)$, obtained by granting a pooling finance loan outweigh the expected losses from that pooling finance loan, $\pi(r_F) = p\big[\big[\gamma(1-\delta_l) + (1-\gamma)(1-\delta_h)\big](1+r_F) - (1+\rho)\big] < 0$.[10] That is, it is optimal that a captive finance company offers the pooling loan contract, $\langle r_F, 1 \rangle$, if and only if it can receive the increased expected marginal profits as follows:

$$\gamma(1-a_L)(p-\kappa) + \pi(r_F) > 0. \tag{16}$$

The necessary and sufficient condition for the emergence of a captive finance company, inequality (16), also implies that the captive finance company's expected combined marginal profits per borrower at the pooling loan contract $\langle r_F, 1 \rangle$ is positive as follows:

[10] Gilligan and Smirlock (1983) show that, in order to maximize the value of the firm, a multiproduct firm can obtain revenues in excess of production costs on goods sold in monopolized market and uses these rents to subsidize the production of goods sold in competitive markets.

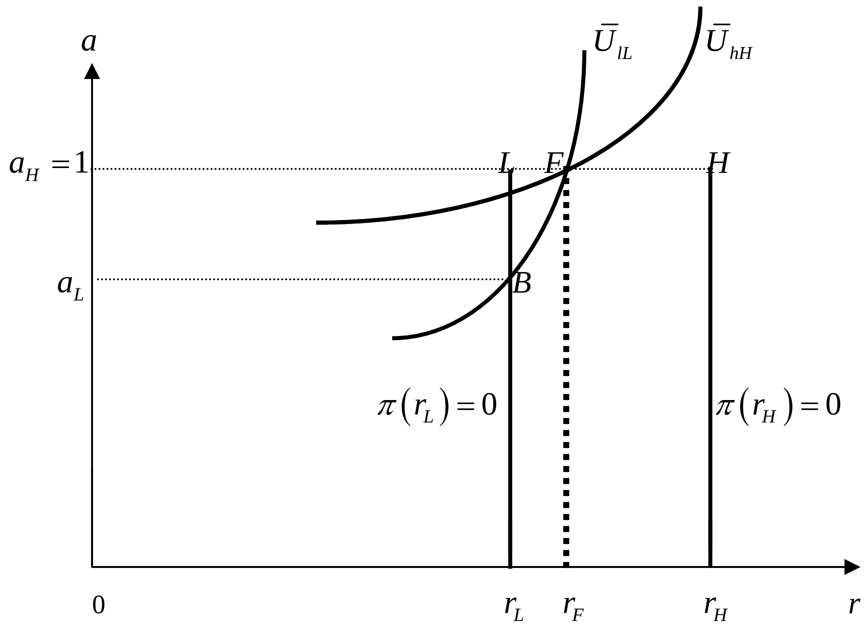
$$\left(p-\kappa\right)+\pi\left(r_F\right)>0\,. \tag{17}$$

When captive finance companies emerge, two competing sets of loan contracts are now offered by banks and captive finance companies in the debt market. As shown in Figure 3, in the equilibrium with the co-existence of banks and captive finance companies, each bank offers separating loan contracts $\langle r_L, a_L\rangle$ to a low-risk borrower and $\langle r_H, 1\rangle$ to a high-risk borrower while each captive finance company offers a pooling loan contract $\langle r_F, 1\rangle$ to a borrower of either low- or high-risk type.[11] Note that the separating bank low-risk loan rate is lower than the pooling finance loan rate or $r_L < r_F$ in Figure 3.

Each high-risk borrower prefers $\langle r_F, 1\rangle$ to $\langle r_H, 1\rangle$ while each low-risk borrower is indifferent between $\langle r_L, a_L\rangle$ and $\langle r_F, 1\rangle$. Since it is publicly known that these two alternative loan contracts, $\langle r_L, a_L\rangle$ and $\langle r_F, 1\rangle$, provide each low-risk borrower with identical expected utility, each low-risk borrower will choose either of these two loan contracts equally likely. All of the high-risk borrowers will choose the pooling finance loan contract, $\langle r_F, 1\rangle$.

[11] One can doubt that, to attract all the low-risk borrowers, a captive finance company can offer a loan rate r_F' which is infinitesimally smaller than r_F since each low-risk borrower will prefer this pooling finance loan contract to both $\langle r_L, a_L\rangle$ and $\langle r_F, 1\rangle$ and then all of the low-risk borrowers will choose only the pooling finance loan contract $\langle r_F', 1\rangle$. However, each lender makes a credit decision on an individual borrower basis without considering its total number of buyers approved for loans. Furthermore, this is also an open set problem. That is, we cannot characterize an equilibrium on an open set of loan rates. To obtain an equilibrium, we have to consider the set of loan rates which must be bounded and closed to guarantee that a maximum loan rate exists on $\left[r_L, r_F\right]$ range. In equilibrium, each captive finance company believes that each low-risk borrower will accept the pooling finance loan contract $\langle r_F, 1\rangle$ since the low-risk borrower is indifferent between $\langle r_L, a_L\rangle$ and $\langle r_F, 1\rangle$. Then, the captive finance company will offer r_F, the maximum loan rate possible on $\left[r_L, r_F\right]$ range.

Figure 3: Co-Existence of Banks and Captive Finance Companies



Let $(1-\alpha) \in (0,1)$ and $\alpha \in (0,1)$ denote the given probability that a low-risk borrower selects $\langle r_L, a_L \rangle$ and $\langle r_F, 1 \rangle$, respectively.[12] Therefore, the expected default rate of a separating bank low-risk loan is $\hat{\delta}_B = (1-\alpha)\gamma a_L \delta_l$ and the expected default rate of a pooling finance loan is $\hat{\delta}_F = \alpha\gamma\delta_l + (1-\gamma)\delta_h$. We now have Proposition 1 which explains the risk segmentation of loan market by banks and finance companies as follows:

Proposition 1: The expected default rate of pooling finance loan is higher than that of separating bank low-risk loan contract or $\hat{\delta}_B < \hat{\delta}_F$.

[12] Since both $\langle r_L, a_L \rangle$ and $\langle r_F, 1 \rangle$ provide a low-risk borrower with the same level of utility and these two loan contracts from the two different types of lending institutions are publicly posted in our mechanism design, $(1-\alpha)$ and α must show the equal likelihood of the selection of $\langle r_L, a_L \rangle$ and $\langle r_F, 1 \rangle$ by a low-risk borrower, i.e., $\alpha = (1-\alpha) = 1/2$.

Proposition 1 confirms the prevailing wisdom in lending practices and previous empirical findings that finance loans are less likely to be repaid than bank loans. The higher expected default rate of a finance loan than that of a bank loan provides the implication that the initial separating equilibrium, obtained with only independent lending institutions or banks, now converts to equilibrium segmentation of the perfectly competitive debt market where finance companies service on average riskier pool of borrowers than banks do.

We assume that each firm does not incur additional fixed costs for establishing its captive finance company. Since the marginal and fixed costs are identical across all the capital good firms, all firms will establish their captive finance companies if and only if $\gamma(1-a_L)(p-\kappa)+\pi(r_F)>0$. With the economy-wide operation of captive finance companies, the new effective number of buyers who are approved for loans and so can purchase the capital goods is given by:

$$\hat{M}=\left[(1-\alpha)\gamma a_L+\alpha\gamma+(1-\gamma)\right]M=M_B+M_F \tag{18}$$

where $M_B=(1-\alpha)\gamma a_L M$ is the number of low-risk borrowers approved at banks and $M_F=\left[\alpha\gamma+(1-\gamma)\right]M$ is the number of low- and high-risk borrowers approved at captive finance companies.

When both banks captive finance companies operate in the debt market, the credit rationing on low-risk borrowers is mitigated due to more risk-taking credit policy of captive finance companies than that of banks. That is, the effective number of buyers with both banks and captive finance companies is greater than that with only banks as follows:

$$\hat{M}=\left[(1-\alpha)\gamma a_L+\alpha\gamma+(1-\gamma)\right]M>\bar{M}=\left[\gamma a_L+(1-\gamma)\right]M \tag{19}$$

The original Chamberlinian tangency condition needs to be adjusted since the expected profits of a firm with its captive finance company include not only profits from selling the capital goods, but also the profits or losses from granting finance loans and more buyers are approved for loans or $\hat{M} > \bar{M}$. Given a unique symmetric zero-profit equilibrium price of products, p, let $\hat{N}$ denote the new number of capital-good sellers when both banks and captive finance companies operate. Since all firms are assumed to be identical in their marginal and fixed costs of production, the new expected demand of each firm is given by equal share of the new effective number of buyers, i.e., $\hat{M} / \hat{N} = M_B / \hat{N} + M_F / \hat{N}$. Then, the zero-profit condition of each firm is given by:

$$\left(\frac{M_B}{\hat{N}}\right)(p-\kappa)+\left(\frac{M_F}{\hat{N}}\right)\left[(p-\kappa)+\pi(r_F)\right]-K=0 \tag{20}$$

where $\pi(r_F) = p\left[\left[\gamma(1-\delta_l)+(1-\gamma)(1-\delta_h)\right](1+r_F)-(1+\rho)\right] < 0$.

The zero-profit condition, equation (20), shows that, given the equilibrium loan contract terms of banks and finance companies in the perfectly competitive debt market and the unique symmetric equilibrium price of capital goods in the monopolistically competitive market, the free entry in the monopolistically competitive capital goods market drives each firm's combined total profits from selling products and lending loans to zero and the number of firms is adjusted from $\bar{N}$ to $\hat{N}$. In particular, the number of firms increases since the effective number of buyers is greater, i.e., $\hat{M} > \bar{M}$, and the combined marginal profits are positive, i.e., $(p-\kappa)+\pi(r_F) > 0$, when banks and captive finance companies co-exist in the debt market. We summarize this in Proposition 2 as follows:

Proposition 2: Given a unique symmetric equilibrium price of capital goods p, $\hat{N} > \bar{N}$.

Until now, a unique symmetric zero-profit equilibrium price p is assumed to be given. However, it would be a more complete modeling if we could characterize an equilibrium with new symmetric zero-profit equilibrium price of capital goods, given r_H and r_L. To characterize the new equilibrium price of products in the monopolistically competitive capital goods market, we have to solve for four unknown variables, pooling finance loan rate, separating bank low-risk approval rate, new equilibrium number of sellers, and new equilibrium price of capital goods. These four unknowns can be solved simultaneously by four equations - the expected utility of a low-risk borrower on separating bank low-risk loan contract $\langle r_L, a_L \rangle$ equal to the expected utility of a high-risk borrower on separating bank high-risk loan contract $\langle r_H, 1 \rangle$, a low-risk borrower's expected utility on separating bank low-risk loan contract $\langle r_L, a_L \rangle$ equal to his expected utility on pooling finance loan contract $\langle r_F, 1 \rangle$, the zero-profit condition, and the new optimal price condition.[13] Given that it is very difficult to characterize the equilibrium in closed form solutions, the only way to characterize the equilibrium is to simulate numerical examples. However, the simulations are dependent upon the appropriate specifications of parameters in the model, which are technically difficult task.

At any rate, the numerical characterization of new equilibrium would be qualitatively identical to conclusion of this paper in that captive finance companies will emerge when the necessary and sufficient conditions for the emergence of captive finance companies hold under the new equilibrium price, separating bank and pooling finance debt contracts will be offered by

[13] The new optimal price condition is given by:
$$\hat{p} = \kappa + \left[1/\hat{N}\left(\hat{N}-1\right)\int\left[G(v)\right]^{N-2}\left[g(v)\right]^2 dv\right] + \left[M_F\left[\left[\gamma(1-\delta_l)+(1-\gamma)(1-\delta_h)\right](1+r_F)-(1+\rho)\right]/\left(\hat{N}-1\right)\hat{M}\int\left[G(v)\right]^{N-2}\left[g(v)\right]^2 dv\right].$$

the two different types of lending institutions, and finance loans are less likely to be repaid than bank loans.

4. Concluding Remarks

This paper examines how banks and finance companies service borrowers with different risk types, how borrowers choose the loan types and lender types, and how the products and loan markets interact. In answering these questions, this paper presents a model economy where screening in lending and searching in purchasing differentiated products are all incorporated, which closely reflects the actual behavior of economic agents in a real economy.

By incorporating the key structural feature of a captive finance company – seeking additional rents by offering loans to riskier borrowers - into the screening process in financial intermediation, this paper successfully explains the prevailing wisdom in lending practices and previous findings that finance companies service, on average, riskier pool of borrowers offering more lenient loan approval rate and higher loan rate than those of banks.

In addition, this paper discusses the characterization of the equilibrium in the monopolistically competitive products market with link to the equilibrium in the debt market.

This paper has some limitations. One possible extension of this paper would be to incorporate the pricing/financing combinations as a scheme for differentiating *ex ante* different types of borrowers in the model, as opposed to the model of this paper with the unique symmetric zero-profit equilibrium price of capital goods. Modeling the pricing/financing discrimination scheme in imperfect product and loan markets might explain more real aspects of those markets.

Furthermore, the theoretical prediction of this paper stimulates empirical investigation of debt markets to examine how the loan contracts vary across different types of lending institutions, how borrowers select lender types and loan types, and what would be the resulting differences in loan repayment performances of the loan contracts from different types of lending institutions.

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