Endogenous Labor/Leisure/Investment Choice with Time Constraint and Asset Returns

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Abstract

We posit the time cost required for managing risky asset investment including conducting research and monitoring its performance. An economic agent, who should allocate a limited amount of time to labor, leisure and risky investment, is subject to the *opportunity time cost*, which is forgone labor or leisure. Our model investigates the change of the equity premium and volatility in the presence of such a time constraint. In particular, we derive the closed-form solutions for the risky asset returns, volatility, and risk-free rate in a simple equilibrium framework wherein agents have log utility. Our model is shown to yield the excess return and the volatility consistent with historical values observed in U.S. stock market even with a small amount of the time cost. In addition, we separate the impact of *endogenous* labor/leisure choice from the total changes on return dynamics by comparing with *exogenous* labor income case.

JEL classification: G11; G12; J22

Keywords: Time cost; Full income approach; Equity premium puzzle; Time allocation; Leisure-labor choice

1 Introduction

Since the seminal work of Mehra and Prescott (1985), many studies have been devoted to resolve the asset pricing puzzles. In spite of numerous endeavors, little could achieve to explain the empirical behaviors of the U.S. stock market such as the *equity premium puzzle, volatility puzzle*, and *predictability puzzle* successfully. The core of the equity premium puzzle is that with assuming reasonable risk aversion, the volatility of consumption growth rate as well as its covariance with stock returns is too low to explain the high equity premium and volatility [See, e.g., Hansen and Singleton (1982), Hansen and Jagannathan (1991), and Campbell and Cochrane (1999)]. The volatility puzzle and the predictability puzzle, on the other hand, can be comprehended through the direct application of the present value formula.

$$R_{t+1} = \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t}$$

Since the volatility of dividend growth is low, the only way for a model to generate a high return volatility is to introduce a variation in P/D ratio. To see this, P/D ratio has a predictive power in the future dividend growth or the future return (dividend discount rate).[1] Empirically, P/D ratio is not a forecaster of dividend growth, thus it has to predict future returns. Therefore a resolution of the volatility puzzle is simultaneously a resolution of the predictability puzzle [Campbell and Shiller (1988), Shiller (1990), and Barberis et al. (2001)].

The object of this paper is to show the importance of endogenous labor-leisure choice and the opportunity cost related to risky investment for explaining the above asset pricing puzzles. Most of modern financial theories have passed over such binding constraints, and hence may mislead the importance of opportunity cost and of human capital.

To prove our argument, we posit the time cost required for managing investment in a risky asset. Time is an unequivocally valuable but scarce resource, thereby being one of the most important binding constraints inherent in economic decision making. Since economic agents are endowed with a limited amount of time, they should allocate this scarce resource to several activities for their benefit. For example, some may want to spend more time in labor for increasing future consumption, whereas others may enhance their utility by going after more leisure. Anyone cannot

[1] When both the dividend growth rate and the discount rate are constant, the price-dividend ratio is also constant.

simultaneously achieve both of them, and should choose the optimal behaviors by taking into account the opportunity cost as well as the trade-off between risk and return. Given the importance of time allocation, it is surprising that little has been studied its impact on the return dynamics. We thereby investigate the impact of the time allocation in the presence of time cost for risky investment.

The model presented in this paper is based on a simplified assumption that an economic agent allocates a limited amount of time into three activities: labor, leisure, and risky investment. With the first two activities which are standard [Becker (1965), Gronau (1977), Prescott (1986), Bodie, Merton, and Samuelson (1992)], we introduce the third alternative usage of time, which is associated with risky investment. Typically risky asset investment requires an in-depth analysis on alternatives and the market environment for stock picking and market timing, i.e. for instance, brokerage time and research time such as reading newspapers, web surfing and consulting with brokerage agents. In addition, after implementing an investment, she should keep track of the performance of her portfolio and decide whether to rebalance her portfolio given the revelation of information over time. These efforts require a significant amount of time, which is coined as "monitoring cost." Furthermore, any adverse performance of one's risky investment can elicit psychological turbulence, which in turn induces negative impact on labor productivity or time measured in net working hours, not in a perfunctory sense.

Overall we can summarize that risky asset investment is likely to result in decreasing either labor time or productivity, which can potentially reduce the worker's total labor income. Herein one thing to clarify is which component of labor income is more fragile to the effect of risky asset investment. Denote her labor income at time t by $w(t)$ and time spent on risky asset management at time t by $m(t)$. The primary prediction implied by the above argument is $dw(t)/dm(t) < 0$. When $F(t)$ and $j(t)$ represent labor productivity and time respectively, the labor income is a product of labor productivity and time, i.e., $w(t) = F(t) \cdot j(t)$ under the assumption of constant return to scale labor production. The ill-effect of risky asset management on labor could be an outcome of $dF(t)/dm(t) < 0$ and/or $dj(t)/dm(t) < 0$. In this paper we assume that $dF(t)/dm(t) = 0$ and $dj(t)/dm(t) < 0$ for simplicity.[2]

[2]This assumption is not as critical as it appears to be. Suppose that the labor productivity is the only component ill-affected by risky asset investment, i.e., $dF(t)/dm(t) < 0$ and $dj(t)/dm(t) = 0$. This occurs when the firm prohibits

To simply implement the property, $dj(t)/dm(t) < 0$, we assume that the sum of labor, leisure and risky asset management is fixed to a unit time. This time constraint is a natural way to induce the opportunity cost of risky asset management: increasing the time spent on risky asset management dictates a decrease in either leisure or labor. This structure is equivalent to Becker (1965)'s '*full income approach.*' To measure the opportunity time cost, he used the term 'full income', which is the income level received when consuming all available time in labor.

In addition, our model is on the series of consumption asset pricing models such as Lucas (1978), Barberis, Huang, and Santos (2001), Santos and Veronesi (2006), and Cochrane, Longstaff, and Santa-Clara (2007). We consider a general equilibrium framework where there is a representative agent and a market is cleared such that the perishable consumption is identical to the sum of dividend and *endogenous* labor income at time t.[3] Thus our economy is associated with two kinds of approaches for resurrecting consumption asset pricing models. One approach is to to investigate the *quality of consumption*. For instance, it is possible that if some investors are away from the stock market, their consumption processes contaminate the aggregate consumption data [Mankiw and Zeldes (1981) and Vissing-Jorgensen (2002a)]. Given the frictions that drive investors away from the stock market as in Basak and Couco (1998), the participants' consumption processes are less correlated with aggregate consumption. Hence it could lead to false rejection of consumption-based asset pricing models. As such, Vissing-Jorgensen (2002a) shows that only the consumption on stockholders fares better to explain the observed equity returns and volatilities. Our study tries to answer the fundamental reason to such consumption contamination by introducing the opportunity cost such as time cost.

The second type of research is related to the *source of consumption*. Many researchers have developed equilibrium models with multiple securities [Menzly, Santos, and Veronesi (2004), Longstaff and Piazzesi (2004), and Cochrane, Longstaff, and Santa-Clara (2007)], and found that multiple income sources can generate more complex return dynamics of risky assets.[4] In particular, human

access to internet surfing. In such a case, the worker could potentially kill time due to worrying about the performance of her stocks. If we measure the actual time of labor, this could mean a decrease in the amount of actual labor time. As such, $dF(t)/dm(t) < 0$ and $dj(t)/dm(t) = 0$ could be consistent with $dF(t)/dm(t) = 0$ and $dj(t)/dm(t) < 0$ by changing the operational definition of $m(t)$.

[3] Labor income is a decision variable given the exogenous labor efficiency and the level of dividend.

[4] Menzly et al. (2004) focus on the linkage between the time-varying risk preference and the fluctuations of dividend

capital above all is the most important income source in the real economy. A majority of households work everyday and receive the wage as a reward.[5] Labor income accounts for 75% of total consumption [Campbell (1999), and Santos and Veronesi (2006)], consequently being a primary factor in decision makings. Since Mayers (1972), many studies have explored the role of human capital in asset pricing.[6] For instance, Santos and Veronesi (2006) introduce the economy with two income sources, i.e., dividend and wage. Pivotal intuitions behind their model are such that the correlation between the consumption (the sum of dividend and labor income) growth and the dividend growth depends on wage-to-consumption ratio, which is thereby connected to the rate of return on a risky asset. This mechanism generates a considerable return predictability over time series and cross section. Gomes and Michaelides (2005) prove numerically that the incomplete risk sharing by labor income in incomplete market increases an equity premium and volatility. In addition, Lettau and Ludvigson (2001a, 2001b) show that only a deviation of consumption from the stable state of wealth, *including human capital*, has a remarkable information about stock returns, and Lettau and Ludvigson (2004) emphasis the importance of information contained in labor income and consumption. Our study, in contrast, permits an endogenous labor/leisure choice and opportunity costs in decision makings, and gives the importance of labor income by the different ways from above extant literatures.

The main results of our paper can be summarized as follows. First, we have derived the standard asset pricing formula in the presence of a time constraint. The difference of this formula with the traditional asset pricing formula without a time constraint illustrates the impact of opportunity time cost on the asset returns subject to a time constraint. Second, we have found out the closed-form solutions for the risky asset price, the risky return, and the riskfree rate. The closed-form solutions are restricted by the assumption about agent's log utility in our economy, but such an assumption does not mitigate the importance of opportunity cost, in fact, our results are conserved even if we

growth, and compare their results with the empirical valuation ratios observed in U.S. stock market. Cochrane et al. (2007) elicit the novel features such as serial correlation and time series/cross-sectional predictability when market is cleared by two financial assets giving a stream of perishable dividends.

[5]Human capital can be conceived of as another asset that gives a labor income as a dividend.

[6]Besides, many researches have investigated the importance of a labor income in a portfolio choice setup. [Merton(1971), Bodie, Merton, and Samuelson (1992), Carroll (1997), Viceira (1999), and Cocco, Gomes, and Manhout(2005)]. They focus on a labor income as a substitute for long-term risk-free asset holdings and/or on the background risk implied on a labor income, which can affect the investment choices of households [Viceira (1999)].

increase the relative risk aversion more than one. That is, the time cost induces the substitution effect with labor income or leisure, which thereby influences the equity premium and volatility. In a equilibrium, only a small amount of time for risky investment elicits a significant increase in equity premium and volatility, which can be a possible resolution of asset pricing puzzles. Third, we investigate the impact of endogenous labor/leisure choice by comparing with exogenous labor income case given no time cost required for risky investment. Most extant literatures focus on the background risk implied on labor income and/or the role constituting an implicit riskfree holding by assuming that labor income is *exogenous*. Such studies cannot reflect an endogenous labor/leisure choice which apparently exists in reality, and make it hard to bring out an exact implication from the consideration of human capital. Hence we consider both the economies with and without the endogenous labor income choice, and separate the effect of endogenous labor/leisure choice from total change of return dynamics. Under the same dividend-consumption ratio, an endogenous labor/leisure choice has a slightly negative impact on the equity premium and volatility, which is consistent to Bodie, Merton, and Samuelson (1992). They have shown that endogenous choice makes investors more aggressive in investing a risky asset under the portfolio choice setup, since the flexibility in choosing labor/leisure creates a kind of insurance against adverse performance of portfolio. Such an insurance effect decreases the risky premium. Totally, despite a negative effect of endogenous choice, the opportunity cost effect by a reasonable amount of time for risky investment dominates the effect by an endogenous choice, thereby increasing total equity premium.

The remainder of the paper is organized as follows. Section 2 explains how to represent the time constraint confronted to investors, and describes the economic framework we will explore. We introduce the endowment economy with endogenous labor/leisure/investment choice in continuous-time framework. In Section 3, we derive a standard asset pricing formula in the general economy, and bring out the closed-form solution to the asset return dynamics when an economic agent has a log utility. Section 4 investigates the impact of time cost required for risky investment on equity return and volatility by comparing with several benchmark models. Section 5 documents the results for sensitivity tests with varying some economic parameters. Lastly, Section 6 concludes and discusses future research issues and a limitation of our model.

2 The Model

In this section, we make a formal definition of time cost associated with risky investment coupled with its corresponding time constraint and investors' preference systems. Such a time cost is measured as a percent of total available time and related with shares of stock held.

2.1 The Time Cost and the Time Constraint

We impose a constraint that an investor is endowed with a limited amount of time at each period t, which is normalized to one unit without loss of generality. The unit time will be allocated to labor activity, j_t, leisure, l_t, or risky investment m_t, and therefore

$$l_t + j_t + m_t = 1.$$

This representation is standard in the real business cycle model, as laid out by Prescott (1986).

Given the constraint on the time available, the investor is inherently subject to two budget constraints: the typical wealth constraint and the time constraint. The extant literatures on transaction cost models such as Constantinides (1986) and Davis and Norman (1990) incorporate monetary transaction costs incurred by rebalancing the risky portfolio, which affects the wealth constraint. These costs come from 'changing' the portfolio rather than holding the portfolio. Additionally, Vissing-Jorgensen (2002) considers fixed participation costs, depending on whether or not holding a risky portfolio as well as the amount of transaction, which also affects the wealth contraint. In contrast, we consider the time opportunity cost required for risky investment, which directly affects the time constraint rather than the wealth constraint. Ultimately this time constraint is binding the choice of labor, leisure and risky investment activities, thereby changing the wealth constraint, but only indirectly through the concatenation between two constraints. The time cost herein is also a portfolio holding cost, not a transaction cost.

Next we set the time cost structure. For simplicity, the time cost is represented as a function of only stock holding shares as follows.

$$m_t(N_t) = m^* \cdot |N_t|^{\zeta},$$

where m_t is time costs associated with risky investment; N_t is shares of stock; m^* is the amount of time required when stock market clears, $N_t = 1$. As the invested shares increase, the economic agent might require more time in research, brokerage implementation, and monitoring. Following the above argument, $\partial m_t / \partial |N_t| > 0$, which results in $\zeta > 0$. Besides, there are a variety of possible specifications on time cost structure. However, it is not important because the risky asset price is affected by only a marginal cost of time required at the equilibrium, $N_t = 1$. The marginal time cost required at the equilibrium is equal to the product of m^* and ζ. Even though the time cost required when market clears, m^*, is large, an increase in the opportunity cost can be negligible if ζ is very small. In contrast, although m^* is small, the opportunity cost can be large with large ζ.

Under the above assumptions, we can summarize the time constraint:

$$l_t + j_t + m_t = 1$$
$$\text{where} \qquad m_t = m^* \cdot |N_t|^{\zeta}$$

2.2 The Economic Framework and the Preference

Our starting point is the traditional consumption-based asset pricing model, isomorphic to Merton (1971, 1973) and Lucas (1978), but we introduce the labor/leisure/investment choice under the time constraint. In particular, labor activity provides an alternative source of increasing wealth. There is a continuum of identical and infinitely lived agents in the economy, with total mass of one, whose preference is characterized by the modified CRRA utility function incorporating the Cobb-Douglas property among the numeraire good consumption and leisure consumption:

$$U(C_t, l_t) = \begin{cases} \frac{1}{1-\delta}\left(C_t^{\gamma} \cdot h(l_t)^{1-\gamma}\right)^{1-\delta} & \text{if} \quad \delta \neq 1 \\ \gamma \log(C_t) + (1-\gamma)\log h(l_t) & \text{if} \quad \delta = 1 \end{cases}, \tag{1}$$
$$\text{where } h(l_t) = B \cdot l_t,$$

where l_t is the amount of leisure time consumed at time t; $h(l_t)$ is leisure consumption measured at time t; B is the appropriate constant parameter for transforming the amount of leisure time to the leisure consumption; C_t represents leisure-unrelated consumption at time t. The properties of consumption and leisure in preference (1) satisfy the standard regular conditions such as the decreasing marginal rate of substitution, and intertemporal and intratemporal substitutability between them. δ (> 0) is RRA parameter and γ ($0 < \gamma < 1$) is the Cobb-Douglas parameter governing the relative importance of non-leisure consumption.[7]

The size of the investor's monetary budget constraint is determined by portfolio performance and labor income. Portfolio performance is related to the risky asset returns, whereas labor income is related to labor time and the 'full labor income'. The full labor income is the amount of income received when allocating total available time (the unit time in our model) into labor, forgoing other activities such as leisure and risky asset investment.[8] The *full labor income*, F_t, is represented as follows:

$$dF_t/F_t = \mu_F dt + \sigma_F dZ_1 \tag{2}$$

where μ_F is the expected growth rate of the full labor income; σ_F is the volatility of the full labor income, both of which are assumed to be constant over time for simplicity.

Her labor income, $w(j_t)$ is the product of the full labor income, F_t, and the labor time consumed, j_t at time t:[9]

$$w(j_t) = F_t \cdot j_t, \tag{3}$$

Regarding the investment opportunity set, we consider two financial assets in the economy: a risk-free asset in zero net supply, paying a interest rate of $r_{f,t}$ between time t and time $t + dt$; one unit of a risky asset, i.e., a stock, paying a return of r_t between time t and time $t - dt$. Usually the risky asset is a claim to a stream of perishable output represented by the dividend sequence D_t, whose process is given by

$$dD_t/D_t = \mu_D dt + \sigma_D dZ_2, \tag{4}$$

where $< Z_1, Z_2 >= \rho$; μ_D is the expected growth rate of the dividend; σ_D is the standard deviation of dividend.

At time t, an investor knows the stock price, P_t, her wealth, W_t, and full labor income, F_t. She then chooses how much to consume, C_t, leisure time, l_t, labor time, j_t, risky investment time, m_t,

[7] Over the postwar period, wages have risen substantially, but hours worked have not declined much. This implies that the budget share of a good does not vary despite the fact that price has gone up sharply. A Cobb-Douglas utility function is a utility function consistent with the lack of trend in hours worked per worker. See Prescott (1986).

[8] See, for example, Becker (1965) and Prescott (1986).

[9] Therefore we implicitly posit a constant wage scheme under the assumption of a constant-return-to-scale labor production function.

the amount of risk-free investment, B_t, and the invested shares of risky asset, N_t subject to the following constraint

$$W_t = N_t P_t + B_t. \tag{5}$$

Thus the wealth process at time t is as follows.

$$dW_t = N_t(dP_t + D_t dt) + r_{f,t} B_t dt + F_t j_t dt - C_t dt \tag{6}$$

Under the above assumptions, the investor's intertemporal optimum scheme can be represented as:

$$\max_{\{C_t, l_t, j_t, N_t\}} E\left[\int_0^\infty \beta^t U(C_t, l_t) ds\right], \tag{7}$$

subject to

$$\begin{aligned}
dW_t &= N_t(dP_t + D_t dt) + r_{f,t} B_t dt + F_t j_t dt - C_t dt \\
dF_t/F_t &= \mu_F dt + \sigma_F dZ_1 \\
dD_t/D_t &= \mu_D dt + \sigma_D dZ_2 \\
l_t + j_t + m_t &= 1, \\
w(j_t) &= F_t \cdot j_t, \\
m_t &= m^* |N_t|^\xi
\end{aligned}$$

Additionally, there is an ad-hoc but crucial assumption in our model. Although the sum of the time allocated into three activities is fixed to the unit one, it does not guarantee the time consumed on each activity as non-negative. This means that an agent is allowed to leverage in a particular activity, i.e., a leisure time ($l_t \geq 1 - m^*$), but not a labor time.[10] When an agent wants to increase her leisure time more than an endowed amount of time (for instance, improve the quality of leisure activity), then she pays for the money as much as the full labor income per unit time. This assumption makes our pricing equation to avoid from corner solutions, thereby helping us bring out the closed-form solutions. Such an assumption, however, does not mitigate the importance of time cost required for managing risky investment. Moreover, if we get rid of a

[10]Since a leisure *directly* affects the utility function, a leisure time is always positive. Whereas, the time consumed in labor can be negative corresponding to the economic condition, for instance, with extremely low full labor income (labor efficiency) and high dividend income.

leisure among activities that the agent can allocate her limited time, their optimal decisions would not bind the constraint: the time consumed into each activity is always positive.

3 Equilibrium Prices

3.1 Standard Asset Pricing Formula (Euler Equation)

To resolve the asset pricing formula in the presence of the time cost required for risky investment, we conjecture the equity process as follows:

$$dP_t = (\mu P_t - D_t)dt + h_1 P_t dZ_1 + h_2 P_t dZ_2 \tag{8}$$

Given the risky asset process and state variables, we also define the indirect utility function.

$$J(W_t, D_t, F_t, t) = \max_{\alpha_t, C_t, l_t} E_t \left[\int_t^{t'} U(C_t, l_t) ds \right], \tag{9}$$

where $\alpha_t = \frac{N_t P_t}{W_t}$. Then we can derive the optimality condition from the well-known Bellman equation.

$$0 = \max_{\alpha_t, C_t, l_t} [U(C_t, l_t) + DJ(\cdot, t)] \equiv \Psi_t \tag{10}$$

From $\max_{\alpha_t, C_t, l_t} \Psi$, the first order conditions(FOCs) with respect to the risky investment weight, consumption, and leisure time are given by

$$\begin{aligned}
\Psi_C &= U_c - J_W \leq 0 \qquad \text{and} \qquad C\Psi_C = 0, & (11)\\
\Psi_l &= U_l - J_W F_t = 0, & (12)\\
\Psi_\alpha &= J_W \left[W_t(\mu - r) - F_t \frac{\partial m_t}{\partial \alpha_t} \right] + J_{WW} W_t^2 (h_1^2 + h_2^2 + 2\rho h_1 h_2) & (13)\\
&\quad + J_{WD} W_t D_t \sigma_D (h_1 + \rho h_2) + J_{WF} W_t F_t \sigma_F (\rho h_1 + h_2) = 0 &\\
&\quad \text{where} \qquad \frac{\partial m_t}{\partial \alpha_t} = \frac{\partial N_t}{\partial \alpha_t} \frac{\partial m_t}{\partial N_t} = \frac{W_t}{P_t} \frac{\partial m_t}{\partial N_t}. & (14)
\end{aligned}$$

At an interior solution, the FOCs satisfy the following relationship between leisure l_t and consumption C_t,

$$l_t = \frac{1-\gamma}{\gamma} \frac{C_t}{F_t}. \tag{15}$$

Definition: *A Rational Expectation Equilibrium is a price function $P(t)$, a dividend process $D(t)$, an allocation process $N(t)$, a full labor income $F(t)$, and consumption $C(t)$ such that investors maximize their utility and markets clear, that is*

$$N(t) = 1,$$
$$C(t) = D(t) + w(t),$$
$$\textit{where } \; w(t) = F(t) \cdot j(t) = F(t)(1 - l_t - m^*(N_t = 1)).$$

Proposition 1: *Under the above conditions, the standard asset pricing formula for a risky asset is given by* [11]

$$P_t = E_t \left[\int_0^\infty \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-\delta} \left(\frac{F_t}{F_{t+s}} \right)^{(1-\gamma)(1-\delta)} \left(D_{t+s} - F_{t+s} \frac{\partial m_{t+s}}{\partial N_{t+s}} |_{N_{t+s}=1} \right) ds \right]. \tag{16}$$

Proof: See the Appendix A.

Generally, it is hard to resolve the standard asset pricing equation (16). Once restricted to the economy wherein agents have log utility, however, we can derive the closed-form solutions for the rate of return and volatility, which will be included in later section.

Let ϕ_t be dividend to consumption ratio D_t/C_t. The introduction of term ϕ_t changes the state variables F_t and D_t to C_t and ϕ_t. That is, the full labor income to consumption ratio can be rewritten as using the first order condition (15),

$$\frac{F_t}{C_t} = \frac{1 - \gamma\phi_t}{\gamma(1 - m_t)}.$$

The amount of the time allocated into leisure and labor are shown as respectively,

$$\begin{aligned} l_t &= \frac{1 - \gamma}{1 - \gamma\phi_t}(1 - m_t), \\ j_t &= \frac{\gamma(1 - \phi_t)}{1 - \gamma\phi_t}(1 - m_t). \end{aligned}$$

[11] Whereas, this standard asset pricing formula in discrete-time framework is modified to

$$P_t = E_t \left[\sum_{s=1}^{\infty} \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-\delta} \left(\frac{F_t}{F_{t+s}} \right)^{(1-\gamma)(1-\delta)} \left(D_{t+s} - F_{t+s} \frac{\partial m_{t+s}}{\partial N_{t+s}} |_{N_{t+s}=1} \right) \right] - F_t \frac{\partial m_t}{\partial N_t} |_{N_t=1}.$$

The proof is trivial by using Bellman equation.

Using the above equations, the consumption process, dC_t, and its corresponding dividend to consumption process, $d\phi_t$, can be simplified as follows

$$\frac{dC_t}{C_t} = [\mu_D\gamma\phi_t + \mu_F(1-\gamma\phi_t)]\,dt + \sigma_F(1-\gamma\phi_t)dZ_1 + \sigma_D\gamma\phi_t dZ_2, \tag{17}$$

$$\begin{aligned} d\phi_t &= \phi_t(1-\gamma\phi_t)\left[\mu_D - \mu_F - \gamma\phi_t\sigma_D^2 + (1-\gamma\phi_t)\sigma_F^2 + (2\gamma\phi_t - 1)\rho\sigma_D\sigma_F\right] dt \\ &\quad -\phi_t(1-\gamma\phi_t)\left[\sigma_F dZ_1 - \sigma_D dZ_2\right]. \end{aligned} \tag{18}$$

3.2 The Special Case: Log Utility Case

Recently assuming multiple income sources coupled with the log utility of a representative agent is popular in asset pricing literatures [Menzly et al. (2004), Santos and Veronesi (2006), and Cochrane et al. (2007)]. Multiple income sources reflect the real economic environment, whereas log utility helps deriving analytical solutions under multiple income sources. Such literatures have shown that dividend to consumption ratio is important in pricing the risky asset and predicting future returns. In line with those, we have shown that dividend to consumption ratio can influence the return dynamics through another mechanism in the presence of time required for risky investment coupled with time constraint.

The technique to solve the equilibrium price is aided by Cochrane et al. (2007). If one financial asset is assumed as human capital which gives a labor income as a dividend in Cochran et al. (2007), our economy is similar with theirs. Our economy, however, is more general in that ours includes the time cost required for managing a risky asset as well as the endogenous labor/leisure choice, thereby embracing Cochrane et al. (2007). Such an economic framework gives a nice and simple implication for the existence of multiple assets, despite some drawbacks.[12]

In particular, specifying two assets as one financial asset and a human capital makes an in-depth analysis on endogenous labor/leisure choice possible, and adding the time cost coupled with a time constraint helps investigating how the risky asset return is linked to the efficiency of other activities. We find out a couple of additional mechanisms for labor opportunity to influence a

[12] First, as pointed in Santos and Veronesi (2006), the assumption that both assets follow geometric Brownian motion as in our model and Cochrane et al. (2007)'s implies that in the long run, one of income sources would dominate the economy with probability one. Next, as mentioned in conclusion of Cochrane et al. (2007), the novel features of return dynamics induced by two assets are diluted by introducing many financial assets.

risky asset return. One is the effect of endogenous labor choice, and the other is the effect of opportunity cost, which is forgone labor/leisure activities. These two impacts of labor opportunity are distinguishable features of our model, compared with most extant literatures which explore labor income. They consider labor income as only a implicit risk-free asset or as another asset with a background risk.

Proposition 2: *In equilibrium, the price of a risky asset wherein an investor has log utility is given by*

$$P_t = C_t \left[\left(1 + \frac{1}{1-m^*} \frac{\partial m}{\partial N} \right) M(t) + \frac{1}{\ln \beta} \frac{\partial m}{\partial N} \frac{1}{\gamma(1-m^*)} \right] \tag{19}$$

where

$$M(t) = \frac{1}{\psi\gamma(1-\xi)} \left(\frac{\gamma\phi_t}{1-\gamma\phi_t} \right) F\left(1, 1-\xi; 2-\xi; -\frac{\gamma\phi_t}{1-\gamma\phi_t}\right) + \frac{1}{\psi\gamma\theta} F\left(1, \theta; 1+\theta; -\frac{1-\gamma\phi_t}{\gamma\phi_t}\right).$$

and

$$\psi = \sqrt{\nu^2 - 2\ln\beta\eta^2}, \quad \xi = \frac{\nu-\psi}{\eta^2}, \quad \theta = \frac{\nu+\psi}{\eta^2}$$

$$\nu = \mu_F - \mu_D - \frac{1}{2}\left(\sigma_F^2 - \sigma_D^2\right), \quad \eta^2 = \sigma_F^2 + \sigma_D^2 - 2\rho\sigma_F\sigma_D.$$

Proof: See the Appendix B.

Let r_t and $r_{f,t}$ denote the instantaneous return on the risky asset and the riskless asset, respectively. Given the explicit processes of price, P_t, consumption, C_t, and dividend-consumption share, ϕ, the risky asset return, r, is expressed by a direct application of Ito's lemma.

$$r = \frac{dP_t + D_t dt}{P_t} = \frac{dC_t}{C_t} + \frac{d(P_t/C_t)}{P_t/C_t} + \frac{dC_t}{C_t}\frac{d(P_t/C_t)}{P_t/C_t} + \frac{D_t}{P_t}dt \tag{20}$$

From equation (70),

$$\begin{aligned} E(r) &= r_f + Cov\left(\frac{dC}{C}, r\right) + \frac{F_t}{P_t}\frac{\partial m}{\partial N} \\ &= r_f + Var\left(\frac{dC_t}{C_t}\right) + Cov\left(\frac{dC_t}{C_t}, \frac{dP_t/C_t}{P_t/C_t}\right) + \frac{C_t}{P_t}\frac{1-\gamma\phi_t}{\gamma(1-m^*)}\frac{\partial m}{\partial N} \end{aligned} \tag{21}$$

Proposition 3: *In equilibrium, the rate of return on the risky asset and the riskless asset wherein an investor has log utility is given by*

$$r_t = \left[-\ln\beta + \mu_D\gamma\phi_t + \mu_F(1-\gamma\phi_t) + \left(\rho\sigma_D\sigma_F - \sigma_F^2 + \eta^2\gamma\phi\right)\frac{A(\phi_t)}{B(\phi_t)} \right. \tag{22}$$

$$\left. + \frac{1-\gamma\phi_t}{\gamma(1-m)}\frac{\partial m}{\partial N}\frac{1}{B(\phi_t)} \right] dt + \sigma_F\left(1-\gamma\phi - \frac{A(\phi_t)}{B(\phi_t)}\right) dZ_1 + \sigma_D\left(\gamma\phi_t + \frac{A(\phi_t)}{B(\phi_t)}\right) dZ_2 \tag{23}$$

where

$$\begin{aligned} A(\phi) &= \left(1 + \frac{1}{1-m^*}\frac{\partial m}{\partial N}\right)\left[\frac{1}{\psi(1-\xi)}\frac{\phi_t}{1-\gamma\phi_t}F\left(1, 1-\xi; 2-\xi; -\frac{\gamma\phi_t}{1-\gamma\phi_t}\right)\right. \\ &\quad - \frac{1}{\psi(2-\xi)}\frac{\gamma\phi_t^2}{(1-\gamma\phi_t)^2}F\left(2, 2-\xi; 3-\xi; -\frac{\gamma\phi_t}{1-\gamma\phi_t}\right) \\ &\quad \left. + \frac{1}{\psi(1+\theta)}\frac{1-\gamma\phi}{\gamma^2\phi}F\left(2, 1+\theta; 2+\theta; -\frac{1-\gamma\phi_t}{\gamma\phi_t}\right)\right] \end{aligned} \tag{24}$$

$$B(\phi) = \left(1 + \frac{1}{1-m^*}\frac{\partial m}{\partial N}\right) M(t) + \frac{1}{\ln\beta}\frac{\partial m}{\partial N}\frac{1}{\gamma(1-m^*)} \tag{25}$$

$$r_{f,t} = -\ln\beta + \mu_D\gamma\phi_t + \mu_F(1-\gamma\phi_t) - \sigma_D^2\gamma^2\phi_t^2 - \sigma_F^2(1-\gamma\phi_t)^2 - 2\rho\sigma_D\sigma_F\gamma\phi_t(1-\gamma\phi_t) \tag{26}$$

Proof: The direct application of Proposition 2.

3.3 The Benchmark Models

The results of our model above are mixed with the impact by the time required for risky investment and the impact by endogenous labor/leisure choice. To separate each impact, we introduce several benchmark models. The Benchmark I does not include the time for risky investment, but an *endogenous* labor/leisure choice still. The Benchmark II does not include the time cost, and has an *exogenous* labor income. Benchmark III is only a financial income case wherein consumption is composed of only a dividend. The difference between our model and Benchmark I indicates the impact of the time required for risky investment, whereas the difference between Benchmark I and Benchmark II indicates the impact of endogenous labor/leisure choice.

Benchmark I : No Time cost case with Labor/Leisure Choice

Benchmark I is identical to our model except setting the time required for managing the risky asset investment as zero, $m(t) = 0$.

Proposition 4: *In Benchmark I where there is no time cost required for risky investment, the price of a risky asset is given by*

$$P_t = C_t\left[\frac{1}{\psi\gamma(1-\xi)}\left(\frac{\gamma\phi_t}{1-\gamma\phi_t}\right)F\left(1,1-\xi;2-\xi;-\frac{\gamma\phi_t}{1-\gamma\phi_t}\right)+\frac{1}{\psi\gamma\theta}F\left(1,\theta;1+\theta;-\frac{1-\gamma\phi_t}{\gamma\phi_t}\right)\right]. \quad (27)$$

Proof: See the Appendix C.

Proposition 5: *In Benchmark I, the rate of returns on the risky asset and the riskless asset are as follows. The rate of return on risky asset is given by*

$$\begin{aligned} r_t &= \left[-\ln\beta+\mu_D\gamma\phi_t+\mu_F(1-\gamma\phi_t)+\left(\rho\sigma_D\sigma_F-\sigma_F^2+\eta^2\gamma\phi\right)\frac{A(\phi_t)}{B(\phi_t)}\right]dt \\ &\quad +\sigma_F\left(1-\gamma\phi-\frac{A(\phi_t)}{B(\phi_t)}\right)dZ_1+\sigma_D\left(\gamma\phi_t+\frac{A(\phi_t)}{B(\phi_t)}\right)dZ_2, \end{aligned} \quad (28)$$

where

$$\begin{aligned} A(\phi) &= \frac{1}{\psi(1-\xi)}\frac{\phi_t}{1-\gamma\phi_t}F\left(1,1-\xi;2-\xi;-\frac{\gamma\phi_t}{1-\gamma\phi_t}\right) \\ &\quad -\frac{1}{\psi(2-\xi)}\frac{\gamma\phi_t^2}{(1-\gamma\phi_t)^2}F\left(2,2-\xi;3-\xi;-\frac{\gamma\phi_t}{1-\gamma\phi_t}\right) \\ &\quad +\frac{1}{\psi(1+\theta)}\frac{1-\gamma\phi}{\gamma^2\phi}F\left(2,1+\theta;2+\theta;-\frac{1-\gamma\phi_t}{\gamma\phi_t}\right) \end{aligned} \quad (29)$$

$$B(\phi) = M(t). \quad (30)$$

The rate of return on the riskfree asset is given by (same to our model)

$$r_{f,t} = -\ln\beta+\mu_D\gamma\phi_t+\mu_F(1-\gamma\phi_t)-\sigma_D^2\gamma^2\phi_t^2-\sigma_F^2(1-\gamma\phi_t)^2-2\rho\sigma_D\sigma_F\gamma\phi_t(1-\gamma\phi_t). \quad (31)$$

Proof: Similar to Proposition 3.

Benchmark II: No Time Cost case with Exogenous Labor Income (No Leisure, $\gamma = 1$)

Benchmark II modifies Benchmark I by omitting the chance to allocate her endowed time into several activities. Labor income is exogenous, thereby same to the model of Cochrane et al. (2007) with conceiving labor income as another financial asset.

Proposition 6: *In Benchmark II where there is no time cost required for risky investment and no labor/leisure choice, the price of a risky asset is given by*

$$P_t = C_t\left[\frac{1}{\psi(1-\xi)}\left(\frac{\phi_t}{1-\phi_t}\right)F\left(1,1-\xi;2-\xi;-\frac{\phi_t}{1-\phi_t}\right)+\frac{1}{\psi\theta}F\left(1,\theta;1+\theta;-\frac{1-\phi_t}{\phi_t}\right)\right]. \quad (32)$$

Proof: See the Cochrane et al. (2007).

Proposition 7: *In Benchmark II where there is no time cost required for risky investment and no labor/leisure choice, the rate of returns on a risky asset is given by*

$$\begin{aligned} r_t &= \left[-\ln\beta + \mu_D\phi_t + \mu_F(1-\phi_t) + \left(\rho\sigma_D\sigma_F - \sigma_F^2 + \eta^2\phi_t\right)\frac{A(\phi_t)}{B(\phi_t)}\right] dt \\ &\quad +\sigma_F\left(1-\phi_t - \frac{A(\phi_t)}{B(\phi_t)}\right) dZ_1 + \sigma_D\left(\phi_t + \frac{A(\phi_t)}{B(\phi_t)}\right) dZ_2, \end{aligned} \tag{33}$$

where

$$\begin{aligned} A(\phi_t) &= \frac{1}{1-\xi}\left(\frac{\phi_t}{1-\phi_t}\right) F\left(1, 1-\xi; 2-\xi; \frac{\phi_t}{\phi_t - 1}\right) - \frac{1}{2-\xi}\left(\frac{\phi_t}{1-\phi_t}\right)^2 F\left(2, 2-\xi; 3-\xi; \frac{\phi_t}{\phi_t-1}\right) \\ &\quad + \frac{1}{1+\theta}\left(\frac{1-\phi_t}{\phi_t}\right) F\left(2, 1+\theta; 2+\theta; \frac{\phi_t - 1}{\phi_t}\right), \\ B(\phi_t) &= \frac{1}{1-\xi}\left(\frac{\phi_t}{1-\phi_t}\right) F\left(1, 1-\xi; 2-\xi; -\frac{\phi_t}{1-\phi_t}\right) + \frac{1}{\theta} F\left(1, \theta; 1+\theta; -\frac{1-\phi_t}{\phi_t}\right). \end{aligned}$$

And the rate of return on the riskfree asset is

$$r_{f,t} = -\ln\beta + \mu_D\phi_t + \mu_F(1-\phi_t) - \sigma_D^2\phi_t^2 - \sigma_F^2(1-\phi_t)^2 - 2\rho\sigma_D\sigma_F\phi_t(1-\phi_t). \tag{34}$$

Benchmark III: Only Financial Income, $C_t = D_t$

Benchmark III represents the single tree model of Lucas (1978) with risk aversion of one, i.e., log utility, which keeps the pivotal features underpinning the traditional asset pricing model. The only income stream is a dividend endowment. This model is known to suffer from an inveterate problem in explaining the observed magnitude of equity premium and volatility.

Proposition 8: *In Benchmark III where there is no time allocation and no labor income, hence consumption is composed of only a dividend stream, the price of a risky asset is given by*

$$P_t = -\frac{C_t}{\ln\beta}. \tag{35}$$

Proof: Straightforward.

Proposition 9: *In Benchmark III where there is no time allocation and no labor income, the rate of return on a risky asset is given by*

$$r = (-\ln\beta + \mu_D)\, dt + \sigma_D dZ_2. \tag{36}$$

The rate of return on a riskless asset is given by

$$r_f = -\ln\beta + \mu_D - \sigma_D^2. \tag{37}$$

Proof: Straightforward.

4 Interpretations

There are a lot of novel features when a market clears by two income source. Most of them are touched in Cochrane et al. (2007), the economy of which is identical except the existence of the optimal time allocation coupled with the time required for risky investment. For instance, we can find a serial correlation of returns and a predictability of price-dividend ratio in the time-series as well as in the cross-section in both papers. However, they are not our main focus in this paper. We focus exclusively on the opportunity cost induced by the time required for risky investment and on the endogenous labor/leisure choice, which are distinct properties of our model.[13] We could divide these two effects by comparing our model to benchmark models. As we mentioned, the comparison between our model and Benchmark I brings out the impact of time required for risky investment, whereas the comparison between benchmark I and benchmark II brings out the impact of endogenous labor/leisure choice.

4.1 Equity Premium and Its Volatility

Despite the closed-form solutions for return dynamics as shown in Equation (23), it is hard to analyze the properties of our solutions due to the hypergeometric function. We thereby assume the specific parameter values, and investigate the return dynamics in detail.

[13]In addition, Cochrane et al. (2007) is not a quantitative model, thus their results cannot be comparable to the observable market data. On the other hand, the results from our model are comparable, since the opportunity time cost can generate the high equity premium and volatility consistent with the observed data.

The based parameters are assumed as Table 1, which are consistent with demographic data. Discount factor (β) is 0.96; the preference for non-leisure consumption (Cobb-Douglas parameter, γ) is set to 0.57[14]; both the expected dividend growth rate (μ_D) and full labor income (μ_F) are 1.5%; the volatilities of dividend and full labor income (σ_D, σ_F) are set to 10% and 3%, respectively[15]; the correlation between dividend sequence and full labor income (ρ) is 0. Finally, the time cost structure for risky investment is as follows

$$m(t) = m^* \cdot N_t^{\zeta} = 0.035 \cdot N_t^2.$$

$m^* = 0.035$ is the time consumed for risky investment in equilibrium which means $N_t = 1$. The time cost set at 0.035 corresponds to 5.8 hours per week, or 12.5 days per year, the values of which are sufficiently small. Moreover, the fixed-labor-productivity assumption (constant full labor income) that we adopt in our model is likely to overestimate the time required for risky investment.[16] $\zeta = 2$ means the convexly increasing time cost with respect to the shares invested.

The time cost of risky investment subject to a limited amount of time has a critical impact on the rate of return and volatility. As shown in Equation (16), the time required for risky investment induces an opportunity cost for forgoing labor/leisure activities, and this opportunity cost reduces the net future payoff at time t, equal to $D_t - F_t \partial m_t / \partial N_t$. The demand for a risky asset falls off and induces a decrease in price level. Thus investors require a higher equity premium for holding a risky asset. In contrast, the risk-free rate is not influenced by the time cost required for risky investment.

Table 2 illustrates the labor/leisure time, dividend consumption ratio, consumption dynam-

[14]This parameter value is selected for matching a labor-leisure time with the demographic data shown in ATUS (2005). According to ATUS (2005), labor time and leisure time are almost same.

[15]Previous researches have reported that the volatility of dividend is from 10% to 12%. [See Barberis, Huang, and Santos (2001), Campbell (1999), Campbell and Cochrane (1999) and others]. We adopt a lower bounded value, since we want to derive higher excess return and volatility even with the low variation of dividend. In contrast, the volatility of full labor income is not clear. So we assume this value as 3%, which generates the equity premium and volatility consistent with the demographic values.

[16]If we introduce the assumption that labor productivity is negatively correlated with risky investment $\frac{\partial F}{\partial N} < 0$ rather than fixed labor productivity $\frac{\partial F}{\partial N} = 0$, the opportunity cost by the time required for risky investment becomes higher. Thus the amount of the time for matching the return and volatility observed in the stock market will become smaller.

ics, and return dynamics according to the level of dividend-full labor income ratio. As expected, our model generates a high return and volatility of the risky asset, compared with traditional consumption-based asset pricing models with log utility and one financial asset, isomorphic to Lucas (1975) and Benchmark III. In particular, the dividend-full labor income of 1/8 elicits the dividend-consumption ratio, the consumption dynamics, and the return dynamics consistent with the demographic data. In particular, such a dividend-full income ratio results in the dividend-consumption ratio about 20%. Only a price-dividend ratio is lower than the value observed in the demographic data.

Figure 1 also show the risk-free rate, excess return of the risky asset, and volatility corresponding to dividend-full labor income ratio. A decrease in dividend-full labor income ratio dramatically increases the excess return and volatility of risky asset, since the low dividend-full labor income ratio (or the high full labor income) induces the relatively high opportunity cost, given a fixed time cost. Contrary to the risky asset return, the risk-free rate is nearly constant over the entire range of dividend-full income ratio. We assume that the growth rates of both income sources are same, and thus only a difference in volatility between dividend and full income is able to make a change in the risk-free rate.[17]

In addition, the composition of consumption can influence the return dynamics of the risky asset.[18] The risky asset, a stock, is a claim to a stream of perishable output represented by the dividend sequence. In consumption-based asset pricing models, hence, the asset returns are affected by the correlation between cash-flow outputs and the stochastic discount factor (consumption). If consumption is mainly composed of dividend income, the correlation between consumption and dividend is high, and then the return becomes high. In contrast, if consumption is mainly composed of labor income, the return becomes low.

[17]If consumption growth rate and its standard deviation are intertemporally invariant in the consumption-based model, the risk-free rate is invariant. In our economy, however, the composition of consumption, consisting of dividend and labor income, changes intertemporally, thereby the expected consumption growth rate and its standard deviation variate with respect to dividend-full labor income ratio. Thus the risk-free rate can be intertemporally changed.

[18]Santos and Veronesi (2006) investigate the importance of the composition of consumption to the asset returns. They focus not on the return and volatility of the risky asset but the return predictability which is determined by the composition of dividend and labor income in consumption.

Therefore, full labor income affects equity returns through two mechanisms. One is the effect by the time required for risky investment. In equation (16), $F_t \frac{\partial m_t}{\partial N_t}|_{N_t=1}$ reflects the opportunity cost. The opportunity effect is directly associated with the product of marginal time cost, $\frac{\partial m_t}{\partial N_t}$, and full labor income, F_t. As full labor income increases, opportunity cost becomes higher and hence the investors require higher premium for holding a risky asset. The other is the effect by the change in consumption composition. An increase in full labor income decreases the correlation between future cash flow and consumption (stochastic discount factor) as aforementioned. A decrease of the correlation results in a decrease of the risky asset return. Above two contrary effects of full labor income coexist on the risky asset price. As shown in Table 1 and Figure 1, the risky asset return increases in the full labor income in our model, given a fixed dividend level. That is, the first effect dominates the second effect, and a full labor income thereby has a positive relationship with the equity premium and volatility.

4.2 The Opportunity Cost Effect of the Time required for Risky Investment

In the former Section, we have shown that our model enables to generate the considerable equity return and volatility comparable to the demographic data. There are two candidates for eliciting our desirable results compared to Cochrane et al. (2007). One is the existence of time required for risky investment, and the other is the endogenous labor/leisure choice. Yet, we have not explored the question of which is more critical to generate such desirable properties. In this Section, we analyze each impact by comparing our model to benchmark I and benchmark I to benchmark II.

The results in Table 2 (the result of our model) and Table 3 (the result of benchmark I) are driven under the identical economic condition except the existence of time cost required for risky investment. Thus the comparison above makes us to extract only impact of the time cost. Additionally, Figure 1 shows (a) the riskfree rate and excess return and (b) volatility in our model and Benchmark I.

The excess return and volatility with the time required for risky investment are higher than those without the time cost at the same dividend-full labor income ratio. In particular, at the row ratio where labor income-consumption ratio is consistent with the empirical values, the excess return and volatility increase to the level as high as the empirical values, whereas the results of Benchmark I still remain at the level far less than the real values. Therefore, we can infer that the opportunity cost of time for risky investment may have a crucial explanatory power for the equity premium puzzle. Additionally, Figure 2 shows the variation of excess return and volatility according to the amount of time required for risky investment. The amount of time cost is correlated with excess return and volatility as expected, more specifically, convexly. Besides, the volatility of the risky asset is increased as the size of price fluctuation is remained same in spite of a fall in the risky asset price. To sum up, the extent of opportunity cost is determined by two ingredients: the amount of time cost and dividend-full labor income ratio. This is manifest as shown in equation (16).

One interesting thing is that the excess return and its volatility are positively correlated with dividend/full labor income ratio in Table 3 (Benchmark I), contrary to Our model. This is entirely due to the correlation with consumption which consists of dividend and wage. An increase in dividend/full labor income results in raising the correlation between dividend and consumption, thus it requires higher excess premium to hold a stock. In contrast, our model is dominated by the opportunity cost effect, thereby the excess return is negatively correlated with dividend/labor income.

4.3 The Effect of Endogenous Labor/Leisure Choice

The other candidate that might generate the sufficient excess return and volatility in our model is the endogenous labor/leisure choice. In this section, we investigate it by comparing benchmark I (endogenous choice) to benchmark II (exogenous). Table 4 shows the results of Benchmark II, which matches the dividend/consumption ratio with the values in Table 3.

The endogenous choice affects the return dynamics through two ways. One is the modification of pricing kernel, and the other is the change of consumption process. First, the pricing kernel in equation (16), $\beta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\delta} \left(\frac{F_t}{F_{t+s}}\right)^{(1-\gamma)(1-\delta)}$, is different from that in traditional consumption-based asset pricing models, $\beta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\delta}$. Because the difference term, $\left(\frac{F_t}{F_{t+s}}\right)^{(1-\gamma)(1-\delta)}$, tends to move oppositely to $\left(\frac{C_{t+s}}{C_t}\right)^{-\delta}$ given the positive drift of μ_F and μ_D, the correlation between the pricing kernel and the dividend in our model is lower than that in traditional asset pricing models. Thus the low correlation decreases the excess return. Only a log utility with $\delta = 1$ brings out the identical pricing kernel.

Second, the endogenous choice changes the relation between dividend process and consumption process.[19] As shown in Table 3 and Table 4, the excess return and its volatility with the endogenous labor/leisure choice are slightly lower than those with the exogenous labor income. On the other hand, the risk free rate is opposite. These return dynamics are consistent with the results in Bodie, Merton and Samuelson (1992). They prove that the ability to vary labor supply *ex post* tends to induce the individual to assume greater risks in her investment portfolio *ex ante*. The flexibility of labor supply creates a kind of insurance against adverse investment outcomes. Consistently, in our general equilibrium setup, such a flexibility in labor/leisure choice decreases the excess return and volatility through smoothing a future consumption stream. The consumption processes of Benchmark I and Benchmark II is as follows.

- Endogenous Labor/Leisure Choice (Benchmark I)

$$C_t = \gamma[D_t + F_t] \tag{38}$$

$$\frac{dC_t}{C_t} = [\mu_D \gamma \phi_t + \mu_F (1 - \gamma \phi_t)]\, dt + \sigma_F (1 - \gamma \phi_t) dZ_1 + \sigma_D \gamma \phi_t dZ_2 \tag{39}$$

- Exogenous Labor Income (Benchmark II)

$$C_t = [D_t + \hat{F}_t] \tag{40}$$

$$\frac{dC_t}{C_t} = [\mu_D \phi_t + \mu_F (1 - \phi_t)]\, dt + \sigma_F (1 - \phi_t) dZ_1 + \sigma_D \phi_t dZ_2 \tag{41}$$

where $\hat{F}_t$ is the exogenous labor income which elicits the same dividend/consumption ratio ϕ_t with the endogenous choice case. The disparity between (39) and (41) is totally due to the endogenous labor income $w(j_t)$ which is affected by the amount of dividend income, D_t, in the equilibrium.

$$w(j_t) = F_t \gamma - D_t (1 - \gamma) \tag{42}$$

$$dw(j_t) = (\gamma \mu_F F_t - (1 - \gamma) \mu_D D_t)\, dt + \gamma \sigma_F F_t dZ_1 - (1 - \gamma) \sigma_D D_t dZ_2 \tag{43}$$

In the presence of endogenous labor/leisure choice, the bad portfolio performance (low dividend) can be hedged by increasing an agent's labor input, and thus the consumption process is affected

[19] Log utility indicates that an agent is myopic. Thus stochastic discount factor is not related to the endogenous choice. However, the endogenous choice has an indirect effect on return dynamics through the modification of consumption process

by $\gamma\phi_t$ rather than ϕ_t. As a result, total consumption process is more stable and less correlated with a dividend stream if labor supply is sufficiently stable, $\sigma_F < \sigma_D$.[20]

The correlations between dividend and consumption in Benchmark I and II are as follows.

- Endogenous Labor/Leisure Choice (Benchmark I)

$$Cov\left(\frac{dC_t}{C_t}, \frac{dD_t}{D_t}\right) = \gamma\phi\sigma_D^2 + \rho\sigma_D\sigma_F(1-\gamma\phi) \tag{44}$$

- Exogenous Labor Income (Benchmark II)

$$Cov\left(\frac{dC_t}{C_t}, \frac{dD_t}{D_t}\right) = \phi\sigma_D^2 + \rho\sigma_D\sigma_F(1-\phi) \tag{45}$$

The correlation is lower at Benchmark I than at Benchmark II because $0 \leq \gamma \leq 1$.[21] This low correlation in Benchmark I is transferred to the low risk premium and volatility.

Overall, we have investigated two novel features implied on our model in Section 4.2 and 4.3: the time cost required for risky investment and the endogenous labor/leisure choice. As shown, two features influence the return dynamics *oppositely*. Even with a small amount of time cost, the first effect dominates the second effect under the range of reasonable dividend-full income ratio, and the total equity premium and volatility in our model are comparable with the observed return values.

5 Sensitivity Test corresponding to Parameters

All results above are based on the fixed parameters in Table 1 in order to generate the equity premium and volatility comparable with the empirical values. However, it is also important to analyze the role of other parameters such as the volatility of full labor income (σ_F), the preference to non-leisure consumption (γ), and the correlation between dividend and full labor income (ρ). In this Section, we investigate how the change of these parameter values influence the return dynamics.

[20]A wage as a proxy for a full labor income has a higher priority than a dividend in sharing a firm's profit, and thereby more stable than a dividend stream. Moreover, a majority of people think labor income is safer than financial income. Thus we assume $\sigma_F < \sigma_D$.

[21]More exactly, under $\sigma_D^2 - \rho\sigma_D\sigma_F > 0$.

5.1 The Volatility of Full Labor Income: σ_F

In previous Sections, we set the value of parameter σ_F at 3%, which is a little smaller than the value of labor income used in portfolio choice problem such as Cocco, Gomes and Maenhout (2005), Viceira (2001), and Yao and Zhang (2004). This relatively low volatility can be justified by two facts. First, the full labor income is not the real labor income, and the connection with a volatile dividend sequence makes the realized labor income more volatile as in equation (43). For $\sigma_F = 0.03, \sigma_D = 0.10, \gamma = 0.57, \rho = 0$, and $D_t : F_t = 1 : 8$, the standard deviation of the labor income growth rate is approximately 7%. Second, the estimate of Cocco et al. (2005) and Viceira (2001) is based on age-dependent data surveyed by PSID. If the labor income growth rate is estimated using the time-series data of total households' average labor income regardless of age, the standard deviation of labor income growth rate may be more stable than the age-dependent estimate.

Nevertheless, clarifying the exact specification of full labor income is very difficult. The full labor income is non-observable, and its process can be measured only indirectly through the concatenation to *real* labor income. Hence there might be mis-specification for σ_F. We thereby try to get out of the problem by investigating over the wide range of σ_F. According to Figure 3, we have found that the volatility of full labor income has critical impacts on return dynamics. Figure 3-(a) shows the rate of return on the assets according to σ_F. An incline to the full income's volatility results in a decline to the risky asset return both in our model and in Benchmark I, but an extent of the decline is different in each model. The difference is totally due to the time required for risky management existing only in our model. A rise in the volatility of full income reduces the opportunity cost on the risky asset return, which is presented on the return's gap between in our model and in Benchmark I.

Besides, the change of opportunity cost according to the volatility level is translated to the complex pattern for excess returns. In our model, the excess return decreases until at 18%, for then it rebounds as shown in Figure 3-(b). On the other hand, the excess return in Benchmark I is monotonically increasing. Unless the risky investment requires the time as in Benchmark I, the excess return increases in the volatility of full labor income because the riskfree return is more sensitive to the volatility than the risky return. Hence, the opportunity cost effect dominates at the lower volatility, whereas the effect on the difference of the sensitivity presented in Benchmark I dominates at the higher volatility. The line in Figure 3-(b) reflects such a relationship. In addition, Figure 3-(c) illustrates the return volatility according to the volatility of full labor income.

5.2 The Correlation b/w Dividend and Full Labor Income: ρ

In previous Sections, we set the value of parameter ρ, meaning the correlation between dividend growth and full labor income growth, at 0 for simplicity. But the value of zero has not been justified by empirical data. Campbell (1999) estimates the correlation between the dividend growth and the consumption growth as 0.15 in a time series of U.S. data spanning the past century; the correlation between dividend growth and excess return as almost 0; the correlation between consumption growth and excess return as 0.33. In addition, the correlation between labor income growth and excess return is also 0 at the occupational level, as documented in Cocco et al. (2005), Davis and Willen (2002), and Heaton and Lucas (1997). Under this circumstance, the pure endowment economy with only two income sources like in our model is hard to satisfy such relations simultaneously. For example, the value of $\rho = -0.2$ satisfies the correlation between dividend growth and consumption growth as about 0.15, but it cannot satisfy other relationships.

In this Section, we investigate the change of return dynamics according to ρ. Figure 4-(a) shows that the expected excess return increases in the level of correlation both in our model and in benchmark I whereas the riskfree return decreases but very slightly in both models as shown in Equation (26). A change of correlation ρ affects the consumption process, and alters the correlation between the dividend and the consumption as Equation (44). The low level of ρ results in the low correlation between dividend and consumption, and hence an economic agent is willing to hold a risky asset even with a low risk premium.

According to Figure 4-(b), the return volatility is negatively correlated with ρ in our model, whereas it is positively correlated in Benchmark I. In Benchmark I, an increase in ρ makes the consumption growth rate more volatile, thereby resulting in the high return volatility. In our model, in contrast, an increase in ρ makes the net payoff, $D_{t+s} - F_{t+s}\partial m/\partial N$, more stable, which decreases the volatility of excess return.

5.3 The Preference to Non-Leisure Consumption: γ

We set the value of γ governing the preference to non-leisure consumption as 0.57 in previous sections. According to ATUS (2005), the amount of time allocated into leisure and labor time is almost half and half. The value of $\gamma = 0.57$ generates the consistent time allocation with the empirical data documented in ATUS (2005).

Then how does the parameter γ influence the return dynamics subject to the time constraint ? Figure 5 which presents the return dynamics according to γ demonstrates the irrelevance of γ with the return dynamics. Since the labor productivity (utility) is fixed as F_t at time t, the marginal utilities of other activities should follow up to the labor's in the equilibrium. Even if γ varies, the productivity for risky investment, which requires a time cost, should be identical to F_t. γ governs only the time allocated to leisure. But if we adopt $\partial F_t/\partial N_t < 0$ rather than $\partial F_t/\partial N_t = 0$, the value of γ will be linked to the return dynamics as well as the time consumed to leisure.

6 Conclusion

We have investigated the equity premium and its volatility in the presence of time required for risky investment subject to the time constraint. An economic agent, who should allocate a limited amount of time to labor, leisure, and risky investment, suffers from the opportunity cost, which is forgone labor or leisure. Our model has two major differences with the traditional consumption-based asset pricing models. One is the existence of a time cost, and the other is the endogenous labor/leisure choice. Such differences highlight our model compared with other asset pricing models.

The main results of our paper can be summarized as follows. First, we have derived the standard asset pricing formula in the presence of time costs. Second, we have found out the closed form solution to the return dynamics in our economy wherein the representative agent has a log utility. The time cost induces the substitution effect with labor income or leisure, and thus only a small amount of the time can induce a significant increase in equity premium and volatility. Third, we have investigated the impact of endogenous labor/leisure choice by comparing with exogenous labor income case. An endogenous labor choice has a slightly negative impact on equity premium and volatility, which is consistent to Bodie, Merton, and Samuelson (1992). Despite the presence of two opposite effects on return dynamics, the opportunity cost by the time required for risky investment dominates the other effect, thereby increasing the total equity premium and volatility.

In particular, we could find out the other mechanisms that the labor opportunity influences the return dynamics. Most extant literatures have focused on the background risk implied on labor income and/or the role constituting an implicit riskfree holding by assuming the exogenous labor income. In reality, besides, labor opportunity affects the asset return dynamics through several mechanisms. The first is the impact by the opportunity time cost required for risky investment, the second is that by the endogenous labor choice, as we discussed. Finally, the constitution of labor income in consumption can significantly influence the return dynamics. The consideration of such properties in the studies about the human capital helps understanding the exact role of labor income on financial decision makings confronted to the economic agents in reality.

Additionally, our model can be extended to more general frameworks. The closed-form solution in our model is restricted by the assumption of log utility, and our analysis is focused on log utility case. But the solution for non-log utility is likely to enrich our model to explain the observed return dynamics. Furthermore, another possible modification of our model is to specify the relation between a dividend stream and a full labor income process in other ways. As pointed in Santos and Veronesi (2006), the assumption that both assets follow geometric Brownian motion implies that in the long run one of income source would dominate the economy with probability one. To avoid such a problem, it is possible to specify two income sources in a different way as in Santos and Veronesi (2006), in which they assume that the dividend-consumption ratio follows a mean-reverting process. Therefore any one income source cannot dominate the total consumption process in equilibrium.

Appendix A : The Proof of Proposition 1

The wealth process is given by

$$\begin{aligned} dW_t &= N_t(dP_t + D_t dt) + rB_t dt + F_t(1 - l_t - m_t)dt - C_t dt \\ &= N_t P_t(\mu dt + h_1 dZ_1 + h_2 dZ_2) + (r(W_t - N_t P_t) + F_t(1 - l_t - m_t) - C_t)dt. \end{aligned} \tag{46}$$

Let $\alpha_t = \frac{N_t P_t}{W_t}$. Then wealth process can be expressed in terms of α_t and W_t.

$$\begin{aligned} dW_t &= [rW_t + (\mu - r)N_t P_t + F_t(1 - l_t - m_t) - C_t]dt + N_t P_t(h_1 dZ_1 + h_2 dZ_2) \\ &= [W_t[(\mu - r)\alpha_t + r] + F_t(1 - l_t - m_t) - C_t]\, dt + \alpha_t W_t(h_1 dZ_1 + h_2 dZ_2). \end{aligned} \tag{47}$$

The optimization problem of a representative agent is as follows.

$$\max_{\alpha_t, l_t, C_t} E_t \left[\int_t^{t'} U(C_s, l_s)ds \right], \tag{48}$$

subject to

$$dW_t = [W_t[(\mu - r)\alpha_t + r] + F_t(1 - l_t - m_t) - C_t]\, dt + \alpha_t W_t(h_1 dZ_1 + h_2 dZ_2), \tag{49}$$

$$dF_t = \mu_F F_t dt + \sigma_F F_t dZ_1, \tag{50}$$

$$dD_t = \mu_D D_t dt + \sigma_D D_t dZ_2. \tag{51}$$

Let's specify the following indirect utility function. From the Bellman's principle of optimality,

$$J(W_t, D_t, F_t, t) = \max_{\alpha_t, C_t, l_t} E_t \left[\int_t^{t'} U(C_s, l_s)ds \right] \tag{52}$$

$$= \max_{\alpha_t, C_t, l_t} E_t \left[\int_t^{t+h} U(C_s, l_s)ds + J(W_{t+h}, D_{t+h}, F_{t+h}, t+h) \right]. \tag{53}$$

Applying *Ito's lemma*,

$$\begin{aligned} J(W_{t+h}, D_{t+h}, F_{t+h}, t+h) &= J(W_t, D_t, F_t, t) + \int_t^{t+h} DJ(\cdot, s)ds \\ &\quad + \int_t^{t+h} J_W W \alpha_t(h_1 dZ_1 + h_2 dZ_2) + \int_t^{t+h} J_F \sigma_F F dZ_1 + \int_t^{t+h} J_D \sigma_D D dZ_2, \end{aligned} \tag{54}$$

where DJ is a Dynkin's operator,

$$\begin{aligned} DJ(\cdot, t) &= J_t + J_W \left[W_t(\alpha_t(\mu - r) + r) + F_t(1 - l_t - m_t) - C_t\right] + J_D \mu_D D_t + J_F \mu_F F_t \\ &\quad + \frac{1}{2} J_{WW} W_t^2 (h_1^2 + h_2^2 + 2\rho h_1 h_2)\alpha_t^2 + \frac{1}{2} J_{DD} \sigma_D^2 D_t^2 + \frac{1}{2} J_{FF} \sigma_F^2 F_t^2 \\ &\quad + J_{WD} W_t D_t \sigma_D(\rho h_1 + h_2)\alpha_t + J_{WF} W_t F_t \sigma_F (h_1 + \rho h_2)\alpha_t + J_{DF} \rho \sigma_D \sigma_F D_t F_t. \end{aligned} \tag{55}$$

Substituting (54) into (53),

$$J(W_t, D_t, F_t, t) = \max_{\alpha_t, C_t, l_t} E_t \left[\int_t^{t+h} U(C_s, l_s) ds + J(W_t, D_t, F_t, t) + \int_t^{t+h} DJ(\cdot, s) ds \right. \tag{56}$$
$$\left. + \int_t^{t+h} J_W W \alpha_t (h_1 dZ_1 + h_2 dZ_2) + \int_t^{t+h} J_F \sigma_F F dZ_1 + \int_t^{t+h} J_D \sigma_D D dZ_2 \right]$$

Since $E_t \left[dZ_1, dZ_2 | W_t, D_t, F_t \right] = 0$,

$$0 = \max_{\alpha_t, C_t, l_t} E_t \left[\int_t^{t+h} U(C_s, l_s) ds + \int_t^{t+h} DJ(\cdot, s) ds \right] \tag{57}$$

Dividing by h and taking limits to zero,

$$0 = \max_{\alpha_t, C_t, l_t} \lim_{h \to 0} E_t \left[\frac{1}{h} \int_t^{t+h} U(C_t, l_t) ds + \frac{1}{h} \int_t^{t+h} DJ(\cdot, s) ds \right]. \tag{58}$$

Hence

$$0 = \max_{\alpha_t, C_t, l_t} \left[U(C_t, l_t) + DJ(\cdot, t) \right] \equiv \Psi_t, \tag{59}$$

which is equivalent to the well-known Bellman equation in the discrete time framework.
From $\max_{\alpha_t, C_t, l_t} \Psi$, we get FOCs.

$$\Psi_C = U_c - J_W \leq 0 \qquad \text{and} \qquad C \Psi_C = 0 \tag{60}$$
$$\Psi_s = U_l - J_W F_t = 0 \tag{61}$$
$$\Psi_\alpha = J_W \left[W_t (\mu - r) - F_t \frac{\partial m_t}{\partial \alpha_t} \right] + J_{WW} W_t^2 (h_1^2 + h_2^2 + 2\rho h_1 h_2)$$
$$+ J_{WD} W_t D_t \sigma_D (\rho h_1 + h_2) + J_{WF} W_t F_t \sigma_F (h_1 + \rho h_2) = 0 \tag{62}$$
$$\text{where} \qquad \frac{\partial m_t}{\partial \alpha_t} = \frac{\partial N_t}{\partial \alpha_t} \frac{\partial m_t}{\partial N_t} = \frac{W_t}{P_t} \frac{\partial m_t}{\partial N_t}$$

In equilibrium, risky asset has net positive supply, normalized to one. And consumption is the sum of dividend and labor income.

$$\alpha_t = N_t = 1$$
$$W_t = P_t$$
$$C_t = D_t + F_t (1 - l_t - m_t)$$

By imposing these equilibrium condition, the rate of return on the riskfree asset is given by

$$r = \mu - \frac{F_t}{P_t} \frac{\partial m_t}{\partial N_t} |_{N_t = 1} + \frac{J_{WW} W_t}{J_W} \left(h_1^2 + h_2^2 + 2\rho h_1 h_2 \right) + \frac{J_{WD} D_t}{J_W} \sigma_D (\rho h_1 + h_2) + \frac{J_{WF} F_t}{J_W} \sigma_F (h_1 + \rho h_2). \tag{63}$$

Examining the locally riskyless interest rate more carefully,

$$\begin{aligned}
\frac{\partial D(J)}{\partial W} &= J_{tW} + J_{WW}\left[\mu W_t + F_t(1-l_t-m_t) - C_t\right] + J_W\left[\mu - F_t\frac{\partial m_t}{\partial W_t}|_{\alpha_t=1}\right] \\
&+\frac{1}{2}J_{WWW}W_t^2(h_1^2+h_2^2+2\rho h_1h_2) + J_{WW}W_t(h_1^2+h_2^2+2\rho h_1h_2) + J_{WD}\mu_D D_t + J_{WF}\mu_F F_t \\
&+\frac{1}{2}J_{WDD}\sigma_D^2D_t^2 + \frac{1}{2}J_{WFF}\sigma_F^2F_t^2 + J_{WWD}W_tD_t\sigma_D(\rho h_1+h_2) + J_{WWF}W_tF_t\sigma_F(h_1+\rho h_2) \\
&+J_{WDF}\rho\sigma_D\sigma_F D_tF_t + J_{WD}D_t\sigma_D(\rho h_1+h_2) + J_{WF}F_t\sigma_F(h_1+\rho h_2) \\
&= 0, \qquad (64)
\end{aligned}$$

where

$$\frac{\partial m_t}{\partial W_t}|_{\alpha_t=1} = \frac{1}{P_t}\frac{\partial m_t}{\partial N_t}|_{N_t=1}. \qquad (65)$$

It's zero since $\frac{\partial \Psi}{\partial W} = \frac{\partial D(J)}{\partial W} = 0$, which is called as an envelope condition. By *Ito's Lemma*,

$$dJ_W = D(J_W)dt + J_{WW}W(h_1dZ_1 + h_2dZ_2) + J_{WF}\sigma_F F_t dZ_1 + J_{WD}\sigma_D D_t dZ_2, \qquad (66)$$

where

$$\begin{aligned}
D(J_W) &= J_{tW} + J_{WW}\left[\mu W_t + F_t(1-l_t-m_t) - C_t\right] + \frac{1}{2}J_{WWW}W_t^2(h_1^2+h_2^2+2\rho h_1h_2) \\
&+J_{WD}\mu_D D_t + J_{WF}\mu_F F_t + \frac{1}{2}J_{WDD}\sigma_D^2D_t^2 + \frac{1}{2}J_{WFF}\sigma_F^2F_t^2 + J_{WWD}W_tD_t\sigma_D(\rho h_1+h_2) \\
&+J_{WWF}W_tF_t\sigma_F(h_1+\rho h_2) + J_{WDF}\rho\sigma_D\sigma_F D_tF_t. \qquad (67)
\end{aligned}$$

From Equations (63), and (64),

$$\frac{\partial D(J)}{\partial W} = rJ_W + D(J_W) = 0. \qquad (68)$$

Thus

$$\frac{dJ_W}{J_W} = -rdt + \frac{J_{WW}W}{J_W}(h_1dZ_1 + h_2dZ_2) + \frac{J_{WF}}{J_W}\sigma_F F_t dZ_1 + \frac{J_{WD}}{J_W}\sigma_D D_t dZ_2 \qquad (69)$$

Further, Equation (63) and (69) give the following relationship.

$$\mu - r_f = Cov\left(\frac{dP_t}{P_t}, -\frac{dJ_W}{J_W}\right) + \frac{F_t}{P_t}\frac{\partial m_t}{\partial N_t}|_{N_t=1} \qquad (70)$$

Equation (70) can be expressed as follows,

$$\begin{aligned}
&\mu P_t - rP_t + Cov\left(dP, \frac{dJ_W}{J_W}\right) - F_t\frac{\partial m_t}{\partial N_t}|_{N_t=1} = 0 \\
&\mu P_t - D_t - rP_t + Cov\left(dP, \frac{dJ_W}{J_W}\right) + D_t - F_t\frac{\partial m_t}{\partial N_t}|_{N_t=1} = 0
\end{aligned}$$

$$
\begin{aligned}
&E_t\left[\frac{d(J_W P_t)}{J_W}\right]/dt + D_t - F_t\frac{\partial m_t}{\partial N_t}|_{N_t=1} = 0\\
&E_t\left[d(J_W P_t)\right] + J_W\left(D_t - F_t\frac{\partial m_t}{\partial N_t}|_{N_t=1}\right)dt = 0\\
&E_t\left[\int_0^h d(J_W(\cdot,t+s)P_{t+s})\right] + \int_0^h J_W(\cdot,t+s)\left(D_{t+s} - F_{t+s}\frac{\partial m_{t+s}}{\partial N_{t+s}}|_{N_{t+s}=1}\right)ds = 0\\
&-J_W(\cdot,t)P_t + E_t\left[\int_0^\infty J_W(\cdot,t+s)\left(D_{t+s} - F_{t+s}\frac{\partial m_{t+s}}{\partial N_{t+s}}|_{N_{t+s}=1}\right)ds\right] = 0
\end{aligned}
$$

Therefore

$$
P_t = E_t\left[\int_0^\infty \frac{J_W(\cdot,t+s)}{J_W(\cdot,t)}\left(D_{t+s} - F_{t+s}\frac{\partial m_{t+s}}{\partial N_{t+s}}|_{N_{t+s}=1}\right)ds\right] \tag{71}
$$

From the first order conditions (FOCs) in equilibrium,

$$
J_W = U_C
$$

Therefore, the risky asset price is

$$
P_t = E_t\left[\int_0^\infty \frac{U_C(C_{t+s},l_{t+s})}{U_C(C_t,l_t)}\left(D_{t+s} - F_{t+s}\frac{\partial m_{t+s}}{\partial N_{t+s}}|_{N_{t+s}=1}\right)ds\right] \tag{72}
$$

Substituting an investor's utility into above pricing equation, U_C and U_l is as follows.

$$
U_C(C_t,l_t) = \beta^{t-t_0}B'\gamma C_t^{\gamma(1-\delta)-1}l_t^{(1-\gamma)(1-\delta)}, \tag{73}
$$

$$
U_l(C_t,l_t) = \beta^{t-t_0}B'(1-\gamma)C_t^{\gamma(1-\delta)}l_t^{(1-\gamma)(1-\delta)-1}. \tag{74}
$$

At optimal condition, the following relation between leisure l_t and consumption C_t is always satisfied.

$$
\begin{aligned}
&\frac{U_C(C_t,l_t)}{U_l(C_t,l_t)} = \frac{1}{F_t},\\
&l_t = \frac{1-\gamma}{\gamma}\frac{C_t}{F_t}.
\end{aligned} \tag{75}
$$

Marginal rate of substitution of consumption between time t and time $t+s$

$$
\begin{aligned}
\frac{U_C(t+s)}{U_C(t)} &= \beta^s\left(\frac{C_{t+s}}{C_t}\right)^{\gamma(1-\delta)-1}\left(\frac{l_{t+s}}{l_t}\right)^{(1-\gamma)(1-\delta)}\\
&= \beta^s\left(\frac{C_{t+s}}{C_t}\right)^{-\delta}\left(\frac{F_t}{F_{t+s}}\right)^{(1-\gamma)(1-\delta)}
\end{aligned} \tag{76}
$$

Therefore, Standard Asset Pricing Formula (Euler Equation) is given by

$$
P_t = E_t\left[\int_0^\infty \beta^{s-t}\left(\frac{C_{t+s}}{C_t}\right)^{-\delta}\left(\frac{F_t}{F_{t+s}}\right)^{(1-\gamma)(1-\delta)}\left(D_{t+s} - F_{t+s}\frac{\partial m_{t+s}}{\partial N_{t+s}}|_{N_{t+s}=1}\right)ds\right]. \tag{77}
$$

Appendix B : The Proof of Proposition 2

From Proposition 1, the price of the risky asset is given by

$$
\begin{aligned}
P_t &= C_t E_t \left[\int_0^\infty \beta^s \left[\left(1 + \frac{\partial m}{\partial N} \frac{1}{1-m^*} \right) \phi_{t+s} - \frac{\partial m}{\partial N} \frac{1}{\gamma(1-m^*)} \right] ds \right] && (78) \\
&= C_t \left[1 + \frac{\partial m}{\partial N} \frac{1}{1-m^*} \right] E_t \left[\int_0^\infty \beta^s \phi_{t+s} ds \right] + C_t \cdot \frac{1}{\ln \beta} \cdot \frac{\partial m}{\partial N} \cdot \frac{1}{\gamma(1-m^*)}, && (79)
\end{aligned}
$$

where the representative agent has a log utility.

Let $\Lambda_t = \frac{F_t}{D_t}$. Then the process of full labor income to dividend ratio is given by

$$
\begin{aligned}
d\Lambda_t / \Lambda_t &= \left(\mu_F - \mu_D + \sigma_D^2 - \rho \sigma_F \sigma_D \right) dt + \sigma_F dZ_1 - \sigma_D dZ_2, && (80) \\
d \ln(\Lambda_t) &= \left(\mu_F - \mu_D - \frac{1}{2} \left(\sigma_F^2 - \sigma_D^2 \right) \right) dt + \sigma_F dZ_1 - \sigma_D dZ_2. && (81)
\end{aligned}
$$

Since

$$
\begin{aligned}
C_t &= D_t + F_t(1 - l_t - m^*), \\
l_t &= \frac{1-\gamma}{\gamma} \frac{C_t}{F_t} \quad \text{(from FOC)},
\end{aligned}
$$

then

$$
C_t = \gamma \left(D_t + F_t(1-m^*) \right).
$$

The expectation term in the price function is given by

$$
E_t \left[\int_0^\infty \beta^s \phi_{t+s} ds \right] = E_t \left[\int_0^\infty \beta^s \frac{1}{\gamma \left(1 + \Lambda_t \exp(u)(1-m^*) \right)} ds \right] \qquad (82)
$$

where Λ_t is the initial full labor income to dividend ratio; u is a normally distributed random variable with mean $\nu\tau$ and variance $\eta^2\tau$, where

$$
\begin{aligned}
\nu &= \mu_F - \mu_D - \frac{1}{2} \left(\sigma_F^2 - \sigma_D^2 \right) && (83) \\
\eta &= \sigma_F^2 + \sigma_D^2 - 2\rho\sigma_F\sigma_D. && (84)
\end{aligned}
$$

Note that $\nu dt = E_t[d \ln \Lambda_t]$ and $\eta^2 dt = \mathrm{Var}[d \ln \Lambda_t]$. Introducing the density for u into the last integral gives

$$
\begin{aligned}
& E_t \left[\int_0^\infty \beta^s \phi_{t+s} ds \right] && (85) \\
&= \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi\eta^2}} \frac{1}{\gamma(1 + \Lambda_t \exp(u)(1-m^*))} \exp\left(\frac{\nu u}{\eta^2} \right) \int_0^\infty s^{-\frac{1}{2}} \exp\left(-\frac{u^2}{2\eta^2 s} - \frac{\nu^2 - 2\ln\beta\eta^2}{2\eta^2} s \right) ds du. && (86)
\end{aligned}
$$

From Equation (3.471.9) of Gradshteyn and Ryzhik (2000), this expression becomes,

$$
= \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi\eta^2}} \frac{1}{\gamma(1 + \Lambda_t \exp(u)(1-m^*))} \exp\left(\frac{\nu u}{\eta^2} \right) \left(\frac{u^2}{\nu^2 - 2\ln\beta\eta^2} \right)^{1/4} K_{1/2} \left(2\sqrt{\frac{u^2(\nu^2 - 2\ln\beta\eta^2)}{4\eta^4}} \right) du \quad (87)
$$

where $K_{1/2}(\cdot)$ is the modified Bessel function of order 1/2 (see Abramowitz and Stegum (1970) Chapter 9). From the identity relations for Bessel functions of order equal to an integer plus one half given in Gradshteyn and Ryzhik Equation (8.469.3) ($K_{\pm 1/2}(z) = \sqrt{\frac{\pi}{2z}}\exp(-z)$), however, the above expression can be expressed as,

$$E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right] = \frac{1}{\psi\gamma}\int_{-\infty}^{\infty} \frac{1}{1+\Lambda_t \exp(u)(1-m^*)} \exp\left(\frac{\nu u}{\eta^2}\right) \exp\left(-\frac{\psi}{\eta^2}|u|\right) du \tag{88}$$

where $\psi = \sqrt{\nu^2 - 2\ln\beta\eta^2}$.

In turn, above expression can be written

$$\begin{aligned} E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right] &= \frac{1}{\psi\gamma}\int_0^\infty \frac{1}{1+\Lambda_t \exp(u)(1-m^*)} \exp(\xi u)\, du \\ &+ \frac{1}{\psi\gamma}\int_{-\infty}^0 \frac{1}{1+\Lambda_t \exp(u)(1-m^*)} \exp(\theta u)\, du \end{aligned} \tag{89}$$

where

$$\xi = \frac{\nu-\psi}{\eta^2} \quad \text{and} \quad \theta = \frac{\nu+\psi}{\eta^2}.$$

Define $\omega_1 = \exp(-u)$ and $\omega_2 = \exp(u)$ respectively. By a change of variables, Equation can be written

$$\begin{aligned} E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right] &= \frac{1}{\psi\gamma\Lambda_t(1-m^*)}\int_0^1 \frac{1}{1+\omega_1/(\Lambda_t(1-m^*))}\omega_1^{-\xi} d\omega_1 \\ &+ \frac{1}{\psi\gamma}\int_0^1 \frac{1}{1+\Lambda_t(1-m^*)\omega_2}\omega_2^{\theta-1} d\omega_2, \end{aligned} \tag{90}$$

From Abramowitz and Stegum Equation (15.3.1), this expression becomes

$$\begin{aligned} E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right] &= \frac{1}{\psi\gamma\Lambda_t(1-m^*)(1-\xi)} F\left(1, 1-\xi; 2-\xi; -\frac{1}{\Lambda_t(1-m^*)}\right) \\ &+ \frac{1}{\psi\gamma\theta} F\left(1, \theta; 1+\theta; -\Lambda_t(1-m^*)\right) \end{aligned} \tag{91}$$

$F(\alpha, \beta; \gamma; z)$ is the standard hypergeometric function (see Abramowitz and Stegum (1970) Chapter 15). The hypergeometric function has an integral representation, which can be used for numerical evaluation and as an analytic continuation beyond $\|z\| < 1$,

$$F(\alpha, \beta; \gamma; z) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 w^{\beta-1}(1-w)^{\gamma-\beta-1}(1-wz)^{-\alpha} dw; \quad Re(\gamma) > Re(\beta) > 0. \tag{92}$$

The derivative of the hypergeometric function, needed for Ito's lemma calculation, has the simple form

$$\frac{d}{dz}F(\alpha, \beta; \gamma; z) = \frac{\alpha\beta}{\gamma}F(\alpha+1, \beta+1; \gamma+1; z). \tag{93}$$

Since $\phi = \frac{1}{\gamma[1+\Lambda(1-m^*)]}$, then $\Lambda = \frac{1}{1-m^*}\left(\frac{1}{\gamma\phi} - 1\right)$. Therefore,

$$\begin{aligned} E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right] &= \frac{1}{\psi\gamma(1-\xi)}\left(\frac{\gamma\phi_t}{1-\gamma\phi_t}\right) F\left(1, 1-\xi; 2-\xi; -\frac{\gamma\phi_t}{1-\gamma\phi_t}\right) \\ &+ \frac{1}{\psi\gamma\theta} F\left(1, \theta; 1+\theta; -\frac{1-\gamma\phi_t}{\gamma\phi_t}\right) \end{aligned} \tag{94}$$

where

$$\phi_t = D_t/C_t$$

Define $E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right]$ as $M(t)$. Finally, then, asset price is as follows.

$$\begin{aligned} P_t &= C_t\left(1+\frac{1}{1-m^*}\frac{\partial m}{\partial N}\right)M(t)+C_t\frac{1}{\ln\beta}\frac{\partial m}{\partial N}\frac{1}{\gamma(1-m^*)} \\ &= C_t\left[\left(1+\frac{1}{1-m^*}\frac{\partial m}{\partial N}\right)M(t)+\frac{1}{\ln\beta}\frac{\partial m}{\partial N}\frac{1}{\gamma(1-m^*)}\right] \end{aligned} \tag{95}$$

where

$$M(t) = \frac{1}{\psi\gamma(1-\xi)}\left(\frac{\gamma\phi_t}{1-\gamma\phi_t}\right)F\left(1,1-\xi;2-\xi;-\frac{\gamma\phi_t}{1-\gamma\phi_t}\right)+\frac{1}{\psi\gamma\theta}F\left(1,\theta;1+\theta;-\frac{1-\gamma\phi_t}{\gamma\phi_t}\right)$$

Appendix C : The proof of Proposition 3

The price of the risky asset with the time required for risky investment is given by

$$P_t = C_t E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right] \tag{96}$$

$$= C_t\frac{1}{\psi\gamma}\int_{-\infty}^{\infty}\frac{1}{1+\Lambda_t\exp(u)}\exp\left(\frac{\nu u}{\eta^2}\right)\exp\left(-\frac{\psi}{\eta^2}|u|\right)du \tag{97}$$

where $\psi = \sqrt{\nu^2 - 2\ln\beta\eta^2}$.

Similar with time costs (the opportunity cost) case, this expression becomes

$$P_t = C_t E_t\left[\int_0^\infty \beta^s \phi_{t+s} ds\right] \tag{98}$$

$$= C_t\left[\frac{1}{\psi\gamma\Lambda_t(1-\xi)}F\left(1,1-\xi;2-\xi;-\frac{1}{\Lambda_t}\right)+\frac{1}{\psi\gamma\theta}F\left(1,\theta;1+\theta;-\Lambda_t\right)\right] \tag{99}$$

Since $\phi = \frac{1}{\gamma[1+\Lambda]}$, then $\Lambda = \frac{1}{\gamma\phi} - 1$. Therefore,

$$P_t = C_t\left[\frac{1}{\psi\gamma(1-\xi)}\left(\frac{\gamma\phi_t}{1-\gamma\phi_t}\right)F\left(1,1-\xi;2-\xi;-\frac{\gamma\phi_t}{1-\gamma\phi_t}\right)+\frac{1}{\psi\gamma\theta}F\left(1,\theta;1+\theta;-\frac{1-\gamma\phi_t}{\gamma\phi_t}\right)\right] \tag{100}$$

References

[1] Abel, A., 1990, "Asset Prices under Habit Formation and Catching up with the Joneses," *American Economic Review*, 80, 38-42.

[2] Ahn, D., and I. Kim and S. Yoon, 2006, "Time Costs of Risky Asset Management: Life-Cycle Portfolio Choice and Limited Participation," *Working Paper.*

[3] Barberis, N., M. Huang and T. Santos, 2001, "Prospect Theory and Asset Prices," *Quaterly Journal of Economics*, 116, 1-54.

[4] Basak, S., 1999, "On the Fluctuations in Consumption and Market Returns in the presence of Labor and Human Capital: An Equilibrium Analysis," *Journal of Economic Dynamics and Control*, 23, 1029-1064.

[5] Basak, S. and D. Couco, 1998, "An Equilibrium Model with Restricted Stock Market Participation," *Review of Financial Studies*, 11, 771-792.

[6] Becker, G., 1965, "A Theory of the Allocation of Time," *Economic Journal*, 75, 493-517.

[7] Bodie, Z., C. Merton and W. Samuelson, 1992, "Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model," *Journal of Economic Dynamics and Control*, 16, 427-449.

[8] Breeden, D., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7, 265-296.

[9] Campbell, J., 1999, "Asset Prices, Consumption, and the Business Cycle," in *Handbooks in Economics*, 15, 1231-1304.

[10] Campbell, J., 2000, "Asset Pricing at the Millennium," *Journal of Finance*, 55, 1515-1567.

[11] Campbell, J. and J. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205-251.

[12] Campbell, J. and R. Shiller, 1988, "Stock Prices, Earnings, and Expected Dividends," *Journal of Finance*, 43, 661-676.

[13] Chan, Y., and L. Viceira, 2000, "Asset Allocation with Endogenous Labor Income: The Case of Incomplete Markets," working Paper.

[14] Cochrane, J., 2000, "New Facts in Finance," *Economic Perspectives*, Federal Reserve Bank of Chicago.

[15] Cochrane, J., F. Longstaff, and P. Santa-Clara, 2007, "Two Trees," *Review of Financial Studies*, forthcoming.

[16] Constantinides, G., 1986, "Capital Market Equilibrium with Transaction Costs," *Journal of Political Economy*, 94, 842-862.

[17] Constantinides, G., 1990, "Habit Formation: A Resolution of the Equity premium Puzzle," *Journal of Political Economy*, 98, 519-543.

[18] Danthine, J., and J. Donaldson, 2002, "Labour Relations and Asset Returns," *Review of Economic Studies*, 69, 41-64.

[19] Davis, M. and Norman, A., 1990, "Portfolio Selection with Transaction Costs," *Mathematics of Operations Research*, 15, 676-713.

[20] Eichenbaum, M., L. Hansen, and K. Singleton, 1988, "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty," *Quarterly Journal of Economics*, 103, 51-78.

[21] Epstein, L., S. Zin, 1989, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937-969.

[22] Gomes, F. and A. Michaelides, 2005, "Asset Pricing with Limited Risk Sharing and Heterogeneous Agents," *Working Paper*, London Business School.

[23] Gronau, R., 1977, "Leisure, Home Production, and Work-the Theory of the Allocation of Time Revisited," *Journal of Political Economy*, 85, 1099-1124.

[24] Hansen, L., and K. Singleton, 1982, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica*, 50, 1269-1286.

[25] Hansen, L., and R. Jagannathan, 1991, "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy*, 99, 225-262.

[26] Heckman,J., 1974, "Shadow Prices, Market Wages, and Labor Supply," *Econometrica*, 42, 679-694.

[27] Jagannathan, R., and Z. Wang, 1996, "The Conditional CAPM and the Cross-Section of Expected Returns," *Journal of Finance*, 51, 3-53.

[28] Kandel, S. and R. Stambaugh, 1996, "On the Predictability of Stock Returns: An Asset-Allocation Perspective," *Journal of Finance*, 51, 385-424.

[29] Keim, D. and R. Stambaugh, 1986, "Predicting Returns in the Stock and Bond Markets," *Journal of Financial Economics*, 17, 357-390.

[30] Lettau, M. and S. Ludvigson, 2001a, "Consumption, Aggregate Wealth, and Expected Returns," *Journal of Finance*, 55, 815-849.

[31] Lettau, M. and S. Ludvigson, 2001b, "Resurrecting the CCAPM: A Cross-Sectional Test of Linear Factor Models When Risk Premia are Time-Varying," *Journal of Political Economy*, 109, 1238-1287.

[32] Lettau, M. and S. Ludvigson, 2004, "Understanding Trend and Cycle in Asset Values: Reevaluating the Wealth Effect on Consumption," *American Economic Review*, 94, 276-299.

[33] Lucas, R., 1978, "Asset Prices in the Exchange Economy," *Econometrica*, 46, 1429-1446.

[34] Mankiw, N., S. Zeldes, 1991, "The Consumption of Stockholders and Non-Stockholders," *Journal of Financial Economics*, 29, 97-112.

[35] Mehra, R. and E. Prescott, 1985, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145-161.

[36] Menzly, L., T. Santos, and P. Veronesi, 2004, "Understanding Predictability," *Journal of Political Economy*, 112, 1-47.

[37] Merton, R., 1971, "Optimal Consumption and Portfolio Rules in a Continuous Time Model," *Journal of Economic Theory*, 3, 373-413.

[38] Merton, R., 1973, "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41, 867-887.

[39] Merton, R., 1992, *Continuous-Time Finance*, Blackwell Publishers, Cambridge, MA

[40] Prescott, M., 1986, "Theory Ahead of Business Cycle Measurement," *Carnegie-Rochester Conference Series on Public Policy*, 25, 11-44.

[41] Santos, T. and P. Veronesi, 2006, "Labor Income and Predictable Stock Returns," *Review of Financial Studies*, 19, 1-44.

[42] Shelley, K., 2005, "Developing the American Time Use Survey activity classification system." *Monthly Labor Review*, June 2005, 3-15.

[43] Shiller, R., 1981, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividend?" *American Economic Review*, 80, 58-62.

[44] Shiller, R., 1990, "Market Volatility and Investor Behavior," *American Economic Review*, 80, 58-62.

[45] U.S. Department of Labor, U.S. Bureau of Labor Statistics, 2005, "American Time Use Survey-2004 Results Announced by BLS," *American Time Use Survey(ATUS).*

[46] Vissing-Jorgensen, A., 2002a, "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, 110, 825-853.

[47] Vissing-Jorgensen, A., 2002b, "Towards an Explaination of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structure," *Working Paper*, Northwestern University.

[48] Weil, P., 1989, "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics*, 24, 401-421.

Table 1: Parameter Values

Common Parameters	Value
Discount factor: β	0.96
Constant Relative Risk Aversion (CRRA) parameter: δ	1 (log utility)
Cobb-Douglas parameter: γ	0.57
Expected Dividend Growth Rate: μ_D	1.5%
Dividend Volatility: σ_D	10%
Expected Full Labor Income Growth Rate: μ_F	1.5%
Full Labor Income Volatility: σ_F	3%
Correlation b/w Dividend and Full Labor Income: ρ	0
Time Required for Risky Investment in Equilibrium : m^*	0.035
Index of Time Cost Structure: ξ	2

Table 1 presents the values of parameters adopted for comparing to empirical data in the stock market. Most parameter values are consistent with those set at Barberis, Huang, and Santos (2001) except the values related to full labor income. The parameters about full labor income, in particular the volatility of full labor income, will be later investigated.

Table 2: Our Model: Results with respect to Dividend/Full Labor Income, D_t/F_t **With Time Costs of Risky Investment**

Dividend/Full Labor Income, D_t/F_t	1/9	1/8	1/7	1/5	1/3	1	Empirical Value
Leisure Time, l_t	46.27%	46.87%	47.64%	50.10%	55.83%	84.49%	50%
Labor Time, $\jmath_t$	50.23%	49.63%	48.86%	46.40%	40.67%	12.00%	50%
Dividend to Consumption rate, D_t/C_t	0.1811	0.2012	0.2262	0.3012	0.4504	0.8928	from 0.2 to 0.25
Expected Consumption Growth Rate, μ_C	1.50%	1.50%	1.50%	1.50%	1.50%	1.50%	1.84%
Consumption Growth Rate Volatility, σ_C	2.88%	2.89%	2.91%	3.02%	3.40%	5.30%	3.79%
Risk-free Rate, $r_{f,t}$	5.50%	5.50%	5.50%	5.49%	5.47%	5.30%	from 2 to 6%
Excess Return Mean, $E[r_t - r_{f,t}]$	7.27%	5.50%	4.42%	2.50%	1.44%	0.87%	6.03%
Excess Return Std., $std[r_t - r_{f,t}]$	27.31%	22.85%	19.61%	15.25%	12.49%	10.76%	20.2%
Price-Dividend Ratio Mean, $E[P_t/D_t]$	8.8361	10.4843	12.1205	15.3414	18.4518	21.4658	25.5

where $m_t = m^* \cdot N_t^{\xi}$, $m^* = 0.035$, $\xi = 2$, $\beta = 0.96$, δ(CRRA) $= 1$, γ(Cobb-Douglas) $= 0.57$, $\mu_D = 0.015$, $\sigma_D = 0.10$, $\mu_F = 0.015$, $\sigma_F = 0$, $\rho_{D,F} = 0$

Table 3: Benchmark I: Results with respect to Dividend/Full Labor Income, D_t/F_t **WithOut Time Costs of Risky Investment**

Dividend/Full Labor Income, D_t/F_t	1/9	1/8	1/7	1/5	1/3	1	Empirical Value
Leisure Time, l_t	47.78%	48.38%	49.14%	51.60%	57.33%	86.00%	50%
Labor Time, $\jmath_t$	52.22%	51.62%	50.86%	48.40%	42.67%	14.0%	50%
Dividend to Consumption rate, D_t/C_t	**0.1754**	**0.1949**	**0.2193**	**0.2924**	**0.4386**	**0.8772**	from 0.2 to 0.25
Expected Consumption Growth Rate, μ_C	1.50%	1.50%	1.50%	1.50%	1.50%	1.50%	1.84%
Consumption Growth Rate Volatility, σ_C	2.88%	2.89%	2.91%	3.00%	3.36%	5.22%	3.79%
Risk-free Rate, $r_{f,t}$	5.50%	5.50%	5.50%	5.49%	5.47%	5.31%	from 2 to 6%
Excess Return Mean, $E[r_t - r_{f,t}]$	0.10%	0.11%	0.12%	0.16%	0.25%	0.50%	6.03%
Excess Return Std., $std[r_t - r_{f,t}]$	9.79%	9.78%	9.77%	9.75%	9.76%	9.96%	20.2%
Price-Dividend Ratio Mean, $E[P_t/D_t]$	24.3026	24.2413	24.1688	23.9755	23.6768	23.2728	25.5

where $m_t = 0$, $\beta = 0.96$, δ(CRRA) $= 1$, γ(Cobb-Douglas) $= 0.57$, $\mu_D = 0.015$, $\sigma_D = 0.10$, $\mu_F = 0.015$, $\sigma_F = 0.03$, $\rho_{D,F} = 0$

Table 4: Benchmark II (when labor income is exogenous): Results with respect to Dividend/Exogenous Labor Income, D_t/F_t

Dividend/Exogenous Labor Income, D_t/F_t	1/4.70	1/4.13	1/3.56	1/2.42	1/1.28	1/0.14	Empirical Value
Leisure Time, l_t	·	·	·	·	·	·	50%
Labor Time, $\jmath_t$	·	·	·	·	·	·	50%
Dividend to Consumption rate, D_t/C_t	**0.1754**	**0.1949**	**0.2193**	**0.2924**	**0.4386**	**0.8772**	from 0.2 to 0.25
Expected Consumption Growth Rate, μ_C	1.50%	1.50%	1.50%	1.50%	1.50%	1.50%	1.84%
Consumption Growth Rate Volatility, σ_C	2.88%	2.89%	2.91%	3.00%	3.36%	5.22%	3.79%
Risk-free Rate, $r_{f,t}$	5.49%	5.49%	5.48%	5.45%	5.36%	4.81%	from 2 to 6%
Excess Return Mean, $E[r_t - r_{f,t}]$	0.17%	0.19%	0.22%	0.29%	0.43%	0.89%	6.03%
Excess Return Std., $std[r_t - r_{f,t}]$	9.75%	9.75%	9.75%	9.78%	9.90%	10.17%	20.2%
Price-Dividend Ratio Mean, $E[P_t/D_t]$	23.9390	23.8623	23.7749	23.5614	23.3147	23.8297	25.5

where $m_t = 0$, $\beta = 0.96$, $\delta(\text{CRRA}) = 1$, $\mu_D = 0.015$, $\sigma_D = 0.10$, $\mu_F = 0.015$, $\sigma_F = 0.03$, $\rho_{D,F} = 0$

Figure 1: The Relation b/w Dividend to Full-Labor-Income ratio and Return Dynamics

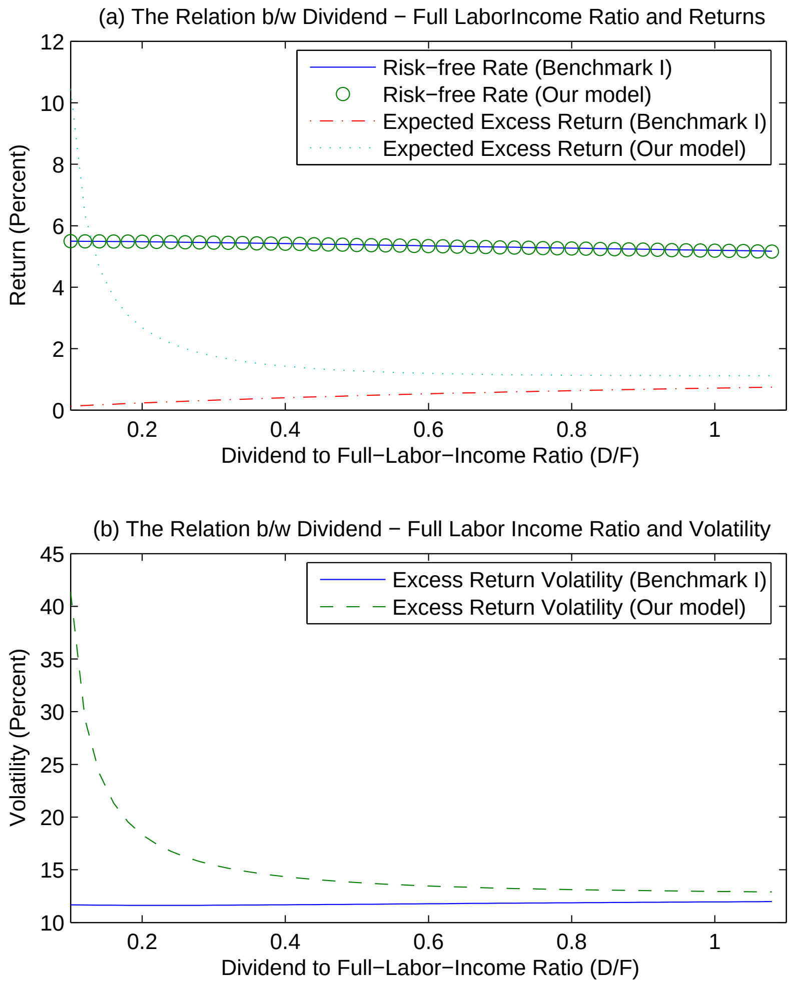

Figure 1 illustrates the relationship between dividend to full-labor-income ratio and return dynamics. Figure 1-(a) shows the impact on expected return whereas Figure 1-(b) shows volatility impacts. In Our model, an increase in D_t/F_t decreases the excess return and the volatility of risky asset, whereas in Benchmark I, an increase in D_t/F_t increases them. This difference reflects the coexistence of two contrary effects: (i) the opportunity cost effect (ii) the consumption composition effect. Given a fixed amount of time m^*, the opportunity cost increases (decreases) in the level of F_t $(1/F_t)$, thereby increasing the risky return and volatility. In contrast, as F_t increases relatively, consumption (or pricing kernel) are less correlated with a dividend stream and less volatile under $\sigma_F < \sigma_D$, thereby decreasing the risky return and volatility. In Our model, both effects coexist but the first effect dominates, whereas in Benchmark I, only second effect exists. In addition, the risk-free rates in our model and Benchmark I are nearly constant over entire range of D_t/F_t, and are same regardless of time cost m^*. The used parameters are as follows: $m^* = 0.035$, $\xi = 2$, $\beta = 0.96$, $\delta(\text{CRRA}) = 1$, $\gamma(\text{Cobb-Douglas}) = 0.57$, $\mu_D = 0.015$, $\sigma_D = 0.12$, $\mu_F = 0.015$, $\sigma_F = 0.03$, $\rho_{D,F} = 0$.

Figure 2: The Relation b/w Time Costs of Risky Investment and Return Dynamics

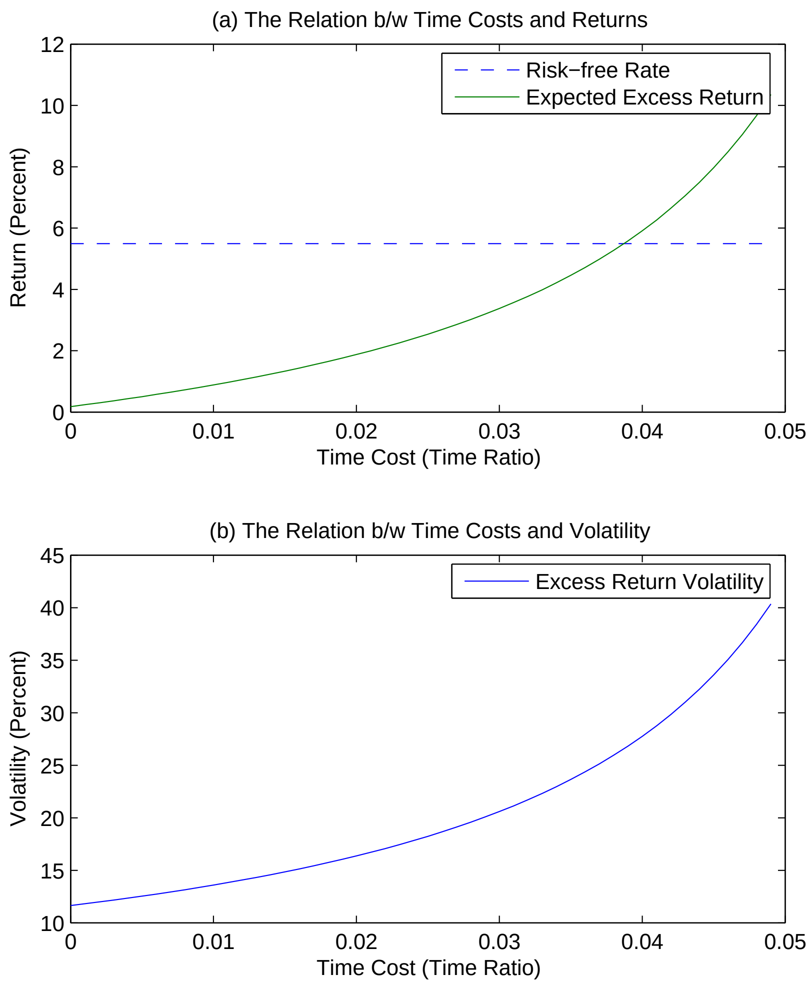

Figure 2 illustrates the relationship between the amount of time costs required for investing in a risky asset and return dynamics. Figure 2-(a) shows the impact on expected return whereas Figure 2-(b) shows volatility impacts. The opportunity cost which dominates in our model are positively related with the amount of time required for risky investment, m^*. Figure 2 (a) and (b) show such effects on the risky return and volatility. On the other hand, the risk-free return is not affected by the level of time cost. The used parameters are as follows: $\xi = 2$, $\beta = 0.96$, $\delta(\text{CRRA}) = 1$, $\gamma(\text{Cobb-Douglas}) = 0.57$, $\mu_D = 0.015$, $\sigma_D = 0.12$, $\mu_F = 0.015$, $\sigma_F = 0.03$, $\rho_{D,F} = 0$, $D_t : F_t = 1 : 8$.

Figure 3: The Relation b/w Full-Labor-Income Volatility and Excess Return Dynamics

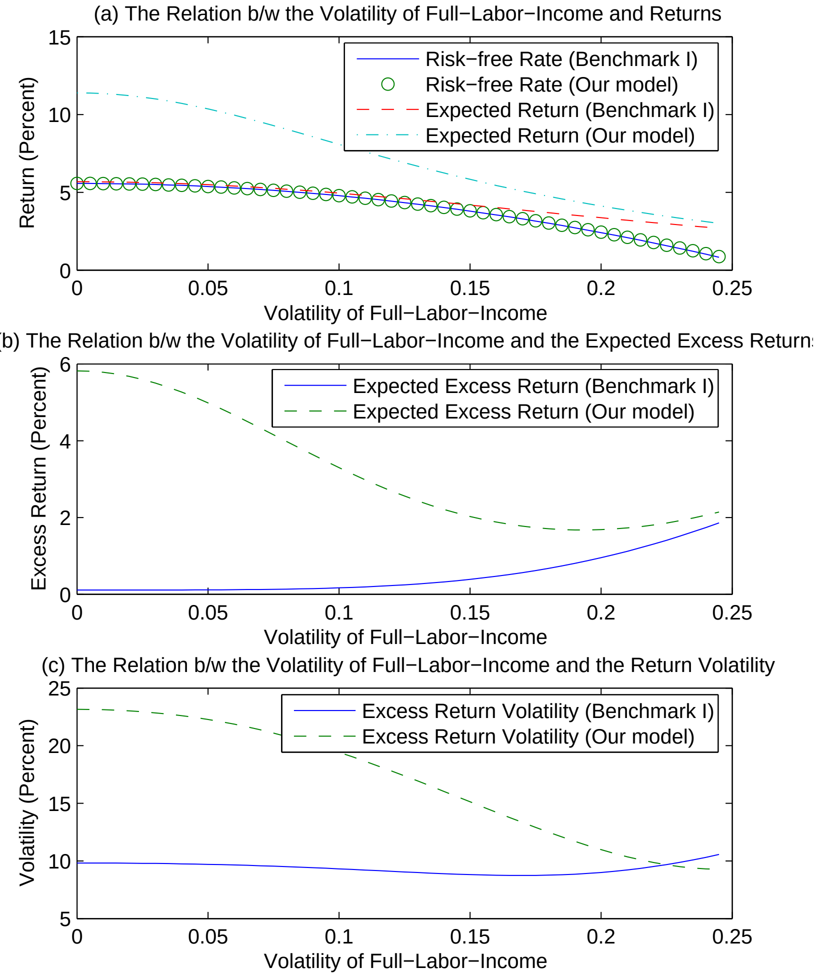

Figure 3 illustrates the relationship between the volatility of full labor income and return dynamics. Figure 3-(a) shows the change on expected return, Figure 3-(b) shows the change of excess return, and figure 3-(c) shows the change of volatility. Since an increase of σ_F results in an increase of σ_C, the risk-free rate decreases in the level of σ_F regardless of the presence of time cost. An increase of σ_F weakens the opportunity cost effect, and thus increases the demand for a risky asset. Additionally, a change of σ_F results in the change of correlation between consumption and the process of the risky asset price, as shown on the line of Benchmark I in Figure 3-(a). Since the opportunity cost effect influences only Our model whereas the correlation effect influences both Our model and Benchmark I, the difference of excess returns between Our model and Benchmark I in figure 3-(b) reflects the opportunity cost effect, whereas the excess return of Benchmark I reflects the correlation effect with consumption. Similarly, the volatility change in Figure 3-(c) reflects the effect of opportunity cost and correlation effect. In particular, at the high level of σ_F the high volatility in Our model is induced by a fall in the risky asset price. The used parameters are as follows: $m^* = 0.035$, $\xi = 2$, $\beta = 0.96$, $\delta(\text{CRRA}) = 1$, $\gamma(\text{Cobb-Douglas}) = 0.57$, $\mu_D = 0.015$, $\sigma_D = 0.10$, $\mu_F = 0.015$, $\rho_{D,F} = 0$, $D_t : F_t = 1 : 8$.

Figure 4: The Correlation b/w Dividend and Full-Labor-Income, and Its Effect on Return Dynamics

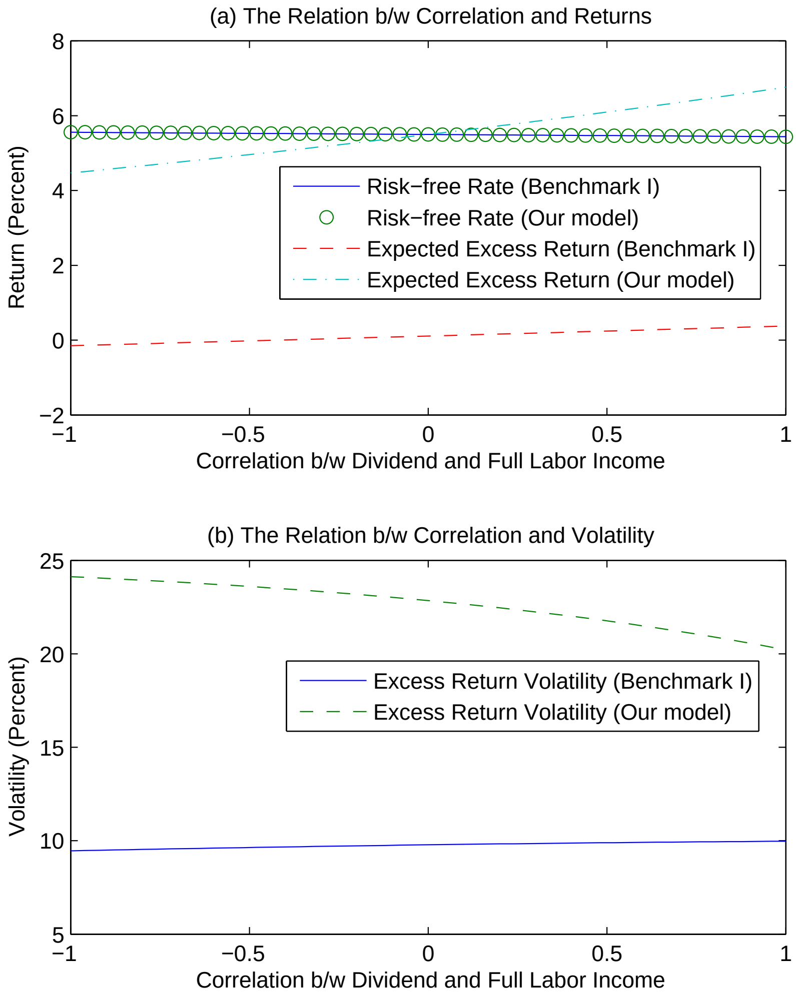

Figure 4 illustrates the relationship between the correlation among dividend process and full labor income process, and volatility of full labor income and return dynamics. Figure 4-(a) shows the impact on expected return whereas figure 4-(b) shows volatility impacts. In Our model, the expected return increases in the level of ρ both in Our model and Benchmark I. The high ρ induces not only the change of pseudo payoff, $D - F\partial m/\partial N$, as well as a high correlation between a dividend stream and consumption, and thus makes changes on risky asset returns. Particularly shown in our model, as ρ increases, the volatility of pseudo payoff decreases, which results in an decrease on the volatility of the risky asset. In contrast, in Benchmark I, an increment of ρ increases the volatilities of consumption (pricing kernel) and risky asset return. The used parameters are as follows: $m^* = 0.035$, $\xi = 2$, $\beta = 0.96$, δ(CRRA) $= 1$, γ(Cobb-Douglas) $= 0.57$, $\mu_D = 0.015$, $\sigma_D = 0.10$, $\mu_F = 0.015$, $\sigma_F = 0.03$, $D_t : F_t = 1 : 8$.

Figure 5: The Relation b/w Preference to Consumption, γ, and Return Dynamics

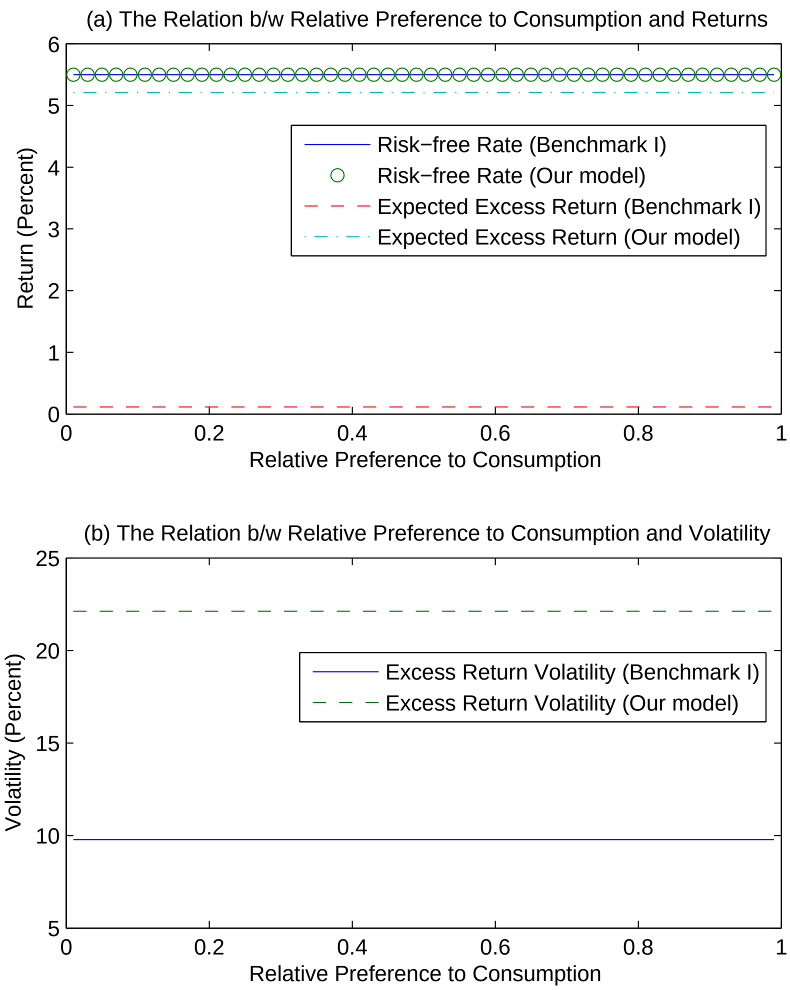

Figure 5 illustrates the relationship between preference to consumption γ and return dynamics. Figure 5-(a) shows the impact on expected return whereas figure 5-(b) shows volatility impacts. Figure 5 demonstrates the irrelevance of γ with the return dynamics. Since the labor productivity is fixed as F_t at time t, the marginal utility of other activities should follow up to the labor's in equilibrium. The change of γ governs only time allocated to leisure. The used parameters are as follows: $m^* = 0.035$, $\xi = 2$, $\beta = 0.96$, $\delta(\text{CRRA}) = 1$, $\mu_D = 0.015$, $\sigma_D = 0.10$, $\mu_F = 0.015$, $\sigma_F = 0.03$, $\rho_{D,F} = 0$, $D_t : F_t = 1 : 8$.