Dynamic Factors and Asset Pricing

Zhongzhi (Lawrence) He[a]

Sahn-Wook Huh[a]

Bong-Soo Lee[b, *]

[Current Version: April 3, 2008]

[a]Faculty of Business, Brock University, St. Catharines, Ontario, Canada L2S 3A1.

[b]College of Business, Florida State University, Tallahassee, Florida 32306, USA. E-mail: blee2@cob.fsu.edu.

[*]*Corresponding Author.*

We gratefully appreciate the constructive and thoughtful comments of an anonymous referee, Stephen Brown (the editor), Michael Brennan, and Amit Goyal, which have substantially contributed to improving the paper. Our special thanks go to Chang-Jin Kim, who provided us with the GAUSS routines described in Kim and Nelson (1999). We also thank Antonio Bernardo, Robert Grauer, Avanidhar Subrahmanyam, Kevin Wang, and seminar participants at Brock University for valuable feedback. Huh gratefully acknowledges the generous financial support from the Social Sciences and Humanities Research Council of Canada (SSHRC). All errors are solely the authors' responsibility.

Dynamic Factors and Asset Pricing

ABSTRACT

In this study, we develop a dynamic factor model that incorporates features of price dynamics across assets as well as through time. With the dynamic factors extracted via the Kalman filter, we formulate two testable asset-pricing models: the risk-adjusted pricing model (RAPM) and the bias-adjusted pricing model (BAPM). We then conduct asset-pricing tests in the in-sample context. In addition, we perform out-of-sample tests for competing models, presenting pair-wise comparisons of the accuracy in one-step-ahead forecasts. We provide evidence that the *ex post* dynamic factors alone do a better job than the Fama-French (FF, 1993) three factors both in-sample and out-of-sample. Our analyses also demonstrate that the *ex ante* factors are a key component in asset pricing and forecasting. By employing the *ex ante* factors together with *ex post* ones, the BAPM further improves upon the explanatory and predictive power achieved by the naive benchmark, the FF 3-factor model, and the RAPM. In particular, the BAPM can even explain and better forecast the momentum portfolio returns, which are mostly missed by the FF 3-factor model.

Since the preeminence of the capital asset pricing model (CAPM) was challenged by many studies, conditional versions of the CAPM with time-varying parameters or the arbitrage pricing theory (APT) were proposed as alternative models.[1] Given the fact that the explanatory power of these models was not satisfactory, yet another series of alternative models was proposed, and the most prominent of them is Fama and French's (1993, hereafter "FF") three-factor model. After the publication of their study, there has been enduring controversy over the effectiveness of this model. For example, Ferson and Harvey (1999) test its empirical performance, concluding that the FF 3-factor model is rejected as a pricing model. Other recent studies (Stambaugh, 1999; Ferson, Sarkissian and Simin, 2003) also find that the forecasting ability of some macroeconomic variables (especially dividend yield) may be spurious. Moreover, Simin (2006) shows that existing conditional and unconditional asset-pricing models produce very poor one-step-ahead forecasts compared to a simple benchmark. His result is intriguing because it implies that existing asset-pricing models provide little guidance to investment practitioners, to whom acquiring more accurate forecasts in future asset returns is of primary concern.

Recognizing the issues mentioned above, in this study we propose a new approach to identifying factors that can incorporate features of price dynamics across assets as well as through time. The controversy notwithstanding, we cannot deny the fact that the FF 3-factor model has been the most popular benchmark in the modern empirical asset pricing literature. Therefore, we start with the same data sets as the FF 3-factor model uses, attempting to develop an econometric method that can extract more informative risk factors, which in turn produce better in-sample pricing performance as well as more accurate step-ahead forecasts than the FF 3 factors. Specifically, in the spirit of Stock and Watson (1989, 1991), we employ the Kalman filter to estimate unobservable state variables: i.e., the *ex ante* factors that capture the time-varying expected factor premia, and the *ex post* factors that are formed as a linear combination of a predictable component and an unpredictable forecast error. Given that as an adaptive process the Kalman filter repeats a sequence of forecasting and updating as the information set develops, we refer to our model as a "dynamic" factor model.

The dynamic factor model has both cross-sectional and time-series features that distinguish this model from the FF 3-factor model or other statistical factor models. Cross-

[1] For example, see Ross (1976), Harvey (1989), Ferson and Harvey (1991), Ferson and Korajczyk (1995), Jagannathan and Wang (1996), and Brennan et al. (2004).

sectionally, the dynamic factors are identified through a prespecified factor-loading matrix, where parameter restrictions are imposed across assets. Through time, the dynamic factors are formed as linear combinations of both contemporaneous and past returns so that they can capture the entire history of the information set. These features of our dynamic factors appear to play an important role in enhancing the performance in in-sample pricing and out-of-sample forecasting. Especially, our finding suggests that the *ex ante* size factor is a good proxy for information uncertainty, effectively capturing the momentum effect missed by the FF 3 factors.

For empirical tests, we formulate two asset-pricing models after extracting the dynamic factors via the Kalman filter. One is a restricted model hypothesizing that the three *ex ante* dynamic factors are irrelevant for asset pricing. Thus, this model uses the three *ex post* factors only and is termed as the risk-adjusted pricing model (RAPM). The other is an unrestricted model, termed as the bias-adjusted pricing model (BAPM), which employs the three *ex ante* dynamic factors as well as the three *ex post* dynamic factors. We then conduct asset-pricing tests in the in-sample context for the two models (RAPM, BAPM) through the two-pass regression procedure as in Cochrane (2005) and Brennan et al. (2004) using the size and book-to-market sorted portfolios as well as the momentum portfolios over the past 600 months (50 years: 1955:01-2004:12), and compare the performance of the two models with that of the FF 3-factor model. More importantly, we perform out-of-sample tests for several competing models, presenting pair-wise comparisons of the accuracy in one-step-ahead forecasts using the 35 extended portfolios as test assets.

Our in-sample test results show that the model which uses the *ex post* dynamic factors only (RAPM) tends to do a better job than the FF 3-factor model in explaining the return variation over time and across assets. For example, the RAPM presents on average higher adjusted $R^{2\prime}$s and lower standard errors of residuals than the FF 3-factor model in the time-series regressions. In the cross-sectional regressions, the RAPM exhibits large pricing errors in the extreme portfolios such as smallest growth, smallest value, largest growth, or largest value portfolios, confirming the general tendency that the extreme size and book-to-market portfolios are most mispriced by any pricing models.[2] However, the absolute

[2]It is well known that the FF 3 factors can explain most portfolios well, but for the smallest growth portfolio, the FF factors systematically misprice its excess returns by over 4% per year. Fama (1998) describes this as a "bad model" problem.

error levels of the RAPM are lower in general than those of the FF 3-factor model in these extreme portfolios.

We also test the relevance of the *ex ante* factors and find that the *ex ante* factors, especially book-to-market and size factors, are a key component in asset pricing. The high explanatory power of the residual loadings on the *ex ante* book-to-market factor results in a strong rejection of the RAPM in favor of the unrestricted model that additionally uses the *ex ante* factors (BAPM). Our asset-pricing tests suggest that the strong incremental power of the BAPM stems from the fact that the loadings on the *ex post* factors alone do not fully incorporate all the past and current information. With the residual explanatory power of the *ex ante* factors, the BAPM can better explain returns than the RAPM as well as the FF 3-factor model.

To alleviate the possible data-snooping biases described in Lo and MacKinlay (1990), we extend the scope of test assets by adding 10 momentum portfolios formed on past returns to the 25 portfolios. The two-pass procedure with the augmented 35 portfolios exhibits that the RAPM again performs quite well, providing more evidence that even the *ex post* factors alone do a better job than the FF 3 factors in explaining time variation in the momentum portfolio returns. Together with the *ex ante* factors, the BAPM further improves upon the performance achieved by the RAPM and the FF 3-factor model. In particular, the cross-sectional regressions with the 35 portfolios show that, owing to the role of the *ex ante* size factor, the BAPM well explains the momentum portfolio returns: None of the pricing errors is statistically significant, and none has a pricing error greater than 0.20% per month. This presents a sharp contrast to the large pricing errors and their high significance levels in the RAPM and the FF 3-factor model. Considering that the FF 3-factor model cannot explain the returns on portfolios formed on short-term past returns (Fama and French, 1996), our results are quite encouraging.

In addition, the out-of-sample tests reassure that the *ex post* factors alone (RAPM) outperform the FF 3 factors in the one-step-ahead forecasts as well, in the sense that the RAPM by and large generates a lower level of forecast errors than the FF 3-factor model. Moreover, when the *ex ante* factors are jointly used, the BAPM provides more accurate forecasts than the naive benchmark (9.35%), the FF 3-factor model, and the RAPM. Again, the better predictive power of the BAPM over the FF 3-factor model occurs mainly in the extreme portfolios such as the loser/winner deciles and the small stock portfolios, which are exactly the ones for which traditional asset-pricing models have difficulties in pricing or forecasting.

Our motivation for using the Kalman filter is to demonstrate that the FF 3 factors miss some important intertemporal information which is vital to explain the momentum effect, and to utilize an econometric technique that can help extract more efficient factors. While the momentum effect remains as a puzzle for the FF 3 factors, the effect can be well explained by our *ex ante* dynamic factors. We attempt to find a reason from the fact that the FF 3 factors are correlated with each other but not serially correlated. In contrast, our dynamic factors are weakly correlated with each other but highly serially correlated. Given that the FF 3 factors, like any factors, are subject to some noise in their measurement, our study shows that the Kalman filter provides us with a recursive process in extracting more efficient unobserved factors.

The remainder of this paper is organized as follows. Section I presents model specification, factor identification, and properties of the dynamic factors. Section II discusses estimation of model parameters, specification tests, descriptive statistics of the factors, and their relations to other variables. Section III formulates the null and alternative hypotheses associated with the models. In Sections IV and V, we conduct in-sample performance tests of the null and alternative models, comparing the results with those of the FF 3-factor model. In Section VI, we discuss the robustness of the above results. In Section VII, we conduct out-of-sample tests, comparing the accuracy of one-step-ahead forecasts among competing models. Section VIII concludes.

I. The Dynamic Factor Model

Stock and Watson (1989, 1991) form a composite index of coincident economic indicators based on the notion that co-movements in observed macroeconomic time series have a common component that can be captured by a single time-varying latent variable. Along the lines of Stock and Watson (1989, 1991), we develop a dynamic multifactor model of stock returns, with each of the dynamic factors being identified within a prespecified factor structure.

Our dynamic factor model may be viewed as a conditional APT with time-varying expected factor premia but with constant factor variances. Stambaugh (1983), Gibbons and Ferson (1985), and Connor and Korajczyk (1989) follow a similar approach of time-varying factor premia and constant betas. A different approach is considered by Harvey and Kirby (1996), Ferson and Harvey (1999), and others in order to express the betas as linear functions of conditioning variables. Ferson and Harvey (1991), Evans (1994), and Ferson and Korajczyk (1995) argue that predictability is primarily driven by changes in risk premia through time, whereas the impact of variation in conditional betas is marginal. Moreover, researchers have raised a concern that conditional beta models are subject to model misspecification (Ghysels, 1998) and variable selection biases (Wang, 2004). Our model can largely circumvent these empirical difficulties, because a model specification test is conducted to ensure that the estimated dynamic factors are unbiased.

Following Fama and French (1993), we begin our discussion on the model specification by looking at the returns of the six portfolios formed on size and book-to-market equity (BTM). Given the fact that most commonly used risk factors in the finance literature are Fama and French's three factors (MKT, SMB, and HML)[3] and these factors are originated from the six size and book-to-market sorted portfolios, we conjecture that presumably useful latent variables may be extracted from the six portfolios formed in this way. Our goal in this study is to develop an econometric method that enables us to extract risk factors more efficiently using the same data sets as the FF 3-factor model employs. That is why we use the six size- and BTM-sorted portfolios as our starting point.

We notate the six size- and BTM-sorted portfolios as SL, SM, SH, BL, BM, and BH.[4] Now let $\underline{R}_t \equiv [\underline{R}_{SL,t}\ \underline{R}_{SM,t}\ \underline{R}_{SH,t}\ \underline{R}_{BL,t}\ \underline{R}_{BM,t}\ \underline{R}_{BH,t}]'$ denote a vector of demeaned excess returns on the six portfolios at month t.[5] Also let $D_t \equiv [D_{mkt,t}\ D_{size,t}\ D_{btm,t}]'$ denote a vector of zero-mean unobserved state variables (termed as dynamic factors: *market* factor, *size* factor, and *BTM* factor, respectively) at month t. The dynamic factor model is then specified as follows. The six observation or measurement equations are given (in

[3] MKT is the excess return on the market portfolio, SMB is the return on a zero net investment portfolio which is long in small firms and short in large firms, and HML is the return on a zero net investment portfolio which is long in high book-to-market firms and short in low book-to-market firms.

[4] As usual, S and B denote 'small' and 'big' in firm size, respectively. Similarly, L, M, and H denote 'low' 'medium' and 'high' in the book-to-market ratio, respectively.

[5] Throughout the paper, we denote $R_{i,t}$ as the excess return (over the one-month T-bill rate) on asset i at time t, $\overline{R}_i$ as the sample mean of $R_{i,t}$, and $\underline{R}_{i,t} \equiv R_{i,t} - \overline{R}_i$ as the demeaned excess return.

the form of deviations from the means) by

$$\begin{bmatrix} \underline{R}_{SL,t} \\ \underline{R}_{SM,t} \\ \underline{R}_{SH,t} \\ \underline{R}_{BL,t} \\ \underline{R}_{BM,t} \\ \underline{R}_{BH,t} \end{bmatrix} = \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} \\ \beta_{6,1} & \beta_{6,2} & \beta_{6,3} \end{bmatrix} \begin{bmatrix} D_{mkt,t} \\ D_{size,t} \\ D_{btm,t} \end{bmatrix} + \begin{bmatrix} w_{SL,t} \\ w_{SM,t} \\ w_{SH,t} \\ w_{BL,t} \\ w_{BM,t} \\ w_{BH,t} \end{bmatrix}, \tag{1}$$

and the three transition or state equations are expressed as

$$\begin{bmatrix} D_{mkt,t} \\ D_{size,t} \\ D_{btm,t} \end{bmatrix} = \begin{bmatrix} \phi_{mkt} & 0 & 0 \\ 0 & \phi_{size} & 0 \\ 0 & 0 & \phi_{btm} \end{bmatrix} \begin{bmatrix} D_{mkt,t-1} \\ D_{size,t-1} \\ D_{btm,t-1} \end{bmatrix} + \begin{bmatrix} v_{mkt,t} \\ v_{size,t} \\ v_{btm,t} \end{bmatrix}, \tag{2}$$

where β's in Eq.(1) are the loadings on the state variables and ϕ's in Eq.(2) are the AR(1) coefficients of the factors. The state-space representation of the dynamics of D_t is also given in a matrix form by the following system of equations:

$$\underline{R}_t = BD_t + w_t, \tag{3}$$

$$D_t = \Phi D_{t-1} + v_t, \tag{4}$$

where B is a prespecified constant factor-loading matrix whose columns have identifying restrictions on the market, size, and BTM factors, w_t is a vector of idiosyncratic returns on the six portfolios, Φ is a constant matrix whose diagonal elements consist of the AR(1) coefficients that capture the time-series predictability of the three factors, and v_t is a vector of factor disturbances. The factor model is *dynamic* in a sense that the state vector D_t follows a stationary vector autoregressive process of order 1, VAR(1). That is, the three equations in Eq.(2) or Eq.(4) describe the dynamics of the state variables, $D_t \equiv [D_{mkt,t}\ D_{size,t}\ D_{btm,t}]'$.

We make the following conventional assumptions for Eq.(3) and Eq.(4): w_t and v_t

follow joint normal distributions, with their covariance matrices given by

$$E[w_t w_\tau'] = \begin{cases} \Delta & \text{for } t = \tau \\ 0 & \text{for } t \neq \tau \end{cases} \tag{5}$$

$$E[v_t v_\tau'] = \begin{cases} \Omega & \text{for } t = \tau \\ 0 & \text{for } t \neq \tau \end{cases} \tag{6}$$

$$E[w_t v_\tau'] = 0 \text{ for all } t \text{ and } \tau \tag{7}$$

where $\Delta \equiv diag[\sigma_{SL}^2, \sigma_{SM}^2, \sigma_{SH}^2, \sigma_{BL}^2, \sigma_{BM}^2, \sigma_{BH}^2]$ is a (6×6) diagonal covariance matrix of idiosyncratic risk for the six portfolios, and $\Omega \equiv diag[\sigma_{mkt}^2, \sigma_{size}^2, \sigma_{btm}^2]$ is a (3×3) diagonal covariance matrix of factor disturbances.

The dynamic factor model has both cross-sectional and time-series features that distinguish this model from the FF 3-factor model or other statistical factor models such as Roll and Ross (1980), Lehmann and Modest (1988), and Connor and Korajczyk (1988). Cross-sectionally, the three dynamic factors are identified through a prespecified factor-loading matrix, in which parameter restrictions are imposed across assets (see the next subsection for details about the parameter restrictions). As a result, the three factors are not only readily interpretable, but they also preserve some desirable properties (e.g., mutually uncorrelated, minimum variances) of purely statistical factors. Through time, the three factors are formed as linear combinations of both contemporaneous and past returns, thereby incorporating the entire history of the information set.

A. Factor Identification

The first factor, D_{mkt}, in Eq.(1) is identified from the first column of the coefficient matrix, B, with no restrictions on the six factor loadings. Using the firm characteristic notations employed above for the six portfolios, we can denote the parameters in the first column of B as follows:

$$\begin{aligned} \beta_{1,1} &= \beta_{SL} \\ \beta_{2,1} &= \beta_{SM} \\ \beta_{3,1} &= \beta_{SH} \end{aligned}$$

$$\begin{aligned}\beta_{4,1} &= \beta_{BL} \\ \beta_{5,1} &= \beta_{BM} \\ \beta_{6,1} &= \beta_{BH}. \end{aligned} \tag{8}$$

Given that the six portfolios consist of market-wide assets, this factor (D_{mkt}) can capture price dynamics common across the six types of assets or portfolios formed on firm characteristics. That is why we term this dynamic factor as the *market* factor. In this sense, the market factor may be interpreted as a composite index.

The size factor, D_{size}, is identified from the second column of matrix B by imposing the restrictions that β's should be the same within each of the small- and big-stock groups. That is, the first set of parameter restrictions are:

$$\begin{aligned}\beta_{1,2} &= \beta_{2,2} = \beta_{3,2} = \beta_S \\ \beta_{4,2} &= \beta_{5,2} = \beta_{6,2} = \beta_B. \end{aligned} \tag{9}$$

These restrictions are reasonable, because assets within the same size group must have the same sensitivity, while assets belonging to different size groups should have different sensitivities to the size-related factor. The restriction ensures that if there exists any systematic variation in returns beyond that captured by the market factor, this factor will pick up the size-related variation.

Similarly, the book-to-market factor, D_{btm}, is identified from the third column of B by imposing another set of restrictions that β's should be the same within each of the low, medium, and high book-to-market groups. That is,

$$\begin{aligned}\beta_{1,3} &= \beta_{4,3} = \beta_L \\ \beta_{2,3} &= \beta_{5,3} = \beta_M \\ \beta_{3,3} &= \beta_{6,3} = \beta_H. \end{aligned} \tag{10}$$

These restrictions ensure that D_{btm} captures the variation that is related to the book-to-market effect beyond that captured by the market factor.

Finally, we normalize the variances so that each of the factors has a unit variance (i.e., $\sigma^2_{mkt} = \sigma^2_{size} = \sigma^2_{btm} = 1$).[6] In addition, because the three factors are, by construction, mutually uncorrelated (i.e., diagonal Φ and Ω), the dynamic factor model uniquely identifies the three common factors with zero means and unit residual variances.[7] Our dynamic factors may be viewed as the ICAPM state variables (Merton, 1973) that contemporaneously span investors' investment opportunity sets, or as the coincident financial indicators (Chauvet and Potter, 2000) that capture investors' contemporaneous beliefs about the cycles of financial markets.

B. The Kalman Filter as a Model of Conditional Expectations

Under the assumptions in Eq.(5)-Eq.(7), the Kalman filter is a statistically optimal procedure to extract the unobserved factors from a finite set of observed returns. Using the Kalman filter, we first estimate the parameters in matrices B, Δ, and Φ specified by Eq.(1) to Eq.(4), with the state vector being initialized by the unconditional means and variances.[8] The Kalman filter is then iterated for $t = 1, \ldots, T$ to recursively extract conditional expectations of the dynamic factors, $D_{t|t-1}$ and $D_{t|t}$, as well as their mean squared error (MSE) matrices, $P_{t|t-1}$, and $P_{t|t}$.

At the beginning of month t, investors make prior assessments about the conditional means and variances of the unobserved factors (D_t) based on the information set I_{t-1}, which have the following characteristics:

Property 1 (Producing a Forecast of D_t Based on I_{t-1}): *In a conditional distribution* $D_t \mid I_{t-1} \sim N(D_{t|t-1}, P_{t|t-1})$ *and from Eq.(4),*

1) A vector of the ex ante (prior) expectations of the unobserved true factors (D_t) is given by $D_{t|t-1} = \Phi D_{t-1|t-1}$.

2) $P_{t|t-1} = \Phi P_{t-1|t-1}\Phi' + \Omega$ *is a function of the population parameters (hence a constant*

[6] This choice of normalization is standard in state-space models, as is also adopted by Stock and Watson (1991).

[7] The restrictions on the factor-loading matrix B are both necessary and sufficient to identify the three factors. If no restriction is imposed, then the total number of free parameters (18 factor loadings + 6 variances) exceeds the number of independent variance-covariance terms of returns (21). In addition, the factor-loading restrictions make the three factors free from arbitrary rotations, thereby keeping the factor identities unique. To see this, consider a (3×3) orthogonal transformation matrix M ($MM' = I$), so that Eq. (3) becomes $\underline{R}_t = (BM)(M'D_t) + w_t$. Any non-zero off-diagonal elements in the second or third row of M destroy the factor-loading pattern and violate the definition of the size or book-to-market factor.

[8] For details, see Kim and Nelson (1999) and Hamilton (1994), for instance.

matrix).

3) $D_{t|t-1}$ *is the minimum mean squared error (MMSE) estimator of* D_t *with respect to* $P_{t|t-1}$*.*

$D_{t|t-1}$ defines a vector of time-varying expected factors, which we term as the *ex ante* factors. The MMSE property of $D_{t|t-1}$ is a well-established result of the Kalman filter.[9] Constant $P_{t|t-1}$ follows from the stationary joint normal distributions of $\underline{R}_t$ and D_t. This characterizes the conditional structure of the dynamic factor model as having time-varying expected factors ($D_{t|t-1}$), constant conditional factor variances ($P_{t|t-1}$), and constant factor loadings (B), as in Stambaugh (1983). The constant and MMSE features of $P_{t|t-1}$ imply that we can define a time-varying maximum *ex ante* squared Sharpe ratio (SSR) as

$$SSR_{t|t-1} \equiv D'_{t|t-1} P^{-1}_{t|t-1} D_{t|t-1}, \tag{11}$$

which reflects investors' optimal forecast of the anticipated reward-risk tradeoff in the factors. Due to the constant property of $P_{t|t-1}$, any intertemporal variation in $SSR_{t|t-1}$ is solely driven by the *ex ante* forecasts, $D_{t|t-1}$, that describe anticipated shifts in the investment opportunity set.

At the end of month t, investors make contemporaneous (real-time) assessments about the conditional means and variances of the unobserved factors as the data ($\underline{R}_t$) are observed, leading to the following characteristics:

Property 2 (Updating the Inference about D_t Based on I_t)*: In a conditional distribution* $D_t \mid I_t \sim N(D_{t|t}, P_{t|t})$*,*

1) A vector of the ex post (posterior) expectations of the true factors (D_t*) is given by*

$$D_{t|t} = D_{t|t-1} + K_t e_{t|t-1}, \tag{12}$$

where $K_t \equiv P_{t|t-1} B' \Sigma^{-1}_{t|t-1}$ *is the Kalman gain matrix (which is a function of the population parameters). In the Kalman gain matrix,* $\Sigma_{t|t-1} \equiv (BP_{t|t-1}B' + \Delta)$ *is the variance-covariance matrix of the forecast error,* $e_{t|t-1} \equiv \underline{R}_t - \underline{R}_{t|t-1}$*.*

2) $P_{t|t} = P_{t|t-1} - K_t B P_{t|t-1}$ *is a function of the population parameters (hence a constant matrix).*

[9] Harvey (1989) and Hamilton (1994) provide details on the MMSE estimators obtained from the Kalman filter.

3) $D_{t|t}$ is the MMSE estimator of D_t with respect to $P_{t|t}$.

$D_{t|t}$ defines a set of *ex post* factors that capture investors' contemporaneous beliefs about the state of financial markets, and $P_{t|t}$ measures the constant real-time uncertainty when making new inferences about the unobserved factors, D_t. As we see in Eq.(12) above, the *ex post* factors ($D_{t|t}$) are formed as a linear combination of the factors predicted at time $t-1$ ($D_{t|t-1}$) and contemporaneous information surprises contained in the forecast error ($e_{t|t-1}$), with the weight being the constant ($3\times$ 6) Kalman gain matrix, K_t. The MMSE property means that $D_{t|t}$ is extracted in a way that the variance of tracking error ($D_t - D_{t|t}$) is minimized. Thus, $D_{t|t}$ is said to mimic D_t in the MMSE sense, so that the *ex post* factors can replace the unobserved true factors within a finite sample of assets for exact arbitrage pricing.[10]

It is worth mentioning some advantages of our approach (that estimates the latent factors using the Kalman filter) relative to other statistical methods such as principal component analysis. For example, principal components are essentially of contemporaneity, and so they do not have dynamic features. Moreover, they are purely statistical factors that cannot be intuitively understood or named. With the cross-sectional restrictions on the factor loading matrix [i.e., Eq.(9) and Eq.(10)], however, our dynamic factors are intuitive and reasonably interpretable, which in turn enables us to call them (D_{mkt}, D_{size}, D_{btm}) market, size and BTM factors. On balance, our employing the Kalman filter is justified, especially because the filter is a useful vehicle as an adaptive process with which a sequence of updating and forecasting becomes available in order to form rational expectations as the information set evolves over time.[11]

Rational expectations carry two testable implications on the relations between $D_{t|t-1}$ and $D_{t|t}$. First, by invoking the law of iterated expectations, we obtain $E[D_{t|t} \mid I_{t-1}] = D_{t|t-1}$, which reveals an essential feature of time-varying expected premia of the *ex post* factors. Whether or not this factor predictability is empirically important depends on the significance of the AR(1) coefficients in Φ. Second, rational expectations imply that $D_{t|t}$ is predictable only to the extent that conditioning information is used to form $D_{t|t-1}$. This entails a testable hypothesis that the difference $\eta_t \equiv D_{t|t} - D_{t|t-1} = K_t e_{t|t-1}$, or equiva-

[10] $D_{t|t}$ is in the spirit of returns on a set of factor-mimicking portfolios analyzed by Grinblatt and Titman (1987) and further characterized by Huberman, Kandel, and Stambaugh (1987). Recently, Sentana (2004) formally discusses and evaluates the mimicking property of Kalman filter portfolios in the MMSE sense.

[11] Engle and Watson (1987) use the Kalman filter to formulate rational-expectations models.

lently the forecast errors $e_{t|t-1}$,[12] should be unpredictable using the existing information set, I_{t-1}. These empirical implications are examined in the next section.

II. Estimation of the Dynamic Factor Model and Specification Tests

A. Data and Estimation Methods

We obtain monthly returns on the six portfolios formed on size and book-to-market equity from Kenneth French's website. To compute excess returns on the six portfolios, we subtract the one-month T-bill rate. The sample used in this study ranges from January 1955 to December 2004, covering the past 600 months (50 years: 1955:01-2004:12) of the U.S. stock market.

With the excess returns on the six portfolios, we first estimate the 20 model parameters in matrices B, Δ, and Φ with the maximum likelihood method. For this purpose, we run the Kalman filter as described in Kim and Nelson (1999) with iterations until all the parameters converge to a high precision.[13] Given these parameter estimates, we run the Kalman filter again to recursively extract the *ex ante* (forecast at month $t-1$) factors $D_{t|t-1} \equiv (D_{mkt,t|t-1}\ D_{size,t|t-1}\ D_{btm,t|t-1})'$ as well as the *ex post* (updated at month t) factors $D_{t|t} \equiv (D_{mkt,t|t}\ D_{size,t|t}\ D_{btm,t|t})'$ for $t =$ 1955:01 to 2004:12.

B. Descriptive Statistics and Estimated Parameters

Means and variances of excess returns (over the 1-month T-bill rate) for the six size- and BTM-sorted portfolios are contained in Panel A of Table I. It shows that mean returns increase monotonically with book-to-market equity within each of the two size groups [small (S), and big (B)], while volatilities of returns are highest for growth stocks (L) within each of the two size groups. The former fact is consistent with the well known book-to-market effect. Considering that growth portfolios are likely to consist of small, tech-oriented stocks, the latter is also a reasonable feature.

[12] Note that K_t is constant.

[13] The maximum likelihood estimates converge quickly to a preset 10^{-5} convergence criterion and are robust to different sets of reasonably chosen starting values of parameters.

Panel B of Table I presents the maximum-likelihood parameter estimates and their standard errors of the model specified in Eq.(1) to Eq.(7).[14] The parameter estimates are all statistically significant at the 5% level: β's are larger than 1.96 standard errors from zero, and mostly are larger by the factor of 10-30; ϕ's are larger than 2.8-4.5 standard errors from zero; and $\sigma^{2\prime}$s are larger than 5 standard errors. These results, including the highly significant factor loadings, suggest that excess returns on the six portfolios conform to our three-dimensional factor structure.[15]

Specifically, the loadings on the market factor (D_{mkt}) in the small-stock group (β_{SL}, β_{SM}, β_{SH}) are significantly larger than those in the big-stock group (β_{BL}, β_{BM}, β_{BH}). This suggests that the market factor captures well the notion that small stocks are relatively riskier than big stocks. As for the loadings on the size factor (D_{size}), the magnitude of loadings in the big-stock group (β_B) is more than twice as large as that in the small-stock group (β_S), indicating that returns are more sensitive to the size factor in the big-stock group. We also find that low book-to-market (L) stocks have a significantly negative loading (-1.09) on the book-to-market factor (D_{btm}), while high book-to-market (H) stocks have a significantly positive loading (1.21), providing evidence of the growth and value effects. The loading on the factor in the medium BTM group (β_M) is small and marginally significant. This is a desirable feature because it implies that in a portfolio with a medium level of book-to-market equity, stock returns are not very sensitive to the BTM factor.

For the predictability of our dynamic factors, we find in Panel B that the AR(1) coefficients on the three factors specified in Eq.(2) are all statistically significant at any conventional level. A change in the factor realizations by one standard deviation in the current month will affect the unobserved factor realizations by 14-21% in the following month. To visualize the level of time variation in investors' optimal forecasts about the risk/reward ratio, we draw a graph of the maximum *ex ante* squared Sharpe ratio ($SSR_{t|t-1}$) based on Eq.(11) over the sample period (1955:01-2004:12). Figure 1 shows that there exists substantial variation in investors' forecasts about their risk-return trade-off. The average value of the *ex ante* squared Sharpe ratio is 0.09, which is comparable

[14] The magnitude of factor loadings in B is with respect to the normalization of unit factor residual variances ($\sigma^2_{mkt} = \sigma^2_{size} = \sigma^2_{btm} = 1$).

[15] We also perform a number of likelihood ratio tests for the hypotheses that the returns on the six portfolios can be explained by any one or two of the three factors. These tests strongly reject one- or two-factor structure at any significance level.

to that of the FF 3 factors (0.06). The ratio is highest (1.76) in March 2000, when the S&P 500 index reached its peak in our sample period.

C. Specification Tests of the Model

A specification test is necessary to ensure that our dynamic factors are unbiased. The key testable hypothesis of the model specification is that forecast errors of portfolio returns, $e_{t|t-1}$, shown in Eq.(12) should be unpredictable based on the available information set, I_{t-1}. Following Stock and Watson (1989, 1991), we test the predictability by regressing one-step-ahead forecast errors of portfolio returns on six lagged returns and forecast errors as in the equation,

$$e_{i,t} = a + \sum_{s=1}^{6} \gamma_{i,t-s} \underline{R}_{i,t-s} + \sum_{s=1}^{6} \theta_{i,t-s} e_{i,t-s} + \varsigma_{i,t}, \tag{13}$$

where $e_{i,t} = \underline{R}_{i,t} - \underline{R}_{i,t|t-1}$denotes forecast errors obtained from the Kalman filter, and $\underline{R}_{i,t-s}$ denotes lagged returns on portfolio i (i = SL, SM, SH, BL, BM, BH). The null hypothesis of the test is that the six coefficients (γ's) of $\underline{R}_{i,t-s}$ (s = 1,..., 6), or the six coefficients (θ's) of $e_{i,t-s}$ (s = 1,..., 6) for each portfolio i are jointly zero.

The p-values of the joint F-test are reported in Table II. Considering that the null hypothesis is rejected at the 5% significance level only for four cases out of the total 72 tests, it is likely that our dynamic factor model is reasonably well specified.[16]

D. Descriptive Statistics and Correlations of the Dynamic Factors

As we have examined above that our dynamic factor model is appropriately specified, we now report in Table III the descriptive statistics and correlations of the three dynamic factors after estimating the *ex post* factors ($D_{mkt,t|t}$, $D_{size,t|t}$, and $D_{btm,t|t}$) as well as the *ex ante* factors ($D_{mkt,t|t-1}$, $D_{size,t|t-1}$, and $D_{btm,t|t-1}$) using the Kalman filter. For notational simplicity, we denote (D^1_{mkt}, D^1_{size}, D^1_{btm}) and (D^0_{mkt}, D^0_{size}, D^0_{btm}) as the time series collection of $D_{t|t}$ and $D_{t|t-1}$, respectively. In general, we let superscript '1' denote

[16] We also estimate several more parameterized models. For example, we test other models, where w_t in Eq.(3) follows an AR(1) process, and/or D_t in Eq.(4) follows a VAR(2) process, and so on. These additional specifications do not significantly improve upon the current parsimonious specification.

ex post and '0' *ex ante*. For comparison purposes, we also report the statistics about the FF 3 factors (MKT, SMB, and HML) and the excess return on the equal-weighted market index (EWM).

The first part of Table III shows that the means of the *ex post* and *ex ante* estimates of the three dynamic factors are all zero by construction. The volatility of the *ex ante* factor estimates is about 14-22% of the *ex post* factor volatility. The middle part of the table presents the correlations between the two types of dynamic factors and the FF 3 factors. Most notable is that the FF 3 factors are highly correlated with each other: The absolute values of the correlation coefficients are as high as 27%-39%. However, both the *ex post* and the *ex ante* factors are very weakly correlated with each other within each of the two groups: The absolute values of the correlation coefficients are only 1%-3%.[17] This implies that each of D_{mkt}, D_{size}, and D_{btm} independently captures specific aspects in return variations, whereas MKT, SMB, and HML tend to jointly explain return variations.

Looking at the cross-correlations between the dynamic factors and the FF 3 factors, the *ex post* BTM factor (D^1_{btm}) is strongly correlated with HML (92%). The *ex post* market factor (D^1_{mkt}) is also highly correlated with MKT (77%), and especially with SMB (82%). It is interesting to see that the *ex post* size factor (D^1_{size}) is negatively correlated with SMB (-55%), while it is positively correlated with MKT (64%). The *ex ante* factors are relatively weakly correlated with the FF 3 factors in general, but there are some statistically significant relations: positive correlation of D^0_{mkt} with SMB, negative correlation of D^0_{size} with SMB, and positive correlation of D^0_{btm} with HML. Another noteworthy aspect is that the correlations of the equal-weighted market index (EWM) with D^1_{mkt} and D^0_{mkt} are 95% and 24%, respectively.

Lastly, both *ex post* and *ex ante* factors are highly autocorrelated, compared to the FF 3 factors. For example, the 1st- and 2nd-order autocorrelation coefficients of D^0_{btm} are 21% and 10%, respectively. This indicates that our two types of the dynamic factors are more predictable than the FF 3 factors. Particularly interesting is that D^0_{size} has a negative and statistically significant 1st-order autocorrelation (-14%), while SMB has a small positive serial correlation (6%). These features may have some implications for the better performance of our dynamic factors, as we will see later.

[17] A formal test does not reject the null hypothesis that the dynamic factors are uncorrelated with each other. However, the test strongly rejects the null that the FF factors are uncorrelated with each other.

III. Formulating the Asset-Pricing Tests

Assume that the excess return on a given asset or portfolio, R_i ($i = 1, \ldots, N$), follows a conditional factor structure with the true factors identified as $D_t \equiv (D_{mkt,t}\ D_{size,t}\ D_{btm,t})'$ whose prior (*ex ante*) expectations are extracted as $D_{t|t-1} \equiv (D_{mkt,t|t-1}\ D_{size,t|t-1}\ D_{btm,t|t-1})'$, and posterior (*ex post*) expectations are extracted as $D_{t|t} \equiv (D_{mkt,t|t}\ D_{size,t|t}\ D_{btm,t|t})'$ using the Kalman filter. Both $D_{t|t-1}$ and $D_{t|t}$ have zero unconditional means. Also assume that the information set (I_{t-1}) contains only the excess returns on the six portfolios formed on size and BTM up to time $t-1$.

Now consider the following return-generating process for asset i conditional on I_{t-1}:

$$R_{i,t} = R_{i,t|t-1} + B_i^1(D_t - D_{t|t-1}) + \epsilon_{i,t}, \tag{14}$$

where $R_{i,t|t-1} \equiv E[R_{i,t} \mid I_{t-1}]$, $D_{t|t-1} \equiv E[D_t \mid I_{t-1}]$, and $B_i^1 \equiv [\beta_{i,mkt}^1\ \beta_{i,size}^1\ \beta_{i,btm}^1]$ is a (1×3) row vector of constant factor loadings on the factor innovations for asset i. Let the error term $\epsilon_{i,t}$ be $E[\epsilon_{i,t} \mid I_{t-1}] = 0$ and $E[\epsilon_{i,t} D_t \mid I_{t-1}] = 0$. Furthermore, conditional asset pricing assumes that $R_{i,t|t-1}$ is linearly related to $D_{t|t-1}$ so that

$$R_{i,t|t-1} = \overline{R}_i + B_i^0 D_{t|t-1}, \tag{15}$$

where $\overline{R}_i$ is the unconditional mean of $R_{i,t}$, and $B_i^0 \equiv [\beta_{i,mkt}^0\ \beta_{i,size}^0\ \beta_{i,btm}^0]$ is a (1×3) row vector of constant factor loadings on $D_{t|t-1}$. For convenience, Eq.(15) can be expressed in a demeaned form as $\underline{R}_{i,t|t-1} \equiv R_{i,t|t-1} - \overline{R}_i = B_i^0 D_{t|t-1}$.

Combining Eq.(14) and Eq.(15), we have the following asset-pricing test equation:

$$R_{i,t} = \overline{R}_i + B_i^* D_{t|t-1} + B_i^1 D_t + \varepsilon_{i,t}, \tag{16}$$

where $B_i^* \equiv B_i^0 - B_i^1 = [(\beta_{i,mkt}^0 - \beta_{i,mkt}^1)\ \ (\beta_{i,size}^0 - \beta_{i,size}^1)\ \ (\beta_{i,btm}^0 - \beta_{i,btm}^1)] \equiv [\beta_{i,mkt}^*\ \ \beta_{i,size}^*\ \ \beta_{i,btm}^*]$ denotes a (1×3) row vector of the differences between the loadings on $D_{t|t-1}$ and those on D_t for asset i. What B_i^* captures is the portion of factor loadings B_i^0 that is left over by B_i^1. Henceforth, B_i^* is referred to as the *residual factor loadings*, and $B_i^* D_{t|t-1}$ is referred to as asset i's *residual predictability* that reflects a bias in the *ex post* risk adjustment. Missing out $B_i^* D_{t|t-1}$ leads to a bias because it is the portion of the predictable return but is not incorporated into the asset's *ex post* expected value.[18] Thus, we call Eq.(16) the *bias-adjusted pricing model* (BAPM), because this equation explicitly adjusts for the bias.

Now, Eq.(16) involves a testable restriction that distinguishes between the null and alternative hypotheses. If we set the null hypothesis as H_0: $B_i^* = 0$, this in turn implies:

$$R_{i,t} = \overline{R}_i + B_i^1 D_t + \varepsilon_{i,t}, \tag{17}$$

which is referred to as the *risk-adjusted pricing model* (RAPM) in our study. Given that the vector of true factors (D_t) is not observable to an econometrician in a finite sample of assets, D_t in Eq.(16) and Eq.(17) is proxied by its factor-mimicking estimate, $D_{t|t}$, for our empirical testing purposes [e.g., see Eq.(20) and Eq.(22) later]. Under the null hypothesis ($B_i^* = 0$), Eq.(16) reduces to Eq.(17) as B_i^1 fully incorporates all the past and present information, thereby making $D_{t|t-1}$ irrelevant in pricing asset i. Under the alternative hypothesis of H_a: $B_i^* \neq 0$, however, D_t (and hence $D_{t|t}$) alone does not fully explain conditional expected returns, and thus $D_{t|t-1}$ has explanatory power beyond what $D_{t|t}$ captures in Eq.(16). Accordingly, the essence of testing the alternative hypothesis (BAPM) is to detect any incremental explanatory power over and above what is explained by the null hypothesis model (RAPM).

For Eq.(16), asset pricing imposes the following restriction on the unconditional mean of excess returns:

$$\overline{R}_i = B_i^* \lambda^0 + B_i^1 \lambda^1, \tag{18}$$

where $\lambda^0 \equiv [\lambda^0_{mkt} \ \lambda^0_{size} \ \lambda^0_{btm}]'$ is a three-vector of *ex ante* factor risk premia (corresponding to $D_{t|t-1}$); and $\lambda^1 \equiv [\lambda^1_{mkt} \ \ \lambda^1_{size} \ \ \lambda^1_{btm}]'$ is a three-vector of *ex post* factor risk premia (corresponding to D_t and hence $D_{t|t}$).

Eq.(18) decomposes the mean excess return into a risk-adjusted return attributed to risk factors, $B_i^1 \lambda^1$, plus a portion of predictable return that is missed by risk adjustments,

[18] Formally, the conditional expectation of $\underline{R}_{i,t|t}$, i.e., $E[\underline{R}_{i,t|t} \mid I_{t-1}] = B_i^1 D_{t|t-1}$, is a biased estimate of $\underline{R}_{i,t|t-1} = B_i^0 D_{t|t-1}$ unless $B_i^0 = B_i^1$. Allowing the bias in risk adjustment addresses Elton's (1999) concern that the *ex post* (realized or expected) returns may not be an unbiased estimate of the *ex ante* returns.

$B_i^* \lambda^0$. Under the null hypothesis ($B_i^* = 0$), Eq.(18) reduces to:

$$\overline{R}_i = B_i^1 \lambda^1. \tag{19}$$

Thus, testing Eq.(18) is essentially to gauge the economic significance of $B_i^* \lambda^0$ over and above $B_i^1 \lambda^1$.

In the next sections, we test whether $B_i^* D_{t|t-1}$ in Eq.(16) and $B_i^* \lambda^0$ in Eq.(18) have any incremental explanatory and predictive power (RAPM vs. BAPM), investigating how the two models compare to the FF 3-factor model. We impose on Eq.(16) the restriction of B_i^* =0, which results in the risk-adjusted pricing model (RAPM) specified in Eq.(17). We relax the restriction later, in which case the model is the BAPM. For expositional convenience, we first present the in-sample test results in the next three sections and then perform the out-of-sample tests in the following section making pair-wise comparisons of the accuracy in one-step-ahead forecasts.

IV. Testing the Risk-Adjusted Pricing Model (RAPM)

A. *Time-Series Regressions*

Following Cochrane (2005, Chapter 12) and Brennan, Wang, and Xia (2004), we run the two-pass regressions in order to test the risk-adjusted pricing model (RAPM). As a first step, we estimate the loadings on our three *ex post* dynamic factors by regressing the excess return on the three *ex post* dynamic factors for the 25 portfolios over the sample period (600 months: 1955:01-2004:12) as in the equation,

$$R_{i,t} = \alpha_i + \sum_{j=1}^{3} \beta_{i,j}^1 D_{j,t|t} + u_{i,t}, \tag{20}$$

where $R_{i,t}$ is the excess return on portfolio i ($i = 1, \ldots, 25$), and $D_{j,t|t}$ is *ex post* factor j ($j = mkt$, $size$, btm) extracted from the Kalman filter. We report in Table IV the factor loadings ($\beta^{1\prime}$s), their t-values [$t(\beta^1)'$s], adjusted R^2, and standard errors of the regressions [$s(u)'$s] for each of the 25 portfolios. For brevity, we do not report all the corresponding results with FF 3 factors, but some parts or averaged values only are reported.

As can be seen in Panel A of Table IV, the loadings on factor D^1_{mkt} (i.e., β^1_{mkt}) are relatively large, and their statistical significance is remarkably strong across all 25 portfolios. Also notable is that the magnitude of its loadings is decreasing with the book-to-market ratio within a given size group and decreasing with size within a given BTM group. Returns are most sensitive to D^1_{mkt} in the portfolio with the smallest size and the lowest BTM, while they tend to be least sensitive to the factor in the portfolios with the biggest size and the highest BTM. This suggests that small, growth stocks are riskier and *vice versa*. On the contrary, although the pattern is not so salient within a given size group, the loadings on D^1_{size} become larger as asset size increases within a given BTM group, presenting that returns are more sensitive to the *ex post* size factor in the larger stock groups.[19] The statistical significance of the loadings is also pronounced. The coefficients of D^1_{btm} are mostly negative in the two lowest BTM groups, and they turn larger and positive as BTM rises within a given size group. But the pattern is not so salient in the other direction. The coefficients are all statistically significant at the 1% level except for one portfolio.[20]

In terms of explanatory power, the adjusted R^2 values are higher than 90% for 21 cases out of the total 25 portfolios, with the average across the portfolios being 92%. This is 2% higher than that from the regressions using the FF 3 factors (see Panel B). The standard error of residuals, $s(u)$, is 1.51% on average, which is 10 basis points lower than the average $s(u)$ computed from the regressions with the FF factors. These results exhibit that our *ex post* dynamic factors (even without the *ex ante* factors) are likely to do a better job than the FF 3 factors in explaining the time variation in stock returns.

[19] We note that this observation is consistent with the pattern of the loadings on the dynamic factors reported in Table I as well as that of the cross-correlation coefficients between the *ex post* factors and the FF 3 factors reported in Table III. For instance, the loading on the size factor (D_{size}) in the big-stock portfolio (i.e., $\beta_B = 2.86$) is substantially larger than that in the small-stock portfolio (i.e., $\beta_S = 1.32$) in Panel B of Table I. Also, the correlation coefficient between D^1_{size} and SMB is *negative* and large (-55%) in Table III. Another notable aspect in Table III is that the correlation coefficient between the equal-weighted market index return (EWM) and D^1_{size} is relatively small (23%) but the coefficient between EWM and D^1_{mkt} is as high as 95%. However, D^1_{size} and D^1_{mkt} are uncorrelated with each other. Given that EWM places greater weights on small firms, this finding suggests that D^1_{mkt} effectively captures the return behavior of small stocks while D^1_{size} is more likely to capture that of large stocks. Thus, the pattern shown in Table IV reflects how the model parameters are estimated and the latent factors are extracted via the Kalman filter.

[20] In Panel B of Table I, we also observe that low book-to-market stocks have a negative loading ($\beta_L = -1.09$) on the book-to-market factor (D_{btm}), while high book-to-market stocks have a positive loading ($\beta_H = 1.21$). These provide evidence of the growth and value effects. The loading on the factor in the medium BTM group ($\beta_M = 0.47$) is small and marginally significant. These loading patterns are again reflected in β^1_{btm} reported in Table IV.

B. Cross-Sectional Regressions

In the second step, based on Eq.(19) we cross-sectionally regress the sample mean of monthly excess returns on the factor loadings (estimated in the first-step above) as in the equation,

$$\overline{R}_i = \sum_{j=1}^{3} \lambda_j^1 \widehat{\beta}_{i,j}^1 + e_i, \tag{21}$$

where $\overline{R}_i$ is the mean excess return on portfolio i,$(i = 1, \ldots, 25)$, $\widehat{\beta}_{i,j}^1$ is an estimated loading on *ex post* factor j ($j = mkt$, $size$, btm), λ_j^1 denotes the coefficient to be estimated as a factor risk premium for *ex post* factor j, and e_i is the residual term that measures the pricing error in portfolio i. From the regression above, we report the estimated factor risk premia and pricing errors together with their test statistics. The t-statistics and the covariance matrix of pricing errors, $\Sigma \equiv Cov(e)$, are computed with Shanken's (1992) correction. For comparison purposes, we also report the results from the FF 3-factor model.

First, we check the risk premia (λ_j^1, $j = mkt$, $size$, btm) for our *ex post* dynamic factors. Panel A of Table V shows that the levels of the risk premia for the dynamic factors are 0.06%-0.20%, with the premia for the market and BTM factors being statistically significant at any conventional level. But the premium for the size factor is not significant. The monthly risk premia for the FF 3 factors are of a similar pattern (see Panel B), but the significance levels of λ_{MKT} and λ_{HML} are lower than the corresponding premia for the RAPM factors.[21]

Next, we examine the pricing errors (e_i's) of the RAPM. It is discernible that large pricing errors tend to fall on the "corner" portfolios such as smallest growth, smallest value, largest growth, or largest value portfolios. For example, the pricing error in the portfolio with the smallest size and the lowest BTM amounts to -0.34% monthly (-4.08% annually). This shows that it is the extreme size and book-to-market portfolios that are most mispriced by the *ex post* dynamic factors. As we see in Panel B, the pricing errors of the FF 3-factor model also show a qualitatively similar pattern, but the absolute levels

[21] Because our *ex post* dynamic factors and the FF factors are different in the scale of variances, the level of risk premiums is not directly comparable. To compare the two sets of factors, one can rescale the *ex post* factors to have the same pair-wise standard deviation as that of the FF factors. For example, $\lambda'_{mkt} \equiv [\lambda^1_{mkt}/\sigma(D^1_{mkt})] \times \sigma(MKT)$, and so on. The rescaled premiums are $\lambda'_{mkt} = 0.56\%$, $\lambda'_{size} = 0.19\%$ and $\lambda'_{btm} = 0.60\%$, which are comparable to but higher than the premiums for the FF 3 factors.

of the errors in the corner portfolios are likely to be higher than those of the RAPM in general. The average absolute error (AAE) of the RAPM (0.099%) is lower by 7.2 basis points annually, and the sum of squared errors (SSE) of the model (0.408) is also smaller than that from the FF 3-factor model. When the null hypothesis of a zero pricing error is formally tested using the quadratic statistic, $e'\Sigma^{-1}e$, which is distributed as $\chi^2(22)$ under the null hypothesis, the hypothesis is strongly rejected for both models, however.

Based on the two-pass test results above, we find that the RAPM appears to perform better than the FF 3-factor model in explaining asset returns. However, the simple approach of Fama and French (1993) also produces comparable results. Given that extracting the dynamic factors is more involved than using the simple FF 3 factors, more questions still remain to be answered: 1) Can the *ex ante* dynamic factors, together with the *ex post* factors, achieve beyond what the RAPM (and hence the FF 3 factors) can do in the in-sample context?; 2) More importantly, can the *ex post* dynamic factors alone (or jointly with the *ex ante* factors) generate more accurate one-step-ahead forecasts than the FF 3 factors? We examine these issues in the next three sections.

V. Testing the Bias-Adjusted Pricing Model (BAPM)

The role of the *ex ante* dynamic factors seems to largely hinge upon the extent to which pricing errors in the corner portfolios are reduced. We first investigate if the *ex ante* factors play an incremental role beyond what the FF 3-factor model and the RAPM do.

A. Time-Series Regressions

We test our alternative, bias-adjusted pricing model (BAPM) specified in Eq.(16). In the first step, we estimate the factor loadings by regressing the excess return on the *ex ante* dynamic factors as well as the *ex post* dynamic factors as in the equation,

$$R_{i,t} = \alpha_i + \sum_{j=1}^{3} \beta_{i,j}^{1} D_{j,t|t} + \sum_{j=1}^{3} \beta_{i,j}^{*} D_{j,t|t-1} + u_{i,t}, \tag{22}$$

where $R_{i,t}$ is the excess return on portfolio i ($i = 1, \ldots, 25$), $D_{j,t|t}$ and $D_{j,t|t-1}$ are *ex post* and *ex ante* factors j ($j = mkt$, $size$, btm), respectively.

Panel A of Table VI contains the estimates of the residual factor loadings ($\beta^{*\prime}_{i,j}$s) ($i = 1, \ldots, 25$) and their t-values from the time-series regressions for the 25 portfolios. We do not report the estimates and their t-values of the loadings on the *ex post* factors ($\beta^{1\prime}_{i,j}$s) to save space, because they are similar to those in Table V. Note that many of the residual loadings are statistically significant at the 10% level, especially in the extreme size or BTM groups. Given the statistical significance levels of the loadings on the *ex post* and *ex ante* factors, it is not likely that our second-step results in the next subsection are subject to the "useless" factor problem described in Kan and Zhang (1999).[22] In particular, the magnitude of the residual loadings and their statistical significance are often much higher in the corner portfolios. For instance, the loadings on the market and the BTM factors in the portfolio with the smallest size and the lowest BTM are 2.14 and -1.45, and their t-values are 3.42 and -2.64, respectively. This suggests that the *ex post* dynamic factors alone (and hence the RAPM) cannot fully explain the return variation over time, especially in the extreme portfolios, and it is the *ex ante* dynamic factors that capture additional explanatory power in those portfolios.

B. Cross-Sectional Regressions

Based on Eq.(18), we cross-sectionally regress the sample mean of monthly excess returns on the *residual* and *ex post* factor loadings (estimated in the first-step above) as in the equation,

$$\overline{R}_i = \sum_{j=1}^{3} \lambda^1_j \widehat{\beta}^1_{i,j} + \sum_{j=1}^{3} \lambda^0_j \widehat{\beta}^*_{i,j} + e_i, \tag{23}$$

where $\overline{R}_i$ is the mean excess return on portfolio i,($i = 1, \ldots, 25$), $\widehat{\beta}^1_{i,j}$ is the loading on *ex post* factor j ($j = mkt$, $size$, btm), $\widehat{\beta}^*_{i,j}$ is the *residual* factor loading on *ex ante* factor j, λ^1_j and λ^0_j denote the coefficients to be estimated as factor risk premia, and e_i is the residual term that measures the pricing error of the BAPM for portfolio i.

Panel B of Table VI reports the estimated factor risk premia and pricing errors with their test statistics.[23] The size of the factor premia for the *ex post* factors ($\lambda^{1\prime}$s) and their

[22] Kan and Zhang (1999) suggest testing the significance of factor loadings as a diagnostic tool to detect useless factors. To formally test the null hypothesis of jointly zero coefficients on the dynamic factors, we conduct the F-test in a seemingly unrelated regression (SUR) system for the 25 portfolios. The test strongly rejects the null hypothesis.

[23] Again, the t-statistics and the covariance matrix of pricing errors, $\Sigma \equiv Cov(e)$, are computed with

statistical significance are similar to those of the RAPM reported in Table V. Notice here that the premium for the *ex ante* BTM factor, λ^0_{btm}, is positive and statistically significant at the 1 % level: i.e., the beta risk of the *ex ante* BTM factor is priced. The strong residual predictability of the *ex ante* BTM factor ($D_{btm,t|t-1}$) is consistent with Lewellen (1999), who documents significant predictability of the book-to-market ratio for time-varying expected returns.

Panel B also shows that in general the pricing errors (e's) of the BAPM become much lower (especially in the extreme portfolios), compared to those of the RAPM reported in Panel A of Table V. It follows that the average absolute error (AAE) is down by 24% from 10 basis points (*bps*) to 7.6 *bps* per month, and the sum of squared errors (SSE) decreases from 0.41 to 0.26. In particular, the decreases in pricing errors mainly occur in the corner portfolios: For instance, the pricing error in the smallest growth portfolio for the BAPM is only -0.14% per month, while its counterparts are -0.34% in the RAPM and -0.37% in the FF 3-factor model. The decreases are mainly because of the residual power of the loading on the *ex ante* BTM factor. As mentioned in the previous section, the RAPM is likely to perform better than the FF 3-factor model. Given the incremental role of the *ex ante* factors, it is obvious that the BAPM in turn performs better than the FF 3-factor model. As we can compare the results, the pricing errors of the BAPM are substantially reduced (with a few exceptions in the non-corner portfolios), relative to those of the FF 3-factor model reported in Panel B of Table V.

Next, we examine the joint test statistics $e'\Sigma^{-1}e$. This quadratic term follows a $\chi^2(19)$ distribution under the null hypothesis. For the BAPM, $e'\Sigma^{-1}e$ is 39.68, which is much smaller than that for the RAPM. However, the null hypothesis of a jointly zero pricing error is still rejected at the 5% level for this model, as the p-value indicates. However, as Fama and French (1993, 1996) suggest, the rejection of the BAPM appears to reflect the high explanatory power of the test. The time series regressions for the 25 portfolios produce an average R^2 of 92%, leading to a small covariance matrix, Σ. Thus, even small pricing errors suffice to generate rejectable test statistics. In the same context, the reason that some recent ICAPM studies cannot reject the null hypothesis may be that the explanatory power of their tests is low (Daniel and Titman; 2005).[24] As Fama and

Shanken's (1992) correction.

[24]For example, The ICAPM of Brennan, Wang and Xia (2004) produces comparable pricing errors to those of the FF 3-factor model, but the ICAPM cannot be rejected due to a larger residual covariance

French (1993, 1996) suggest, therefore, it may be more meaningful to assess the *economic* significance of pricing errors. Judging by this criterion, we see that the BAPM performs better than the RAPM as well as the FF 3-factor model, particularly in the extreme size or BTM quintiles. For the BAPM, only two portfolios have pricing errors larger than 2% per year, and the largest error is about 3% per year.

Overall, our analyses demonstrate that the *ex ante* factors are a key component in asset pricing. The high explanatory power of the residual loadings on the *ex ante* BTM factor results in a strong rejection of the RAPM. Ferson and Harvey (1999) conduct asset-pricing tests of the FF 3-factor model. They find that loadings on lagged macroeconomic variables have significant premia that explain variation in expected returns not captured by the FF model. However, Ferson and Harvey (1999) do not propose an alternative model that can explain expected returns better than the FF 3-factor model. Our asset-pricing tests present that the residual power of the BAPM stems from the fact that loadings on the *ex post* factors alone do not fully incorporate all the past and current information. Once the bias is adjusted by adding the *ex ante* factors, the BAPM can better explain expected returns than the RAPM as well as the FF 3-factor model.

VI. Robustness Checks

A. Analysis with an Extended Set of Portfolios

Lo and MacKinlay (1990) raise an issue of possible data-snooping biases when drawing inferences from samples of characteristic-sorted data. To reduce the potential problem, we now extend the scope of test assets by adding 10 momentum portfolios formed on past returns to the 25 size and book-to-market sorted portfolios (total 35 portfolios). Following Fama and French (1996), we form 10 momentum portfolios by splitting NYSE common stocks into deciles based on continuously compounded returns over the past 11 months (from month $t-1$ to $t-11$) in an ascending order, and then computing equal-weighted excess returns (in excess of the one-month T-bill rate) for each decile over the

matrix (p. 1764). In addition, Petkova (2006) cannot reject an ICAPM for a similar size of pricing errors because the model leaves a large portion of portfolio variance unexplained (compare the R^2 values in Tables III and IV of her paper).

sample period (1955:01-2004:12).[25] We then apply the two-pass regression procedure in the same way as described in the previous sections. As shown in Panel A of Table VII, the 10 portfolio monthly returns, by construction, monotonically increase from 0.11% (decile 1) to 1.38% (decile 10). Note also that the portfolios show U-shaped volatility, with the returns of extreme portfolios (e.g., decile 1 and decile 10) being more volatile.

We first apply the two-pass procedure to the RAPM and the FF 3-factor model with the augmented set of portfolios. For brevity, we report the results for the 10 momentum portfolios only in Panels B-C in Table VII. Comparing the R^2 values and standard errors $[s(u)'\text{s}]$ from the time-series regressions for the RAPM (Panel B) with those for the FF 3-factor model (Panel C), we find that with no exception the RAPM has higher $R^{2\prime}$s and lower standard errors than the FF 3-factor model. The average R^2 is three percentage points higher, and the average $s(u)$ is 17 basis points lower. This provides further evidence that even the *ex post* dynamic factors do a better job than the FF 3 factors in explaining time variation in returns.

Next, we relax the restriction of $B_i^* = 0$ in Eq.(16) and Eq.(18), which results in the BAPM. With the same procedure, we check whether the BAPM has incremental explanatory power beyond what the RAPM or the FF 3-factor model can capture in pricing assets. Panel A of Table VIII presents the first-step results for the 10 momentum portfolios. The loading on the *ex post* market factor, β_{mkt}^1, shows a U-shaped pattern, while the loadings on the *ex post* size and BTM factors exhibit little variation across the deciles. The statistical significance of the loadings on the three *ex post* factors is very strong. The features of the loadings on the *ex ante* factors, $\beta^{*\prime}$s, may be of more interest, however. While β_{mkt}^* shows a U-shaped pattern, this and β_{btm}^* are both statistically insignificant. On the contrary, the residual loading on the *ex ante* size factor, β_{size}^*, increases monotonically with momentum, and it is significant at the 5% level in 8 cases out of the 10 portfolios. This indicates that the *ex ante* size factor plays an important role in describing momentum portfolio returns.

Panel B of Table VIII contains the results from the cross-sectional regressions using the 35 portfolios. The risk premia for the *ex post* factors, $\lambda^{1\prime}$s, are comparable to those reported in Table VI. The size and statistical significance of the premium for the *ex ante* BTM factor (λ_{btm}^0) are also similar to those in Table VI. However, the premium for the

[25] Thus, decile 1 (10) is the portfolio with the lowest (highest) continuously compounded past returns.

ex ante size factor, λ^0_{size}, is 0.16% and statistically significant at the 1% level, which contrasts with its counterpart in Table VI. This significant premium, coupled with the increasing pattern of the loadings described above, suggests that the *ex ante* size factor ($D_{size,t|t-1}$) plays a key role in explaining the momentum portfolio returns. To gauge the incremental impact of this factor, we see that monthly excess returns for the extreme loser (decile 1) and winner (decile 10) portfolios are -0.57% ($= -3.58$ x 0.16%) and 0.32% (=1.99 x 0.16%), respectively. This means that such a large portion of explainable returns is missed by the RAPM and the FF 3-factor model.

Panel B also reports the pricing errors of the BAPM for the 10 momentum portfolios and the 25 size and book-to-market sorted portfolios. We find that once biases are adjusted with the *ex ante* factors, the BAPM completely describes the momentum portfolio returns: None of the pricing errors is statistically significant, and economically, none has a pricing error greater than 0.20% per month. This presents a sharp contrast to the large pricing errors and their high significance levels in the RAPM and the FF 3-factor model reported in Panel B of Table VII. For instance, compare -0.16% of decile 1 in Table VIII with -0.92% (both RAPM and FF 3) in Table VII, and 0.20% of decile 10 in Table VIII with 0.60% (RAPM) or 0.61% (FF 3) in Table VII. For the 25 size and book-to-market portfolios, the largest pricing error is 0.27% per month. Moreover, the average absolute error (AAE) of the 35 portfolios is less than 0.11% in Table VIII, but its counterpart of the FF 3-factor model in Table VII is 0.17%. Finally, given the magnitude of the p-value, we cannot reject the null hypothesis that pricing errors in the 35 portfolios are jointly zero.[26]

Grundy and Martin (2001) and Jegadeesh and Titman (2002) document that risk adjustments have little to do with the momentum effect. The RAPM is more or less in line with their findings. With bias adjustments, however, the BAPM explains the momentum portfolio returns quite well. This result is very encouraging given that the FF 3-factor model misses the continuation of returns for portfolios formed on short-term past returns (Fama and French, 1996). What captures this effect is the residual loading on the *ex ante* size factor. To our knowledge, this is the first study that documents this

[26] However, we do not place much significance upon the non-rejection of the joint test. The joint test with the 35 portfolios may not have the same power as that with the 25 portfolios because of the different residual covariance matrices between the two sets of portfolios. A more meaningful criterion would be the economic significance of the pricing errors and the relative performance between the different models. See Cochrane (2001, Chapter 11) for similar discussion.

feature in the asset-pricing literature.[27]

To further assess the incremental role of the *ex ante* factors, we directly examine to what extent our dynamic factors can explain this anomaly (the momentum effect) in asset pricing. For this purpose, we obtain the time series of the momentum factor, UMD, from the website of Kenneth French. Because our goal is to investigate the incremental explanatory power of the dynamic factors after accounting for the effects of the FF 3 factors, we first estimate the risk-adjusted (against FF 3 factors) momentum factor. Then we test if the dynamic factors can explain the risk-adjusted momentum factor by running a time-series regression,

$$UMD_t^* = a + \sum_j \delta_j^1 D_{j,t}^1 + \sum_j \delta_j^0 D_{j,t}^0 + \zeta_t, \tag{24}$$

where $j = mkt$, $size$, btm and UMD^* is the residual from the regression of UMD on the Fama-French (1993) three factors. As we see in specification 1 of Table IX, none of the *ex post* factors can explain the momentum effect. However, D_{size}^0 has strong explanatory power as shown in specification 2 in the table. When we include both groups in Eq.(24), the result is again similar: The *ex ante* size factor plays an important role in describing the momentum effect.

Previous studies find that information uncertainty plays an important role in explaining the momentum effect, and firm size is often used as a reasonable proxy for information uncertainty. For example, Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) argue that return predictability should be stronger for firms with greater uncertainty because investors tend to be more overconfident when firms' businesses are hard to value. Recently, Zhang (2006) extends their idea and uses firm size as a proxy for information uncertainty, providing evidence that greater information uncertainty produces higher return momentum. In the same context, our finding suggests that the *ex ante* size factor (D_{size}^0) is a good proxy for information uncertainty. What distinguishes D_{size}^0 from the

[27] Avramov and Chordia (2006) show that the conditional FF 3-factor model provides the best result. In their study, it is 'best' in a sense that the conditional FF 3-factor model captures the impact of size and BTM on stock returns. However, the model cannot capture the predictive ability of past returns (momentum variables) even when the momentum factor is included in the first-pass regressions. Moreover, they use returns of *individual* stocks instead of portfolios as test assets. Although our dynamic factor model is within the conditional framework, it is hard to directly compare our model with the conditional FF 3 factor model tested in Avramov and Chordia (2006), given the differences in the approach.

FF 3 factors (especially SMB) is that D^0_{size} is essentially dynamic and more efficient in exploiting available information through the Kalman filter. Therefore, D^0_{size} appears to effectively capture the momentum effect while the FF 3 factors cannot.

B. Further Robustness Checks

In addition to the above robustness checks, we conduct a number of other tests. First, we again extend the scope of assets by forming 10 reversal portfolios based on the continuously compounded returns for the past 13 to 60 months. We then follow the two-pass procedure with the new augmented set of the 35 portfolios to test both the RAPM and the FF 3-factor model over the sample period. Not surprisingly, our *ex post* dynamic factors (RAPM) perform better than the FF 3 factors. Furthermore, we conduct a more comprehensive test with the 45 portfolios after pooling the 25 size and book-to-market sorted, 10 momentum, and 10 reversal portfolios all together. As before, the RAPM exhibits significant pricing errors in the 10 momentum portfolios. Once the *ex ante* factors are added, however, the BAPM has substantial incremental explanatory power. This model explains both momentum and reversal portfolios, with the largest (absolute) pricing error across the 45 portfolios being below 0.30%, and the average absolute error (AAE) below 0.11% per month.

Second, we re-estimate the model specified in Eq.(3) to Eq.(4) to extract another set of *ex ante* and *ex post* dynamic factors over the period from July 1963 to December 1993, which is the same sample range studied by Fama and French (1996). The parameter estimation shows that the magnitude of the AR(1) coefficients is larger for this sample period.[28] The BAPM test results with the new set of factors using the 35 portfolios (the 25 size and book-to-market and 10 momentum portfolios) again confirm the additional explanatory power left out by the RAPM or the FF 3-factor model.[29] The BAPM explains momentum in all the deciles, except for decile 1 whose pricing error is –0.45% per month. Although this level is still economically significant, it is a substantial improve-

[28] In this case, the maximum-likelihood estimates of the AR(1) coefficients and their t-values for the three factors are as follows: $\phi_{mkt} = 0.23$ ($t = 4.37$), $\phi_{size} = -0.18$ ($t = -2.90$) and $\phi_{btm} = 0.26$ ($t = 4.26$). The statistical significance levels of the coefficients are similar to the results reported in Table I, but the magnitude of the coefficients is much higher.

[29] The residual premium for the *ex ante* size factor (λ^0_{size}) is similar (0.15%) to that reported in Table VIII for our whole sample period, and statistically significant.

ment upon the corresponding error of -1.15% reported in Fama and French (1996).[30] The BAPM also explains excess returns on the 25 size and book-to-market sorted portfolios, generating the largest pricing error of 0.33% in the smallest growth portfolio. This is again much lower than that of -0.45% reported in Fama and French (1996).[31]

VII. Out-of-Sample Tests

In the previous sections, we have shown that the RAPM generally outperforms the FF 3-factor model, and that the *ex ante* dynamic factors play key roles in explaining returns. However, these results are in the context of in-sample tests. Simin (2007) argues that while conditional asset pricing models outperform unconditional models in the in-sample tests, they perform poorly in the out-of-sample tests. He also documents that simply using the historical average return (6% annually, or 0.5% monthly) gives the best result in the one-step-ahead forecasts for most of his test assets, compared to other sophisticated models (regardless of conditional or unconditional).[32] From the perspective of practitioners, being able to obtain more accurate forecasts is particularly important in a variety of settings. In this section, therefore, we focus on the performance of our dynamic factors in the out-of-sample context. Specifically, we make pair-wise comparisons of the accuracy in one-step-ahead forecasts.

In comparing the accuracy of the forecasts, we use the following five competing models: 1) CAPM: the capital asset pricing model; 2) 9.35%: a simple benchmark that uses a fixed annual rate of 9.35% (monthly 0.78%), which is the time-series average of the CRSP value-weighted index returns over our 300-month training period (1955:01-1979:12); 3) FF3: the Fama-French (1993) 3-factor model; 4) RAPM; and 5) BAPM. To see how our results compare to those of Simin (2007), we include the low cost forecast (historical average return of 9.35%) and the CAPM in the analysis. We use the 35 portfolios (10 momentum portfolios and 25 size and book-to-market sorted portfolios) as test assets. For this purpose, the first 300 months (1955:01-1979:12) of our whole sample period (600 months: 1955:01-2004:12) are used as a training period in order to estimate the first set of model parameters. The first one-step-ahead forecast is then computed for January 1980 using

[30] See Table VII in Fama and French (1996).
[31] See Table I in Fama and French (1996).
[32] See Table 8a in Simin (2007), for instance.

the estimated model parameters. The one-step-ahead forecasts for the remaining 299 months (1980:02-2004:12) are also computed based on the model parameters estimated using the relevant data from the 300-month rolling windows, resulting in the total 300 forecasts (from 1980:01 to 2004:12) for each model.[33]

For pair-wise comparisons between the competing models, the average of differentials in the mean squared forecast errors (*MSFE*s), $\overline{d}$, is computed as $\overline{d} = \frac{1}{T}\sum_{t=1}^{T}[u_{1,t}^2 - u_{2,t}^2]$, where $u_{i,t}$ is the time t forecast error of model i (i.e., $u_{i,t} = R_{i,t} - \widehat{R}_{i,t|t-1}$, where $\widehat{R}_{i,t|t-1}$ is the forecast of excess return $R_{i,t}$ at time t) and T is the total number of the forecasts (300). In a comparison pair, $BAPM\ vs.\ FF3$, for instance, $u_{1,t}$ is the forecast error of the BAPM and $u_{2,t}$ is the forecast error of the FF3. Following Simin (2007), we compute *DM-stat* based on Diebold and Mariano (1995) to test the null hypothesis of H_0: $\overline{d} = 0$. This test statistic is defined as

$$DM\text{-}stat = \frac{\overline{d}}{\sqrt{\frac{2\pi\ \widehat{f}_d(0)}{T}}},$$

where $\widehat{f}_d(0)$ is a consistent estimator of the spectral density of $[u_{1,t}^2 - u_{2,t}^2]$ at frequency 0 and $2\pi/T$ is the length of time required for the process to repeat a full cycle. To examine if the average of $\overline{d}$'s are zero, we also conduct the overall t-test for each of the eight pairs using $\overline{d}$'s obtained from the 35 portfolios.

The results are contained in Table X. Given the best performance of the simple benchmark (6%) in Simin (2007), we first compare the predictive ability of our fixed benchmark (9.35%) with that of the other four models. As Panels A and B show, the average MSFE differentials ($\overline{d}$'s) of the first two pairs ($CAPM\ vs.$ 9.35% and $FF3\ vs.$ 9.35%) are mostly positive (in both the 10 momentum portfolios and the 25 size- and BTM-sorted portfolios) and many of them are statistically significant at 10% (12 cases for the first pair and 8 cases for the second). The overall t-test results in Panel C exhibit that $\overline{d}$'s are positive on average and statistically significant at 1% for both pairs. This indicates that indeed the naive low cost forecast (9.35%) does a better job than the CAPM or the FF 3-factor model, which is consistent with Simin (2007). In the next pair ($RAPM\ vs.$ 9.35%), the

[33]Note that, unlike our study, Simin (2008) uses the first 60 months (1926:01-1930:12) as the training period for estimating the first set of model parameters and repeats the estimation using the subsequent 60-month rolling windows. In his study, the 6% benchmark is the average of the CRSP value-weighted index returns during the initial 60-month training period (1926:01-1930:12).

results are mixed. Some of $\overline{d}$'s are negative, while others are positive. Though the overall mean of $\overline{d}$'s are negative, it is not significant (Panel C), suggesting that the levels of forecast errors in the RAPM and the historical average are about the same.

However, the last comparison pair ($BAPM$ $vs.$ 9.35%) shows that $\overline{d}$'s in many portfolios are negative (22 cases out of the 35 portfolios) and statistically significant at 5% in the extreme portfolios such as the loser and winner deciles as well as the small (MV1) stock portfolios. The overall t-test presents that $\overline{d}$'s are negative on average and statistically different from zero at the 1% level in this pair. This clearly demonstrates that the BAPM generally performs better than the naive benchmark in the one-step-ahead forecasts.[34]

Next, we turn to the comparisons among the four models (CAPM, FF3, RAPM, and BAPM) in Panels D-F. As we see the average MSFE differentials and the t-test for the first pair ($FF3$ $vs.$ $CAPM$), the FF3 outperforms the CAPM, especially in the winner portfolios (deciles 6-10) and the high book-to-market (BTM3-BTM5) assets. It is interesting to see that our result is not consistent with Simin (2007). Comparing the RAPM and the FF3 in the second pair, we find that $\overline{d}$'s are negative in 22 cases out of the total 35 portfolios and significant at 10% in eight portfolios. The overall t-test in Panel F shows that $\overline{d}$'s are negative and statistically significant at 5%, confirming again that our *ex post* factors alone (RAPM) tend to perform better than the Fama-French (1993) 3-factor model in the one-step-ahead forecasts as well.

Now we examine if the BAPM performs better than the FF3 or the RAPM. As the third comparison pair ($BAPM$ $vs.$ $FF3$) exhibits, the average MSFE differentials are negative with no exception in the 10 momentum portfolios (in Panel D) and significant at 10% in three cases. Panel E also shows that the vast majority of $\overline{d}$'s are negative except for some larger portfolios (MV4-MV5) and six cases are statistically significant at 5%. The overall t-test presents that $\overline{d}$'s are negative on average and statistically significant at the 1% level, demonstrating that the BAPM provides more accurate forecasts than the FF3. Noteworthy is that the better predictive ability of the BAPM over the FF3 occurs mainly in the extreme portfolios such as the loser/winner deciles (deciles 1-2, and 10) and the small stock portfolios (MV1-MV2). Theses portfolios or assets are exactly the ones for which traditional asset-pricing models have difficulties in pricing

[34] The performance of the BAPM is much better if we use other benchmarks, such as 6% used in Simin (2007).

or forecasting. Qualitatively similar results are observed in the last comparison pair (*BAPM vs. RAPM*), which suggests that the BAPM has lower forecast errors than the RAPM. On balance, the out-of-sample tests reassure that the *ex post* factors alone (RAPM) outperform the FF 3 factors in that the RAPM by and large generates a lower level of forecast errors. Furthermore, if the *ex ante* factors are jointly used, there is solid evidence that the BAPM outperforms the naive benchmark (9.35%), the FF3, and the RAPM in the one-step-ahead forecasts.

To summarize, our analyses in Sections IV-VII uncover that our dynamic factors (the *ex post* factors alone and jointly together with the *ex ante* factors) generally perform better than the FF 3 factors out-of-sample as well as in-sample. In addition, the *ex ante* dynamic factors, especially BTM and size factors ($D_{btm,t|t-1}$ and $D_{size,t|t-1}$), play key roles in describing and forecasting asset returns. We conjecture that the better explanatory and predictive power of our factors is related to their lower correlations within each group but higher serial correlations, which may stem from the dynamic features of the Kalman filter. As we have seen in Table III, the FF 3 factors are highly correlated with each other, while the *ex ante* and *ex post* factors are weakly correlated with each other within each of the two groups. However, the 1st-order autocorrelations of the *ex post* and *ex ante* factors are much higher than those of the FF 3 factors. In particular, the 2nd-order autocorrelation of the *ex ante* BTM factor is quite high (10%), compared with that of HML (4%). Moreover, the *ex post* and *ex ante* size factors have negative and substantially high 1st-order autocorrelations (-14%), while SMB has positive and low autocorrelations (6%). These differences may induce the higher explanatory and predictive power of the dynamic factors relative to the FF 3 factors. This is an interesting issue that deserves further investigation in future research.

VIII. Conclusion

The FF 3-factor model has been subject to controversy because its empirical performance in describing stock returns is not unambiguously successful. Researchers also recognize that the forecast ability of some macroeconomic variables may be spurious, while predictive variables are not empirically observable but instead their predictable elements may be reflected on some latent variables. In consideration of this situation, we develop a dynamic factor model that incorporates features of price dynamics across assets as well as through time. These aspects of our factors extracted via the Kalman filter make the model distinctive from the FF 3-factor model or other statistical factor models.

With the extracted dynamic factors, we formulate the two testable asset-pricing models. The first is the risk-adjusted pricing model (RAPM) which uses the three *ex post* factors only. The second is the bias-adjusted pricing model (BAPM) that employs both the *ex ante* factors and the *ex post* factors at the same time. We then conduct in-sample asset-pricing tests for the two models through the two-pass regression procedure using the 25 size and book-to-market sorted portfolios or the more extensive sets of assets. In addition, we perform out-of-sample tests, thereby presenting pair-wise comparisons of the accuracy in one-step-ahead forecasts using the 35 extended portfolios as test assets.

We provide evidence that the *ex post* dynamic factors alone do a better job than the FF 3 factors in describing the return variation over time and across assets, as well as in forecasting one-step-ahead asset returns. Our analyses also demonstrate that the *ex ante* factors, especially book-to-market and size factors, are a key component in asset pricing and prediction. The high explanatory and predictive power of the residual loadings on the *ex ante* book-to-market and size factors leads to the dominance of the BAPM over the other competing models. Together with the *ex ante* factors, the BAPM further improves upon the performance achieved by the naive benchmark, the FF 3-factor model, or the RAPM, especially in forecasting the extreme portfolio returns. In particular, owing to the role of the *ex ante* size factor, the BAPM can even explain and better forecast the momentum portfolio returns, which are mostly missed by other competing models including the FF 3-factor model.

Our results suggest that there is an interesting economic story behind the latent state variables. They are common factors related to the market, firm size, and book-to-market equity that capture stock returns. The results carry implications for performance measurement, risk analysis, and other applications, although many aspects still remain to be explored. We hope future research will continue to address the issues not resolved in this study.

References

Avramov, Doron, and Tarun Chordia, 2006, Asset pricing models and financial market anomalies, *Review of Financial Studies* 9, 1001-1040.

Brennan, Michael J., Ashley Wang, and Yihong Xia, 2004, Estimation and test of a simple model of intertemporal capital asset pricing, *Journal of Finance* 59, 1743-1775.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57-82.

Chauvet, Marcelle, and Simon Potter, 2000, Coincident and leading indicators of the stock market, *Journal of Empirical Finance* 7, 87-111.

Cochrane, John, 2005, *Asset Pricing*, Princeton University Press, Revised Ed., Princeton, New Jersey.

Connor, Gregory, and Robert Korajczyk, 1988, Risk and return in an equilibrium APT: Applications of a new test methodology, *Journal of Financial Economics* 21, 255-289.

Connor, Gregory, and Robert Korajczyk, 1989, An Intertemporal Equilibrium Beta Pricing Model, *Review of Financial Studies* 2, 373-392.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market over- and under-reactions, *Journal of Finance* 53, 1839–1886.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 2001,Overconfidence, arbitrage, and equilibrium asset pricing, *Journal of Finance* 56, 921–965.

Daniel, Kent, and Sheridan Titman, 2005, Testing factor-model explanations of market anomalies, Working Paper, the Kellogg School at Northwestern University.

Diebold, Francis X, and Roberto S. Mariano, 1995, Comparing predictive accuracy, *Journal of Business and Economic Statistics* 13, 253-263.

Elton, Edwin J., 1999, Presidential address: expected returns, realized returns, and asset pricing tests, *Journal of Finance* 54, 1199-1220.

Engle, Robert F., and Mark Watson, 1987, Kalman filter: applications to forecasting and rational expectations models, In *Advances in Econometrics*, Fifth World Congress, of the Econometric Society, ed. Truman Bewley, vol. 1, 245-283.

Evans, Martin D., 1994, Expected Returns, Time-varying Risk, and Risk Premia, *Journal of Finance* 69, 655-679.

Fama, Eugene F., 1998, Market efficiency, long-term returns, and behavioral finance, *Journal of Financial Economics* 49, 283-306.

Fama, Eugene F., and Kenneth R. French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3-25.

Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected stock returns, *Journal of Financial Economics* 25, 23-50.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on bonds and stocks, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene F., and Kenneth R. French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55-84.

Ferson, Wayne E., and Campbell R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385-415.

Ferson, Wayne E., and Campbell R. Harvey, 1999, Conditioning variables and the cross-section of stock returns, *Journal of Finance* 54, 1325-1360.

Ferson, Wayne E., and Robert A. Korajczyk, 1995, Do arbitrage pricing models explain the predictability of stock returns? *Journal of Business* 68, 309-349.

Ferson, Wayne E., Sergi Sarkissian, and Timothy T. Simin, 2003, Spurious regressions in financial economics? *Journal of Finance* 58, 1393-1413.

Ghysels, Eric, 1998, On stable factor structures in the pricing of risk: Do time varying betas help or hurt? *Journal of Finance* 53, 549-574.

Gibbons, Michael R., and Wayne E. Ferson, 1985, Tests of asset pricing models with changing expectations and an unobservable market portfolio, *Journal of Financial Economics* 14, 217-236.

Grinblatt, Mark, and Sheridan Titman, 1987, The relation between mean-variance efficiency and arbitrage pricing, *Journal of Business* 60, 97-112.

Grundy, Bruce D., and Spencer J. Martin, 2001, Understanding the nature of the risks and source of the rewards to momentum investing, *Review of Financial Studies* 14, 29-78.

Hahn, Jaehoon, and Hangyong Lee, 2006, Yield spreads as alternative risk factors for size and book-to-market, *Journal of Financial and Quantitative Analysis* 41:2.

Hamilton, James D., 1994, *Time series analysis*, Princeton University Press, Princeton, New

Jersey.

Harvey, Andrew C., 1989, *Forecasting, structural time series models and the Kalman filter*, Cambridge: Cambridge University Press.

Harvey, Campbell R., and Chris Kirby, 1996, Instrumental variables and estimation of conditional beta pricing models, *Handbook of Statistics*, Vol. 14.

Huberman, Gur, Samuel Kandel, and Robert F. Stambaugh, 1987, Mimicking portfolios and exact arbitrage pricing, *Journal of Finance* 42, 1-9.

Jagannathan, Ravi., and Zhenyu Wang, 1996, The conditional CAPM and the cross-section of expected returns, *Journal of Finance* 51, 3-54.

Jegadeesh, Narasimhan, and Sheridan Titman, 2002, Cross-sectional and time-series determinants of momentum returns, *Review of Financial Studies* 15, 143-157.

Kan, Raymond, and Chu Zhang, 1999, Two-pass tests of asset pricing models with useless factors, *Journal of Finance* 54, 203-235.

Keim, Donald B., and Robert F. Stambaugh, 1986, Predicting returns in the bond and stock markets, *Journal of Financial Economics* 17, 357-390.

Kim, Chang-Jin, and Charles R. Nelson, 1999, *State-space models with regime switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press.

Lehmann, Bruce N., and David M. Modest, 1988, The empirical foundations of the arbitrage pricing theory, *Journal of Financial Economics* 21, 213-254.

Lewellen, Jonathan, 1999, The time-series relations among expected return, risk, and book-to-market, *Journal of Financial Economics* 54, 5-43.

Lo, Andrew, and Craig MacKinlay, 1990, Data-snooping biases in tests of financial asset pricing models, *Review of Financial Studies* 3, 431-468.

Merton, Robert, 1973, An intertemporal capital asset-pricing model, *Econometrica* 41, 867-887.

Petkova, Ralitsa, 2006, Do the Fama-French factors proxy for innovations in predictive variables?, *Journal of Finance* 61:2, 581-612.

Roll, Richard, and Stephen A. Ross, 1980, An empirical investigation of the arbitrage pricing theory, *Journal of Finance* 35, 1073-1103.

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economics*

Theory 13, 341–360.

Sentana, Enrique, 2004, Factor representing portfolios in large asset markets, *Journal of Econometrics* 119, 257-289.

Shanken, Jay, 1992, On the estimation of beta-pricing models, *Review of Financial Studies* 5, 1-34.

Simin, Timothy, 2007, The (poor) predictive performance of asset pricing models, *Journal of Financial and Quantitative Analysis,* Forthcoming.

Stambaugh, Robert F., 1983, Arbitrage pricing with information, *Journal of Financial Economics* 12, 357-369.

Stambaugh, Robert F., 1999, Predictive regressions, *Journal of Financial Economics* 54, 315-421.

Stock, James H., and Mark W. Watson, 1989, New indexes of coincident and leading economic indicators, In *NBER Macroeconomics Annual*, eds. O. Blanchard and S. Fisher, Cambridge: MIT Press, 351-394.

Stock, James H., and Mark W. Watson, 1991, A probability model of the coincident economic indicators, In *Leading Economic Indicators: New Approaches and Forecasting Records*, ed. K. Lahiri and G.H. Moore, Cambridge: Cambridge University Press, 63-89.

Vassalou, Maria, 2003, News related to future GDP growth as a risk factor in equity returns, *Journal of Financial Economics* 68, 47-73.

Wang, Kevin Q., 2004, Variable selection for conditional asset pricing: an out-of-sample analysis, Working Paper, University of Toronto.

Zhang, X. Frank, 2006, Information uncertainty and stock returns, *Journal of Finance* 61:1, 105-136.

Table I
Maximum Likelihood Estimates of the Parameters for the Dynamic Factor Model

Panel A reports the means and variances of excess returns (over the one month T-bill rate, in %) on the six size and book-to-market sorted portfolios over the past 600 months (50 years: 1955:01-2004:12). Panel B reports the maximum-likelihood estimates (Est) of the parameters (Para) and their standard errors (SE) of the dynamic factor model obtained using the Kalman filter. The six size and book-to-market sorted portfolios are notated as SL, SM, SH, BL, BM, and BH, where S and B denote 'small' and 'big' in firm size, respectively, and L, M, and H denote 'low,' 'medium,' and 'high' in the book-to-market ratio, respectively. $\underline{R}_t \equiv [\underline{R}_{SL,t}\ \ \underline{R}_{SM,t}\ \ \underline{R}_{SH,t}\ \ \underline{R}_{BL,t}\ \ \underline{R}_{BM,t}\ \ \underline{R}_{BH,t}]'$ is a (6×1) vector of demeaned excess returns on the six portfolios at month t; and $D_t \equiv [D_{mkt,t}\ \ D_{size,t}\ \ D_{btm,t}]'$ is a (3×1) vector of zero-mean unobserved state vector at month t. The dynamic factor model is specified as 1) an observation equation:

$$\begin{bmatrix} \underline{R}_{SL,t} \\ \underline{R}_{SM,t} \\ \underline{R}_{SH,t} \\ \underline{R}_{BL,t} \\ \underline{R}_{BM,t} \\ \underline{R}_{BH,t} \end{bmatrix} = \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} \\ \beta_{6,1} & \beta_{6,2} & \beta_{6,3} \end{bmatrix} \begin{bmatrix} D_{mkt,t} \\ D_{size,t} \\ D_{btm,t} \end{bmatrix} + \begin{bmatrix} w_{SL,t} \\ w_{SM,t} \\ w_{SH,t} \\ w_{BL,t} \\ w_{BM,t} \\ w_{BH,t} \end{bmatrix},$$

and 2) a state equation:

$$\begin{bmatrix} D_{mkt,t} \\ D_{size,t} \\ D_{btm,t} \end{bmatrix} = \begin{bmatrix} \phi_{mkt} & 0 & 0 \\ 0 & \phi_{size} & 0 \\ 0 & 0 & \phi_{btm} \end{bmatrix} \begin{bmatrix} D_{mkt,t-1} \\ D_{size,t-1} \\ D_{btm,t-1} \end{bmatrix} + \begin{bmatrix} v_{mkt,t} \\ v_{size,t} \\ v_{btm,t} \end{bmatrix},$$

where $\Delta \equiv E[w_t w_t'] = diag[\sigma^2_{SL}, \sigma^2_{SM}, \sigma^2_{SH}, \sigma^2_{BL}, \sigma^2_{BM}, \sigma^2_{BH}]$, $\Omega \equiv E[v_t v_t'] = diag[\sigma^2_{mkt}, \sigma^2_{size}, \sigma^2_{btm}]$ with σ^2_{mkt} = σ^2_{size} = σ^2_{btm} = 1, and $\{w_t, v_t\}_{t=1}^T$ follows multivariate normal distributions, with w_t and v_t uncorrelated at all leads and lags. Other notations are defined as follows: *Para*: the parameters in the above specifications; *Est*: the estimated value for each of the parameters; and *SE*: standard error. Using the firm characteristic notations employed above for the six portfolios, we can denote β's in the first column of the parameter matrix in the observation equation as follows: $\beta_{1,1} = \beta_{SL}$, $\beta_{2,1} = \beta_{SM}$, $\beta_{3,1} = \beta_{SH}$, $\beta_{4,1} = \beta_{BL}$, $\beta_{5,1} = \beta_{BM}$, and $\beta_{6,1} = \beta_{BH}$. In the same context, the restrictions that β's should be the same within each of the small- and big-stock groups can be expressed as: $\beta_{1,2} = \beta_{2,2} = \beta_{3,2} = \beta_S$ and $\beta_{4,2} = \beta_{5,2} = \beta_{6,2} = \beta_B$. The restrictions that β's should be the same within each of the low, medium, and high book-to-market groups can be expressed as: $\beta_{1,3} = \beta_{4,3} = \beta_L$, $\beta_{2,3} = \beta_{5,3} = \beta_M$, and $\beta_{3,3} = \beta_{6,3} = \beta_H$.

Panel A: Descriptive Statistics about the Six Portfolios Formed on Size and BTM

	SL	SM	SH	BL	BM	BH
Mean	0.52	0.88	1.06	0.49	0.60	0.74
Var	45.02	26.22	26.95	21.55	16.93	19.41

PanelB: Parameter Estimates and Standard Errors of the Dynamic Factor Model

Para	Est	SE	Para	Est	SE	Para	Est	SE
β_{SL}	6.26	0.22	β_S	1.32	0.28	σ^2_{SL}	1.51	0.25
β_{SM}	4.79	0.16	β_B	2.86	0.18	σ^2_{SM}	0.38	0.06
β_{SH}	4.72	0.17				σ^2_{SH}	0.60	0.11
β_{BL}	3.14	0.23	β_L	-1.09	0.27	σ^2_{BL}	1.91	0.24
β_{BM}	2.59	0.20	β_M	0.47	0.24	σ^2_{BM}	1.42	0.14
β_{BH}	2.81	0.20	β_H	1.21	0.24	σ^2_{BH}	1.62	0.17
ϕ_{mkt}	0.18	0.04	ϕ_{size}	-0.14	0.05	ϕ_{btm}	0.21	0.05

Table II

Specification Tests of the Dynamic Factor Model

This table reports the specification test results for the dynamic factor model: p-values from the joint *F*-test. The null hypothesis of the test is that the six coefficients (γ′s) of $\underline{R}_{i,t-s}$ (s = 1,..., 6), or the six coefficients (θ ′s) of $e_{i,t-s}$ (s = 1,..., 6) for each portfolio *i* are jointly zero in the regression equation,

$$e_{i,t} = a + \sum_{s=1}^{6} \gamma_{i,t-s} \underline{R}_{i,t-s} + \sum_{s=1}^{6} \theta_{i,t-s} e_{i,t-s} + \varsigma_{i,t},$$

where $e_{i,t} \equiv \underline{R}_{i,t} - \underline{R}_{i,t|t-1}$ denotes forecast errors obtained from the Kalman filter, and $\underline{R}_{i,t-s}$ denotes lagged returns on portfolio *i* (*i*= SL, SM, SH, BL, BM, BH: here S stands for small, B for big, L for low, M for medium, and H for high). In the table below, $\gamma_{SL,t16}$ stands for six coefficients from $\gamma_{SL,t-1}$ to $\gamma_{SL,t-6}$, and $\theta_{SL,t16}$ is also similarly defined. The sample period ranges over the past 600 months (50 years: 1955:01-2004:12) of the U.S. stock market.

	$\gamma_{SL,t16}$	$\gamma_{SM,t16}$	$\gamma_{SH,t16}$	$\gamma_{BL,t16}$	$\gamma_{BM,t16}$	$\gamma_{BH,t16}$	$\theta_{SL,t16}$	$\theta_{SM,t16}$	$\theta_{SH,t16}$	$\theta_{BL,t16}$	$\theta_{BM,t16}$	$\theta_{BH,t16}$
$e_{SL,t}$	0.69	0.73	0.41	0.63	0.43	0.70	0.72	0.80	0.47	0.76	0.57	0.80
$e_{SM,t}$	0.65	0.65	0.45	0.21	0.12	0.37	0.67	0.72	0.53	0.32	0.20	0.49
$e_{SH,t}$	0.72	0.66	0.52	0.06	0.04	0.16	0.75	0.76	0.64	0.12	0.08	0.25
$e_{BL,t}$	0.51	0.31	0.17	0.35	0.07	0.50	0.56	0.40	0.24	0.47	0.11	0.63
$e_{BM,t}$	0.46	0.16	0.08	0.09	0.01	0.09	0.53	0.26	0.14	0.17	0.01	0.19
$e_{BH,t}$	0.54	0.25	0.18	0.13	0.03	0.29	0.59	0.35	0.26	0.20	0.05	0.38

Table III

Descriptive Statistics and Correlations of the Estimated Dynamic Factors and Other Related Variables

This table reports the descriptive statistics and correlations of the estimates of the three dynamic factors and other related variables over the past 600 months (50 years: 1955:01-2004:12). The variables are defined as follows: *Mean*: the mean of the variable in %; *STD*: the standard deviation of the variable; $D_{t|t} = (D^1_{mkt} \quad D^1_{size} \quad D^1_{btm})'$: a vector of the *ex post* (updated at month *t*) dynamic factors; $D_{t|t-1} = (D^0_{mkt} \quad D^0_{size} \quad D^0_{btm})'$: a vector of *ex ante* (forecast at month *t-1*) dynamic factors; $(MKT \quad SMB \quad HML)'$: a vector of the FF 3 factors; *EWM*: the excess return on the equal-weighted market index; and ρ_k: an autocorrelation coefficient of order k.

	Mean	STD	Cross Correlations										Autocorrelations		
			D^1_{mkt}	D^1_{size}	D^1_{btm}	D^0_{mkt}	D^0_{size}	D^0_{btm}	*MKT*	*SMB*	*HML*	*EWM*	ρ_1	ρ_2	ρ_6
D^1_{mkt}	0.00	1.01	1										0.18	0.00	0.04
D^1_{size}	0.00	0.95	0.03	1									-0.14	0.02	-0.03
D^1_{btm}	0.00	0.93	0.02	-0.01	1								0.21	0.10	0.08
D^0_{mkt}	0.00	0.18	0.18	-0.07	0.08	1							0.18	0.00	0.04
D^0_{size}	0.00	0.13	-0.12	0.14	-0.08	-0.03	1						-0.14	0.02	-0.03
D^0_{btm}	0.00	0.20	-0.01	-0.04	0.21	0.02	0.01	1					0.21	0.10	0.08
MKT	0.52	4.33	0.77	0.64	-0.12	0.09	0.00	-0.04	1				0.07	-0.04	-0.03
SMB	0.21	3.05	0.82	-0.55	0.02	0.18	-0.16	0.02	0.27	1			0.06	0.04	0.06
HML	0.40	2.79	-0.33	-0.02	0.92	0.00	-0.03	0.14	-0.39	-0.27	1		0.13	0.04	0.06
EWM	0.80	5.48	0.95	0.23	0.05	0.24	-0.08	-0.03	0.85	0.66	-0.29	1	0.21	-0.01	0.00

Table IV
Time-Series Regression Results with the *Ex Post* Dynamic Factors to Test the Risk-Adjusted Pricing Model (RAPM)

This table (in Panel A) reports the first-step results (time-series regressions) in the two-pass procedure for the risk-adjusted pricing model (RAPM),

$$R_{i,t} = \alpha_i + \sum_{j=1}^{3} \beta_{i,j}^{1} D_{j,t|t} + u_{i,t},$$

where $R_{i,t}$ is the excess return on portfolio i *(i = 1,…, 25)* formed on size and book-to-market equity, and $D_{j,t|t}$ is *ex post* (updated at month t) dynamic factor j (j = *mkt, size, btm*) extracted from the Kalman filter. Other statistics are defined as follows: $\beta_{i,j}^{1}$: the factor loading on *ex post* factor j for portfolio i; $t(\beta_j^1)$: the t-value for the factor loading $\beta_{i,j}^{1}$; R^2: adjusted R-squared; *s(u)*: the standard error of residuals in the regression; *Avg* R^2: the average of R^2's across the 25 portfolios; and *Avg s(u)*: the average of standard errors across the 25 portfolios. The values in Panel B [*Avg* R^2, *Avg s(u)*] are the average R^2 and standard error from the time-series regressions with the Fama-French (1993) three factor model (FF3). The sample range is over the past 600 months (50 years: 1955:01-2004:12) in the US stock market.

Panel A: Time-Series Regression Results for RAPM

	Book-to-Market					Book-to-Market				
Size	1 low	2	3	4	5 high	1 low	2	3	4	5 high
			β_{mkt}^1					$t(\beta_{mkt}^1)$		
1 small	7.22	6.40	5.47	5.06	5.22	63.64	80.28	92.53	96.22	92.34
2	6.55	5.40	4.70	4.44	4.90	109.86	101.98	102.73	98.22	90.93
3	5.82	4.51	3.93	3.69	4.10	100.50	76.87	72.73	71.04	62.37
4	4.80	3.81	3.44	3.32	3.67	81.36	63.66	59.80	56.38	46.61
5 big	2.88	2.66	2.28	2.14	2.50	51.29	51.50	36.69	39.50	29.09
			β_{size}^1					$t(\beta_{size}^1)$		
1 small	0.52	0.37	0.55	0.63	0.71	4.36	4.35	8.78	11.40	11.92
2	1.40	1.36	1.35	1.45	1.47	22.17	24.20	27.93	30.23	25.78
3	1.73	2.00	2.08	2.11	2.10	28.11	32.26	36.33	38.37	30.12
4	2.31	2.72	2.72	2.54	2.78	36.92	42.88	44.66	40.72	33.31
5 big	3.19	3.37	3.20	3.26	3.25	53.67	61.59	48.76	57.03	35.66
			β_{btm}^1					$t(\beta_{btm}^1)$		
1 small	-1.52	-0.46	0.32	0.77	1.38	-12.37	-5.27	5.03	13.56	22.55
2	-1.86	-0.18	0.55	1.09	1.56	-28.79	-3.17	11.10	22.29	26.66
3	-1.97	-0.02	0.79	1.26	1.72	-31.34	-0.38	13.45	22.39	24.16
4	-1.88	0.05	0.74	1.12	1.60	-29.38	0.80	11.93	17.45	18.77
5 big	-1.56	-0.18	0.29	1.13	1.47	-25.65	-3.12	4.30	19.25	15.73
			R^2					*s(u)*		
1 small	0.88	0.92	0.94	0.94	0.94	2.80	1.96	1.46	1.30	1.39
2	0.96	0.95	0.95	0.95	0.94	1.47	1.31	1.13	1.11	1.33
3	0.95	0.92	0.92	0.92	0.90	1.43	1.45	1.33	1.28	1.62
4	0.94	0.91	0.91	0.90	0.86	1.45	1.47	1.42	1.45	1.94
5 big	0.91	0.92	0.87	0.90	0.80	1.38	1.27	1.53	1.33	2.12
			Avg R^2*: 0.92*					*Avg s(u): 1.51*		

Panel B: Summary of Time-Series Regression Results for FF3

Avg R^2*: 0.90*	*Avg s(u): 1.61*

Table V
Cross-Sectional Regression Results to Test the Risk-Adjusted Pricing Model (RAPM)

This table reports the second-step results in the two-pass procedure from cross-sectional regressions to test risk-adjusted pricing model (RAPM). For this, the sample mean of monthly excess returns is regressed on the estimated factor loadings as in the equation,

$$\overline{R}_i = \sum_{j=1}^{3} \lambda_j^1 \hat{\beta}_{i,j}^1 + e_i,$$

where $\overline{R}_i$ is the sample mean of monthly excess returns on portfolio *i* ($i = 1,\ldots,25$), $\hat{\beta}_{i,j}^1$ is the estimated loading on factor *j* (j = *mkt*, *size*, *btm*), λ_j^1 denotes the coefficient to be estimated as a factor premium, and e_i is the residual term that measures the pricing error for portfolio *i*. From the regression, the estimated factor risk premiums and pricing errors (e) are reported in Panel A, together with their test statistics. The *t*-statistics and covariance matrix of pricing errors, $\Sigma \equiv Cov(e)$, are computed with Shanken's (1992) correction. *AAE* stands for the average absolute error, and $SSE = e'e$ is the sum of squared errors. $e'\Sigma^{-1}e$ is the quadratic test statistic, which is distributed as $\chi^2(22)$ under the null hypothesis that pricing errors are zero. For comparison purposes, similar results from the FF 3-factor (*MKT, SMB, HML*) model are also reported in Panel B.

Panel A: for RAPM

	λ_{mkt}^1		λ_{size}^1		λ_{btm}^1	$t(\lambda_{mkt}^1)$		$t(\lambda_{size}^1)$		$t(\lambda_{btm}^1)$
Premium	0.13%		0.06%		0.20%	3.18		1.46		5.27
	Book-to-Market					Book-to-Market				
Size	1 low	2	3	4	5 high	1 low	2	3	4	5 high
	***e** (in %)*					***t(e)***				
1 small	-0.34	0.05	0.03	0.22	0.16	-3.88	0.68	0.62	4.56	3.02
2	-0.15	-0.04	0.07	0.06	0.06	-2.58	-0.71	1.44	1.16	0.89
3	0.03	0.04	-0.06	0.03	-0.03	0.58	0.72	-1.13	0.55	-0.46
4	0.18	-0.10	0.03	0.04	-0.13	3.56	-1.84	0.54	0.62	-1.74
5 big	0.22	0.02	0.06	-0.12	-0.20	3.32	0.39	0.87	-1.97	-2.09
	AAE: 0.099					***SSE***: 0.408				
	$e'\sum^{-1}e$: 70.08					***p*-value:** 0.000				

Panel B: for FF 3-factor Model

	λ_{MKT}		λ_{SMB}		λ_{HML}	$t(\lambda_{MKT})$		$t(\lambda_{SMB})$		$t(\lambda_{HML})$
Premium	0.48%		0.21%		0.44%	2.68		1.61		3.76
	Book-to-Market					Book-to-Market				
Size	1 low	2	3	4	5 high	1 low	2	3	4	5 high
	***e** (in %)*					***t(e)***				
1 small	-0.37	0.04	0.04	0.23	0.17	-4.17	0.57	0.80	4.78	3.26
2	-0.17	-0.03	0.09	0.08	0.06	-2.80	-0.57	1.74	1.50	1.08
3	0.02	0.06	-0.05	0.04	-0.02	0.33	0.92	-0.84	0.76	-0.37
4	0.16	-0.09	0.04	0.03	-0.14	3.28	-1.67	0.73	0.60	-1.89
5 big	0.22	0.02	0.06	-0.14	-0.24	3.27	0.44	0.89	-2.21	-2.53
	AAE: 0.105					***SSE***: 0.463				
	$e'\sum^{-1}e$: 74.52					***p*-value**: 0.000				

Table VI
Two-Pass Regression Results to Test the Bias-Adjusted Pricing Model (BAPM)

This table reports the two-pass regression results to test the bias-adjusted pricing model (BAPM). Panel A contains the first step results that include the estimates of the residual factor loadings (β^*'s), their *t*-values [$t(\beta^*)$'s] for each of the 25 portfolios, and the averaged values of adjusted R^2 (*Avg* R^2) and standard errors [Avg $s(u)$] after running the time-series regression,

$$R_{i,t} = \alpha_i + \sum_{j=1}^{3} \beta^1_{i,j} D_{j,t|t} + \sum_{j=1}^{3} \beta^*_{i,j} D_{j,t|t-1} + u_{i,t},$$

where $R_{i,t}$ is the excess return on portfolio i ($i = 1, \ldots, 25$) formed on size and book-to-market equity, $D_{j,t|t}$ and $D_{j,t|t-1}$ are *ex post* and *ex ante* dynamic factors j (j = *mkt, size, btm*) extracted from the Kalman filter. Panel B contains the results from the second step which involves cross-sectionally regressing the sample mean of monthly excess returns on the estimated factor loadings as in the equation,

$$\overline{R}_i = \sum_{j=1}^{3} \lambda^1_j \hat{\beta}^1_{i,j} + \sum_{j=1}^{3} \lambda^0_j \hat{\beta}^*_{i,j} + e_i,$$

where $\overline{R}_i$ is the sample mean of monthly excess returns on portfolio i ($i = 1,\ldots,25$), $\hat{\beta}^1_{i,j}$ is the estimated loading on *ex post* factor j (j = *mkt*, *size*, *btm*), $\hat{\beta}^*_{i,j}$ is the estimated loading on *ex ante* factor j, λ^1_j and λ^0_j denotes the coefficient to be estimated as factor premiums, and e_i is the residual term that measures the pricing error of the BAPM in portfolio i. From the regression, we report the estimated residual factor premiums, pricing errors (e), together with their test statistics [$t(e)$]. The *t*-statistics and covariance matrix of pricing errors, $\Sigma \equiv Cov(e)$, are computed with Shanken's (1992) correction. *AAE* stands for the average absolute error, and *SSE* = $e'e$ is the sum of squared errors. $e'\Sigma^{-1}e$ is the quadratic test statistic, which is distributed as $\chi^2(19)$ under the null hypothesis that pricing errors are jointly zero.

Panel A: Time-Series Regression Results for BAPM

	Book-to-Market					Book-to-Market				
Size	1 low	2	3	4	5 high	1 low	2	3	4	5 high
	β^*_{mkt}					$t(\beta^*_{mkt})$				
1 small	2.14	1.02	0.91	1.29	2.24	3.42	2.32	2.79	4.51	7.41
2	-0.34	-0.10	-0.65	-0.67	-0.91	-1.02	-0.34	-2.56	-2.67	-3.04
3	-2.09	-0.56	-0.05	-0.80	-0.23	-6.77	-1.70	-0.17	-2.79	-0.62
4	-0.91	0.31	-0.23	-1.25	-0.56	-2.78	0.93	-0.70	-3.84	-1.28
5 big	0.79	0.55	-0.39	-0.50	0.51	2.56	1.94	-1.12	-1.68	1.07
	β^*_{size}					$t(\beta^*_{size})$				
1 small	0.92	1.69	1.20	0.83	0.47	1.06	2.76	2.65	2.09	1.12
2	0.26	0.43	-0.74	-0.12	-0.58	0.57	1.04	-2.10	-0.33	-1.39
3	-1.17	-0.87	-0.08	-0.84	0.48	-2.73	-1.92	-0.19	-2.10	0.94
4	0.15	0.02	0.17	0.04	0.46	0.33	0.04	0.38	0.10	0.75
5 big	-0.73	-0.86	-0.36	0.37	-1.40	-1.70	-2.16	-0.75	0.88	-2.13
	β^*_{btm}					$t(\beta^*_{btm})$				
1 small	-1.45	-0.09	-0.10	-0.69	-0.14	-2.64	-0.20	-0.32	-2.55	-0.49
2	0.18	0.55	-0.05	0.25	0.29	0.57	1.96	-0.20	1.04	1.04
3	1.01	0.10	0.01	0.01	-0.40	3.49	0.32	0.05	0.03	-1.14
4	0.45	0.30	-0.10	-0.54	-0.35	1.44	0.97	-0.32	-1.77	-0.86
5 big	0.79	0.68	0.39	-0.54	-1.52	2.70	2.51	1.19	-1.90	-3.39
	Avg R^2 : 0.92					**Avg $s(u)$**: 1.50				

(Table VI continued: Panel B)

Panel B: Cross-Sectional Regression Results for BAPM

	λ^1_{mkt}		λ^1_{size}		λ^1_{btm}	λ^0_{mkt}		λ^0_{size}		λ^0_{btm}
Premiums	0.14%		0.05%		0.24%	0.04%		0.00%		0.17%
t	3.27		1.24		5.77	1.30		0.35		3.27
	Book-to-Market					Book-to-Market				
Size	1 low	2	3	4	5 high	1 low	2	3	4	5 high
	e *(in %)*					***t(e)***				
1 small	-0.14	0.04	0.00	0.26	0.06	-2.12	0.46	0.05	4.32	1.11
2	-0.12	-0.13	0.07	0.00	-0.04	-1.63	-1.77	1.16	-0.07	-0.51
3	-0.02	0.04	-0.09	0.00	-0.01	-0.30	0.54	-1.27	0.06	-0.17
4	0.20	-0.15	0.04	0.13	-0.10	3.30	-2.15	0.55	1.94	-1.08
5 big	0.14	-0.08	0.02	-0.04	-0.01	1.91	-1.35	0.20	-0.55	-0.12
	AAE: 0.076					***SSE***: 0.255				
	$e'\Sigma^{-1}e$: 39.68					***p*-value**: 0.004				

Table VII

Two-Pass Regression Results to Test the RAPM and the FF 3-factor Model with an Augmented Set of Portfolios Using the 10 More Portfolios Formed on Past Returns

This table reports the two-pass regression results to test the RAPM and the FF 3-factor model using the 25 size and book-to-market sorted portfolios augmented by 10 momentum portfolios (total 35 portfolios). The 10 momentum portfolios are formed by splitting NYSE common stocks into deciles after sorting in ascending order by the continuously compounded return over the past 11 months (from month *t-1* to *t-11*), and then the equal-weighted excess return (in excess of the one-month T-bill rate) is computed for each decile over the sample period (1955:01-2004:12). Decile 1 (10) is a portfolio with the lowest (highest) continuously compounded past returns. Panel A contains the means (Mean) and standard deviations (STD) of the excess returns for each decile. The two-pass regressions are performed with the 35 portfolios in the same way as described in Table VI. The adjusted R^2's (R^2) and residual standard errors [$s(u)$] from the time-series regressions for the 10 momentum portfolios are reported in the upper parts of Panels B and C. The pricing errors (e) and associated statistics [$t(e)$] from the cross-sectional regressions across the 35 portfolios are reported (for the 10 momentum portfolios only) in the middle parts of Panels B and C. In the lower parts in Panels B-C, we report other statistics from the cross-sectional regressions with the 35 portfolios. The t-statistics [$t(e)$] and covariance matrix of pricing errors, $\Sigma \equiv Cov(e)$, are computed with Shanken's (1992) correction. *AAE* stands for the average absolute error, and $SSE = e'e$ is the sum of squared errors. $e'\Sigma^{-1}e$ is the quadratic test statistic, which is distributed as $\chi^2(29)$ under the null hypothesis that pricing errors are jointly zero.

	Decile										
	1 loser	2	3	4	5	6	7	8	9	10 winner	Avg(abs)
Panel A: Descriptive Statistics for the 10 Momentum Portfolios											
Mean	0.11	0.53	0.55	0.64	0.74	0.77	0.89	0.97	1.08	1.38	0.77
STD	7.64	5.65	5.15	4.84	4.68	4.57	4.60	4.80	5.07	6.03	5.30
Panel B: Two-Pass Regression Results for RAPM											
Time-Series Regressions:											
R^2	0.72	0.83	0.87	0.91	0.93	0.93	0.93	0.92	0.90	0.83	0.88
$s(e)$	4.05	2.32	1.85	1.48	1.26	1.19	1.22	1.33	1.64	2.49	1.88
Cross-Sectional Regressions:											
e(in %)	-0.92	-0.34	-0.28	-0.15	-0.03	0.01	0.15	0.22	0.33	0.60	0.30
$t(e)$	-6.22	-4.05	-4.42	-2.92	-0.75	0.21	3.25	4.24	4.86	5.87	
AAE: 0.17					***SSE***: 2.10						
$e'\Sigma^{-1}e$: 107.96					***p*-value**: 0.00						
Panel C: Two-Pass Regression Results for FF 3-Factor Model											
Time-Series Regressions:											
R^2	0.71	0.81	0.84	0.87	0.90	0.90	0.90	0.90	0.88	0.82	0.85
$s(e)$	4.12	2.46	2.06	1.72	1.49	1.44	1.42	1.51	1.78	2.55	2.05
Cross-Sectional Regressions:											
e(in %)	-0.92	-0.33	-0.27	-0.13	-0.02	0.02	0.16	0.23	0.33	0.61	0.30
$t(e)$	-6.23	-3.96	-4.22	-2.65	-0.50	0.44	3.43	4.40	4.96	5.92	
AAE: 0.17					***SSE***: 2.15						
$e'\Sigma^{-1}e$: 112.13					***p*-value**: 0.00						

Table VIII

Two-Pass Regression Results to Test the BAPM with an Augmented Set of Portfolios Using the 10 More Portfolios Formed on Past Returns

This table reports the two-pass regression results to test the BAPM using the 25 size and book-to-market sorted portfolios augmented by 10 momentum portfolios (total 35 portfolios). The 10 momentum portfolios are formed by splitting NYSE common stocks into deciles after sorting in ascending order by the continuously compounded return over the past 11 months (from month *t-1* to *t-11*), and then the equal-weighted excess return (in excess of the one-month T-bill rate) is computed for each decile over the sample period (1955:01-2004:12). Decile 1 (10) is a portfolio with the lowest (highest) continuously compounded past returns. The two-pass regressions are performed in the same way as described in Table VI with the 35 portfolios. Panel A contains the time-series regression results for the 10 momentum portfolios (factor loadings and associated *t*-statistics). Panel B contains the cross-sectional regression results across the 35 portfolios (factor premiums and associated *t*-statistics for the *ex post* and *ex ante* dynamic factors, and pricing errors (e) and associated statistics $t(e)$). *AAE* stands for average absolute error of the 35 portfolios, and $SSE = e'e$ stands for the sum of squared errors of the 35 portfolios, $e'\Sigma^{-1}e$ is the quadratic test statistic, which is distributed as $\chi^2(29)$ under the null hypothesis of jointly zero pricing errors. The *t*-statistics for the factor premiums, the *t*-statistics for the pricing errors, and the covariance-matrix of pricing errors (Σ) are computed with Shanken's (1992) correction.

Panel A: Time-Series Regression Results for BAPM

	Decile									
	1 loser	2	3	4	5	6	7	8	9	10 winner
β^1_{mkt}	5.94	4.50	4.05	3.90	3.78	3.68	3.71	3.93	4.16	5.03
t	*35.57*	*46.71*	*52.99*	*63.90*	*72.44*	*74.07*	*73.04*	*71.11*	*61.09*	*48.74*
β^1_{size}	1.93	2.11	2.25	2.17	2.21	2.22	2.25	2.27	2.26	2.07
t	*11.02*	*20.92*	*28.06*	*33.94*	*40.36*	*42.74*	*42.35*	*39.16*	*31.72*	*19.18*
β^1_{btm}	1.01	0.95	1.04	0.91	0.89	0.87	0.72	0.61	0.50	0.08
t	*5.58*	*9.04*	*12.55*	*13.73*	*15.63*	*16.22*	*13.05*	*10.18*	*6.76*	*0.74*
β^*_{mkt}	1.19	0.76	0.33	0.14	0.15	-0.16	-0.04	0.05	0.06	0.27
t	*1.31*	*1.45*	*0.80*	*0.42*	*0.52*	*-0.60*	*-0.15*	*0.17*	*0.17*	*0.48*
β^*_{size}	-3.58	-1.64	-1.83	-1.74	-0.93	-0.74	-0.06	0.58	1.24	1.99
t	*-2.83*	*-2.26*	*-3.17*	*-3.77*	*-2.37*	*-1.97*	*-0.17*	*1.39*	*2.42*	*2.55*
β^*_{btm}	-1.16	-0.11	-0.27	0.28	0.14	0.12	0.04	0.04	0.04	0.24
t	*-1.35*	*-0.22*	*-0.68*	*0.90*	*0.51*	*0.46*	*0.14*	*0.15*	*0.13*	*0.46*

Panel B: Cross-Sectional Regression Results for BAPM

	λ^1_{mkt}		λ^1_{size}		λ^1_{btm}	λ^0_{mkt}		λ^0_{size}		λ^0_{btm}
Premium	0.12%		0.09%		0.23%	-0.01%		0.16%		0.21%
t	*2.78*		*2.20*		*4.98*	*-1.15*		*3.18*		*2.86*
e (in %)	-0.16	-0.12	-0.02	0.00	0.01	0.02	0.08	0.05	0.05	0.20
t(e)	*-1.28*	*-1.18*	*-0.19*	*-0.02*	*0.16*	*0.25*	*0.96*	*0.62*	*0.56*	*1.43*
	e (in %) for the 25 Portfolios					*t(e)* for the 25 Portfolios				
	1 low	2	3	4	5 high	1 low	2	3	4	5 high
1 small	-0.05	-0.12	-0.08	0.27	0.15	*-0.50*	*-0.91*	*-0.86*	*3.02*	*1.72*
2	-0.14	-0.19	0.20	0.00	0.05	*-1.40*	*-1.81*	*2.10*	*0.05*	*0.48*
3	0.06	0.15	-0.09	0.10	-0.09	*0.80*	*1.46*	*-0.91*	*1.09*	*-0.76*
4	0.09	-0.21	-0.05	0.05	-0.22	*0.96*	*-2.04*	*-0.49*	*0.52*	*-1.61*
5 big	0.16	-0.04	-0.05	-0.19	0.24	*1.44*	*-0.46*	*-0.38*	*-1.63*	*1.64*
AAE: 0.108						***SSE***: 0.599				
$e'\Sigma^{-1}e$: 36.02						***p*-value**: 0.173				

Table IX

Regression Results for FF3-adj Momentum Portfolio Returns (UMD*) Using the Dynamic Factors

This table reports the time-series regression results using the risk-adjusted momentum portfolio return over the past 600 months (50 years: 1955:01-2004:12) from the equation,

$$UMD_t^* = a + \sum_{j=1} \delta_j^1 D_{j,t}^1 + \sum_{j=1} \delta_j^0 D_{j,t}^0 + \zeta_t,\ j = mkt, size, btm,$$

where the dependent variable, *UMD**, is the residual from the regression of the momentum portfolio return (*UMD*) on the Fama-French's (1993) three factors; *UMD* is the winner portfolio (decile 10) return less the loser portfolio (decile 1) return (after the CSRP stocks are sorted into deciles based on the past returns (from month *t-2* to month *t-12*)), which is obtained from the Kenneth French's website; $D^1_j = (D^1_{mkt}\ \ D^1_{size}\ \ D^1_{btm})'$ is a vector of the *ex post* (updated at month *t*) dynamic factors; $D^0_j = (D^0_{mkt}\ \ D^0_{size}\ \ D^0_{btm})'$ is a vector of *ex ante* (forecast at month *t-1*) dynamic factors. The values in the first row for each explanatory variable are the coefficients of the variable and those in the second row are their *t*-statistics. *adj-R²* is the adjusted R-squared in the regressions, and *N* is the number of observations in the regressions. Coefficients significantly different from zero at the significance levels of 1% and 5% are indicated by ** and *, respectively.

Explanatory Var	Dep Var = UMD* 1	2		3	
Intercept	1.032	1.032		1.032	
	6.69	*6.75*		*6.74*	
D^1_{mkt}	-0.035			0.018	
	-0.23			*0.12*	
D^1_{size}	-0.061			-0.138	
	-0.37			*-0.84*	
D^1_{btm}	0.032			0.083	
	0.19			*0.49*	
D^0_{mkt}		0.434		0.338	
		0.52		*0.40*	
D^0_{size}		3.834	**	4.033	**
		3.32		*3.42*	
D^0_{btm}		-0.177		-0.284	
		-0.23		*-0.35*	
adj-R^2	-0.003	0.015		0.012	
N	600	600		600	

Table X
Out-of-Sample Tests: Comparison of Accuracy in One-Step-Ahead Forecasts

This table presents the pair-wise comparison of the accuracy in one-step-ahead forecasts from the competing models using the 35 portfolios (10 momentum portfolios and 25 size and book-to-market sorted portfolios). As in Fama and French (1996), the 10 momentum portfolios in Panels A and D are formed by splitting NYSE common stocks into deciles after sorting in ascending order by the continuously compounded return over the past 11 months (from month *t-1* to *t-11*). Thus, *decile 1* in Panels A and D is the 'loser' portfolio, while *decile 10* is the 'winner' portfolio. Similarly, the 25 portfolios in Panels B and E are formed by first splitting the component stocks into 5 portfolios based on firm size (market capitalization, MV), and then by splitting each of the 5 portfolios again into 5 portfolios based on book-to-market equity (BTM) in ascending order. Thus, *MV1BM5* in Panels B and E denotes the portfolio that includes stocks of size (MV) group 1 (smallest) and book-to-market (BTM) group 5 (highest). The five competing models are as follows: 1) *CAPM*: the capital asset pricing model; 2) *9.35%*: a simple benchmark that uses a fixed annual rate of 9.35% (monthly 0.78%), which is the time-series average of the CRSP value-weighted index returns over the 300-month training period (1955:01-1979:12); 3) *FF3*: the Fama-French (1993) 3-factor model; 4) *RAPM*: the model defined in Section III; and 5) *BAPM*: the model defined in Section III. The first 300 months (1955:01-1979:12) of our whole sample period (600 months: 1955:01-2004:12) are used as a training period in order to estimate the first set of model parameters. The first one-step-ahead forecast is then computed for January 1980 using the estimated model parameters. The one-step-ahead forecasts for the remaining 299 months (1980:02-2004:12) are also computed based on the model parameters estimated using the relevant data from the 300-month *rolling* windows, resulting in the total 300 forecasts (from 1980:01 to 2004:12) for each model. For pair-wise comparison between the competing models, the average of differentials in the mean squared forecast errors (MSFEs), $\bar{d}$, is computed as $\bar{d} = \frac{1}{T}\sum_{t-1}^{T}[u_{1,t}^2 - u_{2,t}^2]$, where $u_{i,t}$ is the time *t* forecast error of model *i* (i.e., $u_{i,t} = R_{i,t} - \hat{R}_{i,t|t-1}$, where $\hat{R}_{i,t|t-1}$ is the forecast of excess return $R_{i,t}$ at time *t*) and *T* is the total number of the forecasts (300). In the comparison pair of *BAPM vs. FF3* in the table below, for instance, $u_{1,t}$ is the forecast error of BAPM and $u_{2,t}$ is the forecast error of FF3. *DM-stat* is the test statistic computed based on Diebold and Mariano (1995) to test the null hypothesis of H_0: $\bar{d} = 0$. The test statistic is defined as *DM-stat* = $\frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{T}}}$, where $\hat{f}_d(0)$ is a consistent estimator of the spectral density of $[u_{1,t}^2 - u_{2,t}^2]$ at frequency 0 and $2\pi/T$ is the length of time required for the process to repeat a full cycle. Panels C and F contain the overall *t*-test results for each of the 8 pairs using $\bar{d}$'s obtained from the 35 portfolios. Average MSFE differentials ($\bar{d}$) in Panels A-B and D-E or overall averages of $\bar{d}$'s in Panels C and F that are statistically different from zero at the significance levels of 1%, 5%, and 10% are indicated by ***, **, and *, respectively.

(Table X continued: Panels A-C)

Portfolio	CAPM vs. 9.35%		DM-stat	FF3 vs. 9.35%		DM-stat	RAPM vs. 9.35%		DM-stat	BAPM vs. 9.35%		DM-stat
Panel A: 10 Portfolios Formed on Past Return (Momentum)												
1 Loser	-0.361	**	*-2.12*	0.529	***	*2.66*	-0.799		*-1.52*	-2.391	***	*-3.32*
2	0.019		*0.10*	0.078		*1.00*	0.068		*0.43*	-0.477		*-1.48*
3	0.036		*0.22*	0.051		*0.79*	0.060		*0.43*	-0.208		*-0.65*
4	0.155		*0.97*	0.042		*0.79*	0.086		*0.87*	-0.194		*-0.88*
5	0.209		*1.58*	0.086		*1.36*	0.084	*	*1.68*	0.000		*0.00*
6	0.215		*1.62*	0.080		*1.26*	0.075	*	*1.81*	0.053		*0.23*
7	0.278	**	*2.06*	0.089	*	*1.68*	-0.002		*-0.04*	-0.056		*-0.23*
8	0.337	***	*2.74*	0.134	**	*2.08*	-0.064		*-0.82*	-0.147		*-0.44*
9	0.356	**	*2.52*	0.126	*	*1.78*	-0.177		*-1.37*	-0.302		*-0.66*
10 Winner	0.316	**	*1.96*	0.142		*1.26*	-0.442		*-1.47*	-1.123	**	*-2.00*
Panel B: 25 Portfolios Formed on Size (MV) and Book-to-Market Equity (BTM)												
MV1BTM1	-0.250		*-1.13*	-0.069		*-0.63*	-0.363		*-0.95*	-4.608	***	*-4.17*
MV1BTM2	0.035		*0.22*	0.084		*1.46*	0.086		*1.35*	-2.462	***	*-4.01*
MV1BTM3	0.194		*1.19*	0.049		*0.84*	0.024		*0.31*	-1.627	***	*-3.31*
MV1BTM4	0.350	**	*2.30*	0.010		*0.22*	-0.140		*-1.20*	-1.959	***	*-4.53*
MV1BTM5	0.381	*	*1.95*	-0.136		*-1.08*	-0.239		*-1.22*	-2.494	***	*-3.85*
MV2BTM1	-0.035		*-0.20*	0.032		*0.18*	0.017		*0.06*	-1.288		*-1.62*
MV2BTM2	0.122		*0.78*	0.113	**	*1.98*	0.095		*1.46*	-0.474		*-1.13*
MV2BTM3	0.331	**	*2.29*	0.087	*	*1.88*	0.007		*0.13*	-0.337		*-1.32*
MV2BTM4	0.421	***	*2.88*	0.025		*0.54*	-0.090		*-0.97*	-0.343		*-1.26*
MV2BTM5	0.358	**	*2.47*	-0.115		*-0.92*	-0.145		*-1.00*	-0.726	**	*-2.09*
MV3BTM1	0.088		*0.51*	0.121		*0.58*	0.102		*0.42*	-0.181		*-0.37*
MV3BTM2	0.216		*1.44*	0.156	*	*1.69*	0.133	**	*2.01*	0.145		*0.57*
MV3BTM3	0.214		*1.38*	0.099	*	*1.76*	0.101	*	*1.87*	-0.020		*-0.09*
MV3BTM4	0.302	*	*1.84*	0.067		*1.28*	0.023		*0.63*	0.026		*0.11*
MV3BTM5	0.442	***	*3.10*	-0.164	*	*-1.92*	-0.175	**	*-2.05*	-0.244		*-0.72*
MV4BTM1	0.074		*0.42*	0.184		*0.71*	0.162		*0.72*	0.087		*0.22*
MV4BTM2	0.068		*0.47*	0.054		*0.47*	0.143		*0.80*	0.381		*1.04*
MV4BTM3	0.194		*1.17*	0.068		*0.74*	0.063		*1.45*	-0.006		*-0.03*
MV4BTM4	0.220		*1.41*	0.017		*0.43*	0.016		*0.62*	0.180		*1.01*
MV4BTM5	0.324	**	*2.19*	-0.041		*-0.55*	0.002		*0.04*	0.126		*0.45*
MV5BTM1	0.114		*0.62*	0.317		*0.88*	0.209		*0.87*	0.151		*0.40*
MV5BTM2	0.165		*0.99*	0.185		*1.06*	0.151		*0.96*	0.423		*1.38*
MV5BTM3	0.093		*0.46*	0.041		*0.28*	0.088		*0.62*	0.404		*1.43*
MV5BTM4	0.142		*0.79*	0.049		*0.64*	0.054		*0.51*	0.278		*0.91*
MV5BTM5	0.255		*1.44*	0.056		*0.80*	0.132	*	*1.70*	0.328		*1.27*
Panel C: Overall *t*-Test												
Mean	0.182***			0.076***			-0.019			-0.545***		
t-value	*6.21*			*3.69*			*-0.55*			*-2.96*		

(Table X continued: Panels D-F)

Portfolio	FF3 vs. CAPM $\bar{d}$	FF3 vs. CAPM DM-stat	RAPM vs. FF3 $\bar{d}$	RAPM vs. FF3 DM-stat	BAPM vs. FF3 $\bar{d}$	BAPM vs. FF3 DM-stat	BAPM vs. RAPM $\bar{d}$	BAPM vs. RAPM DM-stat
Panel D: 10 Portfolios Formed on Past Return (Momentum)								
1 Loser	0.791 **	*2.13*	-1.364 *	*-1.79*	-3.045 ***	*-3.31*	-1.653 ***	*-3.40*
2	-0.043	*-0.21*	0.011	*0.06*	-0.643 *	*-1.92*	-0.522 *	*-1.69*
3	-0.076	*-0.44*	0.026	*0.17*	-0.300	*-1.20*	-0.288	*-0.95*
4	-0.201	*-1.40*	0.067	*0.69*	-0.296	*-1.26*	-0.318	*-1.32*
5	-0.196	*-1.58*	-0.005	*-0.17*	-0.111	*-0.48*	-0.097	*-0.41*
6	-0.213 **	*-2.15*	-0.007	*-0.47*	-0.051	*-0.20*	-0.041	*-0.16*
7	-0.252 **	*-2.42*	-0.085	*-1.40*	-0.121	*-0.47*	-0.066	*-0.28*
8	-0.270 ***	*-2.64*	-0.180 *	*-1.83*	-0.253	*-0.75*	-0.067	*-0.21*
9	-0.295 ***	*-2.97*	-0.297 *	*-1.93*	-0.401	*-0.87*	-0.077	*-0.16*
10 Winner	-0.248 **	*-2.56*	-0.556 *	*-1.64*	-1.198 **	*-1.98*	-0.719 **	*-1.96*
Panel E: 25 Portfolios Formed on Size (MV) and Book-to-Market Equity (BTM)								
MV1BTM1	0.166	*1.01*	-0.385	*-1.06*	-4.477 ***	*-4.46*	-4.488 ***	*-4.32*
MV1BTM2	-0.029	*-0.18*	0.018	*0.33*	-2.574 ***	*-4.22*	-2.575 ***	*-4.17*
MV1BTM3	-0.231	*-1.15*	-0.025	*-0.36*	-1.708 ***	*-3.53*	-1.683 ***	*-3.56*
MV1BTM4	-0.424 **	*-2.15*	-0.130 *	*-1.81*	-1.971 ***	*-4.53*	-1.821 ***	*-4.21*
MV1BTM5	-0.608 **	*-2.09*	-0.082	*-1.36*	-2.437 ***	*-4.02*	-2.381 ***	*-4.40*
MV2BTM1	0.064	*0.64*	-0.025	*-0.24*	-1.295	*-1.53*	-1.353 *	*-1.80*
MV2BTM2	-0.076	*-0.68*	0.003	*0.14*	-0.592	*-1.27*	-0.606	*-1.29*
MV2BTM3	-0.321 **	*-2.24*	-0.078 ***	*-2.94*	-0.415	*-1.62*	-0.351	*-1.36*
MV2BTM4	-0.464 ***	*-2.81*	-0.110 **	*-2.08*	-0.379	*-1.53*	-0.304	*-1.29*
MV2BTM5	-0.560 **	*-2.10*	-0.014	*-0.29*	-0.656 **	*-2.06*	-0.644 **	*-2.22*
MV3BTM1	0.052	*0.48*	-0.010	*-0.27*	-0.376	*-0.79*	-0.377	*-0.78*
MV3BTM2	-0.100	*-1.17*	-0.066 *	*-1.74*	-0.029	*-0.09*	0.012	*0.04*
MV3BTM3	-0.200	*-1.60*	0.005	*0.26*	-0.132	*-0.53*	-0.145	*-0.66*
MV3BTM4	-0.312 **	*-2.01*	-0.028	*-0.71*	-0.054	*-0.22*	-0.025	*-0.11*
MV3BTM5	-0.677 ***	*-3.25*	0.003	*0.10*	-0.137	*-0.45*	-0.136	*-0.43*
MV4BTM1	0.148	*1.63*	-0.070	*-1.32*	-0.211	*-0.48*	-0.158	*-0.38*
MV4BTM2	-0.054	*-1.35*	0.090	*1.33*	0.247	*0.70*	0.173	*0.46*
MV4BTM3	-0.209 *	*-1.82*	-0.005	*-0.12*	-0.095	*-0.40*	-0.112	*-0.52*
MV4BTM4	-0.288 **	*-2.10*	0.001	*0.05*	0.134	*0.77*	0.140	*0.83*
MV4BTM5	-0.432 **	*-2.16*	0.039	*1.21*	0.113	*0.45*	0.084	*0.32*
MV5BTM1	0.229	*1.43*	-0.155	*-1.53*	-0.259	*-0.73*	-0.135	*-0.41*
MV5BTM2	0.020	*1.20*	-0.040	*-1.33*	0.111	*0.38*	0.174	*0.61*
MV5BTM3	-0.072 **	*-2.38*	0.012	*0.59*	0.239	*1.05*	0.213	*0.98*
MV5BTM4	-0.154	*-1.21*	0.016	*0.37*	0.214	*0.63*	0.170	*0.54*
MV5BTM5	-0.281	*-1.42*	0.080	*1.22*	0.255	*0.88*	0.159	*0.55*
Panel F: Overall *t*-Test								
Mean	-0.166***		-0.096**		-0.654***		-0.572***	
t-value	*-3.66*		*-2.21*		*-3.60*		*-3.38*	

Figure 1. Maximum *Ex Ante* Squared Sharpe Ratio over the Past 50 Years

This figure plots the maximum *ex ante* squared Sharpe ratio ($SSR_{t|t-1}$) over the past 600 months (50 years: 1955:01-2004:12). The maximum *ex ante* squared Sharpe ratio ($SSR_{t|t-1}$) is defined as $SSR_{t|t-1} \equiv D_{t|t-1}' P_{t|t-1}^{-1} D_{t|t-1}$, where $D_{t|t-1} \equiv [D_{mkt,t|t-1} \quad D_{size,t|t-1} \quad D_{btm,t|t-1}]'$ is a vector of the *ex ante* expectations of the three dynamic factors estimated using the Kalman filter. $P_{t|t-1}$ is a (3 × 3) matrix of the mean squared error (MSE) of the *ex ante* expected factors, $D_{t|t-1}$.