# **Intertemporal Behavior of Expected Market Returns: Time-Varying and Asymmetry Properties**

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## **Abstract**

The intertemporal behavior of expected market returns is not only driven by predictable market volatility, but also by unexpected volatility changes. Most of the empirical literature ignores the effects of unexpected volatility changes on the intertemporal relation; consequently, the previous empirical results suffer from the omitted variable bias. With the effects of a volatility shock incorporated in the estimation, we find a strong positive intertemporal relation. We also find that the quicker reversion of a negative return is attributable to a negative intertemporal relation. We interpret this negative intertemporal relation under a negative return shock as a reflection of strong optimistic expectations by investors on the future performance of stock prices.

*JEL Classification*: G14; C40; C51

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# **Intertemporal Behavior of Expected Market Returns: Time-Varying and Asymmetry Properties**

The tradeoff between risk and return is a core tenet in financial economics. In particular, the intertemporal risk-return relation is a key assumption used to explain the predictable variation of expected asset returns.  $\frac{1}{1}$  $\frac{1}{1}$  $\frac{1}{1}$  Despite its importance in asset pricing, there has been a longstanding debate on the empirical sign of the intertemporal relation, with findings that are mixed and inconclusive.

Criticisms of the mixed results refer to a lack of conditional information and/or a heavy dependence on parametric models. We believe that the conflicting findings are attributable to an omitting variable bias. If the predetermined conditional information set does not contain an important variable that affects the risk-return tradeoff, the econometric modeling of market expectations suffers from the model misspecification problem, which leads to a wrong conclusion on the empirical nature of the relationship. In this paper, we suggest that a prior unanticipated volatility change (or an unexpected volatility shock) is one of the important factors that induce the intertemporal relation of the expected market returns. With the effects of prior volatility shocks incorporated in the estimation, we find a statistically strong positive intertemporal relation between the expected market returns and their predictable volatility for the US stock market.

Conventional belief about the intertemporal risk-return tradeoff is that a positive relationship is consistent with the time-varying rational expectation hypothesis in the sense that

<span id="page-2-0"></span> $\frac{1}{1}$ <sup>1</sup> Fama and French (1989) argue that systematic patterns in the predictable variations of expected returns are consistent with the intertemporal asset pricing model by Lucas (1978) and Breeden (1979) and the consumption smoothing idea by Modigliani and Brumberg (1955) and Friedman (1957). Ferson and Harvey (1991) and Evans (1994) also document the relative importance of the time-varying risk premia to the conditional betas to explain predictable variations in expected returns.

the predictable variation of the expected risk premium is induced by the risk-averse investors' revision of their expectations in responding to changing volatility. For example, Pindyck (1984) empirically shows that much of the decline in stock prices during the 1970s in the US stock market is attributable to the upward shift in risk premium arising from high stock market volatility. He suggested that a substantial portion of time variation in the expected risk premium is associated with time-varying risk factors in investment opportunities.  $2$  French, Schwert and Stambaugh (1987) also found evidence of a positive relation between expected market returns and predictable market volatility.<sup>[3](#page-3-1)</sup> Studies that support a positive relation include Fama and French (1988), Ball and Kothari (1989), Tuner, Startz and Nelson (1989), Harvey (1989), Cecchehtti, Lam and Mark (1990), Haugen, Talmor and Torous (1991), Campbell and Hentschel (1992), Scruggs (1998), Kim, Morley and Nelson (2001), Ghysel, Santa-Clara and Valkanov (2005), and Ludvigson and Ng (2006).

Although a positive intertemporal relation is also consistent with Merton's (1980) dynamic CAPM, there is another side to the argument, which is that the equilibrium asset pricing does not necessarily imply a positive intertemporal relation. Abel (1988) suggests that a positive relation between the conditional risk and the risk premium is consistent with the general equilibrium model only when the coefficient of relative risk aversion is less than one. Barky (1989) suggests that the directional effect of an increase in riskiness on stock prices depends on the curvature of the utility function. Showing evidence of a strong negative relation for their

<span id="page-3-0"></span> $\overline{\phantom{a}}$  $2$  Poterba and Summers (1986) suggested the volatility irrelevance argument that, due to the low level of volatility persistence, the volatility effect on the expected risk premium dissipates so quickly that it cannot have a major effect on stock price movements.

<span id="page-3-1"></span> $3$  They also found a strong negative relation between unexpected returns and unexpected changes (or volatility shock) in ex post volatility, and interpreted it as evidence supporting a positive intertemporal relation as well.

<span id="page-4-0"></span>sample period (51:04–89:12), Glosten, Jagannathan and Runkle (hereinafter GJR) (1993) suggest that both positive and negative intertemporal relations are consistent with the equilibrium asset pricing theory. They argue that investors may not require a large premium for bearing risk, but rather may reduce the risk premium when they perceive exceptionally optimistic expectations on the future performance of stock prices. Among others, Gennotte and Marsh (1987), Campbell (1987), Pagan and Hong (1989), Backus and Gregory (1989), Breen, Glosten and Jagannathan (1989), Nelson (1991), Harvey (2001), and Brandt and Kang (2004) support a negative intertemporal relation.<sup>[4](#page-4-0)</sup>

This paper considers the effects of an unexpected volatility shock on the intertemporal relation. A price shock causes two sources of forecasting errors, a return forecasting error and a volatility forecasting error. While a return forecasting error is widely used to generate the conditional variance process utilizing various GARCH family models, the volatility forecasting error (or an unanticipated volatility shock) has not been paid much attention by the literature. An unexpected volatility shock is indeed an important factor in investors' pricing behavior in the sense that rational risk-averse investors revise their expectations in responding not only to the stock market volatility, but also to its unexpected volatility shock. Consequently, the intertemporal behavior of the expected market returns should be driven not only by the underlying market volatility but also by the effect of a volatility shock.

Thus, considering the effect of an unexpected volatility shock on the intertemporal relation, we suggest that there are two important channels to induce the intertemporal behavior of the expected market returns. The first channel allows rational investors to revise their expectation in response to the underlying stock market volatility. With this channel, a risk-

 $\frac{1}{4}$ <sup>4</sup> There are some studies that report weak evidence of the intertemporal relation. See Baillie and DeGennaro (1990), Whitelaw (1994, 2000), and Boudoukh, Richardson and Whitelaw (1997).

averse investor requires a higher risk premium to compensate for the perception of a high level of market volatility. Although it is reasonable to expect that the first channel induces a positive intertemporal relation, previous empirical studies of this channel show mixed results. The second channel considers the effects of prior volatility shocks on the rational investors' expectations, resulting in increases in market volatility for the subsequent period, for which a risk-averse investor requires a higher risk premium. Under the conventional belief of risk-return tradeoff, the second channel is also reasonably expected to induce a positive intertemporal relation.

The first channel has been widely investigated in many studies which examine the sign of the covariance between the expected market risk premium and its predicted volatility. However, the second channel has been ignored in the aforementioned literature, which focuses only on the first channel of the relation. Ignoring the effect of an unexpected volatility change in the estimation causes the model misspecification problem, which yields an omitted variable bias. Unfortunately, almost all of the previous empirical studies on this topic ignore the second channel in their estimations, consequently their empirical results reflect only a partial intertemporal risk return trade-off.

This paper presents empirical models capable of incorporating both channels in the estimation and reexamines the sign of the intertemporal relation. Specifically, we consider the asymmetrical effects of a positive and negative volatility shock on the relation. A positive (negative) volatility shock is defined as the case where a conditional volatility is higher (lower) than expected. Our empirical results show that, compared to a negative volatility shock, a positive volatility shock substantially increases the market volatility for the following period, thereby yielding a stronger impact on the intertemporal relation. It should be noted that the effect of a volatility shock as considered by this paper is different from the so-called "volatility

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<span id="page-6-0"></span>feedback effect." While the volatility feedback effect focuses on the *contemporaneous* effect of concurrent volatility shocks on the expected returns, our volatility feedback effect implies the consequence of a *prior* unexpected volatility shock on the intertemporal relation.<sup>[5](#page-6-0)</sup>

Another important factor considered in this paper is the asymmetric reverting behavior of expected market returns. Nam et al. (2001, 2002**)** document that the monthly excess return series of market indexes exhibit a strong reverting behavior under a prior negative return shock, while showing a significant persistence under a prior positive return shock. <sup>[6](#page-6-0)</sup> This implies that negative returns on average revert more quickly to positive returns than positive returns reverting to negative returns.

Interestingly, while the persistence of positive returns is consistent with a positive intertemporal relation, a quicker reversion of negative returns is not explained under a positive

 $\frac{1}{5}$ <sup>5</sup> There are several studies examining the contemporaneous volatility feedback effect on the expected returns. For instance, French, Schwert and Stambaugh (1987) examine the volatility feedback effect on the relation, using the *ex post* unanticipated volatility change. They argue that a negative *ex post* relation between the excess market returns and the unpredictable changes in market volatility is consistent with a positive *ex ante* relation between expected risk premium and predictable volatility. Specifying a Markov-switching process to capture uncertain volatility states, Tuner, Startz and Nelson (1989) show that an unanticipated volatility shock raises expected returns. Campbell and Hentschel (1992) also show that the volatility feedback effect is a critical factor in explaining the non-normality property of stock returns during periods of high volatility. Recently, Kim, Morley and Nelson (2001) document that the volatility feedback effect is attributable to the mean-reverting components of stock prices, incorporating a positive correlation between market volatility and expected returns.

 $6$  For the excess returns of the value-weighted index over the period of 1926:01 - 1999:12, there were 146 four-consecutive-month rises as compared to only 28 four-consecutive-month declines; 219 three-consecutivemonth rises as opposed to only 68 three-consecutive-month declines; and 339 two-month rises against 153 twomonth declines. Nam et al. suggest that the asymmetric reverting property of stock returns is attributable to the relative profitability of "loser" stocks, and that contrarian profit is a consequence of the buying-selling trading rule exploiting the asymmetry property of stock returns.

<span id="page-7-0"></span>relation. A negative return shock is known to cause excess future volatility, which should increase the risk premium under a positive intertemporal relation. An increase in the risk premium in turn reduces the current stock price. This reduction in stock price yields another realization of negative returns. In other words, under a positive intertemporal relation, a negative return should be accompanied by another negative return in the following period. However, the raw data show an opposite pattern of reversion in that a negative return is more likely to be accompanied by a positive return.

We suggest that the quicker reversion of negative returns is attributable to the negative intertemporal relation. We argue that for a negative return shock, investors would have optimistic expectations of the future performance of the stock price, and consequently do not necessarily raise the risk premium. Even with excessive future volatility, rational investors would be more likely to reduce the risk premium if their optimistic expectations dominate the effects of the excessive volatility. In this case, a reduction in risk premium in turn raises the current stock price, causing a quicker reversion of negative returns. Thus, we investigate the possibility that a negative return shock reduces the expected market risk premium. We also examine whether the quicker reversion of a negative return can be attributed to a reduction in the expected market risk premium.

Some of our findings are notable. First, with the effect of an unexpected volatility shock incorporated in the estimation, the intertemporal relation is strongly positive and highly significant. Second, the relation can be time-varying, as rational investors may reduce the risk premium for a certain period.<sup>[7](#page-7-0)</sup> Third, the negative intertemporal relation reported by Glosten et

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 <sup>7</sup>  $\frac{7}{1}$  Glosten, Jagannathan and Runkle (1993) suggest that rational risk-averse investors may not require larger premiums when they have optimistic expectations. Also, see Abel (1988) and Backus and Gregory (1993) for a similar argument.

al. (1993) for their sample period (hereafter the GJR sample period) could be attributed to the omitted variable bias, as they ignore the effect of an unexpected volatility shock on the relation. We find that when the effect of a prior volatility shock is incorporated in the estimation, the GJR period is indeed characterized by a strong positive intertemporal relation. Fourth, the expected returns exhibit strong asymmetrical intertemporal behavior under prior positive and negative return shocks. The intertemporal relation is negative (positive) under a prior negative (positive) return shock. The asymmetrical intertemporal behavior of the expected market returns is attributable to the observed asymmetric reverting pattern that negative returns on average revert more quickly to positive returns than positive returns reverting to negative returns. Fifth, the quicker reversion of negative returns is attributed to the negative intertemporal relation. We interpret this negative intertemporal relation under a prior negative return shock as reflective of strong optimistic expectations as perceived by investors, of the future performance of a stock experiencing a recent price drop.

 The remainder of the paper proceeds as follows. Section 2 discusses a theoretical model of the intertemporal risk return relation, and presents empirical models to be estimated. Section 3 presents estimation results and interpretations, and section 4 presents empirical models and estimation results of the asymmetric reverting behavior of the expected returns. Section 5 concludes the paper with a brief summary.

## **I. Models**

### *A. Intertemporal Relation*

Merton (1973) suggests the intertemporal risk-return tradeoff as a function of stock market volatility, which can be specified in the following general form:

$$
E_t(R_{mt} - rf_t) = f(\sigma_{mt}^p), \qquad p = 1, 2,
$$
\n(1)

<span id="page-9-0"></span>where  $E_t(\cdot)$  is the expectation operator.  $E_t(R_{mt} - rf_t)$  is time-varying expected market risk premium, where  $R_{mt}$  is the return on a stock market index portfolio, and  $rf_t$  is the risk-free interest rate. The stock market volatility is represented by either  $\sigma_{m}$  (portfolio standard deviation) or  $\sigma_{mt}^2$  (portfolio variance). Although  $f > 0$  is consistent with the equilibrium asset pricing theory, there has been a longstanding controversy in the empirical sign of the relation.

Since the intertemporal behavior of the expected market returns is controlled not only by the predictable volatility but also an unexpected volatility shock, f ' should consist of  $\frac{G_{\ell}}{2\hat{\sigma}^p}$  $E_{t}$  $\hat{\sigma}$  $\left(\cdot\right)$ ∂  $f'$  should consist of  $\frac{\partial E_i}{\partial x_i}$ 

*mt*

and 
$$
\frac{\partial E_t(\cdot)}{\partial(\sigma_{mt-1}^p - \hat{\sigma}_{mt-1}^p)}
$$
, where  $\sigma_{mt-1}^p - \hat{\sigma}_{mt-1}^p$  represents the unexpected market volatility shock.

While the first term measures the underlying risk-return relationship, the second term captures the effects of a volatility shock on the relation. Most of the previous empirical studies ignore the second term and focus only on the first term, thereby causing an omitting variable problem. In this paper, we examine the intertemporal risk-return relation by considering both the predictable volatility and an unexpected volatility shock. In particular, we specify a model which captures the asymmetrical effects of a positive and negative volatility shock on the intertemporal relation in the linear form of expected returns:

$$
E(R_{mt} - rf_t | \Omega_{t-1}) = \alpha + \delta \hat{\sigma}_{mt} + \tau \hat{\sigma}_{mt} \cdot d_t + \varepsilon_t,
$$
\n(2)

where  $E(R_{mt} - rf_t | \Omega_{t-1})$  is the expected market return (or risk premium) conditional on the information set  $\Omega_{t-1}$ ,  $\hat{\sigma}_{mt}$  is the conditional standard deviation of market portfolio returns,  $\varepsilon_t$  is a series of white noise innovations.  $8 d_t$  $8 d_t$  is the dummy variable representing the effect of a

 <sup>8</sup> <sup>8</sup> We use  $\hat{\sigma}_{m}$  instead of  $\hat{\sigma}_{m}^2$  to represent the conditional forecasts of stock market volatility. The use of  $\hat{\sigma}_{m}$  is suggested as the slope of the capital market line in Merton (1980), and is expected to yield an improvement in

<span id="page-10-0"></span>volatility shock, taking the value 1 with a prior unexpected positive volatility shock, i.e.,

 $\hat{\sigma}_{t-1}^2 > 0$ , and 0 otherwise. The intertemporal relation is thus measured by  $\varepsilon_{t-1}^2 - \hat{\sigma}_{t-1}^2 > 0$ , and 0 otherwise. The intertemporal relation is thus measured by  $\delta + \tau$  under a prior positive unexpected volatility shock (with  $d_t = 1$ ) or by  $\delta$  otherwise (with  $d_t = 0$ ). The condition  $\tau \neq 0$  confirms that there is an asymmetrical effect of a positive and negative volatility shock on the intertemporal relation. In particular,  $\tau > 0$  indicates that rational risk-averse investors increase risk premium in responding to a positive volatility shock, i.e.,  $\varepsilon_{t-1}^2 - \hat{\sigma}_{t-1}^2 > 0$ .  $\varepsilon_{t-1}^2 - \hat{\sigma}_{t-1}^2 > 0$ .<sup>9</sup>

### *B. Empirical Models*

### *B.1. ANST-GARCH Model for the Time-Varying Conditional Volatility*

We employ the asymmetric GARCH-M model with various extensions to examine the nature of the relationship between the two conditional moments of the expected returns. In particular, our empirical focus is to capture asymmetry in the predictable volatility and/or in the expected returns with a nonlinear specification. For the conditional variance process, we present the asymmetric nonlinear smooth transition ANST-GARCH model to capture the different volatility response to a positive and negative return shock. We use the estimated conditional variance from the ANST-GARCH model as the best conditional forecast of market volatility. For monthly excess return series  $r_t$ , the ANST-GARCH (1,1) model is specified as follows:

the statistical efficiency of the estimates than those for  $\hat{\sigma}_{m}^2$ , due to a reduction in the mean square error of the regression.

<sup>9</sup> There are other empirical model specifications to examine the intertemporal relation. For example, Campbell (1987), French et al. (1987), Pagan and Hong (1989), and Glosten et al. (1993) investigate the relation in a simple linear relation of  $E(R_{m} - rf_{r} | \Omega_{t-1}) = \alpha + \delta \hat{\sigma}_{m}^{2}$ , while Merton (1980), Harvey (1989), and Scruggs (1998) employs a simple proportional relation of  $E(R_{mt} - rf_t | \Omega_{t-1}) = \delta \hat{\sigma}_{mt}^2$  to examine the relation.

$$
h_{t} = [a_0 + a_1 \varepsilon_{t-1}^2 + a_2 h_{t-1}] + [b_0 + b_1 \varepsilon_{t-1}^2 + b_2 h_{t-1}] F(\varepsilon_{t-1}),
$$
\n(3)

<span id="page-11-0"></span>where  $F(\varepsilon_{t-1}) = \{1 + \exp[-\gamma(\varepsilon_{t-1})]\}^{-1}$  and  $\varepsilon_t = r_t - E(r_t | \Omega_{t-1})$ . By definition,  $\varepsilon_t = v_t \cdot \sqrt{h_t}$  with  $\varepsilon_t$  |  $\Omega_{t-1} \sim N(0, \sqrt{h_t})$ , such that  $v_t \sim N(0,1)$ . The key feature of the ANST-GARCH model is its regime-shift mechanism that allows a smooth, flexible transition of volatility between different states of volatility persistence. <sup>[10](#page-11-0)</sup> The logistic transition function  $F(\varepsilon_{t-1})$  is a smooth and *t* continuous function of  $\varepsilon_{t-1}$ , and takes a value between 0 and 1:  $0 < F(\varepsilon_{t-1}) < 0.5$  for  $\varepsilon_{t-1} < 0$ ,  $0.5 < F(\varepsilon_{t-1})$  < 1 for  $\varepsilon_{t-1} > 0$ , and  $F(\varepsilon_{t-1}) = 0.5$  for  $\varepsilon_{t-1} = 0$ . The volatility persistence is measured by  $(a_1 + a_2) + (b_1 + b_2)F$ , and the condition  $b_1 + b_2 < 0$  captures the excess volatility of a negative return shock. For any negative return shock causing  $0 < F(\varepsilon_{t-1}) < 0.5$ , the current volatility is described as a "*high-persistence-in-volatility regime.*" In contrast, for any positive return shock causing  $0.5 < F(\varepsilon_{t-1}) < 1$ , the current volatility is described as a "*low-persistencein-volatility regime.*" When  $\varepsilon_{t-1} = 0$ ,  $F(\varepsilon_{t-1}) = 0.5$ , which implies that the current volatility  $h_t$  is halfway between the upper and lower volatility regimes. The parameter  $\gamma$  governs the speed of transition between volatility regimes. When the value of  $\gamma$  approaches  $\infty$ , the ANST-GARCH  $(1,1)$  with  $b_0 = b_2 = 0$  degenerates into the modified GARCH  $(1,1)$  model suggested by Glosten et al.(1993).

#### *B.2. Empirical Models for the Intertemporal Relation*

 $10$  The volatility transition mechanism has been applied in the several models, such as the modified GARCH model by Glosten et al. (1993), and the Smooth Transition GARCH model by Gonzalez-Rivera (1998), the SVSARCH (Sign- and Volatility-Switching ARCH) model by Fornari and Mele (1997), and the MSVARCH (Markov switching volatility ARCH model by Turner, et al. (1989) and Hamilton and Susmel (1994).

A simple linear form of the intertemporal relation has been widely examined by many studies. We examine the linear relation in the following ANST-GARCH-M (1,1) model for monthly excess return series  $r_t$ .

Model 1:  
\n
$$
r_{t} = \mu + \phi r_{t-1} + \delta \sqrt{h_{t}} + \varepsilon_{t}
$$
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}), \qquad (4)
$$

where  $F(\varepsilon_{t-1}) = \{1 + \exp[-\gamma(\varepsilon_{t-1})]\}^{-1}$ . We include the first order autoregressive term to capture the serial dependence in return dynamics. We focus on the sign of  $\delta$ .

We also evaluate the same linear relation for the GJR sample period (51:04–89:12) by using a dummy variable to represent the GJR sample period. The model to examine the relation for the GJR sample period is specified as follow:

## *Model 1 for GJR Period:*

$$
r_{t} = \mu + \phi r_{t-1} + (\delta + \delta^{G} G_{t}) \sqrt{h_{t}} + \varepsilon_{t}
$$
  
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + \theta r f_{t} + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}),
$$
\n(5)

where  $G_t$  is a time dummy variable that takes a value 1 for the GJR period or 0 otherwise. The coefficient  $\delta^G$  captures the effect of the GJR period on the relation, while  $\delta$  measures the relation for the full period. The negative intertemporal relation reported by Glosten et al. (1993) can be confirmed by  $\delta + \delta^G < 0$  in equation (5). One of the important features of equation (5) is that it makes it possible to efficiently distinguish the intertemporal relation for the GJR sample period from that for the entire sample period. We also estimate the same model for the GJR

<span id="page-13-0"></span>sample period with or without the one-month T-bill returns  $rf_t$  included in the conditional variance equation. <sup>[11](#page-13-0)</sup>

As mentioned earlier, the intertemporal relation is induced not only from the underlying volatility but also from an unexpected volatility shock. However, ignoring the effect of an unexpected volatility shock on the relation, Model 1 is subject to the omitting variable problem. The estimate of  $\delta$  in Model 1 thus measures only a partial intertemporal relation. To measure the full intertemporal relation we present Model 2:

Model 2:  
\n
$$
r_{t} = \mu + \phi r_{t-1} + [\delta_{1} + \tau_{1} M_{t}] \sqrt{h_{t}} + \varepsilon_{t}
$$
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}), \qquad (6)
$$

where  $M_t$  is a dummy variable that captures the asymmetric effect of an unexpected volatility shock on the intertemporal relation. It takes a value 1 if  $\varepsilon_{t-1}^2 > h_{t-1}$  (a prior positive volatility shock) or 0 otherwise. The sign of the intertemporal relation is measured by  $\delta_1 + \tau_1$  under  $\varepsilon_{t-1}^2 > h_{t-1}$  $\varepsilon_{t-1}^2 > h_{t-1}$  or by  $\delta_1$  otherwise.  $\tau_1$  measures the differential in the effect of a positive and negative volatility shock on the intertemporal relation. Positive value of  $\tau_1$  implies that the expected risk premium increase in responding to a prior positive volatility shock. <sup>[12](#page-13-0)</sup> We also evaluate the full intertemporal relation for the GJR sample period with the following model:

 *Model 2 for GJR Period:* 

$$
r_{t} = \mu + \phi r_{t-1} + [(\delta_{1} + \tau_{1} M_{t}) + (\delta_{1}^{G} + \tau_{1}^{G} M_{t}) G_{t}] \sqrt{h_{t}} + \varepsilon_{t}
$$

<sup>&</sup>lt;sup>11</sup> Several studies show that the estimation results are sensitive to the inclusion of one-month T-bill return in the conditional variance equation. See Campbell (1987), Glosten et al. (1993), and Scruggs (1998).

<sup>&</sup>lt;sup>12</sup> Note that  $\tau_1 = 0$  supports the volatility irrelevance argument by Poterba and Summers (1986).

$$
h_{t} = [a_0 + a_1 \varepsilon_{t-1}^2 + a_2 h_{t-1}] + \theta r f_{t} + [b_0 + b_1 \varepsilon_{t-1}^2 + b_2 h_{t-1}] F(\varepsilon_{t-1}), \tag{7}
$$

where  $G_t$  is a time dummy variable that takes the value 1 for the GJR period or 0 otherwise. The intertemporal relation for the GJR period is measured by  $\delta_1 + \delta_1^G + \tau_1 + \tau_1^G$  under a prior positive volatility shock or by  $\delta_1 + \delta_1^G$  otherwise, such that the differential effect of the GJR period on the relation is captured by  $\delta^G + \tau_1^G$ . We also estimate the model for the GJR sample period with or without the one-month T-bill returns  $rf_t$  included in the conditional variance equation.

 Model 3 is specified to capture the asymmetric effect of a prior positive and negative volatility shock on the conditional volatility.

*Model 3:* 

$$
r_{t} = \mu + \phi r_{t-1} + (\delta_{1} + \tau_{1} M_{t}) \sqrt{h_{t}} + \varepsilon_{t}
$$
  
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + \pi M_{t} + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}).
$$
\n(8)

The only difference between Model 3 and Model 2 is the term  $\pi M$ , in the conditional variance equation, which captures the asymmetric effect of a prior positive and negative volatility shock on the conditional volatility. The level of the conditional volatility process shifts up (or down) by  $\pi$  under a prior positive (negative) volatility shock. Our empirical results confirm the positive effect ( $\pi$  > 0) of a positive volatility shock on the conditional volatility.

We also evaluate Model 3 for the GJR sample period with the following model:  *Model 3 for GJR Period:* 

$$
r_{t} = \mu + \phi r_{t-1} + [(\delta_{1} + \tau_{1} M_{t}) + (\delta_{1}^{G} + \tau_{1}^{G} M_{t}) G_{t}] \sqrt{h_{t}} + \varepsilon_{t}
$$
  
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + \theta r f_{t} + \pi M (d_{t-1}) + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}),
$$
\n(9)

where  $G_t$  is a time dummy variable that takes the value 1 for the GJR period or 0 otherwise. The coefficients  $\delta^G$  and  $\tau_1^G$  capture the differential effect of the GJR period on the relation. The intertemporal relation for the GJR period is measured by  $\delta_1 + \delta_1^G + \tau_1 + \tau_1^G$  under a prior positive volatility shock or by  $\delta_1 + \delta_1^G$  otherwise. We also estimate the model for the GJR sample period with or without the one-month T-bill returns  $rf_t$  included in the conditional variance equation.

## **II. Empirical Results**

### *A. The Data*

We employ the excess market returns as the expected market risk premiums. To generate return series is computed as percentage returns. The analysis employs three sample periods, the 89:12). We evaluate the GJR period by using a time dummy variable to represent the period in weighted market index exhibit significant excess kurtosis and positive first-order autocorrelation, characterizing the nonnormality of the short-horizon stock returns. the excess returns, we use the monthly nominal returns of the value-weighted market portfolio index of the NYSE, AMEX and NASDAQ from the CRSP data files from 1926:01 to 1999:12. The monthly excess return series is constructed by subtracting the one-month US Treasury bill returns reported by Ibbotson Associates from the monthly nominal index returns. The excess full-period (26:01–99:12), the pre-87 Crash period (26:01–87:09), and the GJR period (51:04– the estimation of the full period. Table 1 reports the summary statistics for the data. The descriptive statistics indicate that both the nominal and the excess returns series of the value-

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[*Insert Table I about here*]

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### B. Estimation Results, Interpretations and Diagnostics

 Model 1 is designed to examine the sign of the simple linear relation between the expected risk premium and the predictable market volatility. We employ the maximum likelihood method with the analytical derivatives of each parameter provided in the Gauss cod e. All the statistical inferences are based on the Bollerslev-Wooldrige's (1992) robust standard errors. Note that Model 1 measures only a partial intertemporal relation as it ignores the effect of a volatility shock on the relation. Estimation results for Model 1 are presented in Table 2, which reports a positive intertemporal relation. Table 2 shows that for both the full period, and the pre-87 Crash period, the estimated value of  $\delta$  is positive (0.084 and 0.099, respectively, for the two periods) and statistically significant at the 1% level. A notable finding is that for the GJR sample period, the estimated value of the coefficient  $\delta^G$  is strongly negative (-0.092 without  $r f_t$  and -0.103 with *rf<sub>t</sub>*) and highly significant with  $\delta + \delta^G < 0$ . Note that  $\delta^G$  measures the differential effect of the GJR sample period on the relation, such that the partial intertemporal relation for th e GJR sample period is measured by  $\delta + \delta^G$ . This result is consistent with that of Glosten et al. (1993). We thus confirm that the GJR sample period is characterized by a strong negative partial intertem poral relation when the effect of a volatility shock is not incorporated in the estimation.

With regard to the conditional variance equation, the asymmetric volatility response of a positive and negative return shock is well captured by  $b_1 + b_2 < 0$  with a statistical significance. Also, the estimation results show a high estimated value of the transition parameter  $\gamma$ , which indicates that the transition between volatility-regimes occurs very quickly. The volatility regi me is divided into only two extreme regimes, the upper and lower volatility regimes. The upper (lower) regime induced by a negative (positive) return shock is the high-volatility-persistence (low-volatility-persistence) regime.

[*Insert Table II about here*]

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<span id="page-17-0"></span>---

not well specified. The Ljung-Box Q statistics on the *squared* normalized residuals ascertains if effect of a negative return shock on the conditional variance process. <sup>[13](#page-17-0)</sup> The Ljung-Box  $Q(10)$ captured by Model 1. The negative sign bias test shows insignificant *t*-values for all estimations, and indicates that the asymmetric volatility response to a positive and negative return shock is well captured by the ANST-GARCH model. Table 3 reports the summary of diagnostics for the estimation results of Model 1, such as skewness, kurtosis, the Jarque-Bera normality test, and the Ljung-Box Q test on the normalized and the squared normalized residuals. The Ljung-Box Q statistics on the normalized residuals checks serial correlation in the residuals. Rejection of the null hypothesis of no autocorrelation up to a certain lag length, indicates that either the dynamic structure of the mean equation or the lag structure of the conditional variance equation is not well specified, or that both equations are the serial dependence in the conditional variance is well captured by the conditional variance equation. We also perform the *Negative Sign Bias Test* (NSBT) suggested by Engle and Ng (1993) to examine the ability of the ANST-GARCH-M model to capture the so-called leverage test indicates that the serial dependence of the conditional mean and variance process is well

 $\overline{a}$ 

<sup>&</sup>lt;sup>13</sup> The negative sign bias test is performed with the regression equation  $v_t^2 = a + bS_{t-1} - \varepsilon_{t-1} + \pi \varepsilon_t^* + e_t$ , where  $v_t^2 = (\varepsilon_t / \sqrt{h_t})^2$ .  $S_{t-1}^- = 1$  if  $\varepsilon_{t-1} < 0$ , and  $S_{t-1}^- = 0$  otherwise. Also,  $z_t^* = \tilde{h}(\Psi)/h_t$ , where  $\tilde{h}(\Psi) = \partial h_t / \partial \Psi$ evaluated at the values of maximum likelihood estimates of parameter Ψ . The test statistic of the NSBT is defined as the *t*-ratio of the coefficient *b* in the regression. A statistically significant *t*-value implies the failure of the model to absorb the effect of sign bias, and indicates that the volatility model considered is misspecified.

#### [*Insert Table III about here*]

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Model 1 suffers from an omitted variable bias, as the estimate of the coefficient  $\delta$  in Model 1 measures only a partial relation. The full intertemporal relation is measured by Model 2 which captures the effect of the predictable volatility and a volatility shock simultaneously on the relation. Estimation results of Model 2 are reported in Table 4. There are several notable findings. First, there is a significant asymmetrical effect of a prior positive and negative volatility shock on the intertemporal relation. Second, the result of  $\tau_1 = 0.040$  (0.031 for the pre-87 Crash period) implies that the expected risk premium increases under a prior positive volatility shock (a higher conditional volatility than expected, i.e.,  $\varepsilon_{t-1}^2 > h_{t-1}$ ). Third, due to the significant positive effect of a positive volatility shock, the full intertemporal relation measured by  $\delta_1 + \tau_1$  is strongly positive and highly significant. For the full period and the pre-87 Crash period,  $\delta_1 + \tau_1 = 0.098$  (0.107 for the pre-87 Crash period) under a prior positive volatility shock and  $\delta_1 = 0.058$  (0.076 for the pre-87 Crash period) otherwise. Fourth, the estimation results for the GJR sample period provide some important information about the empirical nature of the relation: (a) the estimation result of  $\delta_1^G + \tau_1^G = -0.065$  (-0.088 with  $r f_t$ ) confirms that the GJR period is characterized by a significant reduction in risk premium, which is consistent with the result of Glosten et al. (1993); (b) the estimation result of  $\tau_1 + \tau_1^G = 0.137$  (0.113 with  $r f_t$ included) implies that there is a strong positive effect of a prior positive volatility shock on the intertemporal relation for the GJR sample period; (c) due to the strong positive effect of a prior positive volatility shock on the relation, the GJR period indeed exhibits a strong positive intertemporal relation ( $\delta_1 + \delta_1^G + \tau_1 + \tau_1^G = 0.081$  without  $r f_t$  and 0.061 with  $r f_t$ ); (d) the

estimates are all highly significant and are not sensitive to the inclusion or exclusion of  $r f_t$  in the conditional variance equation. The estimation results for the GJR sample period indicates that a negative intertemporal relation reported by Glosten et al. (1993) could be attributed to the omitted v ariable bias resulting from ignoring the effect of an unexpected volatility shock on the relation.

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[*Insert Table IV about here*]

effect of a prior positive and negative volatility shock on the conditional volatility process. The Table 4 also reports the estimation results of Model 3, which captures the asymmetrical estimated value of  $\pi$  is positive, implying that a prior positive volatility shock amplifies the conditional volatility. The estimated value of  $\tau_1$  is also positive and statistically significant at the 1% level, indicating that a prior positive volatility shock raises the risk premium. Thus, the estimated result of  $\pi > 0$  and  $\tau_1 > 0$  clearly indicates that a prior positive volatility shock increases risk premium due to an increase in the conditional volatility. Interestingly, the results show that the magnitude of  $\tau_1$  is much greater in Model 3 ( $\tau_1 = 0.062$  and 0.070 for the pre-87 Crash period) than in Model 2 ( $\tau_1$  = 0.040 and 0.031 for the pre-87 Crash period). This implies that incorporating an asymmetrical effect of a prior positive and negative volatility shock in th e conditional volatility process yields a more profound asymmetric effect on the intertempora l relation. In other words, a prior positive and negative volatility shock has a critical role in inducing an intertemporal relation. Estimatio n results for the GJR period show similar results with some variation in parameter estimates.

Diagnostic tests reported in Table 5 indicate that all the estimations pass the Ljung-Box Q(10) test on the normalized and squared normalized residuals. This result implies that there is

no serial dependence remaining in the conditional mean and variance processes. The negative sign bias test shows insignificant *t*-values for all estimations, indicating that the estimated conditional va riance process well captures the excess volatility response caused by a negative return shock.

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[*Insert Table V about here*]

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conclusions: (1) with an asymmetrical effect of a prior unexpected volatility shock incorporated period, the magnitude of the risk premium could be dramatically reduced; and (3) a negative variable bias resulting from ignoring the effect of an unexpected volatility shock on the relation. From the estimation results of models 1 through 3, we have come to the following three in the estimation, the intertemporal relation is strongly positive and highly significant; (2) the intertemporal relation can be time-varying, such that for a certain period, i.e., the GJR sample intertemporal relation reported by the previous literature could be attributed to the omitted

## **III. Asymmetric Reverting Property of Expected returns**

pattern, in that negative returns are more likely to revert to positive returns than positive returns raises the expected returns as investors require an additional premium to compensate for the It has been known that the expected market returns exhibit a strong asymmetric reverting reverting to negative returns. Also, a negative return shock is known to generate excess volatility. However, although it seems plausible that a negative return shock raises risk premium due to an excess future volatility, a positive intertemporal relation cannot explain the quicker reversion of negative returns. Under a positive intertemporal relation, a negative return shock excess volatility. An increase in risk premium reduces the current stock price, such that a negative return should be accompanied by another negative return. Therefore, it is important to

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<span id="page-21-0"></span>investigate what causes such an asymmetric reverting behavior of the expected market returns. First, we specify a model to capture the asymmetric reverting pattern of the expected returns. Since the asymmetric reverting pattern is not captured by a l inear autoregressive specification, we specify the following nonlinear autoregressive model:

*Model 4:* 

$$
r_{t} = [\mu_{1} + \mu_{2} F(\varepsilon_{t-1})] + [\phi_{1} + \phi_{2} F(\varepsilon_{t-1})]r_{t-1} + \varepsilon_{t}
$$
  
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}]F(\varepsilon_{t-1}).
$$
\n(10)

model allows the return serial correlation coefficient to vary with a prior positive and negative return shock, such that the asymmetric reverting property is captured by the changing serial The main feature of Model 4 is that it captures the asymmetry in both the conditional volatility and the conditional mean processes caused by a prior positive and negative return shock. The correlation  $\phi_1 + \phi_2 F(\varepsilon_{t-1})$ . The value of the serial correlation varies between  $\phi_1$  and  $\phi_1 + \phi_2$ depending on the value of the transition function  $F(\varepsilon_{t-1})$ . For an extreme negative return shock causing  $F(\varepsilon_{t-1}) = 0$ , serial correlation is measured by  $\phi_1$ , while it is measured by  $\phi_1 + \phi_2$  for an extreme positive return shock making  $F(\varepsilon_{t-1}) = 1$ . <sup>[14](#page-21-0)</sup> Quicker reversion of a negative return is thus captured by  $\phi_2 > 0$  (=  $\phi_1 + \phi_2 > \phi_1$ ). Note that the condition  $\phi_1 < 0$  with  $\phi_2 > 0$  indicates a negative serial correlation under a prior n egative return shock, thereby indicating a stronger reverting tendency of a negative return.

Estimation results of Model 4 are reported in Table 6, which shows that the estimated value of  $\phi_2$  is positive and highly significant for both the full period and the pre-87 Crash period.

<sup>&</sup>lt;sup>14</sup> Stationarity condition of  $r_t$  is satisfied with  $|\phi_1 + \phi_2 F(\varepsilon_{t-1})|$  < 1, *i.e.*,  $|\phi_1|$  < 1 for  $\varepsilon_{t-1}$  < 0 or  $|\phi_1 + \phi_2|$  < 1 for  $\varepsilon_{t-1} > 0$ .

The measured serial correlation is negative ( $\phi_1 = -0.091$  for both periods) under a prior negative return shock, while it is positive ( $\phi_1 + \phi_2 = 0.059$  and 0.124, respectively, for the two periods) under a prior positive return shock. This result confirms the asymmetric reverting pattern of the expected returns that a negative return reverts more quickly, while a positive return tends to persist.

specify the following model (Model 5) to examine the possible asymmetry in the intertemporal relation . Now the important question is whether the asymmetric reverting pattern of the expected market returns can be explained by the intertemporal relation. To answer the question, we allow an asymmetrical effect of a positive and negative return shock on the relation, such that the sign of the intertemporal relation can be compared to prior positive and negative return shocks. We

Model 5:  
\n
$$
r_{t} = [\mu_{1} + \mu_{2} F(\varepsilon_{t-1})] + \phi_{1} r_{t-1} + [\delta_{1} + \delta_{2} F(\varepsilon_{t-1})] \sqrt{h_{t}} + \varepsilon_{t}
$$
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}). \tag{11}
$$

The intertemporal relation is measured by the estimated value of  $\delta_1$  under a prior negative return shock causing  $F(\varepsilon_{t-1}) = 0$ , while it is measured by  $\delta_1 + \delta_2$  under a prior positive return shock causing  $F(\varepsilon_{t-1}) = 1$ . Specifically,  $\delta_2$  measures the differential effect of a positive and negative return shock on the intertemporal relation. A positive intertemporal relation implies that, due to its excess future volatility, a negative return shock should raise the risk premium, such that the magnitude of the relation under a prior negative return shock should be greater than that under a positive return shock, i.e.,  $\delta_1 > \delta_1 + \delta_2$  or  $\delta_2 < 0$ . In other words,  $\delta_2 < 0$  is consistent with a positive intertemporal relation. As mentioned earlier, however, a positive intertemporal relation cannot explain the observed asymmetric reverting pattern of the expected returns.

The estimation results of Model 5 are presented in Table 6, which shows two interesting findings. First, the estimated value of  $\delta_2$  is positive and statistically significant at the 1% level  $(\delta_2 = 0.259$  and 0.266 for the pre-87 Crash period). The result indicates that the magnitude of the intertemporal relation under a prior positive return shock is greater than that under a prior negative return shock. Thus, the result of  $\delta_2 > 0$  clearly indicates that a prior negative return shock does not induce a positive intertemporal behavior of the expected market returns. Second, the intertemporal relation is indeed negative ( $\delta_1 = -0.175$  and  $-0.224$  for the pre-87 Crash period) under a prior negative return shock, while it is positive ( $\delta_1 + \delta_2 = 0.084$  and 0.042 for the pre-87 Crash period) under a prior positive return shock. This result implies that while a positiv e return shock complies with the conventional positive intertemporal relation, a negative return shock indeed induces a negative intertemporal behavior of the expected market returns. We interpret this negative intertemporal relation under a prior negative return shock as a result of an optimistic expectation about the future performance of stock price by rational investors who eventually reduce risk premium to a price drop.

More importantly, the quicker reversion of a negative return can be attributed to the negative intertemporal relation under a prior negative return shock. If a negative return shock generates an optimistic expectation on the future performance of stock price, then the quicker reversion of a negative return can be explained by a reduction in premium. A reduction in risk premium in responding to a negative return shock raises the current stock price, thereby causing a quicker reversion of a negative return.

 [*Insert Table VI about here*] ---

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Model 5 ignores the effect of a prior volatility shock in estimation. We thus specify Model 6 to incorporate the effect of an unexpected volatility shock on the asymmetrical intertemporal relation of a prior positive and negative return shock:

Model 6:  
\n
$$
r_{t} = [\mu_{1} + \mu_{2} F(\varepsilon_{t-1})] + \phi_{1} r_{t-1} + [(\delta_{1} + \tau_{1} M_{t}) + (\delta_{2} + \tau_{2} M_{t}) F(\varepsilon_{t-1})] \sqrt{h_{t}} + \varepsilon_{t}
$$
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}). \tag{12}
$$

While  $\delta_2 + \tau_2$  measures an asymmetrical effect of a prior positive and negative return shock,  $\tau_1 + \tau_2$  measures an asymmetrical effect of a prior volatility shock on the intertemporal relation. When a negative return shock is realized, the intertemporal relation is measured by  $\delta_1 + \tau_1$  under a prior positive volatility or by  $\delta_1$  otherwise. With a prior positive return shock, the relation is measured by  $\delta_1 + \delta_2 + \tau_1 + \tau_2$  under a prior positive volatility shock or by  $\delta_1 + \delta_2$  otherwise.

The estimation results of Model 6 are also reported in Table 6. It shows several notable findings. (a) the estimation result of  $\tau_1 + \tau_2 = 0.064$  (0.073 for the pre-87 Crash period) and  $\delta_2 + \tau_2 = 0.236$  (0.290 for the pre-87 Crash period) confirm that both the return shock and the volatility shock have an asymmetrical effect on the intertemporal relation; (b) The results of  $\tau_1 = 0.078$  (0.062 for the pre-87 Crash period) and  $\tau_1 + \tau_2 = 0.064$  (0.073 for the pre-87 Crash period) indicate a positive effect of a positive volatility shock on the intertemporal relation, regardless of the sign of a prior return shock. This result is consistent with the estimation results of Models 2 and 3; and (c) the result of  $\delta_1 + \tau_1 = -0.150$  (–0.138 for the pre-87 Crash period) and  $\delta_1 + \delta_2 + \tau_1 + \tau_2 = 0.094$  (0.092 for the pre-87 Crash period) implies that the intertemporal relation is negative (positive) under a prior negative (positive) return shock. This result is also consistent with the estimation result of Model 5, implying that the asymmetrical effect of a prior

positive and negative return shock is still persistent even under a presence of a prior positive and negative volatility shock. Table 7 reports diagnostic tests. The Ljung-Box  $Q(10)$  test on  $v_t$  and  $v_t^2$  indicates that serial dependence is well captured by the specified conditional mean and variance processes. The negative sign bias test shows insignificant *t*-values for all estimations, confirming the ability of the ANST-GARCH model to capture the excess volatility response caused by a negative return shock.

In sum, the estimation results of Models 4 through 6 provide the following conclusions. First, the expected market returns exhibit not only an asymmetric reverting pattern but also asymmetrical intertemporal behavior under a prior positive and negative return shock. Second, there is always a positive effect of a positive volatility shock on the intertemporal relation, regardless of the sign of the return shock. Lastly, a negative intertemporal relation under a prior negative return shock is attributable to the observed quicker reversion of a negative return**.** Due to their strong optimistic expectations of the future performance of stock prices, rational investors indeed reduce risk premium with an unexpected price drop, and the reduction in risk premium in turn raises the current stock price. This uplifting force on the stock price then induces a quicker reversion of a negative return.

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[*Insert Table VII about here*]

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# **IV. Conclusions**

We suggest that the intertemporal behavior of the expected market returns is driven not only by the underlying market volatility but also by the effect of an unexpected volatility shock. Most of the previous literature on this topic ignores the effect of an unexpected volatility shock on the relation, hence the previous empirical results reflect only a partial intertemporal relation.

Allowing an asymmetrical effect of a prior positive and negative volatility shock on the relation, we find a strong, highly significant intertemporal relation. We believe that the negative intertemporal relation reported by the previous literature could be attributed to the omitted variable bias resulting from ignoring the effect of an unexpected volatility shock on the relation. We also find that the intertemporal relation can be time-varying, such that for a certain period the magnitude of the risk premium could be dramatically reduced.

We show that the expected returns exhibit asymmetrical intertemporal behavior under a prior positive and negative return shock. The intertemporal relation is negative (positive) under a prior negative (positive) return shock. The asymmetrical intertemporal behavior of the expected market returns attributable to the observed asymmetric reverting pattern indicates that a negative return on average reverts more quickly to a positive return than a positive return reverting to a negative return. Specifically, the quicker reversion of a negative return is attributed to a negative intertemporal relation. We interpret this negative intertemporal relation under a prior negative return shock as a reflection of a strong optimistic expectation perceived by investors on the future performance of a stock with a recent price drop.

## **REFERENCES**

- Abel, A. B., 1988, Stock prices under time-varying dividend risk: an exact solution in an infinitehorizon general equilibrium model, *Journal of Monetary Economics* 22, 373-393.
- Backus, D., and A. W. Gregory, 1993, Theoretical Relations between risk premiums and conditional variances, *Journal of Business and Economic Statistics* 11, 177-185.
- Ball, R., and S. P. Kothari, 1989, Nonstationary Expected Returns: Implications for Tests of Market Efficiency and Serial Correlation in Returns, *Journal of Financial Economics* 25, 51-74.
- Baillie, R., and R. P. DeGennaro, 1990, Stock returns and volatility, *Journal of Financial and Quantitative Analysis* 25, 203-214.
- Barsky, R. B., 1989, Why don't the prices of stocks and bonds move together? *American Economic Review* 79, 1132-1145.
- Boudoukh, J., M. Richardson, and R. F. Whitelaw, 1997, Nonlinearities in the relation between the equity risk premium and the term structure, *Management Science* 43, 371-385.
- Brandt, M. W., and Q. Kang, 2004, On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach, *Journal of Financial Economics* 72, 217-257.
- Breeden, D. T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265-296.
- Breen, W., L. R. Glosten, and R. Jagannathan, 1989, Economic significance of predictable variations in stock index returns, *Journal of Finance* 44, 1177-1189.
- Campbell, J. Y., 1987, Stock returns and the term structure, *Journal Financial Economics* 18, 373-399.
- Campbell, J. Y., and L. Hentschel, 1992, No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns, *Journal Financial Economics* 31, 281-318.
- Cecchetti, S. G.; P. Lam; and N. C. Mark, 1990, Mean Reversion in Equilibrium Asset Prices, *American Economic Review* 80, 398-418.
- Engle, R. F., and V. K. Ng, 1993, Measuring and Testing the Impact of News on Volatility, *Journal of Finance* 48, 1749-78.
- Evans, M. D., 1994, Expected returns, Time-varying risk, and risk premia, *Journal of Finance* 49, 655-679.
- Fama, E. F., and K. R. French, 1988, Permanent and Temporary Components of Stock Prices, *Journal of Political Economy* 96, 246-273.
- Fama, E. F., and K. R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics 25*, 23-49.
- Fama, E. F., K. R. French, 1992. The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.
- Fama, E. F., K. R. French, 1993. Common risk factors in the returns on stock and bonds, *Journal of Financial Economics* 33, 3-56.
- Ferson, W., and C. R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385-415.
- Fornari, F., Mele, A., 1997, Sign- and volatility-switching ARCH models: Theory and applications to international stock markets. *Journal of Applied Econometrics* 12, 49-65.
- French, K. R., G. W. Schwert, and R. F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics* 19, 13-29.
- Friedman, M., 1957, A theory of the consumption function (Princeton University Press, Princeton, NJ).
- Gennotte, G., and T. A. Marsh, 1993, Variations in economics uncertainty and risk premiums on capital assets, *European Economic Review* 37, 1021-1041.
- Ghysel, E., P. Santa-Clara, and R. Valkanov, 2005, There is a risk-return trade-off after all, *Journal of Financial Economics*, forthcoming.
- Glosten, L. R., R. Jagannathan, and D. E. Runkle, 1993, On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, *Journal of Finance* 48, 1779-1801.
- Gonzalez-Rivera, G., 1998, Smooth transition GARCH models. *Studies in Nonlinear Dynamics and Econometrics* 3, 61-78.
- Hamilton, J. D., and R. Susmel, 1994, Autoregressive Conditional Heteroskedasticity and Changes in Regime, *Journal of Econometrics* 64, 307-333.
- Harvey, C., 1989, Time-varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics* 24, 289-317.
- Harvey, C., 2001, The specification of conditional expectations, *Journal of Empirical Finance* 8, 573-638.
- Haugen, R. A., E. Talmor, and W. N. Torous, 1991, The Effect of Volatility Changes on the Level of Stock Prices and Subsequent Expected Returns, *Journal of Finance* 46, 985-1007.
- Kim, C., J. C. Morley, and C. R. Nelson, 2001, Does an intertemporal tradeoff between risk and return explain mean reversion in stock price?, *Journal of Empirical Finance* 8, 403-426.

Lucas, R. E., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429-1446.

Ludvigson, S. C., Ng, S., 2006, The empirical risk-return relation: A factor Analysis approach, *Journal of Financial Economics*, forthcoming.

Merton, R. C., 1973, An intertemporal asset pricing model, *Econometrica* 41, 867-888.

- Merton, R. C., 1980, On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics* 8, 323-361.
- Modigliani, F., and R. Brumberg, 1955, Utility analysis and the consumption function, in K. Kurihara, ed., *Post Keynesian Economics*, (G. Allen, London).
- Nam, K., C. S. Pyun, A. C. Arize, 2002, Asymmetric Return Reversals and Contrarian Profits: ANST-GARCH Approach, *Journal of Empirical Finance* 9, 475-608.
- Nam, K., C. S. Pyun, S. Avard, 2001, Asymmetric reverting behavior of short-horizon stock returns: An evidence of stock market overreaction, *Journal of Banking and Finance* 25, 805-822.
- Nelson, D. B., 1991, Conditional Heteroskedasticity in Asset Returns: A New Approach, *Econometrica* 59, 347-370.
- Pindyck, Robert S., 1984, Risk, Inflation, and the Stock Market, *American Economic Review* 74, 335-51.
- Pagan, A. R., and Y. S. Hong, 1989, Nonparametric estimation and the risk premium, in William A. Barnett, James Powell, and George E. Tauchen, eds.: *Semiparametric and Nonparametric Methods in Econometrics and Statistics* (Cambridge University Press, Cambridge), 51-75.
- Poterba, J. M., and L. H. Summers, 1986, The persistence of volatility and stock market fluctuations, *American Economic Review* 76, 1142-1151.
- Scruggs, J. T., 1998, Resolving the puzzling intertemporal relation between the market risk premium and conditional market variance: a two-factor approach, *Journal of Finance* 52, 575-603.
- Turner, Christopher M., Richard Startz, and Charles R. Nelson, 1989, A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market, *Journal of Financial Economics* 25, 3-22.
- Whitelaw, R. F., 1994, Time variations and covariations in the expectation and volatility of stock market returns, *Journal of Finance* 49, 515-541.
- Whitelaw, R. F., 2000, Stock market risk and return: An equilibrium approach, *Review of Financial Studies* 13, 521-547.

#### **Table I Summary statistics for monthly nominal and excess returns**

The nominal return series are the monthly value-weighted market index returns for NYSE, AMEX, and NASDAQ stocks, and were retrieved from the CRSP tapes for the period from 1926:01 to 1999:12. The monthly nominal return series are the value-weighted market indexes retrieved from the CRSP tapes for the period from 1926:01 to 1999:12. The monthly excess return series is computed by subtracting one-month Treasury bill returns as reported by Ibbotson Associates from the nominal returns. All returns are computed as percentage value. The analysis employs the full-period (26:01-99:12) and the pre-87 Crash period (26:01-87:09). The value in the parentheses is the *p*-value for the Ljung-Box Q test.



#### **Table II Estimation of Model 1**

This table presents the maximum likelihood estimates for Model 1 for monthly excess returns of the value-weighted index for the NYSE, AMEX, and NASDAQ stocks. The estimation employs the full period (26:01-99:12) and the pre 87 Crash period (26:01-87:09). For the monthly excess return series  $r<sub>i</sub>$ , model 1 is specified as follows:

Model 1:  
\n
$$
r_{i} = \mu + \phi r_{i-1} + \delta \sqrt{h_{i}} + \varepsilon_{i}
$$
\n
$$
h_{i} = [a_{0} + a_{1} \varepsilon_{i-1}^{2} + a_{2} h_{i-1}] + [b_{0} + b_{1} \varepsilon_{i-1}^{2} + b_{2} h_{i-1}] F(\varepsilon_{i-1}),
$$

where  $F(\varepsilon_{t-1}) = \{1 + \exp[-\gamma(\varepsilon_{t-1})]\}^{-1}$ . The GJR sample period (51:04 – 89:12) is evaluated by incorporating the period as a dummy variable in the mean equation, with the one-month T-bill rates included or excluded in the conditional variance equation. The model to estimate the GJR sample period is as follows:

Model 1 for GJR sample period:  
\n
$$
r_{t} = \mu + \phi r_{t-1} + (\delta + \delta^{G} G_{t}) \sqrt{h_{t}} + \varepsilon_{t}
$$
\n
$$
h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + \theta r f_{t} + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}),
$$

where  $F(\varepsilon_{t-1}) = \{1 + \exp[-\gamma(\varepsilon_{t-1})]\}^{-1}$ , the indicator function  $G_t$  is a dummy variable for the GJR sample period, and  $rf_t$  is the yield on one month T-bill from Ibbotson Associates. The values in parentheses are the Bolleslev-Wooldridge robust *t*-statistics, and LLV is the log-likelihood value.



#### **Table III Diagnostics of Model 1**

This table presents a summary of diagnostics on the normalized residuals and the squared normalized residuals from the estimations. The normalized residual series is defined as  $v_t = \varepsilon_t / \sqrt{h_t}$ . JB-Normality refers to the Jarque-Bera normality test statistic, which is distributed as  $\chi^2$  with two degrees of freedom under the null hypothesis of performed with the regression equation  $v_t^2 = a + bS_{t-1}^- \varepsilon_{t-1} + \pi \varepsilon_t^* + e_t$ , where  $v_t^2 = (\varepsilon_t / \sqrt{h_t})^2$ .  $S_{t-1}^- = 1$  if  $\varepsilon_{t-1} < 0$ , normally distributed residuals. Q(10) is the Ljung-Box Q(10) test statistic for checking serial dependence in the normalized residuals and the squared normalized residuals from the estimations. NSBT refers to the negative sign bias test suggested by Engle and Ng (1993). It is a diagnostic test that examines the ability of the specified model to capture the so-called leverage effect of a negative return shock on the conditional volatility process. The test is and  $S_{t-1}^- = 0$  otherwise. Also,  $z_t^* = \tilde{h}(\Psi)/h_t$ , where  $\tilde{h}(\Psi) = \partial h_t/\partial \Psi$ , is evaluated at the values of the maximum likelihood estimates of parameter Ψ . The test statistic of the NSBT is defined as the *t*-ratio of the coefficient *b* in the regression. The value in the parentheses is the *p*-value of the individual test statistics considered.



#### **Table IV Estimation of Models 2&3**

This table presents the maximum likelihood estimates of Models 2&3 for the monthly excess returns of the value-weighted index for the NYSE, AMEX, and NASDAQ stocks. The estimation employs the full period (26:01-99:12) and the pre 87 Crash period (26:01-87:09). While the same conditional mean equation is specified for models 2&3, different conditional variance equations are specified for each model. For the monthly excess return series  $r<sub>i</sub>$ , the conditional mean and variance equations of models 2&3 are specified as follows:

*Model 2*:  $r_t = \mu + \phi r_{t-1} + (\delta_1 + \tau_1 M_t) \sqrt{h_t + \varepsilon_t}$  $h_{t} = [a_{0} + a_{1}\varepsilon_{t-1}^{2} + a_{2}h_{t-1}] + [b_{0} + b_{1}\varepsilon_{t-1}^{2} + b_{2}h_{t-1}]F(\varepsilon_{t-1}),$ *Model 3*:  $r_t = \mu + \phi r_{t-1} + (\delta_1 + \tau_1 M_t) \sqrt{h_t + \varepsilon_t}.$  $h_{t} = [a_{0} + a_{1}\varepsilon_{t-1}^{2} + a_{2}h_{t-1}] + \pi M_{t} + [b_{0} + b_{1}\varepsilon_{t-1}^{2} + b_{2}h_{t-1}]F(\varepsilon_{t-1})$ 

where  $F(\varepsilon_{t-1}) = \{1 + \exp[-\gamma(\varepsilon_{t-1})]\}^{-1}$ . The indicator function  $M_t$  is specified to capture the asymmetric effect of an unexpected volatility change on the intertemporal relation, such that  $M_t = 1$  if  $\varepsilon_{t-1}^2 > h_{t-1}$  or  $M_t = 0$  otherwise. The intertemporal relation for the GJR sample period (51:04-89:12) is evaluated by using a dummy variable to represent the period in the mean equation, with the one-month T-bill rates included or excluded in the conditional variance equation. The models for the GJR sample period are as follows:

*Model 2 for GJR sample period*:  $r_{t} = \mu + \phi r_{t-1} + [(\delta_{1} + \tau_{1}M_{t}) + (\delta_{1}^{G} + \tau_{1}^{G}M_{t})G_{t}]\sqrt{h_{t}} + \varepsilon_{t}$  $h_{t} = [a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + \theta r f_{t} + [b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1}),$ *Model 3 for GJR sample period*:  $r_{t} = \mu + \phi r_{t-1} + [(\delta_1 + \tau_1 M_t) + (\delta_1^G + \tau_1^G M_t) G_t] \sqrt{h_t + \varepsilon_t}.$  $h_{t} = [ a_{0} + a_{1} \varepsilon_{t-1}^{2} + a_{2} h_{t-1}] + \theta r f_{t} + \pi M_{t} + [ b_{0} + b_{1} \varepsilon_{t-1}^{2} + b_{2} h_{t-1}] F(\varepsilon_{t-1})$ 

where the indicator function  $G_t$  is a dummy variable to represent the GJR sample period, and  $rf_t$  is the yield on a one month T-bill from Ibbotson Associates. Values in parentheses are the Bolleslev-Wooldridge robust *t*-statistics, and LLV is the log-likelihood value.



#### **Table V Diagnostics of Models 2&3**

This table presents a summary of the diagnostics on the normalized residuals and the squared normalized residuals from the estimations. The normalized residual series defined as  $v_t = \varepsilon_t / \sqrt{h_t}$ . JB-Normality refers to the Jarque-Bera normality test statistic, which is distributed as a  $\chi^2$  with two degrees of freedom under the null hypothesis of normally distributed residuals. Q(10) is the Ljung-Box Q(10) test statistic for checking serial dependence in the normalized residuals and the squared normalized residuals from the estimations. NSBT refers to the negative sign bias test suggested by Engle and Ng (1993). It is a diagnostic test examining the ability of the specified model to capture the so-called leverage effect of a negative return shock on the conditional volatility process. The test is performed with the regression equation  $v_t^2 = a + bS_{t-1} - \varepsilon_{t-1} + \pi \varepsilon_t^* + e_t$ , where  $v_t^2 = (\varepsilon_t/\sqrt{h_t})^2$ .  $S_{t-1} = 1$  if  $\varepsilon_{t-1} < 0$ , and  $S_{t-1} = 0$  otherwise. Also,  $z_t^* = \tilde{h}(\Psi)/h_t$ , where  $\tilde{h}(\Psi) = \partial h_t / \partial \Psi$ , is evaluated at the values of maximum likelihood estimates of parameter  $\Psi$ . The test statistic of the NSBT is defined as the *t*-ratio of the coefficient *b* in the regression. The value in the parentheses is the *p*-value of the individual test statistics considered.



#### **Table VI Estimation of Models 4-6**

This table presents the maximum likelihood estimates of models 4 through 6 for the monthly excess returns of the value-weighted index for the NYSE, AMEX, and NASDAQ stocks over the period from 26:01 to 99:12 and the period from 26:01 to 87:09. For the monthly excess return series  $r_t$ , each model is specified as follows:

Model 4: 
$$
r_t = [\mu_1 + \mu_2 F(\varepsilon_{t-1})] + [\phi_1 + \phi_2 F(\varepsilon_{t-1})]r_{t-1} + \varepsilon_t
$$
  
\nModel 5:  $r_t = [\mu_1 + \mu_2 F(\varepsilon_{t-1})] + \phi r_{t-1} + [\delta_1 + \delta_2 F(\varepsilon_{t-1})] \sqrt{h_t} + \varepsilon_t$   
\nModel 6:  $r_t = [\mu_1 + \mu_2 F(\varepsilon_{t-1})] + \phi_1 r_{t-1} + [(\delta_1 + \tau_1 M_t) + (\delta_2 + \tau_2 M_t) F(\varepsilon_{t-1})] \sqrt{h_t} + \varepsilon_t$ ,

where the indicator function  $M_t$  is specified to capture the asymmetric effect of an unanticipated volatility shock on the intertemporal relation, such that  $M_t = 1$  if  $\varepsilon_{t-1}^2 > h_{t-1}$  or  $M_t = 0$  otherwise. The conditional variance equation for models 4-6 is specified as  $h_t = [a_0 + a_1 \varepsilon_{t-1}^2 + a_2 h_{t-1}] + [b_0 + b_1 \varepsilon_{t-1}^2 + b_2 h_{t-1}] F(\varepsilon_{t-1})$ . The values in parentheses are the Bolleslev-Wooldridge robust *t*-statistics, and LLV is the log-likelihood value.



#### **Table VII Diagnostics of Models 4-6**

This table presents a summary of diagnostics on the normalized residuals and the squared normalized residuals from the estimations. The normalized residual series is defined as  $v_t = \varepsilon_t / \sqrt{h_t}$ . JB-Normality refers to the Jarque-Bera normality test statistic, which is distributed as a  $\chi^2$  with two degrees of freedom under the null hypothesis of normally distributed residuals. Q(10) is the Ljung-Box Q(10) test statistic for checking serial dependence in the normalized residuals and the squared normalized residuals from the estimations. NSBT refers to the negative sign bias test suggested by Engle and Ng (1993). It is a diagnostic test for examining the ability of the specified model to capture the so-called leverage effect of a negative return shock on the conditional volatility process. The test is performed with the regression equation  $v_t^2 = a + bS_{t-1}^- \varepsilon_{t-1} + \pi \varepsilon_t^* + e_t$ , where  $v_t^2 = (\varepsilon_t / \sqrt{h_t})^2$ .  $S_{t-1}^- = 1$  if  $\varepsilon_{t-1} < 0$ , and  $S_{t-1}^- = 0$  otherwise. Also,  $z_t^* = \tilde{h}(\Psi)/h_t$ , where  $\tilde{h}(\Psi) = \partial h_t / \partial \Psi$ , is evaluated at the values of maximum likelihood estimates of parameter Ψ . The test statistic of the NSBT is defined as the *t*-ratio of the coefficient *b* in the regression. The value in the parentheses is the *p*-value of the individual test statistics considered.

