Option Pricing with Extreme Events: Using Câmara and Heston(2008)'s Model

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Abstract

For the KOSPI 200 Index options, we examine the effect of extreme events for pricing options. We compare Black and Scholes(1973) model with Câmara and Heston(2008)'s options pricing model that allows for both big downward and upward jumps. It is found that Câmara and Heston(2008)'s extreme events option pricing models shows better performance than Black and Scholes(1973) model for both in-sample and out-of-sample pricing. Also downward jumps are more important factor for pricing stock index options than upward jumps. It is consistent with the empirical evidence that reports the sneers or negative skews in the stock index options market.

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1. Introduction

Since Black and Scholes (1973) published their seminal article on option pricing, numerous empirical studies have found that the Black-Scholes model (henceforth the BS model) results in systematic biases across moneyness and maturity. It is well known that the implied volatility computed from options on the stock index implied from the BS model appears to be different across exercise prices. This is the so-called "volatility smiles." Given the assumptions of the BS model, all option prices on the same underlying security with the same expiration date but with different exercise prices should have the same implied volatility. However, the "volatility smiles" pattern suggests that the BS model tends to misprice deep in-the-money and deep out-of-the-money options.

There have been various attempts to deal with this apparent failure of the BS model: stochastic volatility, stochastic interest rate and jumps etc.[1] In this paper, we choose jumps. The reasons that we examine jumps are as follows. First, it is generally known that the jump component plays an important role in explaining short-term options. Consider the price of an out-of-the-money option close to maturity. If the underlying asset price follows a diffusion process, the chance of exercising the option at maturity may be quite small. However, with a jump process, one jump may be sufficient to move the option to in the money position, implying that a diffusion model will underprice the option. Also Bakshi, Cao, and Chen (1997) found that adding the random-jump feature improves the fit of short-tem options and Kim, Baek, Noh, and Kim(2007) found that return jumps are essential in capturing the volatility smirk effects observed in short-term options. Second, the short-term options for which jumps

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[1] Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), Melino and Turnbull (1990, 1995), Stein and Stein (1991) and Heston (1993) suggest a continuous-time stochastic volatility model. Merton (1976), Bates (1991) and Naik and Lee (1990) propose a jump-diffusion model. Duan (1995) and Heston and Nandi (2000) develop an option pricing model based on the GARCH process. Recently, Madan, Carr and Chang (1998) use a three-parameter stochastic process, termed the variance gamma process, as an alternative model for the dynamics of log stock prices.

can be significant factor are severely mispriced as Bakshi, Cao and Chen (1997) mention that "The volatility smiles are the strongest for short-term options (both calls and puts), indicating that short-term options are the most severely mispriced by the BS and present perhaps the greatest challenge to any alternative option pricing model."

Among several option pricing models that consider jumps, we examine a simple extension of the lognormal-diffusion model of Black and Scholes(1973) and the jump-diffusion model of Merton(1976). Câmara and Heston(2008) derive closed-form solutions for the values of call and put options in a market where stock prices are affected by rare events in addition to small normal movements. This model has several strong points. First, their model can separate from downward jumps fear and upward jumps fear. The big jump downwards captures the fear and the extreme downside risk in the market and leads to a risk-neutral density negatively skewed and leptokurtic. The role of the big jump downwards is to adjust the Black and Scholes(1973) model for biases related with out-of-the-money put options by introducing a bearish parameter into the option pricing model The big jump upwards captures the extreme upside potential of the market and leads to a risk-neutral density with more positive skewness and kurtosis than the density implicit in the Black and Scholes(1973) model. The role of the big jump upwards is to adjust the Black and Scholes(1973) model for biases related with out-of-the-money call options by introducing a bullish parameter into the option valuation formulas. So, the comparison between the bullish parameter and the bearish parameter can be connected to the shape of the risk neutral distribution. Second, their model retains the simple form of the BS model, but with extra parameters. This simplifies the implementation and avoids costly numerical analysis.

The purpose of this paper is to examine the impacts of the downward and upward jumps for pricing stock index options. We compare the BS model with Câmara and Heston(2008)'s options pricing model (henceforth the CH model) that allows both big downward and upward jumps. Also, we examine the relative effect of big downward and upward jumps by comparing the CH model which assumes that the bullish parameter is zero and the CH model which assume that the bearish parameter is zero. If the CH model which assumes that the bullish parameter is zero shows better performance than the CH model which assume that the bearish parameter is zero, the downward jumps can be a significant factor for pricing stock index options. It is closely related to the shape of the risk neutral distribution. If either the bearish parameter or the bullish parameter are positive then the distribution implicit in the extreme events option pricing equations is more leptokurtic than the distribution implicit in the Black and Scholes(1973) model. The distribution has negative skewness, is symmetric or has positive skewness depending on the bearish parameters be greater, equal, or less than the bullish parameter respectively.

This study fills two gaps. First, we compare the pricing performance of the BS model with that of the CH model. To our knowledge, this is the first study to examine the CH model empirically. If the CH model shows better performance than the BS model, the fear of downward jumps or upward jumps is the important factor for pricing options. Second, we examine the KOSPI 200 index options market. Introduced in July 7 1997, the KOSPI 200 options market has become one of the biggest option markets in the world, despite its short history. In terms of trading volume, the KOSPI 200 options market ranked the 1st in the world. Moreover, the liquidity of KOSPI 200 index options market is concentrated in the nearest expiration contract. As mentioned before, the short-term options are the most severely mispriced by the BS model and present perhaps the greatest challenge to any alternative option pricing model. Thus the KOSPI 200 index options market will be an excellent sample market to investigate mispricing of short-term options.

It has been found that the CH model shows better performance than the BS model for both in-sample and out-of-sample pricing. Also downward jumps are more important factor for pricing stock index options than upward jumps. It is consistent with the empirical evidence that reports the sneers or negative skews in the stock index options market.

The outline of this paper is as follows. Câmara and Heston(2008)'s option pricing model is reviewed in section 2. Section 3 describes estimation methods. The data used for analysis are described in section 4. Section 5 describes parameter estimates of each model and evaluates pricing performances of alternative models. Section 6 concludes our study by summarizing the results.

2. Model

Câmara and Heston(2008) assume that the stock price follows an actual jump-diffusion stochastic differential equation:

$$dS = [\alpha - \lambda(Y-1)]Sdt + \sigma Sdz + (Y-1)Sdq \tag{1}$$

where α is the instantaneous expected return on the stock, σ is the instantaneous volatility of the return, conditional on the no occurrence of jumps, λ is the mean number of jumps per unit of time, dz is the actual standard Gauss Wiener process, dq is the Poisson process. This Poisson process has the value 1 with probability λdt or 0 otherwise. dz and dq are independent. $Y-1$ is the percentage change in the stock price if the jump occurs. This is the limiting case of the general model of Merton(1976), who assumes that the magnitude of the jump is a random variable.

First, Câmara and Heston(2008) assume that $Y=0$. The possibility of a jump of the stock price to zero is obtained when $Y=0$. In this case, the rate of return on the stock if a jump occurs is given by $Y-1=100\%$. If the stock price can jump to zero then the price of a European option is given by the Black and Scholes(1973) European option pricing equation, where the interest rate is substituted by $r+\lambda$ as follows.

$$C = SN(d_1) - Ke^{-(r+\lambda)(T-t)}N(d_2) \tag{2}$$

$$P = Ke^{-r(T-t)}\left(1 - e^{-\lambda(T-t)}\right) + Ke^{-(r+\lambda)(T-t)}N(-d_2) - SN(-d_1) \tag{3}$$

where $d_1 = \dfrac{\ln(S/K) + \left(r + \lambda + \dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

$d_2 = d_1 - \sigma\sqrt{T-t}$

The C and P denote call and put options respectively. The K is the exercise price, $N(\cdot)$ is the cumulative standard normal random variable, $T-t$ is the period of time to maturity. Because this model only captures big downward jumps or the bearish fear, this model is called the CH_{bear} model henceforth.

Second, Câmara and Heston(2008) assume that there are infinite jumps in terms of log-returns, that is, $Y \to +\infty$. If Y approaches infinity while λ is small then $Y\lambda$ remain constant. Then let $\delta = \lambda Y = \lambda(Y-1)$. If the stock price can jump to infinity then the price of a European option is given by the Black and Scholes(1973) European option pricing equation, where the original interest rate is replaced by δ and the original dividend rate is replaced by r as follows.

$$C = Se^{-\delta(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2) + S\left(1 - e^{-\delta(T-t)}\right) \tag{4}$$

$$P = Ke^{-r(T-t)}N(-d_2) - Se^{-\delta(T-t)}N(-d_1) \tag{5}$$

where $d_1 = \dfrac{\ln(S/K) + \left(r - \delta + \dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

$d_2 = d_1 - \sigma\sqrt{T-t}$

Because this model only captures big upward jumps or the bullish fear, this model is called the CH_{bull} model henceforth. Lastly, Câmara and Heston(2008) derive closed-form solutions for European call and put options when both a large downward jump and a large upward jump might occur as follows.

$$C = Se^{-\delta(T-t)}N(d_1) - Ke^{-(r+\lambda)(T-t)}N(d_2) + S\left(1 - e^{-\delta(T-t)}\right) \quad (6)$$

$$P = Ke^{-r(T-t)}\left(1 - e^{-\lambda(T-t)}\right) + Ke^{-(r+\lambda)(T-t)}N(-d_2) - Se^{-\delta(T-t)}N(-d_1) \quad (7)$$

where $d_1 = \dfrac{\ln(S/K) + \left(r + \lambda - \delta + \dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Because this model captures both the bullish fear and the bearish fear, this model is just called the CH model henceforth. Using the CH model, we can examine the relative influence of the bullish parameter δ and the bearish parameter λ on the shape of the risk neutral distribution. If $\delta = 0$ than these equations yield the case where there is a positive probability of immediate ruin which is given by equation (2) and (3). If $\lambda = 0$ then they yield the case of a big jump upwards, which is given by equation (4) and (5). If both $\lambda = 0$ and $\delta = 0$, then the BS model obtains. If either the bearish parameter λ or the bullish parameter δ is positive then the distribution implicit in the extreme events option pricing equations (6) and (7) is more leptokurtic than the distribution implicit in the BS model. The distribution implicit in equations (6) and (7) has negative skewness is symmetric, or has positive skewness depending on the bearish parameters be greater, equal, or less than the bullish parameter respectively.

3. Estimation Procedure

In applying option pricing models, one always encounters the difficulty that spot volatilities and structural parameters are unobservable. We follow the estimation method similar to standard practices (e.g. Bakshi, Cao, and Chen(1997, 2000), Bates (1991, 2000), Kim and Kim(2004, 2005)) and estimate the parameters of each model every sample day. Since closed-form solutions are available for an option price, a natural candidate for the estimation of parameters in the pricing formula is a non-linear least squares procedure, involving a minimization of the sum of squared percentage errors between the model and the market prices. Estimating parameters from the asset returns can be an alternative method, but historical data reflect only what happened in the past. Furthermore, the procedure using historical data is not capable of identifying risk premiums, which must be estimated from the options data conditional on the estimates of other parameters. The important advantage of using option prices to estimate parameters is to allow one to use the forward-looking information contained in the option prices.

To estimate parameters for each model, we minimize the sum of squared percentage errors between the model and the market prices as follows:

$$\min_{\sigma,\lambda,\delta} \sum_{i=1}^{N} \left[\frac{O_i^*(t,\tau;K) - O_i(t,\tau;K)}{O_i(t,\tau;K)} \right]^2 \quad (t = 1,\cdots,T) \tag{8}$$

where $O_i^*(t,\tau;K)$ denote the model price of the option i on day t and $O_i(t,\tau;K)$ denote the market price of option i on day t. N denotes the number of options on day t, and T denotes the number of days in the sample.

For the BS model, the volatility parameter, σ, is estimated. For the CH_{bull} model, we estimate the structural parameters, δ and the volatility parameter, σ. For the CH_{bear} model, we estimate the structural parameters, λ and the volatility parameter, σ. For the CH model, we estimate the structural parameters, $\{\lambda, \delta\}$ and the volatility parameter, σ.

4. Data

We choose KOSPI 200 options market as a sample market. Introduced in July 7 1997, the KOSPI 200 options market has become one of the biggest option markets in the world, despite its short history. During the nine-year duration from 1999 to 2007, In terms of trading volume, the KOSPI 200 options market has ranked the 1st in the world.

The KOSPI 200 options market has three consecutive near-term delivery months and one additional month from the quarterly cycle (March, June, September and December). The expiration day is the second Thursday of each contract month. Each options contract month has at least five strike prices. The number of strike prices may, however, increase according to the price movement. Trading in the KOSPI 200 index options is fully automated. The exercise style of the KOSPI 200 options is European and thus contracts can be exercised only on the expiration dates. Hence our test results are not affected by the complication that arises due to the early exercise feature of American options. Moreover it is important to note that liquidity is concentrated in the nearest expiration contract.

We use out-of-the-money options for calls and puts. First of all, since there is only a very thin trading volume for the in-the-money(henceforth ITM) options, the reliability of price information is not entirely satisfactory. Therefore, we use price data regarding both put and call options that are near-the-money and out-of-the-money (henceforth OTM). Second, if both call and put option prices are used, ITM calls and OTM puts which are equivalent according to the put-call parity are used to estimate the parameters.

The sample period extends from January 4, 2000 through June 30, 2006. Minute-by-minute transaction prices for the KOSPI 200 options are obtained from the Korea Stock Exchange. The

91-day certificate of deposit (CD) yields are used as risk-free interest rates.[2] The following rules are applied to filter data needed for the empirical test. First, for each day in our sample, only the last reported transaction price prior to 2:50 p.m.[3], of each option contract is employed in the empirical test.[4] Second, an option of a particular moneyness and maturity is represented only once in the sample. In other words, although the same option may be quoted again during the time window, only the last record of that option is included in our sample. Third, as options with less than 6 days to expiration may induce biases due to low prices and bid-ask spreads, they are excluded from the sample. Because the liquidity of KOSPI 200 option contracts is concentrated in the nearest expiration contract, we only consider single maturity of options. Fourth, to mitigate the impact of price discreteness on option valuation, prices lower than 0.02 are not included. Lastly, prices not satisfying the arbitrage restriction are excluded:

We divide the option data into several categories according to the moneyness, S/K. Table 1 describes certain sample properties of the KOSPI 200 option prices used in the study. Summary statistics are reported for the option price and the total number of observations, according to each moneyness-option type category. Note that there are 10886 call- and 12039 put-option observations, with deep OTM[5] options respectively taking up 55% for call and 64% for put.

Table 2 presents the "volatility smiles" effects for thirteen consecutive subperiods. We employ six fixed intervals for the degree of moneyness, and compute the mean over alternative subperiods of the implied volatility. It is interesting to note that the Korean options market seems to be "sneer" independent of the subperiods employed in the estimation. As the S/K increase, the implied volatilities decrease to near-the-money but, after that, increase to

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2 Korea does not have a liquid Treasury bill market, so the 91-day certificate of deposit (CD) yield is used in spite of the mismatch of maturity of options and interest rates.

3 In the Korean stock market, there are simultaneous bids and offers from 2:50 p.m.

4 Because the recorded KOPSI 200 index values are not equivalent to the daily closing index levels, there is no non-synchronous price issue here.

5 For call option, deep OTM options are options in S/K < 0.94. For put option, deep OTM options are options in 1.00 < S<K < 1.03.

out-of-the-money put options. Also the implied volatility of deep out-of-the-money puts is larger than that of deep out-of-the-money calls. That is, a volatility smile is skewed towards one side. The skewed volatility smile is sometimes called a 'volatility smirk' because it looks more like a sardonic smirk than a sincere smile. In the equity options market, the volatility smirk is often negatively skewed - where lower strike prices for out-of-the money puts have higher implied volatilities and, thus, higher valuations.[6] So, we recognize the need of other option pricing model to mitigate this effect.

5. Empirical Findings

5.1. Parameters

Table 3 the mean, standard deviation, maximum and minimum of the parameter estimates which are estimated daily for each model. The implicit parameters are not constrained to be constant over time. First, the implied volatility estimated from each model is similar to each other. But the implied volatilities of the models considering the extreme event are smaller than that of the BS model. This can be explained by the distribution of the explanatory power of the implied volatility. That is, the explanatory power of the implied volatility is distributed to other structural parameters of the models considering the extreme event. Second, for the CH model, bearish parameter, λ, is greater than bullish parameter, δ. This result says that the risk-neutral distribution is negatively skewed. This is consistent with the leverage effect documented by Black (1976) and Christie (1982), whereby lower overall firm values increase the volatility of equity returns, and the volatility feedback effects of Porterba and Summers (1986) whereby higher volatility assessments lead to heavier discounting of future expected dividends and thereby lower equity price.

Lastly, the estimates of each model's parameters have excessive standard deviations of daily

[6] See Rubinstein(1994) and Bakshi, Cao, and Chan(1997).

parameters. If the parameters change significantly over time, it would be tempting to dismiss the model as being a "throw-away" of no practical value. However, as discussed by Hull and White(1990), it is important to distinguish the goal of developing a model that adequately describes the stock price movements from the goal of developing a model that adequately values most of the contingent claims that are encountered in practice. Also, as pointed out by Bates(1991), while re-estimating the parameters daily is admittedly inconsistent with the assumption of slowly changing or constant parameters used in deriving option pricing model, such estimation will be valuable for the following reasons. First, the estimated parameters can be generated by indicating market sentiment on a daily basis. Second, the estimated parameters may suggest the future specification of more complicated dynamic models.

5.2 Pricing Performance

In this section, we compare empirical performances of each model with respect to in-sample pricing performance and out-of-sample pricing performance. The analysis is based on two measures: mean absolute percentage errors (henceforth MAPE), and mean squared errors (henceforth MSE) as follows.

$$\text{MAPE} = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{N}\sum_{i=1}^{N}\left|\frac{O_i(t,\tau;K)-O_i^*(t,\tau;K)}{O_i(t,\tau;K)}\right| \tag{9}$$

$$\text{MSE} = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{N}\sum_{i=1}^{N}\left[O_i(t,\tau;K)-O_i^*(t,\tau;K)\right]^2 \tag{10}$$

where, $O_i^*(t,\tau;K)$ denote the model price of the option i on day t and $O_i(t,\tau;K)$ denote the market price of option i on day. N denotes the number of options on day t, and T denotes the number of days in the sample. MAPE measures the magnitude of pricing errors, while MSE measures the volatility of errors. That is, MAPE measures the distance between zero and pricing errors, and MSE measures the adhesion of pricing errors.

5.2.1. In-sample pricing performance

We evaluate the in-sample pricing performance of each model by comparing market prices with model's prices computed by using the parameter estimates from the current day.

Table 4 reports in-sample valuation errors for the alternative models computed over the whole sample of options. First, with respect to all measure, the CH model shows the best performance followed by the CH_{bear} model. This is rather an obvious result when the use of larger number of parameters in the CH model is considered. But it is interesting that the CH_{bear} model shows better performance than the CH_{bull} model although two models have the same number of parameters. This can be explained as follows. Many empirical papers show that the risk-neutral distribution of the stock index options is negatively skewed.[7] As mentioned before, in the CH model, when bearish parameter, λ, is greater than bullish parameter, δ, the risk-neutral distribution is negatively skewed. The CH_{bear} model assume that the bullish parameter is zero. For the in-sample pricing, the better performance of the CH_{bear} model than the CH_{bull} model can be explained by the negative skewness of the risk-neutral distribution.

Second, for MSE measures, the CH_{bull} model does not show better performance than the BS model when $S/K > 1.00$, that is for OTM put options. As mentioned before, the CH_{bull} model only captures big upward jumps or bullish fear and the role of the big jump upwards is to adjust the BS model for biases related with OTM call options by introducing a bullish parameter into the option valuation formulas. So, the CH_{bull} model dose not adjust the biases related with OTM put options. On the other hand, it is interesting that the CH_{bear} model shows better performance than the BS model for all moneyness although the CH_{bear} model adjust the BS model for biases related with OTM put options by introducing a bearish parameter into the option valuation formulas.

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[7] See Shiratsuka (2001), Weinberg (2001), Anagnou, Bedendo, Hodges and Thompkins (2002), Bliss and Panigirtzoglou (2002), Bakshi, Kapadia, and Madan(2003), Kim and Kim (2003).

Lastly, all models show moneyness-based valuation errors. The models exhibits the worst fit for the out-of-the-money options. The fit, as measured by MAPE, steadily improves as we move from out-of-the-money to near-the-money options. Overall, the CH model performs the best for in-sample pricing.

5.2.2 Out-of-sample pricing performance

In-sample pricing performance can be biased due to the potential problem of over-fitting to the data. A good in-sample fit might be a consequence of having an increasingly larger number of structural parameters. To lower the impact of this connection to inferences, we turn to examine the model's out-of-sample cross-sectional pricing performance. In the out-of-sample pricing, the presence of more parameters may actually cause over-fitting and have the model penalized if the extra parameters do not improve its structural fitting. This analysis also has the purpose of assessing the stability of each model's parameter over time. To control the parameters' stability over alternative time periods, we analyze out-of-sample valuation errors for the following day (week). We use the current day's estimated structural parameters to price options for the following day (week).

Table 5 and table 6 respectively report one-day and one-week ahead out-of-sample valuation errors for alternative models computed over the whole sample of options. For one day ahead out-of-sample pricing, the CH model generally shows the best performance, closely followed by the CH_{bear} model. The CH model also exhibits better fit for the one week ahead out-of-sample pricing except the MSE measure. Also similar to MSE measures of in-sample pricing, the CH_{bull} model does not show better performance than the BS model for OTM put options. This is also explained by the absence of the bearish parameter. With respect to moneyness-based errors, similar to the case of in-sample pricing, MAPE steadily decreases as we move from deep out-of-the-money to near-the-money options for all models. Generally, the CH model outperforms all the other models.

Second, pricing errors increase from in-sample to out-of-sample pricing. The average of MAPE of all the models is 0.2249 for the in-sample pricing, and grows to 0.2871 for one-day ahead out-of-sample pricing. There is not a significant contrast between the errors of in-sample pricing and one-day ahead out-of-sample pricing. However, one-week ahead out-of-sample pricing errors grow to 0.3890 almost twice as much as in-sample pricing errors. Although the CH model continues to outperform other models for out-of-sample pricing, the relative margin of performance is significantly less when compared to that of in-sample pricing case. The difference of the BS and the CH models becomes smaller in the out-of-sample pricing. The ratio of the BS model to the CH model for MAPE is 2.0725 for in-sample pricing errors. This ratio decreases to 1.5160 and to 1.1794 for one-day ahead and one-week ahead out-of-sample errors, respectively. As the term of the out-of-sample pricing gets longer, the difference between the two models becomes smaller. Also, the strong pricing performance of the CH model is not maintained as the term of out-of-sample pricing gets longer, implying that the market consensus about the jump and volatility fear is volatile and structural parameters must be changed frequently.

Finally, we consider the relative effect of the structural parameters. The CH_{bull} model reduces MAPE of BS by 0.0083 and 0.0055 for one day and one week ahead pricing errors respectively. The CH_{bear} model reduces MAPE of BS by 0.1055 and 0.0624 for one day and one week ahead pricing errors respectively. In other words, the effects of the reduction of pricing errors for the CH_{bear} model are much better compared with those for the CH_{bull} model. On the other hand, we consider the CH model that adds the CH_{bear} model to the bullish parameter. The CH model reduces the pricing errors of BS by 0.1174 and 0.0642 for one day and one week ahead pricing errors respectively. The difference between the performance of the CH_{bear} model and the CH model is smaller than that between the performance of the CH_{bull} model and the CH model. In view of the results so far, the bullish parameter or the upward jump has only the marginal effects. This is consistent with the negative skewness of risk-neutral distribution.

Summarizing all the findings, the CH model performs better than any other models. However, conjecturing that the differences between the CH and the CH_{bear} models are negligible and that the CH_{bull} model shows the worst performance, the bearish parameter or the downward jump is more important component for pricing stock index options than the bullish parameter.

5.2.3. Validation

Different from what is done when using two measures (MAPE and MSE), we compare the models by using the statistics to draw the concrete results. Figure 1 summarizes the pair-wise comparison results among the models by providing the t-statistics of the probability that one model is better than the other. In regards to the in-of-sample pricing performance, the t-statistics of the difference between each model's absolute pricing errors are shown in panel A. Regarding the one day ahead and one week ahead out-of-sample pricing performance, the t-statistics of the difference between each model's absolute pricing errors are shown in panel B and panel C, respectively.

The comparison results are very clear and similar with those using MAPE. For both in-sample pricing and out-of-sample pricing performance, the CH_{bear} model is exceedingly superior to the BS model. And the difference between CH_{bull} model and the BS model is not significant. The CH_{bear} model shows better performance than the CH_{bull} model. Also the CH model outperforms the CH_{bear} model significantly for in-sample pricing. But, for out-of-sample pricing, the difference between two models is not significant. In the statistic analysis, the bullish parameter shows slightly the marginal effects for in-sample pricing but not for out-of-sample pricing. In other words, the downward jump or the bearish fear is the most important factor for pricing stock index options.

6. Conclusion

For the KOSPI 200 Index options, we examine the effect of extreme events for pricing options. We compare Black and Scholes(1973) model with Câmara and Heston(2008)'s options pricing model that allows both big downward and upward jumps. Also we examine the relative effect of big downward and upward jumps for pricing stock index options by comparing the model which assume that the bullish parameter is zero and the model which assume that the bearish parameter is zero.

It is found that Câmara and Heston(2008)'s extreme events option pricing models shows better performance than Black and Scholes(1973) model for both in-sample and out-of-sample pricing. That is, the jumps fear can be a significant factor for pricing options. Also the CH_{bear} model that only captures downward jumps shows better performance than the CH_{bull} model that only captures upward jumps. That is, a downward jump is more important factor for pricing stock index options than an upward jump. It is consistent with the empirical evidence that reports the sneers or negative skews in the stock index options market.

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Table 1: KOSPI 200 Options Data

This table reports average option price, and the number of options, which are shown in parentheses, for each moneyness and type (call or put) category. The sample period is from January 4, 2000 to June 30, 2006. Daily information from the last transaction prices (prior to 2:50 pm) of each option contract is used to obtain the summary statistics. Moneyness of an option is defined as S/K where S denotes the spot price and K denotes the strike price.

Call Options			Put Options		
Moneyness	Price	Number	Moneyness	Price	Number
S/K<0.94	0.4020	5996	1.00<S/K<1.03	2.5380	2519
0.94<S/K<0.96	1.1416	2149	1.03<S/K<1.06	1.3406	1829
0.96<S/K<1.00	2.4325	2741	S/K>1.06	0.3629	7691
Total	1.0593	10886	Total	0.9666	12039

Table 2: Implied Volatility

This table reports the implied volatilities calculated by inverting the Black-Scholes model separately for each moneyness category. The implied volatilities of individual options are then averaged within each moneyness category and across the days in the sample. Moneyness is defined as S/K where S denotes the spot price and K denotes the strike price. 2000 01-06 is the period from January, 2000 to June, 2000.

	S/K<0.94	0.94<S/K<0.96	0.96<S/K<1.00	1.00<S/K<1.03	1.03<S/K<1.06	S/K>1.06
2000 01-06	0.4263	0.4072	0.4077	0.4261	0.4199	0.4660
2000 07-12	0.5291	0.4799	0.4869	0.5186	0.5169	0.5214
2001 01-06	0.3830	0.3771	0.3726	0.3900	0.3856	0.4051
2001 07-12	0.3540	0.3185	0.3137	0.3702	0.3576	0.4383
2002 01-06	0.3889	0.3705	0.3680	0.3745	0.3753	0.4239
2002 07-12	0.3685	0.3471	0.3437	0.3910	0.3872	0.4150
2003 01-06	0.3307	0.3086	0.3123	0.3415	0.3532	0.3840
2003 07-12	0.2358	0.2265	0.2309	0.2560	0.2634	0.3157
2004 01-06	0.2656	0.2306	0.2431	0.2699	0.2728	0.3051
2004 07-12	0.2252	0.2154	0.2192	0.2827	0.2887	0.3191
2005 01-06	0.1798	0.1640	0.1665	0.1914	0.2028	0.2475
2005 07-12	0.1991	0.1814	0.1864	0.2295	0.2354	0.2747
2006 01-06	0.2060	0.1903	0.1991	0.2325	0.2433	0.2765

Table 3: Parameter Estimates

The table reports the mean, standard deviation, maximum and minimum of the parameter estimates for each model. BS is the Black-Scholes(1973) option pricing model. CH_{bull} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bearish parameter λ is zero. CH_{bear} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bullish parameter δ is zero. CH is Câmara and Heston's (2008) extreme events option pricing model. Each parameter is estimated by minimizing the sum of percentage squared errors between model and market option prices every day.

	BS	CH_{bull}		CH_{bear}		CH		
	σ	σ	δ	σ	λ	σ	λ	δ
Mean	0.3157	0.3118	0.0020	0.3046	0.0181	0.2963	0.0233	0.0051
Standard Deviation	0.1150	0.1127	0.0068	0.1118	0.0274	0.1072	0.0374	0.0171
Maximum	0.7198	0.6943	0.1168	0.7198	0.3169	0.6913	0.5181	0.3289
Minimum	0.1169	0.1148	0.0000	0.1159	0.0000	0.1078	0.0000	0.0000

Table 4: In-Sample Pricing Errors

This table reports in-sample pricing errors for the KOSPI 200 option with respect to moneyness. S/K is defined as moneyness where S denotes the asset price and K denotes the strike price. Each model is estimated every day during the sample period and in-sample pricing errors are computed using estimated parameters from the current day. MAPE denotes mean absolute percentage errors and MSE denotes mean squared errors. BS is the Black-Scholes(1973) option pricing model. CH_{bull} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bearish parameter λ is zero. CH_{bear} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bullish parameter δ is zero. CH is Câmara and Heston's (2008) extreme events option pricing model.

	Moneyness	BS	CH_{bull}	CH_{bear}	CH
MAPE	S/K<0.94	0.2111	0.1634	0.1684	0.0966
	0.94<S/K<0.96	0.2093	0.1955	0.1436	0.1175
	0.96<S/K<1.00	0.1097	0.1027	0.0835	0.0720
	1.00<S/K<1.03	0.1371	0.1363	0.1224	0.1232
	1.03<S/K<1.06	0.2350	0.2342	0.1966	0.1981
	S/K>1.06	0.5321	0.5348	0.2113	0.2106
	Total	0.3003	0.2867	0.1675	0.1449
MSE	S/K<0.94	0.0165	0.0140	0.0092	0.0056
	0.94<S/K<0.96	0.0775	0.0733	0.0337	0.0269
	0.96<S/K<1.00	0.1256	0.1200	0.0706	0.0657
	1.00<S/K<1.03	0.2992	0.3088	0.2489	0.2555
	1.03<S/K<1.06	0.1849	0.1912	0.1263	0.1276
	S/K>1.06	0.0394	0.0411	0.0179	0.0172
	Total	0.0874	0.0878	0.0575	0.0559

Table 5: One Day Ahead Out-of-Sample Pricing Errors

This table reports one day ahead out-of-sample pricing errors for the KOSPI 200 option with respect to moneyness. S/K is defined as moneyness where S denotes the asset price and K denotes the strike price. Each model is estimated every day during the sample period and one day ahead out-of-sample pricing errors are computed using estimated parameters from the previous trading day. MAPE denotes mean absolute percentage errors and MSE denotes mean squared errors. BS is the Black-Scholes(1973) option pricing model. CH_{bull} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bearish parameter λ is zero. CH_{bear} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bullish parameter δ is zero. CH is Câmara and Heston's (2008) extreme events option pricing model.

	Moneyness	BS	CH_{bull}	CH_{bear}	CH
MAPE	S/K<0.94	0.3341	0.3055	0.2975	0.2554
	0.94<S/K<0.96	0.2569	0.2469	0.1970	0.1792
	0.96<S/K<1.00	0.1283	0.1227	0.1047	0.0954
	1.00<S/K<1.03	0.1441	0.1440	0.1332	0.1346
	1.03<S/K<1.06	0.2457	0.2449	0.2130	0.2151
	S/K>1.06	0.5444	0.5471	0.2950	0.2996
	Total	0.3449	0.3366	0.2394	0.2275
MSE	S/K<0.94	0.0343	0.0319	0.0262	0.0232
	0.94<S/K<0.96	0.1180	0.1140	0.0723	0.0677
	0.96<S/K<1.00	0.1552	0.1492	0.1014	0.0934
	1.00<S/K<1.03	0.3216	0.3323	0.2820	0.2923
	1.03<S/K<1.06	0.2021	0.2081	0.1506	0.1531
	S/K>1.06	0.0448	0.0464	0.0282	0.0297
	Total	0.1051	0.1055	0.0782	0.0779

Table 6: One Week Ahead Out-of-Sample Pricing Errors

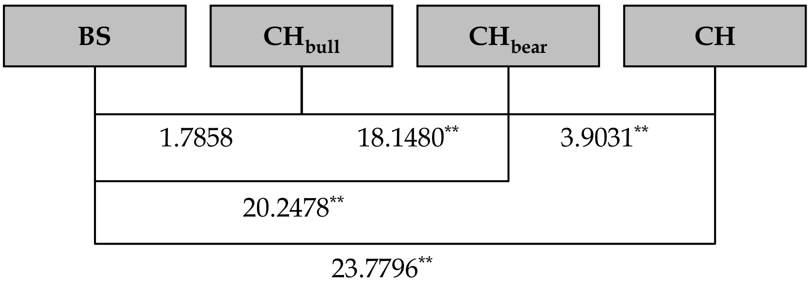
This table reports one week ahead out-of-sample pricing errors for the KOSPI 200 option with respect to moneyness. S/K is defined as moneyness where S denotes the asset price and K denotes the strike price. Each model is estimated every day during the sample period and one week ahead out-of-sample pricing errors are computed using estimated parameters from one week ago. MAPE denotes mean absolute percentage errors and MSE denotes mean squared errors. BS is the Black-Scholes(1973) option pricing model. CH_{bull} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bearish parameter λ is zero. CH_{bear} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bullish parameter δ is zero. CH is Câmara and Heston's (2008) extreme events option pricing model.

	Moneyness	BS	CH_{bull}	CH_{bear}	CH
MAPE	S/K<0.94	0.5215	0.5023	0.4740	0.4497
	0.94<S/K<0.96	0.3387	0.3301	0.2844	0.2694
	0.96<S/K<1.00	0.1699	0.1655	0.1518	0.1447
	1.00<S/K<1.03	0.1686	0.1694	0.1591	0.1616
	1.03<S/K<1.06	0.2751	0.2755	0.2452	0.2470
	S/K>1.06	0.5755	0.5776	0.4582	0.4772
	Total	0.4220	0.4165	0.3596	0.3578
MSE	S/K<0.94	0.0651	0.0604	0.0557	0.0513
	0.94<S/K<0.96	0.1763	0.1666	0.1349	0.1266
	0.96<S/K<1.00	0.2504	0.2399	0.2022	0.1938
	1.00<S/K<1.03	0.3697	0.3787	0.3398	0.3572
	1.03<S/K<1.06	0.2394	0.2435	0.1921	0.1969
	S/K>1.06	0.0598	0.0600	0.0448	0.0483
	Total	0.1433	0.1412	0.1191	0.1196

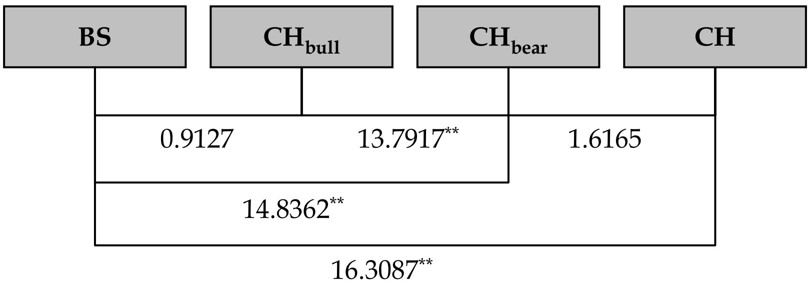
Figure 1: Differences between the Errors of Each Model

This figure reports the differences among the absolute errors of each model. The t-statistics of the difference between each model's absolute pricing errors are shown. Panel A reports t-statistics between in-sample pricing errors of each model. Panel B reports t-statistics between one day ahead out-of-sample pricing errors of each model. Panel C reports t-statistics between one week ahead out-of-sample pricing errors of each model. BS is the Black-Scholes(1973) option pricing model. CH_{bull} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bearish parameter λ is zero. CH_{bear} is Câmara and Heston's (2008) extreme events option pricing model which assume that the bullish parameter δ is zero. CH is Câmara and Heston's (2008) extreme events option pricing model. "**" and "*" indicate the test statistic value that is significantly different from 1% and 5%, respectively.

Panel A: In-sample pricing errors



Panel B: One Day Ahead Out-of-Sample Pricing Errors



Panel C: One Week Ahead Out-of-Sample Pricing Errors