A Comparison of Empirical Performance: Pricing Models VS. Pricing Kernels*

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Abstract

This study examines the empirical performance of the option pricing models and the pricing kernel-based models in the KOSPI 200 options market. We evaluate the pricing, forecasting, and hedging performance of the GARCH option pricing model, the pricing kernel-based GARCH option model, Black-Scholes option pricing model, and the parametric pricing kernel models proposed by Rosenberg and Engle (2002) under the unified framework in which the underlying process follows the extended Duan (1995)-GARCH process. We extend the Duan-GARCH process to reflect the asymmetric volatility phenomenon. We find that empirical performance of the models is fairly improved when we estimate the parameters of models using the option data compared to the results when we only use the underlying data. The parametric pricing kernel of which functional form is a Chebyshev polynomial function shows the best pricing (forecasting) performance for all options among models we investigate. Unlike the pricing (forecasting) results, the difference of hedging performance between the models is relatively small.

JEL Classification:

Keywords: Pricing kernels; Empirical performance; Hedging; GARCH option pricing model; KOSPI 200 options

Current Version: 2008.8.24

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1. Introduction

Hedging derivative assets is an important issue in financial economics. We often derive the relationship between derivatives and hedging instruments by using regression analysis or modeling the GARCH process. We can hedge derivatives based on this relationship through these statistical approaches. If we hedge derivatives which have linear payoffs, such as futures, these statistical approaches are useful tools. However, when we try to hedge derivatives which have non-linear payoffs, such as options, or when to hedge derivatives whose price changes don't follow a stationary process, these statistical methods may not be appropriate. Instead it is better to hedge derivatives based on financial models. Although the hedge result may be sensitive to the selected models, it is quite useful when hedging the derivative whose payoff is non-linear.

This study compares the empirical performance of hedging methods based on option pricing models with the performance of hedging methods based on the pricing kernel. Since these two classes of hedging methods are all based on financial models, we can also examine the empirical pricing and forecasting performance of each model used for hedge. The pricing model can be derived if the pricing kernel is given, and we can identify the pricing kernel if the pricing model is given. They have an one-to-one relationship. Thus, at least theoretically, the empirical performance seems to be same regardless we hedge (price) the option after estimating the pricing kernel or we hedge (price) the option after estimating the parameters of the option pricing model. However, the empirical performance can be quite different because of many practical issues when estimating the parameters of pricing models or pricing kernels.

To investigate the pricing, forecasting and hedging performance of the pricing kernel, we define the functional form of the pricing kernel first and estimate it using financial data. The pricing kernel provides the information on the investors' preference about the payoff depending on the future state. Also, the price of financial assets is determined in case that the pricing kernel, which is the function of the future state, and the probability model on the future state are given. Ross (1978) presents the linear valuation operator which determines the price of asset by discounting the future payoff of asset considering compensation for the risk. This Ross (1978) study is the beginning of the theoretical research on the pricing kernel.

There are many studies which try to correctly estimate the theoretical pricing kernel. A strand of research considers the pricing kernel as the function of consumption, and estimates it using consumption data. After Hansen and Singleton (1982, 1983) estimated the pricing kernel using the aggregate U.S. consumption data, many studies, such as Hansen and Jaganathan (1991) and Chapman (1997) have investigated the relationship between the asset price and the consumption data. However, this approach has a defect in that the aggregate consumption data that is used to estimate the pricing kernel is measured inaccurately and the daily consumption data often doesn't exist.

On the other hand, we can estimate the pricing kernel using the price or the return on assets without consumption data through the no-arbitrage approach that doesn't require any assumption on the investors' risk preference. Ait-Sahalia and Lo (2000) present the concept of the projected pricing kernel that is a function of the asset return. The projected pricing kernel can be derived by projecting the original pricing kernel that is a function of the state variables into the space of the asset return. Rosenberg and Engle (2002) point out that Ait-Sahalia and Lo (2000) estimate the projected pricing kernel on a yearly basis, and they fail to detect any monthly change of the pricing kernel. Rosenberg and Engle (2002) assume that the underlying process follows the asymmetric GARCH process, and estimate the parametric pricing kernels on a monthly basis. Pan (2002) introduces the parametric pricing kernel that incorporate the jump process of the S&P 500 index to determine the option prices. While Ait-Sahalia and Lo (2000) or Rosenberg and Engle (2002) estimate the pricing kernel implied by the return of the same asset class, i.e., the S&P 500 index and the S&P 500 option return, Chernov (2003) estimates the projected pricing kernel by simultaneously considering the return on multiple assets, such as, the S&P 500 index, individual stocks, T-bills, and gold futures.

Instead of considering the pricing kernel, another group of research focuses on the form of the option pricing model itself to explain observed option prices well and hedge them successfully. Since the Black-Scholes (1973) option pricing model, many option pricing models have evolved by relaxing some of the restrictive assumptions of the Black-Scholes model. These models generalize the Black-Sholes option pricing model by introducing jump and stochastic volatility of the underlying process or assuming the stochastic interest rate process. For example, Merton (1973) and Amin and Jarrow (1992)s' models are stochastic interest rate option pricing models and Bates (1996a,c) and Scott (1997)s' option pricing models can be classified as stochastic volatility jump diffusion models. However, more advanced and complicated option pricing models do not always guarantee greater hedge performance. Bakshi et al (1997) derive the option pricing model that can allow stochastic volatility, interest rates and jumps. They find that the pricing performance of option models that simultaneously incorporate the stochastic volatility and jump is fairly good, but the option pricing models that only incorporate stochastic volatility alone show the best hedge performance.

Some studies introduce GARCH framework for option pricing. Duan (1995) develops a GARCH option pricing formula of which the underlying process follows the GARCH-type process. Using the locally risk-neutral valuation relationship, he presents an option pricing formula under the locally risk-neutral measure (measure Q). Yung and Zhang (2003) insist that the exponential GARCH option pricing model performs better than the ad hoc Black-Scholes model proposed by Dumas et al (1998) in terms of the in-sample valuation and the out-of-sample forecasting[1]. Heston and Nandi (2000) suggest a GARCH option pricing model that provides analytical solution for European option pricing and Hsieh and Ritchken (2005) confirm the empirical performance of the Heston and Nandi model in terms of explaining the volatility smile phenomenon. (관련 literature 더 추가가능, Yung and Zhang등의 reference 참고.)

As we can see above, although there are many studies that investigate separately the empirical

[1] However, they find more complicated exponential GARCH models perform poorly relative to the ad hoc Black-Scholes model.

performance of option pricing models or that of models based on the pricing kernel assumption, to the best of our knowledge, this study is the first attempt to compare the empirical performance of option pricing models with the performance of the models based on the pricing kernel under the unified framework. To compare the option pricing model and the model based on the pricing kernel under the same framework, we adopt the Duan (1995)'s GARCH process to describe the dynamics of the underlying process. We call it the Duan-GARCH process in this paper and extend the Duan-GARCH process to reflect the asymmetric volatility phenomenon.

The asymmetric volatility phenomenon is based on the empirical regularity that return and volatility are negatively correlated. Positive and negative return shocks tend to imply a different magnitude of future volatility, or the return shocks respond asymmetrically to the (conditional) change in volatility. Many papers report this asymmetric volatility phenomenon on the individual stock level or the market index level. [Black (1976), Christie (1982), French, Schwert and Stambaugh (1987), Schwert (1990), Campbell and Hentschel (1992), Duffee (1995)]. They explain the asymmetric volatility phenomenon by the leverage hypothesis or the time-varying risk premium theory. Bekaert and Wu (2000) suggest the unified framework where these two hypotheses can be tested and Wu (2001) insists that both the leverage effect and the volatility feedback are the cause of the asymmetric volatility phenomenon. After Nelson (1991) develops the asymmetric ARCH process and Glosten, Jagannathan, and Runkle (1993) present GJR-GARCH to explain the asymmetric volatility, most studies tries to examine the asymmetric volatility phenomenon under the extended GARCH framework. They find their model outperforms other models that don't accommodate the asymmetric volatility phenomenon. Thus, we incorporate the asymmetric volatility by changing the conditional variance in the Duan-GARCH process so that the conditional volatility is affected differently by the positive lag error term and the negative lag error term like the GJR-GARCH process.

In the KOSPI 200 options market, we illuminate whether there is a difference in the empirical performance between the option pricing models and the models based on the pricing kernel under the unified framework, which is the extension of the Duan-GARCH framework. Due to the abundant liquidity of the KOSPI 200 options, many traders and academicians have great interest in this market. The KOSPI 200 options market entered the world's top five derivatives markets in 2000 and took over the top spot in 2001 in terms of trading volume. The KOSPI 200 options had a volume of 2.4 billion contracts traded in 2006, which was about five times larger than the trading volume of the Eurodollar futures, which was the second most active derivative product that year.

We estimate the following models using the option data (and the underlying return data) and investigate the empirical performance of each case. We examine the GARCH option pricing model, the GARCH option model based on the pricing kernel assumption and the parametric pricing kernels which directly assume the functional form of the pricing kernels. We adopt the parametric pricing kernels as Rosenberg and Engle (2002) models and extend them.

On the other hand, the pricing kernel can be estimated using only the underlying return data. So we examine whether the performance deterioration of the pricing kernel-based models exists when we estimate the pricing kernel using only the underlying return data instead of option data. Finally, we check whether the above models improve in terms of the empirical performance, compared to the Black-Sholes model in the extended Duan-GARCH framework.

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2. Theory and previous research

2.1. Duan (1995)'s GARCH option pricing model

Duan (1995) proposes the GARCH option pricing model. He assumes that the one period rate of return on the underlying asset is conditionally log-normally distributed under the probability measure P (physical measure). That is, he suggests the following mean equation of the GARCH(p,q) process.

$$\ln\frac{S_t}{S_{t-1}} = r + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t \qquad \text{where } \varepsilon_t \mid I_{t-1} \sim N(0,h_t) \text{ under measure P} \qquad (1)$$

S_t is the asset price at time t, r is the one period risk-free rate which is constant, λ is the unit risk premium and ε_t is the error term which has zero mean and conditional variance, h_t, under the probability measure P. I_t is the information set accumulated at time t. Duan (1995) also assumes the error term of the mean equation, ε_t, follows a standard GARCH(p,q) process under measure P. The variance equation follows.

$$h_t = \alpha_0 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{p}\beta_i h_{t-i} \qquad (2)$$

where $p \geq 0$, $q \geq 0$; $\alpha_0 > 0, \alpha_i \geq 0$, $i = 1,\ldots,q$, $\beta_i \geq 0$, $i = 1,\ldots,p$.

For the GARCH(p,q) process to become a covariance stationary process and have a finite long run variance, $\sum_{i=1}^{\max(p,q)} \alpha_i + \beta_i$ should be less than 1.

He defines a pricing measure Q which satisfies the locally risk-neutral valuation relationship with measure P. $S_t / S_{t-1} \mid I_{t-1}$ is log-normally distributed under this measure Q, and the probability measure Q is mutually absolutely continuous with respect to the probability measure P defined above. The conditional expectation and variance of the asset return under measure Q follows.

$$E^Q[S_t / S_{t-1} \mid I_{t-1}] = e^r \qquad (3)$$

$$\operatorname{var}^Q[\ln(S_t / S_{t-1}) \mid I_{t-1}] = \operatorname{var}^P[\ln(S_t / S_{t-1}) \mid I_{t-1}] \quad (4)$$

This locally risk-neutral valuation relationship between measure P and measure Q implies that the mean equation changes like equation (5) under the probability measure Q.

$$\ln\frac{S_t}{S_{t-1}} = r - \frac{1}{2}h_t + \xi_t \quad \text{where } \xi_t \mid I_{t-1} \sim N(0, h_t) \text{ under measure Q} \qquad (5)$$

And the variance equation changes like equation (6)

$$h_t = \alpha_0 + \sum_{i=1}^{q}\alpha_i(\xi_{t-i} - \lambda\sqrt{h_{t-i}})^2 + \sum_{i=1}^{p}\beta_i h_{t-i} \qquad (6)$$

where $p \geq 0$, $q \geq 0$; $\alpha_0 > 0, \alpha_i \geq 0$, $i = 1,\ldots,q$, $\beta_i \geq 0$, $i = 1,\ldots,p$.

For this GARCH process under measure Q to become a covariance stationary process and have a finite long run variance, $\sum_{i=1}^{\max(p,q)} \alpha_i(1+\lambda^2) + \beta_i$ should be less than 1.

Now, we can price European options because we identify the underlying process under the locally risk neutral measure (measure Q).

For call options, the option pricing equation follows,

$$C_t^{GH} = e^{-(T-t)r}E^Q[\max(S_T - K, 0) \mid I_t] \qquad (7)$$

For put option, the option pricing equation follows,

$$P_t^{GH} = e^{-(T-t)r}E^Q[\max(K - S_T, 0) \mid I_t] \qquad (8)$$

where I_t is the information set at time t.

2.2. Projected pricing kernel and Duan (1995)'s GARCH process

The original pricing kernel is the function of the state variable and the projected pricing kernel is the function of the asset return or asset payoff. Even though they have different functional forms, they play the same role when pricing financial assets. The projected pricing kernel can be derived by the original pricing kernel. If we utilize the projected pricing kernel instead of the original pricing kernel, we don't have to be concerned about what state variable we should choose and can just use the asset return or payoff data we can easily get. The projected pricing kernel also gives us the similar implication on the risk aversion implied by the original pricing kernel. Thus, we adopt the projected pricing kernel instead of the original pricing kernel in this paper.

Equation (9) shows how to derive the projected pricing kernel from the original pricing kernel.

$$P_t = E_t[M_t(Z_t, Z_{t+1})X_{t+1}] = E_t[E_t[M_t(Z_t, Z_{t+1})X_{t+1} \mid X_{t+1}]] = E_t[E_t[M_t(Z_t, Z_{t+1}) \mid X_{t+1}]X_{t+1}]$$

$$= E_t[M_t^{**}(X_{t+1})X_{t+1}] \quad \text{where } M_t^{**}(X_{t+1}) = E_t[M_t(Z_t, Z_{t+1}) \mid X_{t+1}] \qquad (9)$$

The original pricing kernel $M_t(Z_t, Z_{t+1})$ is a function of a current state variable (Z_t) and a future state variable (Z_{t+1}). The projected pricing kernel $M_t^*(X_{t+1})$ is a function of the future asset price (payoff), X_{t+1}. The first equality holds because the asset price at time t, P_t, should be equal to $E_t[M_t(Z_t, Z_{t+1})X_{t+1}]$ by the definition of the original pricing kernel. The second equation holds by the law of the iterated expected operation.

By the definition of the Arrow-Pratt (Arrow(1964), Pratt(1964))'s relative risk aversion measure, we can derive the relative risk aversion measure from the projected pricing kernel.

$$\gamma_t^* = -X_{t+1}\frac{M_t^{*\prime}(X_{t+1})}{M_t^*(X_{t+1})} \qquad (10)$$

From now on, since we always use the projected pricing kernel instead of the original pricing kernel, we call the projected pricing kernel the pricing kernel in this paper.

We show how one can derive the pricing kernel when the option pricing model is given. The pricing kernel and the pricing model have a one to one relationship. So the pricing kernel implied by the option pricing model varies in each option pricing model. We start with the case of the continuous time Black-Scholes option pricing model.

Under the Black-Scholes assumptions, the stock price, S, will follow the stochastic process:

$$dS = \mu S dt + \sigma S dw \quad \text{where } dw \text{ is a standard Brownian motion.} \qquad (11)$$

There is a riskless asset, B, following the process

$$dB = rBdt \qquad (12)$$

Then the Girsanov theorem shows that the Radon-Nikodym derivative of the risk neutral probability measure Q with respect to the physical probability measure P will be

$$\frac{dQ}{dP} = \exp\left(-\gamma w(T) - \frac{1}{2}\gamma^2 T\right) \quad \text{where } \gamma = \frac{\mu - r}{\sigma} \qquad (13)$$

Thus, the pricing kernel, $m_{BS,T}$ under the Black-Scholes assumption will be

$$m_{BS,T} = e^{-rT}\frac{dQ}{dP} = \exp\left(-\gamma w(T) - rT - \frac{1}{2}\gamma^2 T\right) \qquad (14)$$

Since $\ln\frac{S_T}{S_0} = \left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma w(T)$, We can derive that the pricing kernel implied by the continuous time Black-Scholes option pricing model as a function of the asset (stock) return.

$$m_{BS,T} = \exp\left(-\gamma \frac{\ln\frac{S_T}{S_0} - \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma} - rT - \frac{1}{2}\gamma^2 T \right) = \exp\left(-\frac{\gamma}{\sigma}\ln\frac{S_T}{S_0} + \delta \right) = e^{\delta}\left(\frac{S_T}{S_0}\right)^{-\frac{\gamma}{\sigma}}$$

where $\delta \equiv \gamma \frac{\left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma} - rT - \frac{1}{2}\gamma^2 T$ (15)

We apply this technique to the GARCH option pricing model and derive the pricing kernel implied by the GARCH option pricing model, which is a function of asset return. Even though the Duan-GARCH process for the option pricing model is a discrete time version, we can successfully derive the approximated pricing kernel using the Radon-Nikodym derivative of the locally risk-neutral measure Q with respect to measure P.

If we normalize the each error terms of the Duan's GARCH process under measure P and measure Q so that they follow a standard normal distribution, the normalized error terms correspond to the standard Brownian motion of the continuous time case.

$e_t = \frac{\varepsilon_t}{\sqrt{h_t}}$, where $e_t \mid I_{t-1} \sim \mathrm{N}(0,1)$ (16)

$v_t = \frac{\xi_t}{\sqrt{h_t}}$, where $v_t \mid I_{t-1} \sim \mathrm{N}(0,1)$ (17)

e_t and v_t correspond to the standard Brownian motion.

The Radon-Nykodym derivative in the continuous time finance framework follows

$$Z(t,T) = \exp\left[-\int_t^T \theta(u)dW(u) - \frac{1}{2}\int_t^T \theta^2(u)du \right] \quad (18)$$

θ is the unit risk premium, and λ in the Duan's GARCH option pricing model can correspond to the θ. Thus, we can define the Radon-Nykodym derivative in the Duan's GARCH option pricing model as equation (19).

$$Z^{GH}(t,T) = \exp\left[-\lambda \sum_{k=t+1}^{T} e_k - \frac{1}{2}\lambda^2 (T-t) \right] \quad \text{where } e_t = \frac{\varepsilon_t}{\sqrt{h_t}} \sim N(0,1) \quad (19)$$

e_t is equal to $\left(\ln\frac{S_t}{S_{t-1}} - r - \lambda\sqrt{h_t} + \frac{1}{2}h_t\right) / \sqrt{h_t}$ by the mean equation under measure P.

Thus, the Radon-Nykodym derivative is transformed into equation (20),

$$Z^{GH}(t,T) = \exp\left[-\lambda\sum_{k=t+1}^{T}\left(R_k - r - \lambda\sqrt{h_k} + \frac{1}{2}h_k\right) / \sqrt{h_k} - \frac{1}{2}\lambda^2(T-t)\right] \text{ where } R_k = \ln\frac{S_k}{S_{k-1}} \quad (20)$$

The pricing kernel is the product of the risk-neutral discount factor and the Radon-Nykodym derivative. Equation (21) shows that the pricing kernel implied by the Duan's GARCH option pricing model can be derived as a function of the asset return.

$$\begin{aligned} m^{GH}(t,T) &= e^{-r(T-t)}Z(t,T) \\ &= \exp\left[-r(T-t) - \lambda\sum_{k=t+1}^{T}\left(R_k - r - \lambda\sqrt{h_k} + \frac{1}{2}h_k\right) / \sqrt{h_k} - \frac{1}{2}\lambda^2(T-t)\right] \end{aligned} \quad (21)$$

2.3. The parametric pricing kernel: Rosenberg and Engle (2002)

In the previous section, we show how one can derive the pricing kernel implied by the GARCH option pricing model. Now, we adopt the Rosenberg and Engle (2002) model to investigate the parametric pricing kernel of which the functional form is explicitly modeled. Rosenberg and Engle (2002) explicitly design a functional form of the pricing kernel and estimate it using the observed option data and the generated return distribution of the underlying process. They suggest two specifications for the pricing kernel. One pricing kernel is a power function of the underlying asset's gross return. We call it a power pricing kernel and the functional form of the power pricing kernel follows.

$$M^*(r_T;\theta_t) = \theta_{0,t}(r_T)^{-\theta_{1,t}} \quad (22)$$

Since the power pricing kernel just has two parameters, we can easily estimate it. And we can derive the projected risk aversion factor from the estimated parameter of the power pricing kernel. The first parameter, $\theta_{0,t}$, is a scaling factor and the second parameter, $\theta_{1,t}$, is relative risk aversion as shown in equation (23). The relative risk aversion varies as the second parameter varies over time.

$$\gamma_t^* = -r_T\frac{M_t^{*\prime}(r_T)}{M_t^*(r_T)} = \theta_{1,t} \quad (23)$$

Another pricing kernel is the polynomial function of the underlying asset's gross return. Rosenberg and Engle (2002) use the generalized Chebyshev polynomial to specify the pricing kernel and we call it as a polynomial pricing kernel. It permits more flexibility in the form of the pricing kernel, but it is relatively difficult to estimate and needs some restriction on the return distribution to estimate the pricing kernel successfully.

$$M^*(r_T;\theta_t)=\theta_{0,t}T_0(r_T)+\theta_{1,t}T_1(r_T)+\theta_{2,t}T_2(r_T)+\ldots+\theta_{N,t}T_N(r_T) \quad \text{where,} \quad T_n(x)=\cos(n\cos^{-1}(x)) \tag{24}$$

To estimate these parametric pricing kernels, we need to specify the underlying asset's return process. To reflect the asymmetric volatility phenomenon of the underlying return, Rosenberg and Engle (2002) assume the underlying return process follows the asymmetric GARCH model based on Glosten et al. (1993).

Mean equation: $\ln(S_t/S_{t-1})-r_{f,t-1}=\mu+\varepsilon_t+\theta\varepsilon_t,\quad \varepsilon_t \mid I_{t-1}\sim N(0,h_t)$ (25)

Variance equation: $h_t=w+\alpha\varepsilon_{t-1}^2+\beta h_{t-1}+\delta Max[0,-\varepsilon_{t-1}]^2$ (26)

The variance equation of this asymmetric GARCH model should satisfy the stability condition to guarantee the existence of long-run variance.

Stability condition[2]: $w>0,\alpha\geq 0,\beta\geq 0,\alpha+\delta\geq 0,\alpha+\beta+\frac{1}{2}\delta<1$ (27)

The estimation procedure follows. The first step is to estimate the asymmetric GARCH model with the restriction for the stability condition using the underlying return data.

Next, we use the Monte-Carlo simulation method to determine the future underlying return distribution. The one period log return is determined by the mean equation of the asymmetric GARCH model as equation (28)

$$\ln(S_{t+1}/S_t)=r_{f,t}+\mu+\varepsilon_{t+1} \tag{28}$$

Equation (29) shows m-period log return and equation (30) shows m-period gross return.

$$\ln(S_{t+m}/S_t)=\sum_{i=1,\ldots,m}\left(r_{f,t-1+i}+\mu+\varepsilon_{t+i}\right) \tag{29}$$

$$S_{t+m}/S_t=\exp\left[\sum_{i=1,\ldots,m}\left(r_{f,t-1+i}+\mu+\varepsilon_{t+i}\right)\right] \tag{30}$$

We get the distribution of one period return by generating many one period returns. We can obtain multi-period gross return distribution by the similar generating procedure. ε_{t+i} is extracted from the normal distribution, $N(0,h_{t+i})$, and the conditional variance, h_{t+i}, is obtained during the estimation

[2] See Appendix A.

procedure.

3. Data

3.1. KOSPI 200 options

The KOSPI 200 options market has grown to one of the most liquidity-fluent derivative markets in the world. From January 2001 to July 2006, which is the sample period in this study, the trading activity at the KOSPI 200 options market is best among all derivative markets in the world. Table 1 shows the trading volume in terms of the number of contract for the top ten derivative products in the world during our sample period. As seen in the table, KOSPI 200 options dominate the other top 10 derivatives markets in a view that their trading volume exceeds the combined volume of all other products in the top 10 list.

The active participation of the domestic individual traders is another characteristic of KOSPI 200 options market. While institutional traders or foreign traders are the major traders in the most derivative markets in the developed countries, domestic individual traders are the most active traders group in the KOSPI 200 options market. The trading activity of the domestic individual traders explains more than half of the total transaction in this market irrespective of whether it is measured by the number of transactions, the trading volume, or the dollar trading value. For example, the trading volume of the individual traders is the 65.8% (54.8%) of the total trading volume at the year 2002 (2003)[3].

The great trading activity and the unique market participation rate of each investor type make the KOSPI 200 options market remarkable. Moreover, we may find the empirical results in this market, which is different to other option markets in the world.

3.2. Sample Data

On every trading day, while the KOSPI 200 options are traded until 3:15 p.m.[4], the value of KOSPI 200 index, which is the underlying asset of KOPSI 200 options, is updated per a minute until 3 p.m. Thus, we make daily data by extracting KOSPI 200 options and the KOSPI 200 index at 2:50 p.m. per every trading day to synchronize the option and underlying price.

We use the call rate as the proxy of the one day risk-free rate and calculate the price of short term risk free bonds with the three month CD91 rate. The dividend yield over the life of each option is determined as the present value of future dividends on the KOSPI 200 index until the option maturity date divided by the current index level. Table 2 reports the descriptive statistics for the daily KOSPI 200 log return. The sample period of the daily log-return of KOSPI 200 index is from January, 2001 to

[3] Source: Korean Financial Supervisory Service (http://www.fss.or.kr)
[4] The only exception is the maturity date of option. The KOSPI 200 options are traded until 2:50 p.m. on that day.

October, 2006. (Three month longer than the option sampling period.)

When we estimate the parameters of the option pricing model or the model based on the pricing kernel, we select option data with several screening criteria to eliminate data errors and make the estimates reliable.

First, we include only the nearest maturity options in our sample because other maturity options are barely traded. The trading volume of the nearest maturity options is more than five times larger than that of the second-nearest maturity options even at the liquidation date of the nearest maturity option. There is no comparison between the trading volume of the nearest and the second-nearest maturity options at other trading dates.

Second, to be included in the sample, the option price should be greater than no arbitrage lower bound and it should be laid between 0.03P and 15P. Equation (31) (equation (32)) represents the no arbitrage lower bound for call (put) options.

$$Max(S_t - D_{t,T} - Ke^{-r_f(T-t)}, 0) \quad (31)$$

$$Max(Ke^{-r_f(T-t)} - (S_t - D_{t,T}), 0) \quad (32)$$

S_t is the current KOSPI 200 index price, K is the exercise price, r_f is the risk free rate, T is the option maturity date and $D_{t,T}$ is the present value of the lump-sum dividend over the life of the option.

Third, Black-Sholes implied volatility (equation (33)) should be greater than 0.05 and less than 0.95.

$$\text{Implied volatility} = \sigma(O_t; S_t - D_{t,T}, r_f, \tau) \quad (33)$$

O_t is the currently observed option price, τ is the time to maturity and calculated based on the trading days not calendar days.

Fourth, the absolute value of option moneyness defined as $(K/S_t - 1)$ is less than 0.1.

$$|K/S_t - 1| \leq 0.1 \quad (34)$$

Finally, we include the trading day only if the number of options series that satisfy the above four conditions is more than eight, because we need enough option contracts to estimate the parameters of the models on the specific day. We extract the option series with the time to maturity closest to 20 trading days and estimate the models on a monthly basis. We can find the option series with exactly 20 days to maturity every month during the whole sample period, which is 67 months. In other words, there are 67 months for which the number of option contracts that satisfy our screening criteria is enough to estimate the parameters of the models. Table 3 is the summary of option data that satisfy the screening criteria. We find the volatility smile phenomenon in this sample. The Black-Sholes implied volatility calculated using ATM options data is less than the implied volatility from ITM options or OTM options data. Figure 1 depicts the six cross sections of call and put option of which the time to maturity is 20 trading days. We pick up the options at June per each year and graph the option prices against the option moneyness from the year 2001 to the year 2006. This figure shows that the call option premia decrease with the exercise price while the put option premia increase with the exercise price.

4. Models for estimation

In this paper, we basically try to illuminate the empirical performance of the option pricing model and that of the model based on the pricing kernel. They are estimated using option data. We adopt the GARCH option pricing model as the option pricing model and consider the pricing kernel implied by the GARCH option pricing model. We choose the pricing kernels suggested by Rosenberg and Engle (2002) as the parametric pricing kernels in this study. We also compare the GARCH option pricing model with the Black-Sholes option pricing model in the GARCH framework.

On the other hand, since the pricing kernel can be estimated using only underlying return data, we investigate the empirical performance of the pricing kernels estimated without option data.

We examine the empirical performance of the following six cases.

Case 1. Estimation of the pricing kernel using only the underling return data.

Case 1.1. GARCH model for the underlying process.

Case 1.2. Black-Sholes model (in the GARCH framework) for the underlying process.

Case 2. Estimation using the option data.

Case 2.1. GARCH option pricing model.

Case 2.2. Pricing kernel-based GARCH option model.

Case 2.3. Black-Sholes option pricing model (in the GARCH framework).

Case 3. Parametric pricing kernels. (the extended version of Rosenberg and Engle (2002)).

To fairly compare the performance of each case, we need to set up the unified framework for the underlying process. We select the Duan-GARCH process as our unified framework and extend it to reflect the asymmetric volatility. The extended Duan-GARCH process under physical measure (measure P) follows.

Mean equation: $\ln\frac{S_t}{S_{t-1}} - r_{f,t-1} = \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t$ where $\varepsilon_t \mid I_{t-1} \sim N(0,h_t)$ (35)

Variance equation: $h_t = w + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} + \delta[Max(0,-\varepsilon_{t-1})]^2$ (36)

Stability condition: $w > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \delta \geq 0$ and $\alpha + \beta + \frac{\delta}{2} < 1$ (37)

$r_{f,t-1}$ is the short term risk-free rate which applies from time t-1 to time t. For the extended Duan-GARCH process to become a covariance stationary and have a finite long run variance, the estimated parameter should satisfy the stability condition (equation (37)).

Option pricing is carried out under the risk neutral measure not under the physical measure. The extended Duan-GARCH process under the locally risk neutral measure (measure Q) follows.

Mean equation: $\ln\frac{S_t}{S_{t-1}} - r_{f,t-1} = -\frac{1}{2}h_t + \xi_t$ where $\xi_t \mid I_{t-1} \sim N(0,h_t)$ (38)

Variance equation: $h_t = \alpha_0 + \alpha_1(\xi_{t-1} - \lambda\sqrt{h_{t-1}})^2 + \beta_1 h_{t-1} + \delta[Max(0,-(\xi_{t-1} - \lambda\sqrt{h_{t-1}}))]^2$ (39)

Stability condition[5]: $w > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \delta \geq 0$,

and $\beta_1 + \alpha_1(1+\lambda^2) + \delta\left[\frac{\lambda}{\sqrt{2\pi}}e^{-\frac{\lambda^2}{2}} + (1+\lambda^2)\Phi(\lambda)\right] \leq 1$

where Φ is the cumulative standard normal distribution (40)

Table 4 shows the properties of the estimated standardized innovations of the extended Duan-GARCH process under measure P. The standard innovation series is calculated by dividing each ordinary innovation (ε_t) into its corresponding conditional standard deviation ($\sqrt{h_t}$). Figure 2 depicts the KOSPI 200 annualized conditional volatility implied by the extended Duan-GARCH process under measure P. It generally decrease as time goes by.

We explain how to estimate the pricing kernel or the option pricing model and how to measure pricing, forecasting and hedge performance of the models in each case.

Case 1. Estimation of the pricing kernel using only the underling return data.

Case 1.1. GARCH model for the underlying process.

First, from the underlying return data, we estimate the fixed parameters of the extended Duan-GARCH process and calculate the conditional variance series. The parameter set to be estimated is $\Theta = [w, \alpha, \beta, \lambda, \delta]$ and the conditional variance series is $\{h_k\}$, $k = t+1, 2, \ldots, T$.

[5] See Appendix A

We estimate parameters using the maximum likelihood estimation method assuming the innovation density, $N(0,h_t)$, is normal density. Even though the true innovation density is non-normal, Bollerslev and Wooldridge (1992) show the conditions for which this method provides consistent estimators.

The extended Duan-GARCH process implies the following pricing kernel (equation (41)). As we show at the section 2.2, we can derive it under measure P. The pricing kernel can be calculated given the estimated parameters and the conditional variance series from time t+1 to time T. Since the conditional variance and the short rate varies over time, the pricing kernel changes over time.

$$m^{GH}(t,T) = e^{-\sum_{k=t+1}^{T} r_{f,k-1}} Z(t,T) = \exp\left[-\sum_{k=t+1}^{T} r_{f,k-1} - \lambda \sum_{k=t+1}^{T}\left(R_k - r_{f,k-1} - \lambda\sqrt{h_k} + \frac{1}{2}h_k\right) / \sqrt{h_k} - \frac{1}{2}\lambda^2 (T-t)\right]$$

(41)

$m^{GH}(t,T)$ stands for the pricing kernel implied by the extended Duan-GARCH process and this pricing kernel applies from time t to time T.

Next, we explain how the pricing kernel explains the observed option price and measure its empirical performance. We only deal with the call option in this section. The procedure for the put option is similar and straightforward.

$$C^{Model}(t,T) = E[m^{GH}(t,T;\hat{\Theta}) \times payoff] \approx \frac{1}{J}\sum_{j=1}^{J}\left[m^{GH}(t,T,j;\hat{\Theta}) \max(S_{T,j} - K, 0)\right]$$

$$= \frac{1}{J}\sum_{j=1}^{J}\left[m^{GH}(t,T,j;\hat{\Theta}) \max(S_t R(t,T,j) - K, 0)\right]$$

$$\text{where } R(t,T,j) = \exp\left(\sum_{k=t+1}^{T}\left[r_{f,k-1} + \lambda\sqrt{h_k} - \frac{1}{2}h_k + \varepsilon_k\right]\right)\Bigg|_j \qquad (42)$$

$C^{Model}(t,T)$ is the theoretical call option price implied by the pricing kernel at time t, which matures at time T. The second approximated equality of equation (42) implies that the call option price is calculated using the Monte-Carlo simulation. K is the exercise price of the option. J is the number of simulations and the index j means the value at the j-th simulation. For instance, $m^{GH}(t,T,j;\hat{\Theta})$ means the pricing kernel which applies from time t to time T at the j-th simulation given the parameter estimates. $R(t,T,j)$ is the generated underlying gross return at the j-th simulation.

Using the pricing kernel at time t+1, the theoretical call option price at time t+1 can also be derived in a same manner.

$$C^{Model}(t+1,T) = E[m^{GH}(t+1,T;\hat{\Theta}) \times payoff] \qquad (43)$$

We measure the pricing and one-day forecasting performance by comparing the theoretical price and the observed price. Equation (44) (equation (45)) represents the measure of pricing (forecasting) performance in this study. They are the relative pricing errors and we can judge the pricing (forecasting) performance of a model is good if this value is small.

Pricing performance measure: $\left|\dfrac{C^{Model}(t,T) - C^{Market}(t,T)}{C^{Market}(t,T)}\right|$ (44)

Forecasting performance measure: $\left|\dfrac{C^{Model}(t+1,T) - C^{Market}(t+1,T)}{C^{Market}(t+1,T)}\right|$ (45)

Finally, we investigate hedge performance. We perform delta and gamma hedge, and compare each result. We explain the detailed hedging procedure in the later section and just show how to estimate option delta and gamma using the theoretical option price in this section. We generalize the Engle and Rosenberg (1995) methodology. They adopt the finite difference method to calculate the option delta and gamma. Like Rosenberg and Engle (2002), we assume that there are three possible one-day underlying price changes. Namely, the underlying price rises or falls by the symmetric size or remains the same.

$$S_{t+1} = \begin{cases} S_t + \varepsilon \\ S_t \\ S_t - \varepsilon \end{cases} \qquad (46)$$

Delta: $\dfrac{\partial C_{t+1}}{\partial S_{t+1}} \cong \dfrac{C_{t+1|S_t+\varepsilon} - C_{t+1|S_t-\varepsilon}}{2\varepsilon}$ (47)

Gamma: $\dfrac{\partial^2 C_{t+1}}{\partial S_{t+1}^2} \cong \dfrac{C_{t+1|S_t+\varepsilon} - 2C_{t+1|S_t} + C_{t+1|S_t-\varepsilon}}{\varepsilon^2}$ (48)

We define the size of ε as the one-day underlying price change when the error term, ε_{t+1}, increases by one standard deviation, $\sqrt{h_{t+1}}$. When the error term increases by one standard deviation, we can calculate the underlying price in the next period using the mean equation of the extended Duan-GARCH process under measure P (equation (48)).

$$S_{t+1} = S_t \exp(r_{f,t} + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}) \qquad (48)$$

The difference between the underlying price in the next period and current underlying price determine the size of ε.

$$\varepsilon = S_{t+1} - S_t = S_t[\exp(r_{f,t} + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}) - 1] \quad (49)$$

The one-day future theoretical call option price given the underlying price in the next period follows.

$$\begin{aligned} C_{t+1|S_{t+1}} &= E_{t+1|S_{t+1}}\left[m^{GH}(t+1,T;\hat{\Theta}_{t+1|S_{t+1}})Max(S_T - K, 0)\right] \\ &= E_{t+1|S_{t+1}}\left[m^{GH}(t+1,T;\hat{\Theta})Max(S_{t+1}R(t+1,T) - K, 0)\right] \end{aligned}$$

$$\text{where } R(t+1,T) = \exp(\sum_{k=t+2}^{T}\left[r_{f,k-1} + \lambda\sqrt{h_k} - \frac{1}{2}h_k + \varepsilon_k\right]) \quad (50)$$

The expectation operator $E_{t+1|S_{t+1}}[.]$ means the expectation of the time t+1 random variable given the value of the underlying price at time t+1. Since the parameter at time t+1 given the value of the t+1 underlying price, $\Theta_{t+1|S_{t+1}}$, is equal to the constant value, Θ, in this case, the second equality in equation (50) holds. We already estimate the value of Θ in the first step. Equation (51) means that we can calculate the value of $C_{t+1|S_{t+1}}$ by the Monte-Carlo simulation as previously explained.

$$C_{t+1|S_{t+1}} \cong \frac{1}{J}\sum_{j=1}^{J}\left[m^{GH}(t+1,T,j;\hat{\Theta})\max(S_{T,j} - K, 0)\right] \quad \text{where } S_{T,j} = S_{t+1}R(t+1,T,j) \quad (51)$$

Using the value of $C_{t+1|S_{t+1}}$, we derive the option delta and gamma as in equation (47) and equation (48). Now we can conduct the delta and gamma hedge using the delta and gamma value.

Case 1.2. Black-Sholes model (in the GARCH framework) for the underlying process.

We measure the empirical performance of the pricing kernel implied by the Black-Sholes model in the GARCH framework, which is estimated using only underlying return data. The underlying process of the Black-Sholes model in the GARCH framework corresponds to the Duan-GARCH(0,0) process. This process has a constant variance while the general Duan-GARCH process has a time-varying conditional variance. The underlying process of the Black-Sholes model under the physical measure (measure P) is expressed by equation (51).

$$\ln\frac{S_t}{S_{t-1}} - r_{f,t-1} = \lambda\sigma - \frac{1}{2}\sigma^2 + \varepsilon_t \quad \text{where } \varepsilon_t \mid I_{t-1} \sim N(0,\sigma^2) \quad (51)$$

$$e_t = \frac{\varepsilon_t}{\sigma} \quad \text{where} \quad e_t \mid I_{t-1} \sim N(0,1) \qquad (52)$$

Under the locally risk-neutral measure (measure Q), the underlying process follows.

$$\ln\frac{S_t}{S_{t-1}} - r_{f,t-1} = -\frac{1}{2}\sigma^2 + \xi_t \qquad \text{where} \quad \xi_t \mid I_{t-1} \sim N(0,\sigma^2) \qquad (53)$$

$$v_t = \frac{\xi_t}{\sigma} \quad \text{where} \quad v_t \mid I_{t-1} \sim N(0,1) \qquad (54)$$

(여기서 Q-measure 설명하는 것은 생략가능 P-measure만 설명해도 될 듯)

Except that we use the Black-Sholes process (under measure P) for the underlying process instead of the extended Duan-GARCH process, the entire procedure is the same as in Case 1.1.

Now, we investigate how to estimate the models and measure the empirical performance of the models based on the pricing kernel or the option pricing models using the observed option data. To eliminate the overlap of information due to put-call parity, we estimate the parameters of the models using only OTM (out-of-the-money) call options whose moneyness has positive values and OTM put options whose moneyness has negative value.

Case 2. Estimation with the option data.

Case 2.1. GARCH option pricing model.

In this Case 2.1, we estimate the parameters of the option pricing model and investigate the empirical performance of the model without the consideration of the pricing kernel. We adopt the option pricing model as the GARCH option pricing model for which the underlying process follows the extended Duan-GARCH process. By minimizing the difference between the theoretical price suggested by the GARCH option pricing model and the observed option price, we estimate the time-varying parameters of the underlying process under measure Q.

Under measure Q, the extended Duan-GARCH process follows.

Mean equation: $$\ln\frac{S_t}{S_{t-1}} - r_{f,t-1} = -\frac{1}{2}h_t + \xi_t \qquad \text{where} \quad \xi_t \mid I_{t-1} \sim N(0,h_t) \qquad (55)$$

Variance equation: $$h_t = w + \alpha(\xi_{t-1} - \lambda\sqrt{h_{t-1}})^2 + \beta h_{t-1} + \delta[Max(0, -(\xi_{t-1} - \lambda\sqrt{h_{t-1}}))]^2 \qquad (56)$$

The log return of the underlying asset can be calculated by the mean equation.

$$\ln\frac{S_T}{S_t} = \sum_{k=t+1}^{T}\left[r_{f,k-1} - \frac{1}{2}h_k + \xi_k\right] \qquad (57)$$

First, we derive the theoretical option price implied by the GARCH option pricing model given the parameter estimates.

$$C_t^{GH}(T,K) = e^{-\sum_{i=t+1}^{T} r_{f,i-1}} E_t^Q\left[\max(S_t \exp(\sum_{k=t+1}^{T}\left[r_{f,k-1} - \frac{1}{2}h_k + \xi_k\right]) - K, 0)\right] \quad (58)$$

$C_t^{GH}(T,K)$ is the call option price whose time to maturity is T and exercise price is K. We can calculate this value using the Monte-Carlo simulation given the parameter estimates and the conditional variance series. The parameter is determined by minimizing the sum of the square of the difference between the observed option price and theoretical price by the GARCH option pricing model. We estimate the parameters using the loss function that is defined on dollar pricing error[6].

$$\min_{\Theta_t} \sum_{i=1}^{L}\left(C_{i,t}^{observed} - C_{i,t}^{GH}(\Theta_t)\right)^2 \quad (59)$$

$C_{i,t}^{observed}$ is the i-th observed option price at time t. L is the number of option series classified by the exercise price at time t. We estimate $\Theta_t = [w_t, \alpha_t, \beta_t, \delta_t, \lambda_t]$ at every time t and calculate the conditional variance series $\{h_k\}$, $k = t+1, 2, \ldots, T$. The conditional variance series is calculated using the extended Duan-GARCH process under measure Q during the process of determining the value of parameter estimates.

Next, to measure the empirical performance, we derive the one-day future theoretical price given the underlying price in the next period. We also can calculate the option delta and gamma using this value as in equation (47) and equation (48). But we calculate the size of the one-day underlying asset price change in a different manner. One standard deviation of the error term, ξ_{t+1}, is $\sqrt{h_{t+1}}$ under measure Q. When the innovation term increases by the size of one standard deviation, the underlying price in the next period is determined as in equation (60).

$$S_{t+1} = S_t \exp(r_{f,t} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}) \quad (60)$$

We set the size of change, ε, as the difference between the current price and the underlying price change when the underlying price goes up.

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[6] Besides the loss function that is defined on the dollar pricing error, we can consider other loss functions, such as the loss function that is defined on the percent pricing error or implied volatilities. When we use the parameters, which are estimated from another loss function, the empirical performance of each case a slightly bit deteriorate. Christoffersen and Jacobs(2004) insist that it is more important to maintain the consistency of the loss function between estimating and evaluating the model, rather than the choice of the loss function itself.

$$\varepsilon = S_{t+1} - S_t = S_t[\exp(r_{f,t} - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}) - 1] \quad (61)$$

To measure the empirical performance including forecasting and hedge, we need to derive the one-day future theoretical price in this case. Equation (62) shows how to derive it given the parameter estimates at time t and the underlying price in the next period. We also can obtain the J-set of the generated underlying process under measure Q by the Monte-Carlo simulation. This J-set of the generated underlying process can be used as the distribution of the underlying return, which applies from time t+1 to T.

$$C^{GH}_{t+1|S_{t+1}}(T,K) = e^{-\sum_{k=t+2}^{T} r_{f,k-1}} E^{Q}_{t+1}\left[\max(S_{t+1}\exp(\sum_{k=t+2}^{T}\left[r_{f,k-1} - \frac{1}{2}h_k + \xi_k\right]) - K, 0)\right] \quad (62)$$

$C^{GH}_{t+1|S_{t+1}}(T,K)$ is the theoretical call option price implied by the model at time t+1, which matures at time T and has exercise price K, given the underlying price at time t+1.

$$C^{GH}_{t+1|S_{t+1}} \cong e^{-\sum_{k=t+2}^{T} r_{f,k-1}} \frac{1}{J}\sum_{j=1}^{J}\left[\max(S_{T,j} - K, 0)\right] \quad (63)$$

Equation (63) shows we can calculate the theoretical call option price by the Monte-Carlo simulation as previously explained. $S_{T,j}$ means the j-th generated value of underlying price at time T under measure Q, given the parameter estimates at time t and the underlying price at time t+1. Now, as in equation (47) and (48), we can calculate the option delta and gamma because we derive the current and future call option price implied by the model and determine the size of the one day underlying price change, ε, using equation (61).

Case 2.2. Pricing kernel-based GARCH option model.

In this case, we still estimate the time-varying parameters using only the observed option data like in the previous case, Case 2.1. But, we derive the theoretical option price suggested by the pricing kernel, which is implied by the GARCH option pricing model in this case. When the underlying process follows the extended Duan-GARCH process, we know the pricing kernel can be derived under measure P as in equation (64).

$$m^{GH}(t,T) = e^{-\sum_{k=t+1}^{T} r_{f,k-1}} Z(t,T) = \exp\left[-\sum_{k=t+1}^{T} r_{f,k-1} - \lambda\sum_{k=t+1}^{T}\left(R_k - r_{f,k-1} - \lambda\sqrt{h_k} + \frac{1}{2}h_k\right)/\sqrt{h_k} - \frac{1}{2}\lambda^2(T-t)\right]$$
(64)

Equation (65) shows the theoretical value of the call option at time t. It is the expected value of the product of the pricing kernel and future payoff under measure P. This option price is calculated by the Monte-Carlo simulation as previously shown.

$$C_t^{GH}(T,K) = E_t^P\left[m^{GH}(t,T)\max(S_t \exp(\sum_{k=t+1}^{T}\left[r_{f,k-1} + \lambda\sqrt{h_k} - \frac{1}{2}h_k + \varepsilon_k \right]) - K, 0)\right] \quad (65)$$

We also estimate the time varying parameter set, $\Theta_t = [w_t, \alpha_t, \beta_t, \delta_t, \lambda_t]$ and get the conditional variance series by solving equation (66).

$$\min_{\Theta_t} \sum_{i=1}^{L}\left(C_{i,t}^{observed} - C_{i,t}^{GH}(\Theta_t)\right)^2 \quad (66)$$

Equation (67) is the formula for the one-day future call option price in this case.

$$C_{t+1|S_{t+1}} = E_{t+1}\left[m^{GH}(t+1,T;\Theta_{t+1|S_{t+1}})Max(S_T - K, 0)\right]$$
$$\cong E_{t+1}\left[m^{GH}(t+1,T;\Theta_t)Max(S_{t+1}\exp(\sum_{k=t+2}^{T}\left[r_{f,k-1} + \lambda\sqrt{h_k} - \frac{1}{2}h_k + \varepsilon_k \right]) - K, 0)\right] \quad (67)$$

$m^{GH}(t+1,T;\Theta_{t+1|S_{t+1}})$ is the pricing kernel that applies from time t+1 to time T. $\Theta_{t+1|S_{t+1}}$ is the parameter estimates at time t+1 given the underlying price in the next time period. To compare with other cases in a consistent manner, we replace $\Theta_{t+1|S_{t+1}}$ with Θ_t which is estimated at time t. In other words, we use the parameters of the pricing kernel estimated today (time t) for obtaining the pricing kernel at the next date. Now we calculate the pricing kernel at time t+1 using the estimated parameters and the conditional variance series in a similar manner with equation (64).

Using the generated distribution of the underlying process, The one-day future call option price can be calculated given the pricing kernel and the underlying price at the next date (time t+1). The procedure for measuring the empirical performance using the current and the next theoretical call price is similar to the previous cases and straightforward.

Case 2.3. Black-Sholes option pricing model in the GARCH framework.

We examine the discrete time version of Balck-Sholes option pricing formula in the GARCH framework. This is a kind of benchmark case for the GARCH option pricing model. The underlying process of the Black-Sholes model corresponds to the Duan-GARCH(0,0) process which has no lag-error term and no conditional variance term in the variance equation of the GRACH model. Equation (68) represents the underlying process of the Black-Sholes model in the GRACH framework under measure Q.

$$\ln \frac{S_t}{S_{t-1}} - r_{f,t-1} = -\frac{1}{2}\sigma^2 + \xi_t \qquad \text{where} \quad \xi_t \mid I_{t-1} \sim N(0, \sigma^2) \qquad (68)$$

$$v_t = \frac{\xi_t}{\sigma}, \quad v_t \mid I_{t-1} \sim N(0,1) \qquad (69)$$

Equation (69) shows that the normalize error is distributed as standard normal.

The parameter to be estimate is just σ in this case. The whole procedure for estimating and evaluating is similar to that of Case 2.1 (the GARCH option pricing model) except that the underlying process under measure Q has been changed into equation (68).

Case 3. Parametric pricing kernels

We investigate the parametric pricing kernels that have explicitly functional forms in this case. Instead of using the pricing kernel implied by the underlying process like in Case 2.2, we adopt two parametric pricing kernels suggested by Rosenberg and Engle (2002). But we assume a different process for the underlying process. While Rosenberg and Engle (2002) model the underlying process as the simple asymmetric GARCH model like Glosten et al (1993), we adopt the extended Duan-GARCH process to compare it with other cases under the unified framework.

First, like in Case 1.1, we estimate the constant parameters of the extended Duan-GARCH process under measure P using only underlying return data. Then we generate the return distribution based on the parameter estimates and the conditional variance series.

Second, we estimate two parametric pricing kernels by minimizing the theoretical price given this generated underlying return distribution under measure P and the observed option price (equation (70)). The specification of the power pricing kernel is equation (71) and that of the polynomial pricing kernel is equation (72). We transform the polynomial pricing kernel to guarantee it always has positive value.

$$\min_{\theta_t} \sum_{i=1}^{L} \left(C_{i,t}^{observed} - C_{i,t}^{RE}(\theta_t) \right)^2 \qquad (70)$$

Power pricing kernel: $M(r_{t+1};\theta_t) = \theta_{0,t}(r_{t+1})^{-\theta_{1,t}}$ (71)

Polynomial pricing kernel:

$$M^*(r_T;\theta_t) = \theta_{0,t} T_0(r_T) \exp\left[\theta_{1,t} T_1(r_T) + \theta_{2,t} T_2(r_T) + \ldots + \theta_{N,t} T_N(r_T) \right]$$

where, $T_n(x) = \cos(n \cos^{-1}(x))$ (72)

θ_t stands for the parameter set of the pricing kernel. In the case of the polynomial pricing kernel, the domain of the inverse Cosine function should be [-1,1]. So we impose the restriction on the underlying return distribution when we estimate it. We set the generated return at time t, r_T, to be inside the interval $[a,b]$ and define the x as equation (73).

$$x = ((2r_T - a - b)/(b - a)) \qquad (73)$$

We set the generated underlying return domain for the polynomial pricing kernel equal to [-30, 30%][7] in this study. The remaining procedure for deriving the current and future theoretical price is similar to that of case 2.2 except that we use the parametric pricing kernel estimates in this case. We can measure the empirical performance in a same manner. In addition, we investigate how the empirical performance changes if we impose restriction that the pricing kernel exactly calibrates the short term bond price as in Rosenberg and Engle (2002).

5. Empirical Results

In this section, we present the parameter estimates and the empirical performance of the models in each case. We report the pricing, forecasting and hedging results as the empirical performance. In each case, we estimate the parameters of the pricing model or the pricing kernel on a monthly basis. Since there are 67 months in our sample, we measure the empirical performance 67 times in each case.

5.1. The parameter estimates

Table 5 shows the descriptive statistics of the parameter estimates in each case. We estimate the extended Duan-GARCH process using the underlying return data in Case 1.1. The delta coefficient that catches the asymmetric volatility phenomenon is significantly estimated. We estimate the time-varying parameters of the models using option data in Case 2. We report the time-series mean, median and standard deviation of the estimates over 67 months. We present the time-series mean, median and standard deviation of the pricing kernel parameter estimates over 67 months in Case 3 (Panel C). (A) and (C) of Panel C shows the descriptive statistics on the parametric pricing kernel estimates and (B) and (D) of Panel C show the estimation results when we impose the bond pricing restriction. As we can see in (B), the variation of the scaling parameter ($\theta_{0,t}$) is very small compared to (A) which is estimated without the bond pricing restriction. This is because the scaling parameter is determined to exactly fit the short term bond price in (B).

Figure 3 compares the variation for the estimated parameter values of the Duan-GARCH based

[7] Rosenberg and Engle (2002) restrict the return domain [-10, 10%]. Our case is more general.

models (Case 1.1, Case 2.1, and Case 2.2). Since the parameter of the Case 1.1 is constant, we can't detect any movement of parameter in Case 1.1. Except the unit risk premium parameter, λ , all other parameter estimates of the GARCH option pricing model (Case 2.1) and the pricing kernel-based GARCH option model (Case 2.2) move similarly as time goes by. While the λ estimates of the GARCH option pricing model is relatively stable, that of the pricing kernel-based GARCH option model swings.

Figure 4 graphs the movement of parameter estimates of the parametric pricing kernels. We compare the variation for the parameters estimated with and without bond pricing restriction. We can see that the scaling parameter ($\theta_{0,t}$) of the power pricing kernel estimated with bond pricing restriction remains stable around 1. The risk aversion parameter ($\theta_{1,t}$) shows greater variation as time goes by. The risk aversion parameters estimated with and without bond pricing restriction move similarly. This is because there is no reason for the drift of risk aversion change substantially even if we suppose bond pricing restriction. Three parameters ($\theta_{1,t}$ $\theta_{2,t}$ $\theta_{3,t}$) of the polynomial pricing kernel remains relatively stable before the year 2004 and swing after the 2004 year. This pattern is seemingly detected in the case of the risk aversion parameter of the power pricing kernel.

5.2. The pricing and forecasting performance

As previously explained, we measure the pricing performance by comparing the current theoretical option price suggested by the model with the currently observed option price and measure the forecasting performance by comparing the one-day future theoretical option price suggested by the model with the observed option price in the next period. We evaluate the performance the across option moneyness. We define the OTM (ITM) option as the call option for which moneyness $(K/S_t - 1)$ is closest 0.05 but greater (lesser) than 0.05 and choose the call option for which moneyness $(K/S_t - 1)$ is closest to 0 as in the ATM option.

To examine not only pricing performance but also one-day forecasting and hedge performance in a consistent manner, the OTM (ITM) option in the current day and the OTM (ITM) option in the next day should be the same option series. Namely, they have same exercise price. If we can't find one day's OTM (ITM) option in the sample of the next day, we select that day's OTM (ITM) option again. We redefine the OTM (ITM) option as the call option for which moneyness $(K/S_t - 1)$ is closest 0.05 but can be lesser (greater) than 0.05. We can always find the ATM option defined above in the sample on the next date. So we don't have to think about this issue in the case of ATM options. While we can always define OTM option following above method, we can't select suitable ITM option for 2 months during our sample period.

Table 6 shows the pricing and forecasting performance of each case using the following measures, the average and dispersion of the relative pricing and forecasting error. If these values are small, we can understand the option pricing model or the pricing kernel based model price and forecast well.

Pricing performance measures follows.

$$mean\left(100\times\left|\frac{C^{Model}(t,T)-C^{Market}(t,T)}{C^{Market}(t,T)}\right|\right),\quad std\left(100\times\left|\frac{C^{Model}(t,T)-C^{Market}(t,T)}{C^{Market}(t,T)}\right|\right) \quad (74)$$

Forecasting performance follows,

$$mean\left(100\times\left|\frac{C^{Model}(t+1,T)-C^{Market}(t+1,T)}{C^{Market}(t+1,T)}\right|\right),\quad std\left(100\times\left|\frac{C^{Model}(t+1,T)-C^{Market}(t+1,T)}{C^{Market}(t+1,T)}\right|\right) \quad (75)$$

We evaluate the empirical performance based on the average value of absolute pricing and forecasting error over the whole sample period. When we use option data, the pricing and forecasting performance is much improved compared to using only the underlying data. When we estimate the pricing kernel only using the underlying return data (Case 1), the performance of the extended Duan-GARCH process (Case 1.1) is far better than that of the Black-Sholes underlying process (Case 1.2). This may be because the time-varying conditional variance describes the characteristics of the underlying process better than the constant variance assumption. The pricing (forecasting) performance of BS models (Case 1.2) shows greater variation across option moneyness compared to the performance of GARCH model (Case 1.1). This relates to the volatility smile phenomenon when we use Black-Sholes approach.

However, when we use the option data, the Black-Sholes option pricing model (Case 2.3) shows the best the pricing and forecasting performance for ATM options. Of course, the pricing and forecasting errors of the Black-Sholes option pricing model shows relatively larger dispersion than other cases considering the standard deviation. This may be because the stability on the pricing and forecasting ability of the constant volatility model is relatively low. A slight bit poor pricing performance for OTM and ITM options still relates to the volatility smile phenomenon when we use Black-Sholes approach.

Under the GARCH option pricing model framework, we find the model based on the pricing kernel (Case 2.2) shows slightly better performance than the option pricing model (Case 2.1) except for the ITM option. These models perform better than Black-Sholes option pricing model when pricing and forecasting the OTM option of which liquidity is very fluent compared to ATM or ITM options. Although we derive the pricing kernel with approximation (Case 2.2), its pricing (forecasting) performance is quite good.

When we price (forecast) the options using the parametric pricing kernels (Case 3), the performance is fairly improved compared to other cases. They seem to fit the observed option price well because the parametric pricing kernels have flexible functional structure. Or this is because we model the underlying process and the pricing kernel separately and estimate them using the underlying data and option data each. This may provide more accuracy[8].

Furthermore, the pricing (forecasting) performance of the polynomial pricing kernel, which has a more complicated functional form than the power pricing kernel, is remarkable. It prices (forecasts) pretty well on average and the small dispersion of the pricing and forecasting error means that it prices (forecasts) in a stable manner.

The pricing (forecasting) performance for OTM and ATM options worsens if we impose the restriction that exactly calibrates the short term bond price on the pricing kernel as in Rosenberg and Engle (2002). However, while the degree of deterioration is large in the case of the power pricing kernel, the degree of deterioration is very small in the case of the polynomial pricing kernel, which has relatively flexible functional form. Furthermore, the pricing (forecasting) performance on ITM options is rather enhanced with the short-term bond pricing restriction. ((?)This result may be attributable to the fact that the liquidity of ITM options is not abundant. parameter 추정시 ITM option 자료는 이용하지 않았고 (because of put-call parity, Rosenberg and Engle(2002) 처럼), 따라서 pricing 할 때 bond-pricing 제약 주면 단기채권가격의 정보까지 이용하는 셈이니, 이것과 관련이 있는 것인가? 혹은 만기가 한달 남은 ITM 옵션과 단기채권의 가격과의 관계가 있는가? 이도 저도 아니면 ITM 에 대한 pricing 결과는 hedge section과의 일관성을 위해서도 빼버릴 수도 있다.)

5.3. The hedge performance

We measure the hedge performance by creating hedge portfolio for a 100 point position in OTM call options because we want to fairly evaluate the hedge performance for each month during the whole sample period. For example, let's assume the OTM call option price is 2 points in January and 2.5 points in February. Then we hedge 50 options in January and 40 options in February. We define the OTM option and the ATM option in a same manner as in the previous section (section 5.2).

The hedge performance is evaluated by the following hedge methods. First, we conduct the delta hedge using only the underlying asset. Second, we also conduct the delta hedge but using only the ATM call option. Third, we conduct the gamma hedge using the underlying asset and the ATM call option simultaneously.

Hedge method 1: Delta hedge using the stock index portfolio

Hedge method 2: Delta hedge using the ATM call option

Hedge method 3: Gamma hedge using the stock index portfolio and the ATM call option

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[8] In contrast to Case 3, we estimate the parameters of underlying process implied by option data in Case 2.

When we define the hedge error and measure hedge performance, we follow the convention of Bagchi et al (1997). The hedge error is defined as the difference between the OTM option price and the value of our hedge portfolio. We report the average and standard deviation of the hedge error series, and the average and standard deviation of the absolute hedge error series.

$$\frac{1}{M}\sum_{l=1}^{M} H(t+l\Delta t),\ \ std(H(t+l\Delta t)), \frac{1}{M}\sum_{l=1}^{M}\left|H(t+l\Delta t)\right|,\ \ std(|\,H(t+l\Delta t)\,|)$$

where $l = 1,2,...,M-1, M \equiv (T-t)/\Delta t$ (76)

The $H(t+l\Delta t)$ stands for the hedge error at time $t+l\Delta t$. T is the end of time period. We examine the one-day hedge error. Δt is equal to 1 in our study. Among these measures, we especially have interests in the last two hedge performance measures. If they have small value, we can judge the hedge performance as being good. We explain how we can hedge when we are in a long position of one OTM call option. If we want to hedge for the 100 point OTM call option long position, we can just hedge for call options whose amount is determined by dividing 100 points by the current OTM call option price.

We define the notations first. $C_{OTM,t}$ is the OTM call price that is observed at time t. P_t is the value of portfolio at time t. $w_{S,t}$ is the position on the underlying asset at time t and $w_{ATM,t}$ is the position on the ATM call option at time t. X_t is the amount that is invested in the money market account. $\Delta_{OTM,t}$, $\Delta_{ATM,t}$, $\Gamma_{OTM,t}$ and $\Gamma_{ATM,t}$ are option Greeks calculated by the finite difference method as previously shown, r_f is the continuous compounding risk-free rate and Δt is the time interval (one day). The detail hedge process for each hedge method follows.

Hedge method 1: Delta hedge using the stock index portfolio

P_t, the value of portfolio at time t, is defined as equation (77). Since the delta value of the hedge portfolio should be equal to the OTM call option delta value ($\Delta_{OTM,t}$), we set $w_{S,t}$ as $\Delta_{OTM,t}$.

$$P_t = w_{S,t}S_t + X_t = \Delta_{OTM,t}S_t + X_t \quad (77)$$

X_t is determined so that the OTM call option price, $C_{OTM,t}$ is equal to hedge portfolio value P_t at time t. At the next date, the value of hedge portfolio follows.

$$P_{t+1} = \Delta_{OTM,t}S_{t+1} + X_t \exp(r_f \Delta t) \quad (\Delta t = 1/250) \quad (78)$$

We define the hedge error as the difference between the OTM call option price which is observed at the market and the value of hedge portfolio at this time step.

Hedge error= P_{t+1} - $C_{OTM,t+1}$ (79)

Hedge method 2: Delta hedge using the ATM call option

Equation (80) shows the hedge portfolio value at time t. X_t is also determined so that the OTM call option premium, $C_{OTM,t}$ is equal to hedge portfolio value P_t at time t.

$$P_t = w_{ATM,t} C_{ATM,t} + X_t \quad (80)$$

The delta value of the hedge portfolio should be the same as the call option delta value ($\Delta_{OTM,t}$). Thus $w_{ATM,t}$ is determined by equation (81).

$$\Delta_{OTM,t} = w_{ATM,t} \Delta_{ATM,t} \quad (81)$$

The value of hedge portfolio on the next day follows.

$$P_{t+1} = w_{ATM,t} C_{ATM,t+1} + X_t \exp(r_f \Delta t) \quad (82)$$

We define the hedge error as in equation (79).

Hedge method 3: Gamma hedge using the stock index portfolio and the ATM call option

P_t, the value of portfolio at time t, is defined as equation (83) in the hedge method 3.

$$P_t = w_{S,t} S_t + w_{ATM,t} C_{ATM,t} + X_t \quad (83)$$

We decide the position in the underlying asset, $w_{S,t}$ and the position in the ATM call option using equation (84) and (85) because the delta and gamma value of the hedge portfolio should be equal to the OTM call option delta and gamma value.

$$\Delta_{OTM,t} = w_{S,t} + w_{ATM,t} \Delta_{ATM,t} \quad (84)$$

$$\Gamma_{OTM,t} = w_{ATM,t} \Gamma_{ATM,t} \quad (85)$$

The value of hedge portfolio on the next day follows.

$$P_{t+1} = w_{S,t} S_{t+1} + w_{ATM,t} C_{ATM,t+1} + X_t \exp(R_f \Delta t) \quad (86)$$

We also define the hedge error as in equation (79).

Table 7 shows the empirical results on the hedge performance of each case evaluated by each hedge method. We evaluate the hedge performance based on the mean and standard deviation of absolute hedge errors. When we use option data (Case 2 or 3), we find the hedge performance is generally improved compared to when only the underlying data is used (Case 1). But the degree of improvement is not so remarkable compared to the pricing (forecasting) performance cases.
(hedge를 할 때는 pricing과 달리 underlying만 사용해서 추정한 경우도 성과가 (나빠지기는 나빠지나) 크게 떨어지지 않는다. 추가적인 설명과 이유는 무엇이 있을까?)

When we estimate the pricing kernel only using the underlying return data (Case 1), the hedge performance of the extended Duan-GARCH process (Case 1.1) is still better than that of the Black-Sholes underlying process (Case 1.2) like the pricing (forecasting) performance. We may attribute this result to the fact that the time-varying conditional variance describes the characteristics of the underlying process better than the constant variance assumption.

In the three Case 2 models (Case 2.1, Case 2.2, Case 2.3,), the hedge performance of the pricing kernel-based GARCH option model (Case 2.2) is better than the performance of the GARCH option pricing model (Case 2.1) when we conduct the delta hedge with the ATM options but we find the opposite result when we conduct the gamma hedge. Unlike result of the pricing (forecasting) performance, these two models (Case 2.1 and Case 2.2) show a greater hedge performance than the Black-Sholes option pricing model (Case 2.3) regardless of the chosen hedging method. But, the difference of the hedge performance is not so remarkable among these cases.
(블랙 숄즈 모형의 헤지 성과는 일반적으로 상당히 좋은 것으로 알려져 있다. 본 논문의 결과는 Case 2사이에서 헤지 성과의 큰 차이가 없는 것으로 나타난다. 오히려 다른 경우가 블랙 숄즈보다 약간 더 좋음. 이것은 이 논문에서 사용된 Black-Sholes는 discrete time version(under GARCH(0,0) framework) 이라서 그런가? 이에 대한 추가적인 설명?)

Comparing the hedge performance of the parametric pricing kernel models (Case 3) with that of the models under the extend Duan-GARCH option pricing model framework (Case 2), we have a different conclusion depending on the hedge method we use. For instance, for the power pricing kernel, while the hedge performance is slightly improved in Case 3, if we conduct the delta hedge using the underlying asset, it deteriorates if we evaluate the hedge performance by the gamma hedge result.

The hedge performance of the polynomial pricing kernel is much better than that of the power pricing kernel when we conduct the delta hedge with the ATM call option and the gamma hedge. And the hedge performance of the polynomial pricing kernel is dominant over the performance of the power pricing kernel when we impose the short term bond pricing restriction.((B) and (D) in Panel C.). This result contrasts with Rosenberg and Engle (2002). They insist that the power pricing kernel is superior to the polynomial pricing kernel in terms of the hedge performance.

As we show in the previous section (section 5.2), the pricing (forecasting) performance for OTM options worsens if we impose the restriction that the pricing kernel exactly calibrate the short term bond price. However, the hedge performance sometimes improves depending on the hedge method. For example, when we conduct the gamma hedge, the mean of absolute hedge error is 9.539 if we estimate the power pricing kernel with bond pricing restriction and 13.251 if we estimate the power pricing kernel without restriction.

6. Concluding Remarks

Our paper investigates the empirical performance of the option pricing models and the pricing-kernel based models in the unified framework. We use KOSPI 200 option price and/or KOSOI 200 return to estimate the option pricing models and the pricing kernel-based models. We compare the pricing and forecasting ability across option moneyness and examine the hedge performance using the three different hedge methods.

The models estimated using option data are far better than the models estimated using only the underlying return data in terms of the empirical performance. The parametric pricing kernels generally outperform the GARCH option pricing models and the pricing kernel-based GARCH option models, of which the hedge performance is dominant to the Black-Scholes model under the GRACH framework.

While the pricing and forecasting performance across the models are prominent, the hedge performance shows relatively less difference among the models. We conclude that the empirical evidence on the performance of the models is mixed in the KOSPI 200 options market.

The pricing and forecasting performance across the models is prominent.

Appendix A

Duan (1997) introduces an augmented GARCH(1,1) process that can include many GARCH type models and derive the stability condition to guarantee the existence of the long run variance of the GARCH model. We can derive the stability conditions of the GARCH class process used in this paper through the stability condition of the augmented GARCH process. We show how one can derive the various GARCH type models from the augmented GARCH process and how the stability condition can be determined. We assume that error term follows standard Normal distribution.

Augmented GARCH(1,1)process.

(1)Mean equation: $X_t = \mu_t + \sqrt{h_t}\varepsilon_t$ where $\varepsilon_t \mid I_{t-1} \sim N(0,1)$

(2)Variance equation: $\phi_t = \alpha_0 + \phi_{t-1}\xi_{1,t-1} + \xi_{2,t-1}$

$$h_t = \begin{cases} |\lambda\phi_t - \lambda + 1|^{1/\lambda}, if\, \lambda \neq 0 \\ \exp(\phi_t - 1), if\, \lambda = 0 \end{cases}$$

$$\xi_{1,t} = \alpha_1 + \alpha_2 |\varepsilon_t - c|^{\delta} + \alpha_3 \max(0, -(\varepsilon_t - c)^{\delta})$$

$$\xi_{2,t} = \alpha_4 f(|\varepsilon_t - c|; \delta) + \alpha_5 f(\max(0, -(\varepsilon_t - c)); \delta)$$

(3)Stability condition

Assuming that $\alpha_1 \geq 0$, $\alpha_2 \geq 0$ $\alpha_2 + \alpha_3 \geq 0$,

(a) $\alpha_1 + \alpha_2 E\left[|\varepsilon_t - c|^{\delta}\right] + \alpha_3 E\left[\max(0, -(\varepsilon_t - c))^{\delta}\right] \leq 1$ if $\alpha_2 > 0$ or $\alpha_3 \neq 0$

(b) $\alpha_2 < 1$ if $\alpha_2 = \alpha_3 = 0$

Nested GARCH Models

1. GARCH (1,1)

$\lambda = 1, c = 0, \delta = 2, \alpha_3 = 0, \alpha_4 = 0, \alpha_5 = 0, \alpha_0 > 0, \alpha_1 \geq 0, \alpha_2 \geq 0$

(1)Variance equation

$h_t = \alpha_0 + \alpha_1 h_{t-1} + \alpha_2 h_{t-1}\varepsilon_{t-1}^2$

(2)Stability condition

Using the fact that, $E[\varepsilon_t^2]=1$, we can derive, $\alpha_1+\alpha_2 \le 1$ [9].

2.NGARCH (1,1)

$\lambda=1, \delta=2, \alpha_3=0, \alpha_4=0, \alpha_5=0, \alpha_0>0, \alpha_1\ge 0, \alpha_2\ge 0$

(1)Variance equation

$h_t=\alpha_0+\alpha_1 h_{t-1}+\alpha_2 h_{t-1}(\varepsilon_{t-1}-c)^2$

(2)Stability condition

$\alpha_1+\alpha_2(1+c^2)\le 1$

3.GJR-GARCH (1,1)

$\lambda=1, c=0, \delta=2, \alpha_4=0, \alpha_5=0, \alpha_0>0, \alpha_1\ge 0, \alpha_2\ge 0, \alpha_2+\alpha_3\ge 0$

(1)Variance equation

$h_t=\alpha_0+\alpha_1 h_{t-1}+\alpha_2 h_{t-1}\varepsilon_{t-1}^2+\alpha_3 h_{t-1}\max(0,-\varepsilon_{t-1})^2$

(2)Stability condition

$$\alpha_1+\alpha_2+\frac{1}{2}\alpha_3\le 1$$

4.GJR-NGARCH (1,1)

$\lambda=1, \delta=2, \alpha_4=0, \alpha_5=0, \alpha_0>0, \alpha_1\ge 0, \alpha_2\ge 0, \alpha_2+\alpha_3\ge 0$

(1)Variance equation

$h_t=\alpha_0+\alpha_1 h_{t-1}+\alpha_2 h_{t-1}(\varepsilon_{t-1}-c)^2+\alpha_3 h_{t-1}\max(0,-(\varepsilon_{t-1}-c))^2$

(2)Stability condition

$\alpha_1+\alpha_2 E[|\varepsilon_t-c|^2]+\alpha_3 E[\max(0,-(\varepsilon_t-c))^2]\le 1$

$$\rightarrow \alpha_1+\alpha_2(1+c^2)+\alpha_3\left[\frac{c}{\sqrt{2\pi}}e^{-\frac{c^2}{2}}+(1+c^2)\Phi(c)\right]\le 1$$

(proof)

Let $X=\varepsilon_t$ where $X\sim N(0,1)$

$E[X]=0,\ E[X^2]=1,$ and $\Phi(c)=\int_{-\infty}^{c} f(x)dx=\int_{-\infty}^{c}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$

$\alpha_1+\alpha_2 E[(X-c)^2]+\alpha_3 E[\max(0,c-X)^2]$

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9 The strict condition, $\alpha_1+\alpha_2<1$, guarantee the existence of stationary variance

$$= \alpha_1 + \alpha_2(1+c^2) + \alpha_3\left\{c^2\int_{-\infty}^{c} f(x)dx - 2c\int_{-\infty}^{c} xf(x)dx + \int_{-\infty}^{c} x^2 f(x)dx\right\}$$

$$= \alpha_1 + \alpha_2(1+c^2) + \alpha_3\left[\frac{c}{\sqrt{2\pi}}e^{-\frac{c^2}{2}} + (1+c^2)\Phi(c)\right]$$

The second equality holds by the integration by part,

$$\int_{-\infty}^{c} x^2 f(x)dx = \left[x(-\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}})\right]_{-\infty}^{c} + \int_{-\infty}^{c} \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$$

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Table 1. The World's Top 10 Derivative Contracts

This table shows the ten most active derivative contracts, measured in millions of contracts from the year 2001 to the year 2006. The rank is determined based on the trading volume of the year 2005 because the rank of the TIIE 28-Day Interbank Rate Futures, which is 5th at the year 2006, is below 20th before the year 2002.

Rank	Contract	2001	2002	2003	2004	2005	2006
1	KOSPI 200 options, Korea Exchange	823.3	1,889.80	2,837.70	2,521.60	2,535.20	2,414.42
2	Eurodollar Futures, CME	184	202.1	208.8	297.6	410.4	502.08
3	Euro-Bund Futures, Eurex	178	191.3	244.4	239.8	299.3	319.89
4	10-year T-Note futures, CBOT	57.6	95.8	146.5	196.1	215.1	255.57
5	E-mini S&P500 Index Futures, CBOT	39.4	115.7	161.2	167.2	207.1	257.93
6	Eurodollar Options, CME	88.2	105.6	100.8	130.6	188	268.96
7	Euribor Futures, Euronext.liffe	91.1	105.8	137.7	157.8	166.7	202.09
8	Euro-Bobl Futures, Eurex	99.6	114.7	150.1	159.2	158.3	167.31
9	Euro-Schatz Futures, Eurex	92.6	108.8	117.4	122.9	141.2	165.32
10	DJ Euro Stoxx 50 Futures, Eurex	37.8	86.4	116	121.7	140	213.51

Source: Futures Industry Association (http:\\www.futuresindustry.org)

Table 2. KOSPI 200 daily log-return descriptive statistics

The sample period of the daily log-return of KOSPI 200 index is from January, 2001 to October, 2006. (Three month longer than the option sample period.) The annualized mean and annualized standard deviation is percentage value. We report Jarque-Bera (1980) test statistic and p-value for the normality test.

Number of observations	1441
Annulized mean	17.577
Annulized std. dev.	26.687
Skewness	-0.356
Kurtosis	6.844
Jarque-Bera test statistics	917.816
Jarque-Bera test p-value	0.000
Sum	1.025
Sum sq. dev.	0.415

Table 3. Summary of option data that satisfy the screening criteria

We extract our sample data from the cross-section of options per every single month to estimate the parameters of the models. The time to maturity is 20 trading days. In our sample, there are 67 estimation dates. When we estimate the parameters of the model, we use only the OTM call (moneyness is greater than zero, K/St-1>=0) and the OTM put (moneyness is less than zero, K/St-1<0) among this sample data.

moneyness(K/S-1)		Call options			Put options		
From	To	No. of observation	price	implied vol.	No. of observation	price	implied vol.
-0.1	-0.06	78	9.001	26.21	110	0.784	30.08
-0.06	-0.03	85	6.567	24.65	89	1.512	28.72
-0.03	0	81	4.217	24.62	82	2.620	28.10
0	0.03	87	2.546	25.16	86	4.338	28.44
0.03	0.06	88	1.397	25.08	86	6.655	29.02
0.06	0.1	99	0.686	25.56	94	9.202	29.96

Figure 1. KOSPI 200 option prices of which time-to-maturities are 20 trading days

This figure depicts the six cross-section of the call and put KOSPI 200 option prices. We sample the value at every June per year. We graphs the call price for the positive option moneyness and put price for the negative option moneyenss. The option moneyenss is caculated by dividing the option exercise price into the current KOSPI 200 index value and distracting 1.

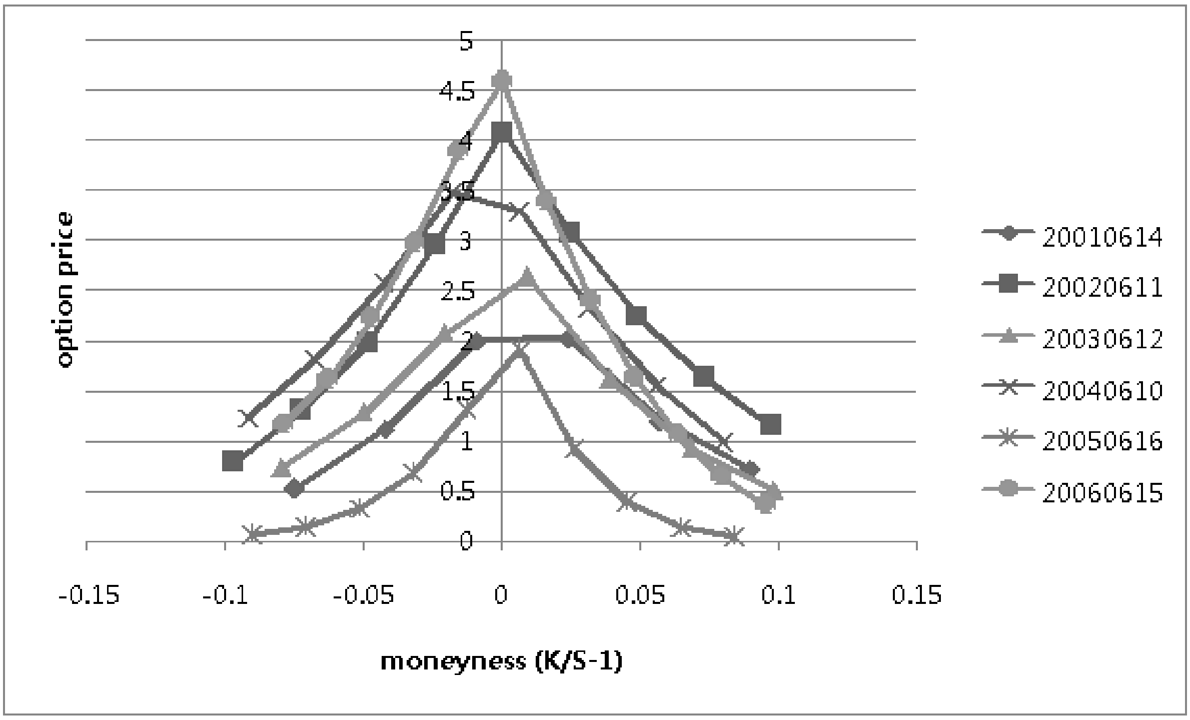


Table 4. Descriptive statistics for the estimated standard innovations of the extended Duan-GARCH process under measure P.

This table shows the properties of the estimated standardized innovations of the extended Duan-GARCH process under measure P. The standard innovation series is calculated by dividing each ordinary innovation by its corresponding conditional standard deviation. The sample period of the daily log-return of KOSPI 200 index is from January, 2001 to October, 2006. (Three month longer than the option sampling period.) We report Jarque-Bera (1980) test statistic and p-value for the normality test.

Observations	1441
Mean	0.013
Median	0.058
Maximum	3.921
Minimum	-7.332
Std. Dev.	0.977
Skewness	-0.463
Kurtosis	5.552
Jarque-Bera	442.522
Probability	0.000

Figure 2. Annualized KOSPI 200 log-return conditional volatility, estimated using the extended Duan-GARCH process under measure P.

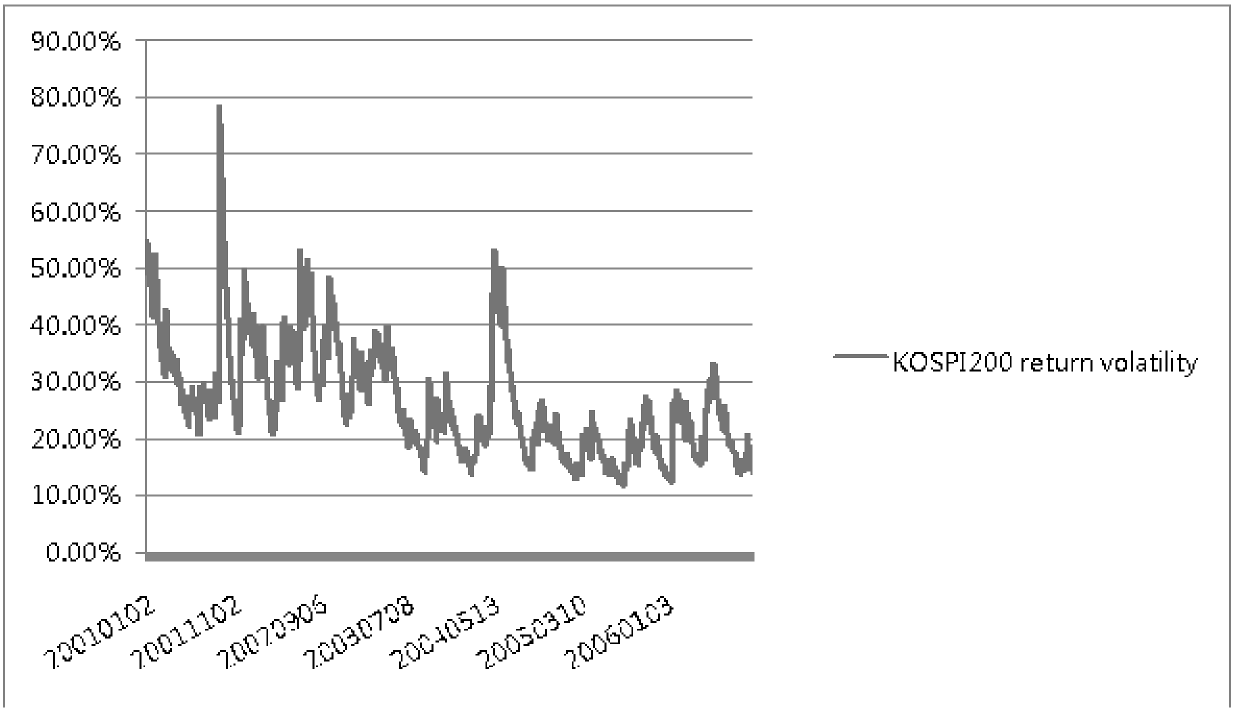


Table 5. Descriptive statistics for the parameter estimates of each case.

This table shows the descriptive statistics for parameter estimates. We report the estimated coefficients, standard deviation and T-statistics for Case 1 because we estimate the constant parameters. We report the time-series mean, median, and standard deviation for Case 2 and Case 3. (A) and (C) of Pancel C shows the descriptive statistics on the parametric pricing kernel estimates without bond pricing restriction and (B) and (D) of Panel C shows the result with bond pricing restriction.

Panel A. Case 1. Estimation of the pricing kernel using only the underling return data.

Case 1.1. GARCH model

	coeff.	std	T-statistics
a0	2.56E-06	1.01E-06	2.54
a1	0.03823	0.01559	2.45
b1	0.91416	0.01727	52.94
lambda	0.03326	0.02636	1.26
delta	0.09280	0.02591	3.58

Case 1.2. Black-Sholes model

	coeff.	std	T-statistics
lambda	0.03975	0.02636	1.51
sigma2	0.00029	0.00001	26.36

Panel B. Case 2. Estimation with the option data.

Case 2.1.GARCH option pricing model

	mean	median	std
a0	0.00011	0.00009	0.00005
a1	0.14798	0.14833	0.01845
b1	0.40861	0.39820	0.05102
lambda	0.03054	0.03056	0.00009
delta	0.05493	0.04075	0.06117

Case 2.3. Black-Sholes model

	mean	median	std
sigma2	0.00035	0.00028	0.00020

Case 2.2. Pricing kernel-based GARCH option model

	mean	median	std
a0	0.00011	0.00009	0.00006
a1	0.12307	0.11923	0.03220
b1	0.39387	0.38692	0.08645
lambda	0.01907	0.01730	0.01512
delta	0.12170	0.11908	0.07903

Panel C. Case 3. Parametric pricing kernel

(A)power pricing kernel

	mean	median	std
theta0	0.9458	0.9313	0.1970
theta1	0.5800	0.3813	0.9457

(B)power PK with bond

	mean	median	std
theta0	0.9935	0.9954	0.0065
theta1	0.5493	0.3429	0.8615

(C)polynomial pricing kernel

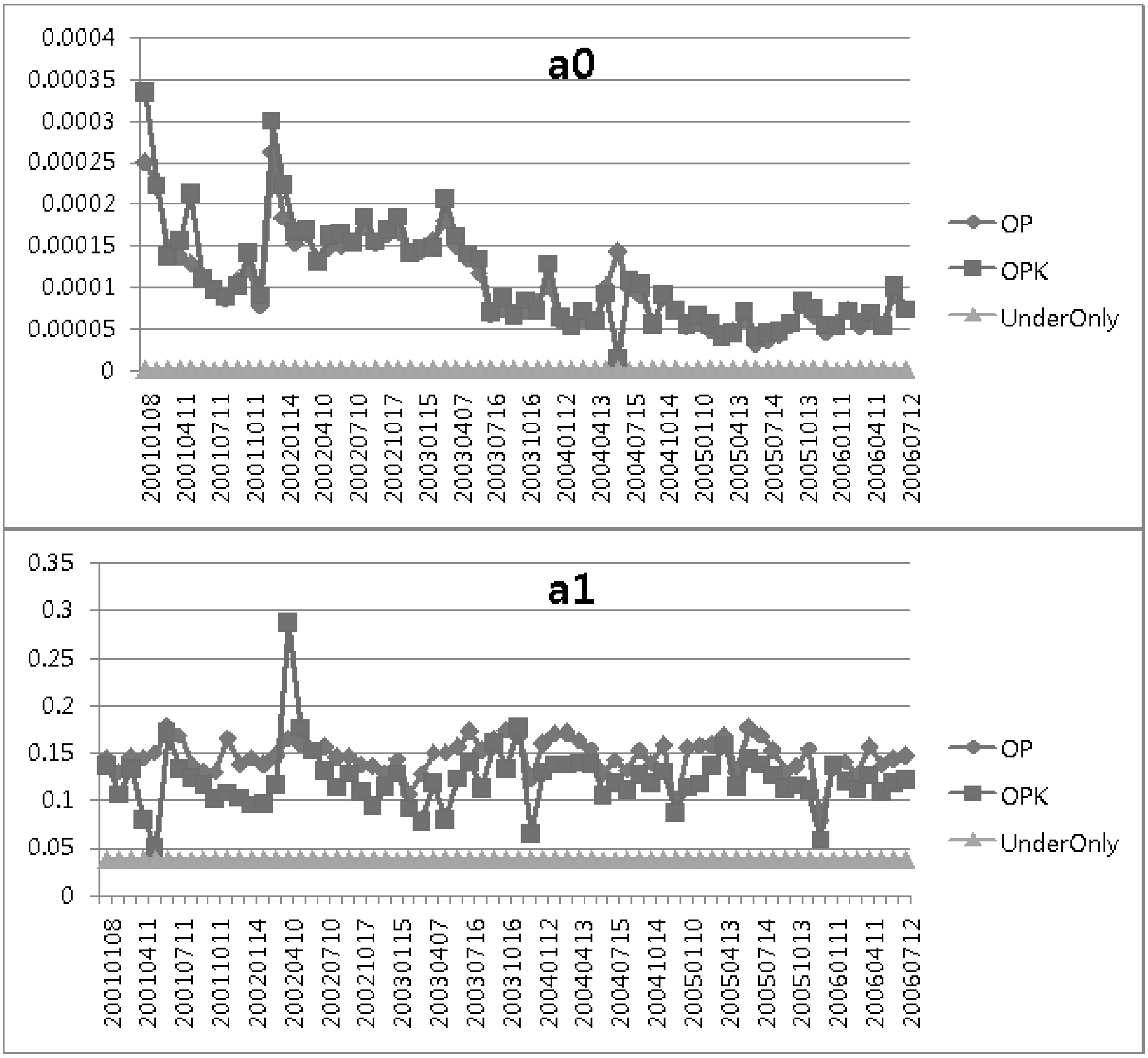
	mean	median	std
theta0	0.8498	0.7201	0.6170
theta1	-0.4637	0.0908	1.4492
theta2	-0.5292	-0.4224	0.9947
theta3	-0.0109	0.1772	0.5874

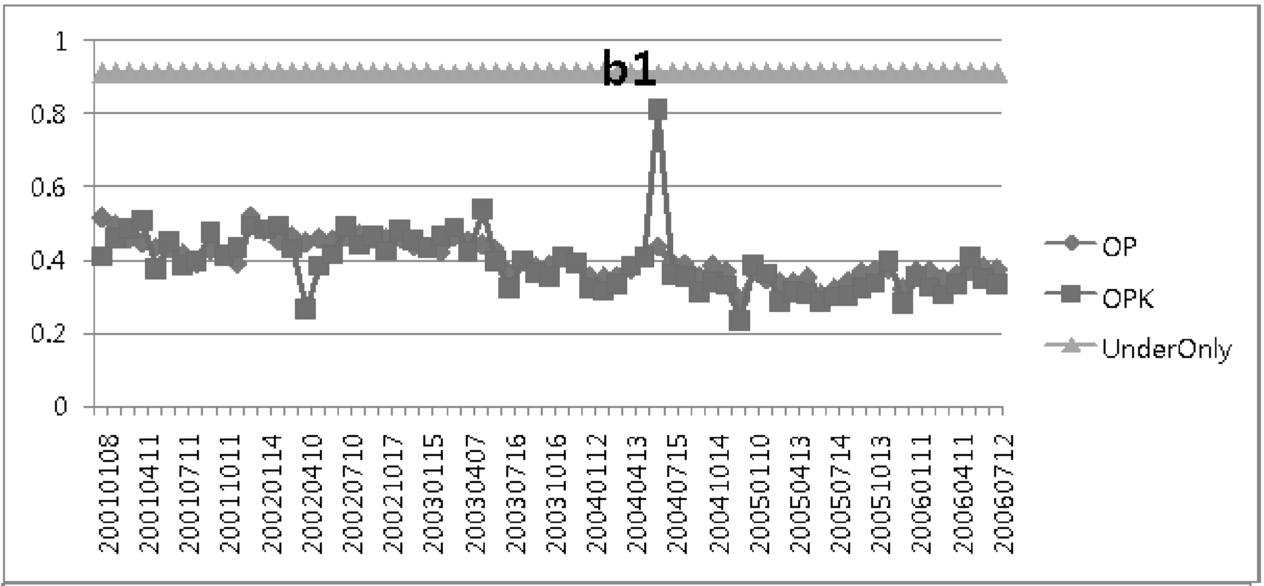
(D)polynomial PK with bond

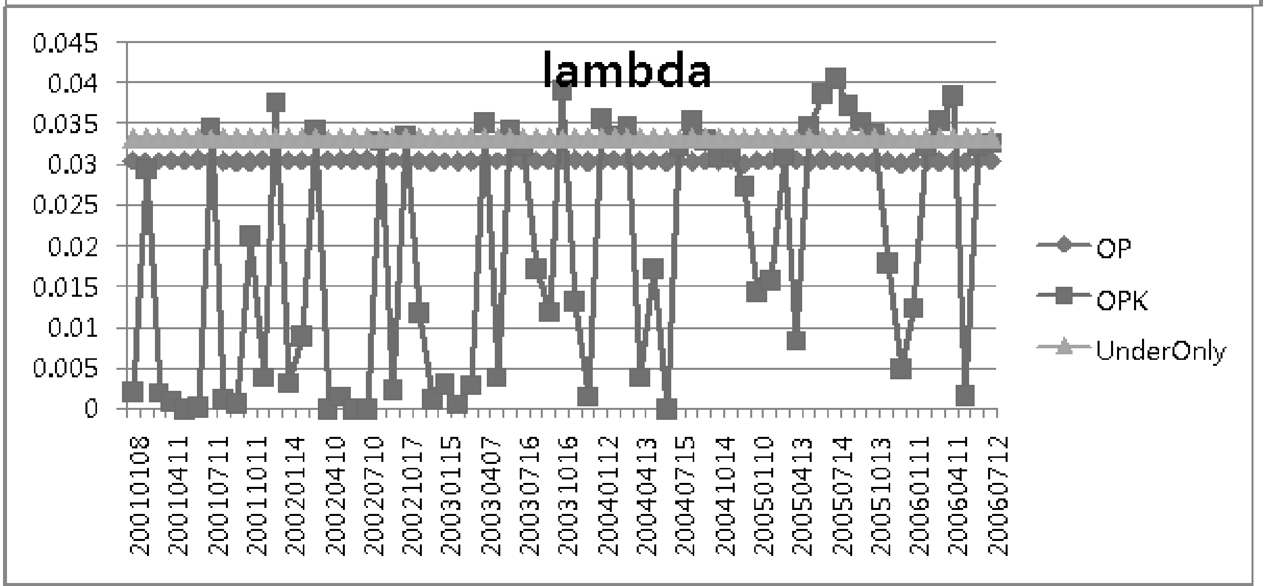
	mean	median	std
theta0	0.9283	0.7893	0.7133
theta1	-0.3677	0.1404	1.3412
theta2	-0.4040	-0.2556	1.0042
theta3	0.0209	0.1868	0.5519

Figure 3. The trend of parameter estimates of the Case 1.1, Case 2.1, and Case 2.2.

This figure shows the trend of five prameter estimates for the extended Duan-GARCH process (Case 1.1, UnderOnly), the GARCH option pricing model (Case 2.1, OP) and the pricing kernel-based GARCH optoin model (Case 2.2, OPK).




b1
1
0.8
0.6
0.4
0.2
0
20010108
20010411
20010711
20011011
20020114
20020410
20020710
20021017
20030115
20030407
20030716
20031016
20040112
20040413
20040715
20041014
20050110
20050413
20050714
20051013
20060111
20060411
20060712
OP
OPK
UnderOnly


lambda
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
20010108
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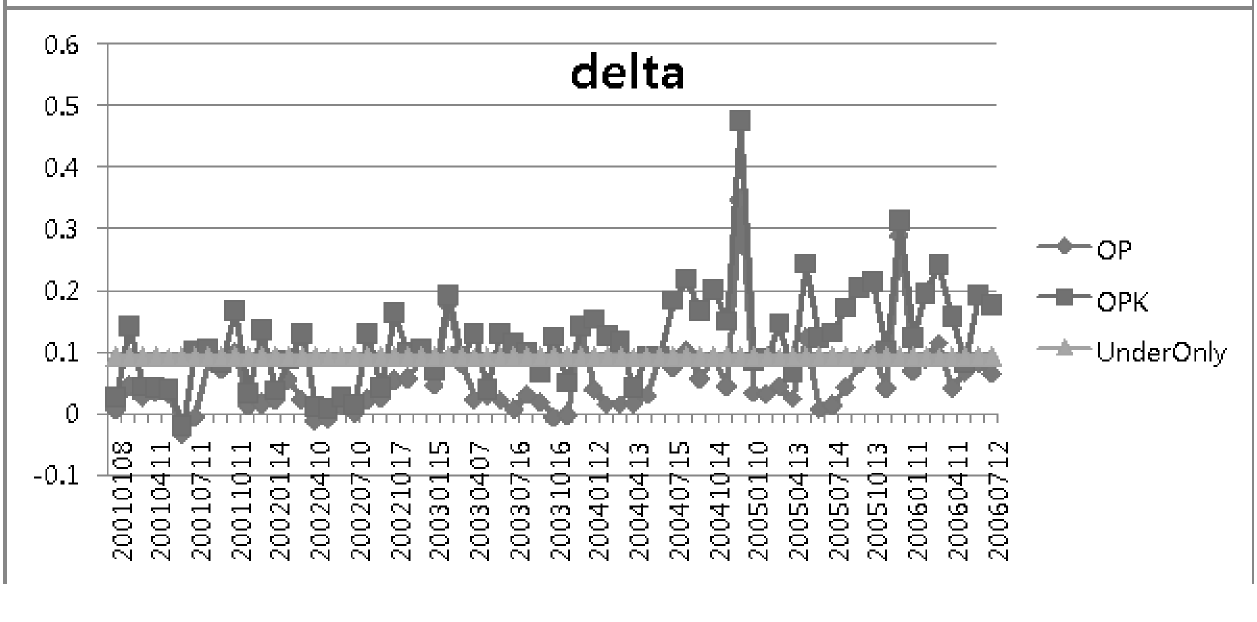
20010711
20011011
20020114
20020410
20020710
20021017
20030115
20030407
20030716
20031016
20040112
20040413
20040715
20041014
20050110
20050413
20050714
20051013
20060111
20060411
20060712
OP
OPK
UnderOnly


delta
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
20010108
20010411
20010711
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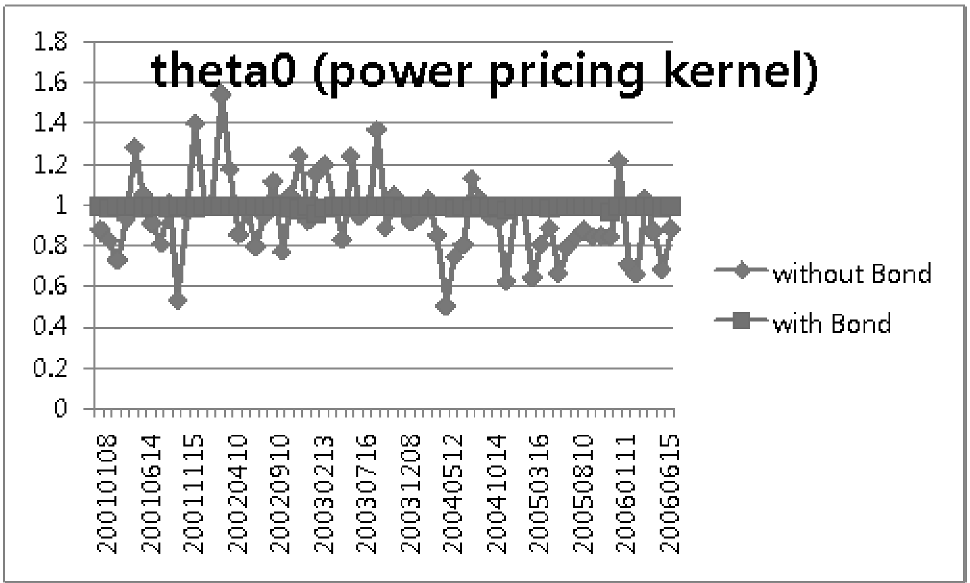
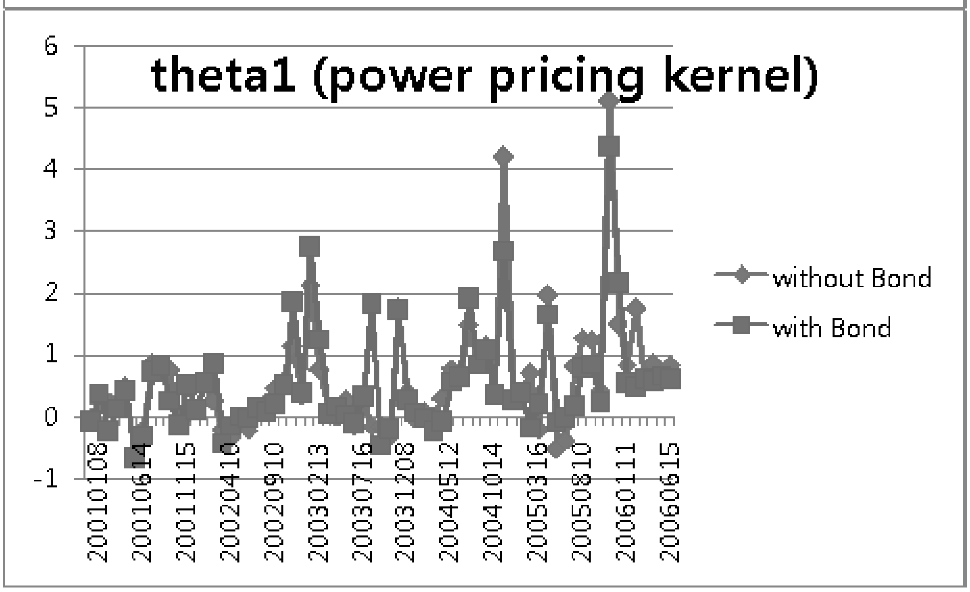
20020410
20020710
20021017
20030115
20030407
20030716
20031016
20040112
20040413
20040715
20041014
20050110
20050413
20050714
20051013
20060111
20060411
20060712
OP
OPK
UnderOnly

Figure 4. The trend of parameter estimates of the parametric pricing kernel (Case 3)

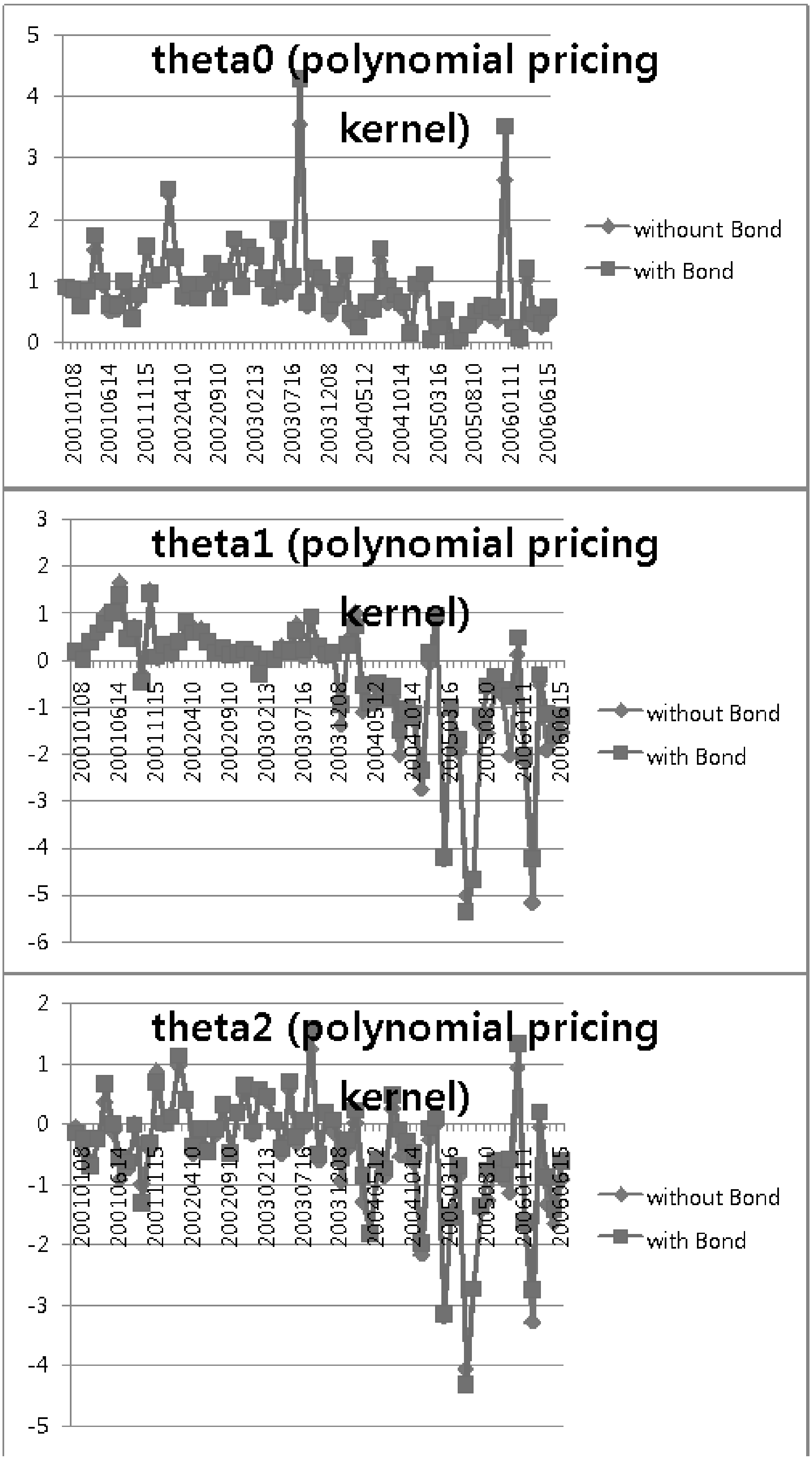
This figure shows the trend of parameter estimates for the parametric pricing kernels in Case 3. Panel A shows the parameter estimates of the power prcing kernel and Panel B shows the parameter estiamtes of the polynomial pricing kernel. In each panel, we compare the parameter value eatimated with and without bond pricing restriction.

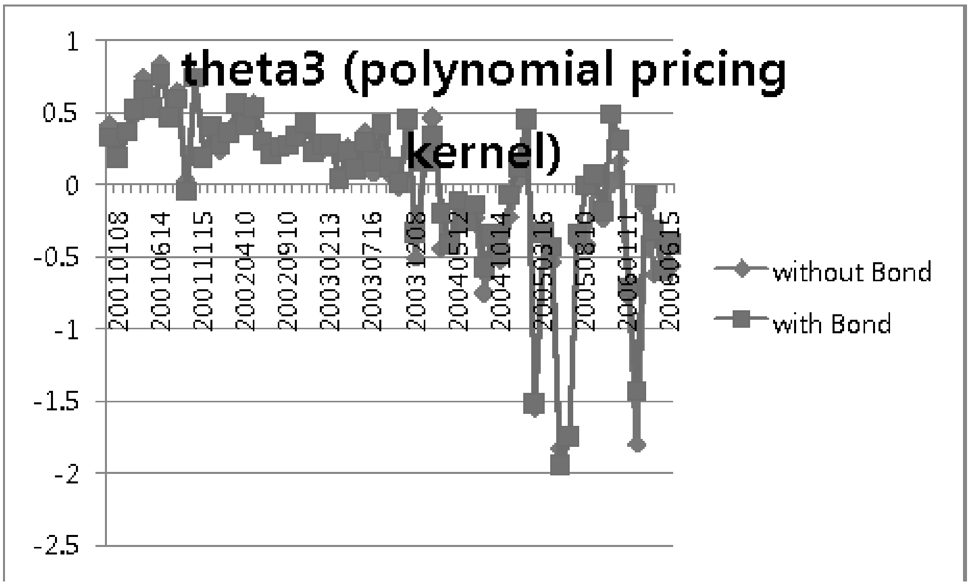
Panle A. power pricing kernel





Panle B. polynomial pricig kernel





theta3 (polynomial pricing kernel)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
20010108
20010614
20011115
20020410
20020910
20030213
20030716
20031208
20040512
20041014
20050316
20050810
20060111
20060615
without Bond
with Bond

Table 6. Pricing and forecasting performance

This table shows the pricing and one-day forecasting performance across option moneyness in each case. We evaluate the empirical performance by the mean and standard deviation of the relative pricing and forecasting errors. (A) and (C) of Panel C shows the pricing and forecasting performance of the parametric pricing kernel models without bond pricing restriction and (B) and (D) of Panel C shows the results with bond pricing restriction.

Pricing performance measures are

$$mean\left(100\times\left|\frac{C^{Model}(t,T)-C^{Market}(t,T)}{C^{Market}(t,T)}\right|\right) \text{ and } std\left(100\times\left|\frac{C^{Model}(t,T)-C^{Market}(t,T)}{C^{Market}(t,T)}\right|\right)$$

Forecasting performance measures are

$$mean\left(100\times\left|\frac{C^{Model}(t+1,T)-C^{Market}(t+1,T)}{C^{Market}(t+1,T)}\right|\right) \text{ and } std\left(100\times\left|\frac{C^{Model}(t+1,T)-C^{Market}(t+1,T)}{C^{Market}(t+1,T)}\right|\right)$$

Panel A. Case 1. Estimation of the pricing kernel using only the underling return data.

Case 1.1. GARCH model

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	48.57	22.88	17.19	45.56	21.82	15.80
std	53.00	21.89	14.49	49.67	21.49	14.03

Case 1.2. Black-Sholes model

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	149.18	48.03	26.21	137.08	44.65	24.37
std	179.52	39.31	23.00	171.78	40.17	21.82

Panel B. Case 2 Estimation with the option data.

Case 2.1.GARCH option pricing model

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	24.01	10.55	4.98	27.53	12.17	5.56
std	23.12	10.16	5.89	25.62	11.41	5.80

Case 2.3. Black-Sholes model

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	26.05	9.11	6.08	26.25	9.65	5.47
std	32.53	13.31	8.19	32.01	13.28	7.24

Case 2.2. Pricing kernel-based GARCH option modell

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	18.89	9.77	8.31	25.06	12.09	8.60
std	22.23	12.93	8.42	22.89	13.54	8.13

Panel C. Case 3. Parametric pricing kernel

(A)power pricing kernel

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	15.16	5.98	13.09	16.16	9.28	14.00
std	17.27	5.98	10.64	15.61	7.76	12.81

(B)power PK with bond

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	39.87	14.58	7.42	37.64	13.75	5.60
std	45.02	13.69	6.55	41.16	14.41	6.62

(C)Polynomial pricing kernel

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	4.74	2.37	3.83	10.95	5.44	4.19
std	4.23	1.77	2.71	8.49	4.86	3.94

(D)polynomial PK with bond

	Pricing			ForeCasting		
	OTM	ATM	ITM	OTM	ATM	ITM
mean	5.67	2.04	1.92	11.64	5.40	3.12
std	5.15	1.97	1.97	9.06	4.99	3.58

Table 7. Hedge performance

This table shows the hedge performance across three hedge method. We evaluate the hedge performance through the mean and standard deviation of the absolute hedge error series in each hedge method. (A) and (C) of Panel C shows the hedge performance of the parametric pricing kernel models without bond pricing restriction and (B) and (D) of Panel C shows the results with bond pricing restriction.

Panel A. Case 1. Estimation of the pricing kernel using only the underling return data.

Case 1.1. GARCH model

	Error		Absolute Error	
	mean	std	mean	std
Stock Only	-3.125	23.050	16.417	16.359
ATM only	-0.533	12.711	8.515	9.395
Stock and ATM	2.489	14.502	10.772	9.940

Case 1.2. Black-Sholes model

Error		Absolute Error	
mean	std	mean	std
-2.010	34.732	22.836	26.097
0.187	22.198	14.450	16.758
2.651	23.300	16.558	16.484

Panel B. Case 2. Estimation with the option data.

Case 2.1.GARCH option pricing model

	Error		Absolute Error	
	mean	std	mean	std
Stock Only	-2.902	16.602	13.029	10.576
ATM only	-1.318	8.773	6.522	5.962
Stock and ATM	0.535	8.274	6.544	5.027

Case 2.3. Black-Sholes model

Error		Absolute Error	
mean	std	mean	std
-1.492	17.826	13.750	11.317
0.370	9.351	6.901	6.264
2.332	10.591	8.081	7.169

Case 2.2. Pricing kernel-based GARCH option model

	Error		Absolute Error	
	mean	std	mean	std
Stock Only	-1.943	16.677	13.293	10.127
ATM only	-0.634	8.038	5.978	5.360
Stock and ATM	1.110	9.617	7.572	5.961

Panel C. Case 3. Parametric pricing kernel

(A)power pricing kernel

	Error		Absolute Error	
	mean	std	mean	std
Stock Only	-2.712	15.611	12.219	9.981
ATM only	-0.925	12.270	8.226	9.095
Stock and ATM	1.232	19.286	13.251	13.974

(B)power PK with bond

Error		Absolute Error	
mean	std	mean	std
-3.071	19.956	14.321	14.128
-0.823	12.391	8.297	9.184
1.669	13.103	9.539	9.063

(C)Polynomial pricing kernel

	Error		Absolute Error	
	mean	std	mean	std
Stock Only	-3.290	16.266	12.502	10.812
ATM only	-1.327	8.661	6.124	6.224
Stock and ATM	0.683	8.123	6.134	5.317

(D)polynomial PK with bond

Error		Absolute Error	
mean	std	mean	std
-3.420	16.362	12.535	10.959
-0.996	8.368	5.882	5.992
1.360	8.191	6.142	5.539