Basket Option Pricing under the Jump-Diffusion Process

Kwangil Bae, Jangkoo Kang and Hwa-Sung Kim

ABSTRACT

This paper derives an approximate pricing formula for basket options when the dynamics of each asset price included in the basket follow the jump-diffusion process. To obtain an approximation for a basket option price, we adopt the Taylor expansion method by Ju (2002). We show that the Taylor expansion method suggested in this paper provides the best pricing performance among the log-normal, the four moments and the Taylor expansion approximations. Performance improvement using the Taylor expansion method becomes bigger, as the time to maturity becomes longer or when jumps are asymmetric.

1. Introduction

Like other exotic options, basket options containing many assets are difficult to price analytically. Even when each asset included in the basket is generally assumed to follow the log-normal diffusion process, the basket price of assets does not follow the log-normal diffusion process. Therefore, one needs to find out an approximation method to price basket options, unless a simulation method is used. There are many ways to approximate the prices of basket options. The first one is the log-normal approximation. It approximates the probability density function of the basket as the probability density function of log-normal distribution. The second method is the four moment method. It approximates the probability density function of the basket by matching the first four moments of the basket with those of the Johnson (1949) family function. The third is an Edgeworth expansion method by Jarrow and Rudd (1982), and Turnbull and Wakeman(1991). In addition, Ju (2002) develops a basket option pricing method using the Taylor expansion. This Taylor expansion by Ju shows the better performance compared with the other approximation methods.

All of the above approximation methods have been applied to the basket option pricing under the diffusion process. However, it is well known that the asset prices in the capital market are affected by jump risks. The derivative price may fluctuate due to the jump occurrence of the underlying assets. Furthermore, if the changes in the prices of assets are correlated in the basket, it is more important to consider correlated jump risks when we valuate basket options.

Recently, Flamouris and Giamouridis (2007) consider basket options under the jump-diffusion process. They derive the price of basket options using the log-normal approximation like Levy (1992). Like them, this paper derives an approximate pricing formula for basket options when the dynamics of each asset price included in the basket follow the jump-diffusion process. However, there are some differences between this paper and theirs. First, we use the Taylor's expansion to calculate the price of basket options rather than the log-normal approximation. Second, while Flamouris and Giamouridis adopt the Bernoulli process as the jump process of asset prices as in Ball and Torous (1983), we utilize the Poisson process. That is, under their assumption, the number of the jump occurrence in asset prices is only one until the basket option's maturity date. By adopting the Poisson process assumption, we allow the number of the jump occurrences to be more than one. When considering the jump movements of asset prices, this Poisson process is used in many studies such as Merton (1976), Ball and Torous (1985), Jorion (1988), Anderson et al. (2002).

In this paper, we use the Taylor expansion approximation method to derive the price of basket options under the jump-diffusion process, and compare the performance of our method with that of other approximation methods under the jump-diffusion. To evaluate the performance of each method, we calculate the price of basket options through Monte Carlo simulations. The results show that the Taylor expansion approximation gives the better performance than any other approximation methods compared. In particular, the performance of our method shows big improvement over those of other approximation methods. In addition, a difference in performances appears more significantly when the jump risks are larger.

This paper is organized as follows. In the next section we derive the approximate pricing formula for basket option prices when each asset included in the basket follows the jump-diffusion process. In Section 3, we evaluate the performance of our method compared with the other approximation methods. In Section 4, we conclude the paper.

2. Approximation for the Basket Options under the Jump-Diffusion Process

Let the asset prices be

$$S_i(t)=S_i \exp\left(g_i t+\sigma_i w_i(t)-\frac{1}{2}\sigma_i^2 t+\sum_{\tau=1}^{\tau(t)}\ln V_{i,\tau}-\lambda t\left(\exp(\gamma_i+\delta_i^2/2)-1\right)\right) \tag{1}$$

for i = 1, 2, ⋯, N.

Here, g_i and σ_i denote the expected return and its volatility of ith asset, respectively (i = 1,⋯, N). We assume that these values are constants. Also, $w_i(t)$ is a standard Brownian motion and ρ_{ij} is a correlation between $w_i(t)$ and $w_j(t)$. Let $V_{i,\tau}$ be the jump size of the ith asset. We assume that $\ln V_{i,\tau}$ has a normal distribution with mean γ_i and variance δ_i^2 and δ_{ij} is a correlation between $\ln V_{i,\tau}$ and $\ln V_{j,\tau}$. $\tau(t)$ is the Poisson process with the intensity λ.

As in Ju (2002), we consider that all the volatilities are scaled by z and all jumps are scaled by y. We use the Taylor expansion around z = 0 and y = 0. Define A(z,y) as

$$A(z,y)\equiv\sum_{i=1}^{N}\chi_i S_i \exp\left(g_i T+z\sigma_i w_i(T)-\frac{1}{2}z^2\sigma_i^2 T+y\sum_{\tau=1}^{\tau(t)}\ln V_{i,\tau}-\lambda T\left(\exp(y\gamma_i+y^2\delta_i^2/2)-1\right)\right) \tag{2}$$

where χ_i is the weight on asset i.

The payoff at basket option's maturity is

$$BC(T)=(A(1,1)-K)^+$$

where K is the exercise price of the option.

As in Ju (2002), for convenience, let $\overline{S}_i=\chi_i S_i e^{g_i T}$, $\overline{\rho}_{ij}=\rho_{ij}\sigma_i\sigma_j T$. Define U_a and U_b as follows.

$$U_a\equiv E[A(z,y)]=A(0,0)=\sum_{i=1}^{N}\overline{S}_i \tag{3}$$

$$U_b\equiv E[A^2(z,y)]=\sum_{i,j=1}^{N}\overline{S}_i\overline{S}_j \exp\left(z^2\overline{\rho}_{ij}+\lambda T\left\{e^{(\gamma_i+\gamma_j)y+\frac{1}{2}(\delta_i^2+2\delta_{ij}+\delta_j^2)y^2}-e^{\gamma_i y+\frac{1}{2}\delta_i^2 y^2}-e^{\gamma_j y+\frac{1}{2}\delta_j^2 y^2}+1\right\}\right) \tag{4}$$

In equation (3), A(0,0) is the expected value of the basket at time T and A(1,1) is value at time T.

Let $Y(z,y)$ be a normal random variable with mean $m(z^2,y)$ and variance $v(z^2,y)$.

$$m(z^2,y)\equiv E[Y(z,y)]=2\log U_a-0.5\log U_b(z^2,y) \tag{5}$$

$$v(z^2,y)=\log U_b(z^2,y)-2\log U_a \tag{6}$$

Also, we define $\log(A(z,y))$ be $X(z,y)$ and then decompose the characteristic function of X as follows.

$$E[e^{i\phi X(z,y)}]=E[e^{i\phi Y(z,y)}]\frac{E[e^{i\phi X(z,y)}]}{E[e^{i\phi Y(z,y)}]}=E[e^{i\phi Y(z,y)}]f(z,y) \tag{7}$$

where

$$E\left[e^{i\phi Y(z,y)}\right]=e^{i\phi m(z^2,y)-\phi^2 v(z^2,y)/2} \tag{8}$$

and

$$f(z,y)=\frac{E[e^{i\phi X(z,y)}]}{E[e^{i\phi Y(z,y)}]}=E[e^{i\phi X(z,y)}]e^{-i\phi m(z^2,y)+\phi^2 v(z^2,y)/2}. \tag{9}$$

That is, be $f(z,y)$ the ratio of the characteristic function of be $X(z,y)$ to that of $Y(z,y)$.

We will expand $f(z,y)$ around $(z,y)=(0,0)$ up to $z^{2m}y^n$ where $m+n\le 3$. Since Ju uses the Taylor expansion up to z^6 and the above function $f(z,y)$ is an even function, there appear three terms with respect to the function of z.[1]

Using $v_i(z^2,y)=-2m_i(z^2,y)$ allows us to approximate the denominator of equation (9) as follows.

$$\begin{aligned}
&e^{-i\phi m(z^2,y)+\phi^2 v(z^2,y)/2}\\
&\approx e^{-i\phi m(0,0)+\phi^2 v(0,0)/2}\Big[1-(i\phi+\phi^2)a_1+\big((i\phi+\phi^2)^2a_1^2-(i\phi+\phi^2)a_2\big)/2\\
&\qquad+\big(3(i\phi+\phi^2)^2a_1a_2-(i\phi+\phi^2)a_3-(i\phi+\phi^2)^3a_1^3\big)/6\\
&\qquad-(i\phi+\phi^2)a_4/2+\big((i\phi+\phi^2)^2a_1a_4-(i\phi+\phi^2)a_6\big)/2\\
&\qquad-(i\phi+\phi^2)a_5/6\Big]
\end{aligned} \tag{10}$$

where $a_i(z,y)$ for $i=1,\cdots,6$ is in Appendix.

Now we examine the characteristic function of $E\left[e^{i\phi X(z,y)}\right]$.

By the equation

$$E\left[\frac{\partial^{i+j}}{\partial z^i\partial y^j}A(0,0)\right]=0,$$

we can express the characteristic function of X:

$$g(z,y)=E\left[e^{i\phi X(z,y)}\right]\approx e^{i\phi X(0,0)}+\frac{z^2}{2}g_{2,0}+\frac{z^4}{4!}g_{4,0}+\frac{z^6}{6!}g_{6,0}+\frac{y^2}{2}g_{0,2}+\frac{y^3}{3!}g_{3,0}+\frac{z^2y^2}{4}g_{2,2} \tag{11}$$

[1] We also other cases, however, we obtain worse performance results compared to the performance when $m+n\le 3$.

According to Ju (2002),

$$\frac{z^2}{2}g_{2,0}=e^{i\phi X(0,0)}\left(i\phi+\phi^2\right)a_1\,, \tag{12}$$

$$\frac{z^4}{4!}g_{4,0}=e^{i\phi X(0,0)}\left(-(i\phi-3)(i\phi-2)(i\phi+\phi^2)a_1^2/2-(i\phi-2)(i\phi+\phi^2)b_1-(i\phi+\phi^2)b_2\right) \tag{13}$$

$$\frac{z^6}{6!}g_{6,0}=e^{i\phi X(0,0)}\begin{pmatrix}-(i\phi-5)(i\phi-4)(i\phi-3)(i\phi-2)(i\phi+\phi^2)\left(-\frac{a_1^3}{6}\right)\\ -(i\phi-4)(i\phi-3)(i\phi-2)(i\phi+\phi^2)c_1-(i\phi-3)(i\phi-2)(i\phi+\phi^2)c_2\\ -(i\phi-2)(i\phi+\phi^2)c_3-(i\phi+\phi^2)c_4\\ \end{pmatrix} \tag{14}$$

where the functions b_i $(i=1,2)$ and c_j $(j=1,2,3,4)$ are defined by

$$b_1(z,y)=\frac{z^4}{4A^3(0,0)}E\left[A_{1,0}^2A_{2,0}\right]$$

$$b_2(z,y)=a_1^2-\frac{1}{2}a_2$$

$$E\left[A_{1,0}^2A_{2,0}\right]=2\sum_{i,j,k=1}^{N}\bar{S}_i\bar{S}_j\bar{S}_k\bar{\rho}_{ik}\bar{\rho}_{jk}$$

$$c_1(z,y)=-a_1b_1\,,$$

$$c_2(z,y)=\frac{z^6}{144A^4(0,0)}\left(9E\left[A_{1,0}^2A_{2,0}^2\right]+4E\left[A_{1,0}^3A_{3,0}\right]\right),$$

$$c_3(z,y)=\frac{z^6}{48A^3(0,0)}\left(4E\left[A_{1,0}A_{2,0}A_{3,0}\right]+E\left[A_{2,0}^3\right]\right),$$

$$c_4(z,y)=a_1a_2-\frac{2}{3}a_1^3-\frac{1}{6}a_3$$

$$E\left[A_{1,0}^2A_{2,0}^2\right]=8\sum_{ijkl=1}^{N}\bar{S}_i\bar{S}_j\bar{S}_k\bar{S}_l\bar{\rho}_{il}\bar{\rho}_{jk}\bar{\rho}_{kl}+2U_{b1,0}U_{b2,0}\,,$$

$$E\left[A_{1,0}^3A_{3,0}\right]=6\sum_{ijk=1}^{N}\bar{S}_i\bar{S}_j\bar{S}_k\bar{\rho}_{il}\bar{\rho}_{jl}\bar{\rho}_{kl}\,,$$

$$E\left[A_{1,0}A_{2,0}A_{3,0}\right]=6\sum_{ijk=1}^{N}\bar{S}_i\bar{S}_j\bar{S}_k\bar{\rho}_{ik}\left(\bar{\rho}_{jk}\right)^2\,,$$

$$E\left[A_{2,0}^{3}\right]=8\sum_{ijk=1}^{N}\bar{S}_i\bar{S}_j\bar{S}_k\bar{\rho}_{ik}\bar{\rho}_{jk}\,.$$

Also, we need the below equations.

$$\frac{y^2}{2}g_{0,2}=-(i\phi+\phi^2)\frac{y^2}{2}e^{i\phi X(0,0)}E\left[\frac{A_{0,1}^2}{A^2(0,0)}\right]=e^{i\phi X(0,0)}\frac{i\phi+\phi^2}{2}a_4 \tag{15}$$

$$\begin{aligned}\frac{y^3}{3!}g_{0,3}&=\frac{y^3}{3!}e^{i\phi X(0,0)}E\left[-(i\phi-2)(i\phi+\phi^2)\frac{A_{0,1}^3}{A^3(0,0)}-3(i\phi+\phi^2)\frac{A_{0,2}A_{0,1}}{A^2(0,0)}\right]\\&=\frac{1}{3!}e^{i\phi X(0,0)}E\left[-(i\phi-2)(i\phi+\phi^2)d_1+(i\phi+\phi^2)a_5\right]\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{z^2y^2}{4}g_{2,2}&=\frac{z^2y^2}{4}e^{i\phi X(0,0)}E\left[\begin{matrix}-(i\phi-3)(i\phi-2)(i\phi+\phi^2)\dfrac{A_{0,1}^2A_{1,0}^2}{A^4(0,0)}\\-4(i\phi-2)(i\phi+\phi^2)\dfrac{A_{1,0}A_{1,1}A_{0,1}}{A^3(0,0)}-2(i\phi+\phi^2)\dfrac{A_{1,1}^2}{A^2(0,0)}\end{matrix}\right]\\&=\frac{1}{4}e^{i\phi X(0,0)}E\left[\begin{matrix}-2(i\phi-3)(i\phi-2)(i\phi+\phi^2)a_1a_4\\-4(i\phi-2)(i\phi+\phi^2)d_2-2(i\phi+\phi^2)(2a_1a_4-a_6)\end{matrix}\right]\end{aligned} \tag{17}$$

where the function $d_i\ (i=1,2)$ is defined by

$$d_1(z,y)=\frac{E\left[A_{0,1}^3\right]}{A^3(0,0)}=\frac{\sum_{i,j,k=1}^{N}\bar{S}_i\bar{S}_j\bar{S}_k\lambda T(\gamma_i\gamma_j\gamma_k+3\delta_{ij}\gamma_k)}{A^3(0,0)}$$

$$d_2(z,y)=\frac{E\left[A_{1,1}A_{1,0}A_{0,1}\right]}{A^3(0,0)}=\frac{\sum_{i,j,k=1}^{N}\bar{S}_i\bar{S}_j\bar{S}_k\bar{\rho}_{ij}\lambda T(\gamma_i\gamma_k+\delta_{ik})}{A^3(0,0)}$$

Let d_3 define as

$$d_3(z,y)=d_1/6-2a_1a_4+d_2\,.$$

Then we can obtain the approximate for $f(z,y)$ as follows.

$$f(z,y) \approx 1 - i\phi e_1 - \phi^2 e_2 + i\phi e_3 + \phi^4 e_4 \tag{18}$$

where

$$e_1(z,y) = \frac{1}{2}\left(6a_1^2 + a_2 - 4b_1 + 2b_2\right) - \frac{1}{6}\left(120a_1^3 - a_3 + 6\left(24c_1 - 6c_2 + 2c_3 - c_4\right)\right) - 2d_3$$

$$e_2(z,y) = \frac{1}{2}\left(10a_1^2 + a_2 - 6b_1 + 2b_2\right) - \left(128a_1^3/3 - a_3/6 + 2a_1b_1 - a_1b_2 + 50c_1 - 11c_2 + 3c_3 - c_4\right) - 3d_3$$

$$e_3(z,y) = \left(2a_1^2 - b_1\right) - \frac{1}{3}\left(88a_1^3 + 3a_1\left(5b_1 - 2b_2\right) + 3\left(35c_1 - 6c_2 + c_3\right)\right) - d_3$$

$$e_4(z,y) = \left(-20a_1^3/3 + a_1\left(-4b_1 + b_2\right) - 10c_1 + c_2\right)$$

The approximate for the characteristic function of the basket is given by

$$E\left[e^{i\phi X(1,1)}\right] \approx e^{i\phi m(1,1) - \phi^2 v(1,1)/2}\left(1 - i\phi e_1(1,1) - \phi^2 e_2(1,1) + i\phi e_3(1,1) + \phi^4 e_4(1,1)\right) \tag{19}$$

The density function of X(1,1) is

$$\begin{aligned} h(x) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\phi x + i\phi m(1,1) - \phi^2 v(1,1)/2}\left(1 - i\phi e_1(1,1) - \phi^2 e_2(1,1) + i\phi e_3(1,1) + \phi^4 e_4(1,1)\right) d\phi \\ &= p(x) + \left(d_1(1,1)\frac{d}{dx} + d_2(1,1)\frac{d^2}{dx^2} + d_3(1,1)\frac{d^3}{dx^3} + d_4(1,1)\frac{d^4}{dx^4}\right)p(x) \end{aligned} \tag{20}$$

where the function p is given by

$$p(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\phi x + i\phi m(1,1) - \phi^2 v(1,1)/2}\, d\phi = \frac{1}{\sqrt{2\pi v(1,1)}} e^{-\frac{(x - m(1,1))^2}{2v(1,1)}} \tag{21}$$

Since $e_1(1,1) - e_2(1,1) + e_3(1,1) - e_4(1,1) = 0$, the call price of the basket is

$$\begin{aligned} BC &= e^{-rT} E\left[e^{X(1)} - K\right]^+ \\ &= \left[U_1 e^{-rT} N(y_1) - K e^{-rT} N(y_2)\right] + \left[e^{-rT} K\left(z_1 p(y) + z_2 \frac{dp(y)}{dy} + z_3 \frac{d^2 p(y)}{dy^2}\right)\right] \end{aligned} \tag{22}$$

where

$$y = \log(K),\ y_1 = \frac{m(1)-y}{\sqrt{v(1,1)}} + \sqrt{v(1,1)},\ y_2 = y_1 - \sqrt{v(1,1)}$$

$$z_1 = e_2(1,1) - e_3(1,1) + e_4(1,1), z_2 = e_3(1,1) - e_4(1,1), z_3 = e_4(1,1)$$

3. Performance Comparison

In this Section we compare the performance of various approximation methods. We use the Monte Carlo simulation to obtain the benchmark prices of basket options. Each approximation method's accuracy is measured by the root mean squared error (RMSE) and the maximum absolute error (MAE). We evaluate and compare the approximation performance of the log-normal, the four moments and the Taylor expansion approximation.

<Insert Table 1>

Table 1 illustrates the prices of basket options when the expected jump size of assets is 0 (that is, symmetric jumps) and the time to maturity is relatively short (T = 1). The the pricing errors of the log-normal approximation are greatest, and those of the Taylor expansion are smallest. RMSEs of the log-normal and the four moments approximations are about twenty two times and five times greater than those of the Taylor expansion method, respectively.

<Insert Table2>

Table 2 provides the prices of basket options when the expected jump size of assets is zero (that is, symmetric jumps) and the time to maturity is relatively long (T = 3). Compared to the case when the time to maturity is relatively short, the pricing errors increase irrespective of an approximation method. Interestingly, we show that RMSE of the four moments approximation increases sharply. As the time to maturity gets longer, the performance of the four moments goes worse compared to others.

<Insert Table 3>

In Table 3 we explore the prices of basket options when the expected jump size is negative (that is, asymmetric jumps) and the time to maturity is relatively short (T = 1). In this case the Taylor expansion method still delivers the smallest pricing errors compared to Table 1 where the expected jump size is zero. When the volatility of jump size is relatively low, RMSE of the log-normal approximation increases by over ten times. On the contrary, RMSE of the four moments stays at the same level and RMSE of the Taylor expansion increases by approximately twice compared to Table 1.

<Insert Table 4>

Table 4 illustrates the prices of basket options when the expected jump size is negative (that is, asymmetric jumps) and the time to maturity is relatively long (T = 3). In contrast with the result of Table 3, RMSEs increase slightly compared to Table 2. If we combine Table 3 with Table 4, we can infer that jump risks have a bigger impact on the pricing performance when the time to maturity is relatively short. This is because as time goes by, the effects of jump risks get weaker.

In sum, in terms of RMSE and MAE, the Taylor expansion method is the most accurate among the approximation methods considered.

4. Conclusion

In this paper we price basket options when each asset price included in the basket follows the jump-diffusion process. To obtain an approximate pricing formula for the basket option price, we adopt the Taylor expansion method by Ju (2002). We show that the Taylor expansion method provides the best pricing performance among the log-normal, the four moments and the Taylor expansion approximations. Compared to the other methods, the pricing performance of the Taylor expansion is better when the time to maturity is relatively long or when jumps are asymmetric.

Appendix – function $a_i(z,y)$

We arrange the function $a_i(z,y)$ as following

$$a_1(z,y)=z^2 m_{1,0}=-\frac{z^2 U_{b1,0}}{2U_b},$$

$$a_2(z,y)=z^4 m_{2,0}=2a_1^2-\frac{1}{2}\frac{U_{b2,0}}{U_b}z^4,$$

$$a_3(z,y)=z^6 m_{3,0}=6a_1a_2-4a_1^3-\frac{1}{2}\frac{U_{b3,0}}{U_b}z^6$$

$$a_4(z,y)=m_{0,2}y^2=\left\{\frac{1}{2}\left(\frac{U_{b0,1}}{U_b}\right)^2-\frac{1}{2}\frac{U_{b0,2}}{U_b}\right\}y^2 ,$$

$$=-\frac{y^2}{2}\frac{U_{b0,2}}{U_b}$$

$$a_5(z,y)=m_{0,3}y^3=\left\{-\left(\frac{U_{b0,1}}{U_b}\right)^3+\frac{3}{2}\frac{U_{b0,1}U_{b0,2}}{U_b}-\frac{1}{2}\frac{U_{b0,3}}{U_b}\right\}y^3 ,$$

$$=-\frac{y^3}{2}\frac{U_{b0,3}}{U_b}$$

$$a_6(z,y)=m_{1,2}z^2y^2=\left\{-\left(\frac{U_{b0,1}}{U_b}\right)^2\frac{U_{b1,0}}{U_b}+\frac{U_{b0,1}}{U_b}\frac{U_{b1,1}}{U_b}+\frac{1}{2}\frac{U_{b1,0}}{U_b}\frac{U_{b0,2}}{U_b}-\frac{1}{2}\frac{U_{b1,2}}{U_b}\right\}z^2y^2$$

$$=\left\{\frac{1}{2}\frac{U_{b1,0}}{U_b}\frac{U_{b0,2}}{U_b}-\frac{1}{2}\frac{U_{b1,2}}{U_b}\right\}z^2y^2$$

$$=2a_1a_4-\frac{z^2y^2}{2}\frac{U_{b1,2}}{U_b}$$

where

$$U_{bi,j}=\left.\frac{\partial^{i+j}U_b(z^2,y)}{\partial(z^2)^i\partial y^j}\right|_{(z,y)=(0,0)}$$

$$U_b=\sum_{i,j=1}^{N}\bar{S}_i\bar{S}_j,$$

$$U_{bk,1}=0,$$

$$U_{bk,2}=2\sum_{i,j=1}^{N}\bar{S}_i\bar{S}_j\left(\bar{\rho}_{ij}\right)^k\lambda T(\delta_{ij}+\gamma_i\gamma_j),$$

$$U_{bk,3}=6\sum_{i,j=1}^{N}\bar{S}_i\bar{S}_j\left(\bar{\rho}_{ij}\right)^k\lambda T(\gamma_i^2\gamma_j+\gamma_i\delta_j^2+2\delta_{ij}\gamma_i),\quad k\in\{0,1,2,3\}$$

Table 1: Values of basket calls

K	r	σ	ρ	MC	SD	LN	FM	TEJ
90	0.05	0.2	0	14.6324	0.0005	14.6372	14.6312	14.6328
100	0.05	0.2	0	6.8368	0.0008	6.8308	6.8382	6.8372
110	0.05	0.2	0	2.2300	0.0007	2.2016	2.2329	2.2306
90	0.05	0.5	0	18.3491	0.0027	18.5035	18.3260	18.3447
100	0.05	0.5	0	12.6581	0.0028	12.7871	12.6723	12.6588
110	0.05	0.5	0	8.4346	0.0026	8.5011	8.4733	8.4381
90	0.1	0.2	0	18.6310	0.0004	18.6342	18.6302	18.6313
100	0.1	0.2	0	10.3233	0.0006	10.3255	10.3224	10.3233
110	0.1	0.2	0	4.2642	0.0009	4.2466	4.2673	4.2649
90	0.1	0.5	0	21.3193	0.0027	21.4717	21.2843	21.3157
100	0.1	0.5	0	15.2458	0.0028	15.3912	15.2403	15.2422
110	0.1	0.5	0	10.5287	0.0027	10.6303	10.5553	10.5297
90	0.05	0.2	0.5	15.6528	0.0010	15.6494	15.6566	15.6567
100	0.05	0.2	0.5	8.9095	0.0014	8.8947	8.9082	8.9081
110	0.05	0.2	0.5	4.4115	0.0013	4.3967	4.4126	4.4124
90	0.05	0.5	0.5	22.8846	0.0045	22.8899	22.8785	22.8795
100	0.05	0.5	0.5	17.9089	0.0045	17.9159	17.9095	17.9087
110	0.05	0.5	0.5	13.8968	0.0043	13.8918	13.8909	13.8886
90	0.1	0.2	0.5	19.2211	0.0009	19.2163	19.2209	19.2210
100	0.1	0.2	0.5	11.9320	0.0012	11.9215	11.9321	11.9321
110	0.1	0.2	0.5	6.5416	0.0014	6.5279	6.5433	6.5431
90	0.1	0.5	0.5	25.3823	0.0044	25.3975	25.3843	25.3862
100	0.1	0.5	0.5	20.2043	0.0045	20.2165	20.2075	20.2076
110	0.1	0.5	0.5	15.9319	0.0044	15.9395	15.9356	15.9341
	RMSE					0.0652	0.0134	0.0029
	MAE					0.1544	0.0387	0.0082

To evaluate the performance, we select the parameter values: the time to maturity, $T = 1$, the standard deviation of jump size, $\delta=0.01$, the expected jump size, 0, the jump intensity, $\lambda=5$ and the correlation between jump sizes is 0. Let K, r, σ, and ρ be the exercise price, interest rate, the return volatility and the correlation between standard Brownian motions, respectively. Let MC, LN, FM and TEJ denote the Monte Carlo simulation, the log-normal approximation, the four moments and the Taylor expansion, respectively. Also, SD is denoted by the standard error in Monte Carlo simulation.

Table 2: Values of basket calls

K	r	σ	ρ	MC	SD	LN	FM	TEJ
90	0.05	0.2	0	23.0284	0.0013	23.0561	23.0175	23.0283
100	0.05	0.2	0	15.7108	0.0015	15.7425	15.7034	15.7094
110	0.05	0.2	0	9.8452	0.0017	9.8546	9.8497	9.8437
90	0.05	0.5	0	30.0241	0.0071	30.8485	29.5416	30.0008
100	0.05	0.5	0	25.1542	0.0069	26.0042	24.8184	25.1614
110	0.05	0.5	0	21.0432	0.0068	21.8495	20.8532	21.0675
90	0.1	0.2	0	33.3727	0.0011	33.3810	33.3684	33.3730
100	0.1	0.2	0	26.1800	0.0012	26.2005	26.1696	26.1793
110	0.1	0.2	0	19.4580	0.0014	19.4894	19.4463	19.4563
90	0.1	0.5	0	37.2459	0.0071	37.9690	36.6759	37.2115
100	0.1	0.5	0	32.1399	0.0071	32.9523	31.6265	32.1210
110	0.1	0.5	0	27.6513	0.0070	28.4928	27.2313	27.6396
90	0.05	0.2	0.5	24.8294	0.0023	24.8172	24.8262	24.8270
100	0.05	0.2	0.5	18.6067	0.0026	18.5875	18.6034	18.6036
110	0.05	0.2	0.5	13.5177	0.0028	13.4954	13.5177	13.5172
90	0.05	0.5	0.5	36.8518	0.0104	36.9131	36.7879	36.8361
100	0.05	0.5	0.5	32.7409	0.0104	32.8050	32.6909	32.7293
110	0.05	0.5	0.5	29.1228	0.0102	29.1871	29.0867	29.1151
90	0.1	0.2	0.5	34.0152	0.0019	34.0140	34.0166	34.0176
100	0.1	0.2	0.5	27.5598	0.0021	27.5519	27.5585	27.5595
110	0.1	0.2	0.5	21.7777	0.0024	21.7664	21.7785	21.7791
90	0.1	0.5	0.5	42.7664	0.0103	42.8455	42.7124	42.7713
100	0.1	0.5	0.5	38.6071	0.0104	38.6742	38.5456	38.5975
110	0.1	0.5	0.5	34.8583	0.0104	34.9267	34.8062	34.8499
	RMSE					0.4070	0.2203	0.0121
	MAE					0.8500	0.5701	0.0344

To evaluate the performance, we select the parameter values: the time to maturity, T = 3, the standard deviation of jump size, $\delta=0.01$, the expected jump size, 0, the jump intensity, $\lambda=5$ and the correlation between jump sizes is 0. Let K, r, σ, and ρ be the exercise price, interest rate, the return volatility and the correlation between standard Brownian motions, respectively. Let MC, LN, FM and TEJ denote the Monte Carlo simulation, the log-normal approximation, the four moments and the Taylor expansion, respectively. Also, SD is denoted by the standard error in Monte Carlo simulation.

Table 3: Values of basket calls

K	r	σ	ρ	MC	SD	LN	FM	TEJ
90	0.05	0.2	0	15.0600	0.0015	14.6372	15.0546	15.0583
100	0.05	0.2	0	7.7845	0.0013	6.8308	7.7852	7.7906
110	0.05	0.2	0	3.1748	0.0010	2.2016	3.1774	3.1784
90	0.05	0.5	0	18.7268	0.0030	18.5035	18.6997	18.7233
100	0.05	0.5	0	13.1060	0.0030	12.7871	13.1207	13.1138
110	0.05	0.5	0	8.8941	0.0027	8.5011	8.9339	8.9044
90	0.1	0.2	0	18.8503	0.0015	18.6342	18.8481	18.8499
100	0.1	0.2	0	11.0395	0.0014	10.3255	11.0393	11.0447
110	0.1	0.2	0	5.2968	0.0012	4.2466	5.3000	5.3038
90	0.1	0.5	0	21.6463	0.0030	21.4717	21.6074	21.6428
100	0.1	0.5	0	15.6663	0.0030	15.3912	15.6600	15.6679
110	0.1	0.5	0	10.9886	0.0029	10.6303	11.0167	10.9977
90	0.05	0.2	0.5	16.1025	0.0017	15.6494	16.1024	16.1046
100	0.05	0.2	0.5	9.5725	0.0017	8.8947	9.5729	9.5765
110	0.05	0.2	0.5	5.0961	0.0015	4.3967	5.0924	5.0949
90	0.05	0.5	0.5	23.1387	0.0046	22.8899	23.1344	23.1364
100	0.05	0.5	0.5	18.1952	0.0046	17.9159	18.1960	18.1964
110	0.05	0.5	0.5	14.1800	0.0044	13.8918	14.1900	14.1890
90	0.1	0.2	0.5	19.5439	0.0017	19.2163	19.5427	19.5439
100	0.1	0.2	0.5	12.5079	0.0017	11.9215	12.5095	12.5127
110	0.1	0.2	0.5	7.2402	0.0016	6.5279	7.2428	7.2461
90	0.1	0.5	0.5	25.6219	0.0046	25.3975	25.6211	25.6238
100	0.1	0.5	0.5	20.4900	0.0046	20.2165	20.4814	20.4826
110	0.1	0.5	0.5	16.2340	0.0045	15.9395	16.2301	16.2298
	RMSE					0.5316	0.0147	0.0052
	MAE					1.0502	0.0398	0.0103

To evaluate the performance, we select the parameter values: the time to maturity, $T = 1$, the standard deviation of jump size, δ=0.03, the expected jump size, -0.02, the jump intensity, λ=10 and the correlation between jump sizes is 0. Let K, r, σ, and ρ be the exercise price, interest rate, the return volatility and the correlation between standard Brownian motions, respectively. Let MC, LN, FM and TEJ denote the Monte Carlo simulation, the log-normal approximation, the four moments and the Taylor expansion, respectively. Also, SD is denoted by the standard error in Monte Carlo simulation.

Table 4: Values of basket calls

K	r	σ	ρ	MC	SD	LN	FM	TEJ
90	0.05	0.2	0	23.7727	0.0028	23.0561	23.7599	23.7742
100	0.05	0.2	0	17.0153	0.0027	15.7425	17.0128	17.0254
110	0.05	0.2	0	11.5582	0.0025	9.8546	11.5618	11.5677
90	0.05	0.5	0	30.6638	0.0076	30.8485	30.1511	30.6848
100	0.05	0.5	0	25.8971	0.0074	26.0042	25.5200	25.9381
110	0.05	0.5	0	21.8297	0.0072	21.8495	21.6136	21.9015
90	0.1	0.2	0	33.5809	0.0029	33.3810	33.5721	33.5808
100	0.1	0.2	0	26.7156	0.0029	26.2005	26.7055	26.7187
110	0.1	0.2	0	20.4638	0.0028	19.4894	20.4569	20.4712
90	0.1	0.5	0	37.7381	0.0077	37.9690	37.1228	37.7329
100	0.1	0.5	0	32.7361	0.0076	32.9523	32.1903	32.7589
110	0.1	0.5	0	28.3379	0.0075	28.4928	27.8882	28.3713
90	0.05	0.2	0.5	25.5737	0.0033	24.8172	25.5773	25.5796
100	0.05	0.2	0.5	19.6111	0.0033	18.5875	19.6093	19.6131
110	0.05	0.2	0.5	14.6871	0.0033	13.4954	14.6887	14.6931
90	0.05	0.5	0.5	37.2620	0.0108	36.9131	37.2048	37.2668
100	0.05	0.5	0.5	33.1916	0.0107	32.8050	33.1510	33.2028
110	0.05	0.5	0.5	29.6087	0.0105	29.1871	29.5797	29.6208
90	0.1	0.2	0.5	34.4186	0.0032	34.0140	34.4127	34.4130
100	0.1	0.2	0.5	28.1941	0.0033	27.5519	28.1966	28.1982
110	0.1	0.2	0.5	22.6618	0.0033	21.7664	22.6571	22.6601
90	0.1	0.5	0.5	43.1136	0.0107	42.8455	43.0606	43.1333
100	0.1	0.5	0.5	38.9881	0.0108	38.6742	38.9427	39.0085
110	0.1	0.5	0.5	35.2879	0.0107	34.9267	35.2446	35.3020
	RMSE					0.6961	0.2366	0.0210
	MAE					1.7036	0.6153	0.0718

To evaluate the performance, we select the parameter values: the time to maturity, T = 3, the standard deviation of jump size, δ=0.03, the expected jump size, -0.02, the jump intensity, λ=10 and the correlation between jump sizes is 0. Let K, r, σ, and ρ be the exercise price, interest rate, the return volatility and the correlation between standard Brownian motions, respectively. Let MC, LN, FM and TEJ denote the Monte Carlo simulation, the log-normal approximation, the four moments and the Taylor expansion, respectively. Also, SD is denoted by the standard error in Monte Carlo simulation.

References

Andersen, T.G., L. Benzoni, J. Lund. 2002 An Empirical Investigation of Continuous-Time Equity Return Models, *Journal of Finance,* 57, 1239-1284

Ball, C.A., Torous, W.N. (1983). A simplified jump process for common stock returns. *Journal of Financial and Quantitative Analysis*, 18, 1, 53–61.

Ball, C.A., Torous, W.N. (1985). On jumps in common stock prices and their impact on call option pricing. *Journal of Finance*, 40, 155–173.

Black, F., M. Scholes, 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81, 637-659.

FLAMOURIS, D., D. GIAMOURIDIS., 2007. Approximate Basket Option Valuation for a Simplified Jump Process, *The Journal of Futures Markets,* 27, No. 9, 819–837

Jarrow, R., A. Rudd, 1982. Approximate Option Valuation for Arbitrary Stochastic Processes. *Journal of Financial Economics*, 10, 347-369.

Johnson, N., 1949. Systems of Frequency Curves Generated by methods of Translation. *Biometrika,* 36, 149-176.

Jorion, P., 1988. On Jump Processes in the Foreign Exchange and Stock Markets. *Review of Financial Studies* 1, 427-445.

Ju,N., 2002. Pricing Asian and Basket Options Via Taylor Expansion, *Journal of Computational Finance,* 5, 79-103.

Levy, E., 1992. Pricing European Average rate Currency Options. *Journal of International Money and Finance,* 11, 474-491.

Merton, R. C., 1976, Option Pricing When Underlying Stock Returns Are Continuous, *Journal of Financial Economics*, 3, 125-144.

Milevsky, M., S. Posner, 1998a. Asian Options, the Sum of Lognormals, and the Reciprocal Gamma Distribution. *journal of Financial and Quantitative Analysis*, 33, 409-422.

Milevsky, M., S. Posner, 1998b. A Closed-Form Approximation for Valuing Basket Options. *Journal of Derivatives*, 5, 54-61.

Posner, S., M. Milevsky, 1998. Valuing Exotic Options by Approximating the SPD with Higher Moments. *Journal of Financial Engineering*, 7, 109-125.

Turnbull, S., L. Wakeman, 1991. A Quick Algorithm for Pricing European Average Options *Journal of Financial and Quantitative Analysis*, 26, 377-389.