ASYMMETRIC VIOLATION OF PUT–CALL PARITY AND OPTION PRICING UNDER SHORT SALES CONSTRAINTS

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Abstract

This paper provides a rational explanation of the previously documented evidence of asymmetric violation of put–call parity under short sales constraints. To accomplish this we set up an option pricing model under short sales constraints in the discrete time binomial framework and reinvestigate the put–call parity relation. Since the cost of short selling is unpredictable and can be enormously high, the upside boundary of the parity relation will not bind. As a result, the stock price may drift away from the implied stock price derived by the options market. For empirical analysis we propose a measure, *implied volatility discrepancy*, which is defined as implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option. If investors in the options market correctly incorporate the short sales constraints, the implied volatility discrepancy should be an increasing function of time to expiry and the cost of short selling, and a U-shaped convex function of the strike price. The empirical results support the prediction of our model–we find that investors in the options market properly take short sales constraints into account.

1. Introduction

An arbitrage opportunity is a zero-investment portfolio with possible riskless profits. Most financial models assume no arbitrage opportunities in the market, and this is the premise for valuation of many financial instruments, especially derivatives. Thus, the existence of arbitrage opportunities has been one of the greatest concerns of financial researchers and practitioners. The most famous arbitrage strategy may be the put–call parity relation. It takes advantage of the differential between stock prices and their synthetic stock prices implied by the options market (hereafter, the implied stock price), both of which promise the same payoff at the expiry of the option. Many empirical studies report a small deviation of one price from the other when there are transaction costs involved in trading the underlying stock.[1]

Recent studies document severe violations of put–call parity when short sales constraints on the underlying stocks bind, i.e., investors find it difficult or impossible to short sell the stocks. Using a small sample of stocks that have gone through an equity carve-out, Lamont and Thaler (2003) find large violations of put–call parity. They regard the equity carve-out as a cause of short sales constraints. Ofek, Richardson, and Whitelaw (2004) collect a large number of stocks whose costs of short selling are considerable compared to other stocks. They find that violations of put–call parity are asymmetric in the direction of short sales constraints. That is, underlying stock prices are more frequently observed above the implied stock price than below it. They also document a strong relation between violations of put–call parity and the cost of short selling, concluding that the stock and options markets are segmented and investors in the stock market are less rational than those in the options market. Although some irrationally optimistic investors bid up the stock price, mispricing in the stock market does not need to carry through to the options market due to limitations in arbitrage, i.e., short sales constraints.

[1] See Klemkosky and Resnick (1979, 1980), Bodurtha and Courtadon (1986), Nisbet (1992), and Kamara and Miller (1995)

In this paper we provide a rational explanation of previously documented evidence of asymmetric violations of put–call parity. To accomplish this we set up an option pricing model under short sales constraints in the discrete time binomial framework similar to Cox, Ross, and Rubinstein (1979) (CRR hereafter) and Boyle and Vorst (1992). Short sales constraints here is in the form of fees charged when stocks are borrowed and short sold. Therefore an option position hedged by short selling the underlying stocks—that is, a long position in call options or a short position in put options—should include the lending fees or the cost of short selling. As a result, the offer price of a put option is higher than the bid price of a put option with the same underlying stock, strike price, and expiry by the exact amount of the discounted and expected cost of short selling. Similarly, the bid price of a call option is lower than the offer price of the respective call option.

Using the option pricing model, we reinvestigate the put–call parity relation under short sales constraints. According to our model, a stock price that is higher than the implied stock price does not represent a true violation of put–call parity. Suppose that an arbitrageur tries to exploit the price difference between the stock and the options market by short selling a share of the stock and simultaneously buying a position of the implied stock. For the arbitrageur to obtain an arbitrage profit, the present stock price must be sufficiently higher than the implied stock price to cover the cost of short selling in every future state. This is theoretically impossible because the cost is unpredictable, and depends on the stock price movement in the future. The cost can be enormously high if the future stock price soars. For this reason we do not consider observations where the stock price is higher than the implied stock price as true violation of put–call parity, but refer to them as *"seeming violations."*

In our empirical analysis we test whether or not investors in the options market correctly incorporate short sales constraints. To measure the impact of short sales constraints on option prices, we use *implied volatility discrepancy*, which is defined as implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option. Tests are designed to address each component of the cost of short selling, which is a multiple of three components: the rate of lending fees, the lending period, and the dollar value of the underlying stocks to be lent. Maintained hypotheses tested are as follows. First, the implied volatility discrepancy is an increasing function of the rate of lending fees. Other things being equal, as the rate of lending fees increases, the implied volatility calculated from the bid (offer) price of a call (put) option falls (rises) while the offer price of a call option and the bid price of a put option do not change. Thus, the difference between the implied volatility calculated from the midpoint of a put option and that of a call option increases. Second, the implied volatility discrepancy is a U-shaped convex function of strike price. The number of shares to be short sold and the related cost of short selling will increase as an option goes deeper in-the-money. As strike price increases, the implied volatility of the bid price of a call option increases at a decreasing rate while that of the offer price of a put option increases at an increasing rate. Consequently, the shape of the implied volatility discrepancy curve across strike price is U-shaped. Third, the implied volatility discrepancy is an increasing function of time to expiry. The longer the time to expiry of an option, the more the cost of short selling will be included in the option premium. Thus, the implied volatility calculated from the bid (offer) price of a call (put) option price falls (rises) as the time to expiry of the option increases. As a result, the implied volatility discrepancy increases as time to expiry increases.

We use a sample of stocks and options under severe short sales constraints from D'Avolio (2002). The empirical results support the prediction of our model—we find that our option pricing model is valid in the sense that investors in the options market properly take short sales constraints into account.

The rest of the paper is organized as follows. In Section 2, we build the option pricing model under short sales constraints in the discrete time binomial framework, and investigate the impact that short sales constraints have on option price by changing parameters. In Section 3, we revisit the put–call parity relation, and derive the lower and upper limit of the stock price under short sales constraints. Section 4 reports our empirical results in support of the option pricing model, and Section 5 concludes.

2. Option Pricing under Short Sales Constraints

The purpose of this section is to build an option pricing model under short sales constraints in a simple binomial tree framework. We then investigate the impact of short sales constraints on option price by changing the rate of lending fees, term to expiry, and strike price of the option. Before going further, we briefly explain the stock lending mechanism.

2.1 Stock lending fees

Continuous trading of the underlying asset is one of the crucial assumptions in the Black and Scholes (1973) option pricing model. When transaction costs are involved in trading the underlying asset, continuous replication of an option is impossible. Hence the cost of replicating an option is no longer equal to the Black-Scholes price; the offer (bid) price of an option is higher (lower) than the Black-Scholes price. Several papers address the issue in both discrete and continuous time frameworks. Leland (1985) is one of the first researchers to develop a modified Black-Scholes formula in a continuous time framework under proportional transaction costs. Boyle and Vorst (1992) use a discrete time framework similar to the binomial tree model of CRR to construct a portfolio of underlying assets and riskless bonds. They show that this portfolio exactly replicates the option payoff at expiry in the presence of proportional transaction costs of the underlying asset, and derive a Black-Scholes–type approximation with an adjusted variance. Hodges and Neuberger (1989) and Davis, Panas and Zariphopoulou (1993) develop a utility-based model to achieve an optimality in replicating an option under proportional transaction costs. Whaley and Wilmott (1997) reduce the optimality problem of Davis, Panas, and Zariphopoulou by applying asymptotic analysis, and derive a simple analytic formula.

The transaction costs addressed in the previous studies are brokerage fees, i.e., commissions that are proportional to the value of the underlying assets traded. The short sales constraints we introduce in this paper are of a different nature from brokerage fees. To demonstrate the difference we briefly summarize the process of short selling stocks described in many papers including D'Avolio (2002), Geczy, Musto and Reed (2002), and Duffie, Garleanu and Pedersen (2002). Stocks must first be borrowed before they can be short sold. A borrower of the stocks posts collateral, mostly cash, to the lender. The lender pays the interest from the collateral to the borrower, which is called a "rebate." The rebate rate is the interest rate that the borrower receives for the collateral. For most stocks, the rebate rates are almost equal to the market interest rate. Some "special" stocks have rebate rates lower than the market interest rate, and can sometimes be negative depending on the negotiation between the borrower and the lender. In case of a negative rebate rate, the borrower must pay the interest to the lender for use of the special stocks. On the special stocks, we say that, short sales constraints are binding. An investor who short sells the special stocks should first borrow them and pay lending fees, which is a multiple of the rate of the lending fee, the lending period, and the dollar value of the underlying stocks to be lent. The rate of lending fees is the rebate rate subtracted from the market interest rate in absolute value. Thus, the lending fees are the cost of short selling that the borrower must pay. We use the cost of short selling and the lending fees interchangeably in this paper.

From the viewpoint of option pricing, lending fees are different from brokerage fees in several ways. An investor who replicates an option position by borrowing and short selling the underlying stocks should include lending fees within the option price in a different way from brokerage fees. First, lending fees are charged proportionally to the period of lending. The longer the investor borrows the shares, the more lending fees he or she should include in the option premium. In contrast, brokerage fees are charged only when a transaction occurs. Second, while brokerage fees charged today are a multiple of the stock price today, lending fees charged today depend on the stock price yesterday. Technically, the former is adapted to the filtration today, but the latter is adapted to the filtration yesterday. Brokerage fees cannot be revealed until a transaction occurs. However, the investor knows today how much lending fees he or she should be charged tomorrow. Third, lending fees can—and usually are—different from one stock to another depending on the specialness of the stock. The rate of lending fees is determined in the individual stock loan market as a unique price.

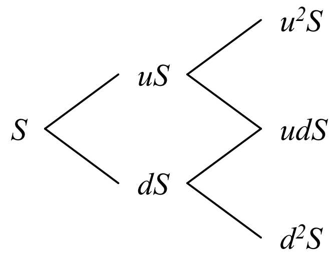
When the demand for borrowing a stock is lower than the supply, the rate of lending fees is only 15 basis points (baseline fees). The stock with baseline fees is called a "general" stock. A "special" stock, where the demand is higher than the supply, will be lent and borrowed with higher lending fees.[2] However, brokerage fees are homogeneous for every stock even though they might be different across the broker or dealers. Fourth, lending fees are included only in a bid price of a call option and an offer price of a put option while brokerage fees are reflected all the option prices. This is because only a long position in a call option and a short position in a put option require short selling the underlying stocks to be hedged.

The characteristics of lending fees we have discussed so far make replicating an option more difficult and costly. In the next subsection, we analyze the price of a portfolio that replicates the payoff of an option in the presence of lending fees.

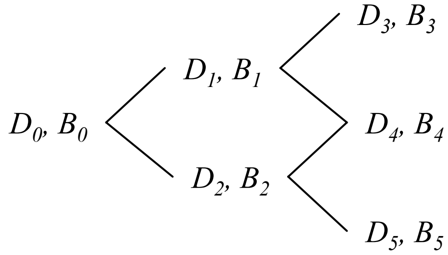
2.2 Model: The cost of replicating an option under short sales constraints

We start with the case of an option investor who writes a put option and hedges it by short selling the underlying asset. The other option positions will be discussed later. For simplicity we assume that there are no transaction costs other than lending fees. We construct a portfolio of the underlying stocks and riskless bonds that exactly replicates the payoff of the put option by using a discrete-time binomial framework. This procedure is similar to that found in CRR and Boyle and Vorst (1992). The evolution of the stock price is given by the following binomial tree:

[2] The equilibrium in the individual stock loan market is discussed in Duffie (1996).



where we assume that $d < R < u$ to preclude any arbitrage opportunity. R is equal to one plus the one-period riskless interest rate. A dynamic replicating strategy at each corresponding node is represented by a pair (D_i, B_i), where D_i and B_i denote the number of shares of the underlying stocks to be short sold and the units of the riskless bonds, respectively. The subscript i represents the location of each node in the tree. Note that the sign of D_i of the put option is always less than or equal to zero.



In a single period case, self-financing portfolios, (D_0, B_0), should satisfy the following two equations:

$$uSD_0 + B_0R = uSD_1 + B_1 - k_L \Delta t D_0 S \quad (1)$$

$$dSD_0 + B_0R = dSD_2 + B_2 - k_L \Delta t D_0 S \quad (2)$$

where k_L is the rate of lending fees of the underlying stock. Equation (1) indicates that the portfolio (D_0, B_0) exactly replicates the option payoff plus the proper amount of lending fees at node 1. Equation (2) can be interpreted in a similar way. Note that the same lending fees are added to the payoff of the option at both nodes 1 and 2. Using equation (1) and (2), the solution, (D_0, B_0), and the price of the put option at initial time, P_0, are:

$$D_0 = D_0^{CRR} \tag{3}$$

$$B_0 = B_0^{CRR} - \frac{1}{R} k_L \Delta t D_0^{CRR} S \tag{4}$$

$$P_0 = P_0^{CRR} - \frac{1}{R} k_L \Delta t D_0^{CRR} S \tag{5}$$

where $D_0^{CRR} = \dfrac{P_1 - P_2}{S(u-d)}$, $B_0^{CRR} = \dfrac{1}{R}\left[P_1 - uSD_0^{CRR}\right]$, $P_0^{CRR} = SD_0^{CRR} + B_0^{CRR}$

P_i denotes the price or the payoff of the put option at node i.[3] We use the CRR model as a benchmark for our model because comparing the differences of the two models yields insights into the results attained by our model. Equation (3) indicates that D_0, the delta of our model at node 0, is the same as that of the CRR model. As the lending fees are independent from the state of the stock price and can be predictable at the initial time, they are not related to the delta. Instead, as shown in equation (4), the lending fees are funded by the riskless bonds. Thus, B_0, the units of riskless bonds of our model, is greater than that of the CRR model by an amount exactly equal to the discounted lending fees. This leads to an increase in the price of the put option as seen in equation (5).

In a multi-period model, the price of a put option can be determined by using the one-period model repeatedly, i.e., equation (1) and (2) are used recursively from the last period to the first period. Take a two-period case for example using the same diagram. In the single-period model that we have just explored, the price of the put option at node 1 and 2 is:

[3] P_i can be calculated by using D_i and B_i; P_i is equal to D_i multiplied by the stock price at node i plus D_i. If node i is at the expiry, we can assume that D_i and B_i of the in-the-money put option are equal to -1 and the strike price, respectively.

$$P_1 = P_1^{CRR} - \frac{1}{R} k_L \Delta t D_1 uS \tag{6}$$

$$P_2 = P_2^{CRR} - \frac{1}{R} k_L \Delta t D_2 dS \tag{7}$$

By substituting equations (6) and (7) for equations (3) and (4), we can find the delta and the price of the put option at the initial time:

$$D_0 = D_0^{CRR} - \frac{1}{R} k_L \Delta t \frac{D_1 u - D_2 d}{u - d} \tag{8}$$

$$P_0 = P_0^{CRR} - \frac{1}{R} k_L \Delta t D_0 S - \frac{1}{R^2} k_L \Delta t \left(p_u uSD_1^{CRR} + p_d dSD_2^{CRR} \right) \tag{9}$$

where $p_u = \frac{R-u}{u-d}, p_d = 1 - p_u$

In equation (8), the delta in our model is no longer equal to the CRR delta. This delta has an additional term that reflects an adjustment to the stock price movement in the following period. The interpretation of equation (9) is interesting. The second term on the right-hand side, $-\frac{1}{R} k_L \Delta t DS$, represents the present value of the lending fees after one period, and the third term, $-\frac{1}{R^2} k_L \Delta t \left(p_u uSD_1^{CRR} + p_d dSD_2^{CRR} \right)$, is the present value of the expected lending fees after two periods. The expectation is taken under the risk-neutral probability measure. That is to say, under short sales constraints, the price of the put option is the CRR price plus the sum of the present values of the expected lending fees in the risk-neutral world. An investor who writes and hedges the put option should offer a price that is higher than the CRR price by the amount of discounted and expected lending fees.

The bid price of a call option can be obtained by using the same model. In this case, however, lending fees are subtracted from the CRR price because the delta of a call option is always equal to or greater than zero. As a result, an investor who buys a call option and hedges it with the underlying stocks should bid at a price that is lower than the CRR price by the amount of discounted and expected lending fees. While a long call position and a short put position involve short selling the underlying stocks, a short call position and a long put position do not require an investor to short sell the stocks to hedge the option positions. In other words, the offer price of a call option and the bid price of a put option are equal to the CRR price even in the presence of short sales constraints.

2.3 Risk of recall

A lender of stocks retains the right to recall the stocks at any time during the lending period. At the lender's notification, the borrower must return the borrowed stocks within three days. To an investor who hedges an option position by short selling the underlying stocks, the abrupt notice of recall is indeed a risk to be taken into account. The investor can choose one of two alternatives at the notice of recall. First, the investor may locate another lender who is willing to lend her the same stocks at the same rate. If she fails to locate the same stocks, the investor must buy the borrowed stocks in the stock market to return them to the original lender, and simultaneously liquidate the option position in the options market. We investigate how the risk of recall can change the price of an option in the latter case.

Even at the event of recall, the bid (offer) price of a call (put) option does not change as long as the rate of lending fees determined in the stock loan market does not change. Take the previous two-period example with a put option. At the initial time, the investor writes a put option, receives the premium, and short sells the appropriate amount of the underlying stocks to hedge the option position as in equation (8). We assume that the stocks borrowed at the initial time are recalled after one period, and the investor fails to locate the same stocks. Hence she buys the borrowed stocks in the stock market to return them to the lender with the lending fees, and liquidates the put option in the options market. The investor's proceeds at each node is:

$$(at\ node\ S) \quad P_0^{recall} - D_0 S$$

$$(at\ node\ uS) \quad -P_1 + D_0 uS + k_L \Delta t D_0 S$$

$$(at\ node\ dS) \quad -P_2 + D_0 dS + k_L \Delta t D_0 S$$

The superscript, "*recall*," emphasizes that it is the option premium for which the risk of recall is considered.

Unless the rate of lending fees changes at the event of the recall, P_1 and P_2 are the same as in equation (6) and (7). The offer price of the put option at the initial time, P_0^{recall}, can be determined by using the risk-neutral expectation argument. That is, the risk-neutral expectation of discounted payoffs after one period should lead to the premium at initial time:

$$\begin{aligned} P_0^{recall} &= D_0 S - \frac{1}{R}\left[p_u\left(-P_1 + D_0 uS + k_L \Delta t D_0 S\right) + p_d\left(-P_2 + D_0 dS + k_L \Delta t D_0 S\right)\right] \\ &= P^{CRR} - \frac{1}{R} k_L \Delta t D_0 S - \frac{1}{R^2} k_L \Delta t S\left(p_u u D_1^{CRR} + p_d d D_2^{CRR}\right) \end{aligned} \tag{10}$$

where $p_u = \dfrac{R-u}{u-d}, p_d = 1 - p_u$

Equation (10) shows that as long as the rate of lending fees does not change, the offer price of the put option remains the same. If the rate of lending fees increases, however, the liquidation prices, P_1 and P_2, will also increase.

2.4 Comparative statics

In this subsection we analyze the price of options under short sales constraints by changing some of the model parameters. Unless noted otherwise, the parameters are as follows: initial stock price = 100, strike price = 100, riskless interest rate = 5%, time to maturity = 0.5 years, volatility = 40%, the rate of lending fees = 5% and the number of revisions = 250.

Number of revisions and option prices

Boyle and Vorst (1992) show that, in the presence of proportional brokerage fees, the offer (bid) price of an option increases (decreases) as number of revisions to the replicating portfolio increases. As stated earlier, this is because brokerage fees are charged whenever the underlying stocks are traded. Unlike brokerage fees, lending fees are charged proportionally to the period of lending. This different characteristic of lending fees is reflected in the option prices in a different way.

We calculate the bid and the offer prices of both the call and put options as increasing the number of revisions to the replicating portfolio. The four prices are calculated by repeatedly using equation (5).[4] To compare the relative level of the four prices, we convert each option price into its implied volatility and present all of them in Figure 1. The implied volatility is calculated by using the Black and Scholes (1973) model. The symbols, "+," "□," "*" and "o" represent the offer price of the call option, the bid price of the call option, the offer price of the put option, and the bid price of the put option, respectively. None of the four prices increase or decrease as the number of revisions increases, but they converge to some constant levels. This shows that lending fees are not related to the number of revisions. It should also be noted that the bid (offer) price of the call (put) option, translated as implied volatility, is lower (higher) than the CRR price by the amount of the cost of short selling. However, the offer price of the call option and the bid price of the put option are the same as the CRR price since they do not involve any lending fees.

[Insert Figure 1 about here]

[4] The lending fees in equation (5) should be omitted if replicating the option does not involve short selling the underlying stocks.

Rate of lending fees and option prices

Figure 2 shows the changes in option prices in response to an increase in the rate of lending fees. It is evident that implied volatility of the bid (offer) price of a call (put) option decreases (increases) as the rate of lending fees increases. The impact that lending fees have on the option price looks tremendous. When the rate of lending fees is 4.3%, the implied volatility of the bid price of the call option drops from 40.0% to 35.6% (a decrease of 4.4%), and the implied volatility of put offer price rises from 40.0% to 43.4% (an increase of 3.4%). When the rate of lending fees is as high as 50%,[5] the implied volatility of the call bid price falls to 5%, and that of the put offer price rises to 90%.

[Insert Figure 2 about here]

Time to expiry and option prices

Figure 3 shows the relation between time to expiry of options and their implied volatilities. As time to expiry increases, implied volatility of the bid (offer) price of a call (put) option decreases (increases). A longer-term option takes a longer period of borrowing and short selling than a shorter-term option. Thus, the longer the term to expiry an option has, the more the costs of short selling will be included in its price. As a result, the implied volatility of the bid (offer) price of a call (put) option decreases (increases).

[Insert Figure 3 about here]

Moneyness and option prices

Figure 4 represents the relation between the moneyness of options and their implied volatilities. We define the moneyness of a pair of call and put options as the delta of the call option.[6] The bid (offer) price of the call (put) option decreases (increases) as the option

[5] The lending fee of Krispy Kreme Doughtnuts Inc. was 55% during February 2001. See D'Avolio (2002), page 287.

[6] Moneyness can be defined in many different ways. We define it as the delta of a call option for empirical reasons that will be discussed later in the paper.

goes deeper in-the-money. Intuitively, the number of shares to be short sold is greater for in-the-money options than for out-of-the-money options. Therefore, the deeper in-the-money an option is, the more the costs of short selling are reflected in its price.

[Insert Figure 4 about here]

To summarize, the bid (offer) price of the call (put) option decreases (increases) as the rate of lending fees increases, time to expiry of the option increases, or the option goes deeper in-the-money.

3. Put–call Parity under Short Sales Constraints

Put–call parity is an arbitrage relation between European call option and put options with the same underlying asset, strike price and expiry. Stoll (1969) first discovered that the payoff of a stock can be constructed by buying a call option, writing a put option, and investing in riskless bonds an amount equal to the present value of the strike price of the options as follows:

$$S = C - P + PV(K) \tag{11}$$

where S is the underlying stock price, C and P are the prices of call and put options, K is the strike price of the options, and PV is the present value operator. Equation (11) can be interpreted as indicating that the underlying stock price in the left-hand side must be equal to the implied stock price derived by put–call parity in the right-hand side. The relation always holds as long as the market is frictionless and there is no arbitrage opportunity. For American style options, equation (11) does not hold. Merton (1973) shows that if there is no dividend payout until the expiry, the stock price can be greater than the implied stock price because the American put option is more valuable than the European one by the amount of the early exercise premium. Thus, put–call parity relation for non-dividend-paying American options can be restated by adding the early exercise premium to the right-hand side of equation (11):

$$S = C - P + PV(K) + EEP \tag{12}$$

where *EEP* is the early exercise premium of the American put option.

An investor, especially an arbitrageur, may try to exploit arbitrage profits between the stock and the options market by using the above put–call parity relation. She can try one of the two strategies: she can buy a share of the underlying stock in the stock market and simultaneously sell the implied stock in the options market, or she can short sell a share of the underlying stock and simultaneously buy the implied stock. We define the former as long arbitrage strategy and the latter as short arbitrage strategy. Considering the bid–offer spread in the options market, the no-arbitrage relation in equation (12) should be modified into the following two inequalities:

$$S \geq S^S = C^{bid} - P^{offer} + PV(K) + EEP \tag{13}$$

$$S \leq S^L = C^{offer} - P^{bid} + PV(K) + EEP \tag{14}$$

where the superscript *offer* (*bid*) points out that the price is the offer (bid) price of the option. We define S^S (S^L) as implied short (long) stock price, which is equal to the cost of replicating a short (long) position in the underlying stock. Equation (13) and (14) indicates that for any investor not to make arbitrage profits, the underlying stock price must be greater than the implied short stock price, S^S, and must be lower than the implied long stock price, S^L. In other words, the stock price is bounded from below and above by the implied short and long stock prices, respectively.

Equation (13) and (14) are derived on the assumption that the investor can short sell the underlying stocks at no cost. We now introduce short sales constraints into the put–call parity relation. There are two lines of research that investigate the behavior of asset prices when short sales constraints are binding. The first group of papers explores the impact that short sales constraints have on the stock price behavior. Diamond and Verrecchia (1987) explore the effects of short sales constraints on the speed of price-adjustment to private information. Hong and Stein (2003) develop a heterogeneous agent model that explains the link between short sales constraints and market crashes. Jones and Lamont (2002) show that stocks that are expensive to short sell have high valuations and low subsequent returns. Ofek and Richardson (2003) show that short sales constraints have considerable and persistent negative impact on subsequent returns of DotCom stocks. Theses studies support the hypothesis that stock prices are frequently over-priced under short sales constraints.

The second group of researchers investigates violations of well-known arbitrage strategies caused by short sales constraints. Geczy, Musto and Reed (2002) use a comprehensive dataset of short sales and find that short sales constraints have a mixed impact on the profitability of well-known arbitrage strategies. Ofek, Richardson and Whitelaw (2004) make an advance in investigating the put–call parity relation of equation (13) and (14) by introducing and directly measuring short sales constraints. They divide their option dataset into two groups according to the "rebate rate spread," which, as they define it, is the deviation of the rebate rate on a particular stock from the standard rebate rate on the majority of stocks as discussed in the previous section. The stocks with a negative rebate rate spread are considered "special." The main evidence can be summarized in three points. First, violations of put–call parity are asymmetric in the direction of short sales constraints, and the frequency of the violations is strongly related to the rebate rate spread. The frequency of violations where the stock price is higher than the implied long stock price is 12.23% while the frequency where the stock price is lower than the implied short stock price is 2.73%. For negative rebate spread stocks, the percentages of put–call parity violations are 19.51% versus 2.65%. The frequency of violations where the stock price is higher than the implied long stock price still remains large after considering the cost of short selling. Second, violations above the implied long stock price are related to time to expiry of options; that is, as time to expiry of options increases, the magnitude of the violations also increases. The mean magnitude of the violations, measured by the log difference between the stock price and the implied long stock price, is 0.86% for long maturity options (183-365 days), versus 0.37% for short maturity options (30-90 days). Third, both the magnitude of put–call parity violations and the cost of short selling are significant predictors of future returns for individual stocks. Stocks with a rebate rate spread less than -0.5% and put-call parity violations greater than 1% yields -12.57% of average returns over the life of the option. Cumulative average returns on portfolios that are long the industry and short the filtered stocks are 66% after taking the cost of short selling into account.

Ofek, Richardson and Whitelaw (2004) explain that these violations are consistent with the behavioral finance theory of overly optimistic investors and market segmentation as discussed in Miller (1977) and Ofek and Richardson (2003). On one hand, the irrationally optimistic investors raise the stock price while, due to short sales constraints, the rational investors do not simultaneously short sell the stock. On the other hand, the over-valued stock price can differ from the implied stock price if the markets are segmented such that the marginal investors across the two markets are different.

We reinvestigate put–call parity relation under short sales constraints. By using our option pricing model, we once again derive the upper and lower limit of the stock price implied by long and short arbitrage strategies. To simplify our argument we assume that the options are European, there are no scheduled dividend payouts until the expiry of the options, and the stocks and riskless bonds are traded without any transaction costs except lending fees.

Long arbitrage strategy under short sales constraints

The object of long arbitrage strategy is to make riskless profit by buying a stock and selling the implied stock simultaneously. A short position in the implied stock can be constructed by writing a call option at the bid price, buying a put option of the same strike price and expiry as the call option at the offer price, and issuing the riskless bond with its par value equal to the strike price of the options. At the expiry of the options, the payoffs from the long position in the stock and the short position in the implied stock will exactly offset each other at any circumstance. The long arbitrage strategy can be terminated even before the expiry by selling the stock and buying back the implied stock, i.e., buying the call option at the offer price, writing the put option at the bid price, and paying the riskless bond back. We assume that, in the previous binomial tree model, an arbitrageur initiates the long arbitrage strategy at the initial time and terminates it after one period. The arbitrageur's proceeds at each node are as follows:

$$(at\, node\ S) \quad -S+\left(C^{bid}-P^{offer}+PV(K)\right)\ (<0) \tag{15}$$

$$(at\, node\ uS) \quad uS-\left(C_1^{offer}-P_1^{bid}+PV(K)\right)\ (=0) \tag{16}$$

$$(at\, node\ dS) \quad dS-\left(C_2^{offer}-P_2^{bid}+PV(K)\right)\ (=0) \tag{17}$$

Equation (16) and (17) show that the proceeds at uS and dS are both zero because the offer price of the call option and the bid price of the put option are equal to the CRR prices. Note that in the CRR model the put–call parity always holds. At node S in equation (15), however, the proceeds are negative because the bid price of the call option is lower than the CRR price and the offer price of the put option is higher than the CRR price. From equation (15) we can derive the lower limit of the stock price; the stock price should be greater than the implied short stock price. The difference between the two prices is:

$$S-\left(C_0^{bid}-P_0^{offer}+PV(K)\right)=\frac{1}{R}k_L\Delta tS \tag{18}$$

Equation (18) shows that the arbitrageur who uses the long stock arbitrage strategy will wind up with an immediate loss in an amount that is equal to the discounted costs of short selling. The sum of the costs of short selling that are included in both the call and put options is exactly equal to the discount costs of short selling because the deltas of the two options add up to one. To summarize, there exists no arbitrage opportunity incurred by the long arbitrage strategy as long as the stock price is above the implied short stock price. However, if the stock price is below the implied short stock price, the arbitrageur can make riskless profit.

Short arbitrage strategy under short sales constraints

Short arbitrage strategy involves short selling the underlying stock and simultaneously buying the implied stock. A long position in the implied stock can be constructed by buying a call option at the offer price, writing a put option of the same strike price and expiry as the call option at the bid price, and investing in riskless bond with its par value equal to the strike price of the options. Using short arbitrage strategy is riskier than using long arbitrage strategy for three reasons. First, the borrowed stock can be recalled any time before the expiry of the options. When recalled, the arbitrageur may finish the strategy at another's discretion. Second, the amount of lending fees that the arbitrageur must pay at the end of each time period is unpredictable. Since the stock price itself is stochastic, lending fees, a multiple of the stock price, are also stochastic. Third, the American put option may be exercised before the expiry even in the absence of dividend payout. In this case the arbitrageur should finish her strategy. To take these aspects into account, we assume that, in the previous binomial framework, an arbitrageur initiates the short arbitrage strategy at the initial time and finishes the strategy after one period. The proceeds at each node are as follows:

$$(at\,node\ S) \qquad S-\left(C_0^{offer}-P_0^{bid}+PV(K)\right)\ (=0) \tag{19}$$

$$(at\,node\ uS) \quad -uS+\left(C_1^{bid}-P_1^{offer}+PV_1(K)\right)-k_L\Delta tS\ \left(=-\left(1+\frac{u}{R}\right)k_L\Delta tS<0\right) \tag{20}$$

$$(at\,node\ dS) \quad -dS+\left(C_2^{bid}-P_2^{offer}+PV_2(K)\right)-k_L\Delta tS\ \left(=-\left(1+\frac{d}{R}\right)k_L\Delta tS<0\right) \tag{21}$$

In equation (19), the short arbitrage strategy does not involve any initial commitment because both the offer price of the call option and the bid price of the put option are the CRR prices and the put–call parity holds. However, as seen in equations (20) and (21), the arbitrageur winds up with a loss that can vary depending on whether the stock price rises or falls. The arbitrageur's loss at node *uS* amounts to the sum of the lending fees during the first period and that during the second period discounted by one period. The lending fees during the second period are embedded in both the call and put options, and are equal to $-(u/R)k_L \Delta t S$. The proceeds and the loss at node *dS* can be interpreted in a similar way. To preclude the arbitrage opportunities, the following equation must hold:

$$S < \left(C_0^{offer} - P_0^{bid} + PV(K)\right) + k_L \Delta t S \cdot \max\left(\left(\frac{1}{R} + \frac{u}{R^2}\right), \left(\frac{1}{R} + \frac{d}{R^2}\right)\right) \qquad (22)$$

Equation (22) indicates that the upper limit of stock price is the implied long stock price at the initial time plus the maximum aggregate discounted cost of short selling charged in the future. The maximum operator is used to make sure that there is no arbitrage opportunity for any possible path of stock price movement in the future. The maximum cost depends on *u*, the size of the upward movement and, in turn, the volatility of the stock.

It is theoretically impossible to make riskless profits with special stocks by the short arbitrage strategy. As time goes by, the maximum of the aggregate discounted lending fees becomes infinite,[7] i.e., the stock price is not bounded from above. This means that violations of equation (13) are not true violations. We instead define those observations where stock price is higher than the implied long stock price as "*seeming violations*" of put–call parity. We can see in equation (22) that it is natural that, other things being equal, the frequency and magnitude of seeming violations increase as the rate of lending fees or time to expiry increases.

[7] The maximum of the sum of the discounted lending fees approaches infinity because $u>R$.

$$k_L \Delta t S\left(\frac{1}{R} + \frac{u}{R^2} + \frac{u^2}{R^3} + \cdots\right) = \infty .$$

In a continuous time framework, the sum of the discounted lending costs can be written as an integral, and its maximum is unbounded.

$$max\left(\int_o^T e^{-rt} k_L S(t) dt\right) = \infty$$

4. Empirical Analysis

In this section we perform an empirical analysis on our option pricing model and put–call parity relation under short sales constraints. The purpose of the empirical analysis is to validate our option pricing model by testing whether investors in the options market correctly incorporate short sales constraints or not, and support our argument that the stock price is not bounded from above by the implied long stock price derived in the options market. If our model is correct, we should be able to find the characteristics shown in Figures 2, 3, and 4—dependency of implied volatility on the rate of lending fees, time to expiry, and moneyness. We can then conclude that investors are rational and put–call parity relation holds even under short sales constraints.

To measure the impact that short sales constraints have on option prices we use *implied volatility discrepancy*. We define implied volatility discrepancy as implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option with the same underlying asset, time to expiry, and strike price, $\sigma_{put} - \sigma_{call}$. Note that without short sales constraints, implied volatilities of call and put options with the same underlying stock, expiry, and strike price must be the same if and only if put–call parity relation holds. We use implied volatility discrepancy because implied volatility may contain other effects and may not be an adequate measure of the impact that short sales constraints have on option price. The shape of implied volatility function across time to expiry can be different from Figure 3 due to the possible influence of volatility term structure. Because of this, we test whether implied volatility discrepancy increases as time to expiry increases. In addition, the shape of implied volatility function may not be a decreasing function of moneyness as shown in Figure 4-(a) due to the well-known volatility smile or smirk phenomenon that is still under debate among scholars and practitioners. Using the implied volatility discrepancy, we expect that it is a U-shaped convex function of moneyness as shown in Figure 4-(b). Testing the dependency of implied volatility on the rate of lending fees in Figure 2 cannot be performed directly because the level of implied volatility can vary from one stock to another. Instead, we see if implied volatility discrepancy is an increasing function of the rate of lending fees. In summary, we test the following hypotheses using implied volatility discrepancy:

(H1) Implied volatility discrepancy is an increasing function of time to expiry. (Figure 3)
(H2) Implied volatility discrepancy is a U-shaped convex function of moneyness. (Figure 4)
(H3) Implied volatility discrepancy is an increasing function of the rate of lending fees. (Figure 2)

Dependency of implied volatility discrepancy on time to expiry, moneyness, and the rate of lending fees can be deduced from Figures 3, 4, and 2, respectively. Although the figures are drawn by European option pricing model, American options should show similarity.

The second hypothesis that implied volatility discrepancy is a U-shaped convex function of moneyness deserves a special remark. Under both the behavioral finance theory and our option pricing model, we should not be able to reject *H1* and *H3*. The behavioral finance theory reports that the magnitude of (seeming) violations is related to time to expiry of options and the rate of lending fees. However, under the behavioral finance theory, there is no reason why implied volatility discrepancy shows moneyness dependency. In the sense, *H2* is more conclusive to our option pricing model than the others.

4.1 Data

D'Avolio (2002) reports a list of 35 negative rebate stocks from April 2000 through September 2001.[8] This list appears in appendix A. Many stocks in the list are related to well-documented events such as initial public offerings, the DotCom crisis, or depository receipt issues, all of which are known to cause the negative rebate rate.[9] The lending fees in the third column of the list range from 10.0% to 79.0%. In the last column, the specific month of the negative rebate rate for each stock is reported. Another source of our dataset is

[8] D'Avolio (2002) does not reveal the source of the data. We could not expand the data since they are inaccessible at this moment.
[9] See Ofek and Richardson (2003), and Bris, Goetzmann and Zhu (2004).

the IVY OptionMetrics. We collect the end-of-the-day bid and offer prices of options by matching the two sources. We assume that all option quotes in the reported month are related to the one and only negative rebate rate although the actual rebate rate may be different on a day-by-day basis. Since it cannot be avoided, we leave this possible misspecification in our analysis.

Initially, 23,650 pairs of call and put option are collected. To filter out undesirable data, we apply a set of screening rules similar to those of Bakshi, Cao and Chen (1997) and Dumas, Fleming and Whaley (1998). First, option quotes less than \$3/8 are dropped from our sample. These quotes may be too small compared to the minimum tick size to reflect the true value of options. Second, options with zero open interest are excluded from the sample. Prices of low liquidity may also show a deviation from the true value. Third, option quotes that violate the lower arbitrage bound are excluded. It is impossible to calculate implied volatility for these options. Fourth, option quotes on the stocks paying dividends before the expiry are removed for the ease of analysis. Fifth, options with time to expiry shorter than 10 days are eliminated. The close-to-expiry options may have microstructural concerns. Option quotes with time to expiry longer than 240 days are also eliminated due to their small sample size.

After applying these screens, the sample is reduced to 4,712 pairs of call and put option quotes. We provide a distributional description of the sample in Table 1. The mean (median) value of expiry of options, T, is 109.85 (101.00) days. The mean (median) of delta of call and put options, Δ_{call} and Δ_{put}, are 51.71% (51.05%) and -45.57% (-45.23%) respectively. Implied volatilities of both call and put options are very high. Implied volatilities are calculated from the midpoint of the bid–offer spread by using the CRR binomial framework to implement the American feature of the options. The implied volatilities of put options are much higher than those of call options. The mean (median) value of the implied volatility of put options is 107.92% (106.86%) while that of call options is 84.91% (81.39%). The maximum values are 188.96% and 223.21% for call and put options, respectively. The open interest, *OI*, of call options is much larger than that of put options. In the last column, the early exercise premium, *EEP*, of American put options over European equivalents is reported after being divided by the option price. The mean of the premium which is 0.81% of the option price on average is negligible.

[Insert Table 1 about here]

To see how implied volatility changes across time to expiry and moneyness, we categorize the sample into four expiry groups of 60 days apart and five moneyness groups similar to Bollen and Whaley (2004) as shown in Table 2. We define the moneyness of a pair of call and put options as the delta of the call option instead of the typical measure of moneyness: exercise price divided by the stock price. The latter cannot truly reflect the likelihood that the option will be in-the-money at expiry because it depends heavily on the volatility and term to expiry. Unlike Bollen and Whaley (2004), we do not categorize call and put options by their own separate deltas because, by doing so, a pair of call and put options with the same underlying asset, expiry, and strike price can be placed in different moneyness groups.

[Insert Table 2 about here]

4.2 Seeming violations of put–call parity

In Table 3 we provide the empirical distribution of the underlying stock prices against the implied stock prices derived from the options market. Specifically, we list the stock prices in comparison to the implied short stock price, S^S, the implied long stock price, S^L, and the stock price derived from put–call parity when all options are assumed to be traded at the midpoint of bid–offer spread, S^M. S^M can be regarded as the average level of the implied stock price. Like Ofek, Richardson and Whitelaw (2004), we report the frequency of observations where the stock prices are below S^S, between S^S and S^M, between S^M and S^L, and above S^L.

A few points are worth elaborating upon. First, there are a large number of observations satisfying $S > S^L$ (73.77% on average) while the observations satisfying $S < S^S$ are rare (0.47% on average). The results are quite similar to those obtained by Ofek, Richardson and Whitelaw (2004), but show more extreme asymmetry. This may be because our sample consists of extremely negative rebate stocks. Second, the frequency of observations satisfying $S < S^S$ represents less than 1% for each expiry group. This shows that there are few arbitrage opportunities to buy the underlying stock and simultaneously short sell the implied stock. Because the observations satisfying $S > S^L$ are not true violations of put–call parity, we can conclude that arbitrage opportunity between the stock market and the options market is negligible. Third, the frequency of seeming violations increases as time to maturity increases. The frequency of observations satisfying $S > S^L$ ranges from 67.29% for expiry group 1 to 79.15% for expiry group 4. In addition, the frequency where the stock prices is greater than the average implied stock price, $S > S^M$, increases from 93.68% to 98.10% as time to expiry increases. As discussed in the previous section, an increase in the frequency of seeming violations as time to expiry increases follows naturally from equation (22). Fourth, we also report the mean value of the log deviation of the stock price from the implied long stock price, $100Ln(S/S^L)$, for each expiry group which measures the magnitude of seeming violations. The standard errors appear in parentheses. As expected, the magnitude of seeming violations rises as time to expiry increases. In summary, although our sample is limited to only 4,712 option pairs, it is large enough to represent the overall features similar in Ofek, Richardson and Whitelaw (2004).

[Insert Table 3 about here]

4.3 Implied volatility discrepancy of negative rebate stocks

In this subsection we present an empirical test of the validity of our option pricing model, that is, whether option investors properly incorporate the cost of short selling into the option price. If our model is correct and investors price options properly, the cost of short selling included in the option prices should depend on the rate of lending fees, time to expiry, and moneyness. Specifically, we test hypotheses *H1*, *H2*, and *H3*.

In Table 4 we present the implied volatility discrepancies of individual stocks across the expiry groups. The stocks are sorted in descending order of the rate of lending fees. The implied volatility discrepancies are positive for almost all the stocks. ERICY is one of the exceptions, with an implied volatility discrepancy that is negative. The exceptions can be observed because we cannot pinpoint the specific time of specialness of the stock. While GM has observations in only one expiry group, other stocks show increasing implied volatility discrepancy with increasing time to expiry. MCDT is a perfect example; its implied volatility discrepancy increases from 13.01% for the near term expiry group all the way to 31.73% for the long-term expiry group.

The average implied volatility discrepancies are listed in Table 5. In panel A, the implied volatility discrepancies averaged across the stocks in expiry groups 1 to 4 are 16.72%, 24.86%, 24.01% and 28.26%, respectively. They are all statistically significant at the 1% confidence level. However, the results in panel A might be misleading due to unequal sample sizes across the stocks. For example, MSTR is over-weighted with a large number of observations, 825, and ABX is practically ignored with only one observation. To circumvent the problem we first take the average of the implied volatility discrepancy for each stock, and then average it across the stocks, so that each stock has the same weight. The results are shown in panel B of Table 5. The average implied volatility discrepancies in panel B are 10.30%, 14.65%, 17.90%, and 22.71% from expiry group 1 to 4, and all statistically significant at the 1% confidence level. These increasing shape of the implied volatility discrepancy are consistent with our option pricing model. We also report the implied volatilities of call and put options across expiry groups, but they are not consistent with the characteristics shown in Figure 3. As previously mentioned this may be due to the term structure of volatility.

[Insert Table 4 about here]

[Insert Table 5 about here]

In Table 6 we provide the implied volatility discrepancies of individual stocks across different moneyness groups. For most stocks the shape of the implied volatility discrepancy curve across moneyness is U-shaped, which is consistent with Figure 4-(b). For example, the implied volatility discrepancy of HAND starts with 18.12% at moneyness group 1, hits the lowest level, 10.33%, at group 3 and goes up to 19.30% at group 5. The average values are shown in panel A of Table 7, and are 36.57%, 25.72%, 19.64%, 20.96%, and 26.56% from moneyness group 1 to 5, respectively. The average shape is also U-shaped with the lowest level at moneyness group 3. The same shape remains even after adjusting for the unequal sample size among stocks, which is shown in panel B of Table 7. The mean values are 27.65%, 14.33%, 12.16%, 14.68%, and 24.60% from moneyness group 1 to 5, respectively, and all statistically significant at the 1 % confidence level.

[Insert Table 6 about here]

[Insert Table 7 about here]

The last column of Table 6 shows the average implied volatility discrepancy for each stock. The stocks are sorted in descending order of the rate of lending fees. As shown in Figure 2, we expect that, all other things being equal, the mean implied volatility discrepancy decreases from top to bottom as the rate of lending fees decreases. However, the numbers do not show a clear decreasing pattern. There may be two explanations for this. First, the empirical results regarding the relation between the implied volatility discrepancy and the rate of lending fees may be misspecified because other variables, such as time to expiry and moneyness, are not controlled for. The second possible explanation can be found in Evans, Geczy, Musto and Reed (2006), who introduce an option to fail. A market maker can short stocks without borrowing, and have an option to fail to deliver stocks to the buyer of the stocks. In this case the marker maker should post the same amount of collateral, but cannot earn interest (rebate) on the posted collateral. With this privilege, the market maker can limit the maximum borrowing costs below the riskless interest on the collateral. Consequently, regardless of the structure of the stock loan market, the rate of lending fees of a marginal arbitrageur can be limited to the riskless interest rate if the options market is competitive. However, if the options market is not competitive, the option price may reflect the higher borrowing costs in spite of the privilege of market markers.

To test hypothesis (*H3*) we control the effect of time to expiry and moneyness with a simple regression model. The regression equation for stock *i* is:

$$IVD_i = \beta_0 + \beta_1 \times Fee_{\mathrm{i}} + \beta_2 \times Expiry_{\mathrm{i}} + \beta_3 \times Moneyness_{\mathrm{i}} + \beta_4 \times Moneyness_i^2 + \varepsilon_i \quad (23)$$

where *IVD* is the implied volatility discrepancy, *Fee* is the rate of lending fees, *Moneyness* is moneyness of an option pair, i.e., the delta of the call option, *Expiry* is the time to expiry in years, ε is the regression error, which is assumed to be normally distributed with zero mean and constant variance. Equation (23) shows the relation between the implied volatility discrepancy and the rate of lending fees after controlling for moneyness and time to expiry of the options. We use the Cochrane-Orcutt estimation procedure to correct for the possible existence of serial correlation in the regression residual. The regression results are shown in Table 8. For regressions on individual stocks we omit the rate of lending fees in the dependent variables. The results on individual stocks show that in most cases, both time to expiry and moneyness are statistically significant at the 1% confidence level. The implied volatility discrepancy of every individual stock shows a U-shaped convex function of moneyness with a negative coefficient for moneyness and a positive coefficient for the square of moneyness. In addition, the coefficient of *Expiry* is statistically significantly positive for most stocks.

The aggregate relation between implied volatility discrepancy and the rate of lending fees is shown to be positive and statistically significant at the 1% confidence level as predicted in Figure 2. Standard errors appear in parentheses. The coefficient of time to expiry and moneyness is positive and statistically significant at the 1% confidence level as well, which confirms the results observed in Figure 3. Altogether, the empirical results are consistent with our option pricing model.

[Insert Table 8 about here]

5. Conclusion

An arbitrage opportunity is a zero investment portfolio with possible riskless profit, and put–call parity is one of the most widely documented arbitrage strategies. In this paper we investigate the put–call parity relation under short sales constraints. For this purpose, we introduce a discrete time binomial tree model similar to that of CRR and Boyle and Vorst (1992). Using our model we reinvestigate put–call parity relation, and show that, under short sales constraints, the stock price that is higher than the implied stock price is not a true violation of the relation. That is to say, an arbitrageur cannot make riskless profits by short selling a share of the underlying stock and buying a position of the implied stock because the cost of short selling that he or she must pay in the future is unpredictable, and can be enormously high. The arbitrageur should consider the maximum lending fees that will discourage him or her to try the strategy.

The empirical results, which employ a sample of extremely negative rebate stocks from D'Avolio (2002), validate our option pricing model. For empirical purpose we define and use implied volatility discrepancy, the implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option with the same underlying stock, time to expiry, and strike price. As predicted in our model, the implied volatility discrepancy of an option pair widens as the time to expiry of the rate of lending fees increases, and is a U-shaped convex function of moneyness. Thus, we conclude that investors in the options market correctly reflect both the stock price and the cost of short selling.

Appendix[10]

Selected negative rebate stocks (April 2000 through September 2001).

Ticker	Company	Fee (%)[a]	Month
CNH	CNH GLOBAL	79.0	200105
GM	GENERAL MTRS CORP	63.0	200005
TOT	TOTAL FINA SA [d]	55.0	200006
KREM	KRISPY KREME DOUGHNUTS INC [b]	55.0	200102
STLW	STRATOS LIGHTWAVE INC [b]	50.0	200009
UN	UNILEVER N V	46.0	200105
PRKR	PARKERVISION INC	45.0	200005
MCDT	MCDATA CORPORATION [b]	40.0	200106
RD	ROYAL DUTCH PETE CO [d]	35.0	200108
PPD	PRE PAID LEGAL SVCS INC	35.0	200109
PLMD	POLYMEDICA CORP	35.0	200109
PLCE	CHILDRENS PL RETAIL STORES INC	35.0	200105
PALM	PALM INC [b]	35.0	200008
ABX	BARRICK GOLD CORP	27.0	200005
BCE	BCE INC	26.5	200006
NOK	NOKIA CORP. [d]	25.0	200103
HAND	HANDSPRING INC [b]	25.0	200102
INRG	INRANGE TECHNOLOGIES CORP [b]	25.0	200010
COH	COACH INC [b]	25.0	200104
AREM	AREMISSOFT CORP DE	22.0	200107
ERICY	ERICSSON [d]	20.5	200104
ABY	ABITIBI CONSOLIDATED INC	20.0	200012
OWC	OWENS CORNING	20.0	200106
RAD	RITE AID CORP	20.0	200103
PCL	PLUM CREEK TIMBER CO INC	20.0	200109
SFP	SALTON INC	20.0	200109
PRCM	PROCOM TECHNOLOGY INC	20.0	200106
PPRO	PURCHASEPRO COM INC [c]	20.0	200107
LNUX	VA LINUX SYS INC [b]	20.0	200006
MSTR	MICROSTRATEGY INC	15.0	200007
WBVN	WEBVAN GROUP INC [b c]	14.0	200009
SWZA	SUIZA FOODS CORP	12.5	200104
PFSW	PFSWEB INC [b c]	10.0	200007
WEBM	WEBMETHODS INC [b c]	10.0	200008
RETK	RETEK INC [b]	10.0	200010

[a] Fee: (defined by Rebate – Fed Funds) the highest recorded by each stock within the sample period.
[b] IPO: stocks within one year of their issue date as provided by the Securities Data Company.
[c] INTERNET: stocks that appear in the portfolio of 400 Internet "pure plays" compiled by Ofek and Richardson (2001).
[d] ADRs

[10] This table is originally reported in D'Avolio (2002), page 287.

References

Bakshi, Gurdip, Charles Cao, and Zhiwu Chen , 1997, Empirical performance of alternative option pricing models, *Journal of Finance* 52, 2003–2049.

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–659.

Bodurtha Jr., James N., and Georges R. Courtadon, 1986, Efficiency tests of the foreign currency options market, *Journal of Finance* 41, 151–162.

Bollen, Nicolas, and Robert E. Whaley, 2004, Does net buying pressure affect the shape of implied volatility functions?, *Journal of Finance* 59, 711−753.

Boyle, Phelim P., and Ton Vorst, 1992, Option replication in discrete time with transaction costs, *Journal of Finance* 47, 271–293.

Bris, Arturo, William N. Goetzmann, and Ning Zhu, 2003, Efficiency and the bear: Short sales and markets around the world, Working paper.

Cox, John C., Stephen A. Ross, and Mark Rubinstein, 1979, Option pricing: A simplified approach, *Journal of Financial Economics* 7, 229–264.

Davis, Mark H., Vassilios G. Panas, and Thaleia Zariphopoulou, 1993, European option pricing with transaction costs, *SIAM Journal of Control and Optimization* 31, 470−493.

D'Avolio, Gene, 2002, The market for borrowing stock, *Journal of Financial Economics* 66, 271–306.

Diamond, Douglas, and Robert Verrecchia, 1987, Constraints on short-selling and asset price adjustment to private information, *Journal of Financial Economics* 18, 277–311.

Duffie, Darrell, Nicolae Garleanu, and Lasse Pedersen, 2002, Securities lending, shorting and pricing, *Journal of Financial Economics* 66, 307–339.

Dumas, Bernard, Jeff Fleming, and Robert E. Whaley, 1998, Implied volatility functions: Empirical tests, *Journal of Finance* 53, 2059–2106.

Geczy, Christopher, David Musto, and Adam Reed, 2002, Stocks are special too: An analysis of the equity lending market, *Journal of Financial Economics* 66, 241–269.

Evans, Richard B., Christopher Geczy, David Musto, and Adam Reed, 2006, Failure is an option: Impediments to short selling and option prices, *Review of Financial Studies*, Forthcoming.

Hodges, Stewart D., and Anthony Neuberger, 1989, Optimal replication of contingent claims under transaction costs, *The Review of Futures Markets* 8, 222–239.

Hong, Harrison, and Jeremy Stein, 2003, Differences of opinion, short sales constraints and market crashes, *Review of Financial Studies* 16, 487–525.

Ingersoll, Jonathan E., 1987, *Theory of financial decision making*, Totowa, NJ: Rowman & Littlefield.

Jones, Charles M., and Owen A. Lamont, 2002, Short-sale constraints and stock returns, *Journal of Financial Economics* 66, 207–239.

Kamara, Avraham, and Thomas Miller, 1995, Daily and intraday tests of European put–call parity, *Journal of Financial and Quantitative Analysis* 30, 519–539.

Klemkosky, Robert, and Bruce Resnick, 1979, Put–call parity and market efficiency, *Journal of Finance* 34, 1141–1155.

Klemkosky, Robert, and Bruce Resnick, 1980, An ex ante analysis of put-call parity, *Journal of Financial Economics* 8, 363–378.

Lamont, Owen, and Richard Thaler, 2003, Can the market add and subtract? Mispricing in tech stock carve-outs, *Journal of Political Economy* 111, 227–268.

Leland, Hayne E., 1985, Option pricing and replication with transaction costs, *Journal of Finance* 40, 1283–1301.

Merton, Robert C., 1973, The theory of rational option pricing, *Bell Journal of Economics* 4, 141–183.

Miller, Edward M., 1977, Risk, uncertainty, and divergence of opinion, *Journal of Finance* 32, 1151–1168.

Nisbet, Mary, 1992, Put–call parity theory and an empirical test of the efficiency of the London–traded options market, *Journal of Banking & Finance* 16, 381–403.

Ofek, Eli, and Matthew Richardson, 2003, DotCom mania: The rise and fall of internet stock prices, *Journal of Finance* 58, 1113–1138.

Ofek, Eli, Matthew Richardson, and Robert Whitelaw, 2004, Limited arbitrage and short sales restrictions: Evidence from the options markets, *Journal of Financial Economics* 74, 305–342.

Stoll, Hans, 1969, The relationship between put and call option prices, *Journal of Finance* 24, 319–332.

Whaley, A. E., and Paul Wilmott, 1997, An asymptotic analysis of an optimal hedging model for option pricing with transaction costs, *Mathematical Finance* 7, 307–324.

Table 1
Descriptive statistics

Table 1 reports the descriptive statistics of the sample after filtering out the undesirable quotes. The sample used here comprises selected negative rebate stocks from April 2000 through September 2001 that originally appeared in D'Avolio (2002). We collect the relevant American option price quotes from IVY OptionMetrics. The sample is made up of 4,712 quotes. The variables T and Δ are the days to expiry, and the delta of an option, respectively. The implied volatility, σ, which is also provided by OptionMetrics, is calculated by using the Cox, Ross and Rubinstein (1979) binomial framework. The variable *OI* is the daily open interest of options. The subscript refers to the type of option. *EEP* (in percent) is the early exercise premium of the American put option over the European equivalent divided by the stock price.

	T	Δ_{call}	Δ_{put}	σ_{call} (%)	σ_{put} (%)	OI_{call}	OI_{put}	*EEP* (%)
Mean	109.85	51.71	-45.57	84.91	107.92	1103	773	0.81
Minimum	10.00	6.49	-95.93	14.17	22.50	1	1	0.06
25th pctl	51.00	32.10	-61.14	63.35	79.10	103	44	0.37
Median	101.00	51.05	-45.23	81.39	106.86	306	143	0.68
75 pctl	173.00	71.78	-29.31	103.46	128.30	961	453	1.12
Maximum	240.00	99.08	-4.70	188.96	223.21	32747	29150	3.81
Std dev	61.65	24.10	20.33	31.14	36.65	2715	2298	0.57
Skewness	0.08	0.03	-0.11	0.40	0.28	7	7	1.29
Kurtosis	1.76	1.93	2.14	2.56	2.57	62	65	5.19

Table 2

Moneyness and expiry groups

Table 2 defines the moneyness and expiry groups of pairs of call and put options in the sample. Moneyness of an option pair is defined as the delta of a call option, Δ_{call}. Option pairs with moneyness below 0.02 or above 0.98 are excluded.

Group	*Labels*	*Range*
Panel A: Moneyness group		
1	Deep in-the-money call Deep out-of-the-money put	$0.875 < \Delta_{call} \leq 0.98$
2	In-the-money call Out-of-the-money put	$0.625 < \Delta_{call} \leq 0.875$
3	At-the-money call At-the-money put	$0.375 < \Delta_{call} \leq 0.625$
4	Out-of-the-money call In-the-money put	$0.125 < \Delta_{call} \leq 0.375$
5	Deep out-of-the-money call Deep in-the-money put	$0.02 < \Delta_{call} \leq 0.125$
Panel B: Expiry group		
1	Nearest term to expiry	$10 \leq T < 60$
2	Short term to expiry	$60 \leq T < 120$
3	Medium term to expiry	$120 \leq T < 180$
4	Long term to expiry	$180 \leq T < 240$

Table 3

Distribution of seeming violations of the put–call parity on selected negative rebate stocks

Table 3 reports the distribution of seeming violations of put–call parity of negative rebate stocks for each group of time to expiry of options. The data used here are selected negative rebate stocks from April 2000 through September 2001 that originally appeared in D'Avolio (2002). The variables S^S, S^M and S^L are the implied short stock price, average implied stock price, implied long stock price derived by the options market defined as:

$$S^S = C^{bid} - P^{offer} + PV(K) + EEP$$
$$S^M = C^{mid} - P^{mid} + PV(K) + EEP$$
$$S^L = C^{offer} - P^{bid} + PV(K) + EEP$$

PV is the present value operator, *EEP* is the early exercise premium of the put option, and S is the underlying stock price. We refer to observations where $S > S^L$ as seeming violations since it is theoretically impossible due to the unpredictable costs of short selling to make riskless profits by short selling the stock and simultaneously buying the implied stock. Below the distribution, the mean of the log deviations of the stock price from the implied long stock price, $100Ln(S/S^L)$, is reported for each expiry group. The standard errors appear in parentheses, and all the means of log deviations of stock prices from the implied long stock prices are significant at the 1% confidence level.

	Expiry group				
	1	*2*	*3*	*4*	*All*
Obs	1330	1450	1036	892	4712
$S < S^S$	0.83	0.34	0.29	0.34	0.47
$S^S \leq S < S^M$	5.49	5.59	3.57	1.57	4.35
$S^M \leq S \leq S^L$	26.39	20.41	18.63	18.95	21.41
$S > S^L$	67.29	73.66	77.51	79.15	73.77
$100Ln(S/S^L)$	0.68*	2.70*	4.04*	5.84*	3.02*
	(0.05)	(0.10)	(0.18)	(0.22)	(0.07)

* Significant at the 1% confidence level.

Table 4

Implied volatility discrepancies of negative rebate stocks across expiry

Table 4 reports the implied volatility discrepancies (in percent) of individual stocks across expiry groups, the range of which is defined in Table 2. We define the implied volatility discrepancy of an option pair, σ_{put} - σ_{call}, as implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option with the same underlying stock, time to expiry, and strike price. The data used comprise selected negative rebate stocks from April 2000 through September 2001 that originally appeared in D'Avolio (2002).

			Expiry group				
Ticker	*Fee*	*Obs*	*1*	*2*	*3*	*4*	*All*
GM	63	25	3.62				3.62
KREM	55	362	19.17	25.79	20.54	28.73	24.21
TOT	55	4	-6.70				-6.70
STLW	50	396	38.30	71.48	84.43	82.59	67.10
UN	46	18	10.30	0.56	-1.59		3.09
MCDT	40	354	13.01	23.55	29.36	31.73	23.72
PLCE	35	194	31.03	25.49	34.80	32.05	31.87
PALM	35	547	1.56	1.07	0.91	1.84	1.35
PLMD	35	274	7.41	12.47	10.12	14.48	10.97
PPD	35	197	30.08	28.18	29.91		29.68
RD	35	3	0.61				0.61
ABX	27	1	7.77				7.77
BCE	26.5	2	7.21				7.21
COH	25	100	12.95	3.21	3.87		7.07
HAND	25	382	8.97	13.35	14.93	12.67	12.48
NOK	25	11	-2.26				-2.26
AREM	22	338	31.23	29.76	34.76	31.51	31.13
ERICY	20.5	44	-8.33	-0.25	-1.61	-2.09	-1.49
PCL	20	32	12.62	15.70			13.19
RAD	20	63	39.18	8.26	17.42	17.79	17.27
SFP	20	16	-38.48	6.15	12.26		4.39
LNUX	20	79	8.12	7.39	11.48	14.41	9.96
MSTR	15	825	17.57	35.50	34.42	41.97	32.26
WBVN	14	40		-19.81	-5.55	11.26	2.26
SWZA	12.5	178	11.41	8.56	12.60		11.55
RETK	10	86	4.50	5.27	4.99	9.40	5.76
WEBM	10	141	6.84	6.03	9.94	12.30	8.81
All		4712	16.72	24.86	24.01	28.26	23.01

Table 5

Average implied volatility discrepancies across expiry

Table 5 reports the average implied volatility discrepancies (in percent) across expiry groups, the range of which is defined in Table 2. We define the implied volatility discrepancy of an option pair, σ_{put} - σ_{call}, as implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option with the same underlying stock, time to expiry, and strike price. Panel B lists the mean values after being adjusted for unequal sample size among stocks while panel A shows the unadjusted mean values. The data used here are made up of selected negative rebate stocks from April 2000 through September 2001 that originally appeared in D'Avolio (2002). Standard errors appear in parentheses, and all implied volatility discrepancies are significant at the 1% confidence level.

	Expiry group				
	1	*2*	*3*	*4*	*All*
Panel A: Unadjusted mean values					
Obs	1330	1450	1036	892	4712
σ_{call}	88.74	88.15	74.19	86.35	84.91
σ_{put}	105.47	113.01	98.20	114.61	107.92
σ_{put} - σ_{call}	16.72*	24.86*	24.01*	28.26*	23.01*
	(0.41)	(0.56)	(0.67)	(0.79)	(0.3)
Panel B: Mean values after being adjusted for unequal sample sizes among stocks					
Obs	26	21	20	15	27
σ_{call}	71.57	75.73	73.16	82.87	67.46
σ_{put}	81.86	90.38	91.06	105.58	80.68
σ_{put} - σ_{call}	10.30*	14.65*	17.90*	22.71*	13.22*
	(3.1)	(3.52)	(3.89)	(3.97)	(2.97)

* Significant at the 1% confidence level.

Table 6

Implied volatility discrepancies of negative rebate stocks across moneyness

Table 6 reports the implied volatility discrepancies (in percent) of individual stocks across moneyness groups, the range of which is defined in Table 2. We use the delta of the call option as the measure of moneyness of an option pair. We define the implied volatility discrepancy of an option pair, $\sigma_{put} - \sigma_{call}$, as implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option with the same underlying stock, time to expiry, and strike price. The data used here are made up of selected negative rebate stocks from April 2000 through September 2001 that originally appeared in D'Avolio (2002).

			Moneyness group					
Ticker	*Fee*	*Obs*	*1*	*2*	*3*	*4*	*5*	*All*
GM	63	25	7.09	2.99	3.73	2.33	7.15	3.62
KREM	55	362	30.97	24.51	23.69	22.37	24.83	24.21
TOT	55	4		-14.19	-5.72	-3.44		-6.70
STLW	50	396	68.36	64.29	65.53	70.28	74.76	67.10
UN	46	18		0.92	1.22	9.85		3.09
MCDT	40	354	35.46	22.27	20.21	22.47	31.40	23.72
PLCE	35	194		30.81	29.64	32.02	34.80	31.87
PALM	35	547	-1.24	0.92	1.25	1.31	3.05	1.35
PLMD	35	274		14.21	9.86	9.42	7.89	10.97
PPD	35	197		29.08	28.23	29.85	38.06	29.68
RD	35	3			0.61			0.61
ABX	27	1				7.77		7.77
BCE	26.5	2				7.21		7.21
COH	25	100		6.56	6.40	5.79	34.22	7.07
HAND	25	382	18.12	13.26	10.33	11.70	19.30	12.48
NOK	25	11		-2.87	-1.47	-2.50		-2.26
AREM	22	338		26.17	27.09	33.11	49.19	31.13
ERICY	20.5	44		-1.85	-2.30	-0.59		-1.49
PCL	20	32		13.96	12.60	13.12	15.97	13.19
RAD	20	63		15.15	10.55	19.55		17.27
SFP	20	16			-0.62	13.72	17.96	4.39
LNUX	20	79			8.40	10.07	12.65	9.96
MSTR	15	825	49.30	36.06	29.50	26.66	20.96	32.26
WBVN	14	40		3.28	2.15			2.26
SWZA	12.5	178		16.08	8.95	9.93	16.18	11.55
RETK	10	86		5.01	5.88	5.76		5.76
WEBM	10	141	13.16	8.55	8.24	9.34	9.79	8.81
Mean		4712	36.57	25.72	19.64	20.96	26.56	23.01

Table 7

Average implied volatility discrepancies across moneyness

Table 7 reports the average implied volatility discrepancies (in percent) across moneyness groups, the range of which is defined in Table 2. We use the delta of an option pair as the measure of moneyness. We define the implied volatility discrepancy of an option pair, $\sigma_{put} - \sigma_{call}$, as implied volatility calculated from the midpoint of the bid–offer spread of a put option minus that of a call option with the same underlying stock, time to expiry, and strike price. Panel B lists the mean values after being adjusted for unequal sample sizes among stocks while panel A shows the unadjusted mean values. The data used here are made up of selected negative rebate stocks from April 2000 through September 2001 that originally appeared in D'Avolio (2002). Standard errors appear in parentheses, and all implied volatility discrepancies are significant at the 1% confidence level.

	Moneyness category					
	1	*2*	*3*	*4*	*5*	*All*
Panel A: Unadjusted mean values						
Obs	197	1311	1525	1309	365	4712
σ_{call}	94.27	89.25	84.38	82.67	75.20	84.91
σ_{put}	130.83	114.97	104.02	103.63	101.76	107.92
$\sigma_{put} - \sigma_{call}$	36.57*	25.72*	19.64*	20.96*	26.56*	23.01*
	(1.57)	(0.60)	(0.48)	(0.53)	(1.20)	(0.30)
Panel B: Mean values after being adjusted for unequal sample sizes among stocks						
Obs	8	22	25	25	17	27
σ_{call}	79.41	71.14	70.07	67.09	68.01	67.46
σ_{put}	107.06	85.47	82.23	81.77	92.61	80.68
$\sigma_{put} - \sigma_{call}$	27.65*	14.33*	12.16*	14.68*	24.60*	13.22*
	(4.46)	(3.23)	(2.96)	(3.00)	(3.46)	(2.97)

* Significant at the 1% confidence level.

Table 8

Regression result of implied volatility discrepancy

Table 8 reports regression results of the implied volatility discrepancy (in percent) on time to expiry, moneyness, and the square of moneyness for each negative rebate stock. For the aggregate sample we regress the implied volatility discrepancy on the rate of lending fees after controlling for the effect of time to expiry, moneyness and the square of moneyness with the following regression equation:

$$IVD_i = \beta_0 + \beta_1 \times Fee_i + \beta_2 \times Expiry_i + \beta_3 \times Moneyness_i + \beta_4 \times Moneyness_i^2 + \varepsilon_i$$

We use the delta of the call option as the measure of moneyness of an option pair. The data used here are made up of selected negative rebate stocks from April 2000 through September 2001 that originally appeared in D'Avolio (2002). The standard errors appear in parentheses.

			Independent variables					
Ticker	*Fee*	*Obs*	*Intercept*	*Fee*	*Expiry*	*Moneyness*	*Moneyness*^{2}	R^2
AREM	22.0	338	0.2191*		-0.0137	-0.5308*	0.7179*	0.5976
			(0.0002)		(0.0006)	(0.0079)	(0.0058)	
PLCE	35.0	194	0.0550*		0.2100*	-0.5071*	0.4141*	0.4954
			(0.0000)		(0.0003)	(0.0047)	(0.0028)	
HAND	25.0	382	0.0739*		0.1187*	-0.5478*	0.5282*	0.5901
			(0.0000)		(0.0001)	(0.0008)	(0.0009)	
KREM	55.0	362	0.0506*		0.1906*	-0.5164*	0.5052*	0.3747
			(0.0000)		(0.0003)	(0.0014)	(0.0014)	
MCDT	40.0	354	0.0658*		0.4796*	-0.7122*	0.7224*	0.7350
			(0.0000)		(0.0004)	(0.0023)	(0.002)	
MSTR	15.0	825	0.0299*		0.6525*	-1.2515*	0.9678*	0.7312
			(0.0000)		(0.0004)	(0.0026)	(0.0024)	
PALM	35.0	547	0.0047**		0.0090	-0.0613*	0.0892*	0.1848
			(0.0000)		(0.0001)	(0.0005)	(0.0004)	
PLMD	35.0	274	0.1630*		0.1072*	-0.5688*	0.4366*	0.1726
			(0.0005)		(0.0011)	(0.0148)	(0.0125)	
PPD	35.0	197	0.1314*		0.0125*	-0.4216*	0.4462*	0.3930
			(0.0000)		(0.0007)	(0.0034)	(0.0027)	
STLW	50.0	396	0.0583*		1.2005*	-0.3535*	0.6266*	0.8424
			(0.0000)		(0.0011)	(0.004)	(0.0033)	
WEBM	10.0	141	0.0642*		0.1149*	-0.3667*	0.3177*	0.6644
			(0.0000)		(0.0001)	(0.0012)	(0.001)	
All		4712	0.0131*	0.3587*	0.2288*	-0.5254*	0.5205*	0.2259
			(0.0000)	(0.0000)	(0.0001)	(0.0005)	(0.0004)	

* Significant at the 1% confidence level.

** Significant at the 5% confidence level.

Figure 1

The convergence of option price under short sales constraints

Four option quotes—call offer, call bid, put offer, and put-bid—are calculated in a discrete time framework similar to that of Cox, Ross Rubinstein (1979) by repeatedly constructing a portfolio of (D_0, B_0) from the following two equations:

$$uSD_0 + B_0R = uSD_1 + B_1 - k_L \Delta t D_0 S$$

$$dSD_0 + B_0R = dSD_2 + B_2 - k_L \Delta t D_0 S$$

D_0 and B_0 are the number of shares and riskless bonds at the initial time, (D_1, B_1) and (D_2, B_2) are those in up-state and down-state, respectively, and k_L is the rate of lending fees. The symbols "+," "□," "*," and "o" represent the offer price of a call option, the bid price of a call option, the offer price of a put option, and the bid price of a put option, respectively. The implied volatilities are calculated by using the Black-Scholes model and are presented to compare the relative level of the four prices. Parameters are as follows: initial stock price = 100, strike price = 100, riskless interest rate = 5%, time to maturity = 0.5 years, volatility = 40%, the rate of lending fees = 5%, and the number of time steps = 250.

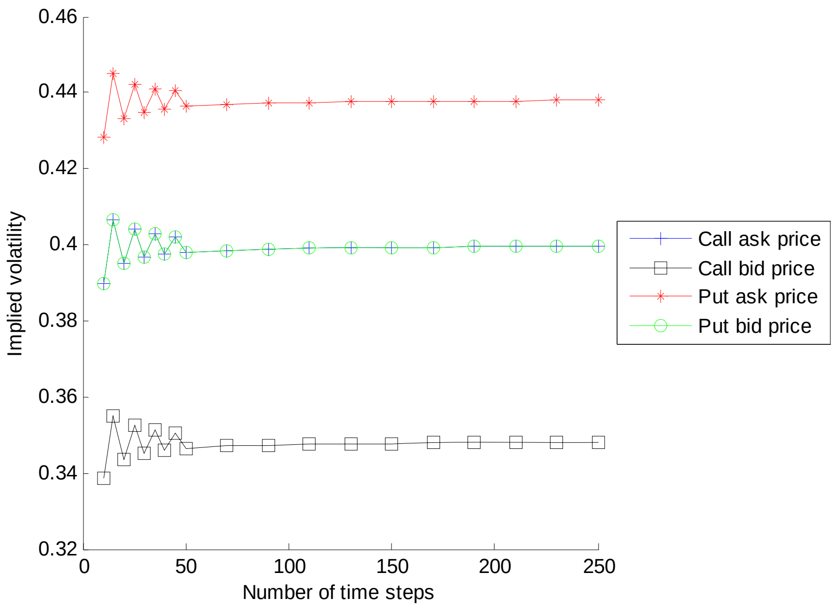


Figure 2

Implied volatility across the rate of lending fees under short sales constraints

Four option quotes—call offer, call bid, put offer, and put bid—are calculated in a discrete time framework similar to that of Cox, Ross Rubinstein (1979) by repeatedly constructing a portfolio of (D_0, B_0) from the following two equations:

$$uSD_0 + B_0 R = uSD_1 + B_1 - k_L \Delta t D_0 S$$

$$dSD_0 + B_0 R = dSD_2 + B_2 - k_L \Delta t D_0 S$$

D_0 and B_0 are the number of shares and riskless bonds at the initial time, (D_1, B_1) and (D_2, B_2) are those in up-state and down-state, respectively, and k_L is the rate of lending fees. The symbols "+," "□," "*," and "o" represent the offer price of a call option, the bid price of a call option, the offer price of a put option, and the bid price of a put option, respectively. The implied volatilities are calculated by using the Black-Scholes model and are presented to compare the relative level of the four prices. Parameters are as follows: initial stock price = 100, strike price = 100, riskless interest rate = 5%, time to maturity = 0.5 years, volatility = 40%, the rate of lending fees = 5%, and the number of time steps = 250.

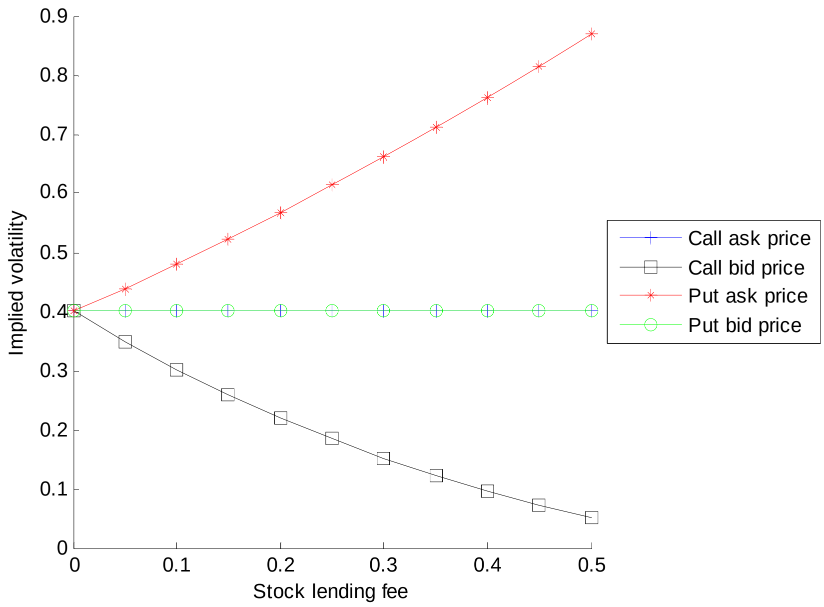


Figure 3

Implied volatility across time to expiry under short sales constraints

Four option quotes—call ask, call bid, put ask, and put bid—are calculated in a discrete time framework similar to that of Cox, Ross Rubinstein (1979) by repeatedly constructing a portfolio of (D_0, B_0) from the following two equations:

$$uSD_0 + B_0 R = uSD_1 + B_1 - k_L \Delta t D_0 S$$

$$dSD_0 + B_0 R = dSD_2 + B_2 - k_L \Delta t D_0 S$$

D_0 and B_0 are the number of shares and riskless bonds at initial time, (D_1, B_1) and (D_2, B_2) are those in up-state and down-state, respectively, and k_L is the rate of lending fees. The symbols "+," "□," "*," and "o" represent the offer price of a call option, the bid price of a call option, the offer price of a put option, and the bid price of a put option, respectively. The implied volatilities are calculated by using the Black-Scholes model and are presented to compare the relative level of the four prices. Parameters are as follows: initial stock price = 100, strike price = 100, riskless interest rate = 5%, time to maturity = 0.5 years, volatility = 40%, the rate of lending fees = 5%, and the number of time steps = 250.

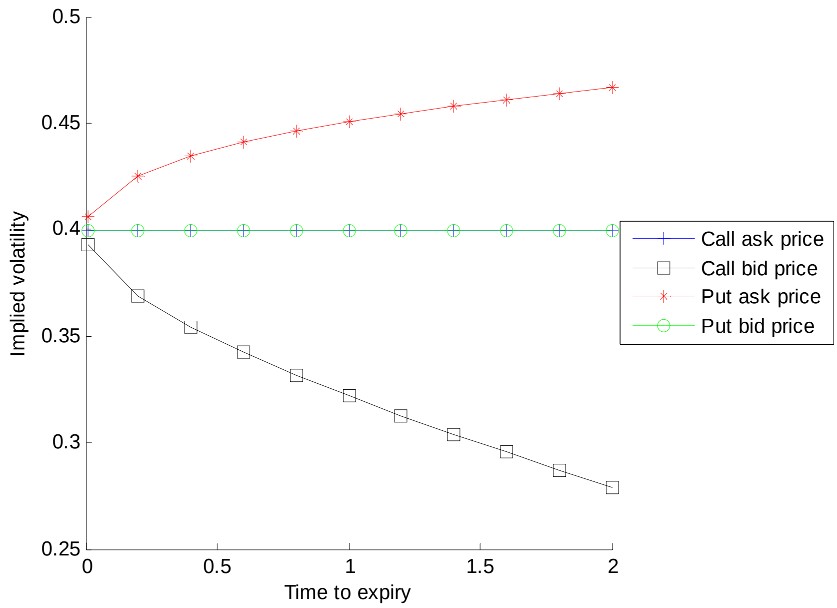


Figure 4

Implied volatility and implied volatility discrepancy across moneyness under short sales constraints

Four option quotes—call ask, call bid, put ask, and put bid—are calculated in a discrete time framework similar to that of Cox, Ross Rubinstein (1979) by repeatedly constructing a portfolio of (D_0, B_0) from the following two equations:

$$uSD_0 + B_0R = uSD_1 + B_1 - k_L\Delta t D_0 S$$

$$dSD_0 + B_0R = dSD_2 + B_2 - k_L\Delta t D_0 S$$

D_0 and B_0 are the number of shares and riskless bonds at initial time, (D_1, B_1) and (D_2, B_2) are those in up-state and down-state, respectively, and k_L is the rate of lending fees. The symbols "+," "□," "*," and "o" represent the offer price of a call option, the bid price of a call option, the offer price of a put option, and the bid price of a put option, respectively. The implied volatilities are calculated by using the Black-Scholes model and are presented to compare the relative level of the four prices in (a). We use the delta of an option pair as the measure of moneyness. The implied volatility discrepancy of an option pair, σ_{put} - σ_{call}, shown in (b) is defined as the implied volatility of a put option minus that of a call option with the same underlying stock, time to expiry, and strike price. Parameters are as follows: initial stock price = 100, riskless interest rate = 5%, time to maturity = 0.5 years, volatility = 40%, the rate of lending fee = 5%, and the number of time steps = 250.