Systematic Risks in the Options Market: Evidence from S&P 500 Index Options

Jaewon Park[*] and Tong S. Kim[†]

Abstract

The assumption of dynamic replication in no-arbitrage option pricing models does not hold in practice due to discreteness of trading hours as well as trading costs, which suggests the potential presence of preference-driven risk premiums. In this paper, we empirically show that discretely hedged S&P 500 index option portfolios are exposed to covariance and coskewness risk with the market portfolio. Using the three-moment CAPM of Kraus and Litzenberger (1976), we find that the rate of return of the portfolio significantly loads on the two risk factors, and their risk premiums are significantly positive. The equilibrium model complements the prevailing approach of the no-arbitrage framework, and reveals that the volatility smile is linked to investors' preference on the unhedged market risks.

JEL classification: G12

Keywords: Discrete hedge; Coskewness; Three-moment capital asset pricing model; Volatility skew

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[*] Corresponding author, Graduate School of Finance, Korea Advanced Institute of Science and Technology, 87 Heogiro, Dongdaemoon-gu, Seoul, Korea, 130-722, Tel: 82-2-958-3689, Fax: 82-2-958-3604, E-mail: rouggh@business.kaist.ac.kr

[†] Graduate School of Finance, Korea Advanced Institute of Science and Technology, 87 Heogiro, Dongdaemoon-gu, Seoul, Korea, 130-722, Tel: 82-2-958-3018, Fax: 82-2-958-3604, E-mail: tskim@business.kaist.ac.kr

Options are redundant in the sense that they can be dynamically replicated by the underlying asset and riskless bonds. In their seminal paper, Black and Scholes (1973) first show that, to preclude arbitrage opportunities, the value of an option should be identical to the cost of dynamic replication. Since then, the no-arbitrage option pricing framework of Black and Scholes has been widely accepted by both academics and practitioners. One challenge to the Black-Scholes model is the well-documented volatility smile. For example, Dumas, Fleming, and Whaley (1998) report that, after the October 1987 market crash, the implied volatility function of S&P 500 options is negatively sloped across the strike price. To improve upon this weakness in the Black-Scholes model, a series of no-arbitrage option pricing models are developed.[1] These option pricing models improve the Black-Scholes model by assuming more realistic stochastic processes of the underlying asset such as stochastic volatility or jumps.

However, many empirical tests reveal that no-arbitrage option pricing models are not very successful in explaining the volatility smile, nor are they consistent with one another. Bakshi, Cao and Chen (1997) report that while incorporating stochastic volatility and jumps can improve pricing performance on S&P 500 options, the fitted parameters take on rather unrealistic values. Chernov and Ghysels (2000) find a small negative correlation estimated from S&P 500 index and its options using the stochastic volatility model, while Bakshi, Cao and Chen (1998) and Pan (2002) report that the correlation is extremely negative when analyzing different sample periods with the same model. Using S&P 500 futures options data in the post-crash period, Bates (2000) finds that the inclusion of a jump component in the stochastic volatility model can improve pricing performance, but the parameters are implausible to rationalize the observed data.

Recently, Bollen and Whaley (2004) present evidence that, while movement of option price may not be fully understood using no-arbitrage framework, supply and demand can play a role in determining the option price. Under excessive net buying pressure, the supply curve in the options market is not horizontal since writing an option incurs various hedging costs and volatility risk exposure. They empirically show that price movement of options are related to the

[1] See Cox and Ross (1976) for the constant-elasticity-of-variance model, Merton (1976), Ball and Torous (1983, 1985), Bates (1991) for the jump diffusion model, Hull and White (1987), Scott (1987), Melino and Turnbull (1990, 1995), Stein and Stein (1991), Heston (1993) for the stochastic volatility model, Bates (1996, 2000), Scott (1997) for the stochastic volatility and jump diffusion model, and Bates (2000), Duffie, Pan, and Singleton (2000), and Eraker, Johannes and Polson (2003) for the jump in volatility model.

net buying pressure, and argue that limits to arbitrage of dynamic replication can permit the relation. It is astonishing that the options market works at least partially in the direction of supply and demand like other equilibrium markets.

If Bollen and Whaley (2004) are correct, then the next step is to determine what drives supply and demand in the options market. Can an equilibrium model that depends on investor preference be applied to the options market? We focus on the point that the dynamic replication of options cannot be successful in practice since the assumptions of Black and Scholes (1973) do not hold in the real world. More specifically, perfect dynamic replication is impossible due to discreteness of trading hours as well as enormous transaction costs. Thus, the discretely hedged option position is no longer risk-free, and the options market is incomplete—particularly at night, during the window from the market close to market open. As Bollen and Whaley point out, "the results of these limits to arbitrage is that market prices can diverge from model values, and the no-arbitrage band within which prices can fluctuate can be quite wide, allowing price to be affected by supply and demand considerations (p713)."

In this paper, we show that the returns of calls, puts, and zero-beta straddles on S&P 500 index can be explained in an extended capital asset pricing framework in which not only variance of investor's portfolio but also skewness and kurtosis are taken into account. Also, we empirically test whether average returns of discretely hedged option portfolios can be explained by systematic risks. The systematic risks are covariance and coskewness with the excess market return. Under the assumption of no-arbitrage models, betas to the excess market return (covariance risk) and its square (coskewness risk) must be zero. If it is found that the average returns reflect the appropriate premiums on the covariance and coskewness risks, we can conclude that the option prices are not preference-free and the equilibrium model complements the prevailing approach of the no-arbitrage framework. Testing this hypothesis is also important because the shape of the implied volatility curve is directly related to the cross-sectional variation in average hedge returns; thus, we indirectly test the hypothesis that the negatively sloped implied volatility curve is linked to investor preference on the systematic risks.

To examine the distributional characteristics of a discrete delta hedge, we perform an empirical simulation using S&P 500 index options. Specifically, we construct a hedge portfolio that consists of writing an option on the market portfolio and buying or selling the delta shares of the underlying asset as a hedge. Since we do not know the exact diffusion process that the underlying stock price follows, the delta values are computed using three alternative models: the

Black-Scholes model, the stochastic volatility model of Heston (1993) and the jump diffusion model of Bates (1991). The hedge portfolio is rebalanced at multiple discrete time intervals over the life of the option. At the expiration of the option, the final payoff of the option is subtracted from the value of the hedge portfolio. The rate of return on the hedge portfolio, referred to as "hedge return," can be interpreted as the deviation from perfect replication.

We find that standard deviation and skewness of the simulated hedge return systematically deviate from zero. More surprisingly, the hedge return covaries with the excess market return and the squared excess market return, strongly depending on the strike price. The hedge portfolios of out-of-the-money options show a higher covariance than those of in-the-money options, and the hedge portfolios of high strike options show a higher coskewness than those of low strike options.

If discretely hedged option portfolios are exposed to covariance and coskewness risks, investors in the options market consider those risks and the option price should reflect an appropriate risk premium. According to the three-moment capital asset pricing model (CAPM hereafter) proposed by Kraus and Litzenberger (1976), an asset with positive covariance or negative coskewness with the market portfolio must provide a positive risk premium. Harvey and Siddique (2000) conduct an empirical analysis on the stock market using the three-moment CAPM, and Dittmar (2002) further extend the standard CAPM by considering cokurtosis of assets.

We apply the three-moment CAPM using the standard two-step regression procedure, which consists of a time-series regression and a cross-sectional regression. For empirical purposes, we propose the Negative coskewness Minus Positive coskewness (NMP) portfolio return as the factor that captures coskewness risk. The NMP portfolio return is defined as the difference in returns between the hedge portfolios of put options with the lowest strikes that exhibit negative coskewness and the hedge portfolios of call options with the highest strikes that exhibit positive coskewness. Thus, the NMP portfolio return represents the spread of risk premiums between a portfolio that is subtracting skewness from a broader portfolio and a portfolio that is adding skewness.

The time-series regression results show that the hedge returns of S&P 500 index options significantly load on the excess market return and the NMP portfolio return. From the cross-sectional regression, the covariance and coskewness risks command premiums of 5.8% and 2.5%, respectively, when the Black-Scholes model is used to compute delta values of one-month-to-expiration options. The risk premiums are also statistically significant when the stochastic volatility model or the jump diffusion model is used. Various goodness-of-fit tests confirm that the excess market return and the NMP portfolio return can explain the cross-sectional variation in average hedge returns of S&P 500 index options.

The rest of the paper is organized as follows. In Section I, we describe the sample data and the empirical distribution of hedge portfolios. Section II presents the related three-moment asset pricing framework. Section III reports the results of the time-series regression and the cross-sectional regression. Section IV offers some concluding remarks and direction for future research.

I. Sample Description

The purpose of this section is to describe the option sample used in this research and to present the general characteristics of both unhedged and (discretely) hedged option returns during the sample period.

A. Data

In this research, we use options on the S&P 500 index from IVY OptionMetrics. The sample is focused on the most recent decade from January 1996 to September 2008. We choose S&P 500 index options because they are the most actively traded European options on a proxy of the market portfolio. The options with time to maturity longer than 30 days and shorter than 100 days are collected once a week, mostly on Fridays at the market close. The related S&P 500 index price, its dividend yield and the risk-free interest rate are drawn from the same source. To proxy the market portfolio, we use the NYSE/AMEX/NASDAQ value-weighted index.

To filter out undesirable data, we apply a set of screening rules. First, option quotes less than \$3/8 are dropped from our sample. These quotes may be too small relative to the minimum tick size to reflect the true value of options. Second, options with zero open interest are excluded from the sample; prices of low liquidity may show a deviation from the true value. Third, option quotes that violate the lower arbitrage bound are excluded since it is impossible to calculate implied volatility for these options. Finally, the options with an absolute delta below 5% or above 95% are excluded.

We categorize the sample into ten moneyness groups, five for each of call and put options.

Following Bollen and Whaley (2004), moneyness of an option is defined as its delta. The typical measure of moneyness that is the strike price divided by the spot price cannot assess the likelihood that an option in in-the-money at expiration since it heavily depends on volatility and time to expiration. On the contrary, delta values better account for the probability since it incorporates volatility and time to expiration. The delta value of each option is computed using the Black-Scholes model and the realized volatility over the most recent 60 trading days.

Table 1 presents description of the sample. Breakpoints of five moneyness groups are reported at the top of the table. For example, moneyness group 1 (M = 1) for call option is comprised of the options with delta value above 75% and below 95%. The mean delta value of the group is reported in the middle of Panel A, 86.14%. For both the call and put options, the lowest moneyness group (M = 1) corresponds to the options with the lowest strike prices, while the highest moneyness group (M = 5) corresponds to the options with the highest strike prices. Panel A shows the number of observations, average daily trading volume per contract, average delta value, average ratio of the strike price divided by the underlying index, average implied volatility (IV), and average implied volatility over realized volatility (RV) for each moneyness group of call options while Panel B presents those statistics for put options.

As shown in Table 1, for call options, the at-the-money options (M = 3) are the most actively traded with average daily trading volume per contract of 6,776, followed by the out-of-the-money options (M = 4). For put options, the out-of-the-money options (M = 1 and 2) show the greatest trading volume on average (9,353 and 8,699, respectively), followed by the at-the-money options. This is consistent with the option sample of Bollen and Whaley (2004) during the period from January 1995 through December 2000.

At the bottom of the each Panel, implied volatilities and realized volatilities are reported. The implied volatilities are computed from the midpoint of bid–ask spread at the market close using the Black-Scholes formula. The general shape of the implied volatility function across moneyness groups is negatively sloped and convex. This is similar to the implied volatility functions estimated in Dumas, Fleming and Whaley (1998) and Bakshi, Cao and Chen (1997). Also, the excess implied volatilities over realized volatilities are generally positive. Specifically, they are all positive for put options, while some are slightly negative for call options. Considering realized volatility as proxy of true volatility, S&P 500 index options in the sample period are overpriced relative to the Black-Scholes assumptions. This finding is also similar to Bollen and Whaley (2004). Therefore, the option sample from January 1996 through September

2008 is comparable to those from previous studies.

[Insert Table 1 about Here]

B. Returns of Calls, Puts, and Zero-Beta Straddles

Coval and Shumway (2001) theoretically show that a call option on market index should have a positive expected return that is increasing in the strike price. On the contrary, a put option, which usually protects market risks, should have an expected return below the risk-free rate and increasing in the strike price. Their empirical evidence using weekly SPX option returns from January 1990 to October 1995 and daily OEX option returns from January 1986 to October 1995 confirms their argument. However, they find that both call and put options earn far too low returns to be rationalized in the standard CAPM framework, suggesting that some factors besides market risk drive the risk premiums of option contracts.

Following Coval and Shumway (2001), we compute weekly returns of S&P 500 index call and put options and present their descriptive statistics in Panel A and B of Table 2, respectively. We record the mean, median, minimum, and maximum weekly returns for each of five moneyness groups. To fairly compare the option returns across five moneyness groups, the weeks when any of five moneyness groups have no observation are excluded. Since the options quotes of deep out-of-the-money options are frequently omitted during the sample period, only the options with absolute delta values between 35% and 75% are reported to prevent excessive exclusion. After the exclusion, the sample is reduced to 535 weeks in time-series. The returns are in weekly percentage forms, and the *t*-statistics test the null hypothesis that the returns are zero.

Looking at mean returns of S&P 500 index calls and puts in Panel A and B of Table 2, we see that call options earn positive average returns while put options have returns that are negative. For example, the at-the-money call options earn more than four percent per week on average, and average return of the at-the-money put options is almost -7.5% per week. Also, both call and put returns are increasing monotonically in the strike price, and they are all statistically significant at the one percent confident level. These results are in accordance with the argument of Coval and Shumway (2001). Not surprisingly, the median, minimum, and maximum statistics demonstrate a substantial degree of positive skewness.

Table 2 also includes the average Black-Scholes betas for each moneyness group of call and put options. We calculate the betas with the Black-Scholes formula as follows:

$$\beta_C = \frac{Se^{-qT}}{C} N\left[\frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right]\beta_S \tag{1}$$
$$\beta_P = -\frac{Se^{-qT}}{P} N\left[-\frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right]\beta_S$$

where N[·] is the cumulative normal distribution function, and β_S and q are the beta and the continuous dividend yield of the underlying index. We use the realized volatility over the past 60 trading days as a proxy of the true volatility. The Black-Scholes betas of both call and put options are, as expected, quite large and increasing across strike prices. The call betas are positive, ranging from 14.6 for the lowest strikes to 25.4 for the highest strikes. The put betas are negative and range from -16.8 to -14.8. However, considering the estimated betas, the option returns appear to be far too low to be consistent with the standard CAPM. Assuming market risk premium of 0.42% per week during the sample period, average return of the at-the-money call options should be around eight percent per week on average, while the at-the-money put options should lose -7.5% per week on average. Given their substantial comovement with the underlying index, call and put options might be expected to earn far greater returns than they actually do.

We also measure the weekly returns of S&P 500 zero-beta straddle positions across moneyness groups in Panel C of Table 2. The straddle positions are constructed by combining call and put options with weights that the betas of the positions are zero. Specifically, the weight θ of the straddle's value in call options is:

$$r_v = \theta r_c + (1-\theta) r_p \tag{2}$$
$$\theta\beta_c + (1-\theta)\beta_p = 0$$

where r_v, r_c, and r_p are the straddle return, the call return, and put return, respectively. We use the Black-Scholes betas computed from Equation (1) to identify the call and put betas in Equation (2). The zero-beta straddles are divided into five moneyness groups on the delta values of the call options in the positions.

According to the standard CAPM, zero-beta straddles should have expected returns equal to the risk-free rate. However, Panel C of Table 2 shows that average returns of zero-beta straddle positions are invariably negative. At-the-money straddles lose two percent per week on average with the lower strike straddles losing more. All the negative returns are highly statistically significant. These negative straddle returns are consistent with the evidence in Coval and Shumway (2001), and strongly suggest that there are additional factors that explain the risk premiums of option contracts.

Frequently mentioned risk factor other than market beta risk in the options market is volatility risk, which is additional risk factor in the stochastic volatility model such as Heston (1993). Since straddles have large, positive volatility betas, they allow investors to hedge volatility risk and, thus, are considered as a less risky asset. Thus, if there is a risk premium for volatility risk, expected returns of zero-beta straddles will be less than the risk-free rate. Recently Bakshi and Kapadia (2003) find that the delta-hedged S&P 500 index option portfolios significantly underperform zero and the underperformance is consistent with a negative volatility risk premium. A negative volatility risk premium implies that index options can play a role to hedge against the market portfolio. On the contrary, Branger and Schlag (2001) argue that tests based on option hedging errors cannot detect additional priced risk factors or identify the sign of their market prices of risk correctly due to discretization error and model mis-specification. Furthermore, many researches including Bakshi, Cao and Chen (1997, 1998), Chernov and Ghysels (2000), Pan (2002) report conflicting results on the stochastic volatility model. In the following sections, we propose additional risk factors in the options market rather than volatility risk and investigate these issues in greater details.

C. Discretely Hedged Option Portfolios

The main idea of Black and Scholes (1973) is that one can remove all of the portfolio's market risk by holding an option contract and continuously hedging it with the underlying asset. The expected return on the portfolio should be equal to the risk-free interest rate. In practice, however, continuous hedging is impossible due to discreteness of trading hours as well as enormous transaction costs. Consequently, the discretely hedged portfolio is no longer risk-free and the options market is incomplete, particularly overnight from market close to market open. In this sense, the assumption of dynamic replication in no-arbitrage option pricing models may not capture all the risk factors included in option contracts. In this subsection, we present empirical distribution of the rate of return on discretely hedged option portfolios.

We define a hedge portfolio as a portfolio of a short position in an option contract plus an appropriated amount of the underlying asset as a hedge. The appropriated amount is equal to the delta of the option. Constructing a hedge portfolio of call options involves writing a call option and buying the underlying stocks so that the hedge portfolio is delta-neutral. Similarly, a hedge portfolio of put options consists of short positions in a put option and the underlying stocks. The hedge portfolio is rebalanced at multiple discrete time intervals over the life of the option. The interval does not have to be equally spaced. We assume that the delta is revised at each trading day by changing the number of shares in the underlying stocks. At the expiration of the option, the final payoff of the option is subtracted from the value of the hedge portfolio. We define hedge return as the rate of return on the hedge portfolio as follows:

$$\textit{Hedge return} = \frac{\left(C_0 e^{rT} - C_T\right) + \sum_{t=0}^{T-1} \Delta_t \left(S_{t+1} + D_t - S_t\right) e^{r(T-t)}}{\left|-C_0 + \Delta_0 S_0\right|} \tag{3}$$

where S_t and C_t are the underlying stock price and option price on day t, respectively, D_t is the dividend payout from the underlying stock on day t, Δ_t is the delta of the option on day t and r is the risk-free interest rate that is assumed to be constant over the life over the option. The subscripts 0 and T represent the time when the option is written and expired, respectively. The terminal payoff of call option on day T is $C_T = max(S_T - K, 0)$, while that of put options is $C_T = max(K - S_T, 0)$.

The first term in the numerator of Equation (3) is the gain or loss on the pure option position. The option premium collected at the outset is assumed to be accrued at the risk-free interest rate until the expiration of the option. The second term is the accumulated mark-to-market gains and losses from adjusting the delta each day, which are also accrued at the risk-free interest rate day-by-day. The denominator is the cost of the position at inception. Note that the hedge return is in the form of an excess return since all the costs or proceeds are financed at the risk-free interest rate.

If revisions of delta occur continuously in time, the hedge return must be degenerated at zero since the hedging scheme is risk-free and completely financed at the risk-free interest rate. In practice, it is impossible to revise the hedge portfolio continuously since the trading hours are discontinuous from the market close to market open, and because the rebalances are costly. Thus, the moments of the hedge return can deviate from zero. In this sense, the hedge return can be interpreted as a measure of deviation from perfect replication.

We compute hedge returns of S&P 500 index options using Equation (3). The hedge return not only measures deviations incurred by discrete rebalancing, but also includes errors induced by possible misspecification of the option pricing model. Unfortunately, we do not know the true option pricing model nor decompose the hedge return into discrete rebalancing errors and model errors. Instead, we employ three different option pricing models to compute the hedge returns: the Black-Scholes model, the stochastic volatility model of Heston (1993) and the jump diffusion model of Bates (1991).

The Black-Scholes model is the simplest and most widely used by market practitioners, but it assumes constant volatility—an assumption that does not conform to evidence from the options market. According to Rubinstein (1994), Dumas, Fleming and Whaley (1998) and Ait-Sahalia and Lo (1998), after the 1987 crash, S&P 500 index options with low strike prices show significantly higher implied volatilities than those with high strike prices, which is referred to as the "volatility smile." Bakshi, Kapadia and Madan (2003) find that a negatively sloped implied volatility curve is related to a negative risk-neutral skewness. Alternatively, the stochastic volatility model and jump diffusion model assume a more complex and – hopefully – more realistic stochastic process for the underlying stock return. While the negatively sloped volatility skew frequently observed in the options market is an anomaly under the Black-Scholes model, advanced models can generate implied volatility function of various shapes and can potentially explain the volatility smile. By comparing the hedge returns computed using these three option pricing models, we could find the difference in hedging performance between the models.

Each option pricing model requires a set of its own parameter values. For the Black-Scholes model, the realized volatility is used to calculate the delta values. For the stochastic volatility model, five parameters are calibrated by minimizing the sum of squared differences (*SSD*) between the market implied volatilities and the model implied volatilities as follows:[2]

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[2] While Bakshi, Cao and Chen (1997) minimize difference in option prices, we minimize difference in implied volatilities. Since ITM options generally have higher prices than OTM options, the estimation

$$SSD(\kappa_v, \theta_v, \sigma_v, \rho, v_0) = \sum_{i=1}^{n} \left(IV_i^{Market} - IV_i^{Model}(\kappa_v, \theta_v, \sigma_v, \rho, v_0) \right)^2 \quad (4)$$

where IV^{Market} and IV^{Model} denote the market implied volatility and the model implied volatility, respectively, v_0 is the current variance rate of the underlying stock, and κ_v, θ_v, σ_v and ρ are the speed of reversion, long-run mean of the variance rate, volatility of variance rate, instantaneous correlation between the Brownian motions from the underlying index and the variance rate, respectively, and n is the number of options available in the options market at the time of calibration. The parameters of the jump diffusion model are estimated in the same way.

On each expiration date, the parameters of each model are estimated using all available options. The same parameter values are used to revise the portfolios for the remaining trading days until the options mature and the hedge portfolios are closed. We do this to fix the information set that investors in the options market have at the date when hedge portfolios are created. Table 3 presents the mean and standard deviation of parameter values. In Panel A, the average realized volatility is 16.32%, which is much less than the average implied volatility of 24.24%. In Panel B, the average parameter values of the stochastic volatility model, κ_v, θ_v, σ_v, ρ and $\sqrt{v_0}$ are, respectively, 5.43, 0.05, 0.58, -0.70 and 0.18. The estimates show values similar to the results of Bakshi, Cao and Chen (1997, 2000) except that the speed of mean reversion, κ_v, is quite high.[3] Panel C presents the estimated parameters of the jump diffusion model. The expected jump size, μ_J, is shown to be negative on average in accordance with the negatively sloped implied volatility curve. The jump frequency per year, λ_J, is estimated to be 3.06 on average.

Table 4 describes the distributional characteristics of empirical hedge returns of different option types, expiration groups and moneyness groups. The expiration group, T, consists of

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scheme employed by Bakshi, Cao and Chen assigns more weights to ITM options than OTM options. However, we are in favor of assigning an equal weight to each option since volatilities are all of comparable magnitude.

[3] The estimate of the speed of mean reversion is 1.15 for the period from June 1998 to May 1991 in Bakshi, Cao and Chen (1997) and 2.18 from March 1994 to August 1994 in Bakshi, Cao and Chen (2000). The difference between our estimate and theirs exists because we exclude options with terms to maturity longer than three months. However, the difference in the mean reversion estimate does not change our empirical results.

options with T months to expiration. Panels A, B and C refer to hedge portfolios where the Black-Scholes model, the stochastic volatility model, and the jump diffusion model are used to compute delta values, respectively. For each hedge portfolio, we present mean, standard deviation and skewness of hedge returns in the third, fourth, and fifth columns, respectively. The standard deviation of call hedge return increases with the moneyness group while that of put hedge return decreases with the moneyness group. Although skewness is not well ordered, the DOTM calls (M = 5) show positive skewness while the DOTM puts (M = 1) demonstrate negative skewness.

The sixth column presents the beta to the excess market return. The market beta is computed from a univariate time-series regression of hedge return on the excess market return. It is particularly interesting that most market betas are positive and statistically significant at the 5% confidence level. This confirms that, contrary to the notion of dynamic replication, daily rebalanced hedge portfolios are exposed to the market risk. This holds true even if the stochastic volatility model or the jump diffusion model is used. The market betas appear to be important since the market betas tend to increase with average hedge returns. Thus, we can deduce that an equilibrium model in which investors' preferences are taken into account can help to explain what no-arbitrage models cannot fully explain. The size of the market beta is typically less than one. In other words, the market risk of an option can be dramatically reduced by daily delta hedging, although it cannot be eliminated. Also, hedge returns of put options show higher market betas when the Black-Scholes model is used. This can be interpreted as the stochastic volatility model or the jump diffusion model being more effective at removing the market risk than the Black-Scholes model.

Next, three measures of coskewness are reported: beta to the squared excess market return, standardized unconditional coskewness and beta to the return of NMP portfolio. Like the market beta, beta to the squared excess market returns is computed from univariate regressions. Following Harvey and Siddique (2000), standardized unconditional coskewness is defined as

$$\textit{Standardized Unconditional Coskewness} = \frac{\mathrm{E}[\varepsilon_{i,t} r_{M,t}^2]}{\sqrt{\mathrm{var}[\varepsilon_{i,t}]\,\mathrm{var}[r_{M,t}^2]}} \qquad (5)$$

where $\varepsilon_{i,t}$ are residuals from regressing hedge return i on the market return, and $r_{M,t}$ are the excess market returns. Beta to the squared excess market return and standardized unconditional coskewness measure the contribution of hedge portfolio *i* to the skewness of a broader portfolio. A negative value means that the hedge portfolio *i* adds negative skewness to the broader portfolio. Since positive skewness is preferred over negative skewness, the risk premiums of the two coskewness measures should be negative.

Looking at Table 3, beta to the squared excess market return and standardized unconditional coskewness seem to increase from moneyness group 1 to moneyness group 5. However, they are not well ordered and are rarely statistically significant at the 5% or 10% confidence level. The confidence level for standardized unconditional coskewness is computed by generating the bootstrap statistic 10,000 times. The bootstrap interval endpoints are given by the acceleration and bias-correction percentiles of bootstrap distribution.

[Insert Table 4 about Here]

We define the return of the NMP portfolio as the difference in the average returns between hedge portfolios of put options with the highest strikes (M = 5) and hedge portfolios of call options with the lowest strikes (M = 1). According to the previous two coskewness measures, the hedge portfolio of put options with the highest strikes exhibit the lowest coskewness and is most negative, while the hedge portfolio of call options with the lowest strikes exhibits the highest coskewness is most positive. Thus, other things being equal, the risk premium of the former portfolio should be higher than the risk premium of the latter portfolio. The return on the NMP portfolio represents the spread of risk premium between a portfolio that is subtracting skewness and a portfolio that is adding skewness, which is the reason we use the term NMP (Negative skewness Minus Positive skewness).

Beta to the NMP return has several advantages to the other two coskewness measures. First, it is analogous to the Fama-French factors and the systemic skewness in Harvey and Siddique (2000); a higher factor loading is associated with a higher expected return.[4] Second, most loadings on the NMP return are statistically significant and well-ordered in strike price. For one-month-to-expiration hedge portfolios in Panel B, the loadings decrease monotonically from

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[4] Our measure is different from that of Harvey and Siddique (2000) in that we construct the NMP portfolio from the options market, while they form the systematic skewness based on the past rakings of coskewness from the stock market.

0.049 to -0.446 for calls, and from 0.448 to -0.036 for puts. Compared to the beta to the NMP return, the previous two coskewness measures appear to be noisy. Third, when the market beta is controlled, beta to the NMP return can explain the cross-sectional difference in average hedge returns. For example, consider the two-month-to-expiration call hedge return in moneyness group 5 and two-month-to-expiration put hedge return in moneyness group 1 in Panel B. While their market betas are close to each other – 0.228 versus 0.233 – their mean returns are quite different – 1.039% versus 3.224%. This difference in means appears to be related to the difference in beta of the NMP factor – -0.301 versus 0.606.

We also compute cross-sectional correlation between average hedge returns and portfolio-specific risk variables, although we do not tabulate the results. These variables include beta to the excess market return, beta to the squared excess market return, standardized unconditional coskewness and beta to the NMP return. For each variable, the correlation is computed for each of the three expiration groups and three option pricing models. The nine correlations are then simply averaged. The average correlations are 0.869, -0.647, -0.544 and 0.813 for the four variables respectively. The market beta and beta to the NMP return show higher correlations in absolute value than the other two variables. Moreover, seven out of nine correlations between the market beta and average hedge returns are statistically significant, while eight out of nine correlations between the NMP beta and average hedge returns are statistically significant.

In summary, the empirical distribution of discretely rebalanced hedge portfolios exhibits deviations from perfect dynamic replication. Specifically, average hedge returns tend to co-move with loadings on the excess market return and the NMP return. While it is not surprising that unhedged option returns have large market betas, the non-zero moments and co-moments of the hedge portfolios suggest that no-arbitrage models cannot fully describe option returns and additional systematic risks related to market return must be considered. In the next section, we formally test higher-moment CAPM in the options market to determine whether or not the market beta and higher-moment betas can explain the cross-sectional variation in average hedged and unhedged option returns.

II. Pricing Test Using Higher-Moment CAPM

In this section, we present an extended capital asset pricing model in which not only variance of investor's portfolio but also skewness and kurtosis are considered. Then, we examine the performance of the asset pricing framework on hedged and unhedged option portfolios. The test follows the standard two-step procedure of a time-series regression and a cross-sectional regression, originally developed by Fama and MacBeth (1973). If the two systematic risk factors explain the cross-sectional variation in average hedge returns, we can conclude that option prices reflect investors' preferences on those factors, and the equilibrium model complements the prevailing approach of using no-arbitrage frameworks to price options.

It is important to note that the shape of the implied volatility curve is directly related to the cross-sectional variations in average hedge returns. Thus, our model indirectly tests the hypothesis that the negatively sloped implied volatility curve is linked to investors' preferences regarding systematic risks. In addition, we extend Bollen and Whaley (2004). Bollen and Whaley provide evidence that, due to the limits to arbitrage, the option price can be influenced by supply and demand in the market. They argue that positive net buying pressure of put options on the S&P 500 index can be related to the negatively sloped implied volatility curve. We specify the risk factors that drive supply and demand in the options market and investigate the relation between the risk factors and the implied volatility curve.

A. Three- and Four-Moment Capital Asset Pricing Model

The rate of return on an asset can be analyzed in the framework of mean, variance and skewness tradeoff. Arrow (1971) argues that desirable utility functions must show positive but decreasing marginal utility of wealth and non-increasing absolute risk aversion. Aversion to variance and negative skewness are general characteristics of utility functions with decreasing marginal utility of wealth and non-increasing absolute risk aversion. Thus, in a market in which the participants have one of those utility functions, a higher expected return is required for an asset with a higher variance, a lower skewness or both.

A formal framework for assessing the relationship between the mean, variance and skewness of an asset return is investigated by Kraus and Litzenberger (1976). Kraus and Litzenberger extend the original version of the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), and develop a three-moment CAPM in which investors consider not only the variance of their portfolios but also the skewness. They expand a utility function defined over the unconditional mean, standard deviation and the third root of skewness in a Taylor series, and derive a stochastic discount factor that is a quadratic function of the excess market return. The quadratic stochastic discount factor prices coskewness of an asset with the excess market return as well as covariance. Harvey and Siddique (2000) explain the cross-asset variation in average stock returns in the conditional coskewness model. Following the notation of Harvey and Siddique, the conditional three-moment CAPM is

$$E_t[r_{i,t+1}] = \lambda_{1,t}\ \mathrm{cov}_t[r_{i,t+1}, r_{M,t+1}] + \lambda_{2,t}\ \mathrm{cov}_t[r_{i,t+1}, r_{M,t+1}^2] \tag{6}$$

$$\lambda_{1,t} = \frac{\mathrm{var}_t[r_{M,t+1}^2]\,\mathrm{E}_t[r_{M,t+1}] - \mathrm{skew}_t[r_{M,t+1}]\,\mathrm{E}_t[r_{M,t+1}^2]}{\mathrm{var}_t[r_{M,t+1}]\,\mathrm{var}_t[r_{M,t+1}^2] - (\mathrm{skew}_t[r_{M,t+1}])^2}$$

$$\lambda_{1,t} = \frac{\mathrm{var}_t[r_{M,t+1}]\,\mathrm{E}_t[r_{M,t+1}^2] - \mathrm{skew}_t[r_{M,t+1}]\,\mathrm{E}_t[r_{M,t+1}]}{\mathrm{var}_t[r_{M,t+1}]\,\mathrm{var}_t[r_{M,t+1}^2] - (\mathrm{skew}_t[r_{M,t+1}])^2}$$

The coefficients $\lambda_{1,t}$ and $\lambda_{2,t}$ are the same across all the assets and are statistically different from zero. Equation (6) can be rewritten as follows:

$$\mathrm{E}_t[r_{i,t+1}] = A_t\ \mathrm{E}_t[r_{M,t+1}] + B_t\ \mathrm{E}_t[r_{M,t+1}^2] \tag{7}$$

$$A_t = \frac{\mathrm{cov}_t[r_{i,t+1}, r_{M,t+1}]\mathrm{var}_t[r_{M,t+1}^2] - \mathrm{cov}_t[r_{i,t+1}, r_{M,t+1}^2]\mathrm{skew}_t[r_{M,t+1}]}{\mathrm{var}_t[r_{M,t+1}]\,\mathrm{var}_t[r_{M,t+1}^2] - (\mathrm{skew}_t[r_{M,t+1}])^2}$$

$$B_t = \frac{\mathrm{cov}_t[r_{i,t+1}, r_{M,t+1}^2]\,\mathrm{var}_t[r_{M,t+1}] - \mathrm{cov}_t[r_{i,t+1}, r_{M,t+1}]\mathrm{skew}_t[r_{M,t+1}^2]}{\mathrm{var}_t[r_{M,t+1}]\,\mathrm{var}_t[r_{M,t+1}^2] - (\mathrm{skew}_t[r_{M,t+1}])^2}$$

where A_t and B_t are functions of the variance and skewness of the excess market return, and covariance and coskewness with the excess market return, respectively. A_t and B_t are analogous to beta in the traditional CAPM.

Kimball (1993) augments the three-moment CAPM by using the restriction of decreasing absolute prudence. Samuelson (1963) proves that if a risk-averse agent had already accepted a bet with a negative expected payoff, she should be unwilling to take another independent bet with a negative expected payoff. Under this restriction, the fourth-order derivative of standard utility function should be negative and the pricing kernel, m_{t+1}, is a cubic polynomial of the market return.

$$m_{t+1} = d_0 + d_1R_{M,t+1} + d_2R^2_{M,t+1} + d_3R^3_{M,t+1} \tag{8}$$

A cubic pricing kernel is consistent with a model in which investors have preference over the first four moments of returns. Dittmar (2002) finds that incorporating cubic market returns substantially improves upon the pricing kernel's ability to describe the cross section of stock returns.

B. Time-Series Analysis

The first-pass time-series regression estimates the loadings on the excess market return, the squared excess market return, and the cubic excess market return. The regression equation is

$$R_{i,t} = \alpha_i + \beta_{i,M}\, R_{M,t} + \beta_{i,SM}\, R^2_{M,t} + \beta_{i,CM}\, R^3_{M,t} + \varepsilon_{i,t} \tag{9}$$

where $R_{M,t}$ is the excess market return at time t and βs signify loadings to the corresponding risk factor of the return, R_i. Table 5 reports loadings on the excess market return and its square and cubic from time-series regression of unhedged option returns. The t-statistics are corrected for autocorrelation and heteroskedasticity using the Newey-West estimator with five legs and are reported below the corresponding loadings. The dependant variables are weekly returns of calls, puts, and zero-beta straddles in Panel A, B, and C, respectively.

In Panel A and B of Table 5, the market betas are large in absolute value and statistically significant. Especially, the estimates of market betas are quite similar in magnitude to the Black-Scholes betas reported in Table 2. Not surprisingly, the market loadings of zero-beta straddles are relatively low, although they are statistically significant. As Coval and Shumway (2001) indicate, the market loadings alone cannot fully explain the option returns. Besides the market loading, the loadings on the squared and cubic excess market returns are also quite large and statistically significant. Thus, coskewness and cokurtosis are important risk factors in the options market. However, the loadings on cubic excess market returns do not show the expected sign. For example, they should be positive and increasing in the strike price for call options. The reason why the loadings are negative is that excess market return and its cubic are highly positively correlated with simple correlation coefficient of 70.1% during the sample period. From a univariate time-series regression of call returns on the cubic excess market return, its loadings are estimated to be positive and increasing in the strike price.

In Table 5, we also test the joint significance of the regression intercepts and present the *F*-statistics of Gibbons, Ross and Shanken (1989) and their corresponding *p*-values in percentage form. The statistic is 21.47 and, thus, the hypothesis that the regression intercepts are jointly zero is rejected. The corresponding *p*-values are equal to or less than 0.01%.

[Insert Table 5 about Here]

Next, we conduct the time-series regression of hedge returns on the excess market return and NMP return as follows. As discussed in Section II, the NMP return is used as a proxy of coskewness risk.

$$R_{i,t} = \alpha_i + \beta_{i,M}\, R_{M,t} + \beta_{i,NMP}\, R_{NMP,t} + \varepsilon_{i,t} \tag{10}$$

where $R_{NMP,t}$ is the NMP portfolio return at time t. Table 6 reports loadings on the excess market return and the NMP return computed in time-series regression for hedge portfolios of one-month-to-expiration options. The results for hedge portfolios of longer-term options will be discussed later. The dependant variable, which is a time series of hedge returns, is computed using the Black-Scholes model in Panel A, the stochastic volatility model in Panel B and the jump diffusion model in Panel C. The size of the loadings is less than one, ranging from 0.018 to 0.246, implying that the market risks are substantially reduced by delta hedging. All the market loadings are positive and statistically significant at the 5% confidence level. The loadings increase in moneyness group for call options, and decrease for put options. In other words, hedge portfolios of OTM options are exposed to a greater market risk than hedge portfolios of ATM options and, in turn, hedge portfolios of ITM options.

The loadings on the NMP return decrease in moneyness group. In general, the hedge portfolios of low moneyness groups (M = 1 or 2) have positive loadings on the NMP return, while the hedge portfolios of high moneyness groups (M = 4 or 5) have negative loadings. In the middle (M = 3), the loadings are close to zero. These mid-group loadings are not statistically significant at the 5% confidence level, i.e., their exposures to the coskewness risk are zero. However, many of the NMP betas from low and high moneyness groups are statistically significant. The *F*-statistics of Gibbons, Ross and Shanken (1989) are presented in the last column of Table 6. As shown in all three panels, the hypothesis that the regression intercepts are jointly zero is rejected. The corresponding *p*-values are equal to or less than 0.01%. The same tests are conducted for the loadings on the excess market return and the NMP return, and are reported separately in Panel A of Table 9. The *F*-statistics are over 100, indicating that both the loadings on the excess market return and the NMP return are jointly significant. Thus, the excess market return and the NMP return are relevant in the sense that hedge portfolios significantly load on them.

[Insert Table 6 about Here]

C. Cross-Sectional Analysis

The objective of the second-pass cross-sectional regression is to test whether the loadings estimated from the time-series regression are important determinants of average options returns. If they are, there should be a significant price of risk associated with each factor. Specifically, we examine the following cross-sectional regression specification:

$$R_{i,t} = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_{SM} \hat{\beta}_{i,SM} + \gamma_{CM} \hat{\beta}_{i,CM} + e_{i,t} \tag{11}$$

where $\hat{\beta}$ stands for the exposure to the corresponding factor estimated from the time-series regression and γ stands for the reward for bearing the risk of the corresponding factor. In each regression, the dependent variable is the time-series average of unhedged option returns.

Result of cross-sectional regression presented in Table 7 shows that the loadings on the excess market return are the most important cross-sectional determinant of average unhedged option returns. The *t*-statistics, which is corrected for the errors-in-variables problem following Shanken (1992), indicate that the hypothesis H_0: $\gamma_M = 0$ is strongly rejected. The risk premium related to the excess market return is estimated as 3.5% to 5.6%. Moreover, loadings on the excess market return can explain most of the cross-sectional variation in the average option returns. The adjusted R^2 which is computed following Jagannathan and Wang (1996) is more than 98%. The loadings on the squared and cubic excess market return are also statistically significant in the presence of market loadings. Under the errors-in-variables correction, the *t*-statistics for the hypotheses, $H_0 : \gamma_{SM} = 0$ and $H_0 : \gamma_{CM} = 0$ are -2.2 and 4.5.

[Insert Table 7 about Here]

Table 8 presents result of cross-sectional regression to test whether the loadings on the excess market return and the NMP return are important determinants of average hedge returns. The regression equation is:

$$R_{i,t} = \gamma_0 + \gamma_M \hat{\beta}_{i,M} + \gamma_{NMP} \hat{\beta}_{i,NMP} + e_{i,t} \tag{12}$$

Like the regression results of unhedged option returns, the loadings on the excess market return are the most important cross-sectional determinant of average hedge returns. From all three panels, the *t*-statistics indicate that the hypothesis H_0: $\gamma_M = 0$ is strongly rejected. Also, loadings on the excess market return can explain most of the cross-sectional variation in the average hedge returns. The adjusted R^2 are 99.24%, 88.83% and 88.81 when hedge returns are computed using the Black-Scholes model, the stochastic volatility model and the jump diffusion model, respectively.

The loadings on the NMP return are also statistically significant in the presence of market loadings. Under the errors-in-variables correction, the *t*-statistics for the hypotheses, H_0 : $\gamma_M = 0$ and H_0 : $\gamma_{NMP} = 0$ are 2.518 and 2.887 in Panel A, 3.884 and 1.928 in Panel B and 3.642 and 2.775 in Panel C. The risk premium related to the excess market return is estimated to be higher than the risk premium related to the NMP return. In Panel A, the risk premium for bearing one unit of the market covariance risk is 5.8% while the price for one unit of the coskewness risk is 2.5%. These premiums are estimated as 6.2% and 1.0% in Panel B and 4.5% and 1.4% in Panel C.

[Insert Table 8 about Here]

D. Goodness-of-Fit Test

Goodness-of-fit tests from various viewpoints are reported in this subsection to determine whether the higher-moment CAPM works in the options market in the correct way. First, we test the hypothesis that the intercept of each cross-sectional regression is zero. Hedge returns are, by nature, in the form of excess returns. Thus, if the regression specification of Equation (12) is correct, the hypothesis that the intercept γ_0 is zero should not be rejected. Table 8 shows that most intercepts are not significantly different from zero irrespective of option pricing models, implying that the hypothesis is not rejected. Also, Table 7 shows that the intercept is not different from zero with the *t*-statistic of 0.15, when average unhedged option returns are regressed on the loadings on excess market return, squared and cubic excess market returns.

Second, comparing adjusted R^2 measures reveals a difference between the Black-Scholes model and the other models. The adjusted R^2 is much higher in Panel A than in Panels B and C of Table 8. It can be deduced that a greater portion of cross-sectional variation in average hedge returns is explained by the systematic risks when the Black-Scholes model is used to hedge the S&P 500 index options rather than when the stochastic volatility model or the jump diffusion model is used. This can be interpreted as evidence that using the advanced models can reduce the amount of exposure to systematic risks.

Third, we test the hypothesis that the pricing errors from the cross-sectional regression are jointly zero, and report the corresponding χ^2-statistics in the last column of Table 7. The χ^2-statistics and their corresponding *p*-values (below the statistics and in percentage form) show that the pricing errors are jointly insignificant when average unhedged option returns are regressed on the loadings on excess market return, squared and cubic excess market returns. For hedge portfolios, the *p*-values for testing the hypothesis are 11.74%, 7.75% and 6.82% in Panels A, B and C in Table 8, respectively. Thus, this χ^2-test confirms that the higher-moment capital asset pricing framework properly spans the options market.

Another way to verify goodness of fit across the option pricing models is to compare the fitted expected return of each hedge portfolio against its realized average return. The fitted expected return is computed using the estimated parameter values from Equations (11) and (12) for unhedged and hedged option portfolios, respectively. The realized average return is the time-series average of the option returns. If the fitted expected return and the realized average return for each portfolio are the same, then their plot should lie on a 45-degree line through the origin. Panels A, B and C of Figure 1 demonstrate the fitted expected returns on the vertical axis versus realized average returns on the horizontal axis for naked option contracts. Each two-digit numbers represents a separate option returns. The first digit refers to option position (1 for call, 2 for put, and 3 for zero-beta straddle), while the second digit refers to moneyness group (1 for the lowest strike price and 5 for the highest). For example, portfolio 21 indicates the DOTM put option. Under the four-moment CAPM in Panel C of Figure 1, the option portfolios are closer to 45 degree line than under the three-moment CAPM in Panel B and, in turn, than under the standard CAPM in Panel A.

[Insert Figure 1 about Here]

Figure 2 show the fitted expected returns on the vertical axis versus realized average returns on the horizontal axis for ten hedge portfolios of one-month-to-expiration options. Panel A differs from Panels B and C in that portfolio 21 is depicted at the upper right corner in Panel A. This means that delta hedging by the stochastic volatility model and the jump diffusion model dramatically reduce the exposure of hedge portfolios of DOTM put options to the excess market return and the NMP portfolio return. However, pricing error tends to increase if the advanced models are employed. For example, the hedge portfolios of DOTM and OTM options (21, 22 and 15) in Panels B and C deviate from the 45-degree line further than in Panel A. In addition, the root mean squared errors are 0.08%, 0.15% and 0.2% from Panels A, B and C, respectively. In summary, various goodness of fit tests confirm that the higher-moment capital asset pricing framework can properly explain the cross-sectional variation in average returns of S&P 500 index options.

[Insert Figure 2 about Here]

III. Robustness

A. Longer-Term Options

So far, we have focused on hedge portfolios of one-month-to-expiration options and drawn a conclusion that the three-moment CAPM works for the portfolios. An interesting extension is to apply the model to other assets and test if the model still works. The natural candidates for “other assets” are hedge portfolios of the same S&P 500 options with longer terms to expiration. Since longer-term options have different characteristics from the shorter-term options (for example, due to volatility term structure), it is worthwhile to test whether the higher-moment CAPM also works for hedge portfolios of longer-term options. Thus, we repeat the two-step regression procedure for two-month-to-expiration options and three-month-to-expiration options. A drawback in examining the longer-term options is that the time-series length of hedge returns is short, and we should keep in mind that this may have an adverse impact on time-series estimates of loadings on the risk factors.

Table 9 summarizes the results of the time-series regression and cross-sectional regression for hedge portfolios of longer-term options. Panel A of Table 9 reports the number of significant loadings on the excess market returns and the NMP return from the time-series regression at the 5% confidence level. Although the length of hedge returns of longer-term options is shorter than the length of hedge returns of one-month-to-expiration options, the number of significant loadings is not very different. The Gibbons-Ross-Shanken *F*-statistics and their corresponding *p*-values in percentage form show that the corresponding factor loadings are jointly significant.

[Insert Table 9 about Here]

Panel B of Table 9 presents factor risk premiums from the cross-sectional regression using the full-sample factor loadings computed in the time-series regression. The overall results are similar to the results of the cross-sectional regression for hedge returns of one-month-to-expiration options in Table 8. The risk premiums of the excess market return and the NMP return are positive and statistically significant at the 5% confidence level after the errors-in-variables adjustment. The intercepts of the cross-sectional regressions are generally insignificant, but they are higher when the Black-Scholes model is employed than when the stochastic volatility model or jump diffusion model is used, which is similar to the results in Table 8. In addition, the adjusted R^2 measures are quite high, ranging from 81.92% to 97.44%.

One caveat of the cross-sectional regression result for hedge portfolios of longer-term options is that the hypothesis that the pricing errors in each regression are jointly zero is strongly rejected in many cases. Using the Black-Scholes model for hedge portfolios of two-month-to-expiration options, the χ^2-statistics and their corresponding *p*-values are 21.13 and 0.36%, respectively, implying that three-moment CAPM is rejected. For hedge portfolio of three-month-to-expiration options, the *p*-value for the joint test is less than 1% regardless of hedge models.

Panels D through I of Figure 2 present plots of the fitted expected returns versus the average realized returns for longer-term options, which are similar to Panels A, B and C. When the Black-Scholes model is used, the hedge portfolio of DOTM put options is located in the far upper right of Panels D and G, and in the middle of the plot when the advanced models are used. The root mean squared errors for longer-term options are generally greater than those for shorter-term options. To summarize, the excess market return and the NMP portfolio return are significant determinants of the hedge returns of longer-term options; however, the loadings on the two factors cannot fully explain the cross-sectional variation in average hedge returns.

B. Finite Sample Distribution of Loadings and Risk Premiums

We are concerned that the results we have obtained thus far may be biased or imprecise due to the small sample size in the first-pass time-series regression. Therefore, to evaluate the empirical evidence on three-moment CAPM over hedge returns, we present a finite sample distribution of loadings and the adjusted R^2 for hedge returns with a one-month horizon. The similar finite sample distribution for hedge portfolios of longer-term options can be obtained from the authors upon request.

The procedure to generate the finite sample distribution is as follows. Under the null hypothesis that the estimated factor loadings and risk premiums are true, we simulate 10,000 time-series of hedge returns using the bootstrap method.

$$R_{i,t}^{*} = \alpha_i + \hat{\beta}_{i,M} R_{M,t} + \hat{\beta}_{i,NMP} R_{NMP,t} + \varepsilon_{i,t}^{*} \qquad (13)$$

where $\hat{\beta}$s are the estimated factor loadings from the time-series regression reported in Table 5, and ε_t^{*} stands for the bootstrapped residual. To recover the dependence structure of the estimated residuals in the re-sampled residuals, we use the stationary bootstrap proposed by Politis and Romano (1994). The stationary bootstrap calls for re-sampling blocks of random length where the length of each block has a geometric distribution. Politis and Romano show that the bootstrap sample generated by this re-sampling scheme is stationary. The average block length for the stationary bootstrap is computed based on Politis and White (2004), and is 1.92. The simulated returns are used to estimate a new set of factor loadings, factor risk premiums and cross-sectional adjusted R^2s. In this way, the finite sample distributions of the betas, the cross-sectional risk premiums and the adjusted R^2s are generated.

Panel A of Table 10 reports the finite sample distributions of the simulated betas, along with the betas estimated under the null model. The two-digit number located on the right of each model indicates a separate hedge portfolio with the first digit referring to option type (1 for call and 2 for put) and the second digit referring to moneyness group. For every hedge portfolio, the loadings on the excess market return and the systematic skewness factor are unbiased; the 50% critical value of the distribution is very close to the value estimated under the null model. The mean difference between the true value and the 50% critical value is 1.9% of the true value for the market beta and -1.2% for the NMP beta. Furthermore, the distributions of the simulated loadings appear almost symmetric. The distance from the 50% critical value to the 2.5% critical value is similar to the distance from the 50% critical value to the 97.5% critical value.

[Insert Table 10 about Here]

Panel B of Table 10 reports the finite sample distributions of the risk premiums and the adjusted R^2s in the cross-sectional regression. The null hypothesis that the risk premiums on the excess market return and the NMP return are equal to zero is strongly rejected at the 5% confidence level from their finite sample distributions; none of the 5% confidence intervals include zero. The adjusted R^2s are also far from zero, but their finite sample distribution displays a downward bias due to the sampling error in the estimated betas. In summary, the finite sample distributions confirm the fact that there are significant risk premiums associated with the excess market return and the NMP portfolio return in the S&P 500 index options market.

IV. Conclusion

We have shown that the distribution of the rate of return of discretely hedged option portfolio exhibits a deviation from perfect dynamic replication. Specifically, the hedge returns are exposed to covariance and coskewness risk with the market portfolio. The co-moments differ widely depending on moneyness of the option hedged. The average hedge returns tend to co-move with loadings on the excess market return and the NMP portfolio return. The NMP portfolio return is defined to proxy the systematic coskewness risk.

A formal test using the three-moment CAPM and the standard two-pass regression is conducted on hedge portfolios of S&P 500 index options over a period of 138 months from January 1996 to June 2007. We find evidence from the time-series regression that the excess market return and the NMP factor are relevant in the sense that hedge portfolios significantly load on them. From the cross-sectional regression on hedge returns of one-month-to-expiration options, the excess market return and the NMP portfolio return can explain the cross-sectional variation in average hedge returns and command a risk premium of 5.8% and 2.5%, respectively, when the Black-Scholes model is used to compute the delta values. Various goodness-of-fit tests and the finite sample distributions of factor loadings and risk premiums confirm that there are significant risk premiums associated with the excess market return and the NMP portfolio returns in the S&P 500 index options market. Since the cross-sectional variation in the average hedge returns of options with different strike prices are directly related to the volatility skew, we can conclude that investors' preferences on systematic risk play a role in determining the negatively sloped implied volatility function.

The empirical issues addressed in this article can also be tested for individual stock options. The shape of the implied volatility function of stock options is known to differ from that of index options. Whether hedge portfolios of stock options also are exposed to the market risk or whether the loadings on other risk factors, such as the Fama-French factors, can be significant in the stock options market is questionable. These are left for further research.

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Table 1

Sample Description of S&P 500 Index Options

This table presents sample description of S&P 500 index options collected on every Fridays from January 1996 through September 2008. The options with time to maturity shorter than 30 days or longer than 100 days and absolute delta values below 5% or above 95% are removed. Both the call and put options are divided into five moneyness groups based on the delta values. Breakpoints of five moneyness groups are reported at the top of the table. The delta value of each option is computed using the Black-Scholes model and the realized volatility over the most recent 60 trading days. The lowest moneyness group (M = 1) comprises options with the lowest strikes while the highest moneyness group (M = 5) comprises options with the highest strikes. Panel A shows the number of observations, average daily trading volume per contract, average delta value, average ratio of the strike price divided by the underlying index, average implied volatility (IV), and average implied volatility over realized volatility (RV) for each moneyness group of the call options while Panel B presents those statistics for the put options. Implied volatilities are computed from the midpoint of the bid–ask spread at the market close using the Black-Sholes formula. The delta values and volatilities are shown in the percentage form.

	Moneyness Groups (M)				
	1	2	3	4	5
Call delta	(75, 95]	(60, 75]	(40, 60]	(25, 40]	(5, 25]
Put delta	(-5, -25]	(-25, -40]	(-40, -60]	(-60, -75]	(-75, -95]
	Panel A: Call Options				
Obs	7,138	4,158	5,653	4,132	7,060
Trading volume	3,862	5,968	6,776	6,477	5,959
Mean delta	86.14	67.47	50.12	32.43	14.22
K/S	0.93	0.97	1.01	1.04	1.09
IV	23.21	20.88	19.17	17.54	15.99
IV-RV	5.92	3.23	1.51	0.02	-1.37
	Panel B: Put Options				
Obs	7,141	4,169	5,653	3,926	3,383
Trading volume	9,353	8,699	6,327	3,538	2,246
Mean delta	-13.89	-32.51	-49.84	-67.35	-83.75
K/S	0.93	0.98	1.01	1.04	1.08
IV	23.06	20.77	19.13	17.69	18.18
IV-RV	5.77	3.13	1.47	0.27	0.49

Table 2

Weekly Returns of S&P 500 Index Call and Put Options

This table presents descriptive statistics for weekly returns of S&P 500 index calls (Panel A), puts (Panel B), and zero-beta straddle positions (Panel C). The sample period is 661 weeks from January 1996 through September 2008. The options with time to maturity shorter than 30 days or longer than 100 days are removed. The options are divided into five moneyness groups based on the delta values, and breakpoints of each moneyness group are reported at the top of the table. The delta value of each option is computed using the Black-Scholes model and the realized volatility over the most recent 60 trading days. The lowest moneyness group (M = 1) comprises options with the lowest strikes while the highest moneyness group (M = 5) consists of options with the highest strikes. The straddle positions are constructed by combining call and put options with such weights that the betas of the positions are zero. The zero-beta straddles are divided based on the delta values of the call options in the positions. To fairly compare the returns across five moneyness groups, the weeks when any of five moneyness groups have no observation are excluded. To prevent excessive exclusion, options with absolute delta values between 35% and 75% are reported. After the exclusion, the sample is reduced to 535weeks. BS β denotes the option beta computed using the Black-Scholes model. The returns are in weekly percentage forms, and the *t*-statistics test the null hypothesis that the returns are zero.

	Moneyness Groups				
	1	2	3	4	5
Call delta	(65, 75]	(55, 65]	(45, 55]	(35, 45]	(25, 35]
Put delta	(-25, -35]	(-35, -45]	(-45, -55]	(-55, -65]	(-65, -75]
	Panel A: Call Options				
Mean return	3.443	3.769	4.162	4.447	4.971
t-Statistic	(2.973)	(2.842)	(2.731)	(2.504)	(2.331)
Median	2.449	1.546	1.120	0.112	-1.807
Minimum	-63.000	-68.159	-72.999	-77.501	-82.005
Maximum	91.656	116.514	155.221	171.494	271.831
Mean BS β	14.610	16.238	18.202	20.964	25.418
	Panel B: Put Options				
Mean return	-9.510	-8.431	-7.497	-6.663	-5.835
t-Statistic	(-6.022)	(-5.649)	(-5.33)	(-5.013)	(-4.728)
Median	-16.256	-13.552	-12.320	-10.609	-8.606
Minimum	-72.735	-71.801	-68.549	-68.692	-62.824
Maximum	166.243	154.829	142.130	129.841	113.705
Mean BS β	-16.813	-17.164	-16.838	-16.088	-14.847
	Panel C: Zero-Beta Straddles				
Mean return	-2.611	-2.234	-1.989	-1.920	-1.866
t-Statistic	(-7.982)	(-7.14)	(-6.075)	(-5.412)	(-4.649)
Median	-3.814	-3.194	-2.959	-3.113	-3.344
Minimum	-17.515	-16.901	-19.019	-21.508	-23.383
Maximum	46.181	40.503	51.526	49.877	65.955

Table 3

Calibrated Model Parameters

This table presents the estimated parameters for the Black-Scholes model, the stochastic volatility model and the jump diffusion model. The sample period is 138 months from January 1996 to June 2007. At each expiration day, a set of call and put options with absolute delta values above 2% and below 98% are collected from the market. The parameters of a given model are estimated by minimizing the sum of the squared differences between the market implied volatilities and the model implied volatilities for the collected options. The average values of parameters are reported with their standard deviations. Panel A reports average realized volatility (*RV*) next to the average implied volatility (*IV*) computed by the Black-Scholes model. In Panel B, the parameters of the stochastic volatility model, κ_v, θ_v, σ_v, ρ and $\sqrt{v_0}$ are the speed of reversion, the long-run mean of volatility, the volatility of variance rate, the correlation between the underlying index and the volatility process and the current level of the variance rate, respectively. For the jump diffusion model in Panel C, the parameters μ_J, σ_J and λ_J are the expected jump size, the standard deviation of the logarithm of one plus the percentage jump size and the frequency of the jumps per year, respectively. The average value of root mean squared errors (RMSE) is also reported in the last column of Panel B and Panel C.

Panel A: Black-Scholes Model						
	RV	IV				
Mean	0.1632	0.2424				
Stdev	0.0641	0.0804				
Panel B: Stochastic Volatility Model						
	κ_v	θ_v	σ_v	ρ	$\sqrt{v_0}$	RMSE
Mean	5.4348	0.0528	0.5823	-0.6995	0.1821	0.0698
Stdev	4.2462	0.0271	0.1822	0.1429	0.0754	
Panel C: Jump Diffusion Model						
	μ_J	σ_J	λ_J	$\sqrt{v_0}$		RMSE
Mean	-0.2104	0.1626	0.7722	0.1304		0.0441
Stdev	0.1038	0.0716	1.0993	0.0322		

Table 4

Descriptive Statistics on Discretely Hedged Option Portfolios

This table describes distributional characteristics of hedge portfolios of options in various expiration groups (T) and moneyness groups (M). The rate of return on hedge portfolios are computed using the Black-Scholes model (Panel A), the stochastic volatility model (Panel B) and the jump diffusion model (Panel C). The rate of return on the market portfolio (R_M) is the value-weighted NYSE/AMEX/NASDAQ index. Skewness of return i is the third central moment around the mean divided by cubic standard deviation. The βs are computed from univariate regressions of the portfolio return on the risk factor. Standardized unconditional coskewness is defined as $E[\varepsilon_{i,t} r_{M,t}^2] \Big/ \sqrt{\mathrm{var}[\varepsilon_{i,t}]\,\mathrm{var}[r_{M,t}^2]}$, where $\varepsilon_{i,t}$ are residuals from regressing the excess return i on the market return and $r_{M,t}$ is the excess market return. Significance levels for skewness and standardized unconditional coskewness are computed by generating the bootstrap statistic 10,000 times. The bootstrap interval endpoints are given by the acceleration and bias-correction percentiles of bootstrap distribution. The rate of return on the NMP portfolio (R_{NMP}) is defined as the difference in the rate of returns between the put options with the lowest strikes (M = 1), which exhibit negative systematic coskewness, and the call options with the highest strikes (M = 5), which exhibit positive systematic coskewness. The sample period is from January 1996 to September 2008.

		Call Options							**Put Options**						
T	**M**	**Mean**	**Stdev**	**Skewness**	**β to R_M - R_f**	**β to $(R_M - R_f)^2$**	**Standardized Unconditional Coskewness**	**β to R_{NMP}**	**Mean**	**Stdev**	**Skewness**	**β to R_M - R_f**	**β to $(R_M - R_f)^2$**	**Standardized Unconditional Coskewness**	**β to R_{NMP}**
	1	0.205	0.626	-2.561**	0.047**	-0.311**	-0.123**	0.056**	3.928	9.850	-3.603**	0.806**	-6.137**	-0.310**	1.048**
	2	0.272	0.806	-1.832**	0.053**	-0.089	0.050	0.050**	1.133	3.031	-2.573**	0.236**	-0.941	-0.048	0.250**
1	3	0.319	1.162	-1.639**	0.052**	0.083	0.102*	0.025**	0.544	1.304	-1.771**	0.075**	-0.103	0.091	0.062**
	4	0.276	2.024	-2.711**	0.070**	0.209	0.100	-0.014	0.259	0.782	-1.399**	0.038**	0.001	0.130	0.012
	5	0.843	3.017	1.785	0.123**	0.320	0.111	-0.046	0.115	0.492	-3.180**	0.020**	-0.010	0.093	0.000
	1	0.372	0.987	-2.765**	0.059**	0.103	-0.148	0.050**	5.474	15.031	-3.631**	0.929**	0.199	-0.234**	0.816**
	2	0.606	1.204	-1.460**	0.064**	0.237	-0.039	0.036**	1.998	4.873	-3.030**	0.293**	0.613	-0.128*	0.155**
2	3	0.852	1.424	-0.737**	0.073**	0.290	-0.024	0.020*	1.155	1.849	-0.842**	0.103**	0.288	-0.083	0.043**
	4	0.777	2.683	-2.104	0.166**	0.906**	0.049	0.005	0.570	1.228	-1.759**	0.076**	0.222	-0.088	0.014
	5	1.369	3.500	0.966**	0.291**	1.634**	0.085	-0.016	0.162	0.884	-3.216**	0.056**	0.303**	0.048	0.001
	1	0.588	1.254	-2.007**	0.074**	-0.135	0.014	0.063**	6.685	17.200	-3.483**	1.065**	-2.838	-0.064	0.997**
	2	0.872	1.443	-0.077	0.063**	-0.033	0.090	0.042**	2.392	4.372	-1.879**	0.277**	-0.559	-0.003	0.180**
3	3	1.209	1.587	-0.474	0.045	0.010	0.083	0.024	1.422	2.148	-1.696**	0.071*	-0.010	0.082	0.044**
	4	1.726	1.698	0.929**	0.092**	0.018	0.173	0.021	1.012	1.151	-0.837**	0.050**	-0.087	0.015	0.018
	5	1.905	4.423	2.930**	0.194**	0.010	0.124	-0.034	0.511	0.606	0.177	0.038**	-0.033	0.109	0.006

T	M	Call Options Mean	Stdev	Skewness	β to $R_M - R_f$	β to $(R_M - R_f)^2$	Standardized Unconditional Coskewness	β to R_{NMP}	Put Options Mean	Stdev	Skewness	β to $R_M - R_f$	β to $(R_M - R_f)^2$	Standardized Unconditional Coskewness	β to R_{NMP}
Panel B: Stochastic Volatility Model															
	1	0.194	0.555	-2.150**	0.033**	-0.265**	-0.134*	0.049**	1.901	3.846	-2.462**	0.284**	-2.170**	-0.158**	0.448**
	2	0.273	0.819	-2.192**	0.058**	-0.269*	-0.048	0.059**	1.083	2.710	-2.393**	0.210**	-1.236**	-0.100*	0.257**
1	3	0.282	1.055	-1.302**	0.087**	-0.160	0.060	0.013	0.658	1.621	-1.512**	0.139**	-0.542*	-0.027	0.075**
	4	0.215	1.549	-0.656**	0.121**	0.000	0.127*	-0.064**	0.268	0.988	-0.701**	0.089**	-0.192	0.051	-0.015
	5	0.873	3.614	1.152	0.177**	0.378	0.124*	-0.446**	0.136	0.633	-0.192	0.041**	-0.075	0.046	-0.036**
	1	0.404	0.753	-3.185**	0.027**	0.041	-0.091	0.078**	3.224	5.395	-4.218**	0.233**	0.153	-0.140	0.606**
	2	0.576	1.236	-1.479**	0.066**	0.161	-0.096	0.080**	1.897	3.935	-1.575**	0.221**	0.391	-0.134	0.272**
2	3	0.766	1.554	-0.703	0.129*	0.392	-0.119	0.062**	1.284	2.621	-1.159**	0.187**	0.498	-0.122	0.139**
	4	0.718	2.387	0.589	0.232**	1.144**	0.040	0.002	0.701	1.840	-0.734	0.174**	0.584*	-0.113	0.046
	5	1.039	3.886	1.681**	0.228**	0.593	-0.099	-0.301**	0.233	1.174	0.266	0.109**	0.607**	0.096	-0.012
	1	0.608	0.958	-2.037**	0.046**	-0.093	-0.002	0.072**	4.170	5.598	-2.579**	0.306**	-0.758	-0.039	0.577**
	2	0.859	1.375	-1.175**	0.073**	-0.151	-0.006	0.080**	2.476	4.076	-2.127**	0.261**	-0.745	-0.087	0.350**
3	3	1.086	1.684	-1.379**	0.076**	-0.094	0.048	0.038	1.679	2.946	-1.996**	0.130**	-0.195	0.030	0.118*
	4	1.501	2.027	0.625**	0.135**	0.075	0.268*	0.009	1.279	1.938	-0.864*	0.109**	-0.138	0.061	0.043
	5	1.675	4.459	2.711**	0.182**	0.216	0.179*	-0.377**	0.746	1.320	1.116**	0.080**	0.005	0.189	-0.035
Panel C: Jump Diffusion Model															
	1	0.209	0.674	-3.182**	0.042**	-0.401**	-0.187**	0.091**	2.186	5.223	-2.995**	0.378**	-3.408**	-0.203**	0.847**
	2	0.271	0.855	-2.042**	0.079**	-0.267	-0.005	0.075**	1.440	3.896	-2.435**	0.350**	-1.801**	-0.086	0.483**
1	3	0.215	1.130	-1.388**	0.093**	0.056	0.159**	0.001	0.600	1.872	-1.279**	0.185**	-0.310	0.083	0.090**
	4	0.287	1.780	-1.523**	0.094**	0.185	0.130*	-0.065**	0.227	1.013	-0.872**	0.074**	0.007	0.120*	-0.015
	5	0.905	2.572	2.142**	0.132**	0.252	0.125	-0.177**	0.135	0.596	-2.116	0.025**	-0.008	0.056	-0.022**
	1	0.393	0.873	-3.761**	0.028	-0.011	-0.127	0.101**	3.171	5.793	-4.354**	0.171	-0.519	-0.183**	0.746**
	2	0.565	1.323	-2.024**	0.078**	0.182	-0.112	0.056**	1.961	4.358	-1.873**	0.237**	0.336	-0.146*	0.233**
2	3	0.758	1.548	-0.161	0.133**	0.539**	-0.040	0.015	1.329	2.698	-0.984**	0.217**	0.719	-0.094	0.072
	4	0.774	2.471	-0.537	0.213**	1.092**	0.048	-0.054	0.645	1.742	-0.316	0.160**	0.667**	-0.035	-0.006
	5	1.321	3.302	1.820**	0.312**	1.744**	0.101	-0.214**	0.190	1.117	-1.707	0.086**	0.405**	0.013	-0.033
	1	0.589	1.043	-2.398**	0.050**	-0.137	-0.050	0.100**	3.959	5.478	-2.635**	0.280**	-0.998	-0.116	0.671**
	2	0.811	1.417	-1.135**	0.077**	-0.143	0.011	0.084**	2.319	4.229	-1.744**	0.278**	-0.911*	-0.133	0.385**
3	3	1.128	1.637	-1.278**	0.071**	0.011	0.130	0.028	1.593	2.973	-2.077**	0.113**	-0.060	0.076	0.100
	4	1.532	1.921	1.053**	0.127**	0.239	0.405**	-0.014	1.177	1.728	-0.618	0.093**	0.009	0.164	0.033
	5	1.457	3.538	2.975**	0.163**	0.030	0.141	-0.288**	0.659	1.039	0.986**	0.065**	0.041	0.250	-0.038

** and * denote *t*-statistics significant at the 5% and 10% confidence levels, respectively.

Table 5

Loadings on the Market Return, the Squared Market Return, and the Cubic Market Return from Time-Series Regressions of Unhedged Option Returns

This table reports loadings on the excess market return, R_M, the squared excess market return, and the cubic excess market return computed in a time-series regression for weekly calls, puts, and zero-beta straddles of five moneyness groups. The sample size is 535 weeks from January 1996 through September 2008 after removing the weeks when any of five moneyness groups have no observation. The *t*-statistics are reported below the corresponding loadings and are corrected for autocorrelation and heteroskedasticity using the Newey-West estimator with five legs. The last column reports the Gibbons-Ross-Shanken *F*-statistics and their corresponding *p*-values (%) testing the joint significance of the regression intercepts. The R^2s from each time series regression are reported in percentage form.

Moneyness Group	Delta	α	Beta to $R_M - R_f$	Beta to $(R_M - R_f)^2$	Beta to $(R_M - R_f)^3$	R^2 (%)	GRS
		Panel A: Call Options					
1	(65, 75]	-0.03	13.56	22.93	-1169.99	85.50	20.61
		(-5.53)	(25.53)	(2.91)	(-6.3)		(<0.01)
2	(55, 65]	-0.03	15.32	34.82	-1295.13	84.84	
		(-6.17)	(24.95)	(3.68)	(-5.74)		
3	(45, 55]	-0.04	17.41	50.36	-1534.08	83.13	
		(-6.66)	(23.69)	(4.48)	(-5.26)		
4	(35, 45]	-0.06	19.77	74.45	-1704.35	81.41	
		(-7.52)	(22.15)	(5.6)	(-4.62)		
5	(25, 35]	-0.08	22.35	110.51	-1776.36	76.64	
		(-8.01)	(19.36)	(6.69)	(-3.52)		
		Panel B: Put Options					
1	(-25, -35]	-0.07	-17.54	90.47	654.34	85.10	
		(-9.98)	(-27.26)	(5.74)	(2.42)		
2	(-35, -45]	-0.05	-16.84	74.34	782.14	85.32	
		(-8.41)	(-26.66)	(4.88)	(2.82)		
3	(-45, -55]	-0.04	-16.15	60.35	876.46	86.17	
		(-7.25)	(-27.33)	(4.64)	(3.52)		
4	(-55, -65]	-0.03	-15.39	48.26	906.06	86.49	
		(-5.95)	(-27.1)	(4.36)	(4.01)		
5	(-65, -75]	-0.02	-14.31	36.80	874.75	86.42	
		(-4.66)	(-26.09)	(3.87)	(4.2)		
		Panel C: Zero-Beta Straddles					
1	(65, 75]	-0.05	-1.04	56.38	-279.95	37.05	
		(-15.22)	(-5.25)	(8.58)	(-2.76)		
2	(55, 65]	-0.04	-0.38	55.42	-296.42	31.55	
		(-14.37)	(-2)	(8.24)	(-2.91)		
3	(45, 55]	-0.04	-0.01	56.20	-349.74	27.97	
		(-13.39)	(-0.06)	(8.21)	(-3.36)		
4	(35, 45]	-0.04	0.03	57.72	-355.46	25.00	
		(-12.22)	(0.11)	(7.55)	(-3)		
5	(25, 35]	-0.04	-0.25	58.25	-337.58	20.44	
		(-10.94)	(-0.79)	(6.61)	(-2.47)		

Table 6

Loadings on the Market Return and the NMP Return from Time-Series Regressions of Discretely Hedged Option Portfolios

This table reports loadings on the excess market return, R_M, and the NMP portfolio return, R_{NMP}, computed in a time-series regression for hedge portfolios of options with one month to expiration. The NMP portfolio return is defined as the difference in the rate of returns between the put options with the lowest strikes (M = 1) and the call options with the highest strikes (M = 5). The returns of hedge portfolios are calculated using the Black-Scholes model (Panel A), the stochastic volatility model (Panel B) and the jump diffusion model (Panel C). The *t*-statistics are also reported below the corresponding loadings and are corrected for autocorrelation and heteroskedasticity using the Newey-West estimator with five legs. The sample size is from January 1996 to September 2008. The last column reports the Gibbons-Ross-Shanken *F*-statistics and their corresponding *p*-values (%) testing the joint significance of the regression intercepts. The R^2s from each time series regression are reported in percentage form.

	Call Options				Put Options				
M	α	β_M	β_{NMP}	R^2 (%)	α	β_M	β_{NMP}	R^2 (%)	GRS
	Panel A: Black-Scholes Model								
1	0.000	0.018	0.052	54.49	0.008	0.246	0.993	75.55	4.11
	(0.559)	(3.267)	(4.145)		(1.154)	(3.479)	(6.488)		(0.01)
2	0.001	0.028	0.044	26.82	0.004	0.109	0.225	46.70	
	(1.214)	(2.131)	(2.579)		(1.268)	(2.556)	(3.965)		
3	0.002	0.043	0.016	4.73	0.004	0.046	0.051	16.23	
	(1.681)	(2.140)	(0.610)		(2.272)	(2.129)	(1.805)		
4	0.003	0.089	-0.034	3.12	0.002	0.036	0.004	4.59	
	(1.561)	(2.912)	(-0.934)		(2.749)	(2.528)	(0.259)		
5	0.010	0.171	-0.084	7.35	0.001	0.023	-0.005	3.47	
	(2.535)	(5.110)	(-1.444)		(2.715)	(3.031)	(-0.872)		
	Panel B: Stochastic Volatility Model								
1	0.001	0.028	0.047	28.44	0.013	0.241	0.426	48.53	4.76
	(2.364)	(3.153)	(3.239)		(4.575)	(4.680)	(5.344)		(<0.01)
2	0.002	0.052	0.055	24.35	0.007	0.186	0.240	37.20	
	(2.308)	(4.331)	(3.091)		(3.393)	(4.917)	(5.133)		
3	0.002	0.087	0.006	16.53	0.005	0.133	0.063	22.22	
	(2.068)	(5.149)	(0.332)		(3.585)	(5.511)	(2.728)		
4	0.002	0.129	-0.076	21.60	0.002	0.091	-0.023	21.23	
	(1.371)	(6.219)	(-2.777)		(2.497)	(6.510)	(-1.631)		
5	0.012	0.224	-0.466	53.30	0.001	0.045	-0.040	20.99	
	(4.537)	(4.540)	(-7.946)		(2.430)	(5.162)	(-2.767)		
	Panel C: Jump Diffusion Model								
1	0.001	0.022	0.085	48.68	0.012	0.189	0.803	71.09	4.51
	(1.538)	(2.879)	(3.214)		(3.399)	(4.638)	(6.064)		(<0.01)
2	0.002	0.065	0.060	33.08	0.008	0.251	0.424	49.25	
	(1.883)	(4.954)	(2.191)		(2.494)	(4.260)	(3.927)		
3	0.002	0.098	-0.022	17.15	0.004	0.173	0.049	25.83	
	(1.525)	(5.745)	(-0.868)		(2.546)	(5.435)	(1.244)		
4	0.003	0.115	-0.093	12.42	0.002	0.082	-0.034	15.28	
	(1.872)	(5.194)	(-2.460)		(2.154)	(6.275)	(-1.608)		
5	0.010	0.184	-0.221	23.69	0.001	0.032	-0.029	8.94	
	(3.439)	(5.680)	(-2.295)		(2.632)	(3.831)	(-1.987)		

Table 7

Cross-Sectional Regressions with Market, Squared and Cubic Market Loadings

This table presents cross-sectional regressions using the weekly returns on calls, puts, and zero-beta straddles. The sample size is 535 weeks from January 1996 through September 2008 after removing the weeks when any of five moneyness groups have no observation. The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time-series regression. The dependent variables in the regressions are the returns of calls, puts, and straddle positions. The *t*-statistics are reported below the corresponding loadings. Since the coefficient estimates, γ_M , γ_{SM}, γ_{CM}, and γ_{QM}, which are the premiums market, squared market, cubic market, and quartic market returns, are too small, they are presented after being multiplied by 10^2, 10^4, 10^6, and 10^8, respectively. The adjusted R^2 follows Jagannathan and Wang (1996) and is reported in percentage form. The *t*-statistics adjust for errors-in-variables and follows Shanken (1992). The last column reports χ^2-statistics and their corresponding *p*-values (%) for the test that the pricing errors in the regression are jointly zero.

γ_0	γ_M (x10^2)	γ_{SM} (x10^4)	γ_{CM} (x10^6)	Adj. R^2	χ^2
-0.023	0.400			98.16	92.55
(-5.751)	(4.023)				(0.00)
-0.006	0.350	-2.380		99.52	58.00
(-3.945)	(3.438)	(-3.263)			(0.00)
0.003	0.557	-1.846	30.676	99.89	16.44
(0.149)	(5.006)	(-2.159)	(4.463)		(12.57)

Table 8
Cross-Sectional Regressions with Market Loadings and NMP Factor Loadings

This table presents cross-sectional regressions using the rate of returns on ten hedge portfolios with one-month horizons. The full-sample factor loadings, which are the independent variables in the regressions, are computed in one multiple time-series regression. The returns of hedge portfolios, which are the dependent variables in the regressions, are computed using the Black-Scholes model (Panel A), the stochastic volatility model (Panel B) and the jump diffusion model (Panel C). Each panel reports results for the standard CAPM, three-moment CAPM and a combination of the three-moment CAPM and Fama-French model. The NMP portfolio returns, R_{NMP}, is defined as the difference in the rate of returns between the put options with the lowest strikes (M = 1) and the call options with the highest strikes (M = 5). The adjusted R^2 follows Jagannathan and Wang (1996) and is reported in percentage form. The t-statistics adjust for errors-in-variables and follows Shanken (1992). The last column reports χ^2-statistics and their corresponding p-values (%) for the test that the pricing errors in the regression are jointly zero. Each panel examines the sample period from January 1996 to September 2008. The intercept is in percentage form.

γ_0	γ_M	γ_{NMP}	Adj. R^2	χ^2
Panel A: Black-Scholes Model				
0.058	0.048		99.24	13.99
(0.434)	(3.245)			(8.19)
0.006	0.058	0.025	99.38	11.52
(0.074)	(2.518)	(2.887)		(11.74)
Panel B: Stochastic Volatility Model				
-0.235	0.066		88.83	12.10
(-1.862)	(3.801)			(14.67)
-0.193	0.062	0.010	90.11	12.79
(-2.126)	(3.884)	(1.928)		(7.75)
Panel C: Jump Diffusion Model				
-0.108	0.052		88.81	11.57
(-0.722)	(3.351)			(17.16)
-0.045	0.045	0.014	88.02	13.16
(-0.810)	(3.642)	(2.775)		(6.82)

Table 9

Longer-Term Options

This table summarizes results of the time-series and cross-sectional regressions for hedge portfolios of options with two and three months to expiration. The returns of hedge portfolios are computed using the Black-Scholes model, the stochastic volatility model and the jump diffusion model. Panel A reports the number of loadings on the excess market returns and the NMP portfolio return from time-series regression that are significant at the 5% confidence level. The NMP portfolio return is defined as the difference in the rate of returns between the put options with the lowest strikes (M = 1) and the call options with the highest strikes (M = 5). The F-statistics and their corresponding p-values (%) test the joint significance of the corresponding loadings. Panel B reports factor risk premiums from cross-sectional regressions using the full-sample factor loadings computed in the time-series regression. The adjusted R^2 follows Jagannathan and Wang (1996) and is reported in percentage form. The t-statistics adjust for errors-in-variables and follows Shanken (1992). The χ^2-statistics and their corresponding p-values (%) for testing the hypothesis that the pricing errors in the model are jointly zero are also reported. The sample period is from January 1996 to September 2008.

Panel A: Loadings on the Skewness Factors from Time-Series Regression at the 5% confidence level

		T = 1			T = 2			T = 3		
Model		α	β_M	β_{NMP}	α	β_M	β_{NMP}	α	β_M	β_{NMP}
Black-Scholes Model	# of Significant Loadings	4	10	4	7	8	3	8	7	5
	χ^2 test	4.11	>100	>100	5.20	>100	>100	7.17	>100	>100
		(0.01)	(<0.01)	(<0.01)	(0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
Stochastic Volatility Model	# of Significant Loadings	9	10	8	7	10	6	9	10	4
	χ^2 test	4.76	>100	>100	3.28	>100	>100	4.11	>100	>100
		(<0.01)	(<0.01)	(<0.01)	(0.30)	(<0.01)	(<0.01)	(0.14)	(<0.01)	(<0.01)
Jump Diffusion Model	# of Significant Loadings	6	10	7	9	10	6	9	9	6
	χ^2 test	4.51	>100	>100	3.57	>100	>100	6.55	>100	>100
		(<0.01)	(<0.01)	(<0.01)	(0.16)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)

Panel B: Cross-Sectional Regressions with the Market and Skewness Factor Loadings

	T = 2					T = 3				
Model	γ_0	γ_M	γ_{NMP}	Adj. R^2	χ^2	γ_0	γ_M	γ_{NMP}	Adj. R^2	χ^2
Black-Scholes Model	0.003	0.042	0.047	97.11	21.13	0.006	0.067	0.041	96.26	33.95
	(1.851)	(2.623)	(1.970)		(0.36)	(3.301)	(2.283)	(1.635)		(<0.01)
Stochastic Volatility Model	-0.001	0.060	0.027	81.92	9.19	0.001	0.104	0.029	92.11	21.54
	(-1.268)	(3.026)	(2.874)		(23.94)	(0.391)	(3.253)	(2.395)		(0.30)
Jump Diffusion Model	-0.002	0.058	0.023	97.44	8.55	0.003	0.085	0.027	83.83	40.01
	(-1.603)	(3.504)	(2.473)		(28.66)	(1.423)	(3.342)	(2.486)		(<0.01)

Table 10

Finite Sample Distribution of Factor Loadings and Risk Premiums

This table presents the finite sample distributions of the factor loadings, risk premiums and adjusted R^2s for hedge portfolios of options with one-month horizons. Using the stationary bootstrap, 10,000 time-series of hedge portfolio returns are generated under the null hypothesis that the estimated factor loadings and risk premiums are true. The average block length for the stationary bootstrap is computed based on Politis and White (2001), and is 1.92. The simulated returns are used to estimate a new set of factor loadings, factor risk premiums and cross-sectional adjusted R^2s. Panel A reports the finite sample distribution of the simulated betas along with the betas estimated under the null model. Panel B reports the small sample distribution of the risk premiums and the adjusted R^2s.

Panel A: Time-Series Regression

		β_M						β_{NMP}					
Model		Null	2.5	10	50	90	97.5	Null	2.5	10	50	90	97.5
Black-Scholes Model	11	0.018	0.007	0.011	0.018	0.026	0.031	0.052	0.022	0.033	0.051	0.067	0.074
	12	0.028	0.004	0.013	0.030	0.047	0.057	0.044	0.002	0.019	0.043	0.066	0.080
	13	0.043	0.007	0.021	0.046	0.072	0.087	0.016	-0.045	-0.021	0.014	0.048	0.067
	14	0.089	0.026	0.051	0.093	0.135	0.156	-0.034	-0.138	-0.092	-0.034	0.006	0.026
	15	0.171	0.097	0.125	0.180	0.237	0.268	-0.084	-0.268	-0.186	-0.082	-0.021	0.006
	21	0.246	0.095	0.152	0.251	0.349	0.408	0.993	0.635	0.764	0.978	1.174	1.259
	22	0.109	0.029	0.058	0.112	0.170	0.201	0.225	0.085	0.138	0.223	0.297	0.334
	23	0.046	0.006	0.021	0.049	0.077	0.091	0.051	-0.012	0.012	0.049	0.089	0.114
	24	0.036	0.010	0.019	0.037	0.055	0.065	0.004	-0.033	-0.017	0.003	0.020	0.030
	25	0.023	0.007	0.013	0.024	0.034	0.039	-0.005	-0.024	-0.015	-0.005	0.001	0.004
Stochastic Volatility Model	11	0.028	0.011	0.017	0.029	0.042	0.049	0.047	0.014	0.025	0.046	0.062	0.071
	12	0.052	0.034	0.040	0.052	0.070	0.080	0.055	0.013	0.027	0.054	0.076	0.086
	13	0.087	0.063	0.071	0.088	0.110	0.120	0.006	-0.041	-0.022	0.006	0.029	0.043
	14	0.129	0.098	0.108	0.130	0.158	0.173	-0.076	-0.165	-0.128	-0.075	-0.042	-0.030
	15	0.224	0.128	0.158	0.221	0.307	0.360	-0.466	-0.595	-0.547	-0.468	-0.403	-0.360
	21	0.241	0.144	0.177	0.242	0.320	0.366	0.426	0.288	0.338	0.423	0.531	0.618
	22	0.186	0.128	0.145	0.187	0.242	0.276	0.240	0.150	0.182	0.239	0.296	0.334
	23	0.133	0.098	0.109	0.135	0.169	0.185	0.063	0.009	0.029	0.064	0.095	0.117
	24	0.091	0.069	0.076	0.092	0.112	0.123	-0.023	-0.064	-0.048	-0.022	-0.005	0.004
	25	0.045	0.029	0.034	0.045	0.060	0.069	-0.040	-0.080	-0.064	-0.039	-0.023	-0.019
Jump Diffusion Model	11	0.022	0.006	0.011	0.022	0.033	0.039	0.085	0.031	0.048	0.085	0.117	0.130
	12	0.065	0.043	0.050	0.065	0.083	0.093	0.060	0.003	0.022	0.061	0.093	0.107
	13	0.098	0.068	0.078	0.099	0.122	0.136	-0.022	-0.079	-0.058	-0.021	0.011	0.028
	14	0.115	0.069	0.085	0.117	0.149	0.169	-0.093	-0.185	-0.151	-0.092	-0.046	-0.027
	15	0.184	0.111	0.136	0.185	0.237	0.268	-0.221	-0.442	-0.362	-0.215	-0.102	-0.064
	21	0.189	0.095	0.127	0.188	0.250	0.283	0.803	0.503	0.611	0.810	0.967	1.029
	22	0.251	0.149	0.184	0.251	0.328	0.372	0.424	0.194	0.270	0.424	0.561	0.622
	23	0.173	0.119	0.138	0.175	0.216	0.238	0.049	-0.040	-0.009	0.050	0.103	0.127
	24	0.082	0.057	0.066	0.083	0.101	0.112	-0.034	-0.086	-0.066	-0.034	-0.006	0.007
	25	0.032	0.013	0.020	0.032	0.043	0.048	-0.029	-0.063	-0.051	-0.029	-0.012	-0.005

Panel B: Cross-Sectional Regression

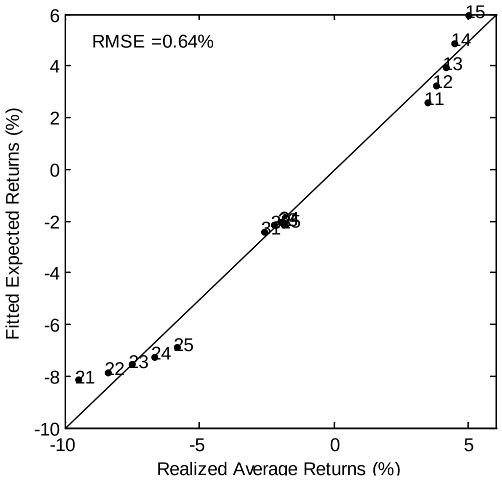
		Null	2.5	10	50	90	97.5
Black-Scholes Model	γ_0	0.000	-0.001	-0.001	0.000	0.002	0.003
	γ_M	0.058	0.019	0.029	0.053	0.084	0.104
	γ_{NMP}	0.025	0.008	0.014	0.026	0.039	0.047
	Adj. R^2	99.38	92.79	95.99	98.58	99.46	99.70
Stochastic Volatility Model	γ_0	-0.002	-0.003	-0.002	-0.002	-0.001	0.000
	γ_M	0.062	0.033	0.041	0.060	0.082	0.097
	γ_{NMP}	0.010	0.003	0.005	0.010	0.015	0.018
	Adj. R^2	90.11	60.16	74.12	88.34	95.03	97.06
Jump Diffusion Model	γ_0	0.000	-0.002	-0.001	0.000	0.001	0.002
	γ_M	0.045	0.012	0.022	0.043	0.069	0.085
	γ_{NMP}	0.014	0.002	0.006	0.014	0.022	0.025
	Adj. R^2	88.02	58.17	71.03	87.34	95.54	97.51

Figure 1

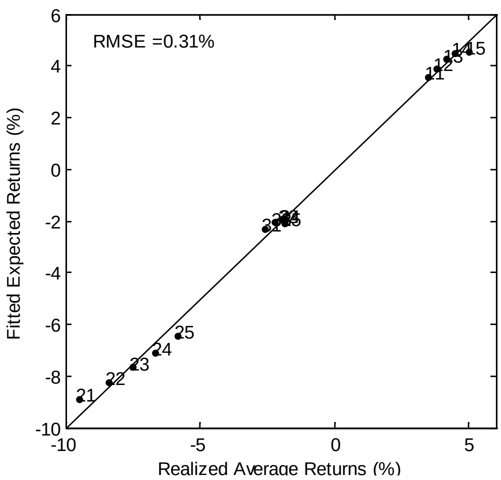
Fitted Expected Returns Versus Average Realized Returns for Naked Options

Each panel in this figure shows realized average returns (%) on the horizontal axis and fitted expected returns (%) on the vertical axis for weekly returns of S&P 500 index options positions. For each hedge portfolio, the realized average return is the time-series average of the option return and the fitted expected return is the fitted value for the expected return found using the standard CAPM (Panel A), the three-moment CAPM (Panel B), and four-moment CAPM (Panel C). Each two-digit number in the figure represents a separate portfolio. The first digit refers to the option type (1 for call, 2 for put and 3 for zero-beta straddle) while the second digit refers to the moneyness group (1 for the lowest strike price and 5 for the highest). The straight line is the 45-degree line from the origin. The sample period is from January 1996 to September 2008.

Panel A: Standard CAPM



Panel B Three-Moment CAPM



Panel A: Four-Moment CAPM

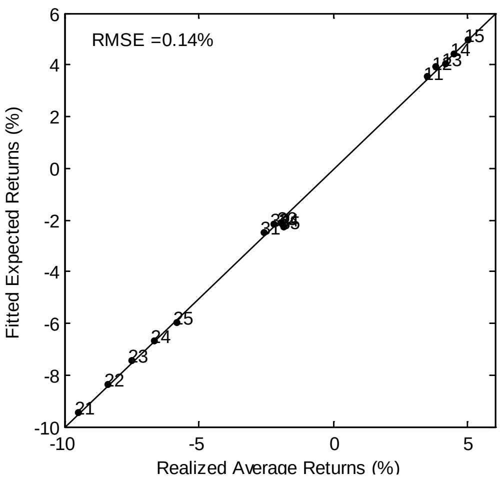


Figure 2
Fitted Expected Returns Versus Average Realized Returns for Discretely Hedged Option Portfolios

Each panel in this figure shows realized average returns (%) on the horizontal axis and fitted expected returns (%) on the vertical axis for ten hedge portfolios of different option types and moneyness groups. The returns on hedge portfolios are computed using the Black-Scholes model, the stochastic volatility model and the jump diffusion model. For each hedge portfolio, the realized average return is the time-series average of the portfolio return and the fitted expected return is the fitted value for the expected return found using the three-moment CAPM. Each two-digit number in the figure represents a separate portfolio. The first digit refers to the option type (1 for call and 2 for put) while the second digit refers to the moneyness group (1 for the lowest strike price and 5 for the highest). The straight line is the 45-degree line from the origin. The sample period is from January 1996 to September 2008.