A study on the forecasting performance of range volatility estimators with nonlinear filters

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Abstracts

This study is on the forecasting performance of three range volatility estimators with ARMA, GARCH, and 2-regime SETAR fiters using the KOSPI 200 daily opening, highest, lowest, and closing prices. RMSE has been used as an evaluation criterion. The results of this analysis can be summarized as follows: First, three range volatility estimators showed relatively inferior forecasting performance in volatile period but relatively superior one in stable period. Second, Parkinson, and Garman and Klass volatility estimators had much better forecasting performance with a linear ARMA filter while Rogers and Satchell volatility estimator did with nonlinear GARCH and 2-regime SETAR filters. Third, a linear ARMA filter contributed to produce superior forecasting performance with Parkinson, and Garman and Klass volatility estimators while a nonlinear 2-regime SETAR filter did with Rogers and Satchell volatility estimator which was designed to reflect trend into price process. It is not surprising that a GARCH filter presented always-not-bad forecasting performance regardless of market conditions and the type of range volatility estimators.

Keywords: range volatility estimators, forecasting performance, nonlinear filters

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Ⅰ. Introduction

The volatility of various asset returns is an important factor in option pricing, asset allocation strategies, risk management and so on. This can make scholars and experts who concern financial markets to have quite a strong interest in how to measure and forecast volatility. Volatility can be conventionally estimated as the standard deviation of returns which are calculated with close prices during a certain period of time. This measurement cannot be the best option due to the noise added to true volatility despite the convenience of measurement(Andersen and Bollerslev, 1998).

One of alternative volatility measurements was proposed by Parkinson(1980). He designed his own volatility estimator in the name of 'range variance' from the very concept of range(maximum-minimum, that is, the highest price-the lowest price), and proved that it was 2.5 to 5 times efficient than conventional volatility measure. Garman and Klass(1980) also proposed their own volatility estimator which corrected Parkinson volatility estimator's downward bias problem caused by discrete trading in the same year. Rogers and Satchell(1991) made a revolution in this field. They included the trend in price process to design much more realistic volatility estimator. These three range volatility estimators were evaluated 5 to 14 times efficient than conventional one(Vipul and Jacob, 2007). They have another benefit of informational content due to the use of the open, high, low, and close prices in the calculation of volatility estimators other than the efficiency.

The use of high and low prices along with open and close ones is not common in academia but in technical analysis of asset prices open, close, high, and low prices are broadly used in the form of open-close-high-low bar charts. These charts are the starting point of graphic/quantitative analysis(Lildholdt, 2002).

This study is to evaluate the forecasting performance of three range volatility estimators using a realized volatility as a benchmark. Three range volatility estimators including Parkinson, Garman and Klass, and Rogers and Satchell volatility estimators were calculated by their own formulas with log-transformed price series of the KOSPI 200 daily open, high, low, and close price series from January 3, 2000 to June 30, 2011. The entire sample was divided into 5 sub-samples after detecting 4 structural change points through the dynamic programming algorithm proposed by Bai and Perron(2003) for more detailed analysis. I used two different kinds of forecasting filters like a linear ARMA(AutoRegressive Moving Average)(1,1) model and nonlinear GARCH(1,1) and 2-regime SETAR(Self-Exciting Threshold AutoRegressive) models. RMSE(Root Mean Square Error) was used as the comparison criterion on the forecasting performance of three range volatility estimators.

Just few previous studies can be found on this topic.

Vipul and Jacob(2007) compared three range volatility estimators with historical volatility estimator in terms of forecasting performance by using two-scale realized volatility proposed by Zhang et al.(2005) as a benchmark. They used Indian Nifty index high-frequency data from January 1, 2001 to December 30, 2005. The same range volatility estimators but somewhat different forecasting filters were used in comparison of those of my study. RW(Random Walk), MA(Moving Average), EWMA(Exponentially Weighted Moving Average), AR(AutoRegressive), GJR-GARCH models were chosen as forecasting filters in their analysis. They evaluated the forecasting performance of all four volatility estimators(three range volatility estimators and one historical volatility estimator) with two different criteria. The former was MSE(Mean Squared Error) with respect to efficiency and the latter was MRB(Mean Relative Bias) with respect to bias. The results of their analysis present that range volatility estimators are superior to historical volatility estimator in terms of short-run and long-run forecasting performance.

Jacob and Vipul(2008) changed their focus of analysis from high frequency level to much lower frequency level, daily data which have much higher accessibility. They used the same realized volatility and comparison criteria as their previous study but only linear forecasting filters such as AR(AutoRegressive), MA(Moving Average), EWMA(Exponentially Weighted Moving Average), ARMA(AutoRegressive Moving Average) models. Their study reached the same conclusions as their previous study in spite of using different frequency data. This can lead people in financial field to use range volatility estimators more often with much stronger confidence of their forecasting performance.

Kim and Park(2009) applied the framework of the two previous studies to Korean stock markets. They used KOSPI 200 tick-by-tick data from January 2, 2006 to May 10, 2008 and limited the scope of forecasting filters to linear ones like AR(AutoRegressive), MA(Moving Average), and ARMA(AutoRegressive Moving Average) models. It is noted that three range volatility estimators tend to show quite different forecasting performance according to the condition of sub-samples. They first seperated the entire sample into two sub-samples by the result of Zivot and Andrews' unit root test. While three range volatility estimators showed better forecasting performance in relatively stable sub-sample, historical volatility estimator did in relatively volatile sub-sample. One surprising finding was Rogers and Satchell volatility estimator presented remarkably superior forecasting performance to the other two range volatility estimators and historical volatility estimator. The reason of this striking result could be the design of Rogers and Satchell volatility estimator, that is, to incorporate trend into price process for making different market conditions structured in this volatility estimator.

This study is organized as follows. How to measure three range volatility estimators and realized volatility is explained in section 2. Three forecasting filters are introduced in section 3. The comparison criterion of forecasting performance, RMSE(Root Mean Square Error) is presented in section 4. The data to be used in this study are described in section 5. Analysis methodologies and their application results are presented in section 6. Section 7 concludes this study.

Ⅱ. Volatility estimators

1. Range volatility estimators

1.1 Parkinson volatility estimator

Parkinson(1980) proposed a range volatility estimator as an alternative to conventional volatility estimators. He introduced the concept of 'range variance' instead of estimating the variance of stock's returns to measure the true volatility. The range as the difference between the highest price and the lowest price was incorporated in the formula of his volatility estimator. Building on the work of Feller(1951), Parkinson showed that, under the assumption of a lognormal diffusion, his volatility estimator tended to have 2.5 to 5 times smaller standard error than conventional volatility estimators. Beckers(1983) wrote that Parkinson's range volatility estimator would be downward-biased due to the approximation of the true high and low values of a Brownian motion by the high and low values of a random walk.

The range volatility estimator Parkinson suggested is as follows.

$$\hat{\sigma}^2_{PK} = \frac{1}{4\ln 2}\frac{1}{n}\sum_{t=1}^{n}(H_t - L_t)^2 \tag{1}$$

where H_t and L_t are the log-transformed high and low prices during the trading day t respectively and n is the number of trading days.

Daily data were used in this study so Parkinson volatility estimator was corrected in the case of $n=1$ like equation (2).

$$\hat{\sigma}^2_{PK} = \frac{1}{4\ln 2}(H_t - L_t)^2 \tag{2}$$

1.2 Garman and Klass volatility estimator

Garman and Klass(1980) pointed out that Parkinson volatility estimator could be downward biased due to discrete trading through simulation from the idea that true high and low prices could not be observed in reality. They added open and close prices into the formula of their volatility estimator other than high and low ones which were inputs of Parkinson volatility estimator like equation (3).

$$\begin{aligned}\hat{\sigma}^2_{GK} = \frac{1}{n}\sum_{t=1}^{n}[0.511(H_t - L_t)^2 - 0.019(C_t - O_t)(H_t + L_t - 2O_t) \\ - 2(H_t - O_t)(L_t - O_t) - 0.383(C_t - O_t)^2]\end{aligned} \tag{3}$$

where H_t, L_t, O_t, C_t are the log-transformed high, low, open and close prices during the trading day t respectively and n is the number of trading days.

The efficiency gain from including open and close prices was remarkable: MSE(Mean Squared Error) decreased by 30% compared with Parkinson's volatility estimator but these two volatility estimators can face overestimation problem caused by not including trend in price process. This matter can be much serious when financial markets have a clear trend of boom or bust.

Daily data were used in this study so Garman and Klass volatility estimator was corrected in the case of $n = 1$ like equation (4).

$$\begin{aligned}\hat{\sigma}^2_{GK} = 0.511(H_t - L_t)^2 - 0.019(C_t - O_t)(H_t + L_t - 2O_t) \\ - 2(H_t - O_t)(L_t - O_t) - 0.383(C_t - O_t)^2\end{aligned} \tag{4}$$

1.3 Rogers and Satchell volatility estimator

Rogers and Satchell(1991) developed their own volatility estimator by including trend in it to settle with the overestimation problem of Parkinson, and Garman and Klass volatility estimators like equation (5).

$$\hat{\sigma}^2_{RS} = \frac{1}{n}\sum_{t=1}^{n}[(H_t - C_t)(H_t - O_t) + (L_t - C_t)(L_t - O_t)] \tag{5}$$

where H_t, L_t, O_t, C_t are the log-transformed high, low, open and close prices during the trading day t respectively and n is the number of trading days.

Daily data were used in this study so Rogers and Satchell volatility estimator was corrected in the case of $n = 1$ like equation (6).

$$\hat{\sigma}^2_{RS} = (H_t - C_t)(H_t - O_t) + (L_t - C_t)(L_t - O_t) \tag{6}$$

2. The measurement of realized volatility

RMSE(Root Mean Square Error) was used as an evaluation criterion on the forecasting performance of range volatility estimators in this study. The predicted value of a certain range volatility estimator and the realized volatility one are used as inputs of RMSE like equation (20). I separated the entire sample into two parts, the first two thirds and the latter one third of it. To find out the exact specification of the fittest forecasting filter, the first two thirds of the entire sample was used for estimation. After getting the specification of three different forecasting filters including ARMA(AutoRegressive Moving Average), GARCH and 2-regime SETAR models, I applied specific specification of three forecasting filters to h-length rolling window in the latter one third of the entire sample to get the estimates of parameters. At last, a one-step-ahead forecast, the $(h+1)$th predicted value of range volatility estimators was calculated with inputs of observations and parameter estimates. I applied the same logic to the measurement of realized volatility as an input of the calculation of RMSE(Root Mean Square Error) so used h-length rolling window to calculate realized volatility like equation (7), which helped solve out noise matter as well.

$$\sigma = \sqrt{\frac{250}{h-1}} \sum_{\tau = t}^{t-1+h} (r_\tau - \mu)^2 \tag{7}$$

where σ is a realized volatility and h is the length of rolling window. $\sqrt{\frac{250}{h-1}}$ is a multiplier to transform daily volatility into yearly one in the case of 250 trading days. μ is the ex post average returns during the following h days from a specific day. r_τ is the returns based on close price of τ day.

Ⅲ. Forecasting filters

Three different forecasting filters were used in this study. I chose them for the following reasons. The same data, KOSPI 200, was used in the previous study of mine(Kim and Park, 2009), but the frequency of data and the type of forecasting filters are different. While only linear models including AR(AutoRegressive), MA(Moving Average), and ARMA(AutoRegressive Moving Average) models were used as forecasting filters on KOSPI 200 tick-by-tick data in Kim and Park(2009), the two nonlinear models including GARCH and 2-regime SETAR(Self-Exciting Threshold AutoRegressive) models were added as forecasting filters on KOSPI 200 daily data in this study. It is noted that it can be the first time to use a regime-shift model as a forecasting filter so how it helps improve the forecasting performance of range volatility estimators can be the main concern.

1. ARMA(AutoRegressive Moving Average) model: ARMA(1,1)

The specification of ARMA(1,1) model is as follows.

$$x_t = \phi_1 x_{t-1} + \epsilon_t - \theta_1 \epsilon_{t-1} \tag{8}$$

The assumption on parameters, $|\phi_1| < 1$(stationarity condition) and $|\theta_1| < 1$(invertibility condition) should be met in this model.

2. GARCH model: GARCH(1,1)

GARCH model is known to be fitted to the data which is characteristic of fat tails or leptokurtosis. The descriptive statistics of three range volatility estimators in this study show this character so GARCH model can be expected to be a good candidate as a forecasting filter. The specification of GARCH(1,1) model is as follows.

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 \epsilon_{t-1}^2 \tag{9}$$

The weak stationarity condition of this model, $\alpha_1 + \beta_1 < 1$, should be met and α_0 should be greater than zero in unconditional variance, $\sigma^2 = \alpha_0 (1 - \alpha_1 - \beta_1)^{-1}$ at all times.

Range-based ML(Maximum Likelihood) estimates were calculated compared to the conventional return-based ML estimation technique. The family of Normal-GARCH models can be defined by

$$r_i = \mu(\theta) + e_i, \qquad e_i \sim \ni d(0, \sigma_i^2) \tag{10}$$

$$\sigma_i^2 = \psi(\Omega_i, \theta) \tag{11}$$

where $\psi(\bullet)$ denotes a deterministic function and Ω_i the information set as of time i,

$$\Omega_i = \{r_{i-1}, r_{i-2}, r_{i-3}, \cdots\}$$

and θ is a parameter vector. As an example, this model nests the popular GARCH(1,1) model of Bollerslev(1986)

$$\sigma_i^2 = \alpha_0 + \alpha_1 e_{i-1}^2 + \beta_1 \sigma_{i-1}^2 \tag{12}$$

where $\theta'' = (\alpha_0, \alpha_1, \beta_1)$

For doing range-based ML estimation, we assume that four different intradaily prices for the Normal-GARCH model are governed by a Brownian motion on a unit interval.

$$dP_i(t) = \mu dt + \sigma_i dW(t), \quad 0 \le t \le 1 \tag{13}$$

We can define the normalized high a_i, the normalized low b_i, and the normalized close r_i relative to the open price by

$$a_i = \sup_{0 \le t \le 1} P_i(t) - P_i(0) \tag{14}$$

$$b_i = P_i(0) - \inf_{0 \le t \le 1} P_i(t) \tag{15}$$

$$r_i = P_i(1) - P_i(0) \tag{16}$$

where $P_i(t)$ denotes the log-price of asset p on day i at time t.

Estimation of Normal-GARCH models are typically performed by Maximum Likelihood. The conditional likelihood function is

$$L(r_1, r_2, \cdots, r_I | \theta) = \prod_{i=m+1}^{I} g(r_i; \sigma_i(\theta), \mu(\theta))$$

where $g(r_i; \sigma_i(\theta), \mu(\theta))$ denotes the density function for a normally distributed variable r_i with mean μ and standard deviation σ_i. $\sigma_i(\theta)$ is computed recursively via equation (11). Estimation proceeds by maximizing ln( ) of the likelihood function above, where presample values of σ_i^2 are set to the unconditional sample variance. We denote return-based ML-estimates of a Normal-GARCH by $\sim$,

$$\tilde{\theta} = \underset{\theta}{arg\ max} \sum_{i=m+1}^{I} \left[-0.5 \ln(2\pi) - 0.5 \left(\frac{r_i - \mu(\theta)}{\sigma_i(\theta)} \right)^2 - 0.5 \ln\left(\sigma_i^2(\theta)\right) \right] \tag{17}$$

In words, the ML procedure chooses the parameter vector θ that makes the observed daily returns, r_i's, most probable. So the standard deviation on day i is inferred from the daily return r_i. Our idea is to incorporate information on daily high's and low's in addition to the daily return r_i. Specifically, we are going to include information on daily high's and low's in the density for r_i:

$$L(r_1, r_2, \cdots, r_I, a_1, a_2, \cdots, a_I, b_1, b_2, \cdots, b_I \mid \theta) = \prod_{i=m+1}^{I} f(a_i, b_i, r_i; \sigma_i(\theta), \mu(\theta))$$

where $f(a_i, b_i, r_i; \sigma_i(\theta), \mu(\theta))$ denotes the joint density of the normalized close r_i, the normalized high a_i, and the normalized low b_i, for a diffusion process with diffusion coefficient $\sigma_i(\theta)$ and drift $\mu(\theta)$. We denote ML-estimates based on open, high, low and close prices of a Normal-GARCH model by $\hat{}$,

$$\hat{\theta} = \underset{\theta}{arg\ max} \sum_{i=m+1}^{I} \ln\left[f(a_i, b_i, r_i; \sigma_i(\theta), \mu(\theta))\right] \tag{18}$$

3. 2-regime self-exciting threshold autoregressive model: 2-regime SETAR(2)

2-regime SETAR model belongs to threshold models proposed by Tong(1978) and has a great virtue that the parameter estimation can be made linearly by regimes but the entire model still has nonlinearity.

If $\{x_t\}$ follows 2-regime SETAR(2) model with x_{t-1} as a threshold variable, the specification of 2-regime SETAR(2) model is as follows.

$$\begin{aligned} x_t &= \phi_0^{(1)} + \phi_1^{(1)} x_{t-1} + \phi_2^{(1)} x_{t-2} + \epsilon_t^{(1)} \qquad (x_{t-1} < \tau) \\ x_t &= \phi_0^{(2)} + \phi_1^{(2)} x_{t-1} + \phi_2^{(2)} x_{t-2} + \epsilon_t^{(2)} \qquad (x_{t-1} \geq \tau) \end{aligned} \tag{19}$$

where x_{t-1} is a threshold variable and τ is a threshold value which can be estimated with data. Although this model is composed of two linear AR models, it can be classified as a nonlinear model with 2 regimes as a whole.

Ⅳ. The evaluation criterion on forecasting performance

RMSE(Root Mean Square Error) were used as an evaluation criterion on forecasting performance of three range volatility estimators. The formula to calculate RMSE is as follows.

$$RMSE = \sqrt{\mathrm{E}\left(\hat{\sigma}_t - \sigma_t\right)^2} \qquad (20)$$

where $\hat{\sigma}_t$ is the square root of range volatility estimates and σ_t is the realized volatility value calculated by equation (7).

Ⅴ. Data

This study is to compare the forecasting performance of three range volatility estimators by sub-samples and forecasting filters. For this analysis, the raw data, KOSPI 200 daily open, high, low, close price series were downloaded from FnGuide version 3.0. Parkinson, Garman and Klass, and Rogers and Satchtell volatility estimators were calculated by equations of (2), (4), and (6) after log-transformation of raw data series. Return series based on close prices were calculated for getting realized volatility as well. These four time series were the dataset for this study. Sample period covers January 3, 2000 to June 30, 2011.

Ⅵ. Empirical analysis

1. Structural change point detection and the division of entire sample period into several sub-sample periods

For more detailed analysis, I divided the entire sample into several sub-samples by structural change point detection method. Although Qu and Perron(2007) method could be effectively used due to multiple number of time series in this study, I applied Bai and Perron(2003) method to each volatility estimator time series just because three range volatility estimators had their own characteristics and level of evolution.

The stationarity test was done on each time series of the entire sample before the application of Bai and Perron(2003) method. The most frequently used unit root tests, ADF(Augmented Dickey-Fuller) and PP(Phillips-Perron) ones were performed. Test results showed that all three range volatility estimator series were stationary so Bai and Perron(2003) method could be applied to them.

<Table 1 > Stationarity test on the entire sample

The stationarity of time series under the study should be met before the application of Bai and Perron(2003)'s structural change point detection method. ADF(Augmented Dickey-Fuller) and PP(Phillips-Perron) unit root test results show that three range volatility estimator series are all stationary. The null hypothesis of these two unit root tests is that 'this time series has one unit root,' that is, 'it is not stationary.' The null hypothesis has been rejected even at 1% significance level.

	PK	GK	RS
ADF test	-7.838***	-7.742***	-7.486***
PP test	-1719.556***	-1390.783***	-1880.813***

The stationarity test results presented that each range volatility estimator series was stationary so no further steps were not needed before the application of Bai and Perron(2003) method. The number of structural change points were estimated by the standard of the minimum SSR(Residual Sum of Squares) and BIC(Bayesian Information Criterion) as a result of dynamic programming algorithm proposed by Bai and Perron(2003).

Test results showed that 5 sub-samples were the most appropriate from the 4 structural change points. The estimation of the number of observations which corresponded to structural change points presented that Parkinson, and Garman and Klass volatility estimators had exactly the same points but Rogers and Satchell volatility estimator did slightly different points compared to the other volatility estimators. This results mean that Parkinson volatility estimator, and Garman and Klass volatility estimator can be grouped in contrast with Rogers and Satchell volatility estimator.

<Table 2> Structural change point detection: Bai and Perron(2003)

The stationarity test results presented that each range volatility estimator series was stationary so no further steps were not needed before the application of Bai and Perron(2003) method. The number of structural change points were estimated by the standard of the minimum SSR(Residual Sum of Squares) and BIC(Bayesian Information Criterion) as a result of dynamic programming algorithm proposed by Bai and Perron(2003). The shaded areas in this table indicate the minimum cases in terms of RSS and BIC.

Panel A: Parkinson volatility estimator						
	0	1	2	3	4	5
RSS	0.000347	0.000331	0.000327	0.000313	0.000310	0.000399
BIC	-37159.42	-37279.42	-37298.50	-37403.96	-37411.46	-37400.59

Panel B: Garman and Klass volatility estimator						
	0	1	2	3	4	5
RSS	0.001147	0.001098	0.001086	0.001042	0.001033	0.001031
BIC	-33761.63	-33869.46	-33885.00	-33986.59	-33995.54	-33983.57

Panel A: Rogers and Satchell volatility estimator						
	0	1	2	3	4	5
RSS	0.000393	0.000378	0.000373	0.000359	0.000356	0.000356
BIC	-36807.37	-36902.89	-36918.24	-37011.00	-37024.14	-37009.78

<Table 3> The observation numbers which correspond to structural change points

The observation numbers which corresponded to structural change points were estimated. The estimation results showed that Parkinson, and Garman and Klass volatility estimators had 100% identical observation numbers but Rogers and Satchell volatility estimator had different observation number which belonged to the second change point compared to the other volatility estimators. This results indicate that Parkinson, and Garman and Klass volatility estimators have lots of similarity in comparison with Rogers and Satchell volatility estimator. PK, GK, RS indicate ParKinson, Garman and Klass, Rogers and Satchell volatility estimators, respectively.

change point	the first	the second	the third	the fourth
PK	426	852	1915	2341
GK	426	853	1915	2341
RS	426	1110	1915	2341

<Table 4> The information on 5 sub-sample periods

Parkinson, and Garman and Klass volatility estimators have uneven numbers of observations in sub-samples. Specifically the number of observations of the third sub-sample is more than double compared with the number of observations of the other sub-samples. Satchell volatility estimator says a different story. It has relatively even number of observations in each sub-sample. SS stands for Sub-Sample in this table.

Panel A: Garman and Klass volatility estimators						
		SS1	SS2	SS3	SS4	SS5
period	start	2000/01/05	2001/10/04	2003/06/27	2007/10/12	2009/07/01
	end	2001/09/28	2003/06/26	2007/10/11	2009/06/30	2011/06/30
obs. interval		1~425	426~851	852~1914	1915~2340	2341~2842
# of obs.		425	426	1063	426	502
ratio(%)		14.95	14.99	37.40	14.99	17.66

Panel B: Rogers and Satchell volatility estimator						
		SS1	SS2	SS3	SS4	SS5
period	start	2000/01/05	2001/10/04	2004/07/16	2007/10/12	2009/07/01
	end	2001/09/28	2004/07/15	2007/10/11	2009/06/30	2011/06/30
obs. interval		1~425	426~1109	1110~1914	1915~2340	2341~2842
# of obs.		425	684	805	426	502
ratio(%)		14.95	24.07	28.33	14.99	17.66

2. The structure and nature of data: 5 sub-samples

The forecasting performance of three range volatility estimators was compared with a linear ARMA(AutoRegressive Moving Average) filter and nonlinear GARCH and 2-regime SETAR(2-regime Self-Exciting Threshold Autoregressive) filters in this study. Hence the nature of data in 5 sub-samples needs to be exactly understood to achieve the goal of this study.

The nature of data can be summarized as follows as in <Table 5>.

First, the three range volatility estimator series of 5 sub-samples don't follow a normal distribution. They have positive skewness and relatively large kurtosis so the shape of their probability density curves is long-tailed on the left and has a sharp top and fat tails. These two statistics tend to be the greatest in the fourth sub-sample.

Second, the average volatility level is the highest in the first sub-sample and the lowest in the fifth sub-sample. The first and fourth sub-samples can be classified as relatively higher volatility group and the second, third, and fifth sub-samples can be classified as relatively lower volatility group by this standard.

Third, in terms of standard deviation the fourth sub-sample is the most volatile. While Parkinson, and Rogers and Satchell volatility estimators are the least volatile in the fifth sub-sample, Garman and Klass volatility estimator are in the third sub-sample.

Fourth, the first and fourth sub-samples can be considered as the most volatile while the fifth sub-sample as the least volatile with respect to the average and standard deviation of volatility.

<표 5> The descriptive statistics on 5 sub-samples

The three range volatility estimator series of 5 sub-samples don't follow a normal distribution. The shape of their probability density curves is long-tailed on the left and has a sharp top and fat tails. Skewness and kurtosis tend to be the greatest in the fourth sub-sample. The first and fourth sub-samples can be considered as the most volatile while the fifth sub-sample as the least volatile with respect to the average and standard deviation of volatility. The shaded areas and underlined areas in this table indicate the largest value and the smallest one, respectively.

Panel A: Parkinson volatility estimator					
	SS1	SS2	SS3	SS4	SS5
mean	0.000366	0.000235	0.000104	0.000327	0.000075
max	0.002677	0.001443	0.002227	0.009178	0.001003
min	0.000022	0.000011	0.000007	0.000011	0.000008
s.d.	0.000358	0.000209	0.000138	0.000710	0.000081
# of obs.	425	426	1063	426	502
skewness	2.253	2.409	6.792	7.508	4.758
kurtosis	6.858	7.057	75.167	73.098	39.929
JB test	1167.1***	1269.3***	256021.2***	96575.8***	34551.81***

Panel B: Garman and Klass volatility estimator					
	SS1	SS2	SS3	SS4	SS5
mean	0.000638	0.000384	0.000174	0.000581	0.000131
max	0.005028	0.002344	0.002378	0.018170	0.002089
min	0.000015	0.000029	0.000007	0.000018	0.000007
s.d.	0.000712	0.000335	0.000138	0.001301	0.000142
# of obs.	425	426	1063	426	502
skewness	2.748	2.201	4.398	8.123	6.348
kurtosis	9.893	5.950	31.323	89.324	73.197
JB test	2218.8***	952.5***	46448.9***	142937.8***	113166.9***

Panel C: Rogers and Satchell volatility estimator					
	SS1	SS2	SS3	SS4	SS5
mean	0.000360	0.000177	0.000085	0.000335	0.000075
max	0.004607	0.001441	0.001006	0.010058	0.001182
min	0.000000	0.000000	0.000000	0.000000	0.000000
s.d.	0.000464	0.000176	0.000087	0.000732	0.000080
# of obs.	425	584	805	426	502
skewness	3.916	2.824	3.547	7.712	6.318
kurtosis	23.266	11.194	23.145	82.004	73.295
JB test	10429.1***	10803.5***	19415.7***	120739.5***	113428.8***

The three forecasting filters such as ARMA, GARCH, 2-regime SETAR models are used in this study. It is necessary to test the stationarity of all time series in each sub-samples before the application of these forecasting filters. <Table 6> shows the stationarity test results. ADF(Augmented Dickey-Fuller) and PP(Phillips-Perron) unit root tests were performed on each range volatility estimator series of each sub-sample. The stationarity of all range volatility estimator series in every sub-sample was guaranteed as a result of two unit root tests as in <Table 6>.

<Table 6> Stationarity test on 5 sub-samples

The stationarity of three range volatility estimators in each sub-sample should be checked before the application of ARMA, GARCH, 2-regime SETAR forecasting filters. ADF(Augmented Dickey-Fuller) and PP(Phillips-Perron) unit root tests were performed for this purpose. The null hypothesis of these two unit root tests is that 'this time series has one unit root,' that is, 'it is not stationary.' The null hypothesis has been rejected even at 1% significance level. Therefore the three forecasting filters can be applied to range volatility estimator series of every sub-sample. *** in this table means the rejection of null hypothesis at 1% significance level.

Panel A: Parkinson volatility estimator					
	SS1	SS2	SS3	SS4	SS5
ADF	-5.383***	-4.564***	-6.660***	-4.468***	-6.561***
PP	-319.462***	-396.939***	-862.583***	-191.833***	-452.419***

Panel B: Garman and Klass volatility estimator					
	SS1	SS2	SS3	SS4	SS5
ADF	-6.292***	-4.804***	-6.425***	-4.695***	-6.719***
PP	-323.218***	-450.572***	-687.255***	-142.111***	-401.729***

Panel C: Rogers and Satchell volatility estimator					
	SS1	SS2	SS3	SS4	SS5
ADF	-6.282***	-5.946***	-5.379***	-4.358***	-6.827***
PP	-390.064***	-637.867***	-734.945***	-178.409***	-419.894***

3. The comparison of forecasting performance

RMSE(Root Mean Square Error) was used as an evaluation criterion on forecasting performance of three range volatility estimators in this study. The two inputs for the calculation of RMSE are the predicted value of range volatility and the value of realized volatility at time t in equation (20). The $(h+1)$th predicted value of range volatility can be attained only with the exact parameter estimates of the model which is chosen by the standard of the highest fitness over the first two thirds of the entire sample. Hence the most important thing in this procedure can be to find out the exact specification of estimation models. As a result, ARMA(1,1) model, GARCH(1,1) model, and 2-regime SETAR(2-regime Self_Exciting Threshold AutoRegressive) model were chosen.

<Table 7> presents the values of RMSE(Root Mean Square Error) as an evaluation criterion on the forecasting performance of three range volatility estimators.

Analysis results can be summarized as follows.

First, RMSE(Root Mean Square Error) varied between relatively volatile periods and relatively stable periods. In other words, the most volatile sub-sample 4 had relatively higher RMSEs which meant relatively lower forecasting performance while the most stable sub-sample 5 had the smallest RMSEs which meant relatively higher forecasting performance. The same conclusions could be reached in the case of groups, not a single period. That is, the volatile group of sub-sample 1 and 4 showed relatively higher RMSEs which indicated relatively lower forecasting performance while the stable group of sub-sample 2, 3, and 5 did relatively lower RMSEs which indicated relatively higher forecasting performance. This is in line with Kim and Park(2009) as well.

Second, Parkinson volatility estimator, and Garkman and Klass volatility estimator can be classified as the first generation of this kind while Rogers and Satchell volatility estimator as the second one. There was a generation gap in terms of forecasting filters with which much superior forecasting performance could be achieved. The former was the winner when ARMA(AutoRegressive Moving Average) filter was used while the latter was when GARCH and 2-regime SETAR(Self-Exciting Threshold AutoRegressive) filters were used. The inclusion of trend into price process is considered to make some synergy effects when Rogers and Satchell volatility estimator is filtered with 2-regime SETAR(Self-Exciting Threshold AutoRegressive) model.

Third, with respect to sub-samples, the most volatile sub-sample 4 showed the highest forecasting performance with a linear ARMA filter but the most stable sub-sample 5 was quite different. More specifically speaking, Parkinson volatility estimator did with a linear ARMA filter and Garman and Klass volatility estimator did with nonlinear 2-regime SETAR filter. This conflicting results might be caused by the characteristics of each range volatility estimator.

Fourth, ARMA(AutoRegressive Moving Average) model was the best functioned with Parkinson, and Garman and Klass volatility estimators but 2-regime model was with Rogers and Satchell volatility estimator in terms of forecasting filters. One interesting thing was GARCH model was always-not-bad forecasting performance regardless of market conditions and the kind of range volatility estimators.

<Table 7> The comparison of forecasting performance by forecasting filters using RMSE

The comparative analysis on the forecasting performance of three range volatility estimators by the criterion of RMSE can be summarized as follows. First, the volatile group of sub-sample 1 and 4 showed relatively higher RMSEs which indicated relatively lower forecasting performance while the stable group of sub-sample 2, 3, and 5 did relatively lower RMSEs which indicated relatively higher forecasting performance. This is in line with Kim and Park(2009) as well. Second, Parkinson, and Garman and Klass volatility estimators were the winners when ARMA(AutoRegressive Moving Average) filter was used while Rogers and Satchell volatility estimator was when GARCH and 2-regime SETAR filters were used. The inclusion of trend into price process is considered to make some synergy effects when Rogers and Satchell volatility estimator is filtered with 2-regime SETAR model. Third, with respect to sub-samples, the most volatile sub-sample 4 showed the highest forecasting performance with a linear ARMA filter but the most stable sub-sample 5 was quite different. More specifically speaking, Parkinson volatility estimator did with a linear ARMA filter and Garman and Klass volatility estimator did with nonlinear 2-regime SETAR filter. This conflicting results might be caused by the characteristics of each range volatility estimator. Fourth, ARMA(AutoRegressive Moving Average) model was the best functioned with Parkinson, and Garman and Klass volatility estimators but 2-regime model was with Rogers and Satchell volatility estimator in terms of forecasting filters. One interesting thing was GARCH model was always-not-bad forecasting performance regardless of market conditions and the kind of range volatility estimators.

Panel A: Parkinson volatility estimator

	SS1	SS2	SS3	SS4	SS5
ARMA	0.020946	0.012855	0.008102	0.022096	0.002913
GARCH	0.018397	0.012975	0.008493	0.022143	0.004184
SETAR	0.020461	0.013374	0.009032	0.022775	0.003491

Panel B: Garman and Klass volatility estimator

	SS1	SS2	SS3	SS4	SS5
ARMA	0.018306	0.009757	0.008151	0.018111	0.006105
GARCH	0.018397	0.009864	0.008268	0.018514	0.006985
SETAR	0.018748	0.010381	0.008755	0.019014	0.005483

Panel C: Rogers and Satchell volatility estimator

	SS1	SS2	SS3	SS4	SS5
ARMA	0.020878	0.012137	0.009495	0.022983	0.003728
GARCH	0.020782	0.012892	0.009334	0.024794	0.004262
SETAR	0.020694	0.012084	0.009422	0.037268	0.003728

VII. Concluding remarks

This study is to compare the forecasting performance of three range volatility estimators including Parkinson, Garman and Klass, and Rogers and Satchell volatility estimators with linear and nonlinear filters. The raw data is the KOSPI 200 daily open, high, low, and close price series which are log-transformed to be the final dataset for this study. Sample period covers January 3, 2000 to June 30, 2011. The conventional return series based on close prices were produced to calculate the values of realized volatility. RMSE(Root Mean Square Erro) was used as an evaluation criterion on the forecasting performance of three range volatility estimators. For more detailed analysis, the entire sample was divided into 5 sub-samples after the application of the dynamic programming algorithm proposed by Bai and Perron(2003). The candidiate models to be used as forecasting filters afterwards were fitted on the two thirds of the entire sample and the specification of the models which recorded the highest goodness of fit was attained. They included ARMA(1,1), GARCH(1,1), and 2-regime SETAR(2) models. The one-step-ahead forecast which was one of two inputs for the calculation of RMSE, was calculated with observations and parameter estimates of the above chosen models using h-length rolling windows in the latter one third of the entire sample.

The conclusions of this study can be summed up as follows.

First, range volatility estimators tend to show relatively higher forecasting performance in relatively stable periods like sub-sample 2, 3, and 5. This is in line with Kim and Park(2009).

Second, Parkinson, and Garman and Klass volatility estimators showed relatively superior forecasting performance with a linear ARMA filter while Rogers and Satchell volatility estimator did nonlinear GARCH and 2-regime SETAR filters. The inclusion of trend into price process is considered to make some synergy effects when Rogers and Satchell volatility estimator is filtered with 2-regime SETAR(Self-Exciting Threshold AutoRegressive) model.

Third, ARMA(AutoRegressive Moving Average) model was the best functioned with Parkinson, and Garman and Klass volatility estimators but 2-regime model was with Rogers and Satchell volatility estimator in terms of forecasting filters. One interesting thing was GARCH model was always-not-bad forecasting performance regardless of market conditions and the kind of range volatility estimators.

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