The Dynamically Leveraged Fund using Optimal Leverage Level : Application to Korea ETF market

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Abstract

There has been a few studies on leveraged Exchange-Traded Funds (ETFs), traded on Korea Exchange (KRX) since it is only introduced to Korea market recently. In this paper, I study the performance of KODEX Leverage, the leveraged ETF with the biggest volume in KRX, over different investment horizon in order to clarify its well-known erratic behavior and various risks. The relationship between the returns of leveraged ETFs and the underlying indices is investigated both analytically and empirically. With understanding of factors which drive the performance of leveraged ETFs, the dynamically leveraged fund is developed. This dynamic strategy includes the optimal leverage level which responds to different market regimes, especially to volatility changes. Finally, I provide the empirical evidence that the dynamically leveraged fund in the study outperform the existing ETFs with constant leverage.
Keywords: ETF, Leveraged ETFs, Inverse ETFs, volatility.

1 Introduction

Exchange-traded funds (ETFs) offer a relatively cheap way for investors to take broad investment positions, often providing access to markets or underlyings that would be otherwise difficult to trade. These typically, open-ended index funds offer comparatively low fees, daily portfolio transparency and secondary trading on stock exchanges. In recent years, leveraged and inverse ETFs, which seek

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to deliver the multiples of the performance of the underlying index or benchmark they're tracking, are introduced to the ETF market. These leveraged and inverse funds have received much press coverage due to issues with their performances. Along the same line, there is some controversy about where and how this new investment tool best fits in the array of choices available to investors.

While classical ETFs track an index or benchmark in one-for-one fashion and are managed passively, leveraged ETFs require active management, involving borrowing funds to purchase additional shares or short-selling and rebalancing the position in order to maintain a constant leverage level. Since most leveraged and inverse ETFs reset on a daily basis, their performances over longer investment periods - over months and years - can differ significantly from the performance of their underlying index during the same period of time. That is, by the construction with daily rebalancing, the long-term returns on a k-times leveraged ETF will not earn k-times the return on the ETF. Indeed, market regulators have expressed concerns over what they perceive to be considerable misunderstanding on the investors' part about risk and return profile of leveraged ETFs. Understanding the different factors which impact the return of leveraged ETFs is important for investors.

Unfortunately, as leveraged ETFs are relatively a new comer, there have been only a few studies on them. Leveraged ETFs have been traded in Korea only for several years. KODEX Leverage with the biggest trading volume among leveraged funds in Korea started only in 2010. Because of the product's popularity among various clients, there is a need to understand properly its characteristics, pricing and performance. The leveraged and inverse ETFs are commonly utilized as short-term tactical trading tool. However, some investors also regularly use leveraged and inverse ETFs as a key component of a long-term portfolio strategy. This paper provides some results that shed light on these issues and long-term performance of leveraged fund. The related literature consists of Lu et al.(2009) and Avellaneda and Zhang(2009). Lu et al.(2009) empirically study the long term performance of leveraged ETF, comparing it to the short term performance, without providing the price process of ETFs. Avellaneda and Zhang(2009) provide an analytic and empirical study of the returns on leveraged ETFs.

KODEX Leverage is used in all empirical studies in this paper. The remainder of the paper is organized as follows. In the Section 2, I look into performance of the leveraged ETF over different investment time periods. The Section 3 introduces a formula for leveraged ETFs, which links the return to the the underlying index returns under simple model assumptions. The optimal leverage ratio is derived from the formula. These results are validated with data from Korea Exchange. The last section concludes the paper with final remarks.

2 Long-Term Performance of leveraged ETF vs. benchmark Index)

Investors consider leveraged ETFs in their investment portfolio since these ETFs allow them to increase their market exposures or to hedge without trading derivatives by themselves. Although leveraged ETFs may serve the purpose, the investors should keep in mind that these ETFs tracks daily returns through daily rebalancing and the performance over a time period longer than one day can be different from the return of the underlying index times the leverage. To look into the long term performance of leveraged ETFs, I consider the following investment horizons: 3 months (63 trading days), 6 months (126 trading days), 9 months (189 trading days) and 12 months (252 business days). The returns considered here is calculated as

$$r_{t,t+j} = S_{t+j}/S_t - 1,$$

where S_i is the fund or the index level at time i and j is the investment holding period. The summary statistics of the data in this study are reported in Table 1. Figure 1 shows the performance of KODEX Leverage and KOSPI 200, which is the underlying index of KODEX Leverage. I consider all overlapping returns due to relatively short history of KODEX Leverage. As the fund holding period gets longer, I observe more discrepancies between the returns of KODEX Leverage and the returns of KOSPI 200. More data points are off the 45 degree line.

In order to see the relationship further, I consider a simple regression model without intercept. The null hypothesis is to see whether the slope parameter is equal to the leverage level, 2. Again, the rolling returns are used in the analysis. Due to potential bias by overlapping observations, I report T-statistics calculated with Newey-West (1987) standard errors. The regression model is defined as

$$r^{LETF}_{t,t+j} = \beta r^{Index}_{t,t+j} + \varepsilon_{t,t+j}, \tag{1}$$

where $r^{LETF}_{t,t+j}$ is the return of the leveraged ETF, $r^{Index}_{t,t+j}$ is the return of the underlying index, t the investment start date and j the holding period. Table 2 reports the results of regression analysis.

For 6 month and 9 month investment holding period, the regression coefficients are different from the leverage level 2 at the 0.05 significance level. However, all 4 regression coefficients are reasonably close to 2. The R-squared values for all regression are above 95%. KODEX Leverage delivers more or less the twice exposure to investors, who holds this investment for longer period. Meanwhile, the cumulative returns of the KODEX Leverage do not grow like the double leveraged fund due to compounding, daily balancing, volatility and some other factors on returns. This is illustrated in Figure 2. and Figure 3. The next section analytically approaches this empirical phenomenon.

3 Dynamically Leveraged Funds

3.1 Model for Leveraged ETF

We consider a continuous time model for a non-dividend paying underlying index S_t on a time interval $[0, T]$ as in Avellaneda and Zhang(2010) :

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dW_t, \tag{2}$$

where W_t is a standard Wiener process and σ_t can be as general as stochastic volatility process. Since funds typically track the total return versions where dividends are assumed to be reinvested, we can neglect the impact of dividends and assume S to be the total return version of an index. L can be positive or negative. When negative, these leveraged ETFs are often called inverse ETFs to distinguish them from positively leveraged ETFs. The process for a leveraged fund F on the underlying index S with leverage L can be characterized as

$$\begin{aligned}
\frac{dF_t}{F_t} &= L\frac{dS_t}{S_t} - ((L-1)r_t + f)dt \\
&= L\left[\mu_t dt + \sigma_t dW_t\right] - ((L-1)r_t + f)dt \\
&= \left[L\mu_t - (L-1)r_t - f\right]dt + L\sigma_t dW_t,
\end{aligned}$$

where r_t denotes the funding rate, f the management cost of the leveraged ETF, which typically ranges between 75bps and 150bps. The management cost of KODEX leverage is 0.93%. Although the stock-borrowing cost could occur in case of inverse ETF, we do not take into account this cost in this setting. By Ito's formula,

$$d\ln F_t = \left[L\mu_t - (L-1)r_t - f - \frac{L^2\sigma_t^2}{2}\right]dt + L\sigma_t dW_t$$

Thus,

$$d\ln F_t - Ld\ln S_t \quad = \quad \left[-(L-1)r_t - f - \frac{L(L-1)\sigma_t^2}{2}\right]dt$$

and

$$\ln\left(\frac{F_t}{F_0}\right) = L\ln\left(\frac{S_t}{S_0}\right) - (L-1)\int_0^t r_u du - ft - \frac{L(L-1)}{2}\int_0^t \sigma_u^2 du.$$

The return of the L-times leveraged ETF over the investment horizon $[0,t]$ is :

$$\frac{F_t}{F_0} = \left(\frac{S_t}{S_0}\right)^L \left[\exp\left(-(L-1)R_t\right)\exp(-ft)\exp\left(-\frac{1}{2}L(L-1)V_t\right)\right], \tag{3}$$

where the accumulated variance $V_t = \int_0^t \sigma_u^2 du$ and the accumulated borrowing cost $R_t = \int_0^t r_u du$. It's worth noting that the equation is different from what one could easily assume

$$\frac{F_t}{F_0} = \left(\frac{S_t}{S_0}\right)^L \left[\exp\left(-(L-1)R_t\right)\exp(-ft)\right],$$

which is just L-th power of the underlying index less financing and management costs. The impact of the underlying volatility on the performance of leveraged ETF is studied in Lu et al. (2009) in a different angle. They did a regression analysis on the return of leveraged ETFs and included the quadratic variation as an independent variable in additional to the return of the underlying indices. For the time horizon $[0,t]$, the L-times leverage ETF log returns is L-times log returns of the underlying index with adjustment factors, which explains the compounding effect. The adjustment factors, which determines the relationship between the return of the leveraged ETF and its underlying index, are the variance of the underlying index, the leverage ratio and the funding rate. This expression allows us to examine the impact of each adjustment factor. For instance, the return of the most commonly traded leveraged ETF with $L = 2$ could be largely reduced by the variance of the underlying index. More generally, there will be a volatility gain to the return of the leveraged fund if $0 < L < 1$, so called "de-leveraging" and a volatility loss if $L < 0$ or $L > 1$. Exposure to the accumulated funding cost and expense cost is relatively small. Another interpretation from this expression is that positions of the underlying index and the money market account in L-time leveraged ETF are dynamic and path-dependent. The combination of all these factors produces a divergence from simply-compounded returns over a long investment time horizon. The equation (3) is validated in Figure 4. It is easily seen that the formula closely fits the real data. The comparison between blue line and brown line emphasizes the impact of the adjustment factor in the equation (3). Figure 5 validates the equation (3) with rolling returns over different investment holding periods. For all periods, it is observable that the equation (3) fits the market data well. Tracking errors are defined as

$$\begin{aligned} \varepsilon_t &= \frac{F_t^{data}}{F_0^{data}} - \frac{F_t}{F_0} \\ &= \frac{F_t^{data}}{F_0^{data}} - \left(\frac{S_t}{S_0}\right)^L \left[\exp\left(-(L-1)R_t\right)\exp(-ft)\right]. \end{aligned} \tag{4}$$

Avellaneda and Zhang(2010) use the variance of the underlying index sampled over a period of 5 business days preceding each trading date for the estimation of V_t. Different from their approach, the variance is calculated with 1 year rolling returns over a period of 5 business days, which gives a better fit. The average of 3-month CD rate over a period of 5 business days is used for the estimation of R_t. Table 3 reports the tracking errors for each holding period.

3.2 Dynamic Strategy using Optimal Leverage Ratio

If a portfolio has an expected return of μ and standard deviation σ over some short time period, the expected return over that time period is approximated as $g = \mu - \sigma^2/2$. From the equation (3), the expected long-term growth of the leveraged fund is given as

$$g_{LETF} = Lu_t - (L-1)r - f - \frac{1}{2}L(L-1)\sigma^2$$

where the expected growth rate of the underlying $u_t = \mu_t - \frac{\sigma_t^2}{2}$. By differentiating the expected long-term growth of the leveraged fund by L, we can obtain the optimal leverage which maximizes the expected long-term growth :

$$L^* = \frac{u_t - r_t}{\sigma_t^2} + \frac{1}{2}.$$

Similar studies are done in Carver (2009). To apply this optimal leverage to the dynamically leveraged fund in practice, the remaining work is to determine rebalancing frequencies, u_t and σ_t. Instead of model estimation, the simple methodology is adopted in this study. The rebalancing frequency has impact on whether the dynamic strategy successes or not and it is not easy to determine. I find monthly rebalancing gives the robust results for all different investment holding periods. u_t is estimated by the average of 1 year rolling returns. For dynamic strategy, a period of 5 business days turns out to be too short and the performance of the dynamically leveraged fund is unstable. 1 year time period is chosen since the estimation u_t responds to the movement of the underlying index and reflect it to the calculated optimal leverage level in a relatively timely manner. σ_t is estimated using 1 year rolling returns over 1 month, the same as the rebalancing frequency. The magnitude of σ has the greatest impact on the size of the optimal leverage level. Using VKOSPI, the variation of KOSPI 200 implied volatilities as an estimate of σ_t is considered as well and the performance was not satisfactory. This result is not reported in the paper. It was a surprising result since it is expected that the forward-looking character of the implied volatilities would improve the reaction of the optimal leverage level to market turbulence. When the optimal leverage levels are applied in the empirical study, they are capped at 4 and floored at -4. Figure 6-9 illustrate how the optimal leverage levels change and how and when the dynamically leveraged funds outperform the funds with constant leverages (KODEX 200 and KODEX Leverage). Figure 6 is an example where the optimal leverage level is maximized to 4 to gear up the performance when the underlying index performs and is gradually reduced to -4 to lower the loss incurred by the negative performance and provide the opposite exposure. Figure 7 and Figure 8 work similarly. On the other hand, Figure 9 depicts the limitation of the current dynamic strategy. The sudden price drop when the optimal leverage level is maximized due to the positive return and low volatility can cause substantial loss. Table 4 summarizes the performance of dynamically leveraged fund, comparing other funds with constant leverages, over different holding periods. One can see that the dynamically leveraged fund outperforms the other two.

4 Conclusion

I try to clarify well-known behavior and risks of the return of leveraged ETF. Both theoretically and empirically, one can see the performance of the leveraged ETF is dependent on volatility and volatility can be destructive for investors' returns from studies done in this paper. Understanding of the mechanism of daily rebalacing of the leveraged ETF and the factors which drives performance of leveraged funds paves a way to create funds with more attractive features. In the current strategy, more stable market condition gives higher leverage to the fund and the volatile market de-levers. I believe the choice of rebalancing frequency, u_t and σ_t can be improved further. The simple model assumption of the underlying index can be improved as well. The current model assuming a normal distribution in returns is known to fail to capture many crucial aspects of real price fluctuations such as market crashes or extraordinary returns. The actual frequency of extreme stock returns is larger than what the model would suggest. Furthermore, the important features of a return distribution - skewness and kurtosis - are not implied in the model. One might want to consider stochastic volatility model or jump model. The longer history of data reflecting various market regimes would help to examine the robustness of the performance of the dynamically leveraged fund, especially for longer holding periods.

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Table 1: Descriptive statistics of KODEX 200 and KODEX Leverage and their daily returns based on market prices Feb 22, 2010 - Oct 26, 2012

Name	Multiple	Listing Date	Mean	Std Dev	Skewness	Kurtosis	Max	Min
KODEX 200	1	Oct 14, 2002	0.0002	0.0130	-0.2606	2.3150	0.05222	-0.0631
KODEX Leverage	2	Feb 22, 2010	0.0002	0.0261	-0.2805	2.2927	0.0977	-0.1287

Table 2: Relationship between long-term performance of leverage ETF and long-term performance of benchmark index

Holding Period		3 months	6 months	9 months	12 months
KODEX Leverage	beta	1.9994	2.0678*	2.083*	2.0744
	(T stat)	(-0.0269)	(1.9994)	(2.2252)	(1.2019)
	R^2	0.9879	0.9866	0.9755	0.9612
	Obs Num	608	545	482	419

This table reports the regression analysis using returns over different holding periods. The null hypothesis for the coefficient is $H_0 = 2$. Newey-West t-statistics are reported in parentheses. * is marked when the null hypothesis is rejected at the 0.05 significance level.

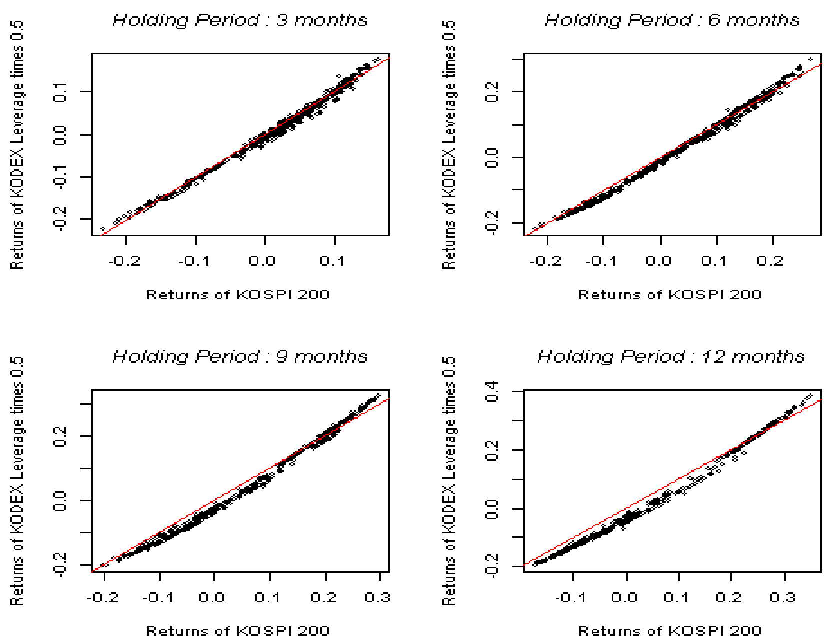
Table 3: Tracking errors(%) from the formula (4) and their standard deviation(%) of rolling returns over different holding periods, starting Feb 22, 2010

Holding Period	Tracking Error (average, %)	Standard Deviations (%)
3 months	0.5511	1.1327
6 months	1.2523	1.4447
9 months	2.0379	1.3819
12 months	2.5839	1.1379

Table 4: Performance of dynamically leveraged fund, comparing other funds with constant leverages

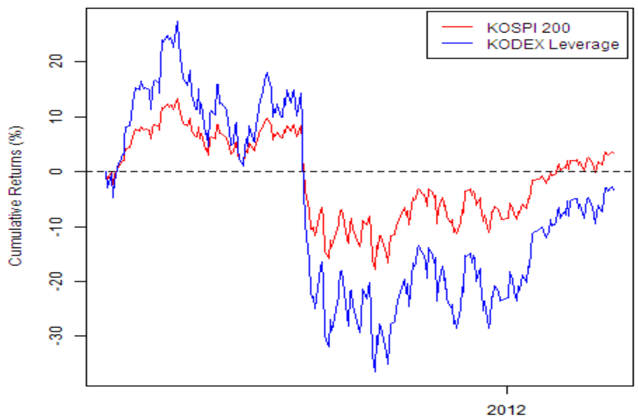
Holding Period	Funds	Mean (%)	Standard Deviation(%)	Max(%)	Min (%)
3 months	Dynamically Leveraged	5.5887	28.7466	72.6009	-67.5213
	Leverage = 2	2.7353	16.5673	35.1701	-44.2353
	Leverage = 1	1.7397	8.4457	17.5799	-23.405
6 months	Dynamically Leveraged	15.3806	47.0867	136.0601	-67.236
	Leverage = 2	5.2385	24.6167	59.8928	-43.4705
	Leverage = 1	3.4595	12.1346	27.6698	-23.2306
9 months	Dynamically Leveraged	20.5981	56.8644	147.3379	-65.5397
	Leverage = 2	7.5506	28.9567	64.9515	-40.1229
	Leverage = 1	5.3377	13.4617	31.2614	-20.4982
12 months	Dynamically Leveraged	12.3323	59.3699	176.2161	-54.2599
	Leverage = 2	4.4618	31.4694	76.9189	-38.5373
	Leverage = 1	4.6659	14.3491	35.4698	-17.6367

Figure 1: Returns of KODEX 200 versus returns of KOSPI 200 over different holding periods



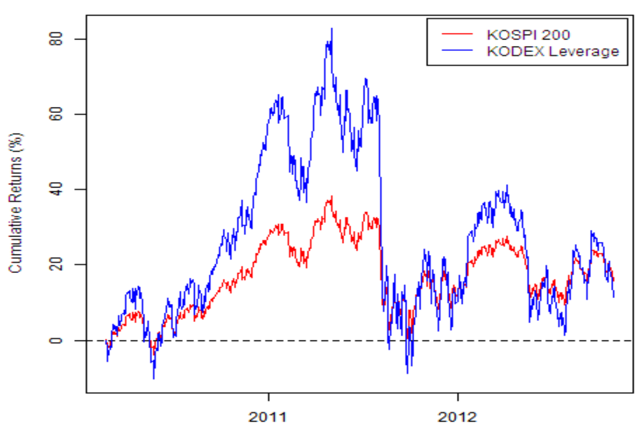
We considered all overlapping returns over different holding periods between Feb 22, 2010 and Oct 26, 2012. The straight red line is the 45-degree line.

Figure 2: The cumulative returns of KODEX 200 versus the cumulative returns of KODEX Leverage between Mar 10, 2011 and Mar 20, 2012



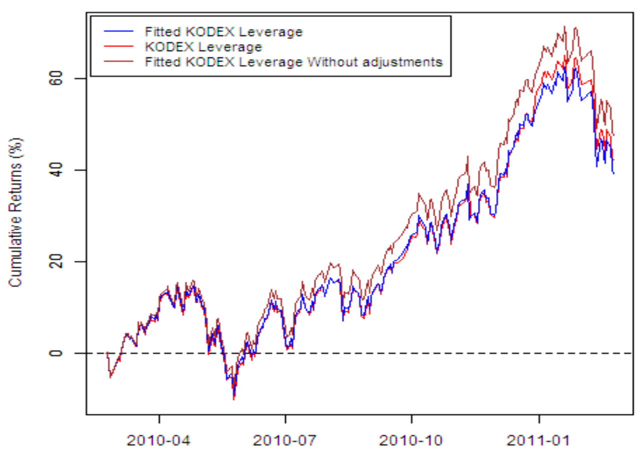
The cumulative returns of KODEX 200 is 3.49% and the cumulative returns of KODEX Leverage is −3.29% over the holding period.

Figure 3: The cumulative returns of KODEX 200 versus the cumulative returns of KODEX Leverage between Feb 22, 2011 and Oct 26, 2012



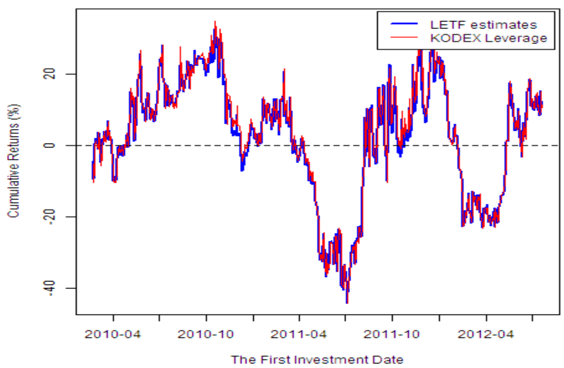
The cumulative returns of KODEX 200 is 15.89% and the cumulative returns of KODEX Leverage is 11.36% over the holding period.

Figure 4: The formula (3) validation with KODEX Leverage market data, investment starting on Feb 22, 2010 and lasting for 1 year

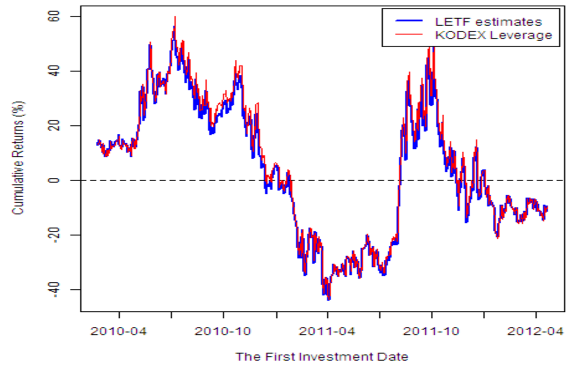


The blue line, which is the formula fitted value, moves closely with the red line, the actual cumulative returns of KODEX Leverage. The brown line, which is the fitted value without adjustment factor, deviates from the red line. We observe the adjustment factor in the formula is necessary.

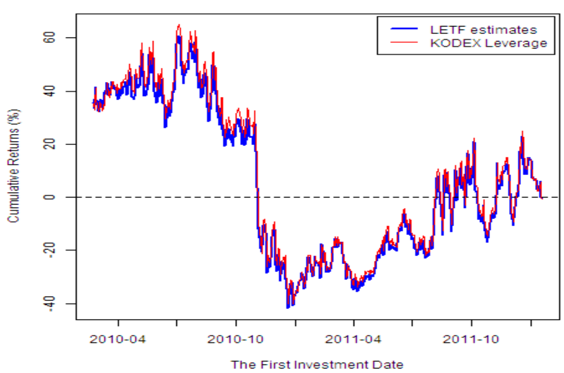
Figure 5: The formula (3) validation using rolling returns over different holding periods



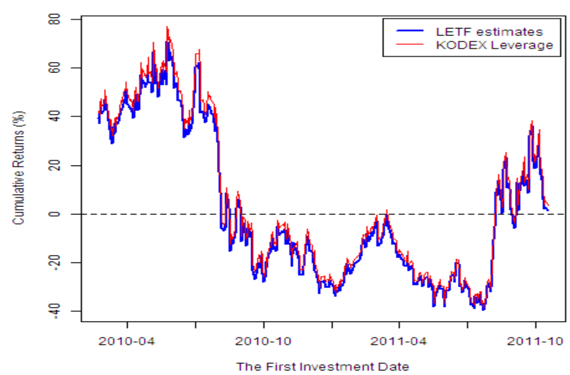
Holding Period : 3 months



Holding Period : 6 months

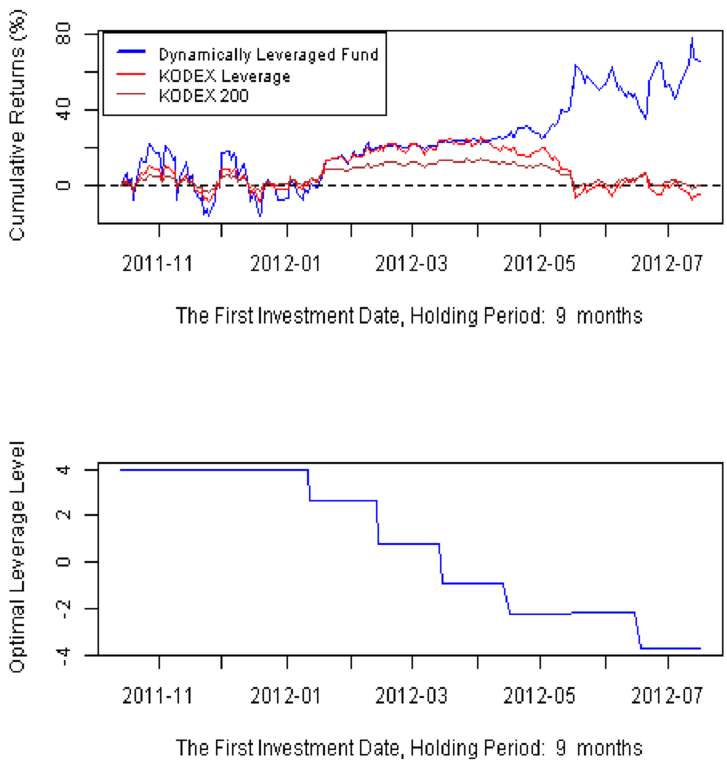


Holding Period : 9 months



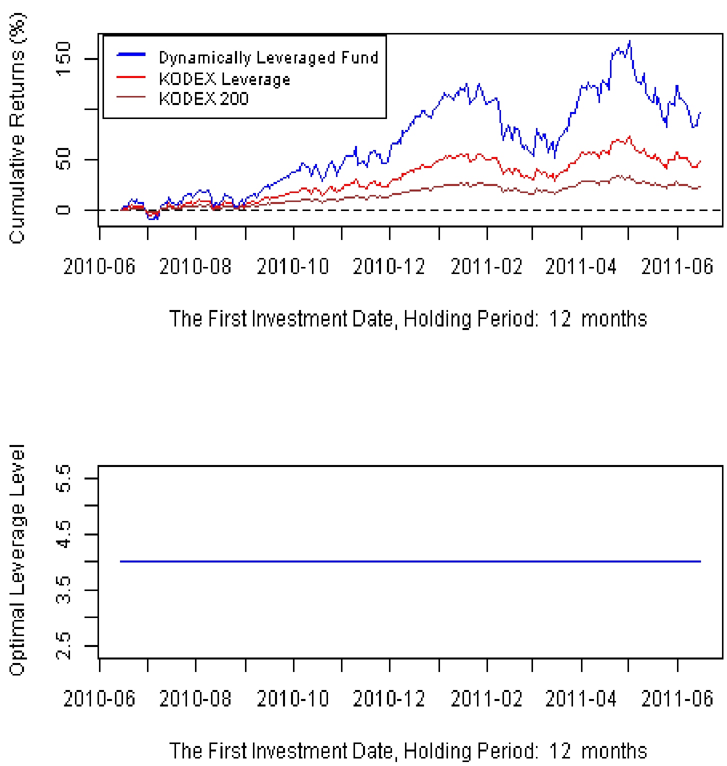
Holding Period : 12 months

Figure 6: Comparison between dynamically leveraged fund and other funds with constant leverage (KODEX Leverage, KODEX 200)



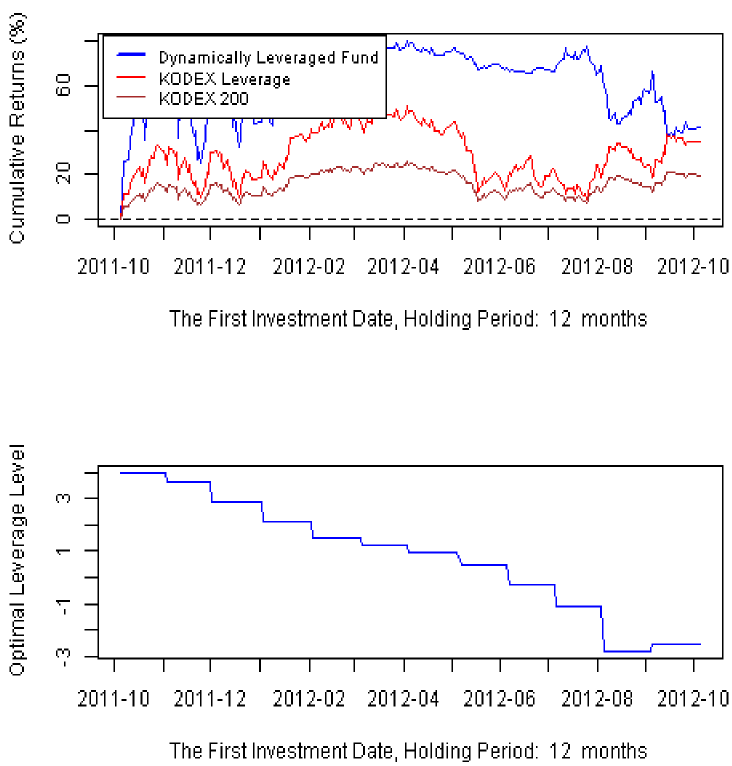
The performance of funds, where the investment starts on Sept 14, 2011 and lasts for 9 months, is compared. The optimal leverage level is reduced when the market becomes turbulent and the volatility is higher. The optimal leverage level goes to negative with the low or negative growth rate of the underlying.

Figure 7: Comparison between dynamically leveraged fund and other funds with constant leverage (KODEX Leverage, KODEX 200)



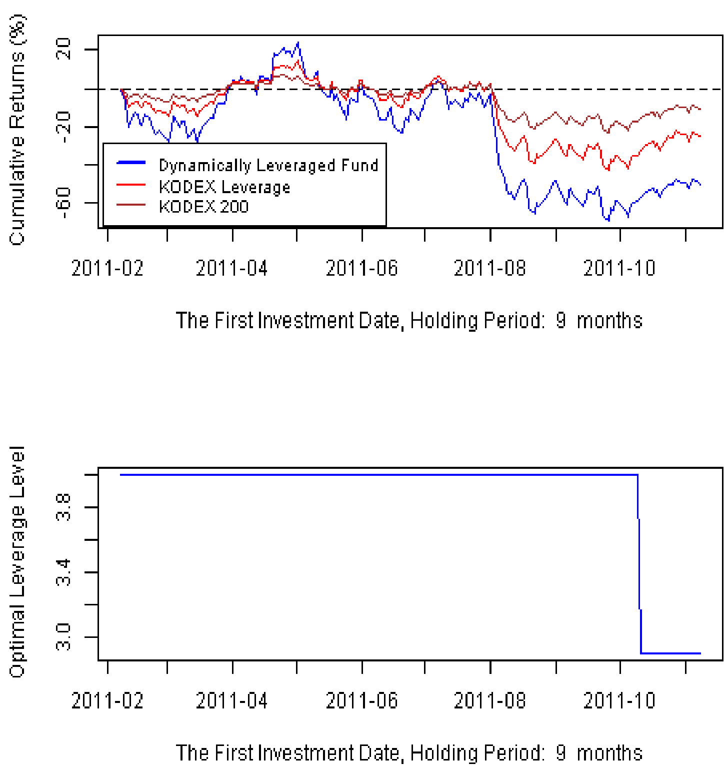
The performance of funds, where the investment starts on June 14, 2010 and lasts for 12 months, is compared.

Figure 8: Comparison between dynamically leveraged fund and other funds with constant leverage (KODEX Leverage, KODEX 200)



The performance of funds, where the investment starts on Oct 05, 2011 and lasts for 12 months, is compared.

Figure 9: Comparison between dynamically leveraged fund and other funds with constant leverage (KODEX Leverage, KODEX 200)



The performance of funds, where the investment starts on Feb 07, 2011 and lasts for 9 months, is compared. This figure depicts the limitation of the current dynamically optimized fund. The sudden price drop following the positive return and low volatility regime is not reflected in the optimal leverage level until the next rebalancing date and incurs substantial loss.