A decomposition of sovereign bond yields : Joint estimation of sovereign CDS and bond data

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We suggest a methodology to decompose sovereign bond yields into four components (risk-free, default risk, risk premium and non-default) using both sovereign CDS and bond data. We find that each fraction varies over time and across bonds. In addition, the default risk accounts for only a small fraction of sovereign yield spreads, and a substantial portion is attributable to risk premium and non-default components. Especially, risk premium is substantially time-varying and increases to account for a major portion of heightened yield spread during the financial crisis, implying that the global investors prefer to hold safer assets and require higher risk premiums per unit risk than before. Our findings provide evidence on flight-to-quality and time-varying risk premium.

Keywords: Sovereign yield spread, reduced-form approach, flight-to-quality, time-varying risk premium.

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1 Introduction

Understanding the determinants of corporate yield spreads (defined as yield difference between corporate bonds and treasury bonds) has long been one of the central research topics in finance. Previous literature has been divided into two strands: Structural-form approach and reduced-form approach. In structural-from models, corporate bonds are treated as a contingent claim on the firm value. In most cases, the model predicts a firm's default time, and allows us to calculate theoretical corporate yield spreads. Prominent examples include the works of Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), Briys and De Varenne (1997), and Collin-Dufresne *et al.* (2001). In contrast, reduced-form models assume that the time of bankruptcy is given as an exogenous process. Thus, the firm's default time is not accessible in reduced-form models. The reduced-form approach is pioneered by Jarrow and Turnbull (1995), and subsequently studied by Jarrow *et al.* (1997), Duffie and Singleton (1999), Madan and Unal (2000), and Jarrow and Yu (2001).

Reviewing the literature reveals that while extant studies have emerged on corporate yield spreads, little attention is devoted to determinants of sovereign yield spreads. This seems natural because when it comes to sovereign yield spreads, there is no theoretical starting point such as the Merton model in the corporate credit risk literature. That is, unlike corporate default risk, sovereign default risk can be hardly measured by historical default rates or accounting variables. Therefore, the structural-form model cannot be employed in investigating the determinants of the sovereign yield spreads.

As an alternative, we study the determinants of the sovereign yield spreads under the reduced-form framework. The key distinction between the present paper and previous studies is threefold, and our paper has benefited from the works of Longstaff *et al.* (2005), and Pan and Singleton (2008). First, inspired by the study of Longstaff *et al.* (2005), we use the sovereign CDS data. One pitfall of the reduced-from approach is that the model assumes that arrival rates of default (or intensity) are exogenous, and hence, is silent on what determines the intensity in an economic sense. Therefore, while fitting the observed credit risk, the reduced-from approach cannot measure the default risk using equity data and accounting data. To overcome this problem, Longstaff *et al.* (2005) use the credit default swaps (CDS) data as proxies for the default risk evaluated by the market. They decompose the observed corporate spreads into default and non-default components, and find that the non-default component is related to liquidity factors. Specifically, we apply the work of Longstaff *et al.* (2005) in corporate spreads to the sovereign yield spreads.

Second, we decompose the yield spread (the difference between the Korean sovereign bond yield and U.S. treasury yield) more specifically. Researchers have decomposed the yield spread into default and non-default components, and have considered default risk as the most important determinant of the yield spread. Important examples include the works of Collin-Dufresne *et al.* (2001), Eom *et al.* (2004), and Huang and Huang (2012). The empirical studies have typically asked how much default risk contributes to yield spreads. Unfortunately, most of the studies fail to compromise the theoretical yield spreads with the observed yield spreads. That is, the expected default risk which is calculated from the model is too small to explain the observed data. Thus, the previous studies conclude that default risk is just one of the determinants and cannot fully explain the observed spreads.

Previous empirical findings call for finer decomposition, and this paper takes one step in that direction. Specifically, following Pan and Singleton (2008), we decompose sovereign yield spread into three parts: default risk, risk premium, and non-default risk. First, the *default risk* is a component that is required by investors due to the sovereign's risk of default on its debt. Second, the *risk premium* is a premium for the unpredictable change in the default risk or intensity. Provided that two sovereigns have the same expected default risk, investors require more premium for the sovereign who has larger variation in the future default risk. Finally, the non-default risk is the rest of the yield which is not explained by the previous two components. A novel idea of Pan and Singleton (2008) enables us to identify risk premium from the default component in CDS spreads.

Third, instead of a non-parametric analysis, we consider a term-structure model-based approach. We do not use the non-parametric analysis because the maturities of the yield spread and CDS spread are not matched. The maturity of yield spread decreases after the issuance of the corresponding bond whereas CDS maturity remains constant at any time. Therefore, direct subtraction of the CDS from the yield spread ignores the term-structure effect of default component. As a result, the non-parametric analysis may induce a bias.

In decomposing the sovereign yield spread, we are particularly interested in one emerging market, Korea, with the following reasons. Emerging markets including Korea have received much attention by academics, practitioners and policy makers alike for several reasons. For practitioners, having been rapidly growing, the emerging sovereign bond and CDS markets have become a new asset class to invest in according to their market views. For academics, subsequently, the analysis on emerging markets can be a unique window to look at the global investors' expectation. For the reason, many existing papers have studied emerging sovereign markets (e.g. Duffie *et al.* (2003), Dittmar and Yuan (2008) in the bond market and Pan and Singleton (2008), Küçük (2009), Zinna (2012) in the CDS market). Emerging sovereign markets are important to policy makers as well because sovereign bonds benefit corporate bonds in emerging markets Dittmar and Yuan (2008) argue that the liquid corporate bond markets in developed countries are due to active government bond issuance and trading. They also find that issuance of sovereign bonds lowers corporate yield and bid-ask spreads in emerging markets.

Hence, we are interested in emerging sovereign markets. Particularly we narrow down the scope of the study by focusing on Korea since global investors have most actively traded Korean bonds and CDS among Asian emerging markets in terms of the number of bonds and the trading size and frequency over the recent decade.

In this paper, we measure the size of each component in the sovereign yield spread under the reduced-form framework using the Korea's sovereign CDS data. Investors require higher compensation for sovereign bonds of emerging markets than the US treasury. Therefore, sovereign yield spread is positive most of time, i.e., sovereign bonds of emerging markets are cheaper than the US treasury. From a theoretical perspective, the positive sovereign yield spreads can be attributable to default risk of the sovereign when we assume that the US treasury is an ideally risk-free asset. Put differently, investors demand higher yields on sovereign bonds because they believe that sovereigns of emerging markets are more likely to default on their debt than the United States. If this is the case, we ask contribution of the default risk to the observed yield spreads. We investigate whether the default risk can fully explain the observed yield spreads, or there are any other components in sovereign yield spreads. We answer to this question by decomposing the sovereign yield spread into the three components: default risk, risk premium, and non-default risk.

Our central findings are easy to summarize. First, our finer decomposition increases the contribution of the default component to the observed yield spread. For Bond I, the default risk component accounts for only about 27% of yield spread when we do not consider the contribution of risk premium to yield spread. When we take the risk premium component into account, about half of the yield spread is explained by the default component (default risk and risk premium). Second, we conclude that the risk premium plays an important role in the sovereign bond market, and the importance of the default component is pronounced during the credit crisis. The risk premium fraction accounts for majority of yield spread as well as total yield during the financial crisis (especially, after Lehman Brothers' bankruptcy on September of 2008). During that time, both the risk-free and non-default components decrease. The default risk component increases, but the increase is modest and not enough to explain such a high yield of the Korean bond. Compared to the default risk component, the risk premium component increases substantially. In sum, the risk premium component significantly varies over time and, in particular, a major portion of the yield spread is attributable to the risk premium during the recent financial crisis. Finally, the non-default component accounts for a significant part and is not a trivial component such as pricing error or idiosyncratic risk.

Our empirical findings suggest that the increase in yield-to-maturity of Korean sovereign bonds during the crisis is more attributable to the substantial increase in risk premium that global investors demand rather than the increase in default risk of Korea. It is well-known that investors extremely seek to purchase safer assets in a period of financial turmoil, which is typically referred to as “flight-to-quality”. Therefore, our finding provides evidence on the flight-to-quality during the crisis, inferred from sovereign bond data.

Our study relates to and contributes to the literature as follows. First, the vast papers have explored the determinants of corporate spreads under a structural-form framework, including Collin-Dufresne *et al.* (2001), Eom *et al.* (2004), Zhang *et al.* (2009) and Huang and Huang (2012). The only exception is Longstaff *et al.* (2005) which use a reduced-form framework. Our methodology is closely related to theirs, but we focus on sovereign yield spreads rather than corporate yield spreads. Furthermore, we differentiate our work from theirs by decomposing the default component into default risk and risk premium using Pan and Singleton (2008)’s idea.

Second, many previous studies have already analyzed the emerging sovereign market. For example, Zhang (2003), Duffie *et al.* (2003) and Dittmar and Yuan (2008) use sovereign bond data while Pan and Singleton (2008), Küçük (2009) and Zinna (2012) use CDS data in emerging markets. However, none of the studies uses both bond and CDS data at the same time. We estimate our model using both the two kinds of financial instruments.

Third, we add an empirical work on the increasing literature on the determinants of sovereign yield spreads such as Dailami *et al.* (2008), Hilscher and Nosbusch (2010) and Maltritz (2012) However, they typically do their work without a model due to the reasons as explained above. Most studies just regress on macroeconomic variables to explain the yield spreads. As our model-free analysis shows that the size of the determinants could be biased, we use a term-structure model to estimate credit curves and measure the size of the determinants.

The remainder of this paper is organized as follows. Section 2 explains the data and a non-parametric analysis to show a simple picture of the relation between yield spread and CDS spread. At the end of Section 2, we emphasize why we consider a term-structure model-based approach instead of the non-parametric analysis. Section 3 introduces our approach based on a term-structure model. Section 4 presents estimation methodology and provides empirical results. Section 5 summarizes and presents our conclusions.

2 The relation between yield spread and CDS spread

In this section, we explain the data and a non-parametric analysis to show a simple picture of the relation between yield spread (defined as Korean sovereign bond yield minus U.S. treasury yield) and CDS spread. At the end of this section, however, we will emphasize why the non-parametric analysis could be biased, and accordingly, why we consider a term-structure model-based approach.

2.1 The Data

We collect the Korean sovereign debt data from the Bloomberg system. Given the US dollar denominated CDS data, we focus on the US dollar denominated *foreign exchange equalization bonds* in consideration of the exchange rate risk factor[1]. Currently, historical bid and ask (dirty) prices are available only for six foreign exchange equalization bonds. The specific information (issue date, maturity, issued amount, coupon rate, etc.) on the bonds is given in Table 1. All bonds pay coupons with semi-annual frequency. For convenience, we name the bonds with numbers in the order of their issued date. For example, KOR Bond I, the oldest in our sample, was issued in June, 2003 and KOR Bond VI was issued in April, 2009. Time-to-maturity at the issued date is reported in 'TTM'. Four bonds were issued with maturities of 10 years and the rest two bonds were 20 years and 5 years, respectively. Before explaining the CDS data, we emphasize that the sovereign bonds in our sample do not have "constant maturity" and their time to maturity decreases as time goes by.

[Insert Table 1 Here]

In contrast with the bond data, CDS data provided from the Markit have "constant maturity"[2], i.e., the maturity of 5 year CDS is constantly 5 years at any time during the sample period. The mismatch of maturity between the bond and CDS data leads us to considering the term-structure model. This issue will be addressed in more detail in the next section. Among the various Korea's sovereign CDS data, we use 5 year-maturity because it is the most liquid among various maturities. The sample period covers from June, 2003 to September, 2012.

We choose the US constant maturity treasury (CMT) as the risk-free benchmark. Also, one- to twenty-years CMT curve is used to bootstrap a zero curve which will be used as the discount factor in pricing the model of bonds and CDS. For the inter-points between the given

[1] Foreign exchange equalization bonds are one of the sovereign bonds issued by Korea.

[2] Actual CDS contracts mature at specific IMM dates. We consider the real structure of CDS contracts in the pricing procedure.

maturities, log values of discount factor are linearly interpolated meaning that we implicitly assume that the forward rates are constant between the given maturity points.

2.2 Non-parametric analysis

Theoretically, it is well-known that yield spreads between defaultable bond and risk-free bond are close to CDS spreads. Roughly speaking, since a CDS contract is an insurance against default on a bond, a defaultable bond holder with the long position on CDS contract is free of default risk. Instead, the return on the investment is decreased by CDS premium which should be paid to the counter-party of the CDS contract. Therefore, yield on the defaultable bond minus CDS premium is close to yield on risk-free bond. In other words, yield spread between defaultable bond and default-free bond is close to CDS spread.

In our study, the Korean sovereign bonds and U.S. treasury bonds are chosen as defaultable bond and default-free benchmark bond, respectively. The following equation shows the relation between sovereign bond yield spread and sovereign CDS spread.

$$y_{KOR} - y_{US} \cong s_{KOR} \tag{1}$$

The major determinant of yield spread ($y_{KOR} - y_{US}$) must be default risk relative to the default-free benchmark. In the study, we assume that the 5-year sovereign CDS spread fully captures the default risk of a sovereign bond without any noise such as liquidity. This assumption is reasonable in that CDS contracts are very liquid compared to bonds and any complicated options or taxes are not involved in the CDS contracts. For that reason, CDS spreads are typically considered as a measure of the default component in the yield spread. However, CDS spread cannot fully explain the yield spread due to non-default factors included in yield spread such as liquidity and tax. The non-default factor, if any, would make the yield spread differ from the CDS spread.

Before the model-based analysis, we simply show the relationship between the two spreads via a non-parametric analysis. We calculate yield-to-maturity of the Korean sovereign bond from its mid (dirty) price by using the following formula:

$$P = \frac{c}{2}\sum_{i=1}^{n} e^{-y_{KOR}T_i} + e^{-y_{KOR}T_n}, \tag{2}$$

where P is bond (dirty) price, c is coupon rate per annum and T_i is time to coupon payment dates at pricing time. We emphasize again that the time to maturity of Korean sovereign bond decreases as time goes by and hence, we bootstrap benchmark yield to matched maturity (y_{US}) from the treasury curve as described in the data.

[Insert Figure 1 Here]

Figure 1 shows the relationship between yield spreads and CDS spreads. Although they are closely related, we also find the significant discrepancy. That is, the yield spread cannot be fully explained by CDS spread only, implying the existence of a non-default factor in yield spread. To take a deeper look into the quantitative relationship, Table 2 reports the summary statistics of yield spread, CDS spread, and basis for each bond. As mentioned earlier, the sample period of each bond is different because the first observation date is different. In order to compare the two spreads, therefore, we report 5-year CDS spread repeatedly along with the 6 bond spreads. We also report the 'basis' defined as yield spread minus CDS spread. Basis statistics are all similar across bonds with some exceptions of skewness and kurtosis. Consistent with Figure 1, the yield spread is higher than the CDS spread, which implies that the positive non-default component is embedded in bond prices. Although the basis occasionally has negative signs, the negative values of skewness of the five bonds except for the KOR Bond IV indicate that the non-default factor is positive most of time. The existence of positive non-default component in "good credit quality" sovereign's bond prices is well-known stylized fact in the emerging markets.

[Insert Table 2 Here]

What is more striking in our study is that the time-series average of basis has similar quantities about 13 bps to 23 bps across the bonds. Therefore, there may be a commonality in the non-default factors across the bonds and the factors may be related to the global market. However, it should be interpreted with caution because the measures might be biased for several reasons. First, the maturities of the two spreads are not matched. The maturity of yield spread decreases after the issuance of the corresponding bond whereas CDS maturity remains constant as 5 years at any time. For example, the time-to-maturity of KOR Bond I is about ten years at issue date around 2003, but it reduces to one year around the end of the sample period. Therefore, direct subtraction of 5-year CDS from the yield spread ignores the term-structure effect of default component. In fact, the time-series average of CDS slope (10-year CDS minus 1-year CDS) during the sample period is about 33 bps which is so significant that we cannot ignore the term-structure effect. Given the positive slope, the basis would be overestimated around the issuance of KOR Bond I because we should have used 10-year CDS spread which is typically greater than 5-year CDS spread. On the contrary to this, the basis tends to be underestimated at the end of the sample period when the time-to-maturity of the bond is about one year. One possible resolution to the problem is the use of credit curve in a similar way that the yield on U.S. bond is bootstrapped from CMT curve. Even if the remedy can be a solution to the "unmatched-maturity" problem, the measures still remain biased for the following second reason.

The second reason is that "yield-to-maturity" approach makes bias on the default-related component and hence, on the non-default component as well. Yield-to-maturity is a constant discount factor which makes the present value of a bond when applied to future cash flows. When we use the 5-year CDS spread as a proxy for credit component of bond price (or yield-to-maturity), there underlies an assumption that the term-structure of credit risk is flat. In other words, regardless of the term of future cash flows are, the constant CDS spread (with risk-free yield) is used for discounting any cash flows of a bond. As noted, the term-structure is not flat and market's expectation about future credit quality may be different for short-term and long-term.[3] As a result, using the 5-year CDS spread as a proxy makes the default component in a bond price biased.

Last, yield-to-maturity obtained by equation (2) ignores the recovery value of a bond on a credit event date,[4] which would make the yield overestimated because the expected cash flows are underestimated, and hence, higher discount rate is required to obtain the same price of a bond. This point becomes clearer in equation (9) of the next section.

3 Alternative approach based on a term-structure model

For the several reasons mentioned in the previous section, we employ a term-structure model which has been widely used in pricing CDSs and defaultable bonds (Duffie *et al.*, 2003; Longstaff *et al*,. 2005; Pan and Singleton, 2008).

3.1 Modeling the default of sovereign debts

We assume that a sovereign defaults on its debt by an arrival of a jump whose arrival rate or intensity is exogenously and randomly determined. In other words, sovereign default occurs at the first jump of the jump process, N_t, and the instantaneous (risk-neutral) probability of the jump is $\widetilde{P}(dN_t = 1) = \lambda_t^Q \ dt$.[5] To complete the model, we now need to specify the intensity. Following Pan and Singleton (2008), we model the intensity process by Black and Karasinski (1991)'s log normal model as follows:

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[3] A steep slope of term-structure is possibly due to default risk premium as well as expected default probability.

[4] A credit event of sovereign debts can be default or (mostly) restructuring and the events cause write-downs in face value or cash distributions of the debt. The nature of sovereign credit risk is well-documented in Duffie *et al.* (2003).

[5] We do not need to model the physical probability of jumps since it does not affect sovereign bond price or sovereign CDS price. For that reason, we cannot infer the risk price on the uncertainty of real default probability λ^P from the price data.

$$d\ln\lambda^Q = \kappa^Q(\theta^Q - \ln\lambda^Q)dt + \sigma_\lambda dW^Q. \quad (3)$$

The log-normal intensity process ensures the probability of default not to be negative while log intensity may be negative. It is well-known that the log-normal intensity process fits CDS spreads better than other processes such as CIR or its variants. One pitfall of log-normal process is that it does not have a closed-form solution to CDS spreads and bond prices. However, we can solve them numerically.

Before pricing sovereign bonds and CDSs, we introduce two building blocks to make the solutions look simple. First, $\mathrm{V(T)}$ is the (risk-neutral) probability that a sovereign will not default on its debt from 0 until T.

$$V(T) = \tilde{E}\left[\exp\left(-\int_0^T \lambda_t^Q dt\right)\right]. \quad (4)$$

Second, $\mathrm{U(T)}$ is the (undiscounted) value of \$1 which will be paid at default between 0 and T.

$$U(T) = \tilde{E}\left[\lambda_T^Q \exp\left(-\int_0^T \lambda_t^Q dt\right)\right]. \quad (5)$$

Unfortunately, we cannot obtain the closed-form solution to $\mathrm{V(T)}$ and $\mathrm{U(T)}$. Instead, we solve them numerically by using the Crank-Nicolson finite difference method.

3.2 Pricing sovereign bonds

In pricing a sovereign debt, we discount defaultable future cash flows of a sovereign bond with hazard rate (h_t^{Bond}) which consists of mutually independent three factors: a riskless short-rate ($\mathrm{r_t}$), a (risk-neutral) default intensity (λ_t^Q) and a non-default factor (γ_t). That is,

$$h_t^{Bond} = r_t + \lambda_t^Q + \gamma_t. \quad (6)$$

Since there indeed exists a non-default factor in the sovereign yield spread, as seen in the previous analysis, we add a non-default factor γ_t to hazard rate for sovereign bond's cash flows. Additionally, we assume that the non-default factor has a normal process as in Longstaff *et al.* (2005).

$$d\gamma = \sigma_\gamma dW_\gamma^Q \quad (7)$$

The present value of sovereign debt which pays coupon c semiannually can be expressed as

$$B(T) = \frac{c}{2}\sum_{i=1}^{n} \tilde{E}\left[\exp\left(-\int_0^{T_i} h_t^{Bond}dt\right)\right] + \tilde{E}\left[\exp\left(-\int_0^{T} h_t^{Bond}dt\right)\right]$$
$$+Rec\int_0^T \tilde{E}\left[\lambda_t^{\mathrm{Q}}\exp\left(-\int_0^t h_s^{Bond}ds\right)\right]dt, \qquad (8)$$

where T_i's are coupon payment dates and Rec is recovery rate of face value at default. Thanks to the independence of three components of $\mathrm{h_t}$ in equation (6), we can easily calculate the expectations of bond price formula (8) and express it as follows:

$$B(T) = \frac{c}{2}\sum_{i=1}^{n} D(T_i)V(T_i)L(T_i) + D(T)V(T)L(T) + Rec\int_0^T D(t)U(t)L(t)dt, \qquad (9)$$

where $\mathrm{L(t)} = \exp\left(\frac{\sigma_\gamma^2 \mathrm{t}^3}{6} - \gamma \mathrm{t}\right)$, $\mathrm{D(t)} = \mathrm{E^Q}\left[\exp\left(-\int_0^t r_s ds\right)\right]$ and V(t) and U(t) are expressed in equation (4) and (5), respectively.

3.3 Pricing sovereign CDS

Now turning to pricing sovereign CDSs, we discount cash flows of sovereign CDS with different hazard rate $\mathrm{h_t^{CDS}}$ which consists of risk-free factor (r_t) and default factor (λ_t^Q) unlike the pricing of bonds in the previous section. That is,

$$h_t^{CDS} = r_t + \lambda_t^Q. \qquad (10)$$

Note that the hazard rate of a CDS, $\mathrm{h_t^{CDS}}$, is different from that of a bond, $\mathrm{h_t}$, in equation (6). This assumption can be seen from two perspectives. First, the model assumes that CDSs are priced without any other factors than the default factor while bonds are priced with both a non-default factor and default factor. This perspective is reasonable in that CDSs are a swap contract and more liquid than bonds. Previous studies have shown that bond spreads have a significant component that is not relevant to the default factor.

However, some recent studies raise questions if CDS spread is a clean measure of credit quality (e.g. see Badaoui *et al.* (2013)). They find evidence that non-default factors (e.g. a liquidity factor) are priced in CDS spreads. This issue can be compromised by the second perspective. Even if there exists non-default factor in CDS spreads, γ_t of bond prices measures the relative size of non-default factors. Thus, we allow γ_t to be both positive and negative by assuming a pure normal diffusion process in equation (7) of the previous section.

As in the pricing of sovereign bonds, it is straightforward to obtain the price formula for CDS spreads maturing at T and quarterly paying premium p. The present value of the premium leg can be obtained by discounting the cash flows with the hazard rate h_t^{CDS}.

$$PREM(T) = \frac{p}{4}\sum_i \tilde{E}\left[\exp\left(-\int_0^{T_i} h_s^{CDS} ds\right)\right], \tag{11}$$

where T_i's are premium payment dates.

The counter-party, protection leg, receives protection (1-Rec) at default. Therefore, the present value of the protection leg is

$$PROT(T) = (1 - Rec)\int_0^T \tilde{E}\left[\exp\left(-\int_0^t h_s^{CDS} ds\right)\right] dt. \tag{12}$$

The CDS premium at time 0 is determined under the condition that the two legs are fair. Similar to the bond price case, we get the semi-closed-form solutions to PREM(T) and PROT(T) and then the CDS premium can be expressed as,

$$p = CDS(T) = 4(1 - Rec)\frac{\int_0^T D(t)U(t)dt}{\sum_i D(T_i)V(T_i)} \tag{13}$$

where D(t), V(t) and U(t) are already defined in the previous section.

4 Estimation methodology and empirical results

4.1 Maximum likelihood estimation

We jointly estimate the term-structure of CDS spread and its reference bond price at the same time by using maximum likelihood (ML) estimation. To obtain the likelihood function under the physical measure, we specify the market prices of diffusion risk of default and non-default factors as $\eta_\lambda = \delta_0 + \delta_1 \ln \lambda^Q$ for the default factor and $\eta_\gamma = \delta_2$ for the non-default factor. That is, $dW^Q = dW^P + (\delta_0 + \delta_1 \ln \lambda^Q)dt$ and $dW_\gamma^Q = dW_\gamma^P + \delta_2 dt$. With this relation, we obtain the dynamics of λ^Q and γ under the physical measure (P-measure) as follows:

$$d\ln\lambda^Q = \kappa^P(\theta^P - \ln\lambda^Q)dt + \sigma_\lambda dW^Q, \tag{14}$$

where $\kappa^P = \kappa^Q - \sigma_\lambda\delta_1$ and $\kappa^P\theta^P = \kappa^Q\theta^Q + \sigma_\lambda\delta_0$.

We also assume that the market price of risk of the non-default factor γ_t by $\eta_\gamma = \delta_2$ to obtain the P-measure dynamics as follows.

$$d\gamma = \mu_\gamma^P dt + \sigma_\gamma dW_\gamma^P \tag{15}$$

where $\mu_\gamma^P = \sigma_\gamma \delta_2$.

Our objective is to maximize the likelihood of the observed data on hand under the P-measure. With the P-measure dynamics of default and non-default factors, we can obtain the P-measure transition density of CDS premium and bond prices by change of variables. Once the likelihood is expressed in terms of λ_t and γ_t, the default and non-default factors can be inferred from CDS and bond price data although they are not explicitly observable. We use a 5-year CDS and one of the Korean sovereign bonds in each estimation procedure to get λ_t and γ_t. In other words, we assume that the 5-year CDS and sovereign bond price data are observed perfectly without any error or noise so that λ_t and γ_t can be inverted from the data using the model price in equation (9) and (13). Given the inverted factors, we can complete the likelihood function since we know the transition density of (λ_t,γ_t).

$$x_t = B(\lambda_t^Q, \gamma_t), \qquad y_t = CDS_5(\lambda_t^Q). \tag{16}$$

In addition to the 5-year CDS data, we use 1, 3, and 10-year CDS spreads as well to estimate the term structure of the credit curve. Since there are not any more factors to fit all term structure data exactly, we assume that the remaining assets are observed with errors which have independent normal distributions with mean of zero and variance of σ_M^2. That is, $z_t(M) = CDS_M(\lambda_t^Q) + \sigma_M \epsilon_t$ for M = 1, 3 and 10 years.

In sum, our objective function to be maximized is

$$\begin{aligned} L(\Theta|data) &= f^P\big(x, y, z(1), z(3), z(10)\big) = f_{x,y}^P(x,y) f_\epsilon^P(\epsilon|y) \\ &= |J| f_{\lambda,\gamma}^P(\lambda,\gamma) f_\epsilon(\epsilon|\lambda). \end{aligned} \tag{17}$$

In the estimations, we do not estimate recovery rate, Rec, as a parameter. We assume that the value is exogenously determined but allowed to be time-varying. Previous studies have used a constant value for Rec or have estimated it as a parameter[6]. In this study, we use recovery data provided by Makit Group. For example, Markit's estimate of recovery rate for

[6] For example, Pan and Singleton (2008) estimate the recovery rate as 0.177 for Korea in their unconstrained estimation procedure and Schneider *et al.* (2011) and Zinna (2012) use a constant value for the revovery rate. In our study, this issue is not important because we jointly price both a bond and CDSs and the recovery rate parameter affects symetrically on the two assets. Furthermore the recovery risk is not in our interest in the study.

the Korean sovereign debt is 33.73% on average and its standard deviation is 5.32% for the sample period of bond I. We use the daily time series estimates as a time-varying exogenous variable.

4.2 Empirical results

Table 3 presents the ML estimates of the model parameters and their asymptotic standard errors. The parameters are well estimated since they are statistically significant. Also, the parameter estimates are consistent and robust across bonds even if their sample periods are different. There are some exceptions for κ^{P} and κ^{Q}, but even in those cases, similar sample periods guarantee the similar estimates. For example, bond I and bond III have similar sample period and their Q mean-reverting speeds are estimated similarly around 0.007. For the remaining bonds, II, IV, V, and VI which have similar sample periods, κ^{Q} are approximately -0.18.

Next, we examine the CDS pricing errors to evaluate estimation results. The pricing errors are plotted in Figure 2, where each subplot corresponds to the bond cross-sections that are used in each joint estimation with the CDS spreads. We note again that a 5-year CDS spread and sovereign bond price are perfectly estimated by the assumption, implying their pricing errors are assumed to be zero. The pricing errors seem to be reasonably small. The errors increase up to 100 bps around the financial crisis period, but this is not quite high given that the CDS spreads surge up to 700 bps at that time. (See Figure 1 and Table 1)

Figure 2 shows that the 3-year CDS spread is the best fit in our model for Bond I and III, which is also supported by the smaller ML estimate of $\sigma_{\epsilon}(3)$ than those of $\sigma_{\epsilon}(1)$ and $\sigma_{\epsilon}(10)$ in Table 3. For other bonds, the 10-year CDS spread is the best fit. In any cases, 1-year CDS has the largest pricing error which is consistent with the finding of Pan and Singleton (2008).[7] The 10-year CDS tends to be overpriced before 2010, and thereafter, to be underpriced. In contrast, 1-year CDS shows a converse tendency. Even though our one-factor intensity model cannot fully capture the movement of CDS term structure, pricing errors are small and the ML estimates for the pricing error parameters are also small and significant.

Thus, our estimation result seems to be reliable and all the parameters, except for mean reversion parameters, are stable and robust. We will further discuss the economic implications on the estimated parameters and estimated factors in the consecutive subsections.

4.2.1 Economic implication of parameter estimates

[7] They argue that it is because speculators typically use a 1-year CDS to reflect their view on the market.

The model parameters related to the default factor (or intensity process) include $\kappa^P, \theta^P, \kappa^Q, \theta^Q,$ and σ_λ. The main findings on those parameters are as follows. First, σ_λ is always stable and significant for any sample period. Second, negative values of θ^P (θ^Q) are not surprising since our model assumes the dynamics of log value of λ^Q and thus, θ implies a long-run mean of $\ln \lambda^Q$. Since λ^Q is typically much smaller than 1 and has the negative log value, it is reasonable that θ has a negative value. Third, mean-reverting parameters show very different results across different sample periods. For interpretation, we divide the sample bonds into two categories, one set (bond I and III) covers the early 2000's and the other one (bond II, IV, V and VI) covers from 2009. We find that, in the longer sample periods starting from the early 2000's and including the financial crisis, mean-reversion parameters are positive under both measures. However, in the shorter sample periods starting from 2009, they have different signs: positive under P and negative under Q. This result is the same for each of the sample bonds within the similar sample period. Thus, it seems that the mean-reversion parameters are rather sensitive to the sample period. Last and most importantly, we find that the estimates for κ and θ are very different under the P- and Q- measures, implying that investors have more pessimistic expectation about the credit environment under Q than P because they require higher compensation for bearing the uncertainty of future credit risk. For example, the long-run mean of arrival rates of credit events θ is greater under Q than under P and the intensity is more persistent under Q than under P (i.e. $\kappa^P > \kappa^Q$), implying that intensity tends to increase more even at good times of credit quality and bad times last longer under Q.[8] Using the difference between Q- and P-expectation, we disentangle the risk premium from the expected default risk component in the yield spread in a similar way that Pan and Singleton (2008) do with CDS spread. We will precisely discuss this point in the next subsection.

4.2.2 Default and Non-default factor in yield spreads

Now, we proceed to explore the implied instantaneous yield spreads, default factor (λ^Q) and non-default factor (γ). The implied values of the factors can be interpreted as yield spreads of a bond which instantaneously matures. Figure 3 depicts the time-series of default and non-default factors implied in Bond I. While the default factor has dominated the non-default factor since the bankruptcy of Lehman, the non-default factor is sizable before the crisis. To conserve the space, we do not plot the factors of the other bonds. Instead, we report the descriptive statistics of factors of all bonds in Table 4. We find that, for bond I, the proportion of the default factor is about 28% during 2003-2005 but is about 67% after 2009. The default

[8] Pan and Singleton (2008) discuss the issue in a similar way.

factor accounts only for about 46% for the full sample period, which is quite different from the result of the non-parametric analysis. Even though not reported explicitly in Table 2, we can see that the default component (CDS spread/yield spread) is about 80%. Therefore, we verify that the non-parametric analysis results in a substantially biased estimate.

[Insert Figure 3 Here]

[Insert Table 4 Here]

Our approach based on a term structure model shows that 64% of instantaneous yield spread of bond I is due to non-default factor on average. Previous studies have shown that default component explains only a small amount of yield spreads and a considerable fraction is attributable to non-default factor such as tax or liquidity. (See Duffie *et al.* (2003), and Küçük (2009) for the sovereign bond market, and Elton *et al.* (2001), Eom *et al.* (2004), Longstaff *et al.* (2005), Chen *et al.* (2007), and Huang and Huang (2012) for the corporate bond market.)

Although the determinants of non-default factors are beyond the scope of our study, it is valuable that we see if the factor is trivial or not. More specifically, our model assumes that the sovereign bonds are priced perfectly or without any observation errors, which may lead the observation errors or idiosyncratic bond risks to be incorporated in the non-default factor. As such, we are interested in the cross-sectional variation of the factors.

Analyzing the cross-sections, we find that the average values of default factors are almost the same across bonds (around 90 bps). However, the non-default factors widely vary across bonds. To compare the cross-sections precisely, the values and proportions of the factors are shown in Panel D during 2009-2012 when all bonds have their market price data and, for more information, time-series of the factors for the overlapping sample period are plotted in Figure 4.

[Insert Figure 4 Here]

The average values of non-default factor for each bond range from -14 bps to 114bps and accordingly the fraction varies from -31% to 55% even for the overlapping period. To verify if such a variation is indeed due to non-default factor (or non-trivial risk component), the cross-sectional mean value can give us a nontrivial factor since mutually independent observation errors or idiosyncratic bond risks, if any, will be *averaged out*. The average value of non-default factors is approximately 39 bps and the fraction of the non-default factor is about 30%

(=39/(90+39)) during 2009-2012. Even after averaging idiosyncratic risks out, there remains a significant amount of non-default factors, implying that the factor is a nontrivial part.

4.2.3 Decomposition of yield-to-maturity

In this subsection, we decompose the yields-to-maturity observed in the market into four components which consist of risk-free, default risk, risk premium, and non-default. The first component, risk-free, is the yield related to the risk-free cash flows of the Korean sovereign bonds. Our risk-free benchmark is the US treasury rates as usual. Yields of USD-denominated Korean sovereign bonds are typically higher than US treasury rates and are strongly related to the benchmark. The second component, default risk, is the yield required by investors due to the expected default probability of the sovereign. When trading Korean sovereign bonds, investors require spread over the US treasury rate because they consider the possibility of default on the bonds and they have some expectation about the default probability. The third component, risk premium, indicates the premium to compensate for the unexpected default probability. Investors' expectation about default probability could change in the future as the credit quality of the sovereign changes. If the future credit quality is stochastic, as our model assumes, investors command spread in addition to the expected default risk component. The more volatile the future credit quality, the more premium the investors would require when the expected default probability is the same. The fourth component, non-default, is the remaining part which is unexplained by the three components above.

To quantify the corresponding fraction of yield-to-maturity, according to the qualitative definition of each component, we first define the four pseudo-bond prices: (i) $B^{RF} = B(T; \lambda^Q = 0, \gamma = 0)$, (ii) $B^{DEF} = B\left(\hat{\lambda}^Q, U^P, V^P, \gamma = 0\right)$, (iii) $B^{RP} = B(\hat{\lambda}^Q, \gamma = 0)$, and (iv) $B^{Mkt} = B(\hat{\lambda}^Q, \hat{\gamma})$. All these prices are calculated by the bond price formula B(T) of equation (9) with the specified inputs and are converted to corresponding yield fractions (y^{RF}, y^{DEF}, y^{RP} and y) using equation (2). Specifically, the risk-free component price B^{RF} is obtained by setting the risk factors zero, i.e. $\lambda = \gamma = 0$, which is the fraction of the observed price determined by the risk-free benchmark. Therefore, y^{RF} measures the fraction of yield related to the risk-free component. To isolate the effect of non-default factor, we set $\gamma = 0$ and use the implied intensity $\hat{\lambda}^Q$ in both (ii) and (iii). The most important part of our decomposition method lies in (ii). To distinguish the risk premium component from the expected default risk, we calculate B^{CREDIT} using U^P and V^P which are calculated by U(T) and V(T) in equation (9) under P-expectation using P-measure parameters κ^P, θ^P. The difference between expectations under P and under Q stems from the risk premium that investors demand for bearing the future change in default probability. Thus, if investors require positive default risk premium, B^{RP} would be cheaper than B^{DEF}, and y^{RP} would be higher than y^{DEF}.

Therefore, $\mathrm{y^{RP}} - \mathrm{y^{DEF}}$ can measure the risk premium fraction and $\mathrm{y^{DEF}} - y^{RF}$ can measure the default risk fraction.

As we perfectly fit the market price of bonds, it is trivial that the calculated bond price with fitted values of the factors $\mathrm{B}(\hat{\lambda}^{\mathrm{Q}}, \hat{\gamma})$ is just the market price. Since $\mathrm{B^{RP}}$ is calculated with $\gamma = 0$, if the non-default factor is positive, the market price of bonds $\mathrm{B}(\hat{\lambda}^{\mathrm{Q}}, \hat{\gamma})$ would be cheaper than $\mathrm{B^{RP}}$. Thus, we interpret the non-default fraction as $\mathrm{y} - \mathrm{y^{RP}}$.

To summarize, the pseudo-bond prices allow us to measure the risk-free fraction as $\mathrm{y^{RF}}$, the default risk fraction as $\mathrm{y^{DEF}} - \mathrm{y^{RF}}$, the risk premium fraction as $\mathrm{y^{RP}} - \mathrm{y^{DEF}}$, and the non-default (or rest) fraction as $\mathrm{y} - \mathrm{y^{RP}}$.

[Insert Figure 5 Here]

First, we focus on the time-series property of yield fraction of Bond I, and then we will analyze the cross-sections next. Figure 5 shows the time-series of yield fraction of Bond I and Figure 6 displays the time-series average of each component (normalized by total yield). Because Figure 5 plots the level of $\mathrm{y^{RF}}, \mathrm{y^{DEF}}$, $\mathrm{y^{RP}}$ and y, the difference between each line means each component, as explained. We find that the yield fractions are very time-varying. The variation of risk free component depending on the US short rate risk is not our interest. We are much more interested in the fractions excluding the risk-free component, that is, sovereign yield spread. The variation of default risk premium fraction ($\mathrm{y^{RP}} - \mathrm{y^{DEF}}$) is noticeable. What is striking is that the risk premium fraction accounts for majority of yield spread as well as total yield during the financial crisis (especially, after Lehman Brothers' bankruptcy on September of 2008). During that time, both the risk-free and non-default components decrease. Of course, the default risk component increases, but the increase is modest and not enough to explain such a high yield of the Korean bond. Compared to the default risk component, the risk premium component increases substantially. For the reason, we argue that the increase in yield-to-maturity of Korean sovereign bonds during the crisis is more attributable to the substantial increase in risk premium that global investors demand rather than the increase in default risk of Korea.

It is well-known that investors extremely seek to purchase safer assets in a period of financial turmoil, which is typically referred to as "flight-to-quality". In fact, the phenomenon that global investors prefer to buy the US treasury and sell risky assets was witnessed during the recent crisis. In Figure 5, we confirm that the risk-free component decreases around 2008, implying that the price of US treasury increased because an increase in demand.

Put differently, "flight-to-quality" implies that investors demand much more premiums for bearing the same level of risk, which is also evidenced by the substantial increase in risk premium component. So, our finding provides evidence on the flight-to-quality during the crisis, inferred from sovereign bond data.

[Insert Figure 6 Here]

In Figure 6, we see that the risk-free component accounts for about 60% of total yield of Bond I in terms of time-series average. We also find that the default risk fraction accounts for only 10% of total yield and 27% of yield spread (i.e. yield excluding risk-free fraction). In the literature on credit spread (defined as yield on corporate bond minus yield on treasury), expected default probability can hardly explain observed credit spread. For example, Huang and Huang (2012) find that only around 20%~30% of credit spread is due to credit risk for investment grade bonds. Although our study differs from theirs in that we deal with sovereign yield spreads rather than corporate yield spreads, the size of fraction due to default risk is similar with their results. Note that the Korean sovereign bonds were in the investment grade during the sample period.

Now, we analyze the cross-sectional variation of each component. We find that the fractions of each component are different for the cross-sections and, in particular, non-default component varies most widely (-3% to 17% with respect to the total YTM and -7% to 50% with respect to the yield spread). It might be due to the different sample period if the portion is significantly time-varying (indeed it is true as shown), but bond II, IV, V, and VI also show different fraction pattern even though their sample periods are the same. Therefore, the variation of portion is not due to the sample period. The cross-sectional variation may be attributable to idiosyncratic bond risk or observation error which can be incorporated in the non-default fraction. However, we find that even after excluding the non-default component, the fraction of risk-free, default risk, and risk premium are cross-sectionally different (See Figure 6): While the risk premium portion is much higher than the credit risk portion for bond IV and VI, credit risk potion is higher for bond II and V.

More precisely, the fractions of the default risk component in yield spread range from 22 to 44 %, risk-premium component 23 to 70%, and non-default risk component -8 to 50%. The standard deviations for each component are 9%, 19% and 23%, respectively. Therefore, we find that the default risk component varies the least across bonds. Furthermore, we verify again that risk premium component plays an important role for all bonds. When we take the risk premium into account, yield spread can be explained up to 45%~100% by default risk and risk premium.

5 Conclusion

We ask the contribution of the default component (default risk and risk premium) to the observed yield spreads. To answer this question, we decompose the yield-to-maturity into four components – risk-free, default risk, risk premium, and non-default component using a reduced-form approach. To distinguish default component and non-default component, we model the instantaneous spread with two independent factors – λ^Q and γ – in line with Longstaff *et al.* (2005). Furthermore, the methodology of Pan and Singleton (2008) allows us to disentangle the risk premium component from the default risk component. To this end, we estimate the model with both sovereign bonds and CDS data.

As a preliminary analysis, we conduct a non-parametric analysis and roughly confirm the relation between yield spreads and CDS spreads. As expected, however, the model-based approach shows that the model-independent approach makes a significant bias on the estimates compared to the implied factors. The default component in the yield spread of Bond I is estimated as 80% and 47% in the non-parametric and model-based analysis, respectively. Although the model-based approach we suggest might have some bias, we believe that our approach outperforms over the non-parametric approach for the several reasons as explained earlier.

In the model-based analysis, we also find that, for Bond I which has the longest sample period, the default risk component accounts for only about 27% of yield spread on average. When we take the risk premium component into account, about half of the yield spread is explained by the default component (default risk and risk premium). Therefore, we conclude that the risk premium plays an important role in the sovereign bond market. Also, in time-series analysis, the risk premium component significantly varies over time and, in particular, a major portion of the yield spread is attributable to the risk premium during the recent financial crisis.

Furthermore, the result shows that the non-default component accounts for a significant part and is not a trivial component such as pricing error or idiosyncratic risk. In the corporate bond market, many previous studies also document that significant non-default component exists in yield spreads and is related to tax or liquidity.

Finally, we report that the cross-sectional variation of each component. Since the characteristics of bonds (e.g., time-to-maturity, age, coupon rate and amount outstanding) are very different, the cross-sectional variation of each fraction is not surprising very much. We remain an analysis about determinants of the cross-sectional variation for each component for further research due to the lack of cross-sections.

Our findings provide an important implication to policy-makers as well as academics and practitioners since the sovereign yield spread is the cost to borrow a foreign currency (typically dollars). The sovereign's borrowing cost is determined not only by her default risk but also by risk premium and non-default factor. Our paper shows that the risk premium and non-default factor are more important in terms of the size of the fractions. Therefore, it is very important to identify the driving forces behind the risk premium and non-default factor. Future research should identify determinants of the each component, particularly the risk premium and non-default component.

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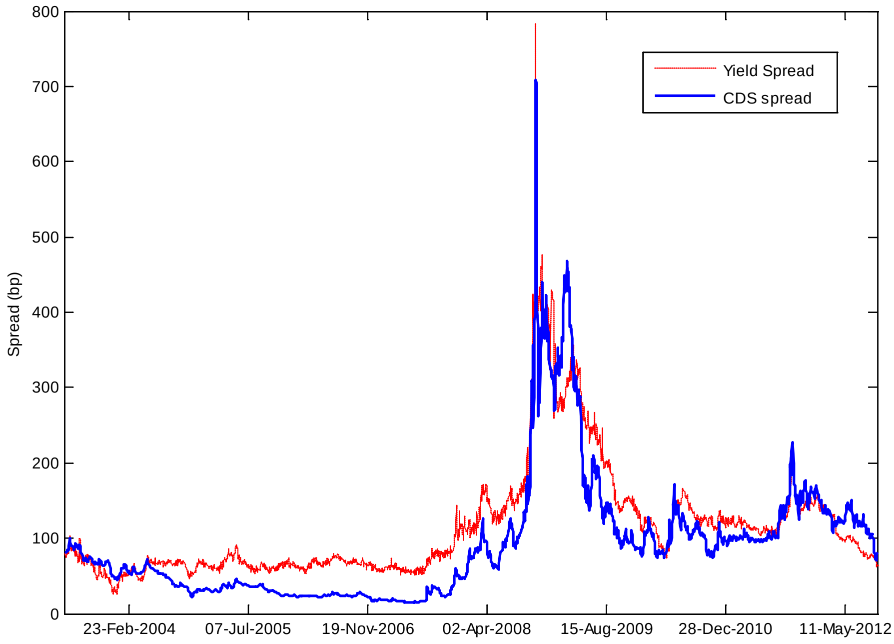

FIGURE 1. YIELD SPREAD AND CDS SPREAD

Notes: The figure plots yield spread (dotted red line) and CDS spread (blue line) from May 1, 2003 to 21 Dec, 2012. The yield spread calculated by the yield difference between KOR Bond I and U.S. CMT. The CDS spread comes from the 5-year CDS contract on the Korean sovereign bond.

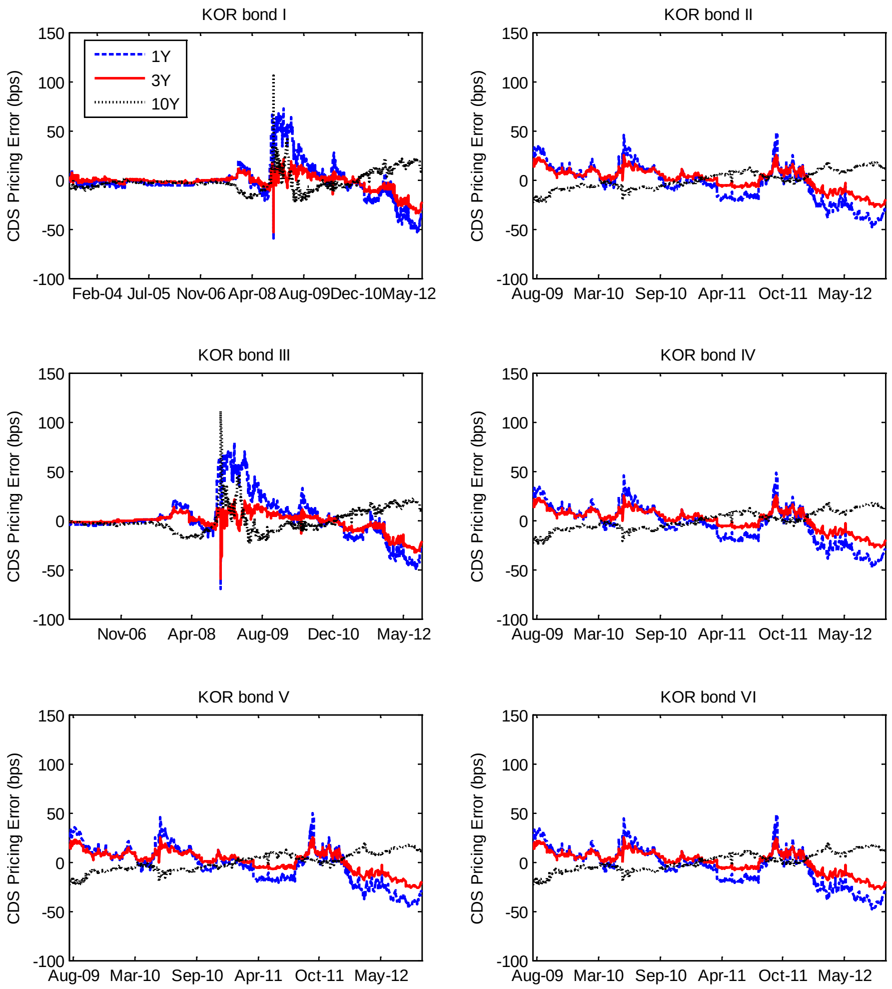

FIGURE 2. THE PRICING ERRORS OF 1-, 3-, AND 10-YEAR CDS

Notes: The pricing errors are measured by market spread minus the model spread. Each plot corresponds to bond cross-section which are used in the joint estimation with CDS. We note that 5-year CDS and sovereign bonds are perfectly priced by the assumption.

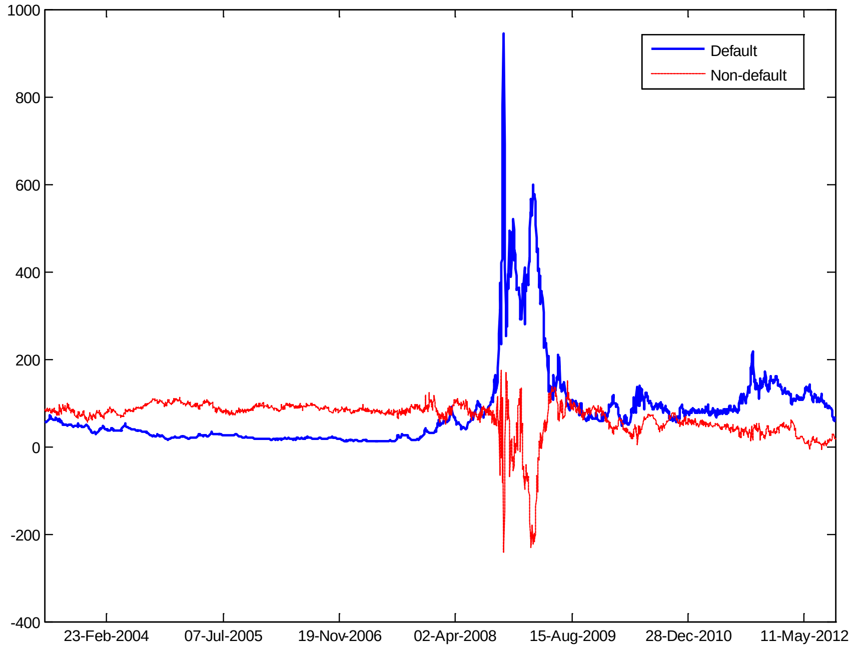

FIGURE 3. TIME-SERIES OF DEFAULT AND NON-DEFAULT FACTOR FOR KOR BOND I

Notes: The figure plots the estimated default factor (λ_t^Q) and non-default factor (γ_t) implicit in the Korean sovereign bond I.

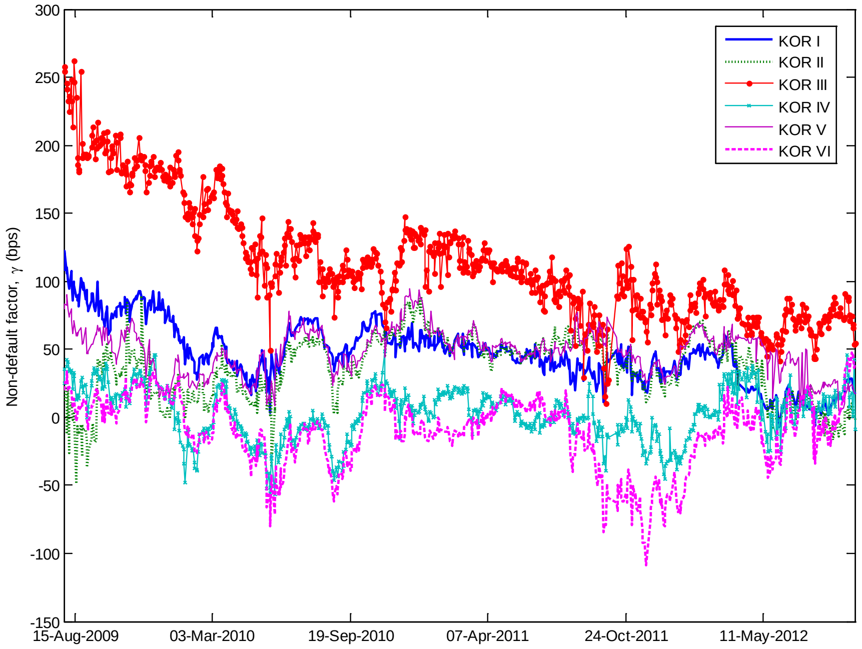

FIGURE 4. TIME-SERIES OF NON-DEFAULT FACTOR

Notes: The figure plots the estimated time-series of non-default factors (γ_t) implicit in the six Korean sovereign bonds from Jul 30, 2009 to Sep 21, 2012.

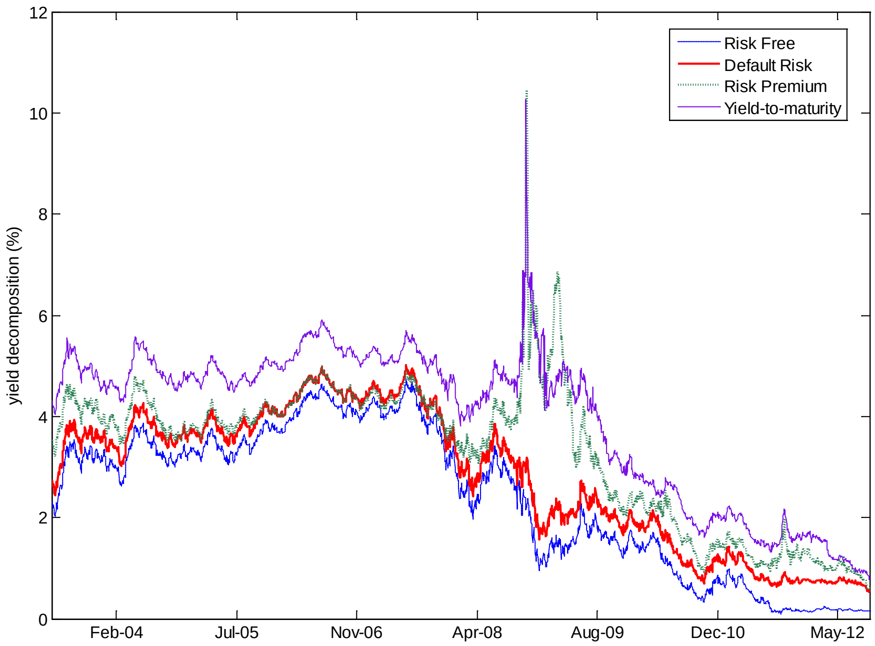

FIGURE 5. DECOMPOSITION OF YIELD-TO-MATURITY OF BOND I

Notes: The figure plots the time-series of yield composition of Bond I. Each line indicates the *cumulative* level of the composition: 'Risk Free' indicates the level of the yield composition explained by the US treasury. 'Default Risk' indicates the composition additionally explained by sovereign default risk, based on the level of 'Risk Free'. 'Risk Premium' adds default risk premium to 'Default Risk'. 'Yield-to-maturity' is the yield of the sovereign bond observed in the market. Thus, the difference between 'Yield-to-maturity' and 'Risk Premium' indicates the yield implied by the non-default factor.

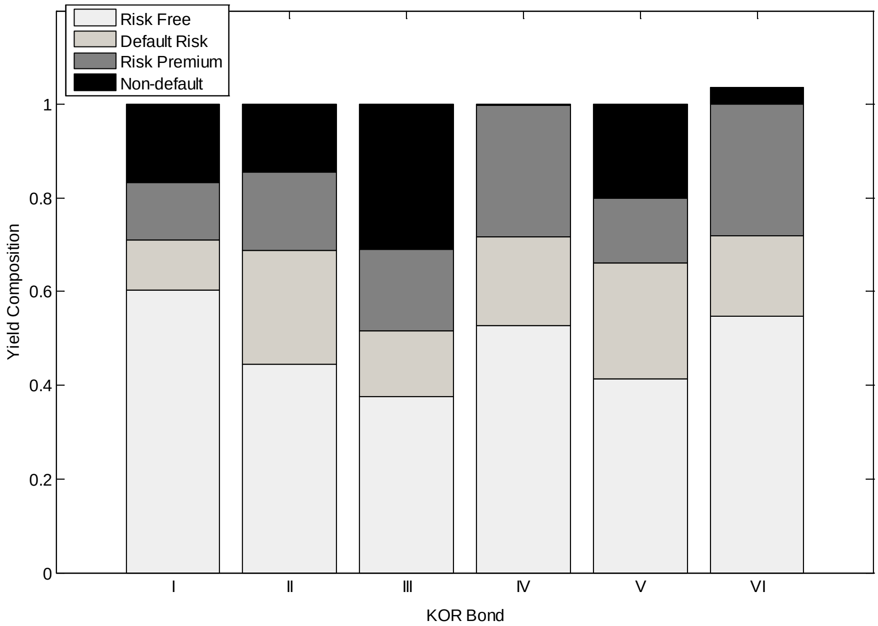

FIGURE 6. YIELD COMPOSITION - AVERAGE

Notes: The figure displays time-series average of yield composition. The value above 1 indicates a negative value in bond VI.

TABLE 1 – INFORMATION ON KOREAN SOVEREIGN BONDS IN THE SAMPLE

KOR Bond	Issue Date	Maturity	Coupon	Amount	TTM	N.Obs	First Obs Date
I	03-Jun-03	01-Jun-13	4.25	1,000,000	10	2331	30-May-03
II	22-Sep-04	22-Sep-14	4.875	1,000,000	10	785	30-Jul-09
III	02-Nov-05	03-Nov-25	5.625	400,000	20	1696	9-Nov-05
IV	07-Dec-06	07-Dec-16	5.125	500,000	10	789	30-Jul-09
V	16-Apr-09	16-Apr-14	5.75	1,500,000	5	791	30-Jul-09
VI	16-Apr-09	16-Apr-19	7.125	1,500,000	10	789	30-Jul-09

Notes: The table summarizes the information on Korean sovereign bonds in the sample. All the bonds are US dollar denominated, semi-annually coupon-paying bonds. 'Amount' indicates the issued amount ($1,000) of the bonds. 'TTM' represents time-to-maturity (years) from the issued date. 'N.Obs' is the number of observations (days). The sample period of each bond is from the 'First Obs Date' to Sep 21, 2012.

TABLE 2 – DESCRIPTIVE STATISTICS FOR YIELD SPREAD, CDS SPREAD, AND BASIS

	MEAN	STD	SKEW	KURT	MIN	MAX
			KOR Bond I			
Yield Spread	114.43	77.10	2.46	7.98	24.70	783.12
CDS Spread	91.17	82.89	2.46	8.14	14.05	708.64
Basis	23.26	33.25	-1.15	5.18	-171.09	167.82
			KOR Bond II			
Yield Spread	127.93	29.57	-0.25	0.46	46.58	227.88
CDS Spread	114.41	25.49	0.97	1.36	70.46	228.24
Basis	13.52	26.00	-0.55	-0.11	-55.10	82.19
			KOR Bond III			
Yield Spread	127.03	90.48	2.04	3.45	7.25	474.88
CDS Spread	104.28	90.54	2.15	6.40	14.05	708.64
Basis	22.74	47.36	-0.31	6.96	-361.56	230.81
			KOR Bond IV			
Yield Spread	130.82	29.60	0.56	0.65	61.78	250.49
CDS Spread	114.47	25.44	0.97	1.36	70.46	228.24
Basis	16.34	15.77	0.33	0.29	-25.44	62.77
			KOR Bond V			
Yield Spread	136.33	27.96	0.25	0.72	67.15	235.10
CDS Spread	114.50	25.42	0.96	1.36	70.46	228.24
Basis	21.83	23.35	-0.15	-0.21	-40.44	84.86
			KOR Bond VI			
Yield Spread	133.89	30.13	0.14	-0.23	50.13	226.43
CDS Spread	114.47	25.44	0.97	1.36	70.46	228.24
Basis	19.42	18.03	-0.11	-0.12	-35.96	64.40

Notes: The table reports the summary statistics for sovereign yield spreads CDS spreads and differences between the two spreads. The yield spreads are calculated as difference between the yield on Korean sovereign bond and the yield on US CMT. The CDS spread comes from the 5-year CDS on Korean sovereign bonds. Since the data-available periods are different for each bond, as seen in Table 1, we report CDS spread along with each bond during the corresponding period in order to show the relationship between the two spreads. 'Basis' is measured by yield spread minus CDS spread. All values are measured in basis points.

TABLE 3- MAXIMUM LIKELIHOOD ESTIMATES OF THE MODEL PARAMETERS

	I	II	III	IV	V	VI
κ^P	1.19695	7.66846	0.56509	7.67734	8.47490	6.53687
	(0.23549)	*(2.02559)*	*(0.25622)*	*(2.06061)*	*(2.03948)*	*(2.11612)*
θ^P	-5.30728	-4.72051	-4.89143	-4.71432	-4.73007	-4.65447
	(0.22339)	*(0.04931)*	*(0.48034)*	*(0.05664)*	*(0.04642)*	*(0.08380)*
σ_λ	0.79682	0.69426	0.84812	0.69275	0.69925	0.69598
	(0.00000)	*(0.00144)*	*(0.00000)*	*(0.00134)*	*(0.00141)*	*(0.00131)*
κ^Q	0.00792	-0.17756	0.00681	-0.17729	-0.17705	-0.17531
	(0.00021)	*(0.00469)*	*(0.00129)*	*(0.00499)*	*(0.00488)*	*(0.00466)*
$\kappa^Q\theta^Q$	-0.06296	0.75035	-0.05853	0.74954	0.75094	0.73981
	(0.00170)	*(0.02066)*	*(0.00533)*	*(0.02170)*	*(0.02148)*	*(0.02021)*
$\sigma_\epsilon(1)$	0.00193	0.00194	0.00223	0.00194	0.00193	0.00195
	(0.00003)	*(0.00018)*	*(0.00005)*	*(0.00018)*	*(0.00018)*	*(0.00019)*
$\sigma_\epsilon(3)$	0.00089	0.00111	0.00099	0.00111	0.00111	0.00111
	(0.00001)	*(0.00009)*	*(0.00002)*	*(0.00009)*	*(0.00010)*	*(0.00009)*
$\sigma_\epsilon(10)$	0.00107	0.00091	0.00127	0.00092	0.00091	0.00092
	(0.00001)	*(0.00004)*	*(0.00002)*	*(0.00005)*	*(0.00004)*	*(0.00005)*
μ_γ^P	0.00100	0.00098	0.00097	0.00104	0.00115	0.00092
	(0.01194)	*(0.01032)*	*(0.01040)*	*(0.01152)*	*(0.00687)*	*(0.01289)*
σ_γ	0.02662	0.01610	0.02210	0.01510	0.01032	0.01486
	(0.00005)	*(0.00014)*	*(0.00012)*	*(0.00021)*	*(0.00012)*	*(0.00023)*
Mean lnL	25.8	25.88	24.3	25.49	26.46	25.16
N.Obs	2331	785	1696	789	791	789

Notes: The table reports ML estimates for the model parameters with their asymptotic standard errors in parentheses. We use 1, 3, 5, and 10 year CDS data and one of the sovereign bonds in each estimation (corresponding to each column). The sample periods are various across the estimations and are noted in Table 1. At the end of the table, we report average value of log likelihood over the sample period and the number of observations.

TABLE 4- DEFAULT AND NON-DEFAULT FACTORS

	Default factor (λ^Q)						Non-default factor (γ)					
	Value (bps)				Proportion (%)		Value (bps)				Proportion (%)	
	Mean	Std	Min	Max	Mean	Std	Mean	Std	Min	Max	Mean	Std
Panel A: Full sample period												
I	79.44	95.91	10.79	943.92	46%	28%	66.36	40.78	-242.66	174.78	54%	28%
II	91.41	17.04	59.60	158.68	74%	18%	36.19	23.89	-50.42	92.94	26%	18%
III	87.93	103.35	9.91	938.19	32%	23%	168.70	69.89	-125.71	352.23	68%	23%
IV	91.59	17.03	59.70	158.80	103%	26%	0.81	19.93	-64.19	47.38	-3%	26%
V	90.37	16.89	58.67	157.26	65%	9%	48.93	16.76	3.04	94.64	35%	9%
VI	91.59	17.14	59.52	159.29	131%	69%	-14.31	29.11	-125.00	47.69	-31%	69%
Panel B: Sub-sample period 2003-2005												
I	34.09	12.65	14.64	71.26	28%	9%	86.21	11.68	56.42	111.10	72%	9%
Panel C: Sub-sample period 2005-2009												
I	95.02	143.25	10.79	943.92	39%	34%	70.00	55.32	-242.66	174.78	61%	34%
III	87.69	138.96	9.91	938.19	22%	23%	215.13	52.45	-125.71	352.23	78%	23%
Panel D: Sub-sample period 2009-2012												
I	96.33	28.05	48.27	217.53	67%	14%	46.72	22.45	-7.45	122.35	33%	14%
II	91.41	17.04	59.60	158.68	74%	18%	36.19	23.89	-50.42	92.94	26%	18%
III	88.22	25.10	44.00	179.04	45%	14%	114.56	43.89	10.59	262.45	55%	14%
IV	91.59	17.03	59.70	158.80	103%	26%	0.81	19.93	-64.19	47.38	-3%	26%
V	90.37	16.89	58.67	157.26	65%	9%	48.93	16.76	3.04	94.64	35%	9%
VI	91.59	17.14	59.52	159.29	131%	69%	-14.31	29.11	-125.00	47.69	-31%	69%

Notes: The table reports descriptive statistics for the values and proportions of the implied factors – default and non-default factors. In panel A, the statistics are calculated with the full sample period for each bond which are specifically reported in Table 1. We also report results for various sub-sample periods in panel B C and D. The subsample periods are indicated