Is the Information on the Higher Moments of the Underlying Returns Correctly Reflected in Option Prices?

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Abstract

This study examines the relations between forward moments and realized moments of underlying asset returns from option prices. Unlike the existing literature, we look at not only the second moment, but also the third and fourth moments. Using the newly suggested model-free moment methods, we document the following: (1) The third and fourth forward measures are not unbiased estimators of the future realized third and fourth moments, though the second forward measure is an unbiased estimator of the future second moment. (2) Long-term option investors, on average, seem to underestimate the third moment and overestimate the fourth moment. (3) The expectation on the future third moment reflected in long-term options is relatively underestimated when the variance is large. (4) The estimation results show that the difference of prices between the cases with and without the biases caused by the underestimated third and overestimated fourth moments are large and economically significant. The third moment effect results in the overpricing of long-term puts and underpricing of long-term calls. The fourth moment effect generates the overpricing of deep out-of-the-money options and underpricing of near-the-money options. (5) The portfolios trading skewness between options with two different maturities show positive and significant (insignificant) returns in case of call (put) options after controlling systematic risks.

Keywords: implied moments, implied forward moments, nearby options, second nearby options

1. Introduction

Option prices contain information about market participants' expected future return distribution of the underlying asset from now to the options' maturity T. So far, researchers have mainly focused on information about the volatility extracted from option prices, that is, the implied volatility. However, the information about the future return distribution that option prices reveal includes not only the volatility of the returns, but also the extent of the asymmetry of the return distribution (skewness), the thickness of the tail distribution (kurtosis) and other information about the return distribution.

After their extensive examination of the first two moments of asset returns, researchers have recently turned their attention to the higher moments of returns. Das and Sundaram (1999) show the relation between option prices and higher moments of their underlying assets. Harvey and Siddique (2000) and Mitton and Vorkink (2007) develop an asset pricing model that incorporates the effect of skewness or idiosyncratic skewness under the assumption of the positive skewness preference. Buraschi and Jiltsov (2006), Xu (2007) and Friesen et al. (2012) also support the importance of skewness in asset pricing by showing that heterogeneous beliefs or variables related to belief differences are strongly related to skewness. These theoretical suggestions are also confirmed by empirical studies; much of the research documents the negative relationship between skewness and future stock returns. For example, Boyer et al. (2010), Xing et al. (2010), Bali et al. (2011), Bali and Murray (2013), Chang et al. (2013) and Conrad et al. (2013) show that the market skewness risk is a systematic risk factor priced in cross-sectional stock returns. In addition, Kozhan et al. (2013) show the negative risk premium using the realized skewness, which is an unbiased estimate of the expected skewness that is proposed by Neuberger (2012).

This study examines whether option prices correctly reflect the information on the moments of the underlying asset returns, not only on the second moment, but also on the third and fourth moments. We estimate the term structure of the moments using options with different maturities, and then examine how consistent they are with the underlying asset return distribution and how consistent the information revealed in the term structure of higher moments is. Out study is different from the previous research on the higher moments in that we analyze the implied moments using the options with different maturities, and extends the literature on volatility overreaction that analyze whether the implied volatilities with different maturities are consistent. Stein (1989) is the first to derive the volatility term structure relations under an assumption of the AR process of volatility and finds that the slope coefficient between implied volatilities of different maturities is statistically higher than the predicted value. He attributes this empirical result to the overreaction of long-term options to volatility news. Heynen et al (1994) confirm Stein's overreaction result using the mean-reverting GARCH and EGARCH models. Poteshman (2001) examines the volatility relation between short-term and long-term options using a stochastic volatility model and finds mixed patterns of long horizon overreaction and short horizon underreaction. Mixon (2007) tests the expectations hypothesis of the term structure of implied volatility for stock market indexes of several countries and finds that the slope of ATM implied volatility over different maturities has predictive ability for future short-term implied volatility. On the other hand, Jiang and Tian (2010) use a model-free implied volatility and show that the overreaction previously documented in the literature can be the result of model misspecification or misestimation.

In this study, we calculate the first to fourth moments from nearby and second nearby options following Bakshi et al (2003). Using the model free higher moments of two different maturities, we estimate forward moments between the two maturities, and it is one of our contribution to the literature that we provide a way to construct these model-free forward moments. Using these model-free forward moments and spot moments, we first examine the relation between the forward moments embedded in option prices and the future realized moments of underlying asset returns. For the second moment, Canina and Figlewski (1993), Christensen and Prabhala (1998), and Jiang and Tian (2005) examine the relation, and the latter two researchers document that the forward volatility is the unbiased estimator of the future realized volatility. In our study, we confirm Jiang and Tian's result for the second moment, but show that the third and fourth forward measures are not unbiased estimators of the future realized third and fourth moments. Even when we examine the implied higher moments from option prices and the calculated risk neutral moments following Bakshi et al. (2006), the calculated risk neutral moments are also biased estimators of the implied higher moments.

Second, we examine whether the future moments implied by long-term options are consistent with the moments implied by short-term options. We document that the unconditional average of the forecasting error (future implied moments – forward moments) for the third and fourth moments are statistically significantly positive and negative for the third and fourth moments, respectively, while it is not statistically significant for the second moment. That is, long-term option investors, on average, seem to underestimate the third moment and overestimate the fourth moment. Next, following Jiang and Tian (2010), we regress the forecasting errors of each moment on the variance, skewness, and kurtosis of nearby options. If the options market is efficient, then all the information regarding the future moments should be reflected in option prices and so the regression results should show that the independent variables (variance, skewness, and kurtosis) cannot predict the forecasting error. However, the variance has predictive power for the forecasting error for the third moment: The variance slope coefficient for the regression is positive and statistically significant, and the realized third moment becomes larger than the forward third moment when the variance becomes larger. This means that the expectation on the future third moment reflected in long-term options is relatively underestimated when the variance is large.

We examine the economic significance of the possible mispricing documented in this study using Corrado and Su’s (1996) option pricing model. Since this model shows how to accommodate the skewness and kurtosis factors in the Black-Scholes framework, it fits our purpose very well. The estimation results show that the mispricing or the difference of prices between the cases with and without the biases caused by the underestimated third and overestimated fourth moments are large and economically significant. The third moment effect results in the overpricing of long-term puts and underpricing of long-term calls. The fourth moment effect generates the overpricing of deep OTM options and underpricing of near-the-money options. The degree of mispricing becomes larger in the large variance period, but percentagewise, the mispricing is larger in the low variance period.

In addition to the analysis based on Corrado and Su’s model, we examine empirically whether the portfolios trading skewness between options with two different maturities produce positive significant returns after controlling systematic risks. Our empirical results show that the portfolio returns are positive and significant (insignificant) in case of call (put) options after controlling systematic risks. This result confirms that the underestimation of the third moment of second nearby options is economically meaningful, consistent with the analysis using Corrado and Su’s model.

The remainder of this paper is organized as follows. In Section II, we describe the data and present the key measures from the option price data. In Section III, we illustrate how to measure forward moments using moments data. In Section IV, we provide the main empirical results of the study, and in Section V, we examine the performance of the portfolios trading skewness. In Section VI, we present the conclusions.

II. Data and Risk neutral measures

II.1 Data description

For the empirical analysis, we use the data on the S&P 500 index options traded on the CBOE from January 4, 1996 to August 30, 2013. The option data are obtained from the OptionMetrics database and include closing bid and ask quotes for each option contract along with the corresponding strike price and maturity. We use the option data of nearby and second nearby options. The nearby call (put) option is defined as that whose remaining time to maturity is closest to 32 days among all available call (put) options. We applied the following filters. First, option quotes are included only when both bid and ask quotes are available. Option data with any missing bid or ask quote are excluded. Second, option quotes whose bid-ask spread is larger than 0.5[1] are excluded, in order to ensure that illiquid options are excluded. Third, option quotes violating no arbitrage bounds with bid and ask quotes are excluded. Specifically, we require that both the bid and ask prices of a call (put) option with a higher (lower) exercise price should be higher than those of a call (put) option with a lower (higher) exercise price, in order to ensure that any non-informative options due to minimum tick size or illiquidity are excluded. We use the three-month CD rate as the risk free rate. The S&P 500 index, obtained from the Center for Research in Security Prices, is also used.

II.2 Estimation of spot moments from option prices

The main variables used in this study are the implied first, second, third, and fourth moments and normalized moments for the future return distribution of the underlying asset. These measures are calculated from option prices as follows.

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[1] The choice of the threshold for the bid-ask spread does not qualitatively change the results reported in this paper.

Bakshi et al (2003) demonstrate that the second, third, and fourth moments from t to maturity T can be calculated as the discounted values of V, W, and X, respectively, with the assumption of continuum strikes and their corresponding option prices with maturity T. The first moment is calculated using the higher moment.

$$\mathrm{V(t,T)} = \int_{\mathrm{S(t)}}^{\infty} \frac{2\left(1-\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)}{\mathrm{K}^2} \mathrm{C(t,T;K)dK} + \int_0^{\mathrm{S(t)}} \frac{2\left(1+\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)}{\mathrm{K}^2} \mathrm{P(t,T;K)dK} \tag{1}$$

$$\mathrm{W(t,T)} = \int_{\mathrm{S(t)}}^{\infty} \frac{6\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]-3\left(\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)^2}{\mathrm{K}^2} \mathrm{C(t,T;K)dK} - \int_0^{\mathrm{S(t)}} \frac{6\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]+3\left(\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)^2}{\mathrm{K}^2} \mathrm{P(t,T;K)dK} \tag{2}$$

$$\mathrm{X(t,T)} = \int_{\mathrm{S(t)}}^{\infty} \frac{12\left(\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)^2-4\left(\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)^3}{\mathrm{K}^2} \mathrm{C(t,T;K)dK} + \int_0^{\mathrm{S(t)}} \frac{12\left(\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)^2+4\left(\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)^3}{\mathrm{K}^2} \mathrm{P(t,T;K)dK} \tag{3}$$

$$\mu(\mathrm{t,T}) = \mathrm{e}^{\mathrm{r(T-t)}} - 1 - \frac{\mathrm{e}^{\mathrm{r(T-t)}}}{2}\mathrm{V(t,T)} - \frac{\mathrm{e}^{\mathrm{r(T-t)}}}{6}\mathrm{W(t,T)} - \frac{\mathrm{e}^{\mathrm{r(T-t)}}}{24}\mathrm{X(t,T)} \tag{4}$$

Using the moments above, their normalized measures, variance, skewness, and kurtosis, can be calculated as below:

$$VAR_t(t,T) = e^{r(T-t)}V(t,T) - \mu(t,T)^2 \tag{5}$$

$$SKEW_t(t,T) = \frac{e^{r(T-t)}W(t,T) - 3\mu(t,T)e^{r(T-t)}V(t,T) + 2\mu(t,T)^3}{VAR_t(t,T)^{\frac{3}{2}}} \tag{6}$$

$$KURT_t(t,T) = \frac{e^{r(T-t)}X(t,T) - 4\mu(t,T)e^{r(T-t)}W(t,T) + 6e^{r(T-t)}\mu(t,T)^2V(t,T) - 3\mu(t,T)^4}{VAR_t(t,T)^2} \tag{7}$$

Here, $\mathrm{T-t}$ is the remaining time to maturity in years, and s(t) is the implied stock price minus the present value of future dividends calculated from the put-call parity relation of ATM options. We calculate the moments of the two nearest maturity options, that is, the nearby options and the second nearby options, using the numerical integration method described by Bakshi et al (2003). The calculation is as follows: ATM options are chosen as the call and put options whose strike price is the closest to the S&P 500 index price. For the numerical integration, the strike price interval is set as one and the domain of integration is from 50% to 150% of s(t). Dennis and Mayhew (2002) show in their simulation that the skewness bias caused by the numerical integration method is less than 0.01 when the strike price interval is one and the skewness bias is pretty close to zero if the half width of the integration domain is set to be larger than 10. For robustness check, we experiment different strike price intervals and domain ranges, but different choices of the integration interval and the integration domain do not qualitatively change our results in this study. When option prices for the numerical integration are not observable, we estimate those option prices using an interpolation. Specifically, we estimate them by the interpolation using the Black-Scholes implied volatilities of available options with adjacent strike prices. That is, when the strike price of the option to be estimated is between the maximum and minimum strike prices of available options, we choose two options with adjacent strike prices between which the strike price of the option to be estimated lies. Then, we linearly interpolate those two implied volatilities and then use the interpolated implied volatilities to obtain the option price of concern using the Black-Scholes formula. When the strike price of the option whose price is to be estimated is larger (smaller) than the maximum (minimum) strike price available, the implied volatility of the option with the maximum (minimum) strike price is used to obtain the option price of concern. In this process, note that we do not assume that the Black-Scholes option pricing model holds. We use the Black-Scholes formula only for the purpose of interpolation.

Figure 1 shows the estimated variance, skewness, and kurtosis measures from the nearby and second nearby options. In this figure, we can see that the skewness is time-varying, as Dennis and Mayhew (2002) show, and so is the kurtosis. Also, we can see that these measures from the nearby options move together with those of the second nearby options. Table 1 shows the descriptive statistics of these measures. We can see in this table that the skewness and kurtosis are strongly negatively related with each other, while the variance and other measures are not as strongly related. In addition, we can see that all of the moments are highly autocorrelated.

[Figure 1 here]

[Table 1 here]

III. Forward moments

The moments from the nearby and second nearby options reveal the future return distributions with different maturities expected in the options market. A goal of our study is to examine whether the relation between the skewness (and kurtosis) implied from the nearby options and that from the second nearby options are consistent with that observed in the market. This goal is motivated by the research on the relation between the short-term volatility and long-term volatility implied by options, such as the works of Stein (1989), Poteshman (2001), and Jiang and Tian (2010). To avoid misspecification or misestimation in the term structures of moments, we apply Jiang and Tian's method in our study. Jiang and Tian (2010) examine the relation between the implied volatilities with different maturities by testing whether the forward implied volatility between two maturities is an unbiased estimator of future spot volatility.

In line with the research above, we need to decide what measure we will use to examine the implied information in the term structure from option prices with different maturities. First of all, we can regard the variance, skewness and kurtosis as the representative measure to express the return distribution, and extract forward variance, skewness, and kurtosis to examine whether the implied future variance, skewness, and kurtosis are consistent with the future realized variance, skewness, and kurtosis. In fact, the literature regarding the variance exactly does that. However, the forward skewness and kurtosis are not appropriate for this purpose, since the forward skewness and kurtosis are not unbiased estimators of their future realized skewness and kurtosis unlike the case of the forward variance. Thus, we use the forward second, third and fourth moments instead of the forward variance, skewness, and kurtosis.

From the definition of the third and fourth moments, we can see that the forward third and fourth moments depend not only on the moments from nearby and second nearby options, but also on the expected cross-product of two different maturities. Since it is not easy to estimate the expected cross-product from the options, we assume that the returns (especially adjacent returns) are independent of each other. Under this assumption, the expected value of the cross-product of two returns becomes the cross-product of two expected returns, the first four moments of a return from time 0 to time 2 can be represented as below.

$$E_t[R(t,T_2)] = E_t[R(t,T_1) + R(T_1,T_2)] \tag{8}$$

$$E_t[R(t,T_2)^2] = E_t\left[\left(R(t,T_1) + R(T_1,T_2)\right)^2\right] = E_t[R(t,T_1)^2] + 2E_t[R(t,T_1)]E_t[R(T_1,T_2)] + E_t[R(T_1,T_2)^2] \tag{9}$$

$$\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_2)^3] = \mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_1)^3] + 3\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_1)^2]\mathrm{E_t}[\mathrm{R}(\mathrm{T}_1,\mathrm{T}_2)] + 3\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_1)]\mathrm{E_t}[\mathrm{R}(\mathrm{T}_1,\mathrm{T}_2)^2] + \mathrm{E_t}[\mathrm{R}(\mathrm{T}_1,\mathrm{T}_2)^3] \tag{10}$$

$$\begin{aligned}\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_2)^4] = {} & \mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_1)^4] + 4\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_1)^3]\mathrm{E_t}[\mathrm{R}(\mathrm{T}_1,\mathrm{T}_2)] + 6\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_1)^2]\mathrm{E_t}[\mathrm{R}(\mathrm{T}_1,\mathrm{T}_2)^2] \\ & +4\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T}_1)]\mathrm{E_t}[\mathrm{R}(\mathrm{T}_1,\mathrm{T}_2)^3] + \mathrm{E_t}[\mathrm{R}(\mathrm{T}_1,\mathrm{T}_2)^4]\end{aligned} \tag{11}$$

where $R(T_1, T_2)$ means a log return between T_1 to T_2.

Using the equation from (8) to (11) above, we can get the first four time-t forward moments of the return from time T_1 to time T_2, $E_t[R(T_1,T_2)]$, $E_t[R(T_1,T_2)^2]$, $E_t[R(T_1,T_2)^3]$, and $E_t[R(T_1,T_2)^4]$, as follows:

$$\mathrm{E_t[R(T_1,T_2)] = E_t[R(t,T_2)] - E_t[R(t,T_1)]} \tag{12}$$

$$\mathrm{E_t[R(T_1,T_2)^2] = E_t[R(t,T_2)^2] - E_t[R(t,T_1)^2] - 2E_t[R(t,T_1)]E_t[R(T_1,T_2)]} \tag{13}$$

$$\mathrm{E_t[R(T_1,T_2)^3] = E_t[R(t,T_2)^3] - E_t[R(t,T_1)^3] - 3E_t[R(t,T_1)^2]E_t[R(T_1,T_2)] - 3E_t[R(t,T_1)]E_t[R(T_1,T_2)^2]} \tag{14}$$

$$\mathrm{E_t[R(T_1,T_2)^4] = E_t[R(t,T_2)^4] - E_t[R(t,T_1)^4] - 4E_t[R(t,T_1)^3]E_t[R(T_1,T_2)] - 6E_t[R(t,T_1)^2]E_t[R(T_1,T_2)^2]}$$
$$\mathrm{-4E_t[R(t,T_1)]E_t[R(T_1,T_2)^3]} \tag{15}$$

Equations (12) to (15) show that we can obtain the first four forward moments from T_1 to T_2 if we have the first four spot moments obtained from two options with maturities of T_1 and T_2, as in equations (1), (2), (3), and (4), and the forward moments of lower orders. Thus, after estimating spot moments, we can recursively get forward moments from the first to the fourth.

[Table 2 here]

Since equations (12) to (15) are derived under the assumption that returns are independent, we check whether the assumption is reasonable in Table 2. Table 2 presents the autocorrelation and partial autocorrelation coefficients of monthly returns on the S&P 500 index in the first and second columns and the Ljung-Box Q statistics and their p-values in the third and fourth columns. This table shows that the first-order serial correlation of monthly returns on the S&P 500 index is 0.09, which is not statistically significant even at the 10% significance level. The Ljung-Box Q statistic on the hypothesis that all the serial correlation coefficients up to the 12th order are zero is 9.04 with a p-value of 0.70, which is not statistically significant at any reasonable significance level. This table shows that our assumption of the independence of returns can be acceptable in the data, and thus, equations (12) to (15) are a reasonable way to estimate the first four forward moments.

Table 3 shows the descriptive statistics of the spot moments estimated from the nearby options and second nearby options and the forward moments for the period between the maturities of the nearby options and second nearby options. All the second and fourth moments estimated are statistically significantly positive on average, as they should be, and all the third moments estimated are negative and statistically significant on average, as has been documented in the literature as in Albuquerque (2012). All the moments are strongly related with each other: The third moment is strongly negatively correlated with the second and fourth moments, while the second moment is strongly positively correlated with the fourth moments. All of the absolute values of the cross-sectional correlations reported in Table 3 are greater than 0.92. The correlations among the moments reported in this table are much larger than those among the variance, skewness, and kurtosis reported in Table 1. Though not reported, the forward moments are also strongly correlated with the spot moments. The serial correlations of the forward moments are almost the same as those of the spot moments reported in Table 1.

[Table 3 here]

IV. Empirical results

The law of iterated expectations guarantees the following relation:

$$F^i(t;T_1,T_2) = E_t\{E_{T_1}[R(T_1,T_2)^i]\} = E_t\{S^i(T_1;T_1,T_2)\} \quad (16)$$

or $$E_t\{\Delta^i M(T_1,T_2)\} \equiv E_t\{S^i(T_1;T_1,T_2) - F^i(t;T_1,T_2)\} = 0 \quad (17)$$

where $F^i(t;T_1,T_2)$ denotes the i-th forward moment at time t for the period T_1 to T_2, $S^i(T_1;T_1,T_2)$ is the i-th spot moment at time T_1 for the period T_1 to T_2, $\Delta^i M(T_1,T_2) \equiv S^i(T_1;T_1,T_2) - F^i(t;T_1,T_2)$ is the forecasting error for the i-th moment, and the expected values are calculated under the risk-neutral probability measure. If options markets are efficient and all the available information is reflected in option prices, equation (16) or (17) implies that the forward moments derived from option prices are the best forecasts of the spot moments given the information at time t. First, we will look into the relation between the implied moments and the realized spot moments in subsection IV.1, and then we will examine the relation between future implied moments and forward moments in subsection IV.2. In subsection IV.3, we will measure the economic significance of the possible mispricing documented in subsection IV.2.

IV.1 The relation between the realized moments and the implied moments

Before examining the relation between forward moments and future spot moments under the risk neutral probability measure in Equation (16), we look into the relation between the implied moments and realized moments under the physical probability measure. This is the topic that has been extensively examined and tested in the literature[2] for the case of i = 2. For example, Canina and Figlewski (1993) reject the hypothesis that the expected value of the future volatility is the implied volatility estimated from the Black-Scholes model, while Christensen and Prabhala (1998) cannot reject the hypothesis and attribute the result of Canina and Figlewski to a measurement error problem. Jiang and Tian (2005) use the model-free implied volatility to test the i = 2 case for equation (16) and do not reject the equation.

This study examines the relation (16) for the third and fourth moments as well as the second moments. The forward measures are calculated using the method provided in the previous section, and the realized moments are calculated as follows:

$$Realized\ N^{th}\ moment = \sum_{i=1}^{s} (R_i)^N$$

where R_i is the daily log return at time i, and s is the number of business days from t to the maturity of options.

[Table 4 here]

Table 4 shows the results. First, let us look at the nearby option case, which is reported in Panel A of Table 4. The slope coefficient for the second moment regression is 0.86, which is not significantly different from one, and the intercept coefficient (= -0.03%) is not statistically significantly different from zero at the 5% significance level, though the joint test for an intercept = 0 and a slope = 1 in the univariate regression is rejected at the 5% significance level (p-value = 0.015). This result is generally consistent with those of Christensen and Prabhala (1998) and Jiang and Tian (2005) in terms of the individual coefficient tests, though not with the joint test. Also, we re-estimate the coefficients using the IV regressions rather than the standard OLS to alleviate the measurement error problem of the independent variable, following the suggestion of

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[2] The expectation operator in (16) or (17) is under the risk-neutral probability measure, but the literature focuses more on equation (16) under the physical probability measure.

Christensen and Prabhala, but the result does not change, though not reported in the paper.

However, the regression tests for the third and fourth moments show a totally different story. The intercept coefficient for the third moment regression for the nearby option case is statistically insignificant, but the slope coefficient, 0.0156, is pretty close to zero, that is, significantly different from one. Also, the joint hypothesis of an intercept of zero and a slope of one is rejected with a p-value close to zero. As with the regression for the third moment, the slope coefficient for the fourth moment regression, 0.0161, is also far from one, with an insignificant constant coefficient. The joint hypothesis is also rejected with a p-value close to zero.

In the multiple regressions including the lagged realized moment, the result does not change much, except that the coefficient of the implied second moment becomes different from one, and the coefficient of the lagged second realized moment is larger than the coefficient of the implied second moment. However, if we use the IV estimator, then the lagged realized second moment becomes insignificant, which shows that the information from the second nearby options regarding the future second moment contains all the information in the past realized second moment.

Panel B of Table 4 shows the regression results for the second nearby options. In general, the results for the second nearby options are similar with those for the nearby options, except that the coefficient of the implied second moment becomes significantly different from one and R^2 becomes smaller. Since this panel with the second nearby options tries to forecast more distant future than Panel A with the nearby option prices, it is not a surprise that R^2 becomes smaller. Panel B shows that the information from the second nearby options regarding the future realized moments is less accurate than the one from the nearby option prices.

However, the regression results themselves in Table 4 do not show whether the option prices reflect correctly the dynamics of underlying asset, because the equation (16) or (17) holds under the risk-neutral probability measure, not under the physical probability measure under which Table 4 is examined. Though the Girsanov theorem guarantees that the implied variance under the risk neutral probability measure should be the unbiased estimator of the future realized volatility in the Black-Scholes model, the implied moments from option prices under the risk-neutral probability measure do not have to be unbiased estimators of the future realized moments under the physical probability measure in general. In particular, Bakshi et al. (2003), and Bakshi and Madan

(2006) show that the implied variance, skewness and kurtosis from option prices are different from those under the physical probability measure with the power utility assumption. They showed that the normalized risk neutral moments are affected by the risk aversion and the normalized physical second to fifth moments as follows.

$$\frac{VAR_{RN,t} - VAR_{P,t}}{VAR_{P,t}} \approx \gamma \times VAR_{p,t}^{0.5} \times SKEW_{p,t} - \frac{\gamma^2}{2} \mathrm{VAR}_{p,t} \times (KURT_{p,t} - 3) \tag{18}$$

$$\mathrm{SKEW}_{RN,t} \approx \mathrm{SKEW}_{p,t} - \gamma(\mathrm{KURT}_{p,t} - 3) \times \mathrm{VAR}_{p,t} \tag{19}$$

$$\mathrm{KURT}_{RN,t} \approx \mathrm{KURT}_{p,t} - \gamma[\left(2(\mathrm{KURT}_{p,t} + 2\right)\mathrm{SKEW}_{p,t} + \mathrm{PKEW}_{p,t}] \times \mathrm{VAR}_{p,t} \tag{20}$$

where the subscript RN and P denote the risk neutral probability measure and physical probability measure, respectively. VAR, SKEW, KURT and PKEW mean the normalized second to fifth moment, respectively, and γ is risk aversion.

Using the equations from (18) to (20), we reexamine whether the calculated risk neutral moments are the same as the implied risk neutral moments from option prices. We estimate the risk aversion parameter as the value to minimize the sum of the squared errors between the risk neutral variance and the calculated risk neutral variance in equation (18), and then regress the calculated risk neutral moments on the implied moments from option prices.

The risk aversions to satisfy the equation (18) for nearby and second nearby option prices are estimated as 2.87 and 2.02 respectively, and the values are pretty close to the estimated risk aversion in Bakshi, Kapadia and Madan (2003). The regressions of the calculated skewness/kurtisis on the implied skewness/kurtosis show pretty similar to the results in table 4. In univariate regressions, calculated skewness has -1.67 and 1.60 as a constant term and 0.055 and 0.148 as a slope for nearby and second nearby options respectively. And calculated kurtosis has 11.22 and 9.19 as a constant term and -0.32 and -0.24 as a slope for nearby and second nearby options respectively. Therefore, all of calculated measures show constant terms that are significantly different from zero and slopes that are significantly different from one, and it means that the implied skewness and kurtosis are not unbiased estimators of calculated measures according to equation (18) to (20).

In summary, the information regarding the third and fourth moments may not be correctly reflected in the option prices, relative to the information regarding the second moment that is reflected in the option prices, especially in the nearby option prices.

IV.2 The relation between the moments implied by short-term and long-term options

In this subsection, we examine the relation (17) following Jiang and Tian (2010). Jiang and Tian examine the following regression for the case of i = 2 to see whether the volatility overreaction phenomenon reported by Stein (1989) and Poteshman (2001) exists. We extend their regression to the cases of i = 3 and 4, as well as i = 2 :

$$\Delta M^{i}(T_1, T_2) = \alpha^{i} + \beta^{i} \times X_t(\mathrm{t}, T_1) + \varepsilon(T_1), \tag{21}$$

We use the variance, skewness, and kurtosis for $X_t(t, T_1)$ calculated from nearby options. We use these measures rather than the non-central second, third, and fourth moments because using the central moments alleviate the multicollinearity problem in the regression, since they have less cross-sectional correlations among themselves than the non-central moments.

If all the available information is reflected in option prices and option prices are consistent with each other, $F^{i}(t; T_1, T_2)$ should be the best predictor of the future i-th moments derived from the nearby option prices at time T_1, which implies that (1) the average forecasting error should be zero and (2) α^i and β^i in equation (21) should be zero. Figure 2 shows the forecasting errors $\Delta^{i} M(T_1, T_2)$. The second, third, and fourth moments' average forecasting errors are 0.0011, 0.00038, and −0.00012, respectively, and their t-values are 0.49, 5.13, and −2.85, respectively. Thus, at the 5% significance level, we can reject that the average forecasting error is zero for the third and fourth moments but cannot reject the hypothesis that the average forecasting error for the second moment is zero. These results show that the information reflected in the nearby options and second nearby options regarding the third and fourth moments may not contain some information available at time t and/or may not be consistent with each other. Since the average forecasting errors of the third and fourth moments are significantly positive and negative, respectively, the forward third (fourth) moment is too small (large). That is, investors in the second nearby options, on average, seem to underestimate the third moment and overestimate the fourth moment.

[Figure 2 here]

Table 5 shows the estimation results for equation (21). Panel A of the table reports the results for the forward second moment case, which is basically what Jiang and Tian (2010) examine in their study. In this case, the hypothesis that the intercept and slope coefficients should be zero cannot be rejected at any reasonable significance level. All the t-statistics using the Newey-West correction with four lags are less than one in absolute values. This result is consistent with that of Jiang and Tian (2010): When we use the model-free second moment as Jiang and Tian do, we do not observe the overreaction phenomenon reported by Stein (1989) and Poteshman (2001).

Panes B and C of Table 5 report the results for the forward third and fourth moment cases. The results for the fourth moment shown in Panel C tell the same story as the results for the second moment. The intercept and slope coefficients are not statistically significant, and thus, the forward fourth moment seems to be the best estimator of the future spot fourth moment. However, the results for the third moment case tell a different story: The slope coefficients of the variance are positive and statistically significant in all the third moment regressions. That is, the realized third moment becomes larger than the forward third moment when the variance becomes larger. This means that the expectation on the future third moment reflected in the second nearby options is relatively underestimated when the variance is large. When the variance is large, investors of relatively long-term options (the second nearby options in our case) seem to put too much weight on the downfall risk and thus, forecast too small value regarding the third moment, which results in the underestimation of the third moment.

These biases can be due to the mispricing of long-term options relative to short-term options caused by some behavioral biases of option investors. Or these biases may be due to the investor preferences and can be justified by an option pricing model. The reasons behind these biases are beyond the scope of this paper, and are left for future research.

[Table 5 here]

In sum, the third and fourth moments embedded in the prices of long-term options seem to be underestimated and overestimated, respectively. In addition, the underestimation of the third moment becomes more severe as the variance of returns grows larger. That is, long-term option investors put too much weight on downside and tail risks, and the tendency to fear the downside risk becomes larger as the volatility of returns becomes larger.

IV.3 Economic significance of the relative bias in the long-term options

In the previous subsection, we document that the forward value of the third moment embedded in the price of the second nearby options may not be consistent with the expected value of the third moment embedded in the prices of the nearby options at a future time and seem to be negatively biased especially when the variance of the underlying return (derived from the nearby option prices) is large. In this subsection, we evaluate the economic significance of this negative bias in the third moment.

To evaluate the effect of this negative bias, we need to calculate the option prices without this bias. However, many of the well-known models are not adequate for our task. For example, the Black-Scholes model assumes a log-normal distribution of the underlying asset price and thus, does not accommodate the third and fourth moment information we want to reflect in option prices. On the other hand, more sophisticated models like those by Heston (1993) and Bates (1997) can accommodate the third and fourth moment information in principle but they are not easy or convenient to use for our purpose and require us to make some assumptions on the underlying return distribution. In this study, we adopt Corrado and Su's (1996) model.

Corrado and Su (1996) suggest an option pricing model to accommodate the skewness and kurtosis of the underlying asset return, extending the Black-Scholes option pricing model to capture the volatility smile. They approximate a non-normal probability density function by a Gram-Charlier series expansion[3] and provide the

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[3] A Gram-Charlier series expansion of the density function f(x) is defined as

$$\mathrm{f(x)} = \sum_{n=0}^{\infty} c_n H_n(x) \varphi(x)$$

where $\varphi(x)$ is a normal density function, $H_n(x)$ are Hermite polynomials derived from successively higher derivatives of $\varphi(x)$, and the coefficients c_n are determined by moments of the distribution function F(x). If we sum the infinite series with infinite moment values, we can generate any distribution. However, we have only following approximate formula for option pricing:

$$\text{Call price} = Call\ price_{BS} + skewness * Q_3 + (kurtosis - 3) * Q_4 \quad (22)$$

where

$$Call\ price_{BS} = S_0 N(d) - Ke^{-rT} N\left(d - \sigma\sqrt{T}\right)$$

$$\mathrm{Q}_3 = \frac{1}{3!} S_0 \sigma \sqrt{t} \left(\left(2\sigma\sqrt{T} - d\right) n(d) - \sigma^2 T N(d)\right)$$

$$\mathrm{Q}_4 = \frac{1}{4!} S_0 \sigma \sqrt{t} \left(\left(d^2 - 1 - 3\sigma\sqrt{T}\left(d - \sigma\sqrt{T}\right)\right) n(d) + \sigma^3 T^{\frac{3}{2}} N(d)\right)$$

$$\mathrm{d} = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

Using equation (22), we can evaluate how much ‘mispricing’ may occur and attribute such mispricing to the biases due to the third or fourth moment, where mispricing means the price difference between the cases with and without the biases. As we mentioned before, ‘mispricing’ may not be mispricing and can be justified by some option pricing models. We use the estimated regression equation (21) without the residual to calculate the bias in the third moment but use the unconditional average bias, −0.00012, to calculate the bias in the fourth moment. Table 6 shows the results. Panel A (B) of Table 6 shows that the median mispricing caused by the third and fourth moments for the cases of OTM puts (calls) range from 2.6% to 16.6% (from −4.1% to −68%) and from −13.7% to 10.5% (from −12.6% to 18.0%), respectively, which is huge in general. As we saw in the previous subsection, on average, the third moment is underestimated, and the fourth moment is overestimated. The underestimated third moment implies that the long-term put options are overestimated, while the long-term call options are underestimated. That effect is shown in the first rows (third moment) of Panels A and B of Table 6. Also, not surprisingly, this effect becomes stronger as the degree of out-of-the-moneyness becomes larger. The deep out-of-the-money put option is overestimated by 16.6%, and the deep out-of-the-money call option is underestimated by 68% in terms of the third moment effect. Given the size of the other moments, the overestimated fourth moment effect implies that deep OTM options are overestimated and near-the-money

fourth moments values and the density function f(x) are truncated to exclude terms beyond the fourth moment.

options are underestimated. The results are consistent with intuition: The deep OTM put (call) in the table is overestimated by 10.5% (18%), and the other values in the table are underestimated in terms of the fourth moment effect. The total effect from the underestimated third and overestimated fourth moments ranges from −10.3% to 27.8% for the put option case and from −16.9% to −27.1% for the call option case. Thus, in general, long-term put options (except near-the-money puts) are overestimated, and long-term call options are underestimated relative to the case without the biases. The degree of the mispricing evaluated by Corrado and Su's model is economically significant.

[Table 6 here]

In sum, the mispricing or the difference of prices between the cases with and without the biases caused by the underestimated third and overestimated fourth moments are large and economically significant. The third moment effect results in the overpricing of long-term puts and underpricing of long-term calls. The fourth moment effect generates the overpricing of deep OTM options and underpricing of near-the-money options.

V. Abnormal Returns from Trading Skewness

In table 5 and Figure 2, we demonstrate that the information regarding the third and fourth moments is not correctly reflected in the option prices, and the possible mispricing from the biases is economically meaningful in Corrado and Su's model. However, since the Corrado and Su's model is an approximation model using the Gram-Charlier series, it is possible that the estimated effect could be far from the real impact of the bias of the moments due to the model specification issue or approximation error. In this section, we reexamine the economic significance of the bias, especially the bias from the underestimation of the third moment by analyzing the skewness portfolio returns suggested by Bali and Murray (2013). In their paper, they show how to construct the portfolios (skewness assets) to have an exposure to skewness. That is, the portfolios (Hereafter skewness assets) have a positive (negative) return if the skewness of the portfolio increases (decreases). The skewness assets are composed of three assets; two options to avoid an exposure to vega risk and its underlying asset to hedge delta risk, The skewness assets[4] are composed as follows.

i) The call skewness assets is a portfolio that is composed of call options and the underlying stock: buying an OTM call option and selling ATM call options as much as $-\text{vega}_{(\text{call OTM})}/\text{vega}_{(\text{call ATM})}$ for vega hedge, and buying the underlying stock as much as $-(\text{delta}_{(\text{call OTM})}$ + call ATM options position * $\text{delta}_{(\text{call ATM})})$ for delta hedge.

ii) The put skewness asset is a portfolio that is composed of put options and the underlying stock: selling an OTM put option and buying ATM put options as much as $\text{vega}_{(\text{put OTM})}/\text{vega}_{(\text{put ATM})}$ to hedge vega risk, and buying the underlying stock as much as $-(\text{delta}_{(\text{put OTM})}$+put ATM option position * $\text{delta}_{(\text{put ATM})})$ to hedge delta risk.

Some more details to construct these skewness assets are as follow. We choose the ATM option as an option having delta closest to 0.5 in absolute terms, and the OTM option is chosen as an option having delta closest to 0.1 (or 0.2) in absolute terms. The skewness assets are created on the following business day of the monthly option expirations, and liquidated at the expiration date of the next month. Delta, vega for hedging are calculated using the Black-Scholes option pricing model. The dividends from the underlying stock of the assets are assumed to be invested in a bank account with the risk-free rate from the dividend payment date to the option expiration date.

We define the excess return on a skewness asset as

$$\text{Excess Return of a skewness asset} = \frac{price_{t+1} - price_t}{price_t} - (e^{rt} - 1)$$

where the price is the sum of the position times the market price of each security comprising the skewnes asset at the time. Especially, the price at t+1 for the skewness asset that is comprised of nearby options is the sum of the position times the payoff of each security including the future value of the received dividends at the expiration.

[4] Bali and Murray also suggest the CallPut skewness asset that is composed of call and put options to trade skewness. In this paper, we do not use the CallPut skewness asset as the portfolio has negative investment money, which makes difficult to interpret the value change in terms of return.

Panel A in Table 7 shows the average returns on the two skewness assets using conditions, |OTM delta| is 0.1 or 0.2 and |ATM delta| is 0.5. The average returns are from -0.4% to 0% and all of the values are statistically insignificantly different from zero. However, the average return insignificantly different from zero does not mean that the skewness assets are priced fairly, since the systematic risk of the skewness assets are not taken into account. Panel B of the table shows the returns after the systematic risks are controlled for the case that the delta of OTM options is 0.2 in absolute terms and the qualitative result does not change when the OTM options are chosen differently. For the risk factors capturing the systematic risks, we use the Fama-French three factors[5] (Market excess return, SMB, HML), delta-hedged excess return as a proxy of the variance risk premium, and the skewness risk premium. The delta-hedged excess return of the S&P 500 index is the one month holding period excess return on the portfolio composed of an ATM call option and ATM put options (delta hedged portfolio) using nearby options as in Goyal and Saretto (2009). The skewness risk premium[6] is the return difference between 10 and 1 decile portfolios in Bali and Murray (2013). The table shows that the constant terms for all skewness assets after controlling the systematic risks are not statistically different from zero, which means that they can be regarded as fairly priced individually under our asset pricing model.

Next, to look into our hypothesis that the implied forward skewness by the second nearby options tend to be lower than the future implied skewness by the nearby options, we examine that the return difference between skewness assets using the two different maturity options. The result is shown in the column of (B)-(A) in panel B. (B)-(A) shows the return on the portfolio buying the skewness asset of the second nearby options and selling the skewness asset of the nearby options. Since we show in section IV.3 that the third moment effect results in the overpricing of long-term puts and underpricing of long-term calls, we expect that the long-short portfolio from the put options has a positive average return and the long-short portfolio from the call options has a negative average return. In addition, the mispricing expected to be larger for the case of the call long-short portfolio than for the put long-short portfolio if we look at Table 6. The empirical results partially confirm our expectation. The average long-short portfolio return constructed by call options is positive and statistically

[5] We obtain the data from Ken French's web site.

[6] The skewness risk premium is obtained from the author of Bali and Murray (2013). The data includes returns from Jan 1996 to Sep 2010.

significant after controlling the systematic risks, which is consistent with our previous analysis. However, the average long-short portfolio return constructed by put options is not significant after controlling systematic risk, and positive, which is not consistent with our previous analysis using the Corrado and Su model. Considering that the extent of the mispricing for put options is smaller than that for call options in Table 6, this is not a big surprise.

In summary, the long-short portfolio consisting of buying long-term skewness and selling short-term skewness using the skewness assets constructed from call options has a positive average return after controlling for the systematic risks. This confirms our result that the underestimated third forward moment results in the underpricing of long-term calls.

VI. Conclusion

If options with every possible strike price exist in the market, we can complete the market or generate any possible payoff returns (Ross (1976)), which means that if we have a sufficient number of options with different strike prices, then we can extract the full information on the distribution of underlying asset returns or moments of asset returns. The existing literature mainly focuses on the information regarding the second moment or the variance of the underlying asset returns and examines the relation between the implied volatility (or model-free volatility) and the realized volatility. Our study extends the literature by investigating the third and fourth moments as well as the second moment.

Our study suggests a way to construct forward moments iteratively and constructs the model-free second, third, and fourth moments. Using these moments, we document the following:

(1) The third and fourth forward measures are not unbiased estimators of the future realized third and fourth moments, though the second forward measure can be regarded as an unbiased estimator of the future second moment.

(2) Long-term option investors, on average, seem to underestimate the third moment and overestimate the fourth moment relative to short-term option investors.

(3) The expectation on the future third moment reflected in long-term options is relatively underestimated when the variance is large.

(4) The estimation results show that the mispricing or the difference of prices between the cases with and without the biases caused by the underestimated third and overestimated fourth moments are large and economically significant. The third moment effect results in the overpricing of long-term puts and underpricing of long-term calls. The fourth moment effect generates the overpricing of deep OTM options and underpricing of near-the-money options.

(5) The long-short portfolio consisting of buying long-term skewness and selling short-term skewness using the skewness assets constructed from call options has a positive average return after controlling for the systematic risks. This confirms that the underestimation of the implied third moment is economically meaningful and it results in the underpricing of long-term calls.

The results reported in this paper show that investors price long-term options as if the underlying asset returns are more negatively skewed and the tails of the return distribution are thicker than they actually are. That is, investors are prudent (Kimbell (1990)) and temperate (Eeckhoudt et al (1995), Kostakis et al (2012)). Further research on these interesting properties of the higher moments implied in option prices will be fruitful.

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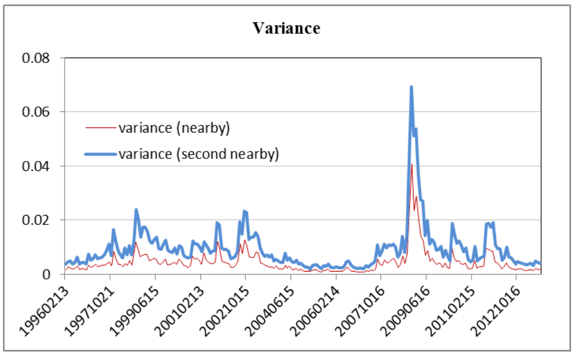
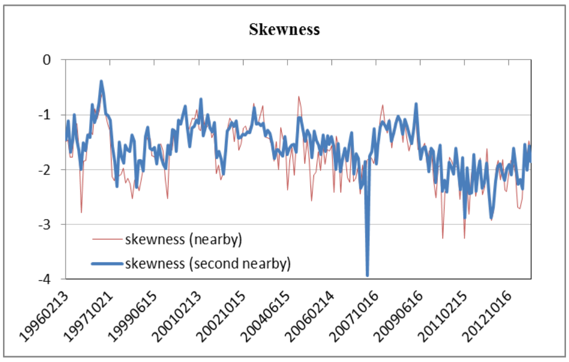
Figure 1. Variance, skewness, and kurtosis

We use the S&P 500 spot index options traded on the CBOE from January 4, 1996 to August 30, 2013. The data are from the OptionMetrics database and include closing bid and ask quotes for each option contract along with the corresponding strike prices and maturity information. We calculate the risk neutral variance, skewness, and kurtosis by the methodology of Bakshi et al. (2003) using option price data. We applied the following filters. First, option quotes are included only when both bid and ask quotes are available. Option data with any missing bid or ask quote are excluded. Second, option quotes whose bid-ask spread is larger than 0.5 are excluded, in order to ensure that illiquid options are excluded. Third, option quotes violating no arbitrage bound with bid and ask quotes are excluded. Specifically, we require that both bid and ask prices of a call (put) option with a higher (lower) exercise price should be higher than those of a call (put) option with a lower (higher) exercise price, in order to ensure that any non-informative options due to minimum tick size or illiquidity are excluded. We use the three-month CD rate as the risk free rate. The S&P 500 index is also used. These data are obtained from the Center for Research in Security Prices.





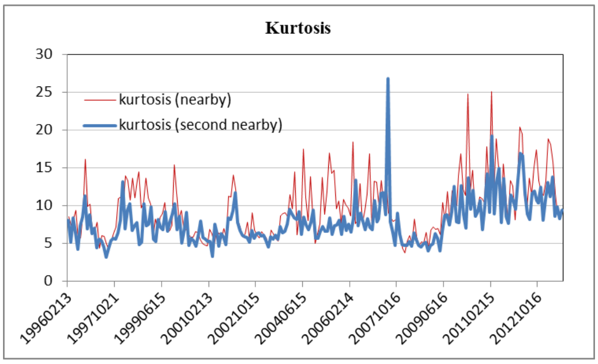


Figure 2. Forecasting errors

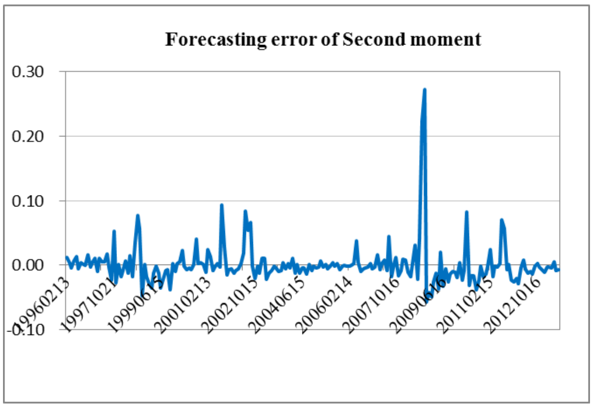
Using spot moments and assuming that the returns of two periods, $R(t, T_1)$ and $R(T_1,T_2)$, $t < T_1 < T_2$, are independent, we calculate the forward moments as below:

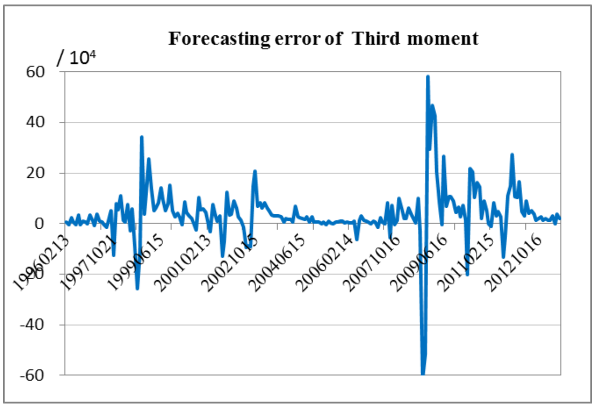
$$E_t[R(T_1,T_2)^2] = E_t[R(t,T_2)^2] - E_t[R(t,T_1)^2] - 2E_t[R(t,T_1)]E_t[R(T_1,T_2)]$$

$$E_t[R(T_1,T_2)^3] = E_t[R(t,T_2)^3] - E_t[R(t,T_1)^3] - 3E_t[R(t,T_1)^2]E_t[R(T_1,T_2)] - 3E_t[R(t,T_1)]E_t[R(T_1,T_2)^2]$$

$$E_t[R(T_1,T_2)^4] = E_t[R(t,T_2)^4] - E_t[R(t,T_1)^4] - 4E_t[R(t,T_1)^3]E_t[R(T_1,T_2)] - 6E_t[R(t,T_1)^2]E_t[R(T_1,T_2)^2] - 4E_t[R(t,T_1)]E_t[R(T_1,T_2)^3]$$

The figures below show the forecasting errors and the future spot moments – the forward moments, when the future spot moments are the moments calculated from the second nearby options at the expiration date of the nearby options.





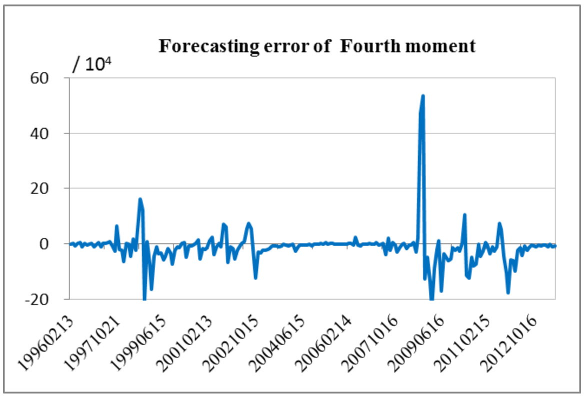


Table 1. Descriptive statistics of the variance, skewness, and kurtosis

This table shows the descriptive statistics of the variance, skewness, and kurtosis from the nearby and second nearby option prices.

Panel A. nearby option

	Variance	Skewness	Kurtosis
mean	0.005	-1.702	9.683
standard deviation	0.005	0.513	3.899
correlation	1.000	0.160	-0.275
	0.160	1.000	-0.932
	-0.275	-0.932	1.000
Autocorrelation	0.808	0.658	0.525
	0.652	0.549	0.493
	0.520	0.478	0.428
	0.370	0.413	0.362
	0.266	0.327	0.317

Panel B. Second nearby option

	Variance	Skewness	Kurtosis
mean	0.009	-1.601	7.920
standard deviation	0.008	0.452	2.966
correlation	1.000	0.141	-0.246
	0.141	1.000	-0.943
	-0.246	-0.943	1.000
Autocorrelation	0.844	0.612	0.436
	0.692	0.571	0.448
	0.554	0.546	0.447
	0.407	0.446	0.303
	0.316	0.375	0.304

Table 2. Autocorrelation of the S&P 500 index monthly returns

This table presents the (partial) autocorrelation and Ljung-Box Q statistics with lag 12 and their p-values. These statistics are calculated with the S&P 500 index monthly returns from January 4, 1996 to August 30, 2013.

Lag	Autocorrelation	Partical autocorrelation	Q statistics	P-values
1	0.09	0.09	1.67	0.20
2	-0.04	-0.04	1.95	0.38
3	0.09	0.10	3.77	0.29
4	0.05	0.03	4.39	0.36
5	0.03	0.03	4.53	0.48
6	-0.06	-0.07	5.41	0.49
7	0.03	0.04	5.59	0.59
8	0.08	0.06	6.84	0.55
9	-0.04	-0.04	7.18	0.62
10	0.01	0.03	7.22	0.70
11	0.02	0.00	7.30	0.77
12	0.09	0.09	9.04	0.70

Table 3. Descriptive statistics of the second, third, and fourth moments and their forward moments

This table shows the descriptive statistics of the risk neutral second, third, and fourth moments described in Figure 1 and their forward moments. Panels A and B show the statistics of the nearby options and second nearby options. Panel C shows the statistics of the forward moments calculated as below, with the assumption of return independence between two periods.

The forward second, third, and fourth moments are calculated as

$$E_t[R(T_1,T_2)^2] = E_t[R(t,T_2)^2] - E_t[R(t,T_1)^2] - 2E_t[R(t,T_1)]E_t[R(T_1,T_2)]$$

$$\mathrm{E_t}[\mathrm{R}(\mathrm{T_1},\mathrm{T_2})^3] = \mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_2})^3] - \mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_1})^3] - 3\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_1})^2]\mathrm{E_t}[\mathrm{R}(\mathrm{T_1},\mathrm{T_2})] - 3\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_1})]\mathrm{E_t}[\mathrm{R}(\mathrm{T_1},\mathrm{T_2})^2]$$

$$\mathrm{E_t}[\mathrm{R}(\mathrm{T_1},\mathrm{T_2})^4] = \mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_2})^4] - \mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_1})^4] - 4\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_1})^3]\mathrm{E_t}[\mathrm{R}(\mathrm{T_1},\mathrm{T_2})] - 6\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_1})^2]\mathrm{E_t}[\mathrm{R}(\mathrm{T_1},\mathrm{T_2})^2] - 4\mathrm{E_t}[\mathrm{R}(\mathrm{t},\mathrm{T_1})]\mathrm{E_t}[\mathrm{R}(\mathrm{T_1},\mathrm{T_2})^3]$$

Panel A. nearby options

	Second moment	Third moment	Fourth moment
mean	0.0046	-0.0006	0.0003
standard deviation	0.0046	0.0013	0.0009
correlation	1.0000	-0.9640	0.9409
	-0.9640	1.0000	-0.9917
	0.9409	-0.9917	1.0000

Panel B. Second nearby options

	Second moment	Third moment	Fourth moment
mean	0.0095	-0.0018	0.0010
standard deviation	0.0084	0.0030	0.0023
correlation	1.0000	-0.9651	0.9454
	-0.9651	1.0000	-0.9921
	0.9454	-0.9921	1.0000

Panel C. forward moments

	Second moment	Third moment	Fourth moment
mean	0.0049	-0.0011	0.0005
standard deviation	0.0039	0.0015	0.0008
correlation	1.0000	-0.9602	0.9286
	-0.9602	1.0000	-0.9884
	0.9286	-0.9884	1.0000

Table 4. The relation between the realized moments and the implied moments from options

This table presents the OLS regression results that show the implied moments from options are good estimators of the realized moments from t to the maturity of options. The standard errors are calculated following the Newey-West method with four lags.

Panels A and B present the OLS regression results of the implied moments from the nearby options and second nearby options, respectively. In these regressions, the realized N-th moments are calculated as

$$Realized\ N^{th}\ moment = \sum_{i=1}^{s} (R_i)^N$$

where R_i is the daily log return at time i, and s is the number of business days from t to the maturity of options.

Panel A: Realized moments and Implied moments of Nearby options

	Nearby options											
	Second moment				Third moment				Fourth moment			
	Constant	Implied Second moment	Lagged realized moment	R^2	constant x 100	Implied Third moment	Lagged realized moment	R^2	constant x 100	Implied Fourth moment	Lagged realized moment	R^2
coefficient	-0.0003	0.8602		0.42	0.0000	0.0156		0.04	0.0001	0.0161		0.18
standard error	0.0004	0.1564			0.0006	0.0100			0.0002	0.0051		
coefficient	0.0005	0.2508	0.5483	0.51	0.0002	0.0222	-0.2325	0.08	0.0002	0.0008	0.5624	0.33
standard error	0.0004	0.1810	0.2075		0.0006	0.0123	0.1205		0.0002	0.0016	0.0738	

Panel B: Realized moments and Implied moments of Secnd nearby options

	Second nearby options											
	Second moment				Third moment				Fourth moment			
	Constant	Implied Second moment	Lagged realized moment	R^2	constant x 100	Implied Third moment	Lagged realized moment	R^2	constant x 100	Implied Fourth moment	Lagged realized moment	R^2
coefficient	0.0008	0.6688		0.27	-0.0006	0.0066		0.02	0.0005	0.0069		0.08
standard error	0.0007	0.1036			0.0011	0.0071			0.0004	0.0018		
coefficient	0.0023	0.2315	0.3703	0.29	-0.0007	0.0061	0.0182	0.02	0.0010	-0.0039	0.5120	0.14
standard error	0.0011	0.1660	0.1969		0.0013	0.0085	0.0826		0.0007	0.0042	0.2035	

Table 5. The forecasting regressions of the difference between the forward moments and future spot moments

This table presents the OLS regression results that test whether the forecasting errors and the future spot moments – forward moments are explained by the stated variables in Jiang and Tian (2010). The OLS regressions are as below:

$$\Delta \mathrm{M}^{\mathrm{i}}(T_1, T_2) = \alpha^{\mathrm{i}} + \beta^{\mathrm{i}} * X_t(\mathrm{t}, T_1) + \varepsilon_t$$

$$\text{where } \Delta \mathrm{M}^{\mathrm{i}}(T_1, T_2) = \mathrm{M}^{\mathrm{i}}(T_1, T_1{:}\, T_2) - \mathrm{M}^{\mathrm{i}}(\mathrm{t}, \mathrm{T1}{:}\,\mathrm{T2})$$

$\mathrm{M}^{\mathrm{i}}(\mathrm{t}, \mathrm{T1}{:}\,\mathrm{T2})$ is the i-th forward moment from T_1 to T_2 calculated at time t by equations (16) and (17), and $\mathrm{M}^{\mathrm{i}}(T_1, T_1{:}\, T_2)$ is i-th future spot moment calculated at time T_1, the monthly option expiration date. $\Delta \mathrm{M}^{\mathrm{i}}(T_1, T_2)$ is the forecasting error for the i-th moment. The explanatory variables in the regression use the variance, skewness, and kurtosis from the nearby options.

Panels A, B, and C show the OLS regression results for the second, third, and fourth moments, respectively. The Newey-West t statistics in parentheses are calculated with four lags.

Panel A: future spot second moment - forward second moment

Constant	Variance	Skewness	Kurtosis	R^2
-0.00002	0.02445			0.001
(-0.05)	(0.24)			
-0.00002		0.02445		0.001
(-0.06)		(0.24)		
-0.00002			0.02445	0.001
(-0.06)			(0.24)	
0.00037	0.02059	0.00022		0.003
(0.52)	(0.21)	(0.55)		
0.00013	0.02118		-0.00001	0.002
(0.21)	(0.22)		(-0.26)	

Panel B: future spot third moment - forward third moment

Constant	Variance	Skewness	Kurtosis	R^2
-0.00006	0.09837			0.173
(-0.59)	(2.91)			
0.00035		-0.00002		0.000
(1.26)		(-0.13)		
0.00035			-0.00002	0.000
(1.26)			(-0.13)	
-0.00035	0.10127	-0.00016		0.178
(-1.56)	(3.02)	(-1.27)		
-0.00021	0.10157		0.00001	0.175
(-1.07)	(3.04)		(0.78)	

Panel C: future spot fourth moment - forward fourth moment

Constant	Variance	Skewness	Kurtosis	R^2
-0.00007	-0.01139			0.006
(-0.89)	(-0.43)			
-0.00007		0.00000		0.001
(-0.44)		(-0.36)		
-0.00007			0.00000	0.001
(-0.44)			(-0.36)	
0.00014	-0.01348	0.00012		0.013
(1.02)	(-0.52)	(1.45)		
0.00003	-0.01357		-0.00001	0.008
(0.26)	(-0.52)		(-0.92)	

Table 6. The price impact of the mispricing on the third and fourth moments

This table presents the median price impact of the mispricing in the third and fourth moments that are described in Table 5. The price impact due to third and fourth moments is calculated using Corrado and Su's model and is defined as the option price with the volatility, skewness, and kurtosis from the second nearby options minus the option price with the adjusted skewness and kurtosis. The adjusted skewness is calculated using the adjusted forward third moment, which is the sum of the forward third moment and −0.00006 + 0.09837 × variance of the nearby options based on the regression results in Table 5. The adjusted kurtosis is calculated using the adjusted forward third moment, which is adjusted as much as the unconditional average bias from the forward third moment.

Panel A. The median mispricing on OTM put options

	(K/S)x100=90		(K/S)x100=92.5		(K/S)x100=95		(K/S)x100=97.5	
	In value	In ratio	In value	In ratio	In value	In ratio	In value	In ratio
Third moment	1.253	0.166	1.283	0.132	1.081	0.075	0.586	0.026
Fourth moment	0.652	0.105	-0.336	-0.022	-1.437	-0.089	-2.821	-0.137
Third & Fourth moments	1.992	0.278	1.121	0.088	-0.272	-0.017	-2.264	-0.103

Panel B. The median mispricing on OTM call options

	(K/S)x100=102.5		(K/S)x100=105		(K/S)x100=107.5		(K/S)x100=110	
	In value	In ratio	In value	In ratio	In value	In ratio	In value	In ratio
Third moment	-0.969	-0.041	-1.548	-0.120	-1.809	-0.293	-1.856	-0.680
Fourth moment	-3.124	-0.126	-2.005	-0.150	-0.430	-0.040	0.222	0.180
Third & Fourth moments	-4.010	-0.169	-3.509	-0.270	-2.147	-0.262	-1.518	-0.271

Table 7. Returns on skewness assets

Panel A shows statistics of the returns on call and put skewness portfolios that are described in Bali and Murray (2013). Panel B shows the portfolio returns after controlling systematic risks using data from Jan 1996 to Sep 2010. Systematic risks are used Fama-French factors, Delta-hedged excess return as a proxy of variance risk premium and skewness risk premium. In details, Delta-hedged excess return of S&P 500 index is the one month holding period excess return on the portfolio composed of ATM call option and ATM put option using nearby options. Skewness risk premium is the return difference between 10 and 1 decile portfolios in Bali and Murray (2013). The portfolio are constructed using ATM and OTM options that whose deltas are closest to 0.5 and 0.2 in absolute terms, respectively. The values in brackets are Newey-West t value with lag 4.

Panel A. Descriptive statistics of the returns on skewness assets

		|OTM delta|=0.1		|OTM delta|=0.2	
		nearby options	second nearby options	nearby options	second nearby options
Call skewness asset	mean	-0.004	0.001	-0.004	0.001
	standard error	0.004	0.002	0.004	0.002
Put skewness asset	mean	0.000	-0.001	-0.003	0.000
	standard error	0.004	0.002	0.003	0.001

Panel B. Returns on skewness assets after controlling risk factors

	Call skewness asset			Put skewness asset		
	nearby options (A)	second nearby options (B)	(B-A)	nearby options (A)	second nearby options (B)	(B-A)
constant	-0.0057	-0.0001	0.0056	-0.0036	-0.0007	0.0043
	(-1.46)	(-0.05)	(2.31)	(-0.7)	(-0.32)	(1.29)
Maket excess return	-0.0025	-0.0007	0.0018	0.0000	-0.0003	0.0022
	(-2.09)	(-1.11)	(2.51)	(0.01)	(-0.51)	(2.16)
SMB	-0.0007	0.0001	0.0007	0.0000	0.0001	0.0009
	(-0.75)	(0.12)	(1.06)	(-0.04)	(0.33)	(1.02)
HML	0.0000	-0.0001	-0.0001	0.0003	0.0003	0.0005
	(0.01)	(-0.07)	(-0.08)	(0.2)	(0.47)	(0.45)
Delta-hedged excess return	-0.0413	-0.0156	0.0257	0.0044	-0.0002	0.0410
	(-5.54)	(-3.81)	(6.18)	(0.5)	(-0.06)	(6.78)
Skewness risk premium	0.0471	0.0096	-0.0375	0.0376	-0.0233	-0.0978
	(1.04)	(0.43)	(-1.11)	(0.26)	(-0.51)	(-1.56)
R^2	29.0%	20.3%	24.4%	0.6%	1.0%	32.7%