Risk Premium Information from Treasury Bill Yields*

Jaehoon Lee†

University of New South Wales

April 24, 2015

Abstract

This paper finds that bond risk premium consists of long-term and short-term components. The long-term factor raises the slope of yield curve, has forecastability horizon of longer than one year, and is related to value, size and momentum premiums in the stock market. In contrast, the short-term factor affects Treasury bill yields but has very little effects on Treasury bonds, has forecastability horizon of less than one quarter, is related to aggregate stock market returns, and is attributed to liquidity premium.

*I am very grateful to an anonymous referee for very helpful suggestions. I am indebted to George Pennacchi, Timothy Johnson, and Neil Pearson for their guidance throughout my PhD program. I also thank Geert Bekaert, Michael Imerman, Robert Kimmel, Andrea Lu, Kwangwoo Park and the seminar participants at the University of Illinois at Urbana-Champaign (UIUC), University of Technology, Sydney (UTS), Korea Advanced Institute of Science and Technology (KAIST), and Korea University. The previous version of this paper was circulated under the title, "Treasury Bills: Overlooked Information."

†Email address: jaehoon.lee@unsw.edu.au. Phone: +61 2 9385 6013

1 Introduction

If an investor buys and holds Treasury bonds (T-bonds) until maturity, his holding returns may vary each year but the overall returns are determined ex ante by bond yields. Therefore, bond yields today are equal to the average of holding returns in the future, or equivalently the sum of two components: the averages of future riskfree interest rates and excess returns.[1]

The excess returns' expectation is called risk premium. Risk premium itself is not observable, but it is found to be predictable by other observable variables. For example, Fama and Bliss (1987) and Campbell and Shiller (1991) document that the risk premium increases with the slope of yield curves. Cochrane and Piazzesi (2005) show that the risk premium can be predicted better if projected not only on the slope but also on five forward interest rates altogether, and Ludvigson and Ng (2009) find that macroeconomic variables are informative of the risk premium that is not spanned by bond yields.

The question is how the risk premium would affect the shape of yield curves. Long-term bonds are riskier than short-term bonds, thus long-term yields may be expected to have higher sensitivity to risk premium than short-term yields. In this case, the slope of yield curves would monotonically increase with risk premium. However, Cochrane and Piazzesi (2005)'s findings are somewhat different. Cochrane and Piazzesi (2005) find that the excess returns of various maturities can be predicted by a single risk premium factor, and this factor "is clearly not related to any of the first three principal components."

To answer this question, we need to revisit the relation between risk premium and bond yields. As will be shown later, forward interest rates are determined by the risk-neutral dynamics of state variables,[2] one of which is the risk premium factor. If a

[1]Equation (4) in Section 2 explains this composition of bond yields.

[2]Equation (19) in Section 4 shows that forward interest rate is equal to the risk-neutral expectation of future riskfree interest rate with Jensen's inequality, and the riskfree rate is a function of state variables.

state variable diverges in the risk-neutral dynamics, long-term forward rates will have higher loadings on the variable than short-term rates. Thus, the slope will increase with it. Otherwise, the slope may decrease if the variable mean-reverts. Therefore, what determines the relation is the risk-neutral dynamics of the state variable.

There is no reason to believe that the risk premium factor would only diverge or only mean-revert. Risk premium is probably made of two latent variables of different dynamics. One is diverging, and the other converging. The diverging one will create variations on long-term yields meanwhile the converging one will create those on short-term yields. In particular, the converging one will affect only the yields of extremely short maturities if its half-life is sufficiently short. This is the main motivation of this paper.

I call them long-term and short-term risk premium factors, rpl and rps, respectively. The setup that the bond risk premium is a function of rpl and rps can explain why risk premium is not necessarily related to any specific principal component of yield curves. I find that rpl raises the slope of yield curve, while rps affects Treasury bills (T-bills) but has almost no effects on T-bonds. rpl predicts excess returns over longer than one year, while rps loses its predictability in one quarter. rpl is related to value, size and momentum premiums in the stock market while rps is related to liquidity premium and aggregate stock market returns.

The contribution of this paper is to suggest a simple orthogonalisation method to estimate rpl and rps from bond yields and inflation data. I assume four state variables, the first two of which are the persistent and temporary components of one-period riskfree interest rates. rpl is estimated as the variations in long-term yields orthogonalised by the two riskfree rate components, and rps the orthogonalised 3-month T-bill yields. The following empirical tests show that rpl and rps outperform all the known risk premium predictors from the literature in the forecast of future excess returns. The paper also shows that an affine term structure model with cross-sectional fit maximisation finds it difficult to estimate risk premium accurately because of the high correlations among bond yields.

This is not the first paper that examines the differences between T-bonds and T-bills. For example, Duffee (1996) documents their market segmentation by using the correlations of T-bond and T-bill yields. Pearson and Sun (1994), who estimate a two-factor Cox, Ingersoll, and Ross (1985) term structure model, also conclude that "estimates based on only bills imply unreasonably large price errors for longer maturities." However, there has been no precedent that systematically compares the risk premium forecast with and without T-bills. Many papers use both T-bonds and T-bills to estimate risk premium but do not investigate how much improvement of forecast performance can be accounted by T-bills independently.

Although three factors can explain most variation of term structure, the need for a fourth factor is stressed by the recent literature. For example, Dai and Singleton (2002) show that the fourth factor is needed to match the moment of projecting the risk-premium adjusted yield changes on the slope of yield curves (i.e., the LPY (ii) condition of the paper) for maturities less than two years. Duffee (2010) also shows that the fourth principal component can explain about a half of the variation of monthly expected excess returns in the bond market. This paper contributes to this literature by showing that T-bills are informative of this fourth factor.

The rest of this paper is organized as follows. Section 2 explains the estimation of state variables from bond yields and inflation. Section 3 compares the qualitative differences of forecastability between long- and short-term risk premium factors. Section 4 estimates affine term structure models with different specifications of state variables and shows that the cross-sectional fit optimisation of the affine model estimation is likely to ignore the risk premium information from T-bill yields. Section 5 combines the state variable estimation and the risk premium forecast into a single GMM framework and finds that the results are robust. Section 6 compares the risk premium factors to other financial market variables such as liquidity and stock market risk factors. Section 7 concludes.

2 Estimation of state variables

Let $p_t^{(n)}$ denote the log price of an n-period maturity discount bond at time t. Its continuously compounded bond yield is

$$y_t^{(n)} = -\frac{1}{n}\, p_t^{(n)}, \tag{1}$$

where $y_t^{(1)} = r_t$ is the one-period riskfree interest rate.

The excess return of holding n-period maturity bonds from time t for h periods is defined as its holding return less the h-period interest rate,

$$\begin{aligned} exr_{t,t+h}^{(n)} &= \left\{ p_{t+h}^{(n-h)} - p_t^{(n)} \right\} - \left\{ 0 - p_t^{(h)} \right\} \\ &= -(n-h)\, y_{t+h}^{(n-h)} + n\, y_t^{(n)} - h\, y_t^{(h)}. \end{aligned} \tag{2}$$

For a unit holding period, $h = 1$, the above equation can be used to derive

$$\begin{aligned} n\, y_t^{(n)} &= r_t + (n-1)\, y_{t+1}^{(n-1)} + exr_{t+1}^{(n)} \\ &= r_t + \left\{ r_{t+1} + (n-2)\, y_{t+2}^{(n-2)} + exr_{t+2}^{(n-1)} \right\} + exr_{t+1}^{(n)} \\ &= \quad \cdots \\ &= \sum_{i=0}^{n-1} r_{t+i} + \sum_{i=0}^{n-2} exr_{t+i+1}^{(n-i)}. \end{aligned} \tag{3}$$

Therefore,

$$\therefore \quad y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} E_t\left[r_{t+i}\right] + \frac{1}{n}\sum_{i=0}^{n-2} E_t\left[exr_{t+i+1}^{(n-i)}\right]. \tag{4}$$

This equation implies that bond yields are essentially the sum of two components: the average of expected future short interest rates (r_{t+i}) and excess returns (exr_{t+i}). Now the question is how to split bond yields into the two components, which is the main theme of the following two subsections. Section 2.1 separates the former component—the average of expected future short rates—into a persistent part of core inflation and a transitory deviation of one-year bond yields. Section 2.2 splits the latter one—the average of expected excess returns—into two risk premium state variables of different frequencies. All state variables are estimated over the sample period from January 1968 through December 2013.

2.1 Decomposition of short interest rates (r_{t+i})

Short interest rates do not revert to a constant mean. Instead, their mean-reversion is toward a time-varying expected value. For example, Fama (2006) concludes that its "mean-reversion is toward a non-stationary (permanent) long-term mean." This finding implies that at least two state variables are needed to model the dynamics of short interest rates: one needs to be persistent and the other transitory.[3]

The literature suggests that the persistent component of short interest rates is primarily determined by the long-run mean of inflation (Kozicki and Tinsley, 2001; Gürkaynak,

[3]In fact, it has long been assumed, either explicitly or implicitly, that short interest rates consist of multiple components with different frequencies. For example, in almost all multi-factor term structure models with at least a level and a slope factor, such as Rudebusch and Wu (2008), short interest rates are implicitly assumed to be driven by persistent and transitory components. The level factor usually appears as a persistent unit-root process as opposed to the slope factor being a stationary mean-reverting variable. Other examples include Campbell, Sunderam, and Viceira (2012), who explicitly model that nominal short interest rates are composed of three components: permanent inflation, transitory inflation, and transitory real interest rate.

Sack, and Swanson, 2005; Atkeson and Kehoe, 2009; Goodfriend and King, 2009). Thus, I estimate the persistent component using the history of realized core inflation, and the transitory component is estimated as the residuals from regressing 1-year T-bond yields on the persistent inflation component. This estimation strategy is borrowed from Cieslak and Povala (2015).

The persistent component (τ_t) is estimated as an exponentially-weighted average of realized core inflation over the past 120 months:

$$\tau_t \equiv \frac{\sum_{i=0}^{120} v^i \, CPI_{t-i}}{\sum_{i=0}^{120} v^i}, \tag{5}$$

where $(1 - v)$ denotes constant gain. Cieslak and Povala (2015) estimate the gain parameter at $v = 0.9868$ (standard error 0.0025) by comparing realized inflation with inflation survey forecasts.

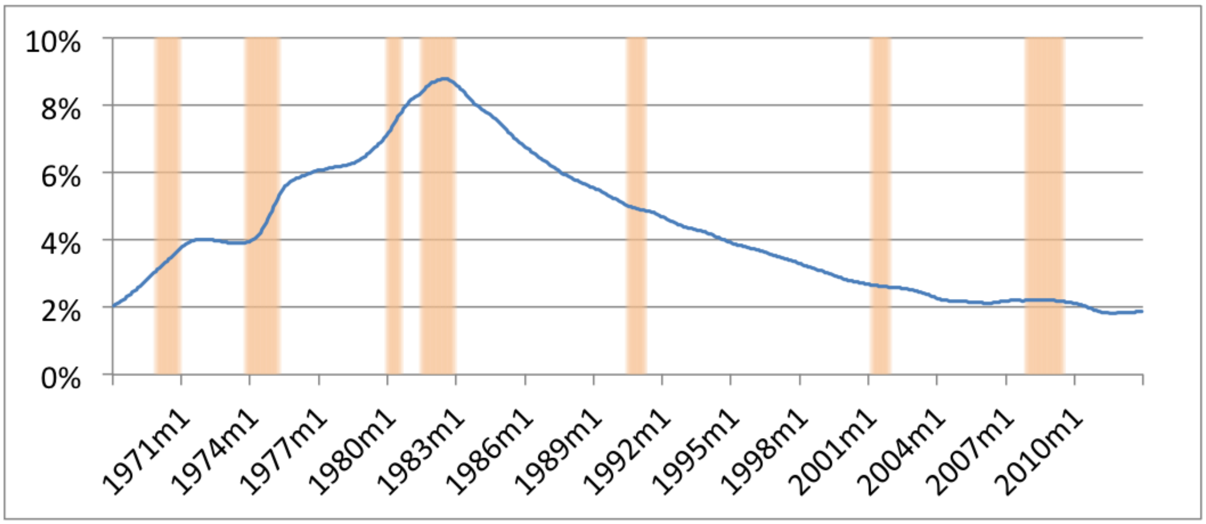
The top panel of Figure 1 shows the time series of the estimated persistent inflation component. Core inflation data are downloaded from the FRED Economic Data.[4] The estimated series start in January 1968 since the database provides core inflation since January 1958 and 10-year histories are used to estimate the variable. According to the figure, the inflation component reached the peak in the early 1980s and has gradually declined since then. The augmented Dickey-Fuller (ADF) test does not reject the null hypothesis for the persistent inflation of being a unit-root process.

Provided that short interest rates are determined by persistent and transitory components, the transitory one can be estimated as the residuals of short interest rates orthogonalized by the persistent inflation component. Following Cieslak and Povala (2015),

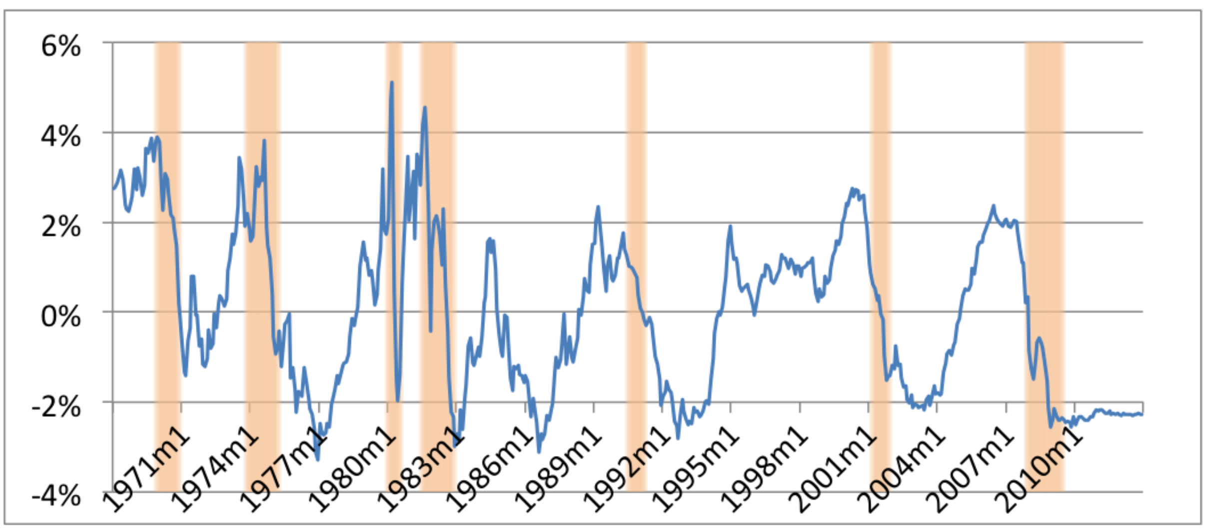
[4] `http://research.stlouisfed.org/fred2/series/CPILFESL?cid=32424`

Figure 1: Decomposition of Short Interest Rates

This figure decomposes one-year Treasury bond yields into two components: one is persistent and the other is transitory. The persistent component is estimated as an exponentially-weighted average of realized core inflation over the past 10 years, and the transitory component as the residuals from the regression of one-year yields on the persistent inflation. Shaded areas denote NBER recessions.



(a) Persistent Inflation (τ_t)



(b) Transitory Short Interest Rates (δ_t)

Fama-Bliss one-year bond yields are used as a proxy of the short interest rates.

$$\delta_t \equiv y_t^{(1y)} - \hat{\beta}_0 - \hat{\beta}_1 \, \tau_t, \tag{6}$$

where δ_t denotes the transitory deviation of short interest rates. $\hat{\beta}_0$ and $\hat{\beta}_1$ are OLS coefficients of regressing $y_t^{(1y)}$ on τ_t.

The bottom panel of Figure 1 shows the time series of the transitory component. It drops rapidly during recession periods, which is consistent with conventional monetary policy that employs federal funds rate to moderate business cycle. The unconditional standard deviation of δ_t is 1.834%.

2.2 Decomposition of risk premium ($E_t\,[exr_{t+i}]$)

Risk premium can be defined as the expected excess returns of risky assets. The early literature of bond risk premium, such as Campbell and Shiller (1991) and Dai and Singleton (2002), shows that the excess returns are predictable, and this finding has two implications. First, it rejects the expectations hypothesis that forward interest rates are equal to expected future short interest rates.[5] Second, it implies that the market price of risk is time-varying and spanned by observable variables.

This paper assumes two risk premium factors. The first one, *rpl*, is estimated following Cieslak and Povala (2015)'s approach. The authors estimate the risk premium factor as the residual from regressing T-bond yields on the two components of short interest rates. They first regress each of 2- to 5-year bond yields on the persistent inflation (τ_t),

$$y_t^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} \, \tau_t + \epsilon_t^{(n)} \quad \text{for } n = 2, \cdots, 5 \text{ years}, \tag{7}$$

[5] Piazzesi (2003) explains the rejection of the expectations hypothesis in detail.

and then regress the average of the residuals $\left(\bar{\epsilon}_t \equiv \frac{1}{4}\sum_{n=2y}^{5y} \hat{\epsilon}_t^{(n)}\right)$ on the transitory interest rate component (δ_t) without an intercept,[6]

$$\bar{\epsilon}_t = \gamma_1 \, \delta_t + u_t. \tag{8}$$

Cieslak and Povala (2015) show that equation (8)'s residual $(\hat{u}_t)$ outperforms both Cochrane and Piazzesi (2005)'s and Ludvigson and Ng (2009)'s risk premium predictors in the predictive regression of excess returns in the bond market. This paper uses the residual as a proxy of *rpl*. The CRSP Fama-Bliss Discount Bond Yields are used for the 2- to 5-year bond yields.

The second risk premium factor, *rps*, is estimated as 3-month T-bill yields that are orthogonalised by the other three state variables,

$$rps_t = -\left\{ y_t^{(3m)} - \hat{\beta}_0 - \hat{\beta}_1 \, \tau_t - \hat{\beta}_2 \, \delta_t - \hat{\beta}_3 \, rpl_t \right\}, \tag{9}$$

where the $\hat{\beta}$s are OLS coefficients. *rps* turns positive when 3-month T-bill yield is lower than the projected value by the three state variables. I specifically choose 3-month T-bill yields for two reasons. First, the 3-month yields do not have mechanical correlation to monthly excess returns while 1-month yields do due to measurement errors. Second, they have the longest history that starts in 1954. In comparison, the secondary market rate of 4-week T-bills is provided only since July 2001. The 3-month T-bill yields are downloaded from the FRED Economic Data.[7]

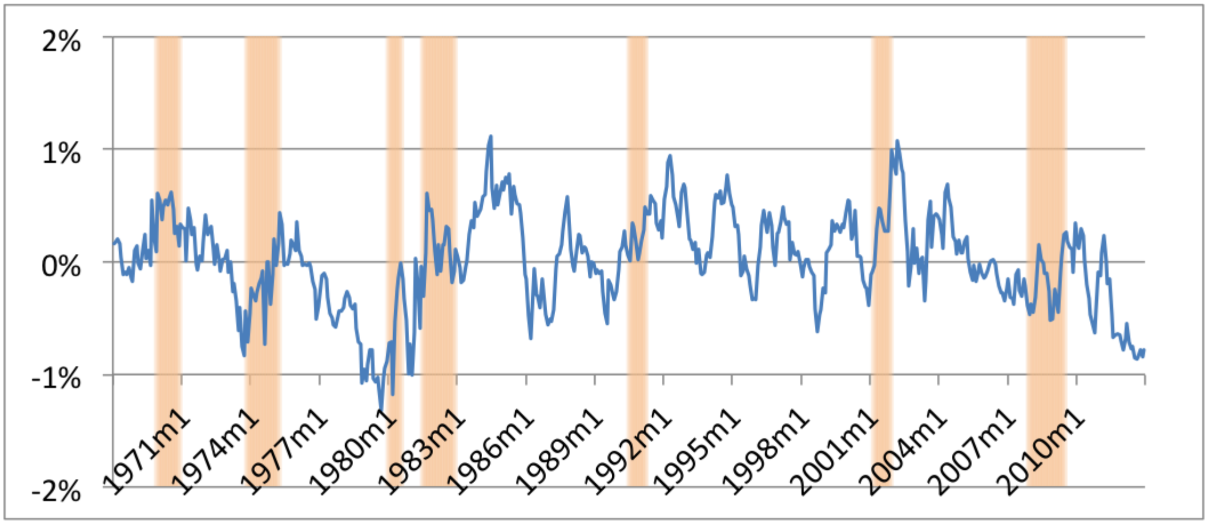
Figure 2 shows the time series of the two risk premium factors, *rpl* and *rps*. The three state variables (τ_t, δ_t, and rpl_t) are based on the replication of Cieslak and Povala (2015),

[6] The latest version of Cieslak and Povala (2015) slightly changes the estimation method of the risk premium factor, but I stick to their original estimation strategy because it is more consistent with the estimation of my second risk premium factor. I also tried their new method and confirmed that the results are not changed.

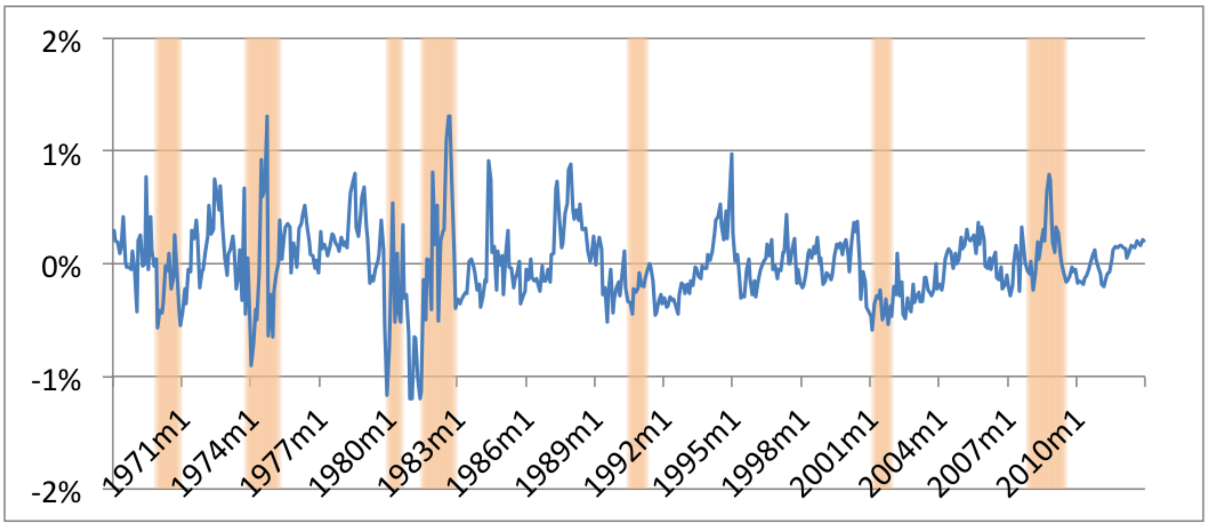
[7] `http://research.stlouisfed.org/fred2/series/DTB3?cid=116`

Figure 2: Time Series of Risk Premium Factors

This figure compares the time series of long- and short-term risk premium factors. The former is estimated from Treasury bond yields meanwhile the latter is from Treasury bill yields. Shaded areas denote NBER recessions.



(a) Risk Premium Factor from Treasury Bonds (rpl_t)



(b) Risk Premium Factor from Treasury Bills (rps_t)

and this paper adds one more state variable—short-term risk premium factor (rps_t)—to the framework and focuses on the comparison of rpl and rps. Although not reported in this paper, other benchmark risk premium factors such as Fama and Bliss (1987)'s forward interest rate slope and Cochrane and Piazzesi (2005)'s tent-shaped factor were also used as an alternative proxy of rpl and the results were not changed. I winsorized rps at 0.5 and 99.5 percentiles, but the un-winsorized version also yields the same results. The unconditional standard deviations of rpl and rps are 0.42% and 0.34%, respectively.

3 Forecast of excess returns in the bond market

Risk premium is not directly observable. However, if future excess returns can be predicted by some observable variables at time t, one can consider that the unobservable risk premium can be projected on the space that is spanned by the observable predictors. Therefore, if rpl and rps were indeed related to risk premium, they would have been able to predict future excess returns.

Table 1 shows the forecast of annual excess returns. The dependent variable is the excess returns of holding T-bonds over the next one year, $exr^{(n)}_{t,t+1y}$, in which riskfree short interest rates are estimated as one-year T-bond yields. Since Fama and Bliss (1987), it has been a norm to use annual excess returns for the test of risk premium factors' forecastability. The excess returns are estimated from the T-bond data provided by the Federal Reserve Board of Governors,[8] which smoothes yield curves using the Svensson curve approximation method. Gürkaynak, Sack, and Wright (2006) explain the construction of the dataset in detail.

Panel A regresses the excess returns on rpl only, replicating Cieslak and Povala (2015)'s predictability test. For comparison, Panel B and C regress the returns on the

[8] `http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html`

Table 1: Forecast of Annual Excess Returns

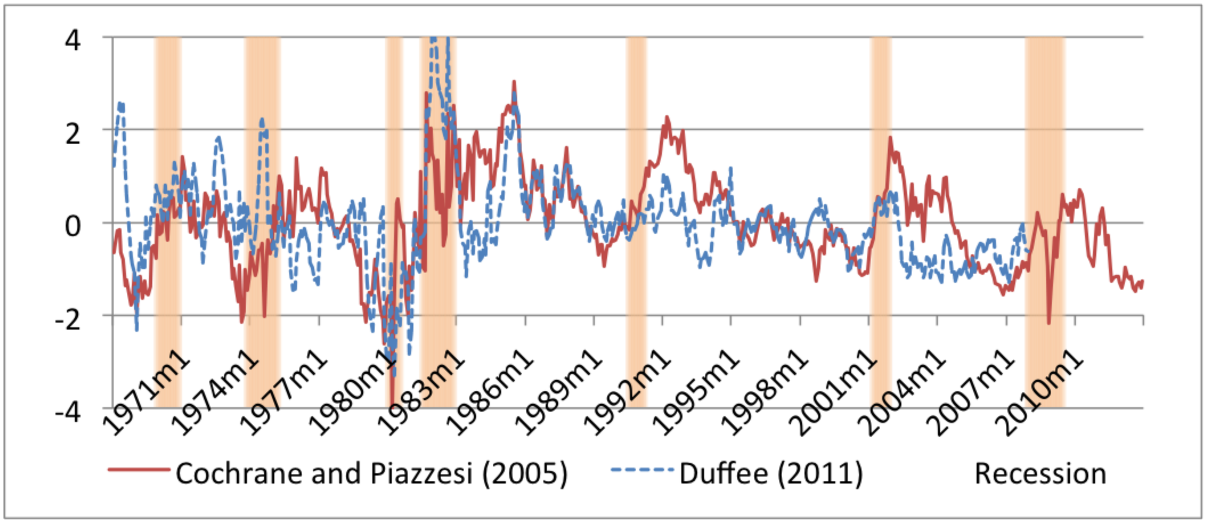
The dependent variable is excess returns of holding Treasury bonds over the next one year, $exr^{(n)}_{t,t+1y}$, whose maturities are specified by the top row. Predictors in each panel are respectively attributed to Cieslak and Povala (2015), Cochrane and Piazzesi (2005), and Duffee (2011). rpl's estimation method is explained in Section 2.2. All risk premium factors are normalized. Numbers in parentheses are Newey–West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level, respectively.

Maturity	2 years	3 years	5 years	7 years	10 years	15 years
Panel A. Forecast by *rpl* (Cieslak and Povala, 2015)						
rpl	2.14***	4.06***	7.43***	10.49***	15.18***	22.26***
	(7.201)	(7.538)	(7.947)	(8.186)	(8.404)	(7.828)
obs	540	540	540	540	497	494
R^2	0.283	0.308	0.340	0.361	0.404	0.420
Panel B. Cochrane and Piazzesi (2005)'s Tent-Shaped Factor						
tent	0.44***	0.84***	1.56***	2.23***	3.05***	4.58***
	(4.393)	(4.424)	(4.588)	(4.749)	(4.376)	(4.403)
obs	540	540	540	540	497	494
R^2	0.165	0.182	0.209	0.226	0.219	0.238
Panel C. Duffee (2011)'s Hidden Factor						
hidden	-7.65***	-13.82***	-24.16***	-33.31***	-50.30***	-74.32***
	(-4.091)	(-4.049)	(-3.951)	(-3.908)	(-4.100)	(-4.066)
obs	480	480	480	480	437	434
R^2	0.158	0.158	0.164	0.170	0.196	0.206

other two benchmark risk premium factors. Panel B uses Cochrane and Piazzesi (2005)'s tent-shaped factor which is estimated from the predicted excess returns projected on five forward interest rates. Panel C uses Duffee (2011)'s hidden factor which is estimated as a higher-order state variable from the Kalman filtering under the restriction that the factor be hidden from the cross-section of bond yields. The hidden factor can be downloaded

Figure 3: Other Benchmark Risk Premium Factors

This figure shows the time series of Cochrane and Piazzesi (2005)'s tent-shaped risk premium factor and Duffee (2011)'s hidden factor. Cochrane and Piazzesi (2005) estimate the tent-shaped factor as the predicted annual excess returns of holding Treasury bonds by five forward interest rates, and Duffee (2011) estimates the hidden factor as a higher-order state variable of the Kalman filtering method. Both factors are normalised, and Duffee (2011)'s hidden factor is multiplied by −1.



from Gregory Duffee's website.[9] Figure 3 shows the time series of the two benchmark risk premium factors.

Table 1 shows that *rpl* outperforms other benchmarks by a large margin. For example, *rpl*'s R^2s in Panel A range from 28.3% to 42.0%. In comparison, the tent-shaped factor's R^2s in Panel B are from 16.5% to 23.8%, and the hidden factor's R^2s in Panel C are from 15.8% to 20.6%. *rpl* also outperforms the benchmarks in terms of statistical significance. *rpl*'s Newey–West t statistics are 7.2 to 8.4 as opposed to the statistics of 4.4∼4.7 in Panel B and 3.9∼4.1 in Panel C. According to Panel A, a 1 standard deviation increase in *rpl* implies 6.4% ($= 15.18 \times 0.42\%$) increase in the annual excess returns from holding 10-year T-bonds.[10]

[9] `http://www.econ2.jhu.edu/people/Duffee/`

[10] For comparison, the unconditional sample standard deviation of the excess returns is 14.6%.

In particular, the large improvement of forecastability from Panel B to Panel A is notable since both predictors are estimated from the same dataset—the cross-section of five Fama-Bliss bond yields[11]—except that *rpl* orthogonalises the yields with inflation rates. R^2s almost double by simply getting rid of the persistent inflation component from predictors. Thus, it implies that the persistent inflation component is not related to the time-varying market price of risk. In a similar vein, Dai, Singleton, and Yang (2004) argue that the tent shape of Cochrane and Piazzesi (2005)'s risk premium factor is due to a mechanical effect to offset the level factor from forward interest rates. Cochrane and Piazzesi (2008) also show that the level factor is not related to the market price of risk although its uncertainty is priced by the market.

However, strong forecastability of annual excess returns does not necessarily mean strong forecastability for shorter investment horizons. First of all, the predictive regression of annual returns with monthly observations is likely to be contaminated by the long-horizon forecast bias since its dependent variable is mechanically autocorrelated. Boudoukh, Richardson, and Whitelaw (2007) show that long-horizon forecast regressions exaggerate R^2 and statistical significance. Moreover, annual excess returns inevitably ignore the risk premium born by one-year T-bonds since one-year bonds are considered a riskfree asset for an annual horizon but a risky asset for a monthly period. One stylized fact in the bond market is that, as shown by Duffee (2010), shorter-maturity bonds offer higher Sharpe ratios. Thus, the act of ignoring one-year bonds' risk premium would have probably undermined the authenticity of excess return forecasts.

Table 2 shows the forecast of monthly excess returns. The dependent variable is the excess returns from holding T-bonds for the next one month over one-month riskfree interest rates, $exr_{t,t+1m}^{(n)}$. The one-month riskfree rates are from the Ibbotson Associates.[12]

[11] I intentionally specify the "Fama-Bliss" bond yields since, as shown by Dai, Singleton, and Yang (2004), the tent shape is turned into a wave pattern if bond yields are estimated from different data sources.

[12] The one-month riskfree interest rates are provided from Kenneth French's website.

Table 2: Forecast of Monthly Excess Returns

The dependent variable is excess returns of holding Treasury bonds over the next one month, $exr^{(n)}_{t,t+1m}$, whose maturities are specified by the top row. *rpl* and *rps* denote risk premium factors which are estimated respectively from Treasury bonds and Treasury bills. $\tilde{y}^{(1m)}_t$ and $\tilde{y}^{(3m)}_t$ denote 1- and 3-month Treasury bills orthogonalized by the persistent and transitory components of riskfree interest rates and *rpl*. Numbers in parentheses are Huber–White t statistics. ***, **, and * denote significances at 1%, 5%, and 10% level, respectively.

Maturity	2 years	3 years	5 years	7 years	10 years	15 years
			Panel A. Forecast by *rpl* Only			
rpl_t	0.314***	0.475***	0.761***	1.003***	1.341***	1.863***
	(2.585)	(2.950)	(3.474)	(3.615)	(3.508)	(3.294)
obs	552	552	552	552	509	506
R^2	0.027	0.031	0.035	0.034	0.035	0.032
			Panel B. Forecast by *rpl* and *rps*			
rpl_t	0.318***	0.480***	0.769***	1.014***	1.333***	1.849***
	(2.654)	(3.030)	(3.581)	(3.751)	(3.604)	(3.414)
rps_t	0.408***	0.549***	0.828***	1.154***	1.750***	2.860***
	(3.130)	(3.110)	(3.250)	(3.484)	(3.777)	(4.157)
obs	552	552	552	552	509	506
R^2	0.055	0.057	0.061	0.063	0.072	0.080

Panel A uses *rpl* as the only predictor. As suspected, its R^2s drop to 2.7%∼3.5%, which are barely one tenth of the R^2s in Table 1. Although not reported here, the R^2s of the other benchmark predictors also drop to 1%∼2%.

Panel B, however, shows that the monthly return forecast is significantly improved by adding *rps* as the second predictor. R^2s almost double to 5.5%∼8.0%, and both *rpl* and *rps* appear significant. *rpl* and *rps* do not have multicollinearity issue since *rps* is orthogonalised by *rpl* and thus they have zero correlation. According to the results, a 1 standard deviation increase in *rpl* and *rps* implies 0.56% and 0.60% increase in the monthly excess returns of holding 10-year T-bonds, respectively. Although not reported here, I also project monthly excess returns on all four state variables and find that neither τ_t nor δ_t is significant. Therefore, this table implies that rps (or equivalently 3-month T-bills) has unique risk premium information that is not spanned by the first three state variables.

These results in Table 1 and 2 reaffirm Duffee (2010)'s finding that monthly and annual expected bond excess returns are driven by two factors and these two factors operate at different frequencies. He shows that the second and fourth principal components are equally responsible for the predictable variation of monthly excess returns meanwhile the second dominates the fourth component in terms of annual excess returns. rpl and rps demonstrate the same pattern as described in the paper. However, the later analysis in the following section will show that rpl and rps do not exactly correspond to the second and fourth principal components.

As Figure 2 shows, rps is more volatile than rpl. In particular, rpl's half-life is estimated to be about 7.0 months meanwhile rps' is 1.8 months. Since rps has a shorter half-life, one can also expect that rps might have a shorter forecast horizon than rpl, which is tested in Table 3.

The dependent variables in each panel of Table 3 are monthly excess returns in 2, 3, and 16 months, respectively ($exr^{(n)}_{t+1m,t+2m}$, $exr^{(n)}_{t+2m,t+3m}$, and $exr^{(n)}_{t+15m,t+16m}$). The table shows that rps is equally significant in two months compared to the forecast in one month but loses most of its forecastability in three months. In contrast, rpl's forecastability remains significant for the horizon of up to 16 months. Particularly, rpl appears significant at 1% confidence level even in 12 months. Thus, the table confirms that rps' forecastability is limited to a short horizon whereas rpl's forecastability remains strong for more than one year.

Every term structure model is essentially a function to convert state variables into

Table 3: Forecast of Monthly Excess Returns in Further Periods

The dependent variable is monthly excess returns of holding Treasury bonds in further periods, $exr^{(n)}_{t+\phi-1,t+\phi}$ for $\phi = 2$, 3 and 16 months. *rpl* and *rps* denote risk premium factors which are estimated respectively from Treasury bonds and Treasury bills. Numbers in parentheses are Huber–White t statistics. ***, **, and * denote significances at 1%, 5%, and 10% level, respectively.

Maturity	2 years	3 years	5 years	7 years	10 years	15 years
	Panel A. Monthly Excess Returns in 2 Months, $exr^{(n)}_{t+1m,t+2m}$					
rpl_t	0.363***	0.551***	0.890***	1.180***	1.584***	2.160***
	(3.494)	(3.929)	(4.481)	(4.664)	(4.683)	(4.499)
rps_t	0.422***	0.589***	0.876***	1.157***	1.587***	2.173***
	(3.303)	(3.422)	(3.539)	(3.564)	(3.518)	(3.174)
obs	552	552	552	552	510	507
R^2	0.066	0.071	0.076	0.076	0.080	0.071
	Panel B. Monthly Excess Returns in 3 Months, $exr^{(n)}_{t+2m,t+3m}$					
rpl_t	0.338***	0.494***	0.760***	0.989***	1.351***	1.990***
	(3.301)	(3.537)	(3.819)	(3.880)	(3.932)	(4.127)
rps_t	0.194	0.263	0.370	0.505	0.869*	1.240*
	(1.410)	(1.438)	(1.423)	(1.500)	(1.878)	(1.790)
obs	551	551	551	551	510	507
R^2	0.037	0.039	0.039	0.039	0.045	0.046
	Panel C. Monthly Excess Returns in 16 Months, $exr^{(n)}_{t+15m,t+16m}$					
rpl_t	0.154	0.240	0.388*	0.505*	0.703*	0.926*
	(1.346)	(1.568)	(1.784)	(1.805)	(1.866)	(1.683)
rps_t	-0.116	-0.173	-0.318	-0.475	-0.869*	-1.326*
	(-0.769)	(-0.861)	(-1.137)	(-1.349)	(-1.839)	(-1.951)
obs	538	538	538	538	510	507
R^2	0.009	0.010	0.013	0.013	0.019	0.018

Table 4: Forecast of State Variables and Bond Yields

The dependent variable is the changes in four state variables: τ, δ, rpl, and rps. τ and δ denote the persistent and transitory components of riskfree interest rates whereas rpl and rps denote long- and short-term risk premium factors. Their estimation is explained in Section 2.1 and 2.2. τ and δ are multiplied by 100 to match with the scales of rpl and rps. Numbers in parentheses are Huber–White t statistics. ***, **, and * denote significances at 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)
dep. var.	$\Delta\tau_{t+1}$	$\Delta\delta_{t+1}$	Δrpl_{t+1}	Δrps_{t+1}
τ_t	0.000	-0.001	0.001	0.000
	(0.095)	(-0.054)	(0.331)	(0.068)
δ_t	0.008***	-0.040**	0.001	-0.008
	(11.517)	(-2.410)	(0.236)	(-1.182)
rpl_t	-0.025***	0.011	-0.095***	0.029
	(-9.877)	(0.137)	(-4.837)	(0.957)
rps_t	-0.004	-0.185**	-0.031	-0.324***
	(-0.909)	(-2.092)	(-1.069)	(-6.197)
obs	551	551	551	551
R^2	0.261	0.035	0.051	0.168

yield curves. In other words, excess returns of bonds can be written as a function of the changes in state variables. Given the previous finding that the excess returns are predicted by rpl and rps, one can expect the risk premium factors to also predict the changes in other state variables. Moreover, it is of a particular interest to see the forecast of the persistent inflation (τ_t) and transitory short interest rate (δ_t) since they account for the level and slope factors (will be shown in the next section) and thus explain almost 99% of the total variation of yield curves.

For this aim, Table 4 estimates the VAR(1) system of the state variables to understand their dynamics. Its dependent variables are the changes in state variables, which are specified in the first row. Note that τ_t's coefficient in column (1) is close to zero, implying that the inflation component is as persistent as a random walk process. In comparison, δ_t's coefficient of -0.040 in column (2) implies that its half-life is about 17.0 $(= -\log(2)/\log(1-0.040))$ months. rpl_t's coefficient of -0.095 in column (3) and rps_t's coefficient of -0.324 in column (4) imply that their half-lives are 7.0 and 1.8 months, respectively. These results are consistent with Figure 1 and 2's findings that τ_t is the most persistent state variable, followed by δ_t, rpl_t, and rps_t. Moreover, δ_t's coefficient in column (1) is significantly positive since the Federal Reserve Board raises federal funds rate in the anticipation of high inflation in subsequent periods.

The most important implication in the table is based on rpl_t and rps_t's coefficients in column (1) and (2), which imply that both risk premium factors predict a decrease in short interest rates. However, they do so in different ways. rpl_t predicts a decrease in the persistent inflation (τ_t) meanwhile rps_t predicts a decrease in the transitory component of short interest rates (δ_t). This result confirms that their predictability is based on different dimensions.

4 Affine term structure model

Several papers in the literature notice the possibility that risk premium consists of two factors of different frequencies. For example, Duffee (2002) explains that "an increase in the slope factor affects the price of risk of the level factor," and the twist (curvature) shock "has a strong effect on instantaneous expected returns, but it is also very short-lived. Thus, this shock is responsible for high-frequency fluctuations in expected excess returns." The short-term risk premium has been often related to high-order principal components but not been pinned down to T-bill yields.

In this section, I will show why it is difficult to estimate rps with an affine term structure model even if T-bill yields are used in the sample. This implication can explain why it has largely eluded the literature that the short-term risk premium factor could be easily estimated from T-bill yields.

4.1 Affine model overview

Let X_t denote a column vector of state variables. The variables are assumed to follow VAR(1) process.

$$X_{t+1} = \mu + \Phi X_t + \epsilon_{t+1}, \qquad \Omega \equiv E\left[\epsilon_{t+1}\epsilon_{t+1}^\top\right]. \tag{10}$$

One-period risk-free interest rate, r_t, is given as a linear function of state variables,

$$r_t = \delta_0 + \delta_1^\top X_t. \tag{11}$$

The market price of risk, λ_t, is a column vector which is also linearly proportional to state variables,

$$\lambda_t = \lambda_0 + \Lambda X_t, \tag{12}$$

where $\lambda_0 \in \mathbb{R}^n$ and $\Lambda \in \mathbb{R}^{n\times n}$. Thus, the log nominal pricing kernel can be derived as

$$m_{t+1} = -r_t - \frac{1}{2}\lambda_t^\top \Omega\, \lambda_t - \lambda_t^\top \epsilon_{t+1}. \tag{13}$$

Note that the price of a discount bond is equal to the expectation of its future value discounted by the pricing kernel. Let $p_t^{(n)}$ denote its log price. This recursive form can be written as

$$\begin{aligned} p_t^{(n)} &= \log E_t\left[\exp\left(m_{t+1} + p_{t+1}^{(n-1)}\right)\right] \\ &= E_t\left[m_{t+1} + p_{t+1}^{(n-1)}\right] + \frac{1}{2}\,\mathrm{var}_t\left(m_{t+1} + p_{t+1}^{(n-1)}\right). \end{aligned} \tag{14}$$

By combining all these equations, the solution of log bond price can be derived as

$$p_t^{(n)} = A_n + B_n^\top X_t, \tag{15}$$

$$A_{n+1} = -\delta_0 + A_n + B_n^\top \mu^Q + \frac{1}{2} B_n^\top \,\Omega\, B_n, \tag{16}$$

$$B_{n+1}^\top = -\delta_1^\top + B_n^\top \Phi^Q, \tag{17}$$

where $\mu^Q \equiv \mu - \Omega\,\lambda_0$ and $\Phi^Q \equiv \Phi - \Omega\,\Lambda$. μ^Q and Φ^Q denote the dynamics of state variables under risk-neutral probability measure. Bond yields are $y_t^{(n)} = -\frac{1}{n}\, p_t^{(n)}$.

The conditional expectation of excess returns can also be derived as a linear function of state variables,

$$E_t\left[exr_{t,t+1}^{(n)}\right] = B_{n-1}^\top \Omega\left(\lambda_0 + \Lambda X_t\right) - \frac{1}{2} B_{n-1}^\top \Omega B_{n-1}. \tag{18}$$

ΛX_t in the above equation drives the time-varyingness of the market price of risk. Each non-zero element of $\Lambda_{i,j}$ denotes that the i-th innovation shock, $\epsilon_{t+1,i}$ is priced by the j-th state variable, $X_{t,j}$. Thus, if some innovation shocks are not priced by the market, their corresponding rows in λ_0 and Λ will be set to zero. Equivalently, if some state variables are not related to the time-varying price, their corresponding columns in Λ will be zero.

Lastly, it can be shown that forward rates are equal to the risk-neutral expectation of future one-period riskfree interest rates after adjusting for Jensen's inequality,

$$\begin{aligned} f_t^{(n)} &\equiv p_t^{(n-1)} - p_t^{(n)} \\ &= \delta_0 + \delta^\top \left\{ \left(I + \Phi^Q + \cdots + \Phi^{Q(n-2)} \right) \mu^Q + \Phi^{Q(n-1)} X_t \right\} - \frac{1}{2} B_{n-1}^\top \Omega B_{n-1} \\ &= E_t^Q\left[r_{t+n-1}\right] - \underbrace{\frac{1}{2} B_{n-1}^\top \Omega B_{n-1}}_{\text{Jensen's inequality}}. \end{aligned} \tag{19}$$

4.2 Affine model estimation

In this section, I estimate and compare four different specifications of the affine term structure model. They all have four factors but use different observations for state variables.

- Model 1: $X_t = (\, \tau_t,\, \delta_t,\, rpl_t,\, rps_t \,)$
- Model 2: $X_t = \left(\tau_t,\, y_t^{(3m)},\, y_t^{(1y)},\, y_t^{(5y)} \right)$
- Model 3: $X_t = \left(\tau_t,\, y_t^{(1y)},\, y_t^{(3y)},\, y_t^{(5y)} \right)$
- Model 4: $X_t = \left(y_t^{(3m)},\, y_t^{(1y)},\, y_t^{(3y)},\, y_t^{(5y)} \right)$

Model 1 uses the four variables that are introduced in the previous section: τ_t, δ_t, rpl_t and rps_t. Model 2 combines τ_t with three-month, one-year and five-year bond yields. Essentially, the state vector of Model 2 can be considered a rotation of Model 1's state vector, and thus spans an identical space.[13] Model 3 replaces Model 2's three-month yields with another T-bond yields to test whether the exclusion of T-bill yields makes a notable difference. The last one, Model 4, uses yields only. The comparison with Model 4 later assures Cieslak and Povala (2015)'s finding that the presence of τ_t significantly improves the predictability of risk premium.

The estimation is done in two steps. First, the parameters for physical dynamics—μ, Φ and Ω—are estimated from running OLS regressions on the state variables. Second, all the other parameters—δ_0, δ_1, λ_0 and Λ—are estimated by numerically minimising the squared errors of observed and model-implied bond yields,

$$\widehat{\delta}_0,\, \widehat{\delta}_1,\, \widehat{\lambda}_0,\, \widehat{\Lambda} = \arg\min \sum_{t=1}^{T} \sum_{n=1\text{m},\cdots,15\text{y}} \left\{ y_t^{(n)} - \widehat{y}_t^{(n)} \right\}^2, \tag{20}$$

[13] Technically, rpl is estimated from the average residuals of 2- to 5-year bond yields. But the difference between Model 1 and Model 2's state space is very small.

where $y_t^{(n)}$ and $\widehat{y}_t^{(n)}$ denote observed and model-implied bond yields, respectively. The squared errors are summed through 11 maturities: 1, 3, 6 months and 1, 2, 3, 4, 5, 7, 10, 15 years. This estimation strategy is motivated by Joslin, Singleton, and Zhu (2011), who show the independence between physical and risk-neutral measures.

The bond yield data are collected from various sources. 1-month yields are from the Ibbotson Associates and downloaded from Kenneth French's website. 3- and 6-month yields are from the FRED Economic Data. 1- to 5-year yields are from the CRSP Fama–Bliss discount bond yields. 7-, 10- and 15-year yields are from the Federal Reserve Board of Governors. The sample horizon spans from January 1968 through December 2013. The 10- and 15-year yields are missing in the first 3$\sim$4 years.

Table 5 compares the cross-sectional fit of bond yields by each model specification. For this comparison, the table shows R^2, which is defined as

$$R^2 \equiv 1 - \frac{SS_{\text{residuals}}}{SS_{\text{total}}} = 1 - \frac{\sum_{t=1}^{T}\left(y_t^{(n)} - \widehat{y}_t^{(n)}\right)^2}{\sum_{t=1}^{T}\left(y_t^{(n)} - \overline{y}^{(n)}\right)^2}, \tag{21}$$

where $y_t^{(n)}$ and $\widehat{y}_t^{(n)}$ denote observed and model-implied bond yields, respectively. $\overline{y}^{(n)}$ is the sample average of observed bond yields. R^2 approaches to one if a model can explain most variations in observed bond yields and thus the sum of squared residuals in the objective function of (20) goes to zero.

According to the table, all four models can explain more than 99% of the total variations on average. This finding is consistent with the literature that three state variables are enough to explain most variations in bond yields.

The table also shows that Model 3 performs slightly worse than others in the fitting of T-bill yields. This result is understandable since Model 3 is the only model that does not use T-bill yields as a state variable. In contrast, Model 1 fits 15-year yields slightly worse

Table 5: Cross-sectional Fit of Bond Yields by the Affine Models

This table shows R^2s of the cross-sectional variations of bond yields that are explained by each model specification,

$$R^2 \equiv 1 - \frac{SS_{\text{residuals}}}{SS_{\text{total}}} = 1 - \frac{\sum_{t=1}^{T}\left(y_t^{(n)} - \widehat{y}_t^{(n)}\right)^2}{\sum_{t=1}^{T}\left(y_t^{(n)} - \overline{y}^{(n)}\right)^2},$$

where $y_t^{(n)}$ and $\widehat{y}_t^{(n)}$ denote observed and model-implied bond yields, respectively. $\overline{y}^{(n)}$ is the sample average of observed bond yields.

maturity (n)	Model 1	Model 2	Model 3	Model 4
1 month	0.980	0.976	0.968	0.976
3 months	0.991	0.999	0.982	0.999
6 months	0.992	0.998	0.989	0.998
1 year	0.997	0.999	0.999	0.998
2 years	0.998	0.999	0.999	0.999
3 years	0.999	0.999	0.999	1.000
4 years	0.999	0.999	0.999	0.999
5 years	0.999	1.000	1.000	1.000
7 years	0.992	0.995	0.994	0.995
10 years	0.985	0.991	0.991	0.992
15 years	0.974	0.983	0.984	0.984
average	0.992	0.994	0.991	0.994

than others (97.4% vs. 98.4%) since its state variables do not directly include long-term T-bond yields. Model 2 and 4 perform the best across all maturities since they use both T-bill and T-bond yields as state variables.

4.3 Model-implied risk premium

All four models are just shown to fit the cross-sectional variations in bond yields equally well. In this section, I will test their predictive power of risk premium.

The first test is to regress realised excess returns on their model-implied conditional expectations,

$$exr_{t,t+1}^{(n)} = \beta_0 + \beta_1 \widehat{E}_t \left[exr_{t,t+1}^{(n)} \right] + \epsilon_{t+1}. \tag{22}$$

The model-implied risk premium, $\widehat{E}_t \left[exr_{t,t+1}^{(n)} \right]$, is derived as a linear function of state variables in equation (18). If the model were correctly specified, the estimated β_1 would appear to be equal to one. This test can be considered a variation from Dai and Singleton (2002), who project the risk-premium-adjusted yield changes on the slope of yield curves.

Figure 4 shows the results of the regression. The upper panel (a) shows the estimated $\widehat{\beta}_1$, and the lower panel (b) shows R^2 from the regression. The regression is run for the excess returns from holding T-bonds with the maturities of 2, 3, 4, 5, 7, 10 and 15 years.

According to panel (a), most estimated $\widehat{\beta}_1$s are close to one. For Model 1, 2 and 3, the standard errors of $\widehat{\beta}_1$ are 0.15∼0.21, and thus none of the estimates are significantly different from one. Only Model 4 shows substantially bigger standard errors, 0.17∼0.33, and its $\widehat{\beta}_1$ for 15-year bond excess returns (0.569) is outside the 95% confidence interval from one. Although not reported in this figure, the results are also similar for $\widehat{\beta}_0$, none of which is significantly different from zero.

Moreover, Model 1 appears to dominate other models in terms of the mean absolute difference (MAD) of $\widehat{\beta}_1$ from one, $\frac{1}{N} \sum_{n=1}^{N} \left| \widehat{\beta}_1^{(n)} - 1 \right|$. The MAD for Model 1 is 0.046 meanwhile those for Model 2, 3 and 4 are 0.147, 0.145 and 0.209, respectively.

The outperformance of Model 1 over others becomes even more conspicuous when it comes to compare R^2s in panel (b). R^2s from Model 1 are 5.5%∼7.2% meanwhile those from Model 2, 3 and 4 are 2.8%∼4.6%, 3.0%∼5.8%, and 1.2%∼3.0%, respectively. The t-statistics of $\widehat{\beta}_1$ also show similar patterns as in R^2s. This result provides several important implications.

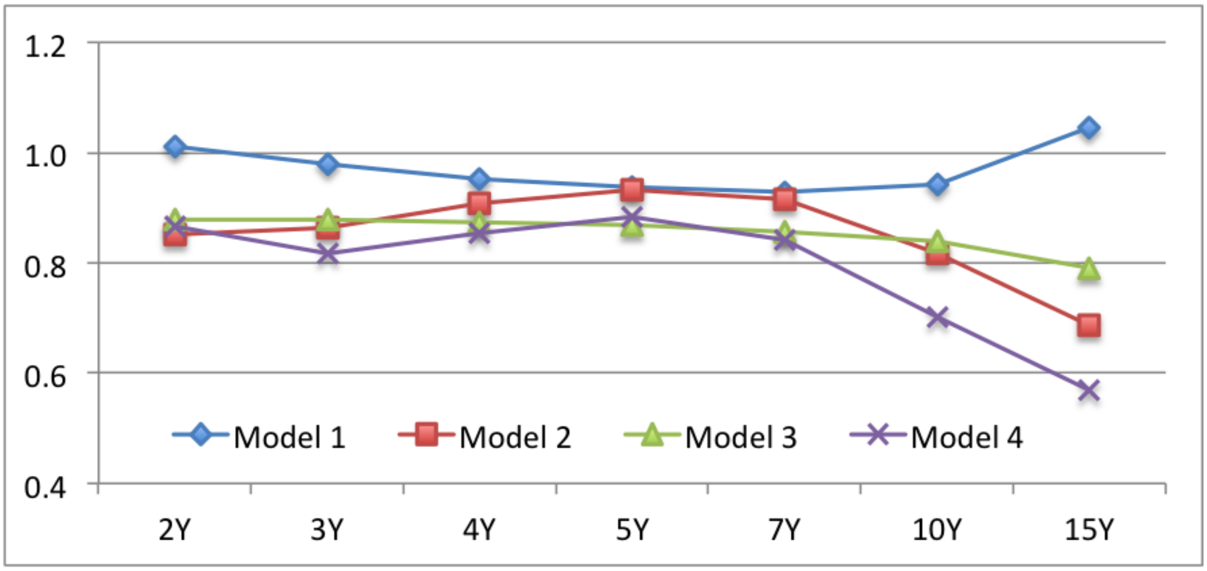
First, the magnitudes of the R^2s in Model 1 (5.5%∼7.2%) are comparable to the R^2s from the unrestricted regressions in Table 2 (5.5%∼8.0%). Not only the magnitudes,

Figure 4: Model-implied and Realised Expected Excess Returns

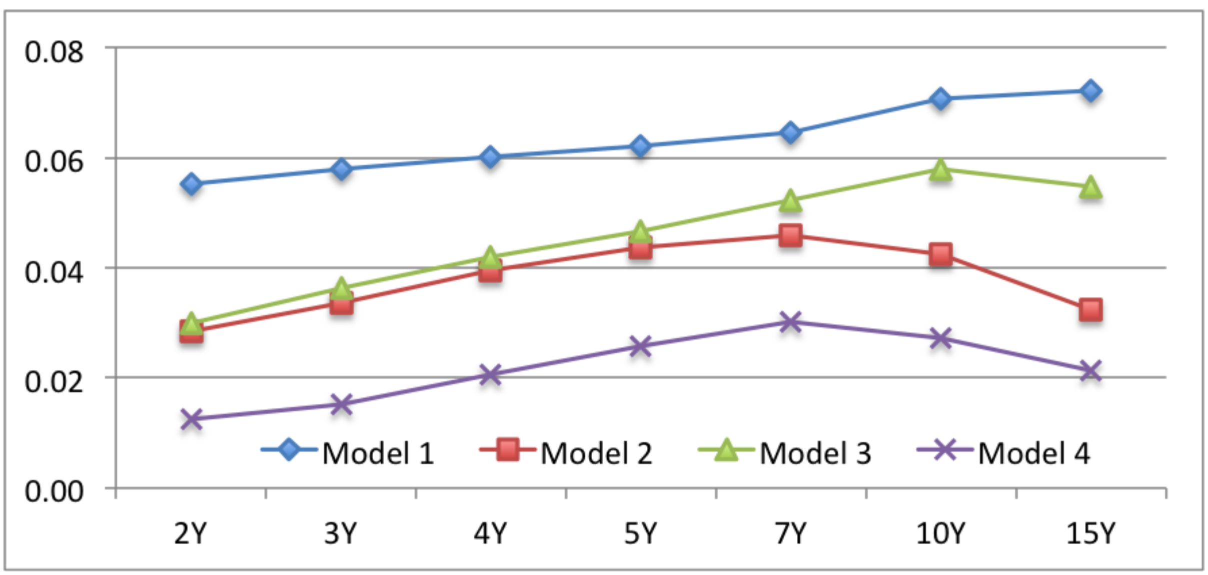
This figure shows the results of regressing realised excess returns on their conditional expectations which are implied by the affine models,

$$exr_{t,t+1}^{(n)} = \beta_0 + \beta_1 \widehat{E}_t \left[exr_{t,t+1}^{(n)} \right] + \epsilon_{t+1}.$$

The model-implied conditional expectations are derived for each model specification, and the maturities n are denoted on the horizontal axis. If the model were correctly specified, β_1 would be equal to one. Panel (a) shows the estimated β_1 and panel (b) compares R^2.



(a) $\widehat{\beta}_1$



(b) R^2

but also both R^2s in this figure and the table increase monotonically with maturity. This result shows that the affine model loses very little information compared to the unrestricted model.

The second notable finding is the humble performance of the yield-only Model 4. Although Model 4 shows the best fitting of the cross-sectional variations, it performs worst in the forecast of the time-series changes in bond yields. The underperformance of Model 4 can be attributed to the fact that the term structure is not a Markov process due to measurement errors or the existence of a hidden factor. This result is consistent with Cieslak and Povala (2015)'s finding that the control of persistent inflation (τ_t) significantly improves the forecastability of excess returns in the bond market.

The last and the most interesting implication is from the difference between Model 1 and Model 2. As explained earlier, the two state vectors are the rotations of each other and thus span almost identical spaces. Therefore, all the differences in their forecastability performance must be attributed to affine model parameter estimation. The question is which part of the affine model estimation—either of the physical dynamics or the risk-neutral dynamics—causes the difference.

To answer this question, suppose the demeaned state variables of Model 1 and 2 have the following relation,

$$X_t^{(M2)} = W\, X_t^{(M1)}, \tag{23}$$

for some invertible rotation matrix, W. Then, the relation between the two models' physical dynamics can be derived as

$$\Phi^{(M1)} = W^{-1}\, \Phi^{(M2)}\, W. \tag{24}$$

I estimate $\Phi^{(M1)}$, $\Phi^{(M2)}$ and W using OLS regressions. The largest eigenvalues of $\Phi^{(M1)}$ and $\Phi^{(M2)}$ are 0.9994 and 0.9996, respectively, due to the persistence of τ_t. However, the residual of equation (24), $\widehat{\Phi}^{(M1)} - \widehat{W}^{-1}\,\widehat{\Phi}^{(M2)}\,\widehat{W}$, is a singular matrix and the absolute value of its largest eigenvalue is 0.0913.

I repeat the same process for risk-neutral dynamics, $\Phi^{Q,(M1)}$ and $\Phi^{Q,(M2)}$. The largest eigenvalue of $\Phi^{Q,(M1)}$ is 1.0671, which implies non-stationarity, whereas the largest eigenvalue of $\Phi^{Q,(M2)}$ is 0.9985, which is still stationary. The largest eigenvalue of their residual, $\widehat{\Phi}^{Q,(M1)} - \widehat{W}^{-1}\,\widehat{\Phi}^{Q,(M2)}\,\widehat{W}$, is 0.1606.

From this simple calculation, one can conclude that the underperformance of Model 2 can be attributed to the measurement errors in both dynamics. First, the physical dynamics is estimated with higher standard errors because of the high correlation among state variables in Model 2. Second, the risk-neutral estimation is also biased by the objective function of minimising the cross-sectional squared errors. This estimation method chooses λ_0 and Λ to optimise the overall yield curve fit while rps is visible only from short-maturity yields. Thus, the variation in 3-month yields by rps might have been ignored to utilise another shape factor from the yields to optimise the overall fit.

Table 6 studies the projection of risk premiums on rpl and rps. In Panel A, the dependent variable is the model-implied conditional expectations of excess returns from holding 5-year T-bonds for each model, $E_t\left[exr_{t,t+1}^{(5Y)}\right]$. 5-year bonds are arbitrarily chosen as a representative example in this table, and similar results are found for other maturities too.

As expected, the risk premium in Model 1 is exclusively driven by rpl and rps. Both of their t-statistics are bigger than 100 and its R^2 is 99.7%. Moreover, even their coefficients—0.840 for rpl and 0.973 for rps—are not significantly different from the coefficients from the unrestricted forecast in Table 2—0.769 for rpl and 0.828 for rps. Thus, one can conclude from this result that the affine model performs on a par with the unrestricted forecast and the time-varying market price of risk in the model is entirely determined by rpl and rps. The two other state variables—τ and δ—do not affect the

Table 6: Projection of Model-implied Risk Premium on *rpl* and *rps*

This table shows the results of regressing risk premiums on *rpl* and *rps*. In Panel A, the dependent variable is the model-implied conditional expectations of excess returns from holding 5-year T-bonds for each model, $E_t\left[exr^{(5Y)}_{t,t+1}\right]$. In Panel B, the dependent variable is risk premium factors which are often used in the literature. Numbers in parentheses are Newey and West t statistics with 12 lags. ***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.

Panel A. dep var: model-implied $E_t\left[exr^{(5Y)}_{t,t+1}\right]$

	Model 1	Model 2	Model 3	Model 4
rpl_t	0.840***	0.889***	0.829***	0.378***
	(147.49)	(74.73)	(15.95)	(7.16)
rps_t	0.973***	0.170***	-0.157***	0.245***
	(174.20)	(10.74)	(-2.60)	(4.77)
obs	552	552	552	552
R^2	0.997	0.955	0.682	0.326

Panel B. dep var: risk premium factors in the literature

dep var	Cochrane and Piazzesi (2005) tent-shaped factor	Duffee (2011) filtered risk premium	Duffee (2011) smoothed risk premium
rpl_t	2.670***	-0.067***	-0.099***
	(11.28)	(-3.85)	(-5.60)
rps_t	-0.305	-0.101***	-0.108***
	(-1.52)	(-5.63)	(-5.12)
obs	552	480	480
R^2	0.531	0.248	0.395

market price although their innovation shocks are priced by the market.

In comparison, *rpl* still plays an important role in the risk premium implied by Model 2. However, both the coefficient and t-statistic of *rps* become much smaller. This result implies that Model 2's risk premium does not successfully capture the information by *rps* despite of its having 3-month yields as a state variable. This implication supports the claim that the information of rps from 3-month yields is somehow ignored in order to improve the overall cross-sectional fit instead.

Interestingly, the coefficients of rpl are similar across Model 1 to 3 but not in Model 4. Thus, this result reassures the finding that it is critical to control for persistent inflation (τ_t) to improve the forecast of risk premium in the bond market.

The results so far suggest that it is difficult to estimate risk premium if the objective function of estimation relies only on the cross-sectional fit of bond yields. The next panel is to study the effects on the estimated risk premium when the estimation is made not only by cross-sectional fit but also by time-series information.

Panel B of the table shows the results of projecting popular risk premium factors in the literature on rpl and rps. The first dependent variable is Cochrane and Piazzesi (2005)'s tent-shaped risk premium factor. The factor is significantly correlated to rpl but not to rps. This result is straightforward since Cochrane and Piazzesi (2005)'s factor is estimated from five T-bond forward rates and annual excess returns while rps is visible from T-bill yields and affects monthly excess returns.

The next two dependent variables in the panel are the two versions of Duffee (2011)'s risk premium factor. Duffee (2011) first shows that yield curves are not necessarily a Markov process due to the possible existence of a hidden factor, and estimate the factor using the Kalman filtering to harness the time-series information since the hidden factor is not spanned by the cross-section of bond yields. The panel shows that both versions of the factor are significantly correlated to rpl and rps.

To sum up, this section offers three suggestions to improve the forecastability of risk premium in the bond market. First, it is important to control for the persistent inflation, τ. Second, T-bill yields contain important information about risk premium. Third, an affine dynamic term structure model may not be able to fully reveal risk premium if its estimation objective function is dependent only on the cross-section of bond yields.

4.4 Yield factor loadings and principal components

Bond yields, as shown by equation (4), are the sum of expected future riskfree interest rates and risk premium. This equation suggests a possibility that the unobservable risk premium can be estimated by the combination of observable bond yields. This intuition motivates many papers in the literature to relate risk premium to the principal components of yield curves, particularly to the slope factor. For example, Fama and Bliss (1987) show that risk premium is proportional to the steepness of yield curve slope. Their finding implies that long-maturity bond yields have higher loadings on risk premium than short-maturity yields. Duffee (2002) documents a similar finding that "an increase in the slope factor affects the price of risk of the level factor."

In addition to the slope factor, the recent literature also sheds light on the importance of higher-order principal components. For example, Cochrane and Piazzesi (2005) estimate a tent-shaped risk premium factor by regressing annual excess returns on five forward interest rates and conclude that "the return-forecasting factor is clearly not related to any of the first three principal components." Duffee (2002) finds that a curvature shock "has a strong effect on instantaneous expected returns, but it is also very short-lived," and Duffee (2010) shows that the fourth principal component can explain about a half of the variation of monthly expected excess returns in the bond market.

I find positive evidence backing all of the aforementioned papers. My results support Fama and Bliss (1987) since the long-term risk premium factor (rpl) indeed raises the slope of yield curves. Duffee (2002) is also supported since rpl explains not only 37% of the slope factor's variations but also 76% of the curvature's variations. The results are also agreeable to Cochrane and Piazzesi (2005) and Duffee (2010) since the short-term risk premium factor (rps) dominates most variations in the fourth and fifth principal components.

Figure 5 shows how bond yields are affected when each state variable is deviated by +/− one standard deviation. Each subfigure is labeled according to a given state variable. The horizontal axis denotes maturities in years from 1 to 15 years, and the vertical axis denotes annualized bond yields. The figure is plotted based on the factor loadings of the affine model in equation (16) and (17).

Panel (a) shows that the persistent inflation (τ_t) shifts the level of yield curves. This effect is intuitive given the near-unit-root persistence of the variable. Panel (b) and (c) show that yield curves' slope is determined by the transitory short interest rate (δ_t) as well as the long-term risk premium factor (rpl_t). High δ_t flattens the slope by raising short-maturity bond yields meanwhile high rpl_t steepens it by raising long-maturity yields.

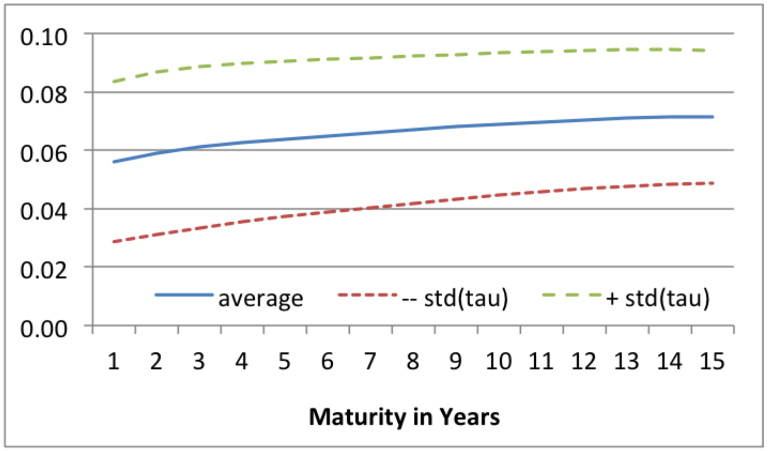
Panel (d) shows that the short-term risk premium factor (rps_t) makes small effects only on short-term bond yields and almost completely hidden from long-term yields. For example, the model-implied one-year yields when rps is +/− one standard deviation are 5.47%—5.75% while the three-year yields are 6.06%—6.14%. The yield spreads by rps become less than 2bps for the maturities of longer than five years. This effect is so small that it is difficult to distilguish rps from measurement errors.

Table 7 shows the contribution of each state variable to the explained variations of principal components (PCs). The contributions are estimated by the Shapley decomposition method. Numbers in any row are summed to one hundred. The estimation is made with 15 T-bond yields (1 year, 2 years, $\cdots$, 15 years) and three T-bill yields (1 month, 3 months and 6 months).

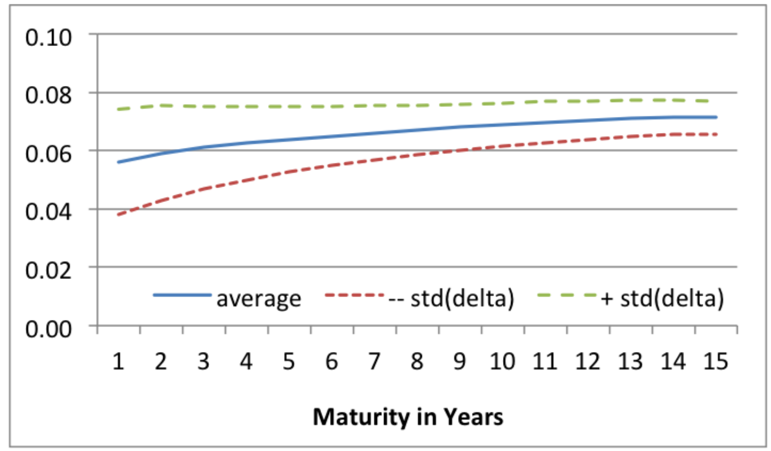
The table shows that the slope factor (PC2)'s variations are largely driven by δ_t and rpl_t, each of which explains 57% and 37%. This result is consistent with the factor loadings of T-bond yields in Figure 5, which shows that the slope becomes steeper when δ_t is low or rpl_t is high. Moreover, it also explains why the slope factor per se is a poor proxy of risk premium. Not only it fails to outperform other benchmark risk premium factors in the predictive regression of bond excess returns, but also it loses all significance

Figure 5: Factor Loadings of Treasury Bond Yields

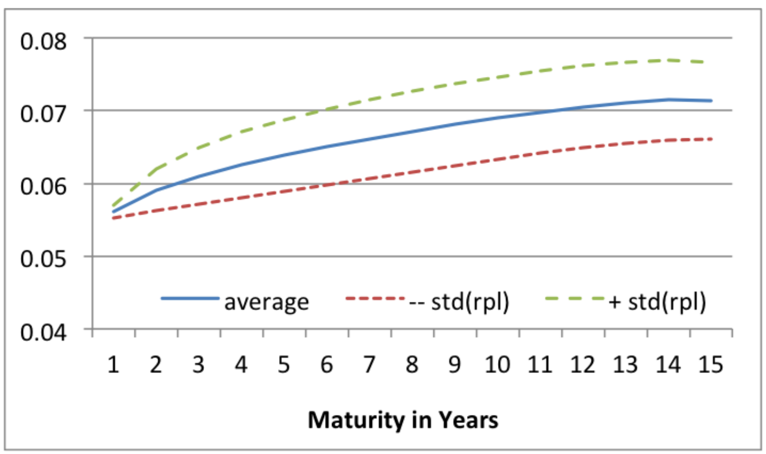
This figure shows how the shapes of Treasury bond yield curves are affected by each state variable. The horizontal axis denotes maturities in years, and the vertical axis denotes annualized bond yields. Each subplot corresponds to the changes of +/− one standard deviations in a given state variable.



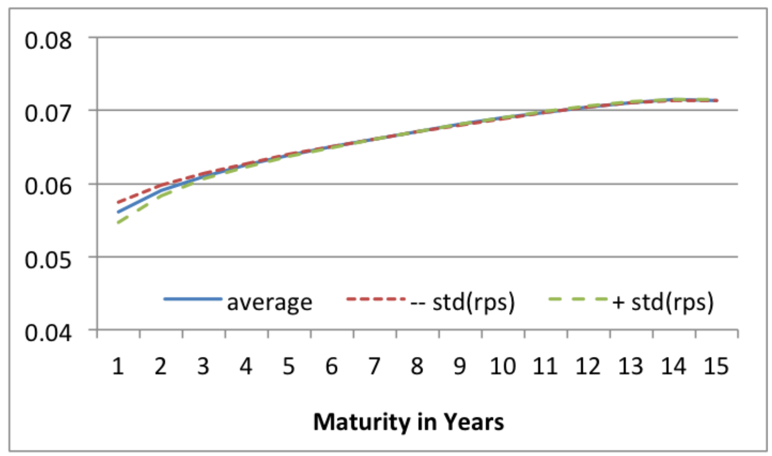
(a) Persistent Inflation (τ_t)



(b) Transitory Short Rates (δ_t)



(c) Long-term Risk Premium Factor (rpl_t)



(d) Short-term Risk Premium Factor (rps_t)

Table 7: Decomposition of Principal Components

This table shows the contributions of the four state variables to the explained variance of the respective principal components. Each state variable's contribution is computed using the Shapley decomposition. Numbers in one row are summed to one hundred. Fifteen Treasury bond yields with the maturities of 1 to 15 years and three Treasury bill yields of 1, 3, and 6-month maturities are used for the estimation of principal components.

	Persistent Inflation (τ)	Transitory Short Rates (δ)	Long-term Risk Premium (rpl)	Short-term Risk Premium (rps)
PC1 (Level)	81.35	16.48	2.16	0.02
PC2 (Slope)	5.13	57.39	36.94	0.53
PC3(Curvature)	0.68	8.48	75.67	15.16
PC4	0.06	0.40	30.87	68.67
PC5	0.46	2.72	0.35	96.46

when monthly excess returns are used as a dependent variable. As the table shows, a large amount of the slope's variations are determined by δ_t, which is not related to risk premium.

The table also shows that higher-order principal components are dominated by the two risk premium factors, rpl_t and rps_t. For example, rpl_t explains 76% of PC3's variations and rps_t does 69% of PC4's and 96% of PC5's variations. This result also explains the literature that higher-order principal components are more informative of risk premium in the bond market than the first three principal components.

5 Combined estimation with GMM

So far, my analysis has largely been made of two separate steps. The first step is to estimate state variables, and the second is to show that the latter two of the variables are risk premium factors. In this section, these two steps are combined into one unified framework by GMM.

Using GMM provides two benefits. First, the GMM estimates are among the most efficient since the method can take into account the covariances among residuals. This feature is important all the more because the T-bond excess returns have strong cross-sectional correlations. For example, the exercise of a simple principal component analysis suggests that around 70% of the variations in excess returns can be explained by the first principal component. Second, over-identifying restrictions can be used as a joint test to show that τ_t and δ_t are not risk premium factors.

Let me briefly recap the state variable estimation with a little simplification. The first state variable, τ_t, is the weighted average of past inflation rates, and the other three state variables are residual bond yields orthogonalised by preceding state variables. The orthogonalisation can be summarised as

$$\delta_t = y_t^{(1y)} - \beta_0^{(\delta)} - \beta_1^{(\delta)}\,\tau_t, \tag{25}$$

$$rpl_t = \overline{y}_t - \beta_0^{(rpl)} - \beta_1^{(rpl)}\,\tau_t - \beta_2^{(rpl)}\,\delta_t, \tag{26}$$

$$rps_t = -y_t^{(3m)} + \beta_0^{(rps)} + \beta_1^{(rps)}\,\tau_t + \beta_2^{(rps)}\,\delta_t + \beta_3^{(rps)}\,rpl_t, \tag{27}$$

where $\overline{y}_t$ denotes the average of 2- to 5-year T-bond yields, $\overline{y}_t = \frac{1}{4}\left(y_t^{(2y)} + y_t^{(3y)} + y_t^{(4y)} + y_t^{(5y)}\right)$. The superscripts of β's are determined by their corresponding dependent variables.

Among these four variables, rpl_t and rps_t are shown to forecast the excess returns from holding T-bonds,

$$exr_{t,t+1}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)}\,rpl_t + \beta_2^{(n)}\,rps_t + \epsilon_{t+1}^{(n)}, \tag{28}$$

for $n = 2$, 3, 5, 7, 10 and 15 years. The superscripts of β's denote the maturities of the excess returns. The sample began in November 1971 since 15-year T-bonds had not been available earlier.

These equations are put into a vector of GMM moments,

$$g_T(\boldsymbol{\beta}) = E_T \begin{bmatrix} \delta_t \otimes \begin{pmatrix} 1 & \tau_t \end{pmatrix}^\top \\ rpl_t \otimes \begin{pmatrix} 1 & \tau_t & \delta_t \end{pmatrix}^\top \\ rps_t \otimes \begin{pmatrix} 1 & \tau_t & \delta_t & rpl_t \end{pmatrix}^\top \\ \boldsymbol{\epsilon}_{t+1} \otimes \begin{pmatrix} 1 & \tau_t & \delta_t & rpl_t & rps_t \end{pmatrix}^\top \end{bmatrix} \in \mathbb{R}^{39\times 1}, \tag{29}$$

where $\boldsymbol{\beta}$ is a vector of 27 parameters,

$$\boldsymbol{\beta} \equiv \Big[\beta_0^{(\delta)} \quad \beta_1^{(\delta)} \quad \beta_0^{(rpl)} \quad \beta_1^{(rpl)} \quad \beta_2^{(rpl)} \quad \beta_0^{(rps)} \quad \beta_1^{(rps)} \quad \beta_2^{(rps)} \quad \beta_3^{(rps)} \\ \beta_0^{(2y)} \quad \beta_1^{(2y)} \quad \beta_2^{(2y)} \quad \cdots \quad \beta_0^{(15y)} \quad \beta_1^{(15y)} \quad \beta_2^{(15y)} \Big]^\top. \tag{30}$$

Note that $g_T(\boldsymbol{\beta})$ has overidentifying restrictions for the orthogonality between $\boldsymbol{\epsilon}_{t+1}$ and τ_t, δ_t. These overidentifying restrictions are to assure that τ_t and δ_t are not risk premium factors.

$\boldsymbol{\beta}$ is estimated by minimising the quadratic form of the sample moments,

$$\hat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \; g_T(\boldsymbol{\beta})^\top W \, g_T(\boldsymbol{\beta}), \tag{31}$$

where the weighting matrix W is an identity matrix ($W = I$) for the first stage and an inversed spectral density matrix ($W = S^{-1}$) in the second stage. The spectral density matrix is estimated as

$$\hat{S} \equiv \sum_{j=-k}^{k} \left(\frac{k - |j|}{k} \right) E_T \left(\hat{u}_t \, \hat{u}_{t+j}^\top \right), \tag{32}$$

where $\hat{u}_t$ denotes the GMM moment residuals. As noted in equation (32), the higher-order correlations are down-weighted as suggested by Newey and West (1987). I choose $k = 2$

since a measurement error may create mechanical autocorrelations between adjacent time periods.

The first-order condition of the minimisation is

$$d^\top W\, g_T(\boldsymbol{\beta}) = 0 \quad \text{where} \quad d \equiv \frac{\partial g_T(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^\top}. \tag{33}$$

The derivations of the matrix d are straightforward but omitted due to the lack of space. They are available upon request. Numerical methods are used to find the optimal $\boldsymbol{\beta}$.

Table 8 shows the results of the GMM estimation. As explained earlier, the GMM

Table 8: GMM Estimates

Panel A shows the correlations of the two estimated time series of state variables, one from the original multi-step orthogonalisation method and the other from the GMM. Panel B shows the GMM estimates of the following forecast regression,

$$exr_{t,t+1}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)}\, rpl_t + \beta_2^{(n)}\, rps_t + \epsilon_{t+1}^{(n)},$$

for n = 2, 3, 5, 7, 10 and 15 years. Panel C shows the results of the J_T test for the over-identifying restrictions.

Panel A. Correlation of state variables from the multi-step orthogonalisation method and the GMM estimates

τ_t	δ_t	rpl_t	rps_t
1.0000	0.9999	0.9999	0.9877

Panel B. Forecast of excess returns in the next month

dep.var.	$exr_{t,t+1}^{(2Y)}$	$exr_{t,t+1}^{(3Y)}$	$exr_{t,t+1}^{(5Y)}$	$exr_{t,t+1}^{(7Y)}$	$exr_{t,t+1}^{(10Y)}$	$exr_{t,t+1}^{(15Y)}$
rpl_t	0.302***	0.475***	0.772***	1.021***	1.373***	1.992***
	(3.021)	(3.496)	(4.023)	(4.176)	(4.186)	(4.157)
rps_t	0.430***	0.579***	0.850***	1.154***	1.686***	2.640***
	(3.597)	(3.527)	(3.534)	(3.710)	(4.062)	(4.345)

Panel C. Over-identifying restriction test

$TJ_T = 20.17$, p–value = 0.064

combines the state variable estimation and the risk premium forecast, each of which is covered by the corresponding panel of the table.

Panel A shows the correlations of the two estimated time series of state variables, one from the original multi-step orthogonalisation method and the other from the GMM. The correlation of τ_t is equal to one since τ_t is directly observed from the past inflation and thus its value is not affected by an estimation method. The variables of interest are the latter three: δ_t, rpl_t and rps_t. According to the panel, the correlations are virtually 100% for δ_t and rpl_t, and 99% for rps_t. Thus, this panel suggests that the multi-step orthogonalisation method and the GMM produce nearly identical state variables.

Panel B shows the results of the risk premium forecast regression. The panel is designed to be matched with the results in Table 2. As shown by the comparison of the two tables, the estimates of the forecast coefficients ($\beta^{(n)}$'s) from GMM are close to those from the OLS regressions, and their differences are less than one standard error. Therefore, one can conclude that the OLS-based multi-step method produces sufficiently close approximates to GMM.

Lastly, Panel C shows the results of the J_T test of the over-identifying restrictions. The test statistic is computed as

$$T\,J_T = T\,g_T(\boldsymbol{\beta})^\top\,S^{-1}\,g_T(\boldsymbol{\beta}) \to \chi^2_{12}, \tag{34}$$

since $g_T(\boldsymbol{\beta})$ has 39 moments and $\boldsymbol{\beta}$ has 27 parameters. According to the table, the over-identifying restrictions are rejected with the p-value of 0.064, implying that the forecast residuals, $\epsilon^{(n)}_{t+1}$'s, are not jointly orthogonal to τ_t and δ_t, although they appear to be orthogonal individually. This weak rejection can be attributed to the effects of risk premium factors on 1-year yields. In the GMM setup, 1-year yields are assumed to be a linear function of τ_t and δ_t but not of rpl_t and rps_t. However, as shown by the yield loadings in Figure 5, 1-year yields are slightly affected by rpl_t and rps_t up to a few basis points. These small effects add up to the rejection of the joint null hypothesis.

6 Relation to the financial market

6.1 Liquidity in the bond market

T-bills are special. Their values rise during a financial crisis since they are considered the safest collaterals in the world. For example, the yield on the three-month T-bill even turned negative on December 9, 2008, three months after the Lehman Brothers' collapse, as investors had sought for a safe haven.[14] One-month T-bill yields briefly went negative again on August 4, 2011, since the European woes had cast a shadow over the market.[15]

Even when compared to other types of credit-riskfree assets such as other government securities, T-bills still look different. For example, Duffee (1996) describes that the market segmentation between T-bills and T-bonds has increased since the early 1980s. Krishnamurthy (2010) also documents that the repo haircut rate of short-term Treasuries had been fixated at 2% in 2007–2009 meanwhile those of long-term Treasuries and investment-grade corporate bonds soared from 5% to 6% and 5% to 20%, respectively.

Considering the role of T-bills as the safest haven and how its price is affected by rps, we can expect rps to be related to liquidity premium. Motivated by the intuition, this section compares rps to various liquidity measures in the bond market.

Fontaine and Garcia (2012) estimate bond liquidity premium as the difference of yields between on-the-run (most recently issued) and off-the-run (seasoned) T-bonds.[16] Since on-the-run bonds are more liquid and thus have higher values as collaterals than

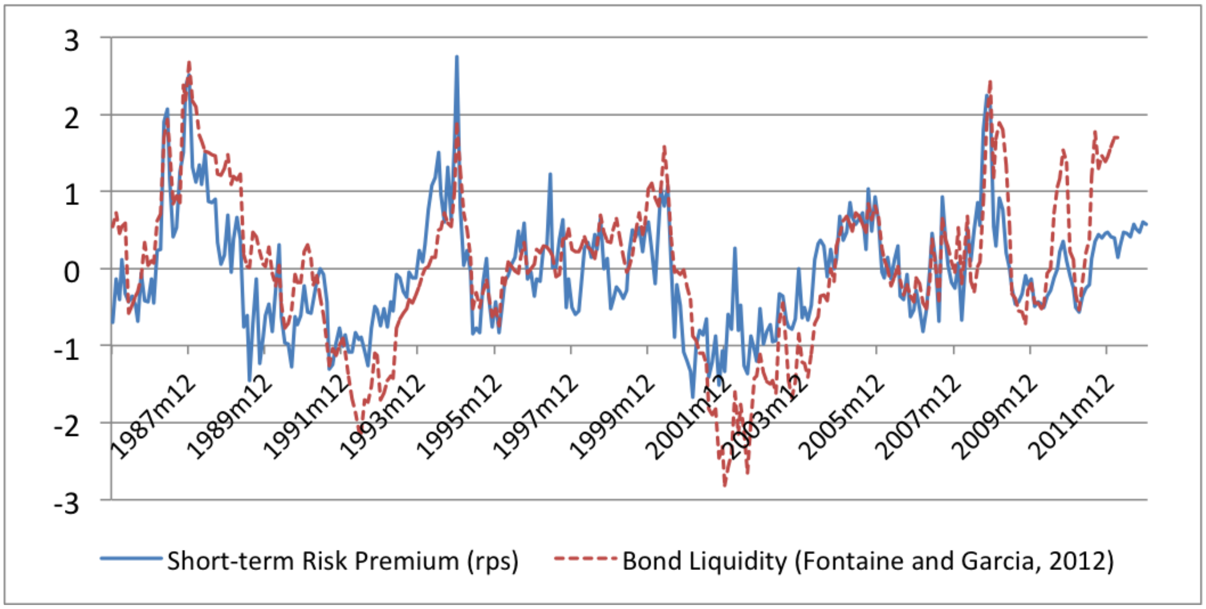
[14] http://blogs.wsj.com/marketbeat/2008/12/09/three-month-bill-yield-goes-negative/

[15] http://blogs.wsj.com/marketbeat/2011/08/04/from-one-crisis-to-another-one-month-t-bill-yields-go-n

[16] I am grateful to Jean-Sebastien Fontaine and Rene Garcia for generously sharing the data.

Figure 6: Fontaine and Garcia (2012)'s Bond Liquidity Premium

Fontaine and Garcia (2012) estimate bond liquidity premium using the difference of yields between on-the-run (most recently issued) and off-the-run (seasoned) Treasury bonds. This figure compares its time series to the short-term risk premium factor (*rps*).



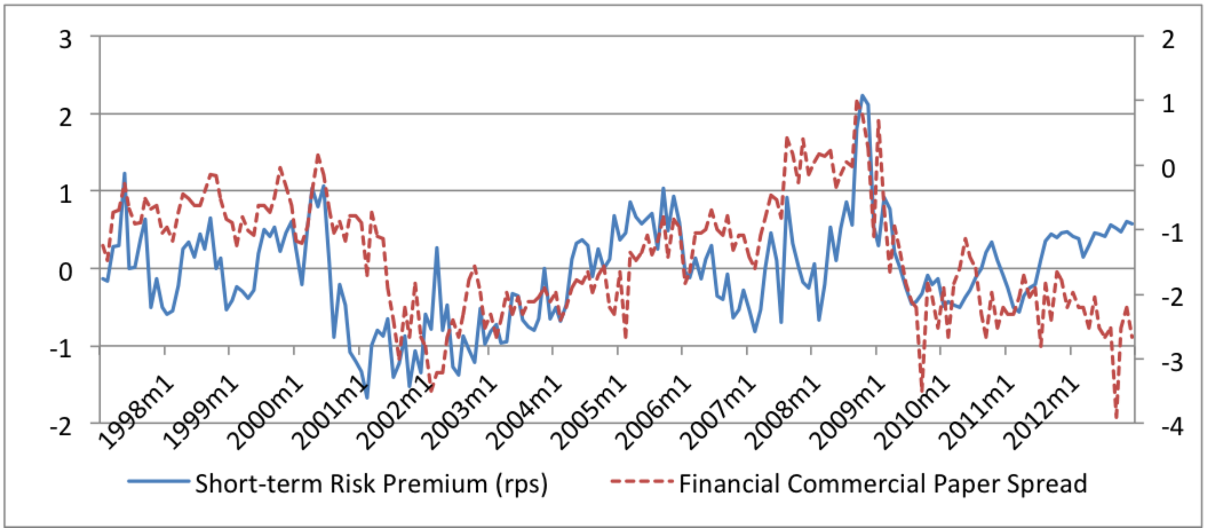
off-the-run bonds, the yield differential of on-the-run bonds over off-the-run ones is likely to increase when there are strong demands for liquidity. Figure 6 compares the time series of the bond liquidity premium to *rps*.

The figure shows that the two time series are remarkably overlapped with each other. They both peaked in 2008 after the Lehman Brothers collapsed and in 1987 after the infamous stock market crash on the Black Monday. They also soared during the Tequila crisis in 1994 and remained high throughout the Asian currency crisis in 1997 and the Russian moratorium and the subsequent demise of the Long Term Capital Management (LTCM) in 1998. The unconditional correlation between the two series is 0.768.

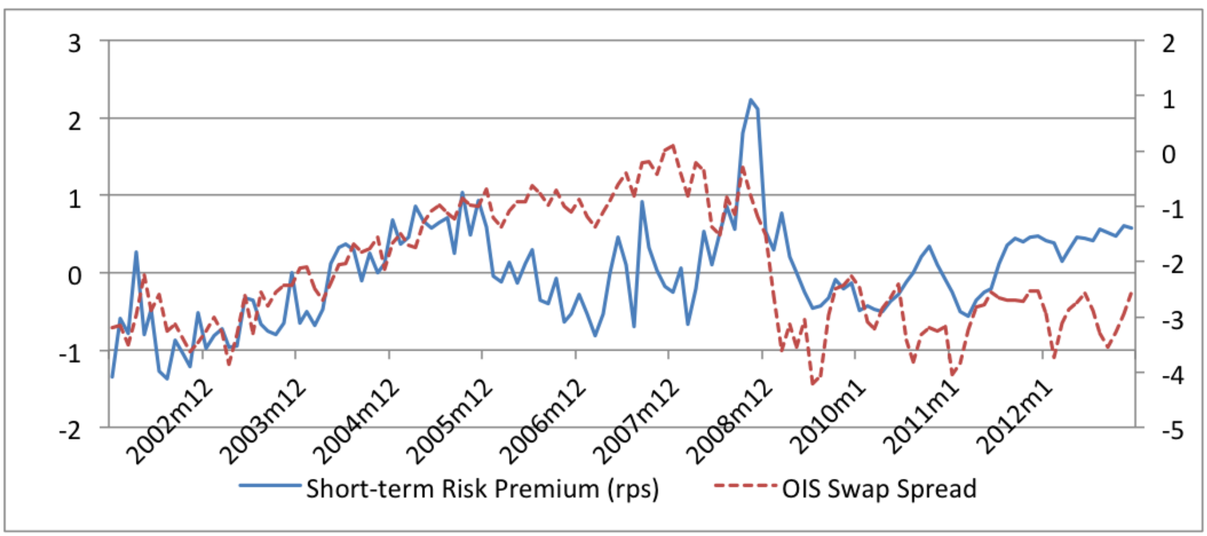
Figure 7 also compares *rps* to two other measures of liquidity in the bond market. Panel (a) uses the 3-month AA financial commercial paper spreads over 3-month T-bill yields, and Panel (b) is based on the 3-month overnight index swap (OIS) spreads. Both

Figure 7: Other Liquidity Measures in the Bond Market

This figure compares the short-term risk premium factor (*rps*) to two other liquidity measures in the bond market. Panel (a) uses 3-month AA financial commercial paper spread, and Panel (b) uses 3-month overnight index swap (OIS) spread. Both figures display the log values of those spreads. The scale of *rps* is on the left axis and the log spreads are on the right axis.



(a) 3-Month AA Financial Commercial Paper Spread



(b) 3-Month Overnight Index Swap (OIS) Spread panels display the log values of the spreads. The commercial paper spreads are downloaded from the FRED Economic Data,[17] and the OIS spreads are from the Bloomberg. Note that the spreads are a close but not perfect liquidity measure since the spreads are determined not only by convenience yields (Grinblatt, 2001) but also by counterparty risk (Duffie and Singleton, 1997).

Again, the figure shows substantial comovement of rps with the bond market spreads. They all had gradually increased since 2002, reached to the peak during the financial crisis, and dropped rapidly and stabilised thereafter. The correlations between rps and the two log spreads are 0.402 and 0.365 respectively.

6.2 Risk factors in the stock market

Risk premium in the bond market has been discussed so far. Naturally it raises a question whether it is also related to the stock market. The no arbitrage model imposes that all risk premium should have come from the covariance between asset returns and innovation shocks to a pricing kernel. If an universal pricing kernel is able to price both bond and stock markets, risk premium factors in the bond market might have been related to the risk premium in the stock market.

This section is motivated by Koijen, Lustig, and Van Nieuwerburgh (2013), who show that "innovations to the nominal bond risk premium price the book-to-market sorted stock portfolios." They find that the joint portfolios of stocks and bonds can be priced by three state variables: the level factor of yield curves, stock market returns, and Cochrane and Piazzesi (2005)'s tent-shaped risk premium factor. Interestingly, growth to value stock portfolio returns are found to have monotonically increasing loadings on the tent-shaped factor, implying that value and bond risk premium might have the same roots.

[17] `http://research.stlouisfed.org/fred2/categories/120`

Let $exr_{t+1}^{(j)}$ denote the excess returns from holding a risky asset j. Under the no arbitrage assumption, all risk premium should have come from the covariance of asset returns and the pricing kernel,

$$E_t\left[exr_{t+1}^{(j)}\right] = -\operatorname{cov}_t\left(exr_{t+1}^{(j)}, m_{t+1}\right) = \Sigma_j\,\Lambda_t, \tag{35}$$

where Σ_j is the covariance between the returns and the innovation shocks to state variables. By taking unconditional expectations on both sides of equation (35),

$$E\left[exr_{t+1}^{(j)}\right] = \Sigma_j\,\bar{\Lambda}. \tag{36}$$

For example, suppose the covariance matrix, Σ_j, is estimated for each of value portfolios. If the value premium were related to the i-th state variable, Σ_{ji} would have shown a monotonically increasing (or decreasing) pattern over j.

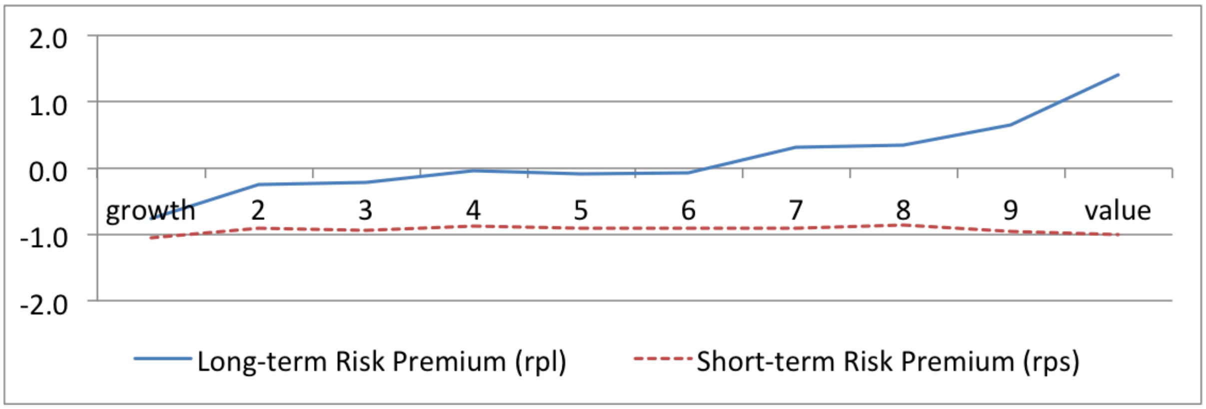
Figure 8 shows the coefficients of regressing the excess returns of value, size, and momentum-sorted decile stock portfolios on the contemporary innovation shocks to the two risk premium factors, $\widetilde{rpl}$ and $\widetilde{rps}$. The innovation shocks are measured as the residuals of estimating *rpl* and *rps*' time series as two independent AR(1) processes. This figure is to illustrate the estimated covariance matrix, Σ_j. Stock portfolio return data are downloaded from Kenneth French's website.[18]

Being consistent with Koijen, Lustig, and Van Nieuwerburgh (2013), the figure shows that the long-term risk premium factor (*rpl*) is related to all three cross-sectional stock risk premiums. $\widetilde{rpl}$'s coefficients show monotonic patterns in all panels, implying that *rpl* is positively related to value and size premiums and negatively to momentum. In contrast, $\widetilde{rps}$' coefficients seem to be flat, but they are all significantly different from zero. Thus, *rps* can be considered closely related to stock market returns.

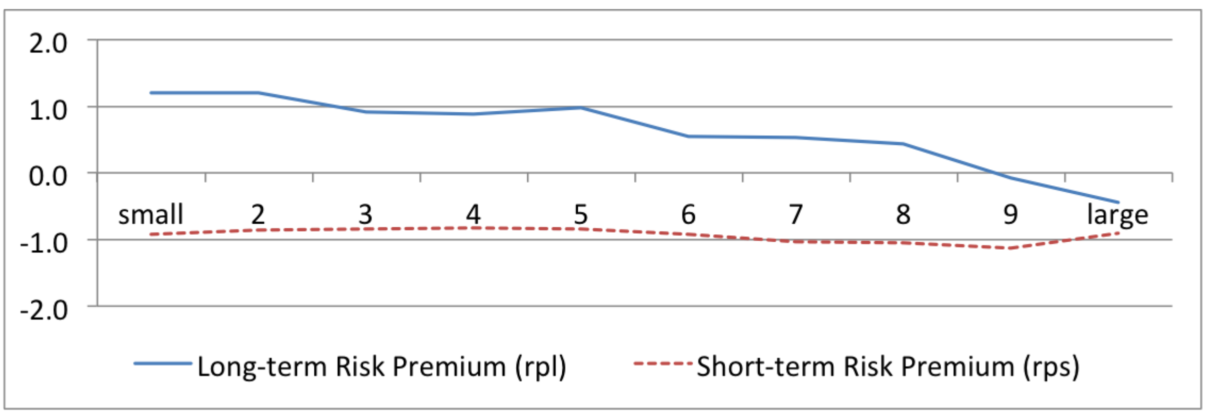
[18] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

Figure 8: Covariance with Stock Portfolio Returns

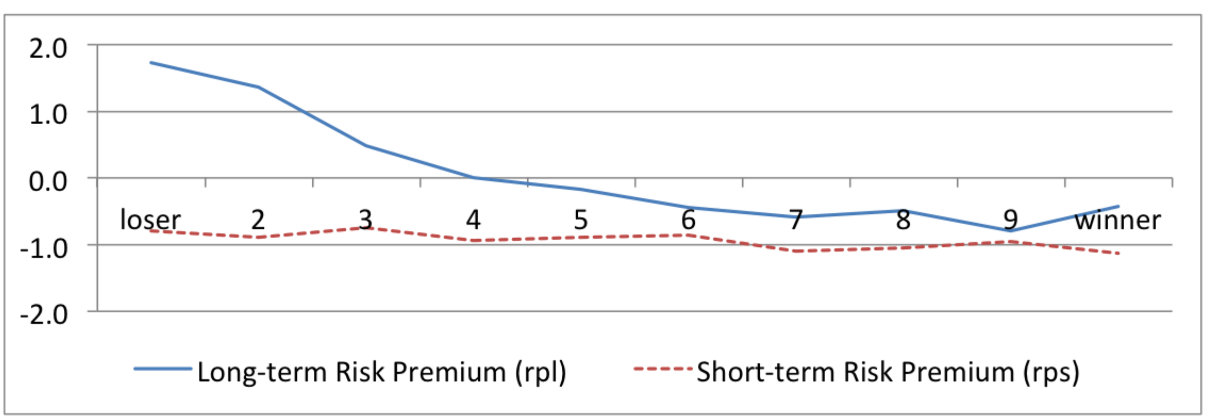
This figure shows the coefficients of regressing stock portfolio excess returns on the contemporary innovation shocks to two risk premium factors, *rpl* and *rps*. Portfolio 1 on the horizontal axis denotes growth / small / loser stocks meanwhile Portfolio 10 denotes value / large / winner stocks. Portfolio returns are downloaded from Kenneth French's website.



(a) Value Portfolios



(b) Size Portfolios



(c) Momentum Portfolios

Table 9: Covariance with Cross-sectional Stock Return Factors

The dependent variables are the Fama-French three factors and the momentum factor, which are denoted on the first row. The explanatory variables are contemporary innovation shocks to the two risk premium factors. Numbers in parentheses are Huber–White t statistics. ***, **, and * denote significances at 1%, 5%, and 10% level, respectively.

	MktRf	SMB	HML	MOM
$\widetilde{rpl}_t$	-0.135	1.089***	0.732**	-1.676***
	(-0.242)	(3.129)	(2.282)	(-2.991)
$\widetilde{rps}_t$	-0.957***	0.057	0.014	-0.293
	(-2.619)	(0.319)	(0.066)	(-1.085)
obs	551	551	551	551
R^2	0.023	0.022	0.011	0.028

As a robustness check, Table 9 directly regresses the Fama-French three factors and the momentum factor on the contemporary innovation shocks to state variables. This table confirms the previous finding that *rps* is related to aggregate stock market returns meanwhile *rpl* is to value, size, and momentum premiums.

7 Conclusion

This paper finds that bond risk premium consist of two factors: one for a long term and the other for a short term. The long-term factor raises the slope of yield curve. In contrast, the short-term factor affects T-bill yields but is almost completely hidden from T-bonds. The long-term factor predicts monthly excess returns over the horizon of even longer than one year meanwhile the short-term factor loses its predictability completely in one quarter. The estimation of the short-term factor is likely to elude an affine term structure model if the model is estimated by the optimisation of cross-sectional fits. The results are robust to the GMM estimation. The long-term factor is found to be related to value, size, and momentum premiums in the stock market whereas the short-term factor is to bond liquidity premium and aggregate stock market returns.

References

Atkeson, Andrew, and Patrick J. Kehoe, 2009, On the Need for a New Approach to Analyzing Monetary Policy, *NBER Macroeconomics Annual*.

Boudoukh, Jacob, Matthew Richardson, and Robert F. Whitelaw, 2007, The Myth of Long-Horizon Predictability, *Review of Financial Studies* 21, 1577–1605.

Campbell, John Y., and Robert J. Shiller, 1991, Yield Spreads and Interest Rate Movements: A Bird's Eye View, *The Review of Economic Studies* 58, 495–514.

Campbell, John Y., Adi Sunderam, and Luis M. Viceira, 2012, Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds, *working paper*.

Cieslak, Anna, and Pavol Povala, 2015, Expected returns in Treasury bonds, *working paper*.

Cochrane, John H., and Monika Piazzesi, 2005, Bond Risk Premia, *The American Economic Review* 95, 138–160.

——— , 2008, Decomposing the Yield Curve, *working paper*.

Cox, John C., Jonathan E. Jr. Ingersoll, and Stephen A. Ross, 1985, A Theory of the Term Structure of Interest Rates, *Econometrica* 53, 385–407.

Dai, Qiang, and Kenneth J. Singleton, 2002, Expectation puzzles, time-varying risk premia, and affine models of the term structure, *Journal of Financial Economics* 63, 415–441.

——— , and Wei Yang, 2004, Predictability of Bond Risk Premia and Affine Term Structure Models, *working paper*.

Duffee, Gregory R., 1996, Idiosyncratic Variation of Treasury Bill Yields, *The Journal of Finance* 51, 527–551.

——— , 2002, Term Premia and Interest Rate Forecasts in Affine Models, *The Journal of Finance* 57, 405–443.

——— , 2010, Sharpe ratios in term structure models, *working paper*.

——— , 2011, Information in (and not in) the Term Structure, *Review of Financial Studies* 24, 2895–2934.

Duffie, Darrell, and Kenneth J. Singleton, 1997, An Econometric Model of the Term Structure of Interest-Rate Swap Yields, *The Journal of Finance* 52, 1287–1321.

Fama, Eugene F., 2006, The Behavior of Interest Rates, *Review of Financial Studies* 19, 359–379.

——— , and Robert R. Bliss, 1987, The Information in Long-Maturity Forward Rates, *The American Economic Review* 77, 680–692.

Fontaine, Jean-Sébastien, and René Garcia, 2012, Bond Liquidity Premia, *Review of Financial Studies* 25, 1207–1254.

Goodfriend, Marvin, and Robert G. King, 2009, The Great Inflation Drift, *NBER Working Paper*.

Grinblatt, Mark, 2001, An Analytic Solution for Interest Rate Swap Spreads, *International Review of Finance* 2, 113–149.

Gürkaynak, Refet S., Brian Sack, and Eric Swanson, 2005, The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models, *American Economic Review* 95, 425–436.

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2006, The U.S. Treasury Yield Curve: 1961 to the Present, *Federal Reserve Board*.

Joslin, Scott, Kenneth J. Singleton, and Haoxiang Zhu, 2011, A New Perspective on Gaussian Dynamic Term Structure Models, *Review of Financial Studies* 24, 926–970.

Koijen, Ralph S.J., Hanno Lustig, and Stijn Van Nieuwerburgh, 2013, The Cross-Section and Time-Series of Stock and Bond Returns, *working paper*.

Kozicki, Sharon, and P.A. Tinsley, 2001, Shifting endpoints in the term structure of interest rates, *Journal of Monetary Economics* 47, 613–652.

Krishnamurthy, Arvind, 2010, How Debt Markets Have Malfunctioned in the Crisis, *Journal of Economic Perspectives* 24, 3–28.

Ludvigson, Sydney C., and Serena Ng, 2009, Macro Factors in Bond Risk Premia, *Review of Financial Studies* 22, 5027–5067.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703–708.

Pearson, Neil D., and Tong-Sheng Sun, 1994, Exploiting the Conditional Density in Estimating the Term Structure: An Application to the Cox, Ingersoll, and Ross Model, *The Journal of Finance* 49, 1279–1304.

Piazzesi, Monika, 2003, Affine Term Structure Models, in *Handbook of Financial Econometrics* (Elsevier).

Rudebusch, Glenn D., and Tao Wu, 2008, A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy, *The Economic Journal* 118, 906–926.