Highlights

- Incorporating uncertainty about risk aids better performance of the volatility forecasting models.
- Uncertainty about risk increases the influence of implied volatility on future volatility.
- Considering uncertainty about risk improves the forecasting performance in both in-sample and out-of-sample analyses.

Effect of uncertainty on volatility forecasting power of option-implied volatility

Abstract

This paper investigates the forecasting performance of new volatility forecasting models exploiting uncertainty about risk. We propose a new class of volatility forecasting models that allow the parameter of implied volatility to vary with uncertainty about risk. By implementing the new models for the S&P 500 index, we find that uncertainty about risk enhances the relation between implied volatility and future volatility. Our findings also suggest that incorporating uncertainty about risk into the volatility forecasting models improves the forecasting powers among both in- and out-of-sample analyses. Furthermore, this effect of uncertainty becomes stronger during financial crisis.

JEL Classification: C13, C22, C53, C58

Keywords: Uncertainty, Realized volatility, Volatility forecasting

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1. Introduction

Modeling volatility process and forecasting volatility are crucial in economic and financial decision making processes, such as asset pricing and risk management. Hence, many researchers have introduced and improved their own versions of the volatility forecasting model. Previous literature on volatility forecasting suggests that the option-implied information is useful for volatility forecasting (Jiang and Tian, 2005; Busch, Christensen, and Nielsen, 2011; Byun and Kim, 2013), because the informed traders first trade in the options market to exploit the financial leverage (Black, 1975).[1] Furthermore, literature has recently provided various empirical evidences of the time-varying relation between future volatility and the option-implied and historical information. Bollerslev, Patton, and Quaedvlieg (2016) and Li, Tsionas, and Izzeldin (2016) find that the volatility forecasting power of past realized volatility varies with the degree of measurement errors and the fluctuation of daily realized volatility compared to the monthly realized volatility, respectively. Seo and Kim (2015) show that the relation between option-implied information and future volatility varies with the level of investor sentiment. Along with this strand of research, we suggest a new class of volatility forecasting models that the forecasting power of option-implied volatility is set to vary with uncertainty.

A strand of the prior literature (Knights, 1921; Keynes, 1937; Ellsberg, 1961) explains that uncertainty is defined as a situation with unknown outcomes and the unknown distribution of the investment, while risk is defined as an environment in which the investors have unknown outcomes of the investment under the known distribution. Thus, a high level of uncertainty about risk indicates that the environment of the investment is unstable and that the future market conditions are likely to be very different from past or present market conditions.

We expect uncertainty about risk extracted from the options market to have a significant effect on the relation between implied volatility and future volatility for the following two reasons: First, a high level of uncertainty about risk leads to a large amount of uninformed trading in the stock market. Since the

[1] The literature on the information of options market for future stock price movements includes Easley, O'Hara, and Srinivas (1998), Pan and Poteshman (2006), Bali and Hovakimian (2009), Cremers and Weinbaum (2010), Doran and Krieger (2010), Xing, Zhang, and Zhao (2010), and An, Ang, Bali, and Cakici (2014).

distribution of the stock market investors' investment returns is more turbulent with higher uncertainty, the investors in the stock market cannot exploit the historical information of the stock market to predict their investment returns. While uncertainty influences on the option market investors, the effect of uncertainty on the stock market investors is larger than that on the option market investors, because the option market investors are relatively more informative than the stock market investors. Therefore, there are more uninformed noise trading in the stock market with higher uncertainty. According to An, Ang, Bail and Cakici (2014), the uninformed noise trading in the stock market strengthens the return predictive power of option-implied information. Similarly, when the level of uncertainty is high, implied volatility is strongly related to future volatility. Second, a high uncertainty about risk changes information content of implied volatility. Prior study by Antonakakis, Chatziantoniou, and Filis (2013) and Bekaert, Hoerova, and Duca (2013) find that implied volatility varies with market uncertainty and that the VIX can be decomposed into two components, namely risk aversion and uncertainty. In this regard, the information contained on the implied volatility can be decomposed into the information on investors' expectations of future volatility (e.g., risk) and disagreement in investors' opinions on future volatility (e.g., uncertainty). Since a high level of uncertainty about risk indicates that the investors' opinions of risk diverge (Diether, Malloy, and Scherbina, 2002; Park, 2005; Andersen, Ghysels, and Juergens, 2009; Beber, Breedon, and Buraschi, 2010; Yu, 2011; Buraschi, Trojani, and Vedolin, 2013), the composition of information content from implied volatility changes with higher uncertainty. Thus, we hypothesize that the predictive power of implied volatility on future volatility depends significantly on uncertainty about risk.

In our empirical investigation, we find that the role of option-implied volatility depends significantly on the market uncertainty. With the in-sample regression analysis, we find that the coefficient on an interaction term between implied volatility and uncertainty measure is significantly positive. In other words, when forecasting volatility, the load on the implied volatility increases as the market becomes more uncertain. The result is robust after accounting for other variables that are known to interact with implied volatility. In addition, we find that the relation between uncertainty and forecasting power of implied volatility is magnified during periods of heightened uncertainty, such as financial crisis.

Furthermore, we find that forecasting power is significantly improved for both in- and out-of-sample analyses when the uncertainty is accounted for. By introducing uncertainty about risk to time-varying parameters, we find that in the in-sample analysis, volatility forecasting models improve their explanatory power for future daily volatility by roughly 1.5%, regardless of which benchmark model we choose. We also document significant improvement of daily volatility forecasts in the out-of-sample analysis using various loss functions and statistical tests.

To the best of our knowledge, this is the first research to study the effect of uncertainty on volatility forecasting. We find that in the volatility forecasting model, the weight on the implied volatility depends significantly on the level of uncertainty about risk. Furthermore, our new models with considering uncertainty about risk outperform the benchmark models in the in- and out-of-sample analyses. Therefore, our findings shed light on improving the volatility forecasting models. Additionally, our research contributes to the increasing literature on uncertainty.[2] Prior research including Drechsler and Yaron (2011), Drechsler (2013), Buraschi, Trojani, and Vedolin (2014), and Konstantinidi and Skiadopoulos (2016) documents the significant role of uncertainty in variance risk premium in particular. Along with this strand of research, we fill a gap in the literature on volatility forecasting with uncertainty by showing that uncertainty about risk has an important impact on the volatility forecasting power of the information on options market.

The remainder of this paper is organized as follows. Section 2 introduces variables and time-varying volatility forecasting models. Section 3 presents the data and examines empirical results. Section 4 describes the application to constant parameter volatility forecasting models and sub-sample analysis in the financial crisis to check the robustness of our empirical results. Lastly, Section 5 concludes the paper.

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[2] The literature including Diether, Malloy, and Scherbina (2002), Park (2005), Buraschi and Jiltsov (2006), Andersen, Ghysels, and Juergens (2009), Ozoguz (2009), Beber, Breedon, and Buraschi (2010), Yu (2011), Barinov (2013), Buraschi, Trojani, and Vedolin (2013), Kouwenberg, Markiewicz, Verhoeks, and Zwinkels (2013), and Baltussen, Van Bekkum, and Van der Grient (2014) investigates the relation between uncertainty and asset pricing in the financial markets, such as the stock market, the options market, the currency market, and the bond market.

2. Variables and volatility forecasting models

There are three main variables in our empirical research: realized volatility, implied volatility, and uncertainty about risk. We use realized variance (RV) and the square of VIX (IV) as our measure of realized volatility and implied volatility, respectively. Our measure of uncertainty about risk is vol-of-vol (VoV) introduced by Baltussen, Van Bekkum, and Van der Grient (2014).

2.1. Realized variance

As our realized volatility measure, we employ realized variance (RV), which is a consistent estimator of the true latent volatility. Let us consider a logarithmic asset price at time t, P_t, for which the return process is determined by a standard jump diffusion process:

$$dP_t = \mu_t dt + \sigma_t dW_t + \kappa_t dq_t, \tag{1}$$

where μ_t and σ_t are the drift with a continuous and locally finite variation sample path and the instantaneous volatility process, respectively. W_t is a standard Brownian motion, κ_t is the jump size, and a pure jump process q_t with time-varying intensity is normalized so that $dq_t = 1$ whenever a jump occurs at time t, and $dq_t = 0$ otherwise.

True latent volatility, or the quadratic return variation (QV) over daily and longer horizons can be written as the sum of the Integrated Variance (ITV) and the cumulative squared jumps over daily and longer horizons. For simplicity and ease of notation, we normalize the unit time interval to a day; the QV over a day is defined as,

$$QV_t = \int_{t-1}^{t} \sigma_s^2 ds + \sum_{t-1<s\leq t} (\kappa_s)^2 = ITV_t + \sum_{t-1<s\leq t} (\kappa_s)^2, \tag{2}$$

where κ_t is non-zero only if there is a jump at time t.

Previous literature (Andersen, Bollerslev, Diebold, and Ebens, 2001; Andersen, Bollerslev, Diebold, and Labys, 2001; Barndorff-Nielsen and Shephard, 2002a, 2002b) shows that the summation of high-frequency returns (e.g., intraday returns) is a consistent estimator of the QV. On day t, the realized variance (RV), the consistent estimator of the QV, is defined by

$$RV_t \equiv \sum_{i=1}^{M} r_{t,i}^2, \tag{3}$$

where M = 1/Δ, and Δ-period intraday return is defined by $r_{t,i} \equiv (P_{t-1+i\times\Delta} - P_{t-1+(i-1)\times\Delta})$. If $\Delta \rightarrow 0$, $RV_t \rightarrow_P QV_t$. For the multi(h)-period, the realized variances are denoted as

$$RV_{t-1,t-h} = (RV_{t-1}+RV_{t-2}+\ldots+RV_{t-h})/h, \tag{4}$$

where h = 5 (weekly) and h = 22(monthly).

2.2. Uncertainty measurement

We employ the uncertainty measure suggested by Baltussen, Van Bekkum, and Van der Grient (2014). To derive the uncertainty measure, Baltussen, Van Bekkum, and Van der Grient (2014) utilize the modeling of the preference with risk and uncertainty in Klibanoff, Marinacci, and Mukerji (2005).[3] Klibanoff, Marinacci, and Mukerji (2005) suggest that the preference V is represented by a function of the double expectational form,

$$V(f) = \int_P \varphi\left(\int_S u(f)d\pi\right)d\psi, \tag{5}$$

where f is a real-valued function defined on a state space S, called "an act", u is a von Neumann-Morgenstern utility function, which is affected by risk, and π is a probability measure on S; φ is the function of the attitude toward uncertainty, and P is the set of possible probabilities, π_is, over S.

Assume that investors want to maximize the expected utility of the mean-variance utility function, they face uncertainty about two parameters; the mean and variance of the returns from their investments. For the case of the investment in the stock market, f is the return from the investment. If there are K possible probability distributions over S, the probability set of possible outcomes is represented by

[3] Examples for uncertain asset, risky asset (not uncertain), and safe asset are the investments in the stock market, lottery, and the government bond market, respectively.

$$P = \{\Pi_1 = (\mu_{S,1}, \sigma_{S,1}), \cdots, \Pi_K = (\mu_{S,K}, \sigma_{S,K})\}, \tag{6}$$

where Π_i is the true probability distribution for future stock market returns with the mean $\mu_{S,i}$ and variance $\sigma^2_{S,i}$. $\Psi = (\psi_1, \ldots, \psi_K)$ represents the investor's assessment of P in that ψ_i is the subjective probability for Π_i. As K tends to infinity, the subjective beliefs about Π_i ($i = 1, \ldots, K$) with finite mean, finite variance, and independence of distribution will be distributed normally by the central limit theorem. Hence, the probability measure Ψ can be represented by a normal distribution over P:

$$P \sim N(\mu_P(\mu_S), \mu_P(\sigma_S), \sigma_P(\mu_S), \sigma_P(\sigma_S)), \tag{7}$$

where $\mu_P(\mu_S)$ ($\sigma_P(\mu_S)$) is the mean (standard deviation) of the distributions of the expected stock market returns from subjective beliefs, and $\mu_P(\sigma_S)$ ($\sigma_P(\sigma_S)$) is the mean (standard deviation) of the distributions of the expected stock market return volatilities from subjective beliefs. Among the four components, Baltussen, Van Bekkum, and Van der Grient (2014) focus on $\sigma_P(\sigma_S)$, which captures uncertainty about risk, and suggest the VoV as a measure of uncertainty about risk.[4] VoV is calculated on day t as follows:

$$VoV_t = \frac{\sqrt{\frac{1}{21}\sum_{i=t-21}^{t}\left(\sigma_i - \bar{\sigma}_t^{IV}\right)^2}}{\bar{\sigma}_t^{IV}}, \tag{8}$$

where $\bar{\sigma}_t^{IV} = \frac{1}{22}\sum_{i=t-21}^{t}\sigma_i$, and σ_i is the implied volatility extracted from options price (i.e., VIX) at day i.[5]

2.3. Time-varying parameter volatility forecasting models

Motivated from the consistency of the RV, Corsi (2009) proposes the heterogeneous autoregressive

[4] Park (2015) utilizes VoV to predict the return on the tail risk hedging options, such as S&P 500 puts and VIX calls and provides empirical evidence supporting uncertainty premiums of a time-varying uncertain belief in volatility. Konstantinidi and Skiadopoulos (2016) employ VoV to explain the variation of the market variance risk premium and find a positive relation between VoV and market variance risk premium.

[5] Baltussen, Van Bekkum, and Van der Grient (2014) use the implied volatilities over the past 20-days (i.e., past month) to calculate the VoV. In this paper, one month corresponds to 22-trading days. Thus, we utilize the implied volatilities over the past 22-trading days for construction of the VoV. The results with the VoV in Baltussen, Van Bekkum, and Van der Grient (2014) are similar to the results in this paper and are available upon request.

realized volatility (HAR) model.[6] Motivated by the ease of implementation, much of the literature suggests the extension of the HAR model.[7]

Recently, the literature on volatility forecasting suggest the HAR model with time-varying parameter depending on conditions, such as the variation of the measurement error and the discrepancy between recent short-term volatility and long-term volatility. Bollerslev, Patton, and Quaedvlieg (2016) suggest the heterogeneous autoregressive realized volatility and quarticity (HARQ) model to consider the variation of the measurement error of RV for ITV in volatility forecasting model. Bollerslev, Patton, and Quaedvlieg (2016) show that as the volatility of the measurement error increases, the magnitude of measurement error of RV for ITV also increases. Thus, the information of the previous day's RV is less important for future volatility forecasting. Additionally, we set the benchmark model, which incorporates the implied volatility into the HARQ model, because we consider the effect of VoV on the volatility forecasting power of the information on options. Hence, the heterogeneous autoregressive realized volatility, quarticity, and implied volatility (HARQ-IV) model as our first benchmark model is defined as,

$$RV_{t+h-1,t} = \beta_0 + \left(\beta_D + \beta_{DQ} RQ_{t-1}^{1/2}\right) RV_{t-1} + \beta_W RV_{t-1,t-5} + \beta_M RV_{t-1,t-22} + \beta_{IV} IV_{t-1}, \tag{9}$$

where $RQ_t \equiv \frac{M}{3}\sum_{i=1}^{M} r_{t,i}^4$ (realized quarticity), and IV_t is the implied volatility (i.e., square of VIX) at day t.

Based on the HARQ-IV model, we introduce heterogeneous autoregressive realized volatility, quarticity, implied volatility, and vol-of-vol (HARQ-IV-VOV) model, which can be expressed as:

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[6] Previous literature on volatility forecasting (Andersen, Bollerslev, and Diebold, 2007; Corsi, 2009; Andersen, Bollerslev, and Huang, 2011; Busch, Christensen, and Nielsen, 2011; Chen and Ghysels, 2011; Byun and Kim, 2013; Liu, Patton, and Sheppard, 2015; Seo and Kim, 2015; Bollerslev, Patton, and Quaedvlieg, 2016; Li, Tsionas, and Izzeldin, 2016) investigates the volatility forecasting models with the RV.

[7] HAR models with realized absolute value (Forsberg and Ghysels, 2007), jump and continuous components of the realized volatility (Andersen, Bollerslev, and Diebold, 2007), past negative returns (Corsi and Renò, 2012), and realized variation depending on the sign of intraday returns (Patton and Sheppard, 2015) have been introduced in the literature.

$$RV_{t+h-1,t} = \beta_0 + \left(\beta_D + \beta_{DQ} RQ_{t-1}^{1/2}\right) RV_{t-1} + \beta_W RV_{t-1,t-5} + \beta_M RV_{t-1,t-22} + (\beta_{IV} + \beta_{IVV} VoV_{t-1}) IV_{t-1}, \quad (10)$$

where VoV_t is the uncertainty measure at day t. In this model, we add an interaction between VoV and implied volatility. In other words, we allow the coefficient on the implied volatility to vary over time depending on the level of uncertainty about risk.

For the second benchmark model, we employ the time-varying parameter HAR (TVP-HAR) model introduced by Li, Tsionas, and Izzeldin (2016). Li, Tsionas, and Izzeldin (2016) allow the autoregressive parameter of daily realized volatility time varying in the HAR model to consider realized volatility's long-term persistence. If the previous day's realized volatility has relatively large increment or decrease compared to its long-term average level (e.g., monthly realized volatility), today's realized volatility tends to revert to long-term average level and hence the parameter of daily realized volatility should decrease. Similar to the HARQ-IV model, we include the implied volatility in the TVP-HAR model. The time-varying parameter HAR and implied volatility (TVP-HAR-IV) model as our second benchmark model is defined as:

$$RV_{t+h-1,t} = \beta_0 + \left(\beta_D + \beta_{TVP} \left| RV_{t-1} - RV_{t-1,t-22} \right| \right) RV_{t-1} + \beta_W RV_{t-1,t-5} + \beta_M RV_{t-1,t-22} + \beta_{IV} IV_{t-1}. \quad (11)$$

Analogous to the first benchmark model, we add an interaction term between VoV and implied volatility. The time-varying parameter HAR, implied volatility, and vol-of-vol (TVP-HAR-IV-VOV) model is expressed as:

$$RV_{t+h-1,t} = \beta_0 + \left(\beta_D + \beta_{TVP} \left| RV_{t-1} - RV_{t-1,t-22} \right| \right) RV_{t-1} + \beta_W RV_{t-1,t-5} + \beta_M RV_{t-1,t-22} + (\beta_{IV} + \beta_{IVV} VoV_{t-1}) IV_{t-1}. \quad (12)$$

3. Empirical analysis

3.1. Data

We focus our empirical analysis on the S&P 500 index. High-frequency S&P 500 index futures trade data are obtained from the Chicago Mercantile Exchange (CME) DataMine, and the five-minute returns are used to calculate the realized variance.[8] VIX, which is used to calculate IV and VoV, is from Chicago Board Options Exchange. Our sample period covers the period from February 1, 1996 to August 29, 2014, yielding a total of 4,671 observations.[9]

[Table 1 about here]

Table 1 provides the summary statistics of the S&P 500 index futures for daily RV, IV, and VoV. On average, IV is larger than RV, which is consistent with Byun and Kim (2013). Variation in the RV (2.2084) is close to that of the IV (2.0829); however, the VoV (0.0412) is less volatile than both RV and IV. RV (10.5372) is more positively skewed than IV (4.4617), and the distributions of the three variables are leptokurtic. The differences between the maximum and minimum of RV, IV, and VoV are 61.2991, 25.5577, and 0.3155, respectively, which is consistent with the pattern of the standard deviations across the three variables. The Ljung-Box statistics of the three variables are very high and significant at the 1% level, which indicates that the time-series of the three variables exhibit a high degree of own serial correlation.

[Figure 1 about here]

8 Sampling frequency is related to accuracy, which is theoretically improved by higher sampling frequency, and microstructure noise, which can occur because of the bid-ask bounce, infrequent trading, and other factors. Liu, Patton, and Sheppard (2015) find empirical evidence that it is difficult to significantly beat five-minute realized variance by any other variation measures from high-frequency data.

9 High-frequency data spans the period from January 1, 1996 to September 30, 2014. Since the first and last months of the data are used to calculate VoV and monthly realized volatility, our sample period is from February 1, 1996 to August 29, 2014.

The time-series of the daily RV, IV, and VoV during the whole sample period is illustrated in Figure 1. As expected, it shows that during the period with large movements in the realized volatility, uncertainty tracks the realized volatility in particular. For example, during the financial crisis (November 2007 to June 2009),[10] the realized volatility reaches a peak with a large increment of uncertainty.[11]

3.2. In-sample analysis

We first consider the in-sample results to evaluate the volatility forecasting power that has been improved by utilizing uncertainty.

[Table 2 about here]

Table 2 reports the coefficient estimates obtained from the regression models of equations (9), (10), (11), and (12). Following Andersen, Bollerslev, and Diebold (2007), all regressions are estimated by the ordinary least square using the Newey and West (1987) robust standard errors with 5, 10, and 44 lags for the daily, weekly, and monthly forecast horizon, respectively. The *t*-statistics based on the standard errors of Newey and West (1987) are reported in parentheses.

As expected, the β_{IVV} coefficient estimates in the HARQ-IV-VOV and TVP-HAR-IV-VOV models are positive and statistically significant at the 5% level in the daily forecast horizon.[12] Furthermore, the

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[10] Following Byun and Kim (2013), we specify the financial crisis as the period from November 2007 to June 2009.

[11] While the correlation between daily realized volatility and VoV during the whole sample period is 0.3086, the correlation between daily realized volatility and VoV during the financial crisis is 0.5970.

[12] In an unreported result, we find that including the interaction terms between the realized volatilities and VoV does not influence the β_{IVV}, and their coefficient estimates are insignificant. Thus, the informativeness of the realized volatilities is not affected by uncertainty about risk. Result is available upon request.

coefficient estimates on IV and the daily, weekly, and monthly RVs are not affected after including the interaction term between IV and VoV. This result suggests that uncertainty provides unique information on the role of implied volatility in volatility forecasting. Consistent with our hypothesis, as the uncertainty about risk increases, the informativeness of the implied volatility for future realized volatilities increases. In addition, we find that the β_{DQ} and β_{TVP} coefficient estimates in the daily forecast horizon are negative and statistically significant at the 1% level, consistent with Bollerslev, Patton, and Quaedvlieg (2016) and Li, Tsionas, and Izzeldin (2016).

Interestingly, compared to our benchmark models, the HARQ-IV-VOV model and TVP-HAR-IV-VOV model improve their adjusted R^2 by 1.47% and 1.71% in the daily forecast horizon, respectively. Therefore, incorporating uncertainty not only provides information on the role of implied volatility, but also improves the forecasting power of the model.

Despite being insignificant, the β_{IVV} coefficient estimates have a positive sign in the weekly and monthly forecast horizons. Generally, the magnitude and *t*-statistics of the β_{IVV} coefficient estimates decreases with the forecast horizon. It is noteworthy that in comparison to the benchmark models, the HARQ-IV-VOV model (TVP-HAR-IV-VOV model) increases adjusted R^2 by 1.02% (1.02%) and 1.01% (1.04%) in the weekly and monthly forecast horizons, respectively.

The predictive power of uncertainty is insignificant in a longer horizon because two opposing forces are in play. On one hand, as the uncertainty increases, the information from the options market increases as we argue in this paper. On the other hand, as the market becomes volatile and sways rapidly, the predictive power of past information deteriorates rapidly. When the market is uncertain (e.g., VoV is high), the information from a week ago gives investors little information on the current or future state of the market. The interaction of these two opposing forces can explain why the coefficient on the interaction term between IV and VoV is insignificant in the weekly and monthly forecast horizons.

[Figure 2 about here]

The averages of the standard deviations of future daily RVs sorted by VoV deciles are depicted in Figure 2. We group our 4,671 daily observations into deciles based on the previous day's VoV. Then, for each observation, we take standard deviation of future five or 22-days' daily RV. The solid (dotted) line depicts the average standard deviations of future five (22) days' daily RV for each decile. Consistent with our hypothesis, we find that as the previous day's uncertainty increases, the fluctuation of future volatility becomes larger, which in turn decreases the predictive power of stale information. Therefore, for forecasting future long-term volatility, while the information on the implied volatility from informed investors' perspective about future volatility is still useful (the significance of the β_{IV} coefficient estimates), the information on the implied volatility from informed investors' perception for uncertain prospects losses the forecasting power (the significance of the β_{IVV} coefficient estimates).

3.3. Out-of-sample analysis

In this subsection, we compare out-of-sample fits to assess the robustness of improved forecasting power that considers uncertainty. Following Patton (2011) and Bollerslev, Patton, and Quaedvlieg (2016), we rely on three loss functions to get results that are robust to noise in the proxy of volatility as follows:

$$MSE \equiv \frac{1}{N}\sum_{t=1}^{N}\left(F_{t+h-1,t} - RV_{t+h-1,t}\right)^2, \tag{13}$$

$$MAE \equiv \frac{1}{N}\sum_{t=1}^{N}\left|F_{t+h-1,t} - RV_{t+h-1,t}\right|, \tag{14}$$

$$QLIKE \equiv \frac{1}{N}\sum_{t=1}^{N}\left[\frac{RV_{t+h-1,t}}{F_{t+h-1,t}} - \log\left(\frac{RV_{t+h-1,t}}{F_{t+h-1,t}}\right) - 1\right], \tag{15}$$

where $F_{t+h\text{-}1,t}$ refers to the forecast of future h-day realized volatility at day t from the models, $RV_{t+h\text{-}1,t}$ is the actual value of future h-day realized volatility at day t, and N is the number of the forecasts. In the rolling window (RW), the forecast of future realized volatility is obtained by re-estimating the coefficient estimates of the models with the observations of the previous 1,000 days. In the increasing window (IW), the models utilize all of the available observations to forecast future realized volatility. While the MSE and MAE are symmetric loss functions, the QLIKE is an asymmetric loss function. Thus, we can consider the symmetry of loss functions.

We employ the model confidence set (MCS) test of Hansen, Lunde, and Nason (2011) and the Diebold-Mariano test of Diebold and Mariano (1995) to confirm the statistical significance. The purpose of the MCS test is to determine the confidence set, M^*, that consists of the best models from the original set of models, M_0. The best models are selected based on the loss functions. The null hypothesis of the MCS test is no difference in the forecasting ability based on the loss function among the models, and the p-values for the null hypothesis are reported. If the model rejects the null hypothesis at the confidence level, the model is excluded from M^*. Thus, given a confidence level, the surviving models in M^* yield the best forecasts based on the specified loss function. The Diebold-Mariano test has the null hypothesis that is no difference in the accuracy of two competing volatility forecasting models, and reports the significant difference among the two models based on the t-statistics. If the t-statistic of the comparison between the two models is negative (positive), the forecasting error of the former (latter) model is smaller than that of the latter (former) model.

[Table 3 about here]

Table 3 reports the values of three loss functions and the p-values of the MCS test. We conduct the MCS test for two groups of the models (the HARQ-IV and HARQ-IV-VOV models, and the TVP-HAR-IV and TVP-HAR-IV-VOV models), and the p-values are calculated based on a block bootstrap procedure with 10,000 resamples. For all loss functions in the daily forecast horizon, the p-values of the HARQ-IV-VOV and TVP-HAR-IV-VOV models are equal to or close to 1 in Panels A (RW) and B

(IW). Thus, the HARQ-IV-VOV and TVP-HAR-IV-VOV models are included in the confidence set. For RW, the HARQ-IV model is excluded from the confidence set at the 1% level with MAE and QLIKE. For IW, the HARQ-IV model is eliminated from the confidence set at the 1% level with QLIKE. In the case of the TVP-HAR-IV model, the model is excluded from the confidence set at the 1% level with MAE and QLIKE of RW and QLIKE of IW. Additionally, the TVP-HAR-IV model is eliminated from the confidence set at the 10% level with MAE of IW. These results are consistent with significant improvement by considering VoV for the daily forecast horizon of the in-sample analysis.

Considering uncertainty shows poor performance in the weekly and monthly forecast horizons. For example, the HARQ-IV-VOV model is eliminated from the confidence set at the 1% level with MAE and QLIKE of IW in the weekly forecast horizon, and the TVP-HAR-IV-VOV model is excluded from the confidence set at the 5% level with MAE and QLIKE of IW in the monthly forecast horizon. These patterns are also consistent with insignificant β_{IVV} coefficient estimates for weekly and monthly forecast horizons of the in-sample analysis.

[Table 4 about here]

Table 4 shows the *t*-statistics of the Diebold-Mariano test. Except for MSE (RW) of the comparison between the HARQ-IV-VOV and HARQ-IV models, the sign of all *t*-statistics is negative in the daily forecast horizon. In addition, 7 out of 12 loss functions are significantly negative at the 1% or 5% levels. On the other hand, the sign of all *t*-statistics in the weekly and monthly forecast horizons is positive, and 10 out of 24 loss functions are statistically significant. These results indicate that in the daily forecast horizon the forecasting errors of the HARQ-IV-VOV and TVP-HAR-IV-VOV models are less than those of the HARQ-IV and TVP-HAR-IV models.

To summarize, the in-sample regressions and the results of the MCS test and the Diebold-Mariano test confirm that the volatility forecasting models considering uncertainty outperform those that do not consider uncertainty in the daily forecast horizon.

4. Robustness checks

4.1. Constant parameter volatility forecasting models

In Section 3, we examined the incremental role of uncertainty using two benchmark models that employ a time-varying parameter. In this subsection, we find that our results are robust under alternative benchmark models with a fixed parameter. In particular, we employ the heterogeneous autoregressive realized volatility and implied volatility (HAR-IV), heterogeneous autoregressive realized absolute value and implied volatility (HARAV-IV), continuous HAR and implied volatility (CHAR-IV), heterogeneous autoregressive realized volatility, jump, and implied volatility (HARJ-IV), and semi-variance HAR and implied volatility (SHAR-IV) models as our alternative benchmark model.[13]

Corsi (2009) proposes the HAR model to consider different volatility components realized over different investment time horizons based on heterogeneity of investors. The HAR model is readily modified in a manner entirely analogous to the HARQ-IV-VOV and TVP-HAR-IV-VOV models, resulting in the heterogeneous autoregressive realized volatility, implied volatility, and vol-of-vol (HAR-IV-VOV) model,

$$RV_{t+h-1,t} = \beta_0 + \beta_D RV_{t-1} + \beta_W RV_{t-1,t-5} + \beta_M RV_{t-1,t-22} + (\beta_{IV} + \beta_{IVV} VoV_{t-1}) IV_{t-1}. \qquad (16)$$

Ghysels, Santa-Clara, and Valkanov (2006) find empirical evidence that the realized absolute value (RAV) outperforms the realized volatility measure in the mixed data sampling (MIDAS) regression for volatility forecasting. Forsberg and Ghysels (2007) apply the RAV to the HAR model, resulting in the heterogeneous autoregressive realized absolute value (HARAV) model. Analogous to previous models,

[13] For brevity, we report results only for the daily forecast horizon. In unreported results, we find that consistent with the results in Section 3, uncertainty about risk plays an insignificant role in weekly and monthly horizons. Results are available upon request.

we define the heterogeneous autoregressive realized absolute value, implied volatility, and vol-of-vol (HARAV-IV-VOV) model as,

$$RV_{t+h-1,t} = \beta_0 + \beta_D RAV_{t-1} + \beta_W RAV_{t-1,t-5} + \beta_M RAV_{t-1,t-22} + (\beta_{IV} + \beta_{IVV} VoV_{t-1}) IV_{t-1}, \quad (17)$$

where $RAV_t = \mu_1^{-1} M^{-1/2} \sum_{i=1}^{M} |r_{t,i}|$ ($\mu_1 = E(|u|), u \sim N(0,1)$), and $RAV_{t-1,t-h} = (RAV_{t-1} + RAV_{t-2} + \ldots + RAV_{t-h})/h$.

Following Andersen, Bollerslev, and Diebold (2007), we include the continuous HAR (CHAR) and heterogeneous autoregressive realized volatility and jump (HARJ) models in constant parameter volatility forecasting models. The CHAR model implements the bi-power variation (BPV), an estimate of the continuous variation in the presence of jumps, introduced by Barndorff-Nielsen and Shephard (2004), instead of realized volatility. With the addition of the implied volatility and the consideration of VoV, the CHAR-IV-VOV model can be expressed as:

$$RV_{t+h-1,t} = \beta_0 + \beta_D BPV_{t-1} + \beta_W BPV_{t-1,t-5} + \beta_M BPV_{t-1,t-22} + (\beta_{IV} + \beta_{IVV} VoV_{t-1}) IV_{t-1}, \quad (18)$$

where $BPV_t = \mu_1^{-2} \sum_{i=1}^{M-1} |r_{t,i}| |r_{t,i+1}|$ ($\mu_1 = E(|u|), u \sim N(0,1)$), and $BPV_{t-1,t-h} = (BPV_{t-1} + BPV_{t-2} + \ldots + BPV_{t-h})/h$.

The HARJ model includes a jump variation measure in the HAR model, and the HARJ-IV-VOV model is defined as,

$$RV_{t+h-1,t} = \beta_0 + \beta_D RV_{t-1} + \beta_W RV_{t-1,t-5} + \beta_M RV_{t-1,t-22} + \beta_J J_{t-1} + (\beta_{IV} + \beta_{IVV} VoV_{t-1}) IV_{t-1}, \quad (19)$$

where $J_t \equiv \max\left[RV_t - BPV_t, 0\right]$.

Patton and Sheppard (2015) investigate the forecasting power of the variation of positive and negative returns (e.g., realized semivariances), proposed by Barndorff-Nielsen, Kinnebrock, and Shephard (2010), for future volatility and find empirical evidence that negative realized semivariance is more important for volatility forecasting than positive realized semivariance. The SHAR model decomposes daily total variations into positive and negative realized semivariances, and the SHAR-IV-VOV models may be expressed as,

$$RV_{t+h-1,t} = \beta_0 + \beta_D^+ RV_{t-1}^+ + \beta_D^- RV_{t-1}^- + \beta_W RV_{t-1,t-5} + \beta_M RV_{t-1,t-22} + (\beta_{IV} + \beta_{IVV} VoV_{t-1}) IV_{t-1}, \quad (20)$$

where $RV_t^+ \equiv \sum_{i=1}^{M} r_{t,i}^2 I_{\{r_{t,i}>0\}}$, and $RV_t^- \equiv \sum_{i=1}^{M} r_{t,i}^2 I_{\{r_{t,i}<0\}}$.

[Table 5 about here]

We report the results of the in-sample regressions for the constant parameter volatility forecasting models in Table 5. With the constant parameter volatility forecasting models, the β_{IVV} coefficient estimates are positive and statistically significant at the 1% or 5% levels. The improvement of adjusted R^2 from considering VoV is 1.16%, 2.17%, and 0.90% for the HAR-IV-VOV, HARAV-IV-VOV, and CHAR-IV-VOV models, respectively. Additionally, the HARJ-IV-VOV and LHAR-IV-VOV models increase by 1.06% and 0.85% in adjusted R^2 compared to their benchmarks. These results confirm that the information of VoV is valuable for volatility forecasting with the constant parameter volatility forecasting models commonly used in the literature.

[Table 6 about here]

Table 6 reports the results of the out-of-sample analysis for the constant parameter volatility forecasting models. Consistent with the results in Table 5, the models that consider uncertainty outperform benchmark models. In Panels A and B of the MCS test, all of the models that consider uncertainty are not rejected at the 10% level. This result indicates that we can include the models that consider uncertainty in the confidence set. On the other hand, 14 out of 30 loss functions of the benchmark models are rejected at the 10% level. Compared to the models that consider uncertainty, the benchmark models show underperformance.

In Panels C and D of the Diebold-Mariano test, 20 *t*-statistics are negative and, in particular, 15 out of 20 negative *t*-statistics are statistically significant. However, there is no significant *t*-statistic among 10 positive *t*-statistics. Since a negative sign in the *t*-statistic means that the forecasting error of the models that consider uncertainty is smaller than that of the benchmark models, we find only evidence that the decrease in the forecasting error with considering uncertainty is statistically significant. The results with constant parameter volatility forecasting models are consistent with the results in Tables 3 and 4. Therefore, we confirm that considering uncertainty is valuable irrespective of whether other parameters in the volatility forecasting models are set as time-varying or constant.

4.2. Financial crisis

In this subsection we examine the interaction between uncertainty and implied volatility during financial crisis to confirm that the interaction is robust to sample periods, and pronounced during times of high uncertainty. We restrict our sample periods to the financial crisis from November 2007 to June 2009, and replicate the in-sample and out-of-sample analyses. The mean of VoV during the financial crisis (0.1008) is larger than that during the whole sample period (0.0857), which confirms that the financial crisis is indeed a highly uncertain period.

[Table 7 about here]

Table 7 provides the results during the financial crisis. In Panel A, the β_{IVV} coefficient estimates are positive and significant at the 5% level. Additionally, since the magnitude of the β_{IVV} coefficient estimates is larger than that in Table 3, the importance of considering uncertainty increases during the financial crisis compared to those during the whole sample period. The increment in the adjusted R^2 by considering VoV is 6.58% and 6.18% in the HARQ-IV-VOV and TVP-HAR-IV-VOV models, respectively. These improvements are about four times larger than those during the whole sample period.

In Panels B and C, the results of the MCS test and the Diebold-Mariano test are similar to those in Tables 4 and 5. For the MCS test, while 5 out of 12 loss functions of the HARQ-IV and TVP-HAR-IV models are rejected at the 1% level, all *p*-values of the loss functions of the HARQ-IV-VOV and TVP-HAR-IV-VOV models are equal to 1.0000. Furthermore, all the *t*-statistics in the Diebold-Mariano test are negative, and 6 out of 12 *t*-statistics are statistically significant at the 1% level. These results are consistent with the results in Section 3. Taken together with Tables 6 and 7, the empirical evidence of the results obtained for the period during the financial crisis confirms the robustness of the improved forecasting performance, taking VoV into consideration.

5. Conclusion

We propose novel volatility forecasting models that exploit uncertainty about risk. First, we find that the forecasting power of option-implied volatility increases as market becomes uncertain. Second, the models improve the accuracy of daily futures volatility forecasts. Compared to the time-varying parameter HAR-class models, in-sample fit and out-of-sample forecasting power significantly increases when the uncertainty is incorporated into the models. Furthermore, we find that our results are more pronounced in the recent financial crisis period. We find our results to be robust to the estimation technique and to the choice of benchmark models.

Our models contribute to both volatility forecasting literature and uncertainty literature by filling the gap between the two. The models can be applied in many other areas, such as forecasting variance risk premium and construction of profitable trading strategies based on volatility forecasts. We leave the further work pertaining to such applications of the volatility forecasting models with considering uncertainty about risk to the future research.

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Table 1. Summary statistics

The table reports the summary statistics of S&P 500 index futures daily RV, IV, and VoV. Summary statistics for all variables are reported based on daily frequency. LB_{10} denotes the Ljung-Box test statistic for up to tenth-order serial correlation. The sample period is from February 1, 1996 to August 29, 2014.

Statistics	RV	IV	VoV
Mean	1.1103	2.0922	0.0857
S.D.	2.2084	2.0829	0.0412
Skewness	10.5372	4.4617	1.9767
Kurtosis	182.9422	30.2573	6.1226
Min	0.0499	0.3881	0.0197
Max	61.3490	25.9458	0.3352
LB_{10}	13,532.1792	38,591.9530	28873.1229

Table 2. In-sample regression

The table provides estimates of the coefficients in the in-sample analysis. Robust Newey and West (1987) *t*-statistics with 5 (daily), 10 (weekly), and 44 (monthly) lags are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Model	β_0	β_D	β_{DQ}	β_{TVP}	β_W	β_M	β_{IV}	β_{IVV}	Adj. R^2
Daily									
HARQ-IV	-0.3149***	0.4563***	-0.0050***		0.2882**	-0.2187*	0.4217***		0.5834
	(-3.32)	(6.33)	(-4.03)		(2.32)	(-1.69)	(5.89)		
HARQ-IV-VOV	-0.1837***	0.4479***	-0.0054***		0.1588*	-0.2041*	0.2549***	1.6952**	0.5981
	(-3.32)	(6.23)	(-3.28)		(1.86)	(-1.84)	(3.30)	(2.00)	
TVP-HAR-IV	-0.3153***	0.4191***		-0.0094***	0.2983**	-0.2330*	0.4400***		0.5803
	(-3.32)	(6.38)		(-7.31)	(2.35)	(-1.66)	(5.73)		
TVP-HAR-IV-VOV	-0.1720***	0.4279***		-0.0109***	0.1531*	-0.2167*	0.2520***	1.8474**	0.5974
	(-3.16)	(6.91)		(-6.53)	(1.79)	(-1.79)	(3.44)	(2.07)	
Weekly									
HARQ-IV	-0.1178*	0.3015***	-0.0029***		0.2664**	0.0074	0.2928***		0.6719
	(-1.70)	(3.71)	(-3.86)		(2.31)	(0.06)	(6.76)		
HARQ-IV-VOV	-0.0272	0.2957***	-0.0031***		0.1769*	0.0175	0.1776**	1.1713	0.6821
	(-0.50)	(3.90)	(-3.49)		(1.75)	(0.15)	(2.29)	(1.59)	
TVP-HAR-IV	-0.1194*	0.2119***		-0.0030**	0.2894**	-0.0060	0.3298***		0.6643
	(-1.65)	(2.86)		(-2.15)	(2.46)	(-0.04)	(6.02)		
TVP-HAR-IV-VOV	-0.0277	0.2170***		-0.0039**	0.1964*	0.0045	0.2093***	1.1831	0.6745
	(-0.50)	(2.94)		(-2.28)	(1.92)	(0.04)	(2.61)	(1.56)	
Monthly									
HARQ-IV	0.1788**	0.2055**	-0.0022**		0.2632***	0.1401	0.1303*		0.5712
	(2.39)	(2.55)	(-2.26)		(2.60)	(1.22)	(1.78)		
HARQ-IV-VOV	0.2586**	0.2004***	-0.0024**		0.1844**	0.1489	0.0288	1.0312	0.5823
	(2.39)	(2.66)	(-2.21)		(2.15)	(1.25)	(0.22)	(1.48)	
TVP-HAR-IV	0.1777**	0.1441**		-0.0025	0.2790***	0.1305	0.1558**		0.5669
	(2.31)	(2.00)		(-1.33)	(2.74)	(1.11)	(2.28)		
TVP-HAR-IV-VOV	0.2591**	0.1486**		-0.0033*	0.1965**	0.1397	0.0490	1.0496	0.5773
	(2.36)	(2.15)		(-1.69)	(2.30)	(1.14)	(0.40)	(1.50)	

Table 3. Out-of-sample test: Model confidence set test

The table reports the results of the model confidence set test. The losses are reported, and the *p*-values of the model confidence set test are reported in parentheses. Panels A and B are based on rolling and increasing window forecasts, respectively.

Horizon	Daily				Weekly				Monthly			
Model	HARQ-IV	HARQ-IV-VOV	TVP-HAR-IV	TVP-HAR-IV-VOV	HARQ-IV	HARQ-IV-VOV	TVP-HAR-IV	TVP-HAR-IV-VOV	HARQ-IV	HARQ-IV-VOV	TVP-HAR-IV	TVP-HAR-IV-VOV
Panel A: RW												
MSE	2.5036	2.3564	2.5683	2.4127	1.3894	1.6103	1.4989	1.6022	1.7641	1.8871	1.7887	1.8897
	(0.5507)	(1.0000)	(0.5406)	(1.0000)	(1.0000)	(0.4157)	(1.0000)	(0.6424)	(1.0000)	(0.3323)	(1.0000)	(0.4416)
MAE	0.5922	0.5471	0.5892	0.5480	0.4487	0.4646	0.4535	0.4637	0.5436	0.5573	0.5434	0.5548
	(0.0001)	(1.0000)	(0.0002)	(1.0000)	(1.0000)	(0.2088)	(1.0000)	(0.3393)	(1.0000)	(0.1085)	(1.0000)	(0.1815)
QLIKE	0.2688	0.2431	0.2637	0.2366	0.1206	0.1469	0.1186	0.1414	0.1999	0.2056	0.1992	0.2003
	(0.0035)	(1.0000)	(0.0005)	(1.0000)	(1.0000)	(0.0541)	(1.0000)	(0.0310)	(1.0000)	(0.4539)	(1.0000)	(0.8924)
Panel B: IW												
MSE	2.2334	2.2393	2.2374	2.2014	1.2632	1.2653	1.3157	1.3198	1.3646	1.5122	1.3862	1.5628
	(1.0000)	(0.9474)	(0.7193)	(1.0000)	(1.0000)	(0.9591)	(1.0000)	(0.9384)	(1.0000)	(0.1940)	(1.0000)	(0.1291)
MAE	0.5254	0.5166	0.5325	0.5194	0.4279	0.4547	0.4354	0.4614	0.4996	0.5159	0.5030	0.5216
	(0.1635)	(1.0000)	(0.0536)	(1.0000)	(1.0000)	(0.0000)	(1.0000)	(0.0000)	(1.0000)	(0.0435)	(1.0000)	(0.0270)
QLIKE	0.2394	0.2195	0.2478	0.2244	0.1046	0.1235	0.1055	0.1255	0.1486	0.1620	0.1485	0.1647
	(0.0083)	(1.0000)	(0.0015)	(1.0000)	(1.0000)	(0.0000)	(1.0000)	(0.0000)	(1.0000)	(0.0316)	(1.0000)	(0.0163)

Table 4. Out-of-sample test: Diebold-Mariano test

The table reports the t-statistics of the Diebold-Mariano test. Panel A and B are based on rolling and increasing window forecasts. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Panel A: RW						
Horizon	Daily		Weekly		Monthly	
Comparison	HARQ-IV-VOV vs. HARQ-IV	TVP-HAR-IV-VOV vs. TVP-HAR-IV	HARQ-IV-VOV vs. HARQ-IV	TVP-HAR-IV-VOV vs. TVP-HAR-IV	HARQ-IV-VOV vs. HARQ-IV	TVP-HAR-IV-VOV vs. TVP-HAR-IV
MSE	-0.63	-0.65	0.97	0.49	1.17	0.94
MAE	-4.51***	-4.15***	1.52	1.09	1.96	1.62
QLIKE	-3.06***	-3.61***	2.27**	2.19**	0.95	0.16
Panel B: IW						
Horizon	Daily		Weekly		Monthly	
Comparison	HARQ-IV-VOV vs. HARQ-IV	TVP-HAR-IV-VOV vs. TVP-HAR-IV	HARQ-IV-VOV vs. HARQ-IV	TVP-HAR-IV-VOV vs. TVP-HAR-IV	HARQ-IV-VOV vs. HARQ-IV	TVP-HAR-IV-VOV vs. TVP-HAR-IV
MSE	0.06	-0.36	0.05	0.09	1.52	1.69*
MAE	-1.56	-2.15**	5.83***	5.66***	2.42**	2.61***
QLIKE	-2.95***	-3.46***	7.19***	7.69***	2.49**	2.78***

Table 5. In-sample regression: Constant parameter volatility forecasting models

The table reports the results of in-sample regressions for the constant parameter volatility forecasting models. The coefficient estimates and adjusted R^2s are reported. Robust Newey and West (1987) *t*-statistics with 5 lags are reported in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Model	β_0	β_D	β_D^+	β_D^-	β_W	β_M	β_J	β_{IV}	β_{IVV}	Adj. R^2
HAR-IV	-0.3205***	0.1507			0.3661**	-0.2533*		0.5441***		0.5616
	(-3.16)	(1.41)			(2.49)	(-1.88)		(5.43)		
HAR-IV-VOV	-0.2048***	0.1237			0.2565**	-0.2427**		0.4043***	1.5010**	0.5732
	(-3.25)	(1.03)			(2.35)	(-2.03)		(5.95)	(1.96)	
HARAV-IV	-0.5118***	0.9794***			1.0002*	-1.7478***		0.6801***		0.5567
	(-5.54)	(3.30)			(1.89)	(-3.30)		(6.05)		
HARAV-IV-VOV	-0.4049***	1.0145***			0.5636	-1.1228***		0.3454***	1.9005***	0.5784
	(-3.60)	(3.32)			(1.42)	(-3.73)		(3.50)	(2.95)	
CHAR-IV	-0.2935***	0.1939*			0.3725**	-0.2473*		0.5109***		0.5705
	(-3.32)	(1.90)			(2.30)	(-1.83)		(6.11)		
CHAR-IV-VOV	-0.1946***	0.1700			0.2660**	-0.2395**		0.3897***	1.3433**	0.5795
	(-3.33)	(1.51)			(2.13)	(-1.96)		(6.16)	(1.98)	
HARJ-IV	-0.2966***	0.2722***			0.3231**	-0.2332*	-0.8868**	0.5177***		0.5732
	(-3.23)	(3.55)			(2.45)	(-1.89)	(-2.35)	(5.98)		
HARJ-IV-VOV	-0.1870***	0.2411***			0.2202**	-0.2238**	-0.8480**	0.3851***	1.4353*	0.5838
	(-3.34)	(2.88)			(2.23)	(-2.04)	(-2.39)	(6.26)	(1.95)	
LHAR-IV	-0.2492***		-0.3169	0.8929***	0.3375***	-0.1662		0.4084***		0.5938
	(-3.37)		(-1.38)	(3.13)	(2.68)	(-1.49)		(5.56)		
LHAR-IV-VOV	-0.1524***		-0.3194	0.8365***	0.2442**	-0.1609		0.2937***	1.2961*	0.6023
	(-3.28)		(-1.35)	(3.15)	(2.55)	(-1.58)		(3.98)	(1.89)	

Table 6. Out-of-sample analysis: Constant parameter volatility forecasting models

The table reports the results of out-of-sample analysis for the constant parameter volatility forecasting models. The losses and the *p*-values of the model confidence set test are reported. In Panels A and B, the results of the model confidence set test based on rolling and increasing window forecasts are reported. In Panels C and D, the results of the Diebold-Mariano test based on rolling and increasing window forecasts are reported. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Model	HAR-IV	HAR-IV-VOV	HARAV-IV	HARAV-IV-VOV	CHAR-IV	CHAR-IV-VOV	HARJ-IV	HARJ-IV-VOV	SHAR-IV	SHAR-IV-VOV
Panel A: Model Confidence Set Test (RW)										
MSE	2.6656	2.6756	2.6787	2.5712	2.6298	2.6507	2.4722	2.5003	2.5026	2.5148
	(1.0000)	(0.9514)	(0.3906)	(1.0000)	(1.0000)	(0.8686)	(1.0000)	(0.8737)	(1.0000)	(0.9493)
MAE	0.6051	0.5779	0.6889	0.5995	0.5825	0.5641	0.5895	0.5648	0.5693	0.5475
	(0.0050)	(1.0000)	(0.0000)	(1.0000)	(0.0266)	(1.0000)	(0.0105)	(1.0000)	(0.0321)	(1.0000)
QLIKE	0.2674	0.2423	0.3343	0.2715	0.2555	0.2310	0.2528	0.2259	0.2428	0.2169
	(0.0004)	(1.0000)	(0.0000)	(1.0000)	(0.0008)	(1.0000)	(0.0000)	(1.0000)	(0.0000)	(1.0000)
Panel B: Model Confidence Set Test (IW)										
MSE	2.3711	2.4083	2.4485	2.3867	2.3106	2.3504	2.2980	2.3310	2.2198	2.2482
	(1.0000)	(0.6900)	(0.4997)	(1.0000)	(1.0000)	(0.6159)	(1.0000)	(0.6932)	(1.0000)	(0.6588)
MAE	0.5398	0.5406	0.6672	0.6229	0.5229	0.5272	0.5228	0.5222	0.5048	0.5039
	(1.0000)	(0.8839)	(0.0000)	(1.0000)	(1.0000)	(0.4151)	(0.9142)	(1.0000)	(0.8462)	(1.0000)
QLIKE	0.2421	0.2293	0.3538	0.3225	0.2295	0.2210	0.2398	0.2162	0.2118	0.2024
	(0.0695)	(1.0000)	(0.0000)	(1.0000)	(0.2229)	(1.0000)	(0.0327)	(1.0000)	(0.1132)	(1.0000)
Comparison	HAR-IV-VOV vs. HAR-IV		HARAV-IV-VOV vs. HARAV-IV		CHAR-IV-VOV vs. CHAR-IV		HARJ-IV-VOV vs. HARJ-IV		SHAR-IV-VOV vs. SHAR-IV	
Panel C: Diebold-Mariano Test (RW)										
MSE	0.05		-0.74		0.15		0.14		0.05	
MAE	-3.16***		-11.63***		-2.55**		-2.84***		-2.30**	
QLIKE	-3.54***		-7.81***		-3.57***		-4.25***		-5.04***	
Panel D: Diebold-Mariano Test (IW)										
MSE	0.34		-0.51		0.44		0.35		0.38	
MAE	0.16		-7.01***		0.92		-0.12		-0.21	
QLIKE	-2.01**		-4.51***		-1.31		-2.23**		-1.69*	

Table 7. Sub-sample analysis: Financial crisis

The table reports the in- and out-of-sample results during the financial crisis. In Panel A, the coefficient estimates and adjusted R^2s are reported. In Panel B-1 (B-2), the losses and the p-values of the model confidence set test based on rolling window (increasing window) forecasts are reported. In Panel C-1 (C-2), the t-statistics of the Diebold-Mariano test based on rolling window (increasing window) forecasts are reported. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. The financial crisis is from November 2007 to June 2009.

Panel A: In-sample regression

Model	β_0	β_D	β_{DQ}	β_{TVP}	β_W	β_M	β_{IV}	β_{IVV}	Adj. R^2
HARQ-IV	-0.9125***	0.5144***	-0.0075***		0.4086**	-0.4668*	0.6019***		0.5954
	(-2.87)	(3.85)	(-4.66)		(2.52)	(-1.84)	(3.40)		
HARQ-IV-VOV	-0.1912	0.4383***	-0.0085***		0.0056	-0.4799**	0.2504*	4.4426**	0.6612
	(-0.77)	(4.08)	(-5.83)		(0.05)	(-2.27)	(1.71)	(2.50)	
TVP-HAR-IV	-0.7945**	0.3792***		-0.0095***	0.4101**	-0.4444*	0.6227***		0.5844
	(-2.57)	(3.23)		(-3.58)	(2.49)	(-1.73)	(3.63)		
TVP-HAR-IV-VOV	-0.0809	0.2824***		-0.0107***	0.0216	-0.4547**	0.2875**	4.3007**	0.6462
	(-0.31)	(3.00)		(-4.03)	(0.16)	(-2.10)	(2.19)	(2.31)	

Model	HARQ-IV	HARQ-IV-VOV	TVP-HAR-IV	TVP-HAR-IV-VOV	Comparison	HARQ-IV-VOV vs. HARQ-IV	TVP-HAR-IV-VOV vs. TVP-HAR-IV
Panel B-1: Model Confidence Set Test (RW)					Panel C-1: Diebold-Mariano Test (RW)		
MSE	15.8088	13.9885	16.5182	14.7543	MSE	-0.91	-0.85
	(0.3904)	(1.0000)	(0.4256)	(1.0000)			
MAE	1.8568	1.5154	1.8910	1.5882	MAE	-4.80***	-4.22***
	(0.0000)	(1.0000)	(0.0001)	(1.0000)			
QLIKE	0.1794	0.1579	0.1805	0.1670	QLIKE	-0.87	-0.62
	(0.3436)	(1.0000)	(0.5083)	(1.0000)			
Panel B-2: Model Confidence Set Test (IW)					Panel C-2: Diebold-Mariano Test (IW)		
MSE	14.1353	13.9489	14.1173	13.5433	MSE	-0.23	-0.68
	(0.7909)	(1.0000)	(0.5023)	(1.0000)			
MAE	1.5708	1.4493	1.5807	1.4363	MAE	-3.43***	-4.65***
	(0.0009)	(1.0000)	(0.0004)	(1.0000)			
QLIKE	0.1271	0.1178	0.1288	0.1226	QLIKE	-3.93***	-5.80***
	(0.0006)	(1.0000)	(0.1853)	(1.0000)			

Figure 1. Daily RV, IV, and VoV

This figure plots the time-series of the S&P 500 index futures daily RV, IV, and VoV. The left scale presents daily RV and IV, and the right scale presents VoV. The sample period is from February 1, 1996 to August 29, 2014.

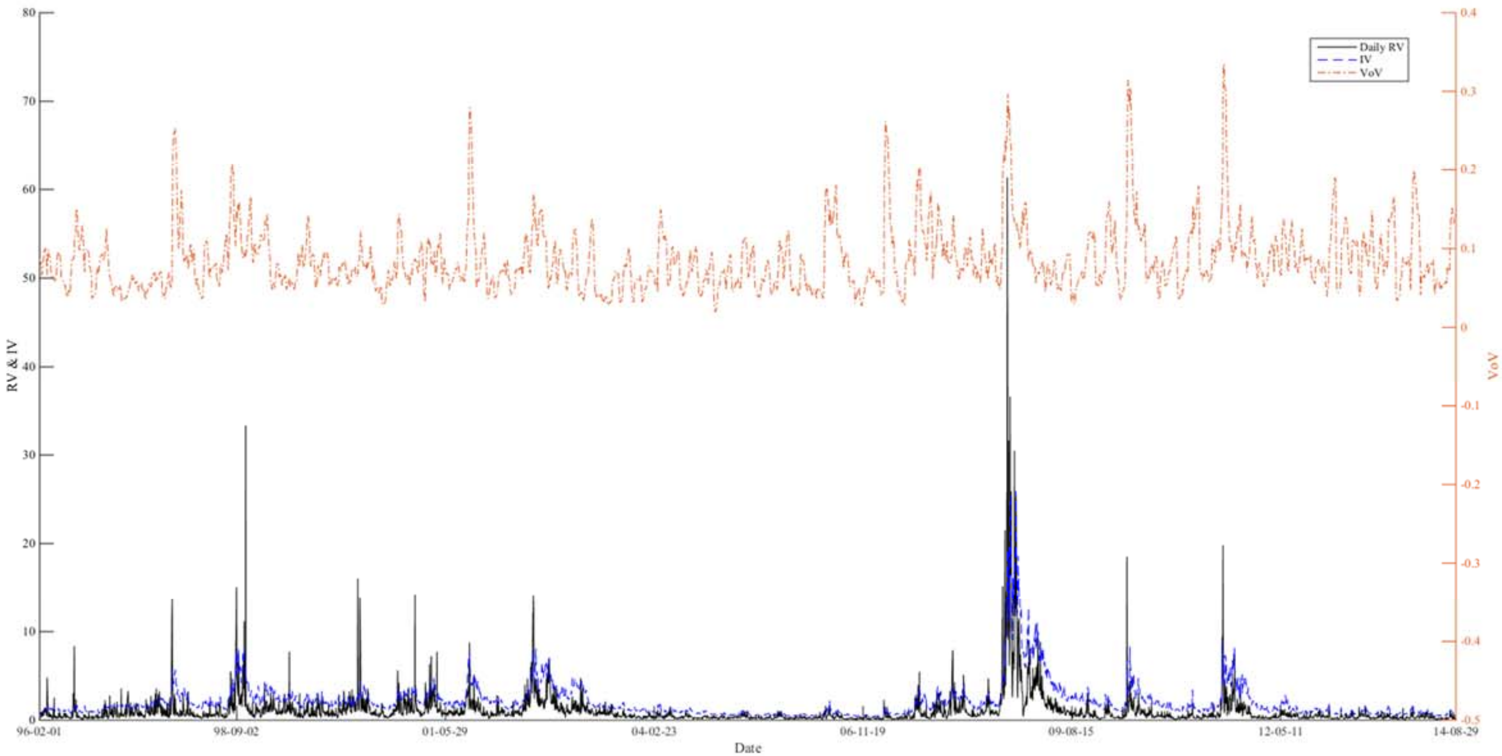

Figure 2. Standard deviation of future daily RVs by VoV deciles

This figure plots the average of the standard deviations of future daily RVs sorted by VoV deciles. Each VoV decile is defined based on the last day's VoV. The dashed (weekly) line is constructed based on future five days daily RVs of S&P 500 index futures. The solid (monthly) line is constructed based on future 22 days daily RVs of S&P 500 index futures. The sample period is from February 1, 1996 to August 29, 2014.