Finding a better momentum strategy from the stock and commodity futures markets

(Job Market Paper)

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Abstract

This paper proposes an improved momentum strategy that efficiently combines the stock momentum and the commodity futures momentum. Using the numeraire portfolio approach, we first find the negative (positive) weights on the commodity futures losers (winners) are optimal for log-utility investors when the investors can invest in the US commodity futures market as well as in the stock market. We also find a positive certainty-equivalent gain of up to 3.45% of wealth from the access to the commodity futures portfolios, and selling the commodity futures loser instead of selling the stock loser can improve the profits while substituting the winner does not. Based on these empirical results, we construct a joint strategy that utilizes the two-market momentum—i.e., buying the stock winner and selling the commodity futures loser—and confirm the improved performance, generating an average monthly return of up to 1.91% and providing 0.77% of the certainty-equivalent gain on average compared to the stock momentum. Moreover, the joint momentum strategy greatly improves the profitability in the contractionary period, which is up to 2.94% on average.

Classification codes:

Keywords: Commodity futures, market integration, momentum

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1. Introduction

According to the index investment data provided by the US Commodity Futures Trading Commission (CFTC),[1] the total value of index investment in the US commodity futures market was about $119 billion at the end of 2008 but was about $187 billion at the end of 2014. At the end of 2010, when the financial markets were affected by the global financial crisis, the total value of index investment in the US commodity futures market was notably high, i.e., $293 billion. Likewise, investments in commodity futures markets have increased rapidly over the last decade, and investing in commodities has drawn great attention because of its hedging effect and low correlation with other assets. Thus, in the literature, diversification benefits or the hedging effects of commodity futures have mainly been investigated, but the results are mixed. Belousova and Dorfleitner (2012) report the diversification benefits of several commodities. Jensen et al. (2000) examine the diversification benefits during the restrictive and expansive monetary policy periods, respectively, and find that the benefits exist only during the restrictive phases. More recently, however, Conover et al. (2010) report the existence of diversification benefits during both restrictive and expansive monetary policy periods. Daskalaki and Skiadopoulos (2011) examine this issue in a more general framework by employing both mean–variance and non-mean–variance tests and comparing both in-sample and out-of-sample settings, and they report that the benefits are not preserved out-of-sample. In terms of profitability, Bessler and Wolff (2015) examine whether investing in commodities enhances the performance of portfolios and find that the portfolio gains largely depend on the types of commodities and periods.

In another stream of the literature on commodity futures, whether there are profitable investment strategies in the commodity futures market has been investigated. In particular, the literature on the commodity futures markets has reported the considerable evidence for the existence of momentum in the

[1] http://www.cftc.gov/MarketReports/IndexInvestmentData/index.htm

commodity futures markets as in other traditional markets (Erb and Harvey, 2006; Miffre and Rallis, 2007; Asness et al., 2013; Kang and Kwon, 2016). In other words, buying contracts with high past returns and selling contracts with low past returns generates significant profits in the commodity futures markets. Pirrong (2005) examines the relation between the stock momentum and the commodity futures momentum and reports the positive and significant relation between them. Kang and Kwon (2016) also document that there is significantly positive relation between the two momentums from the international commodity futures market data, but they report that the correlation is quite low and the commodity futures momentum cannot be explained by the stock momentum.

In this paper, I examine whether the access to the US commodity futures markets is beneficial to investors that invest in the US stock markets using a data set from January 1979 to June 2015. In addition, we explore the possibility of a *better* momentum strategy by combining the effects of the stock momentum with the effects of the commodity futures momentum. We compare the profitability and the certainty-equivalent gain—which is an increase in utility in terms of the current wealth of an investor—of the momentum strategy in the individual market with that of our joint momentum strategy. Lastly, we compare the performance of the momentum strategy in the expansionary and contractionary periods.

The main purpose of this study is to find a better momentum strategy that uses both stocks and commodity futures. To start, we construct the investment universe consisting only of stock portfolios. The stock portfolios in this universe are the portfolios sorted by the firm size, book-to-market ratio, profitability, investment, and the past returns (momentum). Next, we investigate whether the set of commodity futures portfolios sorted by the past returns can be regarded as integrated with the stock portfolios. If it is, we examine whether it is redundant in regard to the stock portfolios, i.e., whether the commodity futures portfolios expand the investment universe of investors relative to the investment universe consisting only of the stock portfolios. Finally, after confirming that the commodity futures portfolios are not redundant to the universe of stock portfolios, we seek a better momentum strategy by combining the stock and the commodity futures momentum portfolios, and then examine the profitability (in terms of the average monthly return and the Sharpe ratio) and the certainty-equivalent wealth gain of the suggested stock-plus-commodity futures momentum strategy relative to the stock-only and the commodity-futures-only portfolios to assess whether the joint strategy has a better performance.

Our study can be distinguished from previous studies in two ways. First, we construct the commodity futures momentum portfolios using 32 commodity futures, and we focus on the beneficial effects of investing in the commodity futures portfolio sorted by the past returns, not in the individual contract or the market index. Previous studies employ only a small number of commodities or a representative commodity index, such as the S&P GSCI index, to examine the benefits from the diversification. Second, we suggest a new investment strategy that combines the stock momentum and the commodity futures momentum and generates larger profits and higher utility for investors. The literature on the commodity futures momentum focuses on the variants of momentum strategies or various long–short strategies only in the commodity futures market or a comparison of the stock momentum and the commodity futures momentum. Combining the profitable strategies in the stock and the commodity futures markets is rarely examined in the literature. For example, Asness et al. (2013) examine the value and the momentum effects in eight diverse markets and asset classes, including stocks and commodity futures, and find consistent value and momentum premiums from the sample markets. They also examine the profitability of the momentum strategy in diverse asset universes, such as non-stock asset classes or all asset classes, but their purpose is not to find a portfolio that generates a better performance than the momentum strategy in only one asset class. They investigate the existence of the momentum effects in various asset universes and report that the Sharpe ratio of the momentum portfolio constructed within global non-stock asset classes is 0.63 while the Sharpe ratio of our joint momentum strategies, buying the stock winner and selling the commodity loser, is up to 0.82.

In our study, we first estimate the stochastic discount factor based on a numeraire portfolio of the commodity futures portfolios sorted by the past returns as well as the stock market portfolios including the portfolios sorted by the past returns. Hentschel et al. (2002) define a set of financial markets as integrated if and only if they do not offer arbitrage opportunities to an investor who has costless access to the markets. They suggest that market integration can be examined by detecting arbitrage opportunities. According to their idea, if there are arbitrage opportunities across markets, then frictions or barriers will separate the markets to prevent arbitrage in practice. Such markets are called "not integrated". Using this analysis framework, in addition to the examination of the integration of two markets, we can obtain the optimal portfolio weights of a log-utility investor on the test portfolios. Thus, based on the estimation of the numeraire portfolio weights, we find that the stock and commodity futures markets are integrated. More interestingly, the optimal weights on the stock and commodity futures portfolios show significant and negative weights on the commodity futures loser and positive weights on the commodity futures winner while both the stock winner and loser have positive weights.

Next, using the stock market portfolios and commodity futures portfolios, we find the beneficial effects of the access to the commodity futures portfolios. We compute the certainty-equivalent wealth gain for a log-utility investor. In specific, we examine the increase of utility by investing in the optimal portfolio of a log-utility investor, which is the numeraire portfolio, in the extended investment universe including both stocks and commodity futures instead of investing in the optimal portfolio only in the stock market. Our results show a positive certainty-equivalent gain, which is up to 3.45% of the wealth of an investor. Moreover, as we find positive weights on the commodity futures winner and negative weights on the commodity futures loser, we investigate the change of returns by buying (selling) the commodity futures winner (loser) instead of the stock winner (loser) in the aspect of a momentum trader. We find the significant improvement in profitability by selling the commodity futures loser instead of selling the stock loser while buying the commodity futures winner does not improve the profitability.

As we find significant and consistent results of selling the commodity futures loser in the extended investment set composed of the stock market portfolios and the commodity futures momentum portfolios, we focus on the joint strategy of the two-market momentum, i.e., buying the stock winner and selling the commodity futures loser. Indeed, we find that the stock winner generally outperforms the commodity futures winner, and the commodity loser greatly underperforms the stock loser, so there is enough motivation to buy the stock winner and sell the commodity loser to generate the larger profits. Thus, we examine this joint strategy with various choices of ranking periods and holding periods. Our results show higher profitability of the joint strategy than the individual market momentum strategy in terms of the average return and the Sharpe ratio. The joint momentum strategies appear to have an average monthly return of up to 1.91% with an annualized Sharpe ratio of 0.82, which is much higher than the stock or bond indices and the stock momentum. In terms of the certainty-equivalent wealth gains from our joint momentum strategies relative to the momentum strategies in the stock market only or the commodity market only, we find positive gains for the log-utility investors (0.77% of wealth gain on average) and also for the power-utility investors except for the extremely risk-averse cases.

Compared with the stock momentum portfolio, our joint strategy substitutes the sell side of the stock momentum portfolio, i.e., the stock loser, with the commodity loser. From the point of view of investors, selling commodity futures is more attractive than selling stocks. Selling commodity futures is a more implementable strategy than selling stocks. Commodity futures offer leverage and they are not subject to short-selling restrictions, unlike stocks. In the stock markets, the short seller is charged interest for the loan of the security, which is called the loan fee, and there are some restrictions, such as the Uptick Rule, which make short selling difficult or sometimes impossible to implement. This difference in the short-sale costs has drawn attention in the literature because Lesmond et al. (2004) document that the profits of the stock momentum mainly stem from the short side and they become insignificant if the short-sale costs are considered. Our joint strategy is free from this issue. Lastly, in this paper, we use the price of the nearby contracts for the return series of the commodity futures that are liquid with relatively small transaction costs. Thus, selling commodity futures rather than selling stocks is much easier to implement and entails smaller costs.

The remainder of the paper is organized as follows. Section 2 describes the data and Section 3 presents the empirical results. In Section 3.1, we compare the stock momentum and the commodity futures momentum. In Sections 3.2 and 3.3, we conduct the integration and redundancy tests in the numeraire portfolio framework, respectively. In Section 3.4, we explore the profitability of the joint momentum strategies, and in Section 3.5, we investigate the performance of the joint momentum strategies in terms of the certainty-equivalent wealth gain. Section 4 concludes.

2. Data

The US commodity futures data, obtained from *Datastream*, comprise daily settlement prices on 32 US commodity futures contracts, and the sample period is from January 1979 to June 2015. We include the commodity futures contracts on butter, feeder cattle, live cattle, corn, dry whey, ethanol, lean hogs, lumber, milk, oats, rough rice, soybeans, soybean meal, No. 2 red wheat, hard red spring wheat, cocoa, coffee "C," cotton seed, orange juice, sugar No. 11, coal, Brent crude oil, light sweet crude oil, heating oil, gasoline, electricity, copper, gold 100 oz, palladium, platinum, silver 5000 oz, and natural gas. To compile the time-series of futures returns, we assume that we hold the nearest contract up to the end of the month prior to the maturity month. At the end of that month, we roll our position over to the second-nearest-to-maturity contract and hold that contract up to the end of the month prior to maturity. This rolling procedure allows us to minimize the problems related to the lack of liquidity and to compute the returns from holding the same contract, instead of switching the contract during the holding month.

In Sections 3.1 to 3.3, we use monthly returns on stock market portfolios, formed on the firm size, book-to-market ratio, profitability, investment, and the past returns (momentum), and these data are obtained from French's website.[2] Most of these portfolio returns are available from July 1926, but we use the data from January 1979 to June 2015 to match the sample period with the commodity futures market data. In Sections 3.4 and 3.5, we use monthly data for all common stocks (share codes 10 and 11) in the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and NASDAQ. The data are obtained from the Center for Research in Security Prices (CRSP). For the sample period of stock portfolio returns, we use the monthly data for common stocks from January 1979 to June 2015.

As opposed to stocks, futures are zero-investment securities, which means no initial investment. Thus, the returns on the commodity futures should be considered as excess returns, which cannot be directly compared to the returns on stocks (Bodie and Rosansky, 1980; Daskalaki and Skiadopoulos, 2011). We use the 3-month Treasury-bill rate provided by *Datastream* as the risk-free rate, and then convert the stock returns into the excess returns to make the returns on stocks and the commodity futures comparable.

3. Results

3.1. Stock momentum and commodity futures momentum

For commodity futures momentum portfolios, we sort commodity futures contracts into quintiles (*Cmom1, Cmom2, Cmom3, Cmom4*, and *Cmom5*) based on the past *J*-month returns (*J* = 1, 3, 6, and 12) in each month and compute the equally weighted monthly return of each quintile. We employ these five portfolios as representative portfolios of the US commodity futures markets to focus on the beneficial effects of investing in the commodity futures momentum.

[2] http://mba.tuck.dartmouth.edu/ pages/ faculty/ken.french/data_library.html

For stock momentum portfolios, we use French's six portfolios that are formed monthly on the basis of the firm size and momentum (2×3). These six portfolios are the intersection of the two portfolios based on the firm size and the three portfolios based on the prior 2- to 12-month return. We compute the average of two portfolios with different sizes in each momentum group, and thus finally construct three momentum portfolios (*Smom1*, *Smom2*, and *Smom3*).

In addition to the stock momentum portfolios, we employ monthly returns on Fama and French's (2015) stock market portfolios based on the firm size (*SIZE*), book-to-market ratio (*BTM*), profitability (*OP*), and investment (*INV*) as stock market portfolios that capture other characteristics. Table 3 in Fama and French (2015) shows the details of the portfolio construction for *SIZE*, *BTM*, *OP*, and *INV*. For each characteristic except *SIZE*, we form three portfolios. For *SIZE*, we form two portfolios (*SIZE1* and *SIZE2*). Consequently, we have 11 portfolios, but among these, we exclude one portfolio due to the linearly dependent relation between the 11 portfolios.[3] Specifically, among the three *BTM* portfolios (*BTM1*, *BTM2*, and *BTM3*), we exclude *BTM2*, which indicates the portfolio of firms with the medium level of the book-to-market ratio. Lastly, using the 3-month Treasury-bill rate as the risk-free rate, we compute the excess return on each portfolio.

Table 1 provides the sample statistics for monthly excess gross returns[4] on the stock portfolios and returns on the commodity futures portfolios. A portfolio with a large number indicates a portfolio with firms that have a high value of the specific characteristic. For example, *INV3* indicates the portfolio with firms that have the largest value of the investment variable (*INV*), and *Smom3* (*Cmom5*) indicates the portfolio with firms (commodity futures contracts) having the highest past returns, which is also called the stock

[3] Specifically, $9 \times (SIZE1 + SIZE2) = 2 \times (BTM1 + BTM2 + BTM3 + OP1 + OP2 + OP3 + INV1 + INV2 + INV3)$ holds.

[4] We report the sample statistics for the gross returns because gross returns are used for the numeraire portfolio tests in Sections 3.2 and 3.3.

winner (commodity futures winner). Likewise, we call *Smom1* (*Cmom1*) the stock loser (commodity futures loser). The table shows the mean, standard deviation, skewness, and kurtosis of the (excess) returns.

[Insert Table 1]

The average returns on stock portfolios confirm the existing effects documented by Fama and French (2015). Small firms (*SIZE1*) outperform big firms (*SIZE2*), and value firms (*BTM3*) outperform growth firms (*BTM1*). In terms of profitability and investment, firms with robust operating profitability (*OP3*) have larger returns than firms with weak operating profitability (*OP1*), and firms with conservative investment (*INV1*) have larger returns than firms with aggressive investment (*INV3*), respectively.

Table 1 confirms the existence of the stock momentum effect and commodity futures momentum effect. The stock (commodity futures) winner shows larger average returns than the stock (commodity futures) loser. The monotonically increasing pattern of the average returns on the commodity futures portfolios is consistently observed for all values of *J*. On the other hand, we can find some differences between the distributions of the stock and the commodity futures portfolio returns sorted on the past returns (*Smom*s and *Cmom*s). First, all three stock portfolios sorted on the past returns have positive average returns (1.0041, 1.0072, and 1.0102) while the commodity futures winners have positive average returns (range from 1.0063 to 1.0100) and the commodity futures losers have negative average returns (range from 0.9945 to 0.9973). When we compare the returns of winners (*Cmom5* and *Smom3*), their average returns seem to be comparable while the commodity futures loser (*Cmom1*) seem to underperform the stock loser (*Smom1*) considerably on average. Next, the commodity futures winner (*Cmom5*) tends to have a notably large kurtosis (range from 2.054 to 6.226) relative to other portfolios, while the stock winner (*Smom3*) has a smaller kurtosis than the stock loser (*Smom1*). These differences may suggest the different natures of the commodity futures momentum and the stock momentum, though the previous studies report a significant and positive relation between them (Pirrong, 2005; Kang and Kwon, 2016).

We also compute the descriptive statistics and correlations of the size (*SMB*), value (*HML*), investment (*CMA*), profitability (*RMW*), stock momentum (*SWML*), and commodity futures momentum (*CWMLJ* for *J* = 1, 3, 6, and 12) factors. As in Fama and French (2015), *SMB*, *HML*, *CMA*, *RMW*, *SWML*, and *CWML* are defined as (*SIZE1* – *SIZE2*), (*BTM3* – *BTM1*), (*OP3* – *OP1*), (*INV1* – *INV3*), (*Smom3* – *Smom1*), and (*Cmom5* – *Cmom1*), respectively.

[Insert Table 2]

As in Table 1, Table 2 shows the positive average returns of all factors and confirms the various risk premiums in the stock market documented in the literature, such as size or value premiums (Carhart, 1997; Fama and French, 2015), and the commodity futures momentum (Kang and Kwon, 2016). Compared to the stock market factors *SMB*, *HML*, *CMA*, *RMA*, and *SWML*, the commodity futures momentum factors (*CWMLJ* for *J* = 1, 3, 6, and 12) show much higher mean returns and larger standard deviations. The commodity futures momentums seem to be highly correlated with each other (correlations range from 0.349 to 0.704) but have low correlation with stock market factors.

The correlations between the stock momentum (*SWML*) and the commodity futures momentum (*CWMLs*) are also quite small. Their absolute values are less than 0.037 and they are even negative in case of *J* = 3 and 6. In case of *J* = 12, *SWML* shows a positive correlation with *CWML*, consistent with Kang and Kwon (2016) and Pirrong (2005), but its value is small as 0.037.

[Insert Figure 1]

Figure 1 presents the time-series of the returns of *SWML* and *CWML12* during the sample period. The time-series patterns of these two returns are quite different, which is not surprising because their correlation is small. This figure shows that the time-series of *CWML12* fluctuates more than *SWML*, which is also consistent with the larger value of *CWML12*'s standard deviation reported in Table 2. Some of the large fluctuations are worth mentioning. In March 1980, the commodity futures momentum shows a large negative return, and this negative return was driven by the commodity futures winner being composed of the precious metals, such as gold, silver, and platinum. In this month, the prices of commodity futures on gold, silver, and platinum dropped by 21.6%, 60.4%, and 39.1%, respectively. During the 1970s, the prices of the precious metals increased gradually, so the commodity futures winner portfolio was mainly composed of these. In particular, the price of silver showed a huge jump from January 1979 to January 1980 of about 200%.[5] However, on March 27, 1980, which is called "Silver Thursday," the Hunt brothers were unable to meet the margin call, which caused a large drop in the price of the silver futures contracts and contributed to the overall panic in the commodity futures markets. The stock momentum also seems to be affected by this event, so thus has a negative return on that month, but the size of the negative return is much smaller than that of the commodity futures momentum. In March 1993, the large loss in the commodity futures momentum is also mainly attributed to the price drop of precious metals, but the stock momentum does not show a notable loss in that month.

In the more recent period, after 1999, the two momentum factor portfolios *SWML* and *CWML12* show different performances during the various financial crises. The literature on the stock momentum has reported that the historical distribution of the stock momentum factor portfolio returns is highly skewed toward the left (Daniel et al., 2012), and the performance of the stock factor momentum portfolio is closely related to the stock market condition (Cooper et al., 2004). In January 2001, during the US recession period, the stock momentum factor portfolio experienced a large negative return (−25%). The 12-month commodity futures momentum factor portfolio (*CWML12*) also experienced a negative return (−15.6%), but the 1-month commodity futures momentum factor portfolio (*CWML1*) experienced a negative return of only −7%. In April 2009, during the global financial crisis, the stock momentum factor portfolio also shows a large

[5] We regard the price of the nearest futures contract as the spot price, and then compute the price change of silver from January 1979 to January 1980.

negative return (−34.6%) but the commodity futures momentum factor portfolios do not show any extreme returns. Likewise, we can expect that the commodity futures momentum factor portfolios are less affected by the financial crises or the business cycle while the stock momentum tends to have a large negative return during the crises. Chordia and Shivakumar (2002) report that the stock momentum in the US markets can be explained by the expected returns predicted by macroeconomic variables, but Kang and Kwon (2016) examine the same macroeconomic model in the US commodity futures markets and find that the commodity futures momentum cannot be predicted by macroeconomic variables. These findings support our prediction. Consequently, in addition to the small correlations between the two momentum strategies, these different performances during crises and the different relations with the business cycle imply that there may be a benefit in including the commodity futures in investment portfolios to avoid the large negative return from the stock momentum.

In summary, based on the small correlations between the stock momentum and commodity futures momentum factor portfolio returns, different exposures of the stock and commodity futures momentum factors to the business cycle and large profitability of the commodity futures momentum factor portfolios, we expect that the commodity futures market may not be redundant in regard to the stock market, and the additional access to the commodity futures market portfolios can be beneficial. In the subsequent sections, we examine the beneficial effects of the commodity futures portfolios.

3.2. Numeraire portfolio test of commodity futures momentum integration

Before assessing the benefits from the additional access to the commodity futures portfolios, we first test whether the commodity futures market is integrated with the stock market. We estimate the stochastic discount factor based on a numeraire portfolio of the stock market portfolios (*SIZE1*, *SIZE2*, *BTM1*, *BTM3*, *OP1*, *OP2*, *OP3*, *INV1*, *INV2*, *INV3*, *Smom1*, *Smom2*, and *Smom3*) and commodity futures portfolios sorted on the past returns (*Cmom1*, *Cmom2*, *Cmom3*, *Cmom4*, and *Cmom5*). Following Long (1990) and Hentschel et al. (2002), we say that there are no arbitrage opportunities in and between the markets, and thus the markets are integrated if and only if there is a numeraire portfolio that satisfies the following conditions:

$$Prob_t[1 + r_{N,t+1} > 0] = 1 \tag{1}$$

$$E_t\left[\frac{1 + r_{i,t+1}}{1 + r_{N,t+1}}\right] = 1 \tag{2}$$

where $r_{N,t+1}$ is the return on the numeraire portfolio and $r_{i,t+1}$ is the return on each asset *i* in the markets.[6] In this setting, the numeraire portfolio stochastic discount factor $\theta_{1,t+1}$ can be written as a non-linear function of the numeraire portfolio returns as follows:

$$\theta_{1,t+1} = \frac{1}{1 + r_{N,t+1}} \tag{3}$$

Following Hentschel et al. (2002), we examine the existence of constant portfolio weights that yield arbitrage profits in our sample of asset returns. If there are constant portfolio weights for the test assets generating a non-negative stochastic discount factor, then it indicates that those markets are integrated. Since our purpose is to examine whether the commodity futures market is integrated with the stock market, we estimate numeraire portfolio weights for the 13 stock portfolios and five commodity futures portfolios.

[Insert Table 3]

Panel A of Table 3 shows the estimated numeraire portfolio weights on each set of test assets. The results for the commodity futures portfolios formed in the past *J*-month returns for *J* = 1, 3, 6, and 12 are

[6] In Equation (1) to (3), *r* indicates the return on the portfolio, not the excess return. Since the returns on commodity futures can be regarded as the excess returns, we add the risk-free rate to them.

reported. The weights on the commodity futures portfolios are not strictly monotonic across the past *J*-month returns, but the overall pattern seems to be consistent with the momentum strategy generating profits by buying winners and selling losers in the commodity futures markets. In specific, we can find a common pattern for all values of *J* that the weights on the portfolios with the low past returns (*Cmom1* and *Cmom2*) are negative in all cases and statistically significant in many cases. The portfolios with the high past returns (*Cmom4* and *Cmom5*) also show positive weights in most cases. In fact, the weights on *Cmom5* are all positive and significant. The weights on *Cmom4*, however, are negative and even significant in case of *J* = 1 and 6. On the other hand, all the stock momentum portfolios formed on the past stock returns carry positive weights in the numeraire portfolio, which is in stark contrast with the commodity futures momentum.

The moment conditions in Equation (2) are also the first-order conditions for a log-utility investor. Thus, the numeraire portfolio weights in Table 3 can be interpreted as the utility-maximizing portfolio weights for a log-utility investor. From the point of view of the utility-maximizing portfolio weights, the weights in the numeraire portfolio show that a log-utility investor should pursue the commodity futures momentum strategy—i.e. buying the commodity futures winner and selling the commodity futures loser—as part of the investor's optimal investment strategy.

Panel B of Table 3 shows the summary statistics of the numeraire portfolio discount factor. Though we do not report the details, the numeraire portfolio discount factor has a higher minimum, maximum, and standard deviation than the linear discount factor suggested by Hansen and Jagannathan (1997). The important thing to note in this panel is that the numeraire portfolio discount factor is always positive, which indicates that it rules out the existence of arbitrage opportunities in and between the two markets.

To summarize, the numeraire portfolio test shows that the commodity futures market comprised of the commodity futures momentum portfolios is integrated with the stock market consisting of momentum and other stock market portfolios.[7] The estimated fixed-weight numeraire portfolio discount factors are always positive values, and these results indicate no arbitrage profit opportunities in and between stock and commodity futures markets. In addition, the weights in the numeraire portfolio, which is the optimal portfolio for log-utility investors, indicate that log-utility investors should carry positive weights on the stock winners and negative weights on the commodity futures losers. On the other hand, the weights on the stock losers in the numeraire portfolio are not negative.

3.3. Gains from diversification

The previous section shows that the commodity futures market is integrated with the stock market in the sense that there are no arbitrage profit opportunities in and between the markets. However, it does not imply that the commodity futures market is 'redundant' or adds benefits to stock market investors. In this section, we examine the economic size of the gains from additional access to the commodity futures market. We measure the gains from diversification by the certainty-equivalent increase in wealth from costless access to the commodity futures markets. The measure is based on the idea that if the additional market is redundant in regard to the existing markets, then the redundant market can be priced with a numeraire portfolio discount factor of the existing markets and produces no gain in the expected log-utility. For a utility function *U* of wealth *W* at time *t*, the certainty-equivalent ψ_t is defined as follows:

$$\mathrm{E_t}\left[U\left(W_t(1+\psi_t)\omega_1' R_{1,t+1}\right)\right] = E_t\left[U\left(W_t\omega_2' R_{2,t+1}\right)\right] \tag{4}$$

[7] We find consistent results if we consider only commodity futures momentum and stock momentum portfolios as test assets excluding other stock portfolios.

The unconditional estimates of the certainty-equivalent gain of a log-utility investor in Equation (4) can be defined as follows (Hentschel et al., 2002):

$$\psi = 100 \times \left[\exp\left\{ \frac{1}{T} \sum_{t=1}^{T} \ln\left(\frac{\omega_2' R_{2,t+1}}{\omega_1' R_{1,t+1}} \right) \right\} - 1 \right] \tag{5}$$

where $R_{1,t+1}$ is the vector of the gross returns on the stock portfolios, $R_{2,t+1}$ is the vector of the gross returns on the stock and commodity futures portfolios, and ω_1 and ω_2 are the optimal log-utility, or numeraire, portfolio weights for those two sets of portfolio assets.

[Insert Table 4]

Table 4 shows significant wealth gains for all values of J (t-statistics = 1.77 to 2.87). These results indicate that there is certain compensation for including the commodity future portfolios to the investment universe. The certainty-equivalent wealth gains are estimated as 1.55% to 3.45% depending on the value of J.

Overall, we find significant and substantial benefits from the additional access to the commodity futures portfolios sorted on the past returns. Our results show that the commodity futures momentum is not redundant to other stock market risk premiums, and significant gains result from the expansion of the investment universe into the one including the commodity futures market.

Based on these findings, in the next section, we extensively examine the profitability of the strategy of including both the stock momentum and commodity futures momentum. In Table 3, we find that, in the aspect of a log-utility investor, the optimal portfolio includes the long position in the commodity futures winner and the short positing in the commodity futures loser. On the other hand, in Table 1, we find that the mean returns on the commodity futures winner and the stock winner are comparable, but the mean returns on the losers are substantially different. The commodity futures loser appears to have much lower average returns that the stock loser, and it indicates that selling the commodity futures loser can be more profitable than selling the stock loser. To briefly see the effects of substituting the long or short side of the stock momentum strategy with the commodity futures, we compute the average monthly returns on the stock-only momentum strategy (*SWML*), the commodity-future-only momentum strategies (*CWMLJ* for $J = 1, 3$, 6, and 12), and the substituted strategies, i.e., *Smom3 − Cmom1* and *Cmom5 − Smom1*.

[Insert Table 5]

Table 5 shows the average monthly returns on various momentum strategies in the stock and commodity futures markets. First of all, Table 5 shows that the commodity futures momentum strategies generate the larger average returns (0.964 to 1.455) compared to the stock momentum strategy (0.607). If we substitute the long side of the stock momentum strategy by buying the commodity futures winner (*Cmom5 – Smom1*), the average monthly returns on this strategy are much lower (0.217 to 0.583) than these of *SWML* and *CWMLs*. On the other hand, if we substitute the short side of the stock momentum strategy by selling the commodity futures loser (*Smom3 – Cmom1*), we can see the larger and more significant returns on this strategy compared to *SWML* and *CWMLs*.

The results in Table 5 show the possibility of improving the profitability of the stock-only or the commodity-futures-only momentum strategies by combining two strategies as buying the stock winner and selling the commodify futures loser. Thus, in the next section, we will focus more on the short position on the commodity futures loser than the long position on the commodity futures winner

3.4. Joint strategy of stock momentum and commodity futures momentum

In this section, we propose a new momentum strategy by combining the effects of the stock momentum and the commodity futures momentum, and investigate the profitability of the joint strategy. Our main goal is to find a better momentum strategy that generates larger profits and utility for investors than a stock-only or commodity-futures-only momentum strategy. To do so, instead of French's momentum portfolios, we use stock market data provided by the Center for Research in Security Prices (CRSP) to construct momentum portfolios with various choices of ranking and holding periods as in the commodity futures momentum.

First, we investigate the profitability of the momentum strategy in each of the stock and commodity futures markets. For commodity futures momentum strategies, we examine the various combinations of ranking periods (*J1*) of 1, 3, 6, and 12 months and holding periods (*K*) of 1, 3, 6, and 12 months. These permutations result in 16 momentum strategies. For stock momentum strategies, we examine the various combinations of ranking periods (*J2*) of 3, 6, and 12 months and holding periods (*K*) of 1, 3, 6, and 12 months. These permutations result in 12 momentum strategies. Unlike the commodity futures momentum strategy, we exclude the 1-month ranking period case for the stock momentum strategy because the stock momentum strategy excludes the prior month's return because of implementable issues or the reversal effect as documented in Section 3.1. For example, for the case of the $J1 = 3$ strategy, the portfolio at month t is formed on the return from month $t-3$ to month $t-1$ while for the case of the $J2 = 3$ strategy, the portfolio at month t is formed on the return from month $t-3$ to month $t-2$.

At the end of each month, futures contracts are sorted into quintiles based on their return over the previous *J1*-month-ranking period, and stocks are sorted into deciles[8] based on their average returns over the previous *J2*-month-ranking period. The top quintile (decile) is assigned as a commodity (stock) winner portfolio and the bottom quintile (decile) is assigned as a commodity (stock) loser portfolio. Momentum strategies indicate buying the winner and selling the loser. We hold that portfolio for the subsequent *K*-month-holding period. We call the resulting strategy the *J1*/*K* or *J2/K* momentum strategy. In computing

[8] We also sort stocks into quintiles, and confirm the qualitatively similar results with deciles.

monthly returns of the portfolio, we follow the approach of Jegadeesh and Titman (1993) forming overlapping portfolios.

[Insert Table 6]

Table 6 presents the monthly excess returns on the momentum portfolios. In Panel A, we report the results of stock momentum strategies, and in Panel B, we report the results of the commodity futures momentum strategies. Panel A shows that the stock market momentum effect is weak during our sample period, although it does exist. Both the stock winner and loser generate positive returns for all cases, and the returns on the stock winners are significant in all cases. The differences between the winner and the loser are positive except for the case of *J2* = 12 and *K* = 12, but they are statistically significant at the 10% significance level for only two cases (*J2* = 6 and *K* = 6; *J2* = 12 and *K* = 1). By contrast, Panel B shows a strong commodity futures momentum effect. The positive returns on winners and the negative returns on losers generate highly significant returns of the momentum strategies (W−L). The average returns on the commodity futures momentum strategies are significant at the 1% significance level in most of the cases, and their annualized Sharpe ratios (SR) are notably high for the short ranking-period strategies (*J1* = 1). Moreover, as in Table 1 and Table 4, Table 6 also confirms that the commodity futures loser greatly underperforms the stock loser. The commodity futures winner also tends to underperform the stock winner, but the difference becomes larger as the holding period (*K*) increases and the ranking period (*J1* for commodity futures and *J2* for stocks) increases. The returns on the commodity futures winner decrease substantially as *J1* or *K* increases, but the returns on the stock winner do not show a big decrease in returns as *J2* or *K* increases as opposed to the commodity futures winner.

Next, we look into a new momentum strategy that combines the stock momentum and the commodity futures momentum strategies and compare the profitability of the joint strategy. In Table 6, we find that the commodity losers greatly underperform the stock loser, and so we suggest a strategy of buying the stock winner and selling the commodity loser, which we will call the *joint momentum* strategy. This strategy is also consistent with our findings in the previous sections. In Table 3, from the estimates of the numeraire portfolio weights, we find negative and significant weights of the commodity futures losers. In Table 5, we confirm the increased average returns from buying the stock winner and selling the commodity futures loser using the French's stock portfolios.

Indeed, from the point of view of investors, selling commodity futures is more attractive than selling stocks. Selling commodity futures is a more implementable strategy than selling stocks. Commodity futures offer leverage and they are not subject to short-selling restrictions, unlike stocks. In the stock markets, the short seller is charged interest for the loan of the security, which is called the loan fee, and there are some restrictions, such as the Uptick Rule, which make short selling difficult or sometimes impossible to implement. As D'Avolio (2002) reports, the short-selling cost is sometimes prohibitively high. This difference in the short-sale costs between stocks and commodity futures is important in the literature because Lesmond et al. (2004) and Stambaugh et al. (2012) document that the profits of the stock momentum mainly stems from the short side and they become insignificant if the short-sale costs are considered. Our joint strategy is free from this issue. Lastly, in this paper, we use the price of the nearest contracts that are liquid with relatively small transaction costs. Thus, selling commodity futures rather than selling stocks is much easier to implement and entails lower costs.

In choosing *J1* and *J2*, as well as the cases of *J1* = *J2*, we also test the case of *J1* = 1 and different *J2* values[9] because we find that the negative return on the commodity is largest for *J1* = 1 in Table 7. The joint strategies indicate buying the stock winner formed in the previous *J1*-month-ranking period and selling the commodity futures loser formed in the previous *J2*-month-ranking period. We call the resulting strategy the "*J1/J2/K* momentum strategy". For comparison, we also investigate the strategy of buying the commodity winner and selling the commodity loser.

[9] Specifically, we test for *J2* = 6 and 12 because the stock momentum shows significantly positive returns for those two cases in Table 6.

[Insert Table 7]

Panel A of Table 7 presents the profitability of the *J1/J2/K* joint momentum strategies. The average monthly returns on the *J1/J2/K* strategies are around 1% and significant for all cases except the 12/12/12 strategy. The shorter-term holding strategies (small *K*) show larger profits, and the 1/12/1 strategy generates the largest average return as 1.907% with the Sharpe ratio of 0.822. The annualized Sharpe ratios and the average returns of these joint strategies are much larger than *J2/K* stock momentum strategies. For example, when we construct stock winners and commodity futures losers based on the past 6-month returns and hold our momentum portfolio for the next month (6/6/1 case), its monthly return is on average 1.463%, which is much larger than the stock-only momentum strategy (0.387%) or the commodity-futures-only momentum strategy (1.131%). The Sharpe ratio of our momentum strategy is also better than those of the corresponding stock strategies, i.e., the Sharpe ratio of our 6/6/1 momentum strategy is 0.646, while those for the stock momentum strategy and the commodity futures momentum strategy are 0.220 and 0.521, respectively. The Sharpe ratios of stock-only momentum strategies never exceed 0.358 in Panel A of Table 6, and those of commodity-futures-only momentum strategies are usually less than those of the joint momentum strategies. The annualized Sharpe ratios of S&P 500 and Russell 2000 for the same sample period are less than 0.3, thus even for other *J1/J2/K* strategies, we can see that these joint strategies greatly outperform other stock indices.

Compared to *J1/K* commodity futures momentum strategies, however, the *J1/J2/K* strategies show larger Sharpe ratios and returns for cases of *J2* = 6 and 12. For *J2* = 1 and 3, the *J1/K* commodity futures strategy seems to outperform the *J1/J2/K* strategies in some cases. These results indicate the combining selling the short-term commodity futures loser with buying the intermediate-term stock winner is effective. In Table 6, we find that he returns on the commodity futures winner decrease substantially as *J1* or *K* increases, while the returns on the stock winner do not show a big decrease in returns as *J2* or *K* increases. Consequently, substituting the commodity futures winner with the stock winner may generate larger profits for a longer ranking or holding period as the differences in returns on the commodity futures winner and the stock winner become larger. Among various combinations of *J1*, *J2*, and *K*, Table 7 shows that for short-term holding strategies (*K* = 1 or 3), the *J1/J2* = 1/12 strategy generates the largest and the most significant returns, and for long-term holding strategies (*K* = 6 or 12), the *J1/J2* = 1/6 strategy generates the highest returns.

In the second column of Panel A, we also report the results of the opposite strategy, i.e., buying the commodity winner and selling the commodity loser. Not surprisingly, this strategy shows negative returns in most cases and some of them are even significant.

In the literature on the commodity futures, the basis—which is the difference between the spot price and the futures price[10]—has been regarded as an important risk factor, and many studies report the significant relation between the basis factor and the momentum factor in the commodity futures markets (Miffre and Rallis, 2007; Gorton et al., 2012; Szymanowska et al., 2014; Kang and Kwon, 2016). We control the basis effect by constructing the double-sorted commodity futures momentum portfolios. We first sort the futures contracts by the basis into three groups and then sort the contracts in each group by the previous *J1*-month returns. After constructing the nine portfolios by double-sorting, we average three winner (loser) portfolios from three basis groups and call the resulting portfolio the commodity futures winner (loser). Panel B of Table 7 presents the joint momentum strategies based on the commodity futures winner and loser portfolios constructed from the double-sorted portfolios, as well as the stock winner and loser portfolios.

[10] The spot-price data are not available for most commodities (Fama and French, 1987; Gorton et al., 2012; Kang and Kwon, 2016). Thus, we employ the price of the nearest futures contract as the spot price and the price of the second-nearest futures contract as the futures price. Then, the basis is defined as the logarithm of the nearest contract's price subtracted by the logarithm of the second-nearest contract's price.

Compared to the results in Panel A of Table 7, the basis-controlled strategies show reduced returns in all cases, but the size of the reduction in returns is small and the returns of these strategies are still highly significant. Thus, Panel B of Table 7 shows that the profitability of the joint strategy of buying the stock winner and selling the commodity futures loser is not driven by the basis effect. Though we do not report in this paper, we also test whether the profitability of the joint strategy can be explained by Fama and French's three factor- and five factor-models (1993, 2015) and Carhart's four factor model (1997).[11] We run the time-series regression with these three test models and examine whether the intercept, which is the unexplained profits of the *J1/J2/K* joint momentum strategy, is significant. The three test models do not show a big difference in explaining the joint momentum profits, and the intercepts appear to be significant in most of cases with some exceptions. For example, for cases of *J1/J2/K* = 6/6/12, 12/12/6, and 12/12/12, the intercepts of the five factor model are 0.255, 0.175, and -0.101 with *t*-values 1.09, 0.65, and -0.41, respectively. However, for cases of *J1/J2* = 1/6 and 1/12 that show the best performance in Table 7, all intercepts appear to be highly significant with only one exception (*t*-values range from 2.09 to 4.40) that the 1/12/12 strategy's intercept is significant at the 10% significance level. These results show that the profitability of the joint momentum strategy is robust to the traditional risk factor models.

In summary, we investigate the profitability of a strategy that utilizes the outperformance of stock winners and the underperformance of commodity losers and find that this joint strategy greatly outperforms the momentum strategy in each individual market in terms of the profits and Sharpe ratios. Specifically, our new strategy shows a notable improvement compared to the stock-only momentum strategy. This higher performance is also consistent with the results in Sections 3.2 and 3.3 in that selling losers in commodity futures is the optimal portfolio strategy for investors that can invest in the commodity futures market as well as in the stock market. This joint strategy may be more attractive to investors in practice as well since

[11] The factors of Fama and French's three- and five-factor models are obtained from French's website. For the stock momentum factor of Carhart's model, we employ the return on the *J2/K* stock-momentum strategy if the test strategy is the *J1/J2/K* joint momentum strategy instead of the stock momentum factor provided by French's website.

commodity futures contracts are much easier and cheaper to take short positions than stocks. These results confirm the benefits of additional access to the commodity futures market documented in Section 3.3.

3.5. Certainty-equivalent wealth gains from investing using the joint momentum strategy

In this section, we evaluate the outperformance of the joint momentum strategy over the stock-only or the commodity-futures-only momentum strategy from a different aspect. We again employ the certainty-equivalent wealth gain as in Section 3.3 to evaluate the gains from investing in the joint momentum strategy. As we denoted in Section 3.3 (Equation (4)), the certainty-equivalent wealth gain can show the increase in utility by investing in the joint momentum strategy instead of the stock-only or the commodity-futures-only momentum strategy. This is also closely associated with the opportunity cost that Daskalaki and Skiadopoulos (2011) use to assess the economic significance of the difference in performance of two portfolios. In this section, we investigate this effect with two utility functions: the log-utility function ($U(W) = \ln(W)$) and the power-utility function ($U(W) = \frac{W^{1-\gamma}-1}{1-\gamma}$ where $\gamma = 2, 4, 8, 10,$ and $20,$ indicating the level of relative risk aversion (*RRA*)). The log-utility case can be regarded as a special case of the power utility with $\gamma = 1$.

[Insert Table 8]

Table 8 shows the certainty-equivalent wealth gains from investing in the joint momentum strategy (*J1/J2/K* strategy) relative to the case of investing in the *J2/K* stock-only (Panel A) or the *J1/K* commodity-futures-only (Panel B) momentum portfolio. The certainty-equivalent percentage gains are computed for the log-utility function and the power-utility function (with *RRA* = 2, 4, 6, 8, 10, and 20).

In Panel A, the certainty-equivalent wealth gain of the log-utility investor is positive for all *J1/J2/K* strategies, and their values are 0.77% on average. For any fixed *J1/J2*, we find that it tends to increase as *K* decreases for both utility functions. For the power-utility function cases, the certainty-equivalent gains are also positive in most cases, but they have some negative values in the case of *K* = 12 and one negative case for *K* = 6. In most of cases, the values of the certainty-equivalent gains tend to increase as *RRA* increases, but for the *K* = 12 cases, we can see the decreasing pattern of the certainty-equivalent gains. The decreasing pattern in case of *K* = 12 indicates that the more risk-averse investors have a lower increase in utility from the joint momentum strategy. For the *K* = 12 cases, our joint strategy shows larger average return and standard deviation than the stock-only strategy, but the increase of the return is smaller, and that of the standard deviation is larger than other cases. In other words, the increase of the Sharpe ratios is smaller for the *K* = 12 cases. For example, for the choice of *J1/J2* = 1/3, the Sharpe ratio of the 1/3/1 joint strategy is increased from 0.173 (1/3 stock-only strategy) to 0.639 while that of the 1/3/12 joint strategy is increased from 0.260 (1/12 stock-only strategy) to 0.553. Thus, as the investor becomes more risk-averse, the joint momentum strategy becomes less attractive. However, for other joint strategies with smaller *K*, the certainty-equivalent gains show different results because the increase in average returns dominates the increase in volatility of returns. Compared to the profitability of the stock-only momentum in Table 6, the profitability of the joint momentum in Table 7 is much improved.

In Panel B, we find much smaller values of the certainty-equivalent gain and more frequent negative cases than those in Panel A. The certainty-equivalent wealth gain of the log-utility investor is positive for all *J1/J2/K* strategies with only three exceptions, but their values are 0.25% on average as opposed to the average value of 0.77% in Panel A. These results indicate that investors using the momentum strategy in the commodity futures market obtain lower utility gains from the joint momentum strategy relative to the momentum investors in the stock market. For example, in the case of the 12/12/1 momentum strategy, the certainty-equivalent gain from the joint momentum strategy of buying stock winners and selling commodity futures losers relative to the stock momentum strategy is 1.033% for a log-utility investor, while the certainty-equivalent gain relative to the commodity futures momentum strategy is 0.345%, which is less than half of 1.033%, for log-utility investors. For a very risk-averse investor case (*RRA* = 20), the former is 57.905%, while the latter becomes −6.419%.

For the power-utility function, we find positive gains for less risk-averse investors. For more risk-averse investors with *RRA* greater than 4 in Panel B, the certainty-equivalent gains appear to be negative in many cases, but the strategies with small *K* (*K* = 1 or 3) show positive results in general. As in Panel A, the certainty-equivalent gains for very risk-averse investors are negative because of the larger return volatility of the joint strategy relative to that of the commodity-futures-only momentum. The difference between the two panels is mainly due to the fact that the increase in the average returns from the commodity-futures-only momentum to the joint momentum is much smaller than the increase in the average returns from the stock-only momentum to the joint momentum. In Panel B, since the increase in the average returns from the commodity futures momentum to the joint momentum is much smaller than that from the stock momentum to the joint momentum in Panel A, the certainty-equivalent gains are smaller and even become negative more frequently.

In terms of utility, overall results in Panel B of Table 8 are weaker than the results in Panel A of the table, but in terms of the mean and variance of returns, our joint momentum strategy seem to outperform the commodity-futures-only momentum strategy. From the mean-variance investor perspective, the Sharpe ratio, which is the ratio of the mean returns to the variance of returns, is an important measure to assess the asset's performance. If we compare the Sharpe ratios of the commodity-futures-only momentum strategy and the joint momentum strategy, that of the *J1/J2/K* joint momentum strategy is larger than the *J1/K* commodity futures momentum strategy in most of the cases. The *J1/K* strategy seems to have larger Sharpe ratios only for *J1/K* = 1/6 and 1/12 cases, and for other cases the *J1/J2/K* joint strategy generates larger Sharpe ratios. For example, the 6/6 commodity futures strategy has the Sharpe ratio of 0.445 but the 6/6/6 joint strategy has that of 0.583. The difference becomes larger as *J1* increases, so the 12/12 commodity futures strategy shows the negative Sharpe ratio of -0.093 but the 12/12/12 joint strategy shows the positive value as 0.242. Moreover, as in Table A1 in Appendix, the Sharpe ratios of the commodity-futures-only momentum strategies decrease substantially after controlling for the basis effect, while Panel B of Table 7 shows that these of the joint strategies are less affected. For example, the Sharpe ratio of the 1/1 commodity futures strategy is reduced from 0.710 to 0.540 by controlling for the basis effect, but that of the 1/12/1 joint strategy shows smaller decrease as from 0.822 to 0.738. Thus, for a mean-variance investor, our joint strategy still can be interpreted as a better strategy than the commodity-futures-only momentum strategy as well as the stock-only momentum strategy.

To summarize, we find the improved average returns and Sharpe ratios from combining the stock momentum and the commodity futures momentum by buying stock winners and selling commodity futures losers, and also the positive certainty-equivalent wealth gains relative to the stock-only momentum and the commodity-futures-only momentum strategies in general except for the highly risk-averse investor cases. . For the log-utility investors, we find positive gains for all strategies relative to both the stock-only and the commodity-futures-only momentum strategies. For the power-utility investors, we find positive gains in general, but for more risk-averse investors, we find some negative gains, probably due to the increase in return volatility for the joint momentum strategy. Moreover, the certainty-equivalent wealth gains for the power-utility investors appear to be larger when the alternative strategy is the stock-only momentum strategy than the commodity-futures-only momentum strategy. These results show that the joint momentum strategy can be attractive to investors in terms of utility as well as its average return or Sharpe ratio, and also confirms that extending the investment universe to the one including the commodity futures in addition to the traditional assets, such as stocks, is beneficial to investors.

3.6. Momentum profits and business cycle

In this section, we explore the beneficial effect of combining the stock momentum and the commodity futures momentum in another aspect. We investigate the relation between the profitability of momentum strategies and the business cycle. Chordia and Shivakumar (2002) divide the sample into two economic environments – expansionary and contractionary (recessionary) periods – using the NBER definition[12] and document that the stock momentum generates significant returns only in the expansionary period. On the other hand, in the literature on the diversification benefits of commodity futures, Jensen et al. (2000) report that in the restrictive monetary policy period, the efficient portfolios show significant weight on the commodity futures, and including the commodity futures even increases the profitability. Here, we examine how the performance of our joint momentum strategy varies according to the business cycle. As opposed to Chordia and Shivakumar (2002)'s findings on the stock momentum, if the strategy generates positive returns even in the contractionary period, then it will be another positive characteristic of the strategy that investors may like.

Following Chordia and Shivakumar (2002), we use the NBER definition for the expansionary and contractionary periods, and then compute the average monthly returns on the stock-only (*J2/K*), commodity-futures-only (*J1/K*), and the joint momentum strategies (*J1/J2/K*). In addition, we also compute the average monthly (excess) returns on the stock winner, stock loser, commodity futures winner, and commodity futures loser in each period.

[Insert Table 9]

First of all, in Panel A of Table 9, the stock momentum strategies generate much larger returns in the expansionary period than in the contractionary period. Though these returns are statistically insignificant in most cases, this pattern is consistent with the findings of Chordia and Shivakumar (2002). If we take a look into the excess returns on the winner and loser in both periods, we can see that the returns on both the

[12] See www.nber.org/cycles.html

winner and loser increase in the contractionary period but the returns on the loser seem to increase more. In some cases, the loser even outperforms the winner, and thus the stock momentum strategy generates the negative average returns (*J2/K* = 3/1, 12/6, and 12/12).

Next, in Panel B of Table 9, the commodity futures momentum strategies show a similar pattern, i.e., the momentum strategies generate the larger and more significant profits during the expansionary period. The positive but less significant profits in the contractionary period can be due to the small number of sample categorized as the contractionary period,[13] but the pattern that the returns in the expansionary period are larger than the returns in the contractionary period seems to be consistent with only 5 exceptions among 16 test strategies. Though the movement of the returns on the stock and commodity futures momentum strategies seems to be similar as in Panel A and B, the returns on the commodity futures winner and loser appear to be different from these on the stock winner and loser. We find that the returns on both the commodity futures winner and loser decrease in the contractionary period as opposed to the increase of returns on the stock winner and loser. Thus, the commodity futures momentum profits become much smaller and less significant in the contractionary period because the returns on the winner decrease much more than the returns on the loser in this period.

The results in Panel A and B shed a light on the improved profitability of our joint strategy in the contractionary period. The stock winner tends to outperform but the commodity futures loser tends to underperform in the contractionary period than in the expansionary period. Panel C of Table 9 shows the monthly average returns of the *J1/J2/K* momentum strategy during the expansionary and contractionary periods, respectively. In the expansionary period, the returns on the joint momentum strategies are positive in all *J1/J2/K* cases and significant at the 10% significance level with only two exceptions. In the contractionary period, the returns are positive in all cases, and their values are much larger than these in the

[13] According to the NBER classification, only 52 of total 377 months are categorized as the contractionary period during our sample period.

expansionary period in all cases. The returns in the contractionary period are less significant but it can be due to the small number of sample for this period as documented before. However, even under this condition, 12 of 24 test strategies show significant returns at the 10% significance level in the contractionary period. More importantly, compared to the stock-only and the commodity-futures-only momentum strategies, the joint momentum strategies much outperform during the contractionary period.

In sum, the returns on the joint momentum strategies in the expansionary and contractionary periods show that the improvement of the momentum profits relative to the stock-only and the commodity-futures-only momentum strategies is notable during the contractionary period. Our results present the beneficial role of the joint momentum strategy generating the positive payoff in the recessionary period, and thus efficiently managing the risk.

4. Conclusion

Momentum strategies have drawn a lot of attention in the literature, and there are some studies regarding how to improve the performance of the momentum strategy first suggested by Jegadeesh and Titman (1993). For example, in stock markets, Barroso and Santa-Clara (2015) note that the risk of momentum is time-varying and predictable, and they thus construct the risk-managed momentum. They report that this risk-managed momentum strategy avoids crashes and almost doubles the Sharpe ratio of the original momentum strategy. In commodity futures markets, Miffre and Rallis (2007) utilize both the momentum signal and the term-structure signal, which is captured by the basis, and report that the double-screening strategy using both signals outperforms the single-signal strategy. More recently, Fuertes et al. (2015) constructed the triple-screen strategy using the momentum, term structure, and idiosyncratic volatility and report that this triple-screening strategy outperforms the single- or double-signal strategies. Asness et al. (2013) examine the profitability of the momentum strategy in diverse asset universes, such as non-stock asset classes or all asset classes, but their purpose is not to find the portfolio that generates a better performance than the momentum strategy in only one asset class. Thus, improving the performance of the momentum strategy by using various asset classes has rarely been examined.

This paper seeks an improved momentum strategy by combining the stock momentum with the commodity futures momentum. We propose momentum strategies that involve buying stock winners and selling commodity futures losers, and show that our suggested momentum strategies provide higher average returns, larger Sharpe ratios, and utility gains relative to the stock-only momentum or commodity-futures-only momentum strategies. In addition, our new momentum strategies appear to outperform the stock-only and commodity-futures-only strategies during the recession and provide stable profits regardless of the business conditions.

Our suggested momentum strategies are motivated by the optimal portfolio for log-utility investors. Using the numeraire portfolio approach, we identify the weights of a log-utility investor's optimal portfolio on each asset in the stock and commodity futures markets and find notable negative weights on the commodity futures loser. In addition, we find that the certainty-equivalent gain from the access to the commodity futures momentum portfolios is estimated as 1.55% to 3.45% depending on how the commodity futures momentum strategy is constructed. All of the evidence regarding the commodity futures momentum portfolios in this paper indicates that taking a short position on the commodity futures losers can be an optimal strategy. On the other hand, the weights of a log-utility investor's optimal portfolio are positive for both the stock market winners and losers when commodity futures portfolios constructed according to their past returns are included in the investment universe. Combining the positive weights on stock winners with negative weights on commodity futures losers in the optimal portfolio of log-utility investors, we suggest a new momentum strategy that involves buying stock winners and selling commodity futures losers in this paper.

The buying-stock-winners and selling-commodity-futures-losers momentum strategies suggested in this paper yield large profits during our sample period. For example, when we construct stock winners and commodity futures losers based on the past 6-month returns and hold our momentum portfolio for the next month, the monthly return on the joint strategy is on average 1.463%, which is much larger than the stock-only momentum strategy (0.387%) or the commodity-futures-only momentum strategy (1.131%). In the case when we construct stock winners and commodity futures losers based on the past 12-month returns instead of 6 months and hold our momentum portfolio for the next month, the monthly return on the joint strategy becomes even higher at 1.709% on average, which is much larger than the stock-only momentum strategy (0.679%) or the commodity-futures-only momentum strategy (1.335%). The Sharpe ratios of our momentum strategies are also higher than those of the stock-only momentum strategies or the commodity-futures-only momentum strategies.

Our new momentum strategy has another advantage over the stock-only momentum strategy: Selling commodity futures is more attractive than selling stocks. Selling commodity futures is a more implementable strategy than selling stocks because they are not subject to short-selling restrictions, unlike stocks, and they even offer leverage. This difference in the short-sale costs is important as Lesmond et al. (2004) document that the profits of the stock momentum mainly stem from the short side and they become insignificant if the short-sale costs are considered. Our joint strategy is free from this issue.

Our paper also confirms that access to commodity futures markets benefits the investors that invest only in the US stock market. The certainty-equivalent wealth gains from the access to commodity futures markets are estimated as 1.55% to 3.45%. This shows that the importance of commodity futures markets cannot be ignored. We expect more research in the future regarding the joint investment strategies that exploit commodity futures in addition to stocks.

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Table 1. Summary statistics of excess gross returns on portfolios

This table shows summary statistics of stock and commodity futures portfolios. For commodity futures portfolios, we sort commodity futures contracts into quintiles (*Cmom1*, *Cmom2*, *Cmom3*, *Cmom4*, and *Cmom5*) based on the past *J*-month returns (*J* = 1, 3, 6, and 12), and for stock momentum portfolios, we use French's six portfolios formed monthly on the basis of the firm size and momentum (2×3). These six portfolios are the intersection of the two portfolios formed on the firm size and the three portfolios formed on the prior 2- to 12-month return. We compute the average of two portfolios with different sizes in each momentum group, and thus finally construct three momentum portfolios (*Smom1*, *Smom2*, and *Smom3*). We also employ Fama and French's (2015) stock market portfolios formed on the firm size (*SIZE*), book-to-market ratio (*BTM*), profitability (*OP*), and investment (*INV*). Table 3 in Fama and French (2015) shows the details of the portfolio construction for *SIZE*, *BTM*, *OP*, and *INV*. Using the 3-month Treasury-bill rate as the risk-free rate, we compute the excess return on each portfolio. This table shows the minimum, maximum, mean, standard deviation, skewness, and kurtosis of monthly excess gross returns on these portfolios. The sample period is from January 1979 to June 2015.

	Portfolio	Min	Max	Mean	SD	Skew	Kurt
	SIZE1	0.7012	1.1927	1.0080	0.0564	-0.749	2.814
	SIZE2	0.7854	1.1221	1.0066	0.0442	-0.626	2.125
	BTM1	0.7170	1.1478	1.0058	0.0546	-0.642	2.041
	BTM3	0.7594	1.1701	1.0086	0.0470	-1.085	3.981
	INV1	0.7395	1.1422	1.0087	0.0486	-0.812	2.841
	INV2	0.7622	1.1360	1.0084	0.0429	-0.968	3.644
	INV3	0.7167	1.1415	1.0055	0.0559	-0.753	2.089
	OP1	0.7200	1.1819	1.0050	0.0562	-0.694	2.295
	OP2	0.7550	1.1440	1.0076	0.0459	-0.888	3.060
	OP3	0.7369	1.1321	1.0086	0.0472	-0.893	3.220
	Smom1	0.7518	1.4011	1.0041	0.0634	0.387	4.849
	Smom2	0.7475	1.1797	1.0072	0.0441	-0.814	4.132
	Smom3	0.7162	1.1640	1.0102	0.0530	-0.801	2.939
J = 1	*Cmom1*	0.8120	1.1589	0.9945	0.0572	-0.250	0.440
	Cmom2	0.7706	1.1542	0.9990	0.0440	-0.131	1.897
	Cmom3	0.7992	1.2017	1.0025	0.0412	0.245	3.355
	Cmom4	0.7790	1.4287	1.0032	0.0501	1.362	13.830
	Cmom5	0.7458	1.2283	1.0091	0.0599	0.170	2.054

J = 3	*Cmom1*	0.8098	1.2130	0.9973	0.0550	-0.123	0.993
	Cmom2	0.7663	1.1639	0.9971	0.0425	-0.182	2.796
	Cmom3	0.7768	1.2020	0.9990	0.0429	-0.091	2.864
	Cmom4	0.8469	1.1684	1.0045	0.0451	-0.057	1.113
	Cmom5	0.5933	1.2791	1.0100	0.0642	-0.158	5.586
J = 6	*Cmom1*	0.8036	1.2150	0.9966	0.0574	0.055	1.171
	Cmom2	0.7936	1.1992	0.9993	0.0462	0.443	2.913
	Cmom3	0.8478	1.1475	1.0030	0.0416	0.004	0.954
	Cmom4	0.7998	1.1525	1.0022	0.0431	-0.030	2.168
	Cmom5	0.5961	1.2661	1.0063	0.0614	-0.581	6.226
J = 12	*Cmom1*	0.8244	1.2341	0.9956	0.0550	0.220	1.286
	Cmom2	0.8339	1.1727	0.9987	0.0432	0.162	1.853
	Cmom3	0.8712	1.1724	1.0001	0.0399	0.269	1.524
	Cmom4	0.7767	1.1809	1.0051	0.0459	-0.233	3.427
	Cmom5	0.5933	1.2242	1.0090	0.0614	-0.723	5.958

Table 2. Summary statistics of risk premiums

This table shows summary statistics of the size (*SMB*), value (*HML*), investment (*CMA*), profitability (*RMW*), stock momentum (*SWML*), and commodity futures momentum (*CWMLJ* for *J* = 1, 3, 6, and 12) factors. As in Fama and French (2015), *SMB*, *HML*, *CMA*, *RMW*, SWML, and *CWML* are defined as (*SIZE1* – *SIZE2*), (*BTM3* – *BTM1*), (*OP3* – *OP1*), (*INV1* – *INV3*), (*Smom3* – *Smom1*), and (*Cmom5* – *Cmom1*). We compute the minimum, maximum, mean, standard deviation, skewness, and kurtosis of each factor and correlations among them. The sample period is from January 1979 to June 2015.

	Min	Max	Mean	SD	Skew	Kurt	*HML*	*CMA*	*RMA*	*SWML*	*CWML 1*	*CWML 3*	*CWML 6*	*CWML 12*
SMB	-14.083	18.477	0.137	2.907	0.472	4.772	-0.204	-0.080	-0.453	0.027	-0.088	-0.029	-0.033	-0.059
HML	-13.110	13.905	0.285	3.031	0.028	2.690	1.000	0.698	0.274	-0.179	0.003	-0.001	-0.005	0.013
CMA	-6.815	9.505	0.318	2.033	0.369	1.937		1.000	0.074	0.025	0.014	0.014	0.016	0.066
RMA	-17.575	12.190	0.363	2.335	-0.529	11.717			1.000	0.090	0.043	0.027	0.015	0.032
SWML	-34.580	18.375	0.607	4.568	-1.461	11.332				1.000	0.015	-0.026	-0.035	0.037
CWML1	-23.479	29.138	1.455	7.640	0.160	0.571					1.000	0.591	0.461	0.349
CWML3	-35.614	30.540	1.262	7.680	-0.142	2.043						1.000	0.704	0.540
CWML6	-32.771	26.744	0.964	7.328	-0.270	1.460							1.000	0.689
CWML12	-29.978	24.822	1.335	7.529	-0.140	0.721								1.000

Table 3. Numeraire portfolio discount factor

This table shows estimates of the numeraire portfolio discount factor for numeraire portfolios that consist of all stock market portfolios and commodity futures portfolios. Panel A shows the weights on each portfolio and Panel B shows the summary statistics for the numeraire portfolio discount factor. In Panel A, the numbers in bold type indicate significance at the 5% level. For commodity futures portfolios, we sort commodity futures contracts into quintiles (*Cmom1*, *Cmom2*, *Cmom3*, *Cmom4*, and *Cmom5*) based on the past *J*-month returns (*J* = 1, 3, 6, and 12), and for stock momentum portfolios, we use French's six portfolios formed monthly on the basis of the firm size and momentum (2×3). These six portfolios are the intersection of the two portfolios formed on the firm size and the three portfolios formed on the prior 2- to 12-month return. We compute the average of two portfolios with different sizes in each momentum group, and thus finally construct three momentum portfolios (*Smom1*, *Smom2*, and *Smom3*). We also employ Fama and French's (2015) stock market portfolios formed on the firm size (*SIZE*), book-to-market ratio (*BTM*), profitability (*OP*), and investment (*INV*). Table 3 in Fama and French (2015) shows the details of the portfolio construction for *SIZE*, *BTM*, *OP*, and *INV*. The sample period is from January 1979 to June 2015.

Panel A. Numeraire portfolio weights

J	*SIZE 1*	*SIZE 2*	*BTM 1*	*BTM 3*	*INV 1*	*INV 2*	*INV 3*	*OP 1*	*OP 2*	*OP 3*	*Smom 1*	*Smom 2*	*Smom 3*	*Cmom 1*	*Cmom 2*	*Cmom 3*	*Cmom 4*	*Cmom 5*
1	**-90.9**	**-98.1**	**-6.1**	4.2	**46.5**	**58.3**	**27.0**	**-14.1**	-4.2	15.7	**26.3**	4.7	**33.5**	**-2.7**	**-1.2**	1.0	**-0.6**	1.8
3	**-104.1**	**-109.5**	**-4.4**	4.7	**46.1**	**58.2**	22.6	**-8.3**	**4.5**	**18.2**	**28.9**	8.8	**37.4**	-0.8	**-3.5**	-1.8	**1.5**	**2.4**
6	**-107.5**	**-112.8**	**-7.9**	1.4	**46.2**	**64.8**	**26.5**	-7.4	5.5	**17.9**	**30.4**	7.2	**38.5**	-1.8	-1.5	**1.0**	**-0.6**	**1.2**
12	**-140.4**	**-146.4**	**-6.9**	**4.4**	**53.3**	**74.3**	**32.8**	-0.8	**17.0**	23.4	**34.9**	15.2	**42.6**	**-1.8**	**-2.0**	**-4.0**	**3.4**	**2.0**

Panel B. Summary statistics

J	Min	Max	Mean	STD
1	0.307	5.281	0.998	0.598
3	0.292	7.120	1.001	0.690
6	0.300	9.198	0.999	0.660
12	0.271	6.809	0.997	0.681

Table 4. Benefits from access to the commodity futures momentum portfolios

This table presents the certainty-equivalent wealth gains from the access to the commodity futures markets. The certainty-equivalent wealth gains are measured as a percentage, and the numbers in parentheses are *t*-statistics corrected by the Newey–West (1987) method. The sample period is from January 1979 to June 2015.

	J		
1	3	6	12
2.356	3.322	1.547	3.449

Table 5. Profitability of joint momentum strategy using market portfolios

This table presents the average monthly return on the stock momentum (*SWML*), commodity futures momentum (*CWMLJ* for *J* = 1, 3, 6, and 12), stock-commodity momentum (*Smom3* – *Cmom1*), and commodity-stock (*Cmom5* – *Smom1*) portfolios. For commodity futures portfolios, we sort commodity futures contracts into quintiles (*Cmom1*, *Cmom2*, *Cmom3*, *Cmom4*, and *Cmom5*) based on the past *J*-month returns (*J* = 1, 3, 6, and 12), and for stock momentum portfolios, we use French's six portfolios formed monthly on the basis of the firm size and momentum (2×3). These six portfolios are the intersection of the two portfolios formed on the firm size and the three portfolios formed on the prior 2- to 12-month return. We compute the average of two portfolios with different sizes in each momentum group, and thus finally construct three momentum portfolios (*Smom1*, *Smom2*, and *Smom3*). As in Fama and French (2015), *SWML* and *CWML* are defined as (*Smom3* – *Smom1*), and (*Cmom5* – *Cmom1*). The numbers in parentheses are *t*-statistics corrected by the Newey–West (1987) method. The sample period is from January 1979 to June 2015.

	SWML	*CWML*				*Smom3* – *Cmom1*				*Cmom5* – *Smom1*			
		J=1	*J*=3	*J*=6	*J*=12	*J*=1	*J*=3	*J*=6	*J*=12	*J*=1	*J*=3	*J*=6	*J*=12
Mean	0.607	1.455	1.262	0.964	1.335	1.564	1.286	1.354	1.456	0.498	0.583	0.217	0.486
	(2.68)	(4.10)	(3.52)	(2.83)	(3.53)	(4.32)	(3.71)	(3.76)	(3.95)	(1.14)	(1.34)	(0.52)	(1.11)

Table 6. Profitability of momentum strategy in stock and commodity futures markets

This table presents the average monthly (excess) return on momentum strategies in stock (Panel A) and commodity futures (Panel B) markets. We report the excess returns for each portfolio to make the stock returns and the commodity futures returns are comparable. We use the 3-month Treasury-bill rate as the risk-free rate. *J1* (*J2*) indicates the ranking period of the commodity futures (stock) momentum portfolios, and *K* indicates the holding period of the stock momentum (Panel A) and the commodity futures momentum (Panel B). SR indicates the annualized Sharpe ratio and std. indicates the standard deviation of returns. The numbers in parentheses are *t*-statistics corrected by the Newey–West (1987) method. The sample period is from January 1979 to June 2015.

Panel A. Momentum in the stock market

		J2 = 3				*J2* = 6				*J2* = 12			
		K = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12
W	Mean	0.930	0.871	0.919	0.935	1.139	1.138	1.130	0.973	1.357	1.210	1.065	0.842
		(2.53)	(2.40)	(2.58)	(2.62)	(3.26)	(3.25)	(3.24)	(2.79)	(3.82)	(3.42)	(3.05)	(2.45)
	std.	6.504	6.472	6.442	6.523	6.397	6.412	6.405	6.471	6.557	6.579	6.567	6.516
	SR	0.495	0.466	0.494	0.497	0.617	0.615	0.611	0.521	0.717	0.637	0.562	0.448
L	Mean	0.693	0.693	0.679	0.723	0.752	0.718	0.682	0.794	0.678	0.717	0.815	1.021
		(1.55)	(1.59)	(1.56)	(1.71)	(1.59)	(1.53)	(1.47)	(1.79)	(1.39)	(1.49)	(1.72)	(2.21)
	std.	8.378	8.233	8.131	7.769	8.872	8.813	8.606	8.116	9.032	8.879	8.690	8.352
	SR	0.287	0.292	0.289	0.322	0.294	0.282	0.275	0.339	0.260	0.280	0.325	0.423
W-L	Mean	0.236	0.178	0.241	0.212	0.387	0.419	0.448	0.179	0.679	0.494	0.250	-0.179
		(1.13)	(0.98)	(1.43)	(1.56)	(1.34)	(1.53)	(1.78)	(0.89)	(2.09)	(1.60)	(0.86)	(-0.72)
	std.	4.737	4.224	3.776	2.825	6.090	5.818	5.230	3.982	6.574	6.146	5.653	4.789
	SR	0.173	0.146	0.221	0.260	0.220	0.250	0.297	0.156	0.358	0.278	0.153	-0.130

Panel B. Momentum in the commodity futures market

		J1 = 1				*J1* = 3				*J1* = 6				*J1* = 12			
		K = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12
W	Mean	1.048	0.745	0.578	0.513	1.203	0.761	0.553	0.549	0.789	0.639	0.652	0.423	0.897	0.570	0.294	0.119
		(3.38)	(2.93)	(2.52)	(2.38)	(3.63)	(2.62)	(2.19)	(2.30)	(2.63)	(2.18)	(2.40)	(1.60)	(3.00)	(1.99)	(1.06)	(0.44)

	std.	6.269	5.188	4.710	4.288	6.778	5.858	5.168	4.697	6.354	6.057	5.433	5.140	6.144	5.724	5.510	5.408
	SR	0.579	0.498	0.425	0.415	0.615	0.450	0.371	0.405	0.430	0.365	0.416	0.285	0.506	0.345	0.185	0.076
L	Mean	-0.550	-0.105	-0.097	-0.129	-0.222	-0.030	-0.013	-0.050	-0.342	-0.119	-0.118	0.026	-0.437	-0.104	0.106	0.270
		(-2.01)	(-0.49)	(-0.46)	(-0.66)	(-0.83)	(-0.12)	(-0.06)	(-0.24)	(-1.24)	(-0.45)	(-0.46)	(0.11)	(-1.59)	(-0.38)	(0.41)	(1.14)
	std.	5.683	4.342	4.114	3.864	5.508	5.044	4.551	4.200	5.732	5.268	5.002	4.550	5.497	5.363	5.066	4.705
	SR	-0.335	-0.084	-0.081	-0.115	-0.140	-0.021	-0.010	-0.041	-0.207	-0.078	-0.082	0.020	-0.276	-0.067	0.073	0.199
W-L	Mean	1.598	0.850	0.674	0.642	1.425	0.791	0.566	0.599	1.131	0.758	0.770	0.397	1.335	0.673	0.188	-0.151
		(4.38)	(3.71)	(3.95)	(4.73)	(3.81)	(2.46)	(2.33)	(2.95)	(3.17)	(2.36)	(2.57)	(1.51)	(3.53)	(1.92)	(0.58)	(-0.53)
	std.	7.800	4.865	3.687	2.756	7.913	6.608	5.083	3.939	7.523	6.749	5.991	4.996	7.529	6.847	6.298	5.608
	SR	0.710	0.605	0.633	0.807	0.624	0.415	0.386	0.527	0.521	0.389	0.445	0.275	0.614	0.341	0.103	-0.093

Table 7. Profitability of joint momentum strategy

This table presents the average monthly return on *J1/J2/K* joint momentum strategies using both stock and commodity futures. *J1* (*J2*) indicates the ranking period of the stock (commodity futures) momentum portfolios, and *K* indicates the holding period of the strategy. In Panel A, commodity futures portfolios are constructed without controlling for the basis, while in Panel B, they are constructed after controlling for the basis. SR indicates the annualized Sharpe ratio and std. indicates the standard deviation of returns. The numbers in parentheses are *t*-statistics corrected by the Newey–West (1987) method. The sample period is from January 1979 to June 2015.

	J1/J2/K Strategy	Stock winner − commodity loser	Commodity winner − stock loser	Stock winner − commodity loser	Commodity winner − stock loser
		Panel A. Uncontrolled		Panel B. Basis controlled	
Mean	1/3/1	1.480	0.355	1.133	-0.002
		(3.49)	(0.69)	(2.84)	(0.00)
std.		8.019	9.726	7.287	8.845
SR		0.639	0.126	0.539	-0.001
Mean	1/3/3	0.976	0.052	0.878	-0.168
		(2.60)	(0.11)	(2.40)	(-0.38)
std.		6.997	8.886	6.681	8.437
SR		0.483	0.020	0.455	-0.069
Mean	1/3/6	1.016	-0.101	0.922	-0.260
		(2.80)	(-0.23)	(2.61)	(-0.60)
std.		6.710	8.475	6.462	8.211
SR		0.525	-0.041	0.494	-0.110
Mean	1/3/12	1.064	-0.209	0.924	-0.307
		(2.95)	(-0.49)	(2.62)	(-0.73)
std.		6.668	8.003	6.494	7.780
SR		0.553	-0.091	0.493	-0.137
Mean	3/3/1	1.127	0.530	1.054	0.146
		(2.75)	(1.01)	(2.70)	(0.31)
std.		7.806	10.198	7.236	9.058
SR		0.500	0.180	0.504	0.056
Mean	3/3/3	0.879	0.088	0.822	-0.086
		(2.22)	(0.18)	(2.18)	(-0.19)
std.		7.414	9.354	6.885	8.629
SR		0.411	0.033	0.413	-0.035
Mean	3/3/6	0.913	-0.103	0.866	-0.213
		(2.43)	(-0.22)	(2.38)	(-0.48)

std.		6.988	8.800	6.654	8.364
SR		0.453	-0.041	0.451	-0.088
Mean	3/3/12	0.964	-0.153	0.886	-0.264
		(2.60)	(-0.34)	(2.48)	(-0.62)
std.		6.872	8.215	6.571	7.868
SR		0.486	-0.065	0.467	-0.116
Mean	6/6/1	1.463	0.062	1.313	-0.288
		(3.53)	(0.12)	(3.48)	(-0.57)
std.		7.841	10.279	7.094	9.504
SR		0.646	0.021	0.641	-0.105
Mean	6/6/3	1.238	-0.055	1.101	-0.367
		(3.07)	(-0.11)	(2.97)	(-0.75)
std.		7.534	9.951	6.909	9.247
SR		0.569	-0.019	0.552	-0.137
Mean	6/6/6	1.228	-0.005	1.133	-0.282
		(3.10)	(-0.01)	(3.03)	(-0.59)
std.		7.300	9.456	6.840	8.875
SR		0.583	-0.002	0.574	-0.110
Mean	6/6/12	0.923	-0.347	0.865	-0.462
		(2.44)	(-0.72)	(2.39)	(-1.01)
std.		7.041	8.719	6.685	8.297
SR		0.454	-0.138	0.448	-0.193
Mean	12/12/1	1.709	0.256	1.451	-0.191
		(4.04)	(0.46)	(3.67)	(-0.37)
std.		7.868	10.378	7.276	9.688
SR		0.752	0.086	0.691	-0.068
Mean	12/12/3	1.231	-0.110	1.078	-0.414
		(2.98)	(-0.20)	(2.79)	(-0.81)
std.		7.764	9.898	7.117	9.332
SR		0.549	-0.038	0.525	-0.154
Mean	12/12/6	0.882	-0.480	0.794	-0.638
		(2.23)	(-0.92)	(2.11)	(-1.29)
std.		7.467	9.477	6.990	9.021
SR		0.409	-0.175	0.393	-0.245
Mean	12/12/12	0.500	-0.854	0.484	-0.896
		(1.35)	(-1.70)	(1.37)	(-1.87)
std.		7.150	9.059	6.707	8.605
SR		0.242	-0.327	0.250	-0.361

Mean	1/6/1	1.689	0.296	1.342	-0.060
		(4.14)	(0.55)	(3.49)	(-0.12)
std.		7.951	10.170	7.200	9.320
SR		0.736	0.101	0.646	-0.022
Mean	1/6/3	1.243	0.027	1.145	-0.193
		(3.42)	(0.05)	(3.23)	(-0.40)
std.		6.906	9.404	6.582	8.976
SR		0.623	0.010	0.603	-0.075
Mean	1/6/6	1.227	-0.105	1.133	-0.264
		(3.40)	(-0.22)	(3.23)	(-0.57)
std.		6.707	8.930	6.447	8.671
SR		0.634	-0.041	0.609	-0.105
Mean	1/6/12	1.102	-0.281	0.962	-0.379
		(3.10)	(-0.62)	(2.77)	(-0.86)
std.		6.625	8.345	6.451	8.123
SR		0.576	-0.116	0.517	-0.161
Mean	1/12/1	1.907	0.370	1.561	0.014
		(4.59)	(0.66)	(3.96)	(0.03)
std.		8.036	10.357	7.321	9.485
SR		0.822	0.124	0.738	0.005
Mean	1/12/3	1.315	0.029	1.218	-0.191
		(3.57)	(0.06)	(3.38)	(-0.39)
std.		7.048	9.475	6.748	9.039
SR		0.646	0.011	0.625	-0.073
Mean	1/12/6	1.162	-0.238	1.068	-0.397
		(3.23)	(-0.49)	(3.05)	(-0.84)
std.		6.834	9.003	6.594	8.753
SR		0.589	-0.092	0.561	-0.157
Mean	1/12/12	0.970	-0.508	0.831	-0.605
		(2.78)	(-1.08)	(2.44)	(-1.32)
std.		6.648	8.568	6.482	8.355
SR		0.506	-0.205	0.444	-0.251

Table 8. Certainty-equivalent wealth gain from joint momentum strategy

This table shows the certainty-equivalent wealth gains from investing in the joint momentum strategy (*J1/J2/K* strategy) instead of the *J2/K* stock-only (Panel A) or the *J1/K* commodity-futures-only (Panel B) momentum strategy. *J1* (*J2*) indicates the ranking period of the commodity futures (stock) momentum portfolios, and *K* indicates the holding period of the strategy. The certainty-equivalent percentage gains are computed for log-utility function (the special case of the power-utility function with *RRA* = 1) and power-utility function (with *RRA* = 2, 4, 6, 8, 10, and 20). The sample period is from January 1979 to June 2015.

J1/J2/K Strategy	Log utility	Power utility					
		RRA = 2	*RRA* = 4	*RRA* = 6	*RRA* = 8	*RRA* = 10	*RRA* = 20
Panel A. Joint momentum vs. stock momentum							
1/3/1	1.051	0.881	0.644	0.716	1.535	3.872	26.132
1/3/3	0.652	0.519	0.319	0.319	0.798	2.278	19.512
1/3/6	0.627	0.486	0.234	0.058	0.048	0.361	8.510
1/3/12	0.669	0.487	0.111	-0.285	-0.717	-1.207	-5.131
3/3/1	0.719	0.561	0.348	0.436	1.265	3.594	25.081
3/3/3	0.527	0.362	0.094	0.014	0.395	1.739	16.961
3/3/6	0.502	0.339	0.035	-0.211	-0.331	-0.197	5.003
3/3/12	0.557	0.360	-0.044	-0.475	-0.953	-1.512	-6.451
6/6/1	0.988	0.973	1.331	3.007	7.807	16.479	45.852
6/6/3	0.730	0.700	0.960	2.333	6.497	14.491	41.378
6/6/6	0.666	0.589	0.613	1.213	3.262	7.846	28.623
6/6/12	0.580	0.418	0.095	-0.233	-0.579	-0.971	-4.191
12/12/1	1.033	1.071	1.711	4.642	13.327	26.981	57.905
12/12/3	0.699	0.655	0.773	1.521	3.766	8.438	27.380
12/12/6	0.567	0.483	0.389	0.460	0.832	1.690	8.161
12/12/12	0.571	0.439	0.178	-0.086	-0.375	-0.729	-4.910
1/6/1	1.213	1.192	1.548	3.241	8.102	16.967	51.442
1/6/3	0.786	0.804	1.172	2.682	7.047	15.413	47.876
1/6/6	0.711	0.678	0.803	1.533	3.776	8.696	34.637
1/6/12	0.786	0.653	0.392	0.140	-0.101	-0.329	-1.193
1/12/1	1.166	1.192	1.815	4.742	13.509	27.601	68.353
1/12/3	0.787	0.795	1.028	1.921	4.384	9.459	34.607
1/12/6	0.854	0.816	0.823	1.024	1.588	2.764	13.984
1/12/12	1.053	0.959	0.784	0.630	0.497	0.379	-0.848
Panel B. Joint momentum vs. commodity futures momentum							
1/3/1	-0.136	-0.158	-0.213	-0.285	-0.381	-0.509	-1.843
1/3/3	-0.002	-0.131	-0.400	-0.687	-0.995	-1.328	-3.056
1/3/6	0.182	0.019	-0.327	-0.706	-1.134	-1.633	-5.995
1/3/12	0.234	0.045	-0.355	-0.796	-1.298	-1.896	-7.659

3/3/1	-0.285	-0.272	-0.225	-0.122	0.080	0.455	5.993
3/3/3	0.030	-0.028	-0.144	-0.248	-0.323	-0.344	0.834
3/3/6	0.230	0.110	-0.144	-0.423	-0.738	-1.104	-3.416
3/3/12	0.204	0.040	-0.310	-0.702	-1.164	-1.739	-7.896
6/6/1	0.307	0.288	0.252	0.225	0.209	0.206	0.112
6/6/3	0.419	0.357	0.214	0.039	-0.185	-0.486	-3.219
6/6/6	0.364	0.266	0.037	-0.259	-0.667	-1.260	-8.164
6/6/12	0.399	0.269	-0.015	-0.346	-0.757	-1.301	-7.656
12/12/1	0.345	0.317	0.243	0.115	-0.117	-0.541	-6.419
12/12/3	0.486	0.413	0.239	0.005	-0.332	-0.844	-6.041
12/12/6	0.614	0.531	0.346	0.120	-0.181	-0.615	-5.140
12/12/12	0.555	0.455	0.244	0.011	-0.262	-0.604	-3.525
1/6/1	0.075	0.057	0.007	-0.065	-0.165	-0.300	-1.689
1/6/3	0.270	0.147	-0.113	-0.394	-0.703	-1.049	-3.281
1/6/6	0.391	0.227	-0.124	-0.517	-0.973	-1.523	-6.739
1/6/12	0.275	0.087	-0.310	-0.749	-1.251	-1.848	-7.543
1/12/1	0.283	0.256	0.180	0.065	-0.105	-0.350	-3.135
1/12/3	0.331	0.196	-0.096	-0.423	-0.803	-1.256	-4.680
1/12/6	0.318	0.144	-0.229	-0.650	-1.140	-1.730	-6.988
1/12/12	0.142	-0.048	-0.453	-0.902	-1.418	-2.033	-7.726

Table 9. Profitability of momentum strategies classified by business cycles

This table presents the average monthly (excess) return on the stock-only momentum (Panel A), the commodity-futures-only, and the joint momentum strategies (Panel C) during the expansionary and contractionary periods. We report the excess returns for each of the winner and loser portfolios to make the stock returns and the commodity futures returns are comparable using the 3-month Treasury-bill rate as the risk-free rate. *J1* (*J2*) indicates the ranking period of the commodity futures (stock) momentum portfolios, and *K* indicates the holding period of the stock momentum (Panel A and Panel C) and the commodity futures momentum (Panel B and Panel C). The numbers in parentheses are *t*-statistics corrected by the Newey–West (1987) method. The sample period is from January 1979 to June 2015.

Panel A. Stock-only momentum portfolio

		J2=3				*J2*=6				*J2*=12			
		K=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12
W	Expansionary	0.873	0.790	0.835	0.845	1.091	1.057	1.035	0.884	1.286	1.131	0.975	0.727
		(2.06)	(1.89)	(2.02)	(2.04)	(2.69)	(2.61)	(2.55)	(2.18)	(3.10)	(2.72)	(2.38)	(1.82)
	Contractionary	0.866	0.920	1.092	1.054	1.091	1.377	1.465	1.101	1.619	1.367	1.163	0.907
		(0.67)	(0.73)	(0.90)	(0.87)	(0.95)	(1.21)	(1.29)	(0.95)	(1.43)	(1.23)	(1.06)	(0.80)
L	Expansionary	0.640	0.613	0.603	0.646	0.705	0.665	0.613	0.724	0.610	0.638	0.736	0.974
		(1.21)	(1.20)	(1.18)	(1.31)	(1.27)	(1.21)	(1.14)	(1.41)	(1.07)	(1.15)	(1.34)	(1.81)
	Contractionary	1.055	0.895	0.810	0.947	0.931	0.775	0.779	1.087	1.015	1.121	1.283	1.456
		(0.70)	(0.60)	(0.53)	(0.63)	(0.57)	(0.47)	(0.47)	(0.68)	(0.57)	(0.63)	(0.73)	(0.88)
W - L	Expansionary	0.234	0.177	0.232	0.200	0.386	0.392	0.422	0.160	0.675	0.493	0.239	-0.247
		(0.92)	(0.83)	(1.12)	(1.24)	(1.09)	(1.16)	(1.39)	(0.67)	(1.74)	(1.36)	(0.71)	-(0.84)
	Contractionary	-0.188	0.025	0.282	0.107	0.160	0.602	0.686	0.014	0.604	0.246	-0.121	-0.549
		-(0.33)	(0.04)	(0.56)	(0.23)	(0.19)	(0.77)	(0.85)	(0.02)	(0.53)	(0.22)	-(0.11)	-(0.62)

Panel B. Commodity-futures-only momentum portfolio

		J1=1				*J1*=3				*J1*=6				*J1*=12			
		K=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12
W	Expansionary	1.072	0.845	0.695	0.613	1.379	0.953	0.691	0.643	0.915	0.800	0.713	0.471	0.872	0.593	0.302	0.106
		(3.12)	(2.94)	(2.69)	(2.52)	(3.60)	(2.80)	(2.37)	(2.34)	(2.54)	(2.27)	(2.21)	(1.53)	(2.58)	(1.84)	(0.97)	(0.36)

	Contractionary	0.908	0.071	-0.309	-0.330	0.319	-0.521	-0.570	-0.238	-0.317	-0.597	-0.147	-0.299	0.778	-0.151	-0.457	-0.581
		(0.92)	(0.09)	-(0.42)	-(0.49)	(0.35)	-(0.58)	-(0.68)	-(0.31)	-(0.38)	-(0.73)	-(0.18)	-(0.35)	(0.78)	-(0.16)	-(0.48)	-(0.56)
L	Expansionary	-0.597	-0.120	-0.091	-0.072	-0.187	-0.060	-0.040	-0.008	-0.358	-0.160	-0.156	0.061	-0.391	0.000	0.224	0.459
		-(1.85)	-(0.50)	-(0.39)	-(0.32)	-(0.61)	-(0.22)	-(0.16)	-(0.03)	-(1.12)	-(0.55)	-(0.55)	(0.24)	-(1.22)	(0.00)	(0.77)	(1.72)
	Contractionary	-1.323	-0.582	-0.614	-0.818	-0.909	-0.510	-0.496	-0.679	-1.397	-0.914	-0.642	-0.587	-1.262	-1.069	-0.726	-0.555
		-(2.02)	-(0.86)	-(0.93)	-(1.36)	-(1.12)	-(0.68)	-(0.67)	-(1.03)	-(1.77)	-(1.10)	-(0.79)	-(0.79)	-(1.42)	-(1.24)	-(0.87)	-(0.73)
W - L	Expansionary	1.669	0.965	0.786	0.685	1.566	1.013	0.732	0.650	1.272	0.960	0.869	0.409	1.263	0.593	0.078	-0.354
		(4.07)	(3.69)	(3.88)	(4.16)	(3.62)	(2.61)	(2.48)	(2.63)	(2.90)	(2.44)	(2.39)	(1.31)	(2.84)	(1.39)	(0.20)	-(1.11)
	Contractionary	2.231	0.653	0.306	0.488	1.228	-0.012	-0.074	0.441	1.080	0.317	0.494	0.289	2.040	0.918	0.268	-0.027
		(2.33)	(0.98)	(0.61)	(1.28)	(1.08)	-(0.01)	-(0.09)	(0.73)	(1.14)	(0.33)	(0.56)	(0.35)	(1.63)	(0.82)	(0.26)	-(0.03)

Panel C. Joint momentum portfolio

		J1/J2=1/3				*J1/J2*=1/6				*J1/J2*=1/12			
		K=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12
W - L	Expansionary	1.471	0.910	0.926	0.918	1.688	1.177	1.127	0.956	1.883	1.251	1.066	0.800
		(2.97)	(2.13)	(2.21)	(2.17)	(3.54)	(2.84)	(2.68)	(2.29)	(3.85)	(2.94)	(2.52)	(1.95)
	Contractionary	2.189	1.502	1.706	1.871	2.413	1.959	2.079	1.919	2.942	1.949	1.777	1.725
		(1.76)	(1.16)	(1.43)	(1.64)	(2.16)	(1.62)	(1.81)	(1.75)	(2.63)	(1.66)	(1.59)	(1.62)
		J1/J2=3/3				*J1/J2*=6/6				*J1/J2*=12/12			
		K=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12	*K*=1	*K*=3	*K*=6	*K*=12
W - L	Expansionary	1.026	0.821	0.849	0.824	1.424	1.191	1.162	0.788	1.558	1.015	0.643	0.168
		(2.20)	(1.79)	(1.95)	(1.88)	(2.92)	(2.54)	(2.47)	(1.74)	(3.06)	(2.03)	(1.34)	(0.38)
	Contractionary	1.775	1.430	1.588	1.732	2.488	2.291	2.107	1.688	2.882	2.436	1.888	1.462
		(1.27)	(1.11)	(1.30)	(1.54)	(2.15)	(1.82)	(1.86)	(1.59)	(2.54)	(2.20)	(1.82)	(1.48)

Figure 1. Time-series of stock momentum and commodity futures momentum

This figure shows the time-series of the stock momentum (*SWML*, dotted line) and the commodity futures momentum (*CWML12*, solid line) constructed by the past 12-month return (J = 12). The sample period is from January 1979 to June 2015.

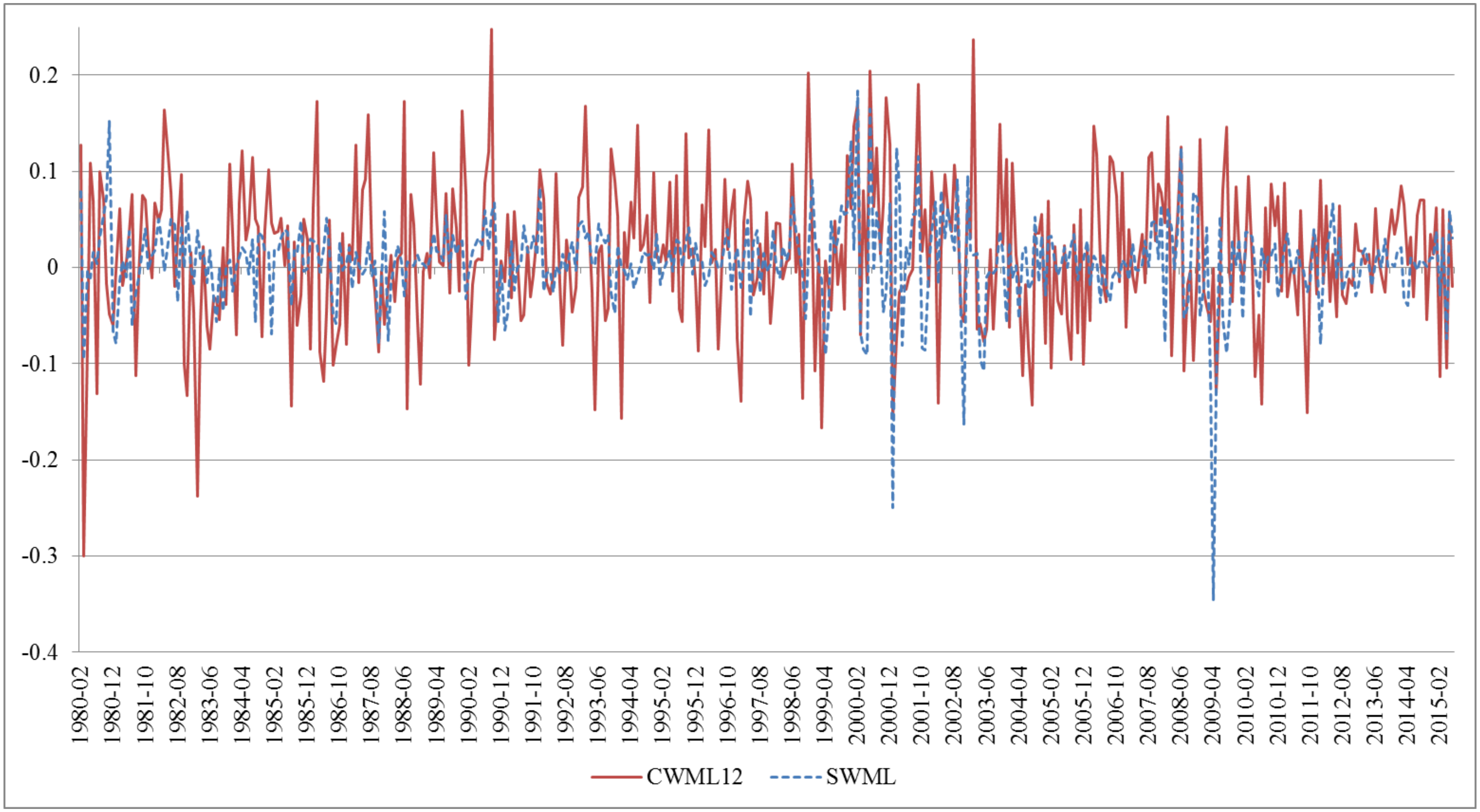

Table A1. Profitability of basis-controlled momentum strategy in commodity futures markets

This table presents the average monthly return on basis-controlled momentum strategies in commodity futures markets. *J* indicates the ranking period, and *K* indicates the holding period of the momentum strategy. We first sort the futures contracts by the basis into three groups and then sort the contracts in each group by the previous *J*-month returns. After constructing the nine portfolios by double-sorting, we average three winner (loser) portfolios from three basis groups and call the resulting portfolio the commodity futures winner (loser). SR indicates the annualized Sharpe ratio and std. indicates the standard deviation of returns. The numbers in parentheses are *t*-statistics corrected by the Newey–West (1987) method. The sample period is from January 1979 to June 2015.

		J = 1				*J* = 3				*J* = 6				*J* = 12			
		K = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12	*K* = 1	*K* = 3	*K* = 6	*K* = 12
W	Mean	0.692	0.525	0.418	0.416	0.820	0.587	0.444	0.438	0.439	0.327	0.375	0.308	0.450	0.266	0.136	0.077
		(2.82)	(2.42)	(2.07)	(2.11)	(3.24)	(2.54)	(2.08)	(2.12)	(1.83)	(1.43)	(1.74)	(1.44)	(1.88)	(1.16)	(0.61)	(0.35)
	std.	5.011	4.333	4.080	3.881	5.172	4.644	4.345	4.081	4.957	4.608	4.268	4.173	4.937	4.521	4.368	4.308
	SR	0.478	0.420	0.355	0.371	0.549	0.438	0.354	0.372	0.307	0.246	0.304	0.256	0.316	0.204	0.108	0.062
L	Mean	-0.203	-0.007	-0.003	0.011	-0.149	0.028	0.034	0.028	-0.192	0.018	-0.023	0.084	-0.180	0.049	0.194	0.286
		(-0.85)	(-0.04)	(-0.01)	(0.06)	(-0.65)	(0.13)	(0.17)	(0.15)	(-0.85)	(0.08)	(-0.11)	(0.43)	(-0.74)	(0.21)	(0.87)	(1.42)
	std.	4.756	3.948	3.756	3.610	4.584	4.174	3.902	3.661	4.628	4.294	4.179	3.835	4.615	4.313	4.123	3.841
	SR	-0.148	-0.006	-0.003	0.011	-0.112	0.023	0.030	0.026	-0.144	0.015	-0.019	0.076	-0.135	0.039	0.163	0.258
W-L	Mean	0.895	0.533	0.421	0.405	0.969	0.559	0.409	0.410	0.631	0.309	0.397	0.224	0.629	0.217	-0.058	-0.210
		(3.37)	(3.19)	(3.61)	(4.22)	(3.55)	(2.51)	(2.34)	(2.88)	(2.39)	(1.39)	(1.93)	(1.28)	(2.29)	(0.90)	(-0.26)	(-1.11)
	std.	5.741	3.520	2.534	1.935	5.766	4.595	3.648	2.775	5.644	4.752	4.171	3.390	5.512	4.766	4.402	3.777
	SR	0.540	0.524	0.576	0.724	0.582	0.421	0.389	0.512	0.387	0.225	0.330	0.229	0.396	0.158	-0.046	-0.192