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Strategic Behaviors and Information Cost under Money Illusion

Min-Jik Kim*

Institute of Finance and Banking, Business School, Seoul National University

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< Abstract >

Money illusion, first documented by Fisher(1928), is the phenomenon that the agent cannot distinguish real monetary value from nominal one. In this paper, money illusion is considered as a matter of information based on the Grossman and Stiglitz(1980), and the agents can resolve it. According to this model, the uninformed investors remain as money-illusioned when deflation occurs, but resolve money illusion when inflation happens for certain conditions of the signal if the cost of calculation to resolve money illusion exists. The inflation calculating cost, as the information cost, makes complementarities in information acquisition. As a result, multiple equilibria and price swings can be generated theoretically in this model.

JEL classification: G11, G12, G14

Keywords: Money Illusion, Strategic Behavior, Information Asymmetry, Information Cost, Information Complementarities, Multiple Equilibria

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* Address: Room 413, Bldg 58, 1 Kwanak-ro, Kwanak-gu, Seoul, Korea. kmj8113@snu.ac.kr.

I. Introduction

Nowadays, the price level is quite stable and inflation rate is maintained at moderate level around the world. However, in recent crises, many major countries have implemented expansionary fiscal and monetary policies, which can lead to high inflation. So, the real economy is more likely to be affected by inflation.

Money is illusionary because it cannot increase the utility level of the agents. On the other hand, the purchasing power of the money, which is the real value of money can increase the level of utility. In the mainstream economics, every economic choice of the rational agents is based on the real value. However, in reality, decisions are often made based on nominal value rather than real one. Fisher(1928) defined this phenomenon as money illusion.

Following Fisher(1928), many studies investigate money illusion. Modigliani and Cohn(1979) assume stock market investors suffer from money illusion and they have difficulty in estimating long-term future growth rates of cash flows, and show a negative relationship between the stock returns and the inflation. Cohen, Polk, and Vuolteenaho(2005) support Modigliani and Cohn(1979) with focusing on safe assets and Treasury bills. Campbell and Vuolteenaho(2004) show the effects of money illusion on stock returns based on time series decomposition of the dividend yield model(Gordon, 1962). Lee(2010) shows that the stock return-inflation relation depends on the business regime. In Basak and Yan(2010), the higher price level, the lower is the real consumption of money-illusioned investor. Recently, Ma, Wang, Cheng, and Hu(2018) develop a dynamic asset pricing model with money illusion and heterogeneous beliefs, and show that stock risk premium is inverse-U shaped as inflation disagreement increases. And Shafir, Diamond, and Tversky(1997) show empirical evidences for existence of money illusion.

Money illusion is also called as inflation illusion. Since money is illusive, there are many kinds of costs associated with inflation for resolving the illusion. According to Ackley(1978), these costs can be categorized into redistributive effects, effects of accounting and taxes, effects on aggregate demand and supply. Mankiw(1985) proposes the concept of menu cost, which is the cost of rewriting prices on the menu to account for price rigidity.

In addition to these types of explicit costs, there are also implicit costs of inflation. One of the implicit costs to resolve money illusion is calculating the inflation rate. And this can be interpreted as one of the information costs. Although money illusion is used to explain major phenomena in economics and finance, it is difficult to find a study that analyzes money illusion in terms of information. In this study, money illusion is considered as a problem of information based on the Grossman and Stiglitz(1980). And it is assume that the uninformed investors can choose to either resolve money illusion by purchasing information or strategically become money-illusioned investors. According to this model, the phenomenon of money illusion can appear or disappear depending on the price levels. Generally, although the uninformed tend to behave as money-illusioned in deflation(if the information cost is higher than certain level, $\underline{c}_{MI}$), but when inflation occurs money illusion is resolved under certain conditions on the compound signal($s \notin [\underline{s}, \overline{s}]$). Moreover, it is dependent on the level of inflation-calculating cost.

In the Grossman and Stiglitz(1980) model, the more informed investors the less valuable the information is. However, Diamond and Verrecchia(1981), Ganguli and Yang(2009), and Manzano and Vives(2011) show that complementarities in information acquisition generate multiple equilibria. In this model, there are information complementarities and multiple equilibria can be derived due to the cost of calculating inflation. Therefore, the equilibrium price are path-dependent and can fluctuate significantly.

The structure of this paper is as follows. Chapter II covers model assumptions and theoretical analysis. The value of information and information acquisition are covered in Chapter III, and concludes with Chapter IV.

II. Model

1. Assumption

Following Grossman-Stiglitz(1980), there are one risky and one riskless asset in economy. The payoff of risky asset f is

$$f = \theta + \epsilon, \qquad \theta \sim N(\mu, \sigma_\theta^2), \qquad \epsilon \sim N(0, \sigma_\epsilon^2)$$

The asset supply z is normally distributed as

$$z \sim N(\mu_z, \sigma_z^2)$$

A riskless asset is also tradable and its rate of return is zero. And supply of riskless asset is perfectly elastic. All investors have negative exponential utility u with constant absolute risk aversion coefficient A, as follows

$$u(W) = E[-e^{-AW}]$$

where the wealth level is defined as W, and its initial value is zero.

In this model, money illusion is considered as a matter of asymmetric information. So, money illusion cannot affect informed investors. When p is the observed asset price, the demand of the informed is

$$x_I(\theta,\, p) = \frac{E(f|\theta, p) - p}{A \cdot Var(f|\theta, p)} = \frac{\theta - p}{A\sigma_\epsilon^2}$$

Meanwhile, uninformed investors can be affected by money illusion. In Section 2 of Chapter II, it is assume that the uninformed participate in the stock market as money-illusioned. In Section 3, they can resolve it strategically with the cost of calculating inflation rate. Finally, Section 4 covers the general equilibrium price when the information cost exists.

The uninformed investors participate in the stock market in two ways. The first is the same as Grossman-Stiglitz type liquidity investor(subscripted as *L*), and the other is governed by money illusion(subscripted as *MI*). Then non-illusioned liquidity investors' demand is

$$x_L(P,\, p) = \frac{E(f|p) - p}{A \cdot Var(f|p)} = \frac{\mu_{f|s} - p}{A\sigma_{f|s}^2}$$

where

$$\mu_{f|s}(s;\, \mu) = (1-\eta)\mu + b_0\eta s, \qquad \eta \equiv \frac{\lambda^2\sigma_\theta^2}{\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4}, \qquad b_0 = \frac{A\sigma_\epsilon^2}{\lambda},$$

$$\sigma_{f|s}^2 = \sigma_\epsilon^2 + \frac{\sigma_\theta^2\sigma_z^2}{\sigma_s^2}, \qquad \sigma_s^2 = \left(\frac{\lambda}{A\sigma_\epsilon^2}\right)^2 \sigma_\theta^2 + \sigma_z^2$$

λ is the proportion of informed investors, and which is exogeneous variable in this Chapter II. It will be determined endogenously in Chapter III.

However, the investors governed by money illusion cannot distinguish the real value from the nominal value. In this model, the uninformed are governed by money illusion only on the payoff(f). And it is assume that the previous price level is normalized to one, then the following price level is the inflation rate Π itself. Since they are not aware of the real values of the payoff, only the nominal value of payoff can be observed as Πf. Then, they choose portfolio holdings $x_{MI}(P,\ p)$ to maximize the expected utility of their final wealth $W_{MI} = (\Pi f - p)x_{MI}$ conditional on the observed price p. So, their demand can be expressed as

$$x_{MI}(P,\ p) = \frac{\Pi E(f|p) - p}{\Pi^2 A \cdot Var(f|p)} = \frac{\Pi \mu_{f|s} - p}{\Pi^2 A \sigma_{f|s}^2}$$

Definition (The deep deflation and deep inflation)
If $\Pi < \underline{\Pi}$, deep deflation occurs. And deep inflation happens when $\Pi > \overline{\Pi}$. For details of $\underline{\Pi}$ and $\overline{\Pi}$, see the Appendix.

In the following of this paper, deflation and inflation are regarded as exclude deep deflation and deep inflation, respectively. Deep deflation and deep inflation are covered in the Appendix.

2. Baseline: Money-illusioned uninformed investors

When the uninformed are governed by money illusion, the equilibrium price is affected by price level, i.e. inflation or deflation. Let conjecture the equilibrium price function as $P(\theta, z) = P\big(s(\theta, z)\big)$, where $s(\theta, z) = \theta/b_0 - (z - \mu_z)$ is the compound signal. From the market clearing condition,

$$\lambda x_I(\theta, p) + (1-\lambda) x_{MI}(P, p) = z$$

the equilibrium price P, which is a function of signal s, can be expressed as

$$P(s) = a_\pi + b_\pi s$$

where

$$a_\pi = \frac{\Pi A \sigma_\epsilon^2 \{(1-\lambda) A \sigma_z^2 \sigma_\epsilon^2 \mu - \Pi(\lambda \sigma_\theta^2 + A^2 \sigma_\theta^2 \sigma_z^2 \sigma_\epsilon^2 + A^2 \sigma_z^2 \sigma_\epsilon^4)\mu_z\}}{\lambda^2 \{\lambda(\Pi^2 - 1) + 1\}\sigma_\theta^2 + \lambda \Pi^2 A^2 \sigma_\theta^2 \sigma_z^2 \sigma_\epsilon^2 + \{\lambda(\Pi^2 - 1) + 1\} A^2 \sigma_z^2 \sigma_\epsilon^4}$$

$$b_\pi = \frac{\Pi A \sigma_\epsilon^2 [\{\lambda(\Pi - 1) + 1\}\sigma_\theta^2 + \Pi A^2 \sigma_\theta^2 \sigma_z^2 \sigma_\epsilon^2 + \Pi A^2 \sigma_z^2 \sigma_\epsilon^4]}{\lambda^2 \{\lambda(\Pi^2 - 1) + 1\}\sigma_\theta^2 + \lambda \Pi^2 A^2 \sigma_\theta^2 \sigma_z^2 \sigma_\epsilon^2 + \{\lambda(\Pi^2 - 1) + 1\} A^2 \sigma_z^2 \sigma_\epsilon^4}$$

When $\Pi = 1$, the equilibrium price is identical to the price of Grossman-Stiglitz(1980), which is

$$P_1(s) = a_1 + b_1 s$$

where

$$a_1 \equiv a_\pi|_{\pi=1} = \frac{A\sigma_\epsilon^2\{(1-\lambda)A\sigma_z^2\sigma_\epsilon^2\mu - (\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)\mu_z\}}{\lambda^2\sigma_\theta^2 + \lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4}$$

$$b_1 \equiv b_\pi|_{\pi=1} = \frac{A\sigma_\epsilon^2(\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)}{\lambda^2\sigma_\theta^2 + \lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4}$$

Lemma 1.

When deflation occurs, $b_\pi < b_1$, and $b_\pi > b_1$ under inflation.

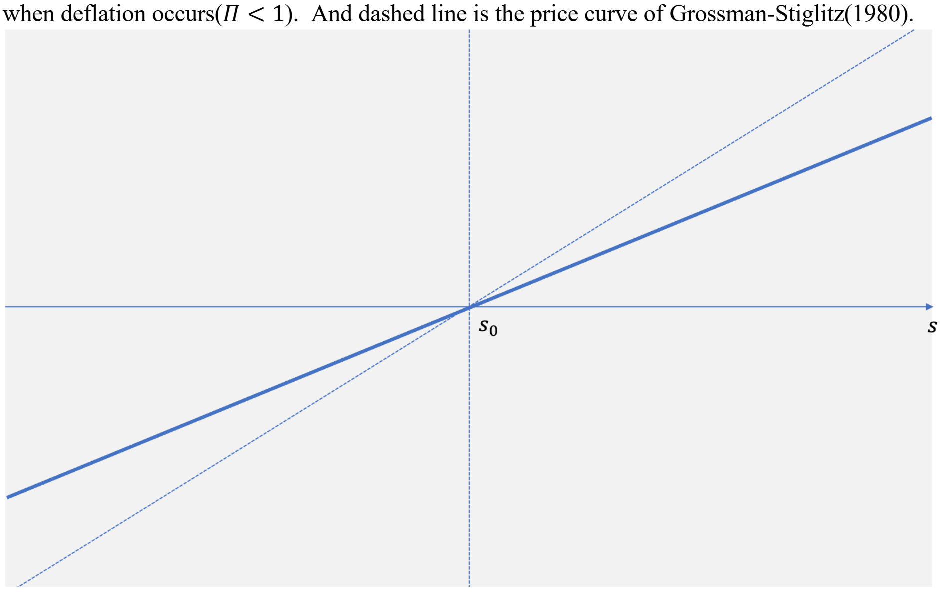
Proof.

See the Appendix.

<Figure 1> and <Figure 2> show the price curves when the uninformed are governed by money illusion under deflation and inflation, respectively. When deflation occurs, the slope of the price curve b_π is less steep than that of Grossman-Stiglitz(1980). However, b_π is steeper than the slope of Grossman-Stiglitz's price curve under inflation.

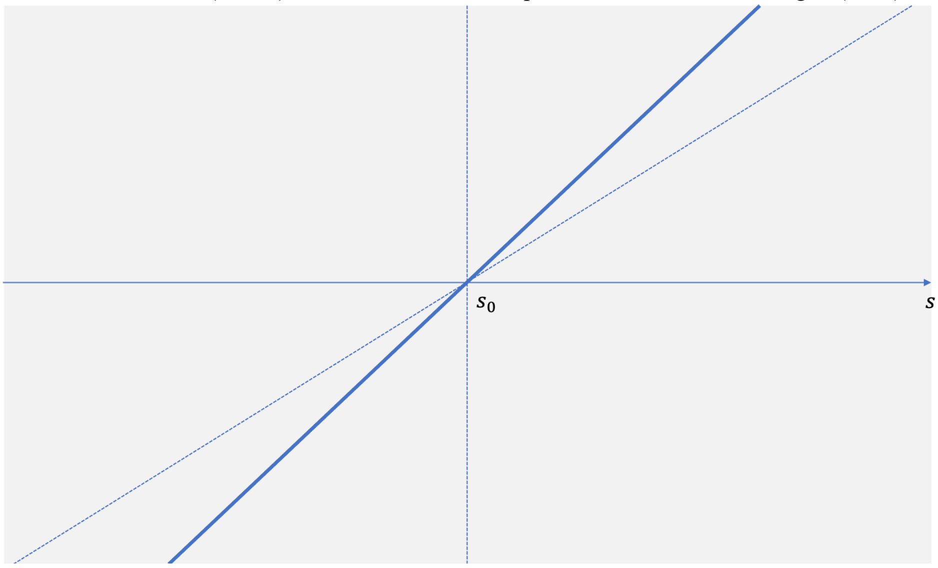
<Figure 1> Price curve with the money-illusioned uninformed investors when deflation happens

Solid line is the price curve over signal s when the uninformed are governed by money illusion when deflation occurs($\Pi < 1$). And dashed line is the price curve of Grossman-Stiglitz(1980).



<Figure 2> Price curve with the money-illusioned uninformed investors when inflation happens

Solid line is the price curve over signal s when the uninformed are governed by money illusion when inflation occurs($\Pi > 1$). And dashed line is the price curve of Grossman-Stiglitz(1980).



3. Strategic behavior under money illusion

In this section, the uninformed can strategically choose their behavior on money illusion after observing the price level or inflation Π. And assume that there is the information cost to resolve money illusion. Let c_{MI} be the cost of calculating inflation rate.

3.1 Deflation

Proposition 1.
Under deflation($\Pi < 1$), the uninformed remain as money-illusioned if $c_{MI} > \underline{c}_{MI}$.

Proof.
Let $u_L(P,\, p)$ and $u_{MI}(P,\, p)$ be the utility of non-illusioned uninformed investors and that of money-illusioned investors, respectively. Given $P(s)$, the uninformed remain as money-illusioned if following is satisfied.

$$u_L(P,\, p) < u_{MI}(P,\, p) \;\Leftrightarrow\; \frac{\left(\mu_{f|s} - P(s)\right)^2}{2A\sigma_{f|s}^2} - c_{MI} < \frac{\left(\Pi\mu_{f|s} - P(s)\right)^2}{2\Pi^2 A\sigma_{f|s}^2}$$

$$\Leftrightarrow\; \left(1 - \frac{1}{\Pi^2}\right)P(s)^2 - \left(1 - \frac{1}{\Pi}\right)2\mu_{f|s}P(s) - 2c_{MI}A\sigma_{f|s}^2 < 0$$

$$\Leftrightarrow\; \frac{\Pi - 1}{\Pi}\left\{\frac{\Pi + 1}{\Pi}P(s)^2 - 2\mu_{f|s}P(s) - c'_{MI}\right\} < 0$$

where

$$c'_{MI} = \frac{2\Pi A\sigma_{f|s}^2}{\Pi - 1}c_{MI}$$

$$\begin{aligned}
\Leftrightarrow\; \frac{\Pi - 1}{\Pi}\Bigg\{ & b_\pi\left(\frac{\Pi + 1}{\Pi}b_\pi - 2b_0\eta\right)s^2 + 2\left\{\frac{\Pi + 1}{\Pi}a_\pi b_\pi - (1 - \eta)b_\pi\mu - a_\pi b_0\eta\right\}s \\
& + \frac{\Pi + 1}{\Pi}a_\pi^2 - 2(1 - \eta)a_\pi\mu - c'_{MI}\Bigg\} \\
\equiv\; & \frac{\Pi - 1}{\Pi}\left\{M_\pi s^2 + 2O_\pi s + \frac{\Pi + 1}{\Pi}a_\pi^2 - 2(1 - \eta)a_\pi\mu - c'_{MI}\right\} < 0
\end{aligned}$$

$$\Leftrightarrow\; M_\pi s^2 + 2O_\pi s + \frac{\Pi + 1}{\Pi}a_\pi^2 - 2(1 - \eta)a_\pi\mu - c'_{MI} > 0 \quad \cdots \quad (*)$$

where

M_π

$$= \frac{\begin{array}{c}\Pi(\Pi + 1)A^2\sigma_\epsilon^4[\lambda\{(\Pi - 1)\lambda + 1\}\sigma_\theta^2 + \Pi A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \Pi A^2\sigma_z^2\sigma_\epsilon^4] \\ \times\begin{bmatrix}\lambda^3\{(\Pi - 1)\lambda + 1\}\sigma_\theta^4 + \Pi\lambda^2A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 + \Pi A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 \\ +\lambda\{(2\Pi - 1)\lambda + 1\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + \Pi A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6\end{bmatrix}\end{array}}{\lambda^2\left(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4\right)[\lambda\{(\Pi^2 - 1)\lambda + 1\}\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2 - 1)\lambda + 1\}A^2\sigma_z^2\sigma_\epsilon^4]^2}$$

O_π

$$\begin{aligned}
= & \frac{\Pi A\sigma_\epsilon^2\begin{bmatrix}(1 - \lambda)(\Pi + 1)A^2\sigma_z^2\sigma_\epsilon^4(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4)[\{(\Pi - 1)\lambda + 1\}\lambda\sigma_\theta^2 + \Pi A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \Pi A^2\sigma_z^2\sigma_\epsilon^4] \\ -\lambda(1 - \lambda)A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4[\{(\Pi^2 - 1)\lambda + 1\}\lambda^2\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2 - 1)\lambda + 1\}A^2\sigma_z^2\sigma_\epsilon^4] \\ -A^2\sigma_z^2\sigma_\epsilon^4[\{(\Pi^2 - 1)\lambda + 1\}\lambda^2\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2 - 1)\lambda + 1\}A^2\sigma_z^2\sigma_\epsilon^4] \\ \times[\{(\Pi - 1)\lambda + 1\}\lambda\sigma_\theta^2 + \Pi A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \Pi A^2\sigma_z^2\sigma_\epsilon^4]\end{bmatrix}}{\left(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4\right)[\{(\Pi^2 - 1)\lambda + 1\}\lambda^2\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2 - 1)\lambda + 1\}A^2\sigma_z^2\sigma_\epsilon^4]^2}\mu \\
+ & \frac{\Pi^2A^2\sigma_\epsilon^4\begin{bmatrix}(\Pi + 1)(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4)(\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4) \\ \times[\{(\Pi - 1)\lambda + 1\}\lambda\sigma_\theta^2 + \Pi A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \Pi A^2\sigma_z^2\sigma_\epsilon^4] \\ -\lambda\sigma_\theta^2(\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4) \\ \times[\{(\Pi^2 - 1)\lambda + 1\}\lambda^2\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2 - 1)\lambda + 1\}A^2\sigma_z^2\sigma_\epsilon^4]\end{bmatrix}}{\left(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4\right)[\{(\Pi^2 - 1)\lambda + 1\}\lambda^2\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2 - 1)\lambda + 1\}A^2\sigma_z^2\sigma_\epsilon^4]^2}\mu_z
\end{aligned}$$

Due to $M_\pi > 0$, the discriminant of $(*)$ is negative if $c_{MI} > \underline{c}_{MI}$, where

$$\begin{aligned}
\underline{c}_{MI} = & \frac{(1 - \Pi)\{(1 - \eta)b_\pi\mu - a_\pi b_0\eta\}^2}{2\Pi A\sigma_{f|s}^2 M_\pi} \\
= & \frac{(1 - \Pi)\Pi\begin{bmatrix}A^2\sigma_z^2\sigma_\epsilon^4(\lambda^2\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)\mu \\ +\lambda\sigma_\theta^2(\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)\mu_z\end{bmatrix}^2}{\begin{array}{c}2\lambda^2\left(\lambda^2\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4\right) \\ \times\Pi(\Pi + 1)A^2\sigma_\epsilon^4[\lambda\{(\Pi - 1)\lambda + 1\}\sigma_\theta^2 + \Pi A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \Pi A^2\sigma_z^2\sigma_\epsilon^4] \\ \times\begin{bmatrix}\lambda^3\{(\Pi - 1)\lambda + 1\}\sigma_\theta^4 + \Pi\lambda^2A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 + \Pi A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 \\ +\lambda\{(2\Pi - 1)\lambda + 1\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + \Pi A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6\end{bmatrix}\end{array}}
\end{aligned}$$

Thus, $(*)$ is positive for all s, and the money-illusioned investors' utility is always higher than that of the non-illusioned under deflation when $c_{MI} > \underline{c}_{MI}$.

Q.E.D.

3.2 Inflation

Proposition 2.

When inflation happens($\Pi > 1$), the uninformed resolve money illusion and take non-illusioned behavior for $s \notin [\underline{s}, \overline{s}]$.

Proof.

Same as Proposition 1, the uninformed choose to remain as money-illusioned if following is satisfied.

$$
\begin{aligned}
u_L(P,\, p) < u_{MI}(P,\, p) \;\Leftrightarrow\; & \frac{\left(\mu_{f|s} - P(s)\right)^2}{2A\sigma_{f|s}^2} - c_{MI} < \frac{\left(\Pi\mu_{f|s} - P(s)\right)^2}{2\Pi^2 A\sigma_{f|s}^2} \\
\Leftrightarrow\; & \left(1 - \frac{1}{\Pi^2}\right) P(s)^2 - \left(1 - \frac{1}{\Pi}\right) 2\mu_{f|s} P(s) - 2c_{MI} A\sigma_{f|s}^2 < 0 \\
\Leftrightarrow\; & \frac{\Pi - 1}{\Pi}\left\{M_\pi s^2 + 2O_\pi s + \frac{\Pi + 1}{\Pi} a_\pi^2 - 2(1-\eta) a_\pi \mu - c'_{MI}\right\} < 0 \\
\Leftrightarrow\; & M_\pi s^2 + 2O_\pi s + \frac{\Pi + 1}{\Pi} a_\pi^2 - 2(1-\eta) a_\pi \mu - c'_{MI} < 0 \quad \cdots \quad (**)
\end{aligned}
$$

Let D_π be the discriminant of $(*)$. Since $D_\pi > 0$, $(**)$ is negative for $s \in [\underline{s}, \overline{s}]$, where

$$
\begin{aligned}
\underline{s} &= \frac{-O_\pi - \sqrt{D_\pi}}{M_\pi} \\
\overline{s} &= \frac{-O_\pi + \sqrt{D_\pi}}{M_\pi} \\
\Delta s \equiv \overline{s} - \underline{s} &= \frac{2\sqrt{D_\pi}}{M_\pi}
\end{aligned}
$$

where

$$
\begin{aligned}
&D_\pi = \{(1-\eta)b_\pi\mu - a_\pi b_0\eta\}^2 + M_\pi c'_{MI} \\
&= \frac{\begin{array}{c}\Pi^4(\Pi-1)A^2\sigma_\epsilon^4\begin{bmatrix} A^2\sigma_z^2\sigma_\epsilon^4(\lambda^2\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)\mu \\ +\lambda\sigma_\theta^2(\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)\mu_z \end{bmatrix}^2 \\ +2\Pi^2(\Pi+1)A^3\sigma_\epsilon^6\begin{bmatrix} (\lambda^2\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4) \\ \times[\lambda\{(\Pi-1)\lambda+1\}\sigma_\theta^2 + \Pi A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \Pi A^2\sigma_z^2\sigma_\epsilon^4] \end{bmatrix} \\ \times\begin{bmatrix} \lambda^3\{(\Pi-1)\lambda+1\}\sigma_\theta^4 + \Pi\lambda^2A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 + \Pi A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 \\ +\lambda\{(2\Pi-1)\lambda+1\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + \Pi A^4\sigma_z^4\sigma_\epsilon^8 \end{bmatrix} c_{MI}\end{array}}{(\Pi-1)\begin{bmatrix} \lambda(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4) \\ \times[\lambda\{(\Pi^2-1)\lambda+1\}\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4] \end{bmatrix}^2}
\end{aligned}
$$

Then $(**)$ is positive for $s \notin [\underline{s}, \overline{s}]$. Therefore, the uninformed resolve money illusion if $s \notin [\underline{s}, \overline{s}]$, while still remain as money-illusioned for $s \in [\underline{s}, \overline{s}]$.

Q.E.D.

As c_{MI} increases, the likelihood of becoming a money-illusioned is increasing because Δs also increases. If $c_{MI} = 0$, then the interval of signals, in which the uninformed behave as money-illusioned, is reduced to $[\underline{s}_0, \overline{s}_0]$, where

$$
\begin{aligned}
\underline{s}_0 &= \min\left\{\frac{(\Pi+1)a_\pi b_\pi + 2\Pi a_\pi b_0\eta}{b_\pi[(\Pi+1)b_\pi - 2\Pi b_0\eta]}, \frac{(\Pi+1)a_\pi + 2\Pi(1-\eta)\mu}{(\Pi+1)b_\pi - 2\Pi b_0\eta}\right\} \\
\overline{s}_0 &= \max\left\{\frac{(\Pi+1)a_\pi b_\pi + 2\Pi a_\pi b_0\eta}{b_\pi[(\Pi+1)b_\pi - 2\Pi b_0\eta]}, \frac{(\Pi+1)a_\pi + 2\Pi(1-\eta)\mu}{(\Pi+1)b_\pi - 2\Pi b_0\eta}\right\} \\
\Delta s_0 \equiv \overline{s}_0 - \underline{s}_0 &= \left|\frac{2\{(1-\eta)b_\pi\mu - a_\pi b_0\eta\}}{M_\pi}\right| = \left|\frac{2\Pi\{(1-\eta)b_\pi\mu - a_\pi b_0\eta\}}{b_\pi\{(\Pi+1)b_\pi - 2\Pi b_0\eta\}}\right| < \Delta s
\end{aligned}
$$

4. Equilibrium under inflation

The equilibrium is dependent on a strategic behavior of uninformed investors in this model. In other words, the equilibrium price is determined after the uninformed choose their behavior on money illusion. Since money illusion cannot play a key role in generating information complementarities under deflation, this chapter analyzes the equilibrium only in inflation to clarify the role of the cost of calculating inflation.

Define $x_U(P, p)$ as the demand of the uninformed when inflation occurs. By Proposition 2, $x_U(P, p)$ is as follows.

$$x_U(P,p)=\begin{cases}\dfrac{\mu_{f|s}-p}{A\sigma_{f|s}^2}, & s<\underline{s}\\ \dfrac{\Pi\mu_{f|s}-p}{\Pi^2 A\sigma_{f|s}^2}, & s\in[\underline{s},\overline{s}]\\ \dfrac{\mu_{f|s}-p}{A\sigma_{f|s}^2}, & s>\overline{s}\end{cases}$$

From the market clearing condition,

$$\lambda x_I(\theta,p)+(1-\lambda)x_U(P,p)=z$$

the equilibrium price $P(s)$ can be expressed as a function of the compound signal $s(\theta,z)$ as follows.

Proposition 3.
When inflation happens($\Pi>1$), the equilibrium price is piecewise linear in the signal $s(\theta,z)$.

$$P^*(s)=\begin{cases}\underline{a}+b_1 s, & s<\underline{s}\\ a_\pi+b_\pi s, & s\in[\underline{s},\overline{s}]\\ \overline{a}+b_1 s, & s>\overline{s}\end{cases}$$

where

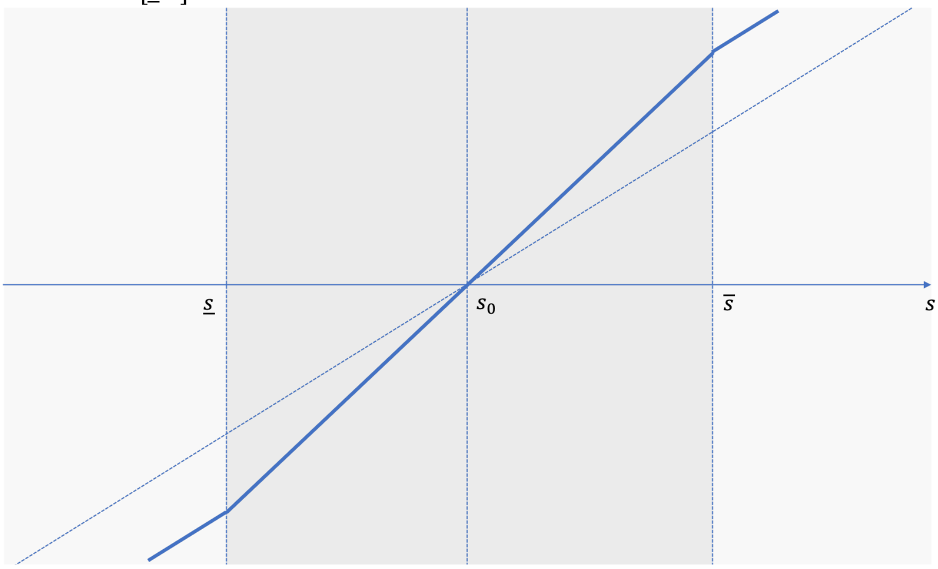
$$\underline{a}=a_\pi+(b_\pi-b_1)\underline{s}\equiv a_\pi-\beta\underline{s}$$
$$\overline{a}=a_\pi+(b_\pi-b_1)\overline{s}\equiv a_\pi-\beta\overline{s}=\underline{a}-\beta(\overline{s}-\underline{s})$$

$$\beta\equiv b_\pi-b_1$$
$$=\frac{A\sigma_\epsilon^2\begin{bmatrix}\lambda^3(1-\lambda)(\Pi-1)\sigma_\theta^4+\lambda\Pi^2(\Pi-1)A^4\sigma_\theta^4\sigma_z^4\sigma_\epsilon^4\\ +(\Pi-1)\{\Pi^2+(\Pi+1)(1-\lambda)\}A^4\sigma_z^4\sigma_\epsilon^8+\lambda(\Pi-1)(\Pi+1-\lambda)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2\\ +(\Pi-1)\{(1+\lambda)\Pi^2+(1+\lambda)\Pi+1+\lambda\}A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6\\ +\lambda(\Pi-1)\{1+\lambda(1+\Pi)(\Pi-\lambda)\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4\end{bmatrix}}{\left(\lambda^2\sigma_\theta^2+\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2+A^2\sigma_z^2\sigma_\epsilon^4\right)\begin{bmatrix}\{(\Pi^2-1)\lambda+1\}\lambda^2\sigma_\theta^2+\Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2\\ +\{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4\end{bmatrix}}$$

<Figure 3> is the equilibrium price curve with the uninformed investors governed by money illusion with the inflation-calculating cost. The slope of the price curve is steeper when $s\in[\underline{s},\overline{s}]$ than when $s\notin[\underline{s},\overline{s}]$. Due to the cost of calculating inflation, being governed by money illusion can increase the utility of uninformed investors even under inflation.

<Figure 3> Price curve with the inflation-calculating cost when inflation happens

Solid line is the price curve over signal s when the uninformed are governed by money illusion and inflation occurs($\Pi > 1$). And dashed line is the price curve of Grossman-Stiglitz(1980). The uninformed behave as money-illusioned if $s \in [\underline{s}, \overline{s}]$(shaded area), whereas they resolve money illusion if $s \notin [\underline{s}, \overline{s}]$.



III. Value of information under inflation

In this chapter, let c_I be the cost of information for being informed investors. Due to money illusion cannot affects the informed investors, c_I should be higher than c_{MI}.
The expected utility of the informed is

$$E\left[-e^{-A(C(\theta,s)-c_I)}\right]$$

where $C(\theta, s)$ is the certainty equivalent of the expected utility of the informed.

$$C(\theta, s) = \frac{\left(\theta - P^*(s)\right)^2}{2A\sigma_\epsilon^2}$$

When deciding whether to purchase the information, the agents can only observe the signal without having the information of θ. So, their *ex ante* utility can be expressed as

$$u_I(c_I,\lambda) = -\frac{\sigma_\epsilon}{\sigma_{f|s}} e^{-A(\overline{C}(s)-c_I)}$$

$\overline{C}(s)$ is the certainty equivalent for a hypothetical uninformed investors who are not governed by money illusion at the trading stage.

$$\overline{C}(s) = \frac{\left(\mu_{f|s} - P^*(s)\right)^2}{2A\sigma_{f|s}^2}$$

$$= \begin{cases} \dfrac{(b_1-b_0\eta)^2}{2A\sigma_{f|s}^2}\left(s - \dfrac{(1-\eta)\mu - a_1}{b_1 - b_0\eta}\right)^2 \equiv \dfrac{Q^2}{2A\sigma_{f|s}^2}(s-R)^2, & s < \underline{s} \\ \dfrac{(b_\pi-b_0\eta)^2}{2A\sigma_{f|s}^2}\left(s - \dfrac{(1-\eta)\mu - a_\pi}{b_\pi - b_0\eta}\right)^2 \equiv \dfrac{Q_\pi^2}{2A\sigma_{f|s}^2}(s-R_\pi)^2, & s \in [\underline{s}, \overline{s}] \\ \dfrac{(b_1-b_0\eta)^2}{2A\sigma_{f|s}^2}\left(s - \dfrac{(1-\eta)\mu - a_1}{b_1 - b_0\eta}\right)^2 \equiv \dfrac{Q^2}{2A\sigma_{f|s}^2}(s-R)^2, & s > \overline{s} \end{cases}$$

where

$$Q \equiv b_1 - b_0\eta$$
$$= \frac{A\sigma_\epsilon^2\{\lambda^3(1-\lambda)\sigma_\theta^4 + \lambda^2(1-\lambda)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 + \lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 + A^4\sigma_z^4\sigma_\epsilon^8\}}{\left(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4\right)\left(\lambda^2\sigma_\theta^2 + \lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4\right)}$$

$$R \equiv \frac{(1-\eta)\mu - a_1}{b_1 - b_0\eta}$$
$$= \frac{A^2\sigma_z^2\sigma_\epsilon^4(\lambda^2\sigma_\theta^2 + \lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4) - (1-\lambda)A^4\sigma_z^2\sigma_\epsilon^4(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4)}{A\sigma_\epsilon^2\{\lambda^3(1-\lambda)\sigma_\theta^4 + \lambda^2(1-\lambda)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 + \lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 + A^4\sigma_z^4\sigma_\epsilon^8\}}\mu$$
$$+ \frac{A\sigma_\epsilon^2(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4)(\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)}{A\sigma_\epsilon^2\{\lambda^3(1-\lambda)\sigma_\theta^4 + \lambda^2(1-\lambda)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 + \lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 + A^4\sigma_z^4\sigma_\epsilon^8\}}\mu_z$$

$$Q_\pi \equiv b_\pi - b_0\eta$$
$$= \frac{A\sigma_\epsilon^2[\lambda^3(1-\lambda)\{\lambda(\Pi^2-1)+\Pi\}\sigma_\theta^4 + \lambda^2\Pi^2(1-\lambda)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2]}{\left(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4\right)[\lambda^2\{(\Pi^2-1)\lambda+1\}\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4]}$$
$$+ \frac{A\sigma_\epsilon^2[\lambda\{\lambda\Pi^2(2-\lambda) + (1-\lambda)(\Pi-1)\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + \Pi^2A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 + \Pi^2A^4\sigma_z^4\sigma_\epsilon^8]}{\left(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4\right)[\lambda^2\{(\Pi^2-1)\lambda+1\}\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + \{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4]}$$

$$R_\pi \equiv \frac{(1-\eta)\mu - a_\pi}{b_\pi - b_0\eta}$$
$$= \frac{A\sigma_z^2\sigma_\epsilon^2\begin{bmatrix}\lambda^2\{(\Pi^2-1)\lambda+1\}\sigma_\theta^2 + \Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 \\ +\{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4 - \Pi(1-\lambda)(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4)\end{bmatrix}}{\begin{bmatrix}\lambda^3(1-\lambda)\{\lambda(\Pi^2-1)+\Pi\}\sigma_\theta^4 + \lambda^2\Pi^2(1-\lambda)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 \\ +\lambda\{\lambda\Pi^2(2-\lambda) + (1-\lambda)(\Pi-1)\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + \Pi^2A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 + \Pi^2A^4\sigma_z^4\sigma_\epsilon^8\end{bmatrix}}\mu$$
$$+ \frac{\Pi^2(\lambda^2\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4)(\lambda\sigma_\theta^2 + A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2 + A^2\sigma_z^2\sigma_\epsilon^4)}{\begin{bmatrix}\lambda^3(1-\lambda)\{\lambda(\Pi^2-1)+\Pi\}\sigma_\theta^4 + \lambda^2\Pi^2(1-\lambda)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2 \\ +\lambda\{\lambda\Pi^2(2-\lambda) + (1-\lambda)(\Pi-1)\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4 + \Pi^2A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6 + \Pi^2A^4\sigma_z^4\sigma_\epsilon^8\end{bmatrix}}\mu_z$$

The *ex ante* utility of a hypothetical uninformed investors who resolve money illusion is $\overline{u}_I(c_I,\lambda)$.

$$\overline{u}_I(c_I,\lambda) = -e^{-A\overline{C}(s)} = \begin{cases} -e^{-\frac{Q^2}{2\sigma_{f|s}^2}(s-R)^2}, & s < \underline{s} \\ -e^{-\frac{Q_\pi^2}{2\sigma_{f|s}^2}(s-R_\pi)^2}, & s \in [\underline{s},\overline{s}] \\ -e^{-\frac{Q^2}{2\sigma_{f|s}^2}(s-R)^2}, & s > \overline{s} \end{cases}$$

Therefore, the expected utility of the informed can be expressed as

$$E\left[-e^{-A(C(s)-c_I)}\right] = e^{Ac_I}\frac{\sigma_\epsilon}{\sigma_{f|s}}E\left[-e^{-A\overline{C}(s)}\right] = e^{Ac_I}\frac{\sigma_\epsilon}{\sigma_{f|s}}\sum_{i=1}^{3}J_i$$

where

$$\begin{cases} J_1 \equiv -\sqrt{2\pi}\frac{\sigma_{f|s}^2}{Q^2}N\left(\frac{Q}{\sigma_{f|s}}(\underline{s}-R)\right) \\ J_2 \equiv -\sqrt{2\pi}\frac{\sigma_{f|s}^2}{Q_\pi^2}\left[N\left(\frac{Q_\pi}{\sigma_{f|s}}(\overline{s}-R_\pi)\right)-N\left(\frac{Q_\pi}{\sigma_{f|s}}(\underline{s}-R_\pi)\right)\right] \\ J_3 \equiv -\sqrt{2\pi}\frac{\sigma_{f|s}^2}{Q^2}\left[1-N\left(\frac{Q}{\sigma_{f|s}}(\overline{s}-R)\right)\right] \end{cases}$$

The utility of the uninformed(subscripted as *U*) is

$$u_U(c_{MI},\lambda) = -e^{-A\hat{C}(s,c_{MI})}$$

and $\hat{C}(s,c_{MI})$ is the certainty equivalent of the expected utility of uninformed investors who are governed by money illusion.

$$\hat{C}(s,c_{MI})=\begin{cases}\dfrac{\left(\mu_{f|s}-P^*(s)\right)^2}{2A\sigma_{f|s}^2}-c_{MI}, & s<\underline{s}\\ \dfrac{\left(\mu_{f|s}-\dfrac{P^*(s)}{\Pi}\right)^2}{2A\sigma_{f|s}^2}, & s\in[\underline{s},\overline{s}]\\ \dfrac{\left(\mu_{f|s}-P^*(s)\right)^2}{2A\sigma_{f|s}^2}-c_{MI}, & s>\overline{s}\end{cases}$$

$$=\begin{cases}\dfrac{(b_1-b_0\eta)^2}{2A\sigma_{f|s}^2}\left(s-\dfrac{(1-\eta)\mu-a_1}{b_1-b_0\eta}\right)^2-c_{MI}\equiv\dfrac{Q^2}{2A\sigma_{f|s}^2}(s-R)^2-c_{MI}, & s<\underline{s}\\ \dfrac{(b_\pi-\Pi b_0\eta)^2}{2A\sigma_{f|s}^2}\left(s-\dfrac{\Pi(1-\eta)\mu-a_\pi}{b_\pi-\Pi b_0\eta}\right)^2\equiv\dfrac{\hat{Q}_\pi^2}{2A\sigma_{f|s}^2}\left(s-\hat{R}_\pi\right)^2, & s\in[\underline{s},\overline{s}]\\ \dfrac{(b_1-b_0\eta)^2}{2A\sigma_{f|s}^2}\left(s-\dfrac{(1-\eta)\mu-a_1}{b_1-b_0\eta}\right)^2-c_{MI}\equiv\dfrac{Q^2}{2A\sigma_{f|s}^2}(s-R)^2-c_{MI}, & s>\overline{s}\end{cases}$$

where

$$\hat{Q}_\pi\equiv b_\pi-\Pi b_0\eta$$
$$=\frac{A\sigma_\epsilon^2[\lambda^3\{\lambda\Pi^2(1-\lambda\Pi)+(1-\lambda)^2\Pi\}\sigma_\theta^4+\lambda^2\Pi^2(1-\lambda\Pi)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2]}{\left(\lambda^2\sigma_\theta^2+A^2\sigma_z^2\sigma_\epsilon^4\right)[\lambda^2\{(\Pi^2-1)\lambda+1\}\sigma_\theta^2+\Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2+\{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4]}$$
$$+\frac{A\sigma_\epsilon^2[\lambda\{\lambda\Pi^2(2-\lambda\Pi)+(1-\lambda)^2\Pi\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4+\Pi^2A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6+\Pi^2A^4\sigma_z^4\sigma_\epsilon^8]}{\left(\lambda^2\sigma_\theta^2+A^2\sigma_z^2\sigma_\epsilon^4\right)[\lambda^2\{(\Pi^2-1)\lambda+1\}\sigma_\theta^2+\Pi^2\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2+\{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4]}$$

$$\hat{R}_\pi\equiv\frac{\Pi(1-\eta)\mu-a_\pi}{b_\pi-\Pi b_0\eta}$$
$$=\frac{A\sigma_z^2\sigma_\epsilon^2\begin{bmatrix}\lambda^2\Pi\{(\Pi^2-1)\lambda+1\}\sigma_\theta^2+\Pi^3\lambda A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2\\+\Pi\{(\Pi^2-1)\lambda+1\}A^2\sigma_z^2\sigma_\epsilon^4-\Pi(1-\lambda)(\lambda^2\sigma_\theta^2+A^2\sigma_z^2\sigma_\epsilon^4)\end{bmatrix}}{\begin{bmatrix}\lambda^3\{\lambda\Pi^2(1-\lambda\Pi)+(1-\lambda)^2\Pi\}\sigma_\theta^4+\lambda^2\Pi^2(1-\lambda\Pi)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2\\+\lambda\{\lambda\Pi^2(2-\lambda\Pi)+(1-\lambda)^2\Pi\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4+\Pi^2A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6+\Pi^2A^4\sigma_z^4\sigma_\epsilon^8\end{bmatrix}}\mu$$
$$+\frac{\Pi^2(\lambda^2\sigma_\theta^2+A^2\sigma_z^2\sigma_\epsilon^4)(\lambda\sigma_\theta^2+A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2+A^2\sigma_z^2\sigma_\epsilon^4)}{\begin{bmatrix}\lambda^3\{\lambda\Pi^2(1-\lambda\Pi)+(1-\lambda)^2\Pi\}\sigma_\theta^4+\lambda^2\Pi^2(1-\lambda\Pi)A^2\sigma_\theta^4\sigma_z^2\sigma_\epsilon^2\\+\lambda\{\lambda\Pi^2(2-\lambda\Pi)+(1-\lambda)^2\Pi\}A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^4+\Pi^2A^4\sigma_\theta^2\sigma_z^4\sigma_\epsilon^6+\Pi^2A^4\sigma_z^4\sigma_\epsilon^8\end{bmatrix}}\mu_z$$

The utility of the uninformed $u_U(c_{MI},\lambda)$ can be expressed as

$$-e^{-A\hat{C}(s,c_{MI})}=\begin{cases}-e^{Ac_{MI}}\times e^{-\frac{Q^2}{2\sigma_{f|s}^2}(s-R)^2}, & s<\underline{s}\\ -e^{-\frac{\hat{Q}_\pi^2}{2\sigma_{f|s}^2}(s-\hat{R}_\pi)^2}, & s\in[\underline{s},\overline{s}]\\ -e^{Ac_{MI}}\times e^{-\frac{Q^2}{2\sigma_{f|s}^2}(s-R)^2}, & s>\overline{s}\end{cases}$$

Therefore, the expected utility of the uninformed is

$$E\left[-e^{-A\hat{C}(s,c_{MI})}\right] = \sum_{i=1}^{3} K_i$$

where

$$\begin{cases} K_1 \equiv -e^{Ac_{MI}} \times \sqrt{2\pi}\dfrac{\sigma_{f|s}^2}{Q^2} N\left(\dfrac{Q}{\sigma_{f|s}}(\underline{s} - R)\right) = e^{Ac_{MI}} \times J_1 \\ K_2 \equiv -\sqrt{2\pi}\dfrac{\sigma_{f|s}^2}{\hat{Q}_\pi^2}\left[N\left(\dfrac{\hat{Q}_\pi}{\sigma_{f|s}}(\overline{s} - \hat{R}_\pi)\right) - N\left(\dfrac{\hat{Q}_\pi}{\sigma_{f|s}}(\underline{s} - \hat{R}_\pi)\right)\right] \\ K_3 \equiv -e^{Ac_{MI}} \times \sqrt{2\pi}\dfrac{\sigma_{f|s}^2}{Q^2}\left[1 - N\left(\dfrac{Q}{\sigma_{f|s}}(\overline{s} - R)\right)\right] = e^{Ac_{MI}} \times J_3 \end{cases}$$

The net gain from becoming informed investors can be defined as

$$\begin{aligned} \mathcal{g}(c_I, c_{MI}, \lambda) &\equiv -\frac{1}{A}\ln\left(\frac{u_I(c_I,\lambda)}{u_U(c_{MI},\lambda)}\right) \\ &= \underbrace{\frac{1}{2A}\ln\left(\frac{\sigma_{f|s}^2}{\sigma_\epsilon^2}\right) - c_I}_{Grossman-Stiglitz\ effect} \quad + \quad \underbrace{\frac{1}{A}\ln\left(\frac{E\left[e^{-A\hat{C}(s,c_{MI})}\right]}{E\left[e^{-A\bar{C}(s)}\right]}\right)}_{Resolving\ money\ illusion\ effect} \end{aligned}$$

and it can be interpreted as the value of information, which can be decomposed into two parts as (i) Grossman-Stiglitz effect and (ii) resolving money illusion effect. λ is determined where the gain $\mathcal{g}(c_I, c_{MI}, \lambda)$ meets the zero, and it makes the expectation of *ex ante* utility of all agents be identical.

In this model, c_{MI} increases the gain from resolving money illusion effect. Moreover, it's trivial that λ increases as c_{MI} increases. Thus, c_{MI} generates the positive correlation between $\mathcal{g}(c_I, c_{MI}, \lambda)$ and λ. On the other hand, λ and gain from Grossman-Stiglitz effect are negatively correlated. Therefore, following Diamond and Verrecchia(1981), Ganguli and Yang(2009), and Manzano and Vives(2011), complementarities in information acquisition can be generated in this model.

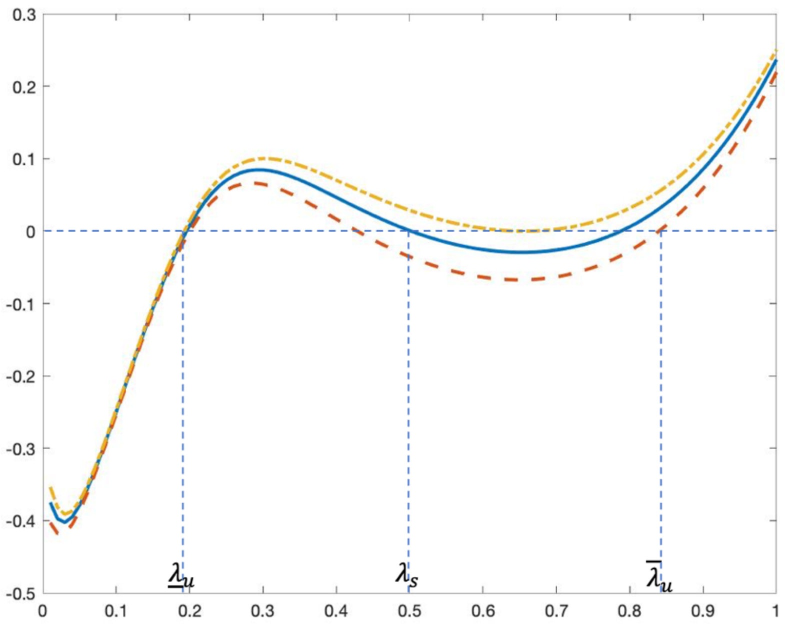
Proposition 4. (Information complementarities)
There exists a inflation calculating cost $c_{MI}^* > 0$ and an absolute risk aversion coefficient $A^* > 0$, such that there are complementarities in information acquisition for all $c_{MI} > c_{MI}^*$ and $A > A^*$, if $\Pi^2(\sigma_\theta^2 + \sigma_\epsilon^2) < 1$.

Proof.
See the Appendix.

The intuition behind the information complementarities lies in the effects of c_{MI} on the proportion of informed investors λ and the value of information $g(c_I, c_{MI}, \lambda)$. As c_{MI} increases, the *ex ante* utility of informed investors can increase faster than that of the uninformed. This means that λ is increasing as c_{MI} increases because gain from resolving money illusion effect is higher than loss due to Grossman-Stiglitz effect. As a result, c_{MI} improves the incentive for information, so c_{MI} generates a positive correlation between λ and $g(c_I, c_{MI}, \lambda)$, i.e. complementarities in information acquisition.

<Figure 4> The value of information $g(c_I, c_{MI}, \lambda)$

Solid blue line depicts the value of information $g(c_I, c_{MI}, \lambda)$, as a function of the ratio of informed investors λ, with the cost of calculating inflation $c_{MI} = 0.15$. $c_{MI} = 0$ for the dashed red line, and $c_{MI} = 0.260$ for the dashed yellow line. The parameter values are $A = 2,\ c_I = 0.5,\ \sigma_\theta^2 = 0.5,\ \sigma_z^2 = 0.3,\ \sigma_\epsilon^2 = 0.15,\ \mu = 3,\ \mu_z = 5$ and $\Pi = 1.03$.



<Figure 4> shows three cases for the value of information $g(c_I, c_{MI}, \lambda)$ as a function of informed investors ratio λ. There are three solutions for $c_{MI} \in [0, 0.260)$, which are $\underline{\lambda}_u$, $\overline{\lambda}_u$, and λ_s. Due to $\underline{\lambda}_u$ and $\overline{\lambda}_u$ are the unstable solutions, $\lambda = 0$ if $\lambda < \underline{\lambda}_u$ and $\lambda = 1$ if $\lambda > \overline{\lambda}_u$. The only stable solution is the middle one, which is λ_s. When $c_{MI} > 0260$, there are only two solutions, both unstable.

<Figure 5> depicts simulation results of multiple equilibria. Panel A shows the proportion of the informed λ as a function of the inflation calculating cost c_{MI}. For a

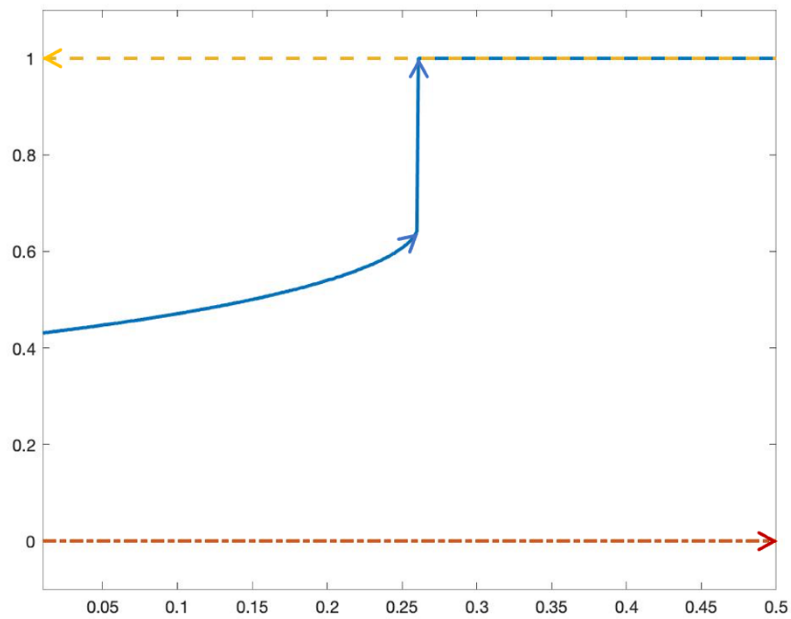
stable solution, λ is increasing from 0.4310 to 0.6420 as c_{MI} increases from 0 to 0.260. When c_{MI} passes 0.260, λ jumps up to 1, i.e. all investors become informed investors, because the inflation calculating cost is too high. And λ can be an unstable solution if the initial λ is less than $\underline{\lambda}_u$. In this case, all investors remain as money-illusioned even if c_{MI} increases. When c_{MI} decreases from a certain high level, all investors resolve money illusion, i.e. $\lambda = 1$.

Panel B of <Figure 5> shows expectation of the equilibrium price $E(P)$ as a function of c_{MI}, when $\mu = 3$ and $\mu_z = 5$. $E(P)$ has the same patterns to λ. If the initial λ is less than $\underline{\lambda}_u$, $E(P)$ is low and when c_{MI} decreases from a high value, $E(P)$ also remains at a high level. However, if the initial λ takes a value between $\underline{\lambda}_u$ and $\overline{\lambda}_u$, $E(P)$ increases from -3.3701 to -2.300 as c_{MI} increases from 0 to 0.260. And when c_{MI} passes 0.260, $E(P)$ jumps up to -1.2300, just same as λ.

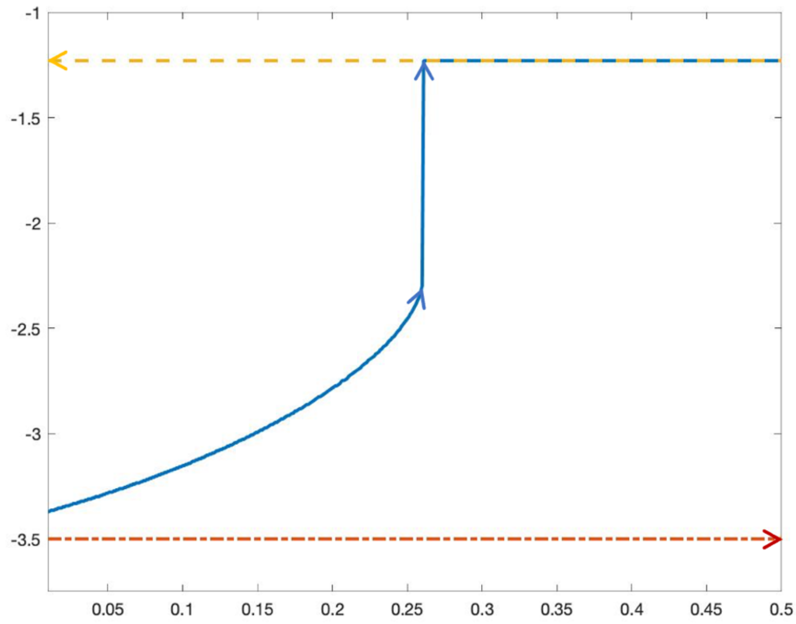
<Figure 5> The multiple equilibria: The proportion of the informed and expectation of the equilibrium price

<Figure 5> depicts the optimal λ(Panel A) and expectation of the equilibrium price(Panel B) as a function of the inflation calculating cost, c_{MI}. Blue line is for increasing c_{MI} under initial $\lambda \in [\underline{\lambda}_u, \overline{\lambda}_u]$, dashed red line is for increasing c_{MI} when initial λ is less than $\underline{\lambda}_u$, and dashed yellow line is for decreasing c_{MI} from a certain high level. The parameter values are $A = 2,\ c_I = 0.5,\ \sigma_\theta^2 = 0.5,\ \sigma_z^2 = 0.3,\ \sigma_\epsilon^2 = 0.15,\ \mu = 3,\ \mu_z = 5$, and $\Pi = 1.03$.

A. The proportion of informed investors λ



B. Expectation of the equilibrium price $E(P)$



IV. Conclusion

Money illusion is a phenomenon that the agents cannot distinguish real monetary value from nominal one. In this paper, money illusion is considered as a matter of information and the agents can resolve it strategically or not. When there exists the cost of calculation to resolve money illusion, uninformed investors remain as money-illusioned under deflation if $c_{MI} > \underline{c}_{MI}$, but resolve money illusion over certain intervals of the signal if inflation happens.

Furthermore, the cost of inflation calculating generates the complementarities in information acquisition and multiple equilibria, so this model can explain price swings and path-dependent price.

This model helps to understand the role of money illusion in economics and finance. Specifically, the perspective of market microstructure, the assumption that considers money illusion as asymmetric information and takes into account the strategic behaviors for the money illusion improves understanding of the money illusion.

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Appendix

A1. Proof of Lemma 1.

Since the denominator is positive, the sign of $b_\pi - b_1$ is the same as the numerator. Let Λ be numerator of $b_\pi - b_1$.

$$\Lambda = (1-\lambda)[\Pi^2(\lambda X + Y + Z)Z + \Pi(\lambda X + \lambda Y + Z) - (X + Y + X)(\lambda X + Z)]$$

where

$$X \equiv \lambda\sigma_\theta^2, \qquad Y \equiv A^2\sigma_\theta^2\sigma_z^2\sigma_\epsilon^2, \qquad Z \equiv A^2\sigma_z^2\sigma_\epsilon^4$$

The slope of Λ is positive when $\Pi > 0$, because

$$\frac{\partial\Lambda}{\partial\Pi} = (1-\lambda)[2\Pi(\lambda X + Y + Z)Z + (\lambda X + \lambda Y + Z)] > 0$$

It's trivial $\Lambda = 0$, if $\Pi = 1$. This implies

$$\begin{cases} \Lambda < 0, & if\ \Pi < 1 \\ \Lambda > 0, & if\ \Pi > 1 \end{cases}$$

Therefore, $b_\pi < b_1$ under deflation, and $b_\pi > b_1$ when inflation occurs.

Q.E.D.

A2. Proof of Proposition 4.

Lemma 2.

There exists a certain level of inflation calculating cost, defined as c_{MI}^*, such that $E\left[e^{-A\bar{C}(s)}\right]$ and $E\left[e^{-A\hat{C}(s,c_{MI})}\right]$ increase in c_{MI} for $c_{MI} > c_{MI}^*$.

Proof.

$\frac{\partial E\left[e^{-A\bar{C}(s)}\right]}{\partial c_{MI}}$ can be expressed in three parts:

$$\frac{\partial J_1}{\partial c_{MI}} = -\sqrt{2\pi}\frac{\sigma_{f|s}^2}{Q^2}n\left(\frac{Q}{\sigma_{f|s}}\left(\underline{s} - R\right)\right)\left(-\frac{1}{2\sqrt{D_\pi}}\right) < 0$$

$$\frac{\partial J_2}{\partial c_{MI}} = -\sqrt{2\pi}\frac{\sigma_{f|s}^2}{Q_\pi^2}\left[n\left(\frac{Q_\pi}{\sigma_{f|s}}(\bar{s}-R_\pi)\right)\left(-\frac{1}{2\sqrt{D_\pi}}\right) - n\left(\frac{Q_\pi}{\sigma_{f|s}}(\underline{s}-R_\pi)\right)\frac{1}{2\sqrt{D_\pi}}\right] > 0$$

$$\frac{\partial J_3}{\partial c_{MI}} = -\sqrt{2\pi}\frac{\sigma_{f|s}^2}{Q^2}\left[-n\left(\frac{Q}{\sigma_{f|s}}(\underline{s}-R)\right)\right]\frac{1}{2\sqrt{D_\pi}} > 0$$

So, $E\left[e^{-A\bar{C}(s)}\right]$ increases in c_{MI} due to Δs increases in c_{MI}.
There exists a certain level of the inflation calculating cost $c_{MI,J}$ which makes

$$\frac{\partial J_1}{\partial c_{MI}} + \frac{\partial J_2}{\partial c_{MI}} + \frac{\partial J_3}{\partial c_{MI}} = 0$$

In the same way, let $c_{MI,K}$ such that $\frac{\partial E\left[e^{-A\hat{C}(s,c_{MI})}\right]}{\partial c_{MI}} = 0$. Then $\sum_{i=1}^{3} J_i$ and $\sum_{i=1}^{3} K_i$ increase in c_{MI} for all $c_{MI} > c_{MI}^*$ by defining $c_{MI}^* = \max\left(c_{MI,J}, c_{MI,K}\right)$.

Q.E.D.

Following Mele and Sangiorgi(2015), there exist the information complementarities in this model if $g(c_I, c_{MI}, 1) > g(c_I, c_{MI}, 0)$. It's because $g(c_I, c_{MI}, 1) > g(c_I, c_{MI}, 0)$ implies that $g(c_I, c_{MI}, \lambda)$ can increase in λ.

$$g(c_I, c_{MI}, \lambda) = \underbrace{\frac{1}{2A}\ln\left(\frac{\sigma_{f|s}^2}{\sigma_\epsilon^2}\right) - c_I}_{Grossman-Stiglitz\ effect} + \underbrace{\frac{1}{A}\ln\left(\frac{E\left[e^{-A\hat{C}(s,c_{MI})}\right]}{E\left[e^{-A\bar{C}(s)}\right]}\right)}_{Resolving\ money\ illusion\ effect}$$

It's well known that Grossman-Stiglitz effect is decreasing in λ. So, it's enough to show that resolving money illusion effect increases in λ under certain conditions. Specifically, I want to show that resolving money illusion effect when $\lambda = 1$ is higher than that effect when $\lambda = 0$ if c_{MI} is high enough. Resolving money illusion effect is equivalent to

$$\frac{E\left[e^{-A\hat{C}(s,c_{MI})}\right]}{E\left[e^{-A\bar{C}(s)}\right]} = \frac{e^{Ac_{MI}} \times J_1 + K_2 + e^{Ac_{MI}} \times J_3}{J_1 + J_2 + J_3}$$

Its denominator is finite scalar as $c_{MI} \to \infty$ and K_2 is also goes to finite number as c_{MI} increases. However, the limit value of $e^{Ac_{MI}} \times J_1$ and $e^{Ac_{MI}} \times J_1$ are dependent on the absolute risk aversion coefficient A. As c_{MI} goes to infinity, $e^{Ac_{MI}}$ also goes to infinity but J_1 and J_3 go to 0. In this case, the velocity of each term plays an important role in determining the limit value of $e^{Ac_{MI}} \times J_1$ and $e^{Ac_{MI}} \times J_3$. Specifically, as c_{MI} increases,

$e^{Ac_{MI}}$ goes to infinity with the velocity of e^{A}, but J_1 and J_3 become zero at the rate of $e^{\frac{1}{M_\pi}}$. Thus, $e^{Ac_{MI}} \times J_1$ and $e^{Ac_{MI}} \times J_3$ are increasing in c_{MI} if $A > \frac{1}{M_\pi}$. In contrast, $e^{Ac_{MI}} \times J_1$ and $e^{Ac_{MI}} \times J_3$ decrease in c_{MI} if $A < \frac{1}{M_\pi}$.

$$\lim_{c_{MI}\to\infty} \frac{E\left[e^{-A\hat{C}(s,c_{MI})}\right]}{E\left[e^{-A\bar{C}(s)}\right]} = \lim_{c_{MI}\to\infty} \frac{e^{Ac_{MI}} \times J_1 + K_2 + e^{Ac_{MI}} \times J_3}{J_1 + J_2 + J_3} = \begin{cases} 0, & if\ A < \dfrac{1}{M_\pi} \\ \infty, & if\ A > \dfrac{1}{M_\pi} \end{cases}$$

Therefore, if $A > \frac{1}{M_\pi|_{\lambda=1}}$ and $A < \frac{1}{M_\pi|_{\lambda=0}}$, then resolving money illusion effect when $\lambda = 1$ is greater than that effect when $\lambda = 0$.

Lemma 3.

There exists a certain level of absolute risk aversion coefficient, defined as A^*, such that $\mathcal{g}(c_I, c_{MI}, 0) < \mathcal{g}(c_I, c_{MI}, 1)$ for all $A > A^*$ if $\Pi^2(\sigma_\theta^2 + \sigma_\epsilon^2) < 1$.

Proof.

The information complementarities exist if

$$\frac{1}{M_\pi|_{\lambda=1}} < A < \frac{1}{M_\pi|_{\lambda=0}}$$

Thus, A exists if

$$\begin{aligned} &\frac{1}{M_\pi|_{\lambda=1}} < \frac{1}{M_\pi|_{\lambda=0}} \iff M_\pi|_{\lambda=1} > M_\pi|_{\lambda=0} \\ \iff &\frac{\Pi+1}{\Pi} A^2\sigma_\epsilon^4 - \frac{2A^2\sigma_\theta^2\sigma_\epsilon^4}{\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4} < \Pi^3(\Pi+1)(\sigma_\theta^2+\sigma_\epsilon^2)^2(\sigma_\theta^2 + A^2\sigma_z^2\sigma_\epsilon^4) \\ \iff &\begin{cases} A^2 < -\dfrac{[2\Pi + (\Pi+1)\{\Pi^4(\sigma_\theta^2+\sigma_\epsilon^2)^2 - 1\}]\sigma_\theta^2}{(\Pi+1)\left\{\Pi^4\left(\sigma_\theta^2+\sigma_\epsilon^2\right)^2 - 1\right\}\sigma_z^2\sigma_\epsilon^4}, & if\ \Pi^2(\sigma_\theta^2+\sigma_\epsilon^2) > 1 \\ A^2 > \dfrac{[2\Pi - (\Pi+1)\{1-\Pi^4(\sigma_\theta^2+\sigma_\epsilon^2)^2\}]\sigma_\theta^2}{(\Pi+1)\left\{1-\Pi^4\left(\sigma_\theta^2+\sigma_\epsilon^2\right)^2\right\}\sigma_z^2\sigma_\epsilon^4}, & if\ \Pi^2(\sigma_\theta^2+\sigma_\epsilon^2) < 1 \end{cases} \end{aligned}$$

and due to $2\Pi > (\Pi+1)\{1-\Pi^4(\sigma_\theta^2+\sigma_\epsilon^2)^2\}$ under inflation, following is satisfied.

$$A > \left[\frac{[2\Pi - (\Pi+1)\{1-\Pi^4(\sigma_\theta^2+\sigma_\epsilon^2)^2\}]\sigma_\theta^2}{(\Pi+1)\left\{1-\Pi^4\left(\sigma_\theta^2+\sigma_\epsilon^2\right)^2\right\}\sigma_z^2\sigma_\epsilon^4}\right]^{\frac{1}{2}} \equiv A^*, \qquad if\ \Pi^2(\sigma_\theta^2+\sigma_\epsilon^2) < 1$$

Therefore, as c_{MI} goes to infinity, resolving money illusion effect goes to infinity when $\lambda = 1$ and resolving money illusion effect becomes zero when $\lambda = 0$, if $A > A^*$ and $\Pi^2(\sigma_\theta^2 + \sigma_\epsilon^2) < 1$. In this case, $g(c_I, c_{MI}, 1)$ is greater than $g(c_I, c_{MI}, 0)$.

Q.E.D.

A3. Deep deflation and the deep inflation

In this section, reanalyze the former results of Section 4 in Chapter II when deep deflation($\Pi < \underline{\Pi}$) and deep inflation($\Pi > \overline{\Pi}$) occur. In this case, M_π can take negative value.

$$M_\pi = b_\pi \left(\frac{\Pi + 1}{\Pi} b_\pi - 2b_0\eta \right)$$

Since b_π and the denominator of M_π are positive, the sign of M_π is the same to the sign of the numerator of M_π, defined as Γ.

$$\begin{aligned}\Gamma \equiv\ & \{\Pi\lambda(1-\Pi\lambda) - \lambda(1-\lambda)\}X^2 + \Pi\lambda(1-\Pi)XY + \{\Pi(1+\lambda) - (1-\Pi)\}XZ \\ & + (1+\Pi)\Pi YZ + (1+\Pi)\Pi Z^2 \\ & = (Z - \lambda X)(\lambda X + Y + Z)\Pi^2 \\ & + (\lambda X^2 + \lambda XY + (1+\lambda)XZ + YZ + Z^2)\Pi - \lambda(1-\lambda)X^2 \\ & - (1-\lambda)XZ\end{aligned}$$

Then there are solutions of $\Gamma = 0$, which are

$$\Pi^* = -\frac{\lambda X^2 + \lambda XY + (1+\lambda)XZ + YZ + Z^2 \pm \sqrt{D_\Gamma}}{2(Z-\lambda X)(\lambda X + Y + Z)}$$

where

$$\begin{aligned}D_\Gamma \equiv\ & \{\lambda X^2 + \lambda XY + (1+\lambda)XZ + YZ + Z^2\}^2 \\ & + 4(Z-\lambda X)(\lambda X + Y + Z)\{\lambda(1-\lambda)X^2 + (1-\lambda)XZ\} > 0\end{aligned}$$

A3.1. Deep deflation

Let Π_1 and Π_2 be the solutions of $\Gamma = 0$, and suppose $\Pi_1 < \Pi_2$. If $Z < \lambda X$, M_π is negative when $\Pi < \Pi_1$. If $Z > \lambda X$, M_π is negative when $\Pi < \Pi_2$. Therefore, if $A^2\sigma_z^2\sigma_\epsilon^4 < \lambda^2\sigma_\theta^2(Z < \lambda X)$, then $\underline{\Pi} = \Pi_1$ and if $A^2\sigma_z^2\sigma_\epsilon^4 > \lambda^2\sigma_\theta^2(Z > \lambda X)$, then $\underline{\Pi} = \Pi_2$.

Proposition 5.

If deep deflation occurs($\Pi < \underline{\Pi}$), the uninformed resolve money illusion when $s \notin [\underline{s}, \overline{s}]$ regardless of the quality of signal.

Proof.

Because of $\Gamma|_{\Pi=0} < 0$ and $\Gamma|_{\Pi=1} > 0$ as follows,

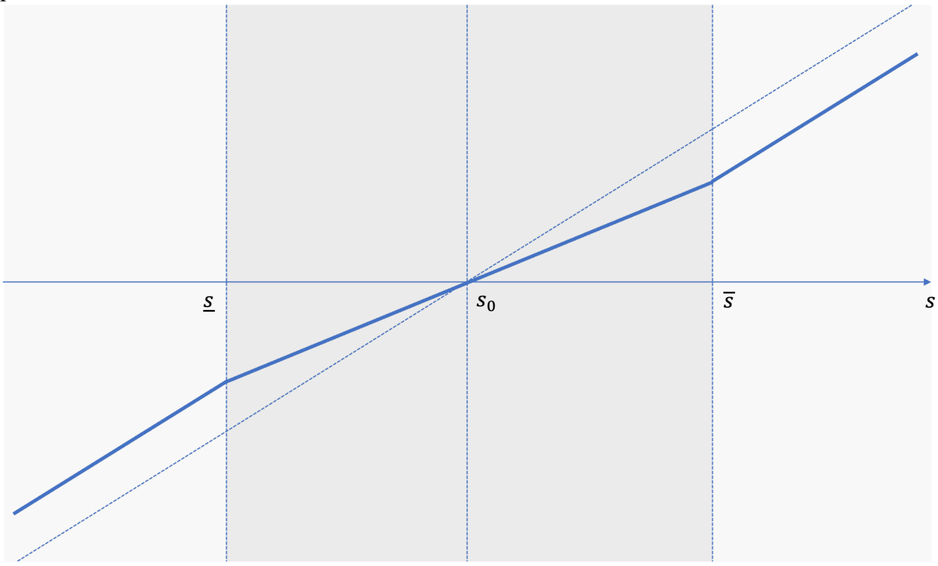
$$\Gamma|_{\Pi=0} = -(1-\lambda)(\lambda X^2 + XZ) < 0$$
$$\Gamma|_{\Pi=1} = 2\lambda XZ + 2YZ + 2Z^2 > 0$$

there exists $\underline{\Pi}$ which makes Γ negative in the interval of $(0, 1)$. Thus, M_π is negative for deep deflation($\Pi < \underline{\Pi}$). In this case, $D_\pi > 0$, because of $c'_{MI} < 0$. It implies that $u_L(P, p) < u_{MI}(P, p)$ when the signal is in the interval of $[\underline{s}, \overline{s}]$.

Q.E.D.

<**Figure 6**> Price curve with the inflation-calculating cost under deep deflation($\Pi < \underline{\Pi}$).

Solid line is the price curve over signal s when the uninformed are governed by money illusion and deep deflation occurs($\Pi < \underline{\Pi}$). And dashed line is the price curve of Grossman-Stiglitz(1980). The uninformed behave as money-illusioned if $s \in [\underline{s}, \overline{s}]$(shaded area), whereas they resolve money illusion if $s \notin [\underline{s}, \overline{s}]$. The behavior of the uninformed is the same as inflation, but the shape of the price curve is different.



This seems to be the same result as what happens in inflation. However, the shape of the equilibrium price curve is different from when inflation occurs. <Figure 6> shows the equilibrium price curve when uninformed investors are governed by money illusion under deep deflation. The behavior of uninformed investors is the same as in inflation, but the shape of the price curve is different from that in inflation.

A3.2. Deep inflation

It's clear that if $A^2\sigma_z^2\sigma_\epsilon^4 < \lambda^2\sigma_\theta^2 (Z < \lambda X)$, M_π is negative when $\Pi > \Pi_2$. Therefore, if $A^2\sigma_z^2\sigma_\epsilon^4 < \lambda^2\sigma_\theta^2 (Z < \lambda X)$, $\overline{\Pi} = \Pi_2$.

Proposition 6.
If the quality of the signal is relatively good($A^2\sigma_z^2\sigma_\epsilon^4 < \lambda^2\sigma_\theta^2$), the uninformed remain as money-illusioned when deep inflation($\Pi > \overline{\Pi}$).

Proof.
If $A^2\sigma_z^2\sigma_\epsilon^4 < \lambda^2\sigma_\theta^2$, M_π is negative for deep inflation($\Pi > \overline{\Pi}$). In this case, D_π is also negative if $c_{MI} > c^*$. It implies $u_L(P,\, p) < u_{MI}(P,\, p)$ for all s.
This is the same result when deflation occurs.

Q.E.D.

However, when the quality of the signal is relatively bad, i.e. $A^2\sigma_z^2\sigma_\epsilon^4 > \lambda^2\sigma_\theta^2$, nothing is changed under deep inflation, because M_π is always positive under inflation due to $\Pi_2 < 1$.

Lemma 4.
If $A^2\sigma_z^2\sigma_\epsilon^4 > \lambda^2\sigma_\theta^2 (Z > \lambda X)$, M_π is always positive for $\Pi > \underline{\Pi}$.

Proof.
If $(Z - \lambda X)(\lambda X + Y + Z) > 0$, then the coefficient of the term of degree 2 in Γ is positive. Therefore, M_π is also positive for $\Pi > \underline{\Pi}$.
When inflation happens($\Pi > 1$), M_π is always positive if $Z > \lambda X$.

Q.E.D.