The Sources of Risk in Credit Portfolio and Their Hedge Possibility

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Abstract

This paper proposes a new method which identifies sources of credit portfolio's default clustering as macroeconomic factors, default contagion (asset correlation) and pure frailty effect under the Basel regulatory framework. Our model estimates time-varying risks of these three sources and their contributions using Hoeffding decomposition. This paper also shows how the credit portfolio's default clustering can be dispersed by investigating the hedge performances of market hedge instruments against the estimated time-varying risk sources.

Our empirical results for the U.S aggregate loan sectors find that the default clustering in each loan portfolio strengthens during economic downturns. The risk contributions to default clustering are large in order of macroeconomic factors, asset correlation and pure frailty effects. On the hedge performance, the risk due to macro factors are most hedgeable, followed by asset correlation and finally pure frailty effect is least hedgeable. We additionally find that the regulatory asset correlation of mortgages and individual sectors are not sufficient even though consideration of model risk during downturn periods.

Keywords: Default clustering, Frailty effect, Asset-correlation, Credit portfolio risk, Time-varying risk parameters, Systamatic risk factor, ASRF model, LHP assumption, Risk source hedge.

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1 Introduction

During the economic downturn, borrowers in the credit portfolio show a default clustering since the risk contagion(spillover) effect resulting doubly default event. Duffie et al. [2009] found an unobservable latent factor that causing default clustering even though control observable macroeconomic and individual-specific covariates. This means the uncertainty of credit loss distribution that cannot be explained by various(common or specific) control variables. This failty effect is unobservable and changeable characteristics over time. Is frailty effect really unknown?

The credit risk is determined by exposure at default (EAD), probability of default (PD) and loss given default (LGD) on individual obligors. The default correlations among obligors are an important parameter in determining the loss distribution for a credit portfolio. Because the greater the correlation is the thicker(or fat) the tail shape of the loss distribution and the bank needs more capital to secure. Thus, this correlation in the risk-weighted requirement capital functions under the Basel regulation as an asset correlation(see Basel [2006]). The Basel's ragulation function is based on Aymptotic Single Risk Factor(ASRF)[1] model by Gordy [2003]. This model shows that the dependence between borrowers can be expressed as their sensitivities of the single common systematic factor. In this regulatory framework, PD and asset correlation are key parameters in determining the shape of credit portfolio loss distribution(see Gordy [2000]).

The in fact, unknown characteristic of the frailty effect is from static assumptions about the coefficients that represent the sensitivity of the risk factor even single or multi-factor models. The Basel's asset correlation [2] is also applied as a static(constant) assumption or a function of the default rate with upper or lower bounds. However, it is well-known that individual asset value sensitivity for market systematic factor changes according to the economic conditions. Due to limited assumptions about asset correlation, the effect is a mixture of the effect for pure common systematic risk factor variation and time varying changes on their sensitivity.

This paper breaks down sources affecting portfolio credit risk and measures the risk contribution of each source under the Basel regulatory frame. We propose methods of expressing the time-varying credit loss distribution that can be caused by the dynamics of three risk sources(observable macroeconomic factors, default contagion(asset correlation), and from pure frailty). For these purposes, our model to the aggregate net charge-off rates of six loan sectors for the U.S. commercial banking system. Each sector loan portfolio is considered to satisfy the Basel model's large homogenous portfolio(LHP) assumption. And we examined the default clustering within the portfolio and cross loan sector spillovers during the economic downturn. In addition, we show the performance of hedging against each risk source in the portfolio.

[1] Model description is in section (2.1).

[2] Appendix 1.Basel's assset correlation criteria.

The default clustering phenomenons are observed during every economic crisis, but its source and hedge method are less investigated. However, accurate measurement of correlated default is also important for portfolio risk monitoring and hedging, as well as pricing for a variety of derivatives such as CDOs that use them as underlying assets. In general, default rates depend on the macro economic conditions(Pesaran et al. [2006], Koopman et al. [2012], Bonfim [2009]). But Das et al. [2007] defined the default correlation among corporate obligors as the effect of frailty using the doubly stochastic intensity model, and Duffie et al. [2009] investigated strong evidence for the default correlation remaining even after controlling the common economic variables and the individual characteristics of borrowers. In particular, Koopman et al. [2011] attempts to explain the default clustering using more than 100 macro variables and firm-specific variables. After that, various studies on the cluster phenomenon of corporate defaults include the common macro economic factors and the frailty effects by credit rating and industry. (Koopman et al. [2012], Kwon and Lee [2018]).

Many studies have been conducted to explain the default clustering not only for individual corporate obligors but also at the portfolio level. Jimenez and Mencia [2009] provides dynamic modeling industry portfolio of the Spanish banking system and shows that the frailty effect is significant. Lee and Poon [2014] proposed a dynamic model that can measure the frailty and spillover effects on three levels(global, parental sector and sector-specific wide) using a state-space model for the U.S. aggregated sector loan portfolio. Babii et al. [2019] shows a strong spatial dependence between commercial and residential defaults in major US cities.

Various approaches to the default clustering are being tried, but most are based on the continuous-time default intensity model for corporate under survival models. Portfolio-based researchs are also attempting multi-factor approaches by expanding the systematic factor based on Basel-based asset correlation. However, due to this limited approach to asset correlation, the frailty effect, which effectively explains the default clustering in economic crisis, is considered a unobservable latent factor.

Basel provides guidance on the risk components(PD,LGD,EAD) and allows internal ratings for regulatory capital in the advanced IRB approach. On the other hand, the asset correlation is provided only conservative criteria for each loan type, and discretion is not exercised. The Basel's asset correlation depends on portfolio asset class by sector(Basel [2019]), since the sensitivity to the overall economic change varies across the characteristics of the borrowers within asset class(Committee et al. [2005]). Thus, the asset correlation is assumed to be a decreasing function of default probability and positive correlation to corporate size. This supervisory asset correlations are based on empirical research from G10 supervisors' data set, and will remain unchanged under Basel III in 2022.

However, if the relationship between the asset correlation and the default rate is not the reverse one, it may be overcharged(undercharged) for a low(high) protfolio loss rate. Although there is study of Lopez [2004] supporting Basel's asset correlation criteria, some empirical studies show different results.[3] In addition, there are attempts used to estimate the asset correlations for various areas differently.[4] But, almost all studies show that Basel's asset correlation is more conservative than their actual estimates (Chernih et al. [2006])[5]. Those approaches sufficiently considering the economic cycles in various regions as using long-period data. But, there are two problems as follows. Firstly, most of the studies are considering only the cross-sectional characteristics of asset correlation as sector,size and PD, etc. Secondly, these reflect a static asset correlation through out the entire period similar to the Through the cycle(TTC) philosophy[6]. But, these problems can deepen the mismatch between real required capital and regulatory capital, even though it reflects the economic downturn sufficiently. In addition, assumptions about the inverse relationship between the default rate and the asset correlation in the actual portfolio have a inherent problem of not being able to reflect defualt clustering during recession.

For this reason, some literature examines how are varying asset correlations during recessions. However, in most studies, asset correlation was estimated by dividing the entire data into parts, such as dummy variables, or using the rolling window method(Lee et al. [2011], Botha and van Vuuren [2010], Siarka [2014], Stoffberg and van Vuuren [2016]). But, these methods also have problems with two aspects. Firstly, it is possible to underestimate the asset correlation that is realized in an actual crisis by the effect of reducing volatility in the estimation of the rolling window method. Secondly, rolling-window estimation requires sufficient data to be obtained within each estimated period, but the default data in the observation period is not enough since observation frequency is quarterly or be more.

Therefore, in this paper, we set the asset correlation and the default threshold of the portfolio as variables that change over time and estimate the values realized in the real loan portfolio for each time point. In particular, in order to build for models of time-varying asset correlation, we revised Patton [2006] dependent model, which estimates the inter-dependence among capital market asset variables. In addition, the dynamic default model(see Hamerle et al. [2003], Rösch [2003], Crook and Bellotti [2010])was developed and used to set a model for the default threshold that changes over time. Through this, we estimate the time-varying

[3]The relation between the asset correlation and the default rate could be positive or U shape according to credit grades or variety categories-size, industry, country, etc. (Perli and Nayda [2004], Bandyopadhyay et al. [2007], Düllmann and Scheule [2003], Dietsch and Petey [2004] etc.).

[4]Using default data(Hamerle et al. [2003], Frey and McNeil [2003], Jobst and de Servigny [2005], Jakubík et al. [2006]).

Using asset returns data(Düllmann et al. [2007], Lopez [2004], Lee et al. [2011] etc.).Those analyses have limitations by conducting only for listed companies because the stock return used an approximation. However, the default frequency data is used for retail portfolios.(see Botha and van Vuuren [2010], Stoffberg and van Vuuren [2016], Siarka [2014], Crook and Bellotti [2010]).

[5]This conservative assumption can be justified to reduce model risk by constant assumptions on LGD or infinite granular portfolio assumption of the Large homogeneous portfolio(LHP) model(Chernih et al. [2006], Hamerle et al. [2003]).

[6]The TTC(Through the cycle) philosophy applied in calculating the obligor's credit assessment model and these transition matrix on Internal ratings-based(IRB) model .

portfolio loss distribution and identify the three risk sources(due to observable macroeconomic factors by default threshold model, default contagion or spillover by asset correlation model, and pure frailty) of portfolio loss under the Basel framework. Moreover using the Hoeffding decomposition applied conditional copula simulation, we propose a method that can measure the contribution of risk sources at each time for the loss distribution. Futhermore we test the effect of spillover between the two sectors[7] from one identified as the cause of the crisis to the other sector during the two crisis periods. Finally, we examine the possibility of hedging using market-tradable assets for each decomposed risk source.

The contributions of this paper are as follows. Firstly, our model proposes new method for time-varying model of loan portfolios loss distribution under Basel framework which the current global stadards of credit risk management. This model can decompose the default clustering to macro economic effect and contagion(asset correlation) effect and pure frailty effect. To the best of our knowledge, this approach to asset correlation through time modeling is the first. These time-varying approach not only presents the existing cross-sectional considerations(eg. PD level, Sector, Region, Credit grade), but also suggests that dynamics in asset correlation over time within the portfolio. These could serve as a practical consideration for the regulatory framework and useful monitoring tools for financial institutions that manage their portfolio losses. Secondly, our methods are useful ways to capture the portfolio risk sources and their contributions point-in-time. The result can build strategies for managing credit risks on a source-by-source basis to ensure portfolio stability. We show more efficient way to hedge each source by tradable market vehicles. It will also provide a new approach to pricing various derivatives on underlying assets of credit loss. Thirdly, our results can be used to imporove the Basel's regulatory criteria for asset correlation in times of economic crisis. Our model also can minimize the model risk that can arise from the assumption of various models and the lack of data that the existing rolling window method must encounter on the Basel frame work. In addition, the advantage of our approach is simple without additional information under the current risk management system.

This remainder of this paper is organized as follows. Section 2 Develops the time-varying loss distribution using the ASRF model under the LHP assumption by time-varying sources and suggested a methodology to estimate the contribution of risk sources point in time. In the empirical analysis of Section 3 using the aggregate charge-off rate of the U.S. banking system, each portfolio risk source is decomposed and it's contribution to the portfolio is measured. Also, we test the hedge possibility for each risk source and evaluate portfolio asset correlation to Basel's conservative criteria. Section 4 conclues with comments.

[7]Business sector-DBC crisis and Mortgages-sector -GFC crisis

2 Methodology

2.1 Basel's portfolio credit risk model

2.1.1 Model for obligor level default

As a well-known structured model by Merton (1974), obligor's default event occurs when the value of asset becomes less than the value of unexpired liabilities during a certain period of time. That is, what happens when the net asset value is negative. In particular, Black and Cox (1976) proposed that a specific threshold to replace a liability, because default event can reveal before asset value drops below the debt value in debt maturity.

Let V_i and h_i are asset return and default threshold for borrower i,

$$Y_i = \begin{cases} 1, & V_i < h_i \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Y_i denotes the default indicator for i obligor, taking either the value of one for default or the value of zero for non-default. Assume that the distribution of obligor's asset return is standard normal distribution, the unconditional default probability of obligor i is given by

$$p(y_i = 1) = p(V_i < h_i) = p_i(y) = \Phi(h_i) \tag{2}$$

where Φ is a cumulative standard normal distribution all time periods.

Let obligor i's standard asset return V_i is a function of single systematic risk factor F,

$$V_i = \sqrt{\rho_i}F + \sqrt{1-\rho_i}\varepsilon_i, \qquad for \qquad i = 1, \cdots, N \tag{3}$$

where $F \sim N(0,1)$ and $\varepsilon_i \sim N(0,1)$, then $V_i \sim N(0,1)$. F and ε_i are assumed to indepenent for all i and $Cov(\varepsilon_i, \varepsilon_j)$s are zero when $i \neq j$. Factor F is the composite systematic risk factor and ε_i is the idiosyncratic factor for asset return. Then $\sqrt{\rho_i}$ illustrates obligor's sensitivity to the systematic factor as a linear correlation between V_i and F. We can write that the conditional default probability of obligor i is given by

$$p_i(y) = \mathbb{P}(V_i < h_i | F = f) = \Phi(\frac{h_i - \sqrt{\rho_i}f}{\sqrt{1-\rho_i}}) \tag{4}$$

where f is realized systematic risk factor. F is a common risk factor of default that can't be diversified and observable. Both the Merton-type model as well as the default intensity model try to break down systematic factor into observable macroeconomic covariate and individual obligors' characteristics, which are called the multi-factor model approach.

2.1.2 Model for portfolio level default

Vasicek (1991) and Gordy (2003) proposed Asymptotic Single Risk Factor(ASRF) model that approximate credit portfolio loss distribution on three assumptions. This model used IRB approch for Basel II regulatory capital calculation. Assumption 1. Homogeneity: Individual assets within a portfolio have the same characteristics. Assumption 2. Large portfolio: There are countless(infinite) borrowers in the portfolio, so there is no contribution of individual loans on the entire portfolio. Assumption 3. Fine grained: Loan size is so evenly distributed that no individual loan dominates the portfolio. This asymptotic model known as large homogeous portfolio(LHP) is based on large number theory. All borrowers in each portfolio are affected solely by a single common factor F and their idiosyncratic risks are diversified away. The portfolio multi-factor approach is to break down latent single systematic factor under these assumptions into observable macroeconomic common covariate and various unobservable frailty effects. Also asset correlation ρ_i can be replaced by a portfolio-wide correlation ρ.

We can omit subscript i in eq (4) on three assumption for simplicity and write default probability as,

$$p(y) = \Phi(\frac{h - \sqrt{\rho}f}{\sqrt{1-\rho}}) \tag{5}$$

where the default threshold h and the sensitivity $\sqrt{\rho}$ are same value in each portfolio. These mean all obligors have the same default probability on these LHP assumption 1 and 3. We denotes for the portfolio of size n, it's default rate ,L_n as

$$L_n = \frac{1}{n}\sum_{i=1}^{n} I_{(V_i<h)} \tag{6}$$

where $I_{(V_i<h)}$ is the defalut indicator under the value 1 if $V_i < h$, and 0 otherwise. Thus L_n converses to $p(y)$ on assumption 2 by law of large numbers, $n \to \infty$.

$$L = \lim_{n\to\infty} L_n \to p(y) \tag{7}$$

.

The cumulative distribution of unconditional portfolio default rate on three assumption is

$$F(\ell) = \mathbb{P}[L = p(y) < \ell] = 1 - \Phi\left(\frac{h - \sqrt{1-\rho}\Phi^{-1}(\ell)}{\sqrt{\rho}}\right) \tag{8}$$

where $F(\ell)$ is a function with the parameters of ρ and h. This paper analyzed credit loss distribution for portfolio level as the sector under ASRF on LHP assumptions. Under the Basel framwork, the loss distribution for portfolio g as,

$$F_g(\ell_g) = 1 - \Phi\left(\frac{h_g - \sqrt{1-\rho_g}\Phi^{-1}(\ell_g)}{\sqrt{\rho_g}}\right) \tag{9}$$

where h_g and ρ_g are constant value for certain period of time by static assumption model.

Portfolio asset correlation about systematic risk ρ_g is an important parameter of credit risk valuation. In addition, a common static default threshold h_g is also needed to reflect the economic situation. But, the systematic common single factor is integrated and disappears as in this process. These models represent changes by the systematic factor according to economic conditions as a distribution of expected losses and are described as unexpected losses for extreme conditions.

2.2 Time-varying risk parameters

Phiosophy of probalblity of default(PD) rating on IRB rules covers downturn economic cycle by the Through the cycle(TTC) method that used a long-term stressed default rate. This approach is only a conservative approach for credit rating stability and economic compliance, and there is a limit to actually reflecting the risk dynamics of the portfolio by economic conditions(see Catarineu-Rabell et al. [2005], Kashyap et al. [2004]). Although PD rating grade covers downturn periods, it does not reflect volatility of PD through the cycle within grades. This phenomenon is due to asset returns in economic downturn periods that realized more volatility and sensitivity than stable periods. In the credit market, the increase of unexpected variability under the standard credit model is perceived as default clustering called uncertainty. But, Basel provides conservative standards for asset correlations in order to secure the stability of a portfolio of financial institutions against asymmetric asset movements in each economic conditions. However, portfolio managers need a default model according to the economic situation. For this, the establishment of a model based on the PIT philosophy has been proposed. In particular, Rösch [2003] defined the setting of the default threshold as a model affected by observable systematic factors, such as macroeconomic variables, unlike conventional static models. However, these models focus on modeling the default rate of individual borrowers. So, our model targets a aggregate portfolio's PD that can be used by the bank's portfolio managers and regulators. In this paper we don't consider individual obligor's co-variate(idiosyncratic factor) within portfolios because of the loss distribution of portfolios assumption based on LHP assumtions in section 2.1.1. In addition, macroeconomic factors affecting the portfolio are set differently among the candidate variables because the characteristics of obligors differ by loan sector.

The purpose of setting the default threshold and asset correlation as a time-varying model is to separately examine the effect of the change of expected default rate from macroeconomic variables and the change of the asset correlation among assets in the portfolio accoding to the fluctuation of the economic situation. Indeed, Hamerle et al. [2003] investigaged the absolute value of the correlation between assets is sharply reduced in the model that assumes dynamic PD compared to the model that assumes static PD. So we propose time-varying models for default threshold and asset correlation, respectively.

2.2.1 Time-varying default threshold(PD)

To model the expected default level according to macroeconomic conditions, we assume that defaults of the portfolio are independent given the observable time-lagged macro variables at time t.

$$\begin{aligned} p_{g,t|t-\tau}\,|\boldsymbol{z}_{t-\tau} &= \Phi\left(\beta_0 + \textstyle\sum_{k=1}^{q_z} z_{k,t-\tau}\beta_k^z\right) \\ &= \Phi\left(h_{g,t|t-\tau}\right), \end{aligned} \tag{10}$$

where $z_{k,t-\tau}$ is the $k-th$ macroeconomic variable at time $t-\tau$ for $k=1,2,\cdots,q_z$, and $h_{g,t|t-\tau}$ is a default threshold at time t as predicted at time $t-\tau$. And we denote a vector of observable common macro factors as $\boldsymbol{z}_{t-\tau} = (z_{1,t-\tau}, z_{2,t-\tau},\cdots,z_{q_z,t-\tau})'$.

Also,we denote default threshold for portfolio g as

$$h_{g,t|t-\tau} = \beta_0 + \sum_{k=1}^{q_z} z_{k,t-\tau}\beta_k^z \tag{11}$$

The intercept β_0 and the parameter β_k^z are the sensitivities of portfolio g to common macroeconomic factors. The macro economic factors are time-lagged in eq (11) as each of the portfolios are generally foreclosed[8] after a certain period of borrower distress. Hence, macroeconomic factors is expected to lead portfolio defaults by time τ. This property is of great practical importance, as it implies that portfolios default probabilities can be forecast using well-given information that is available at the time of forecast. Therefore this paper is interested in four quaters relation of default threshold and lagged macro variables as $\tau = 1,2,3,4$. This model examines the dynamics of expected loss for the portfolio level impliled by observable macroeconomic variables. The credit rating model of the individual obligors may be an important thing in the monitoring of loans or the early warning of the possibility of insolvency of borrowers, but the dynamics of the portfolio's loss distribution is more important for the portfolio managers of financial institutions and supervisors. In addition, the characteristics of the distribution of loan portfolio losses may differ depending on the sector of the loan classified by the financial institution. Therefore, our default threshold model defines the time-varying type by portfolio units, and each portfolio's model selects macroeconomic variables and thier leading time τ that respond sensitively and have predictive power on default rates.

This process is attempted to secure the safety of the model through three step tests. After

[8] Foreclosure is the default criterion in the empirical analysis.

selecting the first candidate variables through the previous research, in order to understand the time lag (leading time) relationship between each economic variable and default probability. We examine cross-correlation and the second candidate variables are selected by considering the significance of statistics and explanation of the economic model simultaneously. After that, the multicollinearity and model explanatory power are checked for regression analysis before finding our model, and then the third candidate variables are selected and optimized simultaneously when estimating the final time-varying loss distribution.

2.2.2 Time-varying asset correlation

The homogeneous asset correlation within portfolio under the Basel framework is the static value or the function of PD across economic cycles including the stressed conditions. This approach, along with the PD model based on the TTC philosophy, able to supervise the portfolio of assets sufficient stability, but has the weakness of not affecting the actual risk. Our model can estimate the asset correlation reflecting the economic condition and default rate uncertainty over time without collect additional data in the current supervisory system.

We define $\rho_{g,t}$ is a time-varing asset correlation of portfolio g, at time t for $t = 1, 2, \cdots, T$. We can write

$$\rho_{g,t|t-s} = \tilde{\Lambda}\left(\alpha_{g,0} + \alpha_{g,1}\rho_{g,t-1} + \alpha_{g,2}\frac{1}{S}\sum_{s=1}^{S}\left(\Phi^{-1}\left(u_{g,t-s}\right)\right)^2\right) \tag{12}$$

where $\tilde{\Lambda}(x) = (1 + e^{-kx})^{-1}$,$k > 0$,$-\infty < \alpha_0 < \infty$, $\alpha_1 \geq 0$, and $\alpha_2 \geq 0$. The logistic transformation is intended to limit the divergence of estimates for the time-varying asset correlation, which is the second moment over time. The univariate variance term $u_{g,t-s}$ is a estimated portfolio loss on sector g at time $t - s$ given $h_{g,t-s|t-s-\tau}$ and $\rho_{g,t-s}$. The time-varying asset correlation in eq (12) has similar type as ARMA(1,S) process. We modify the original specification of Patton (2006) that modelling asymmetric conditional dependnece for high-frequency market price. But our time-varing model correlaton is different in use univariate variance term instead of variables covariance term. This model can be defined by the homogeneity assumption of assets in the portfolio on LHP model described section 2.1.2.

This model assumes that asset correlation at time t consists of two factors. First, through the auto-correlation term of asset correlation just before(time $t - 1$), it intends to reflect the long-term memory of the relationships among assets in the loan portfolio. The second term, the moving average of the univariate volatility term from the estimated endogenous loss distribution at time $t - s$, is used to model the persistence of the impact of portfolio loss due to the economic fluctuations. The impact persistence of these economic shocks also varies depending on the characteristics of the portfolio, so it is set up separately, and the optimal duration s is also selected simultaneously by Akaike's Information Criterion(AIC) and Schwarz-Bayesian Infomation Criterion(SBC) statistics.

2.2.3 Time-varying loss distribution

We can represent cumulative distribution function for time-varing portfolio loss rate with time-varying default threshold equeition in eq (11) and time-varying asset correlatton in eq (12) to eq (13) as

$$F_{g,t}(\ell_{g,t}) = 1 - \Phi\left(\frac{h_{g,t} - \sqrt{1-\rho_{g,t}}\Phi^{-1}(\ell_{g,t})}{\sqrt{\rho_{g,t}}}\right) \tag{13}$$

where $\sqrt{\rho_{g,t}}$ are time-varying asset correlations as sensitivity of common systematic risk factor F at time t and $\ell_{g,t}$ is a observable charge-off rate data for sector g at time t. This distribution drives by all of the time-varying variables, so we can examine the dynamics of the loss distribution at each observation point.

2.2.4 Parameter estimation

To estimate time-varying asset correlatton $\rho_{g,t|t-s}$ in eq (12) and time-varing default threshold $h_{g,t|t-\tau}$ in eq (11), we sequentially follow next four stages.

In the first stage, we estimate cross-correlations through 24 macro variables(raw data and differencial term) and sector's charge-off rate in order to select economic variables to be used in the $h_{g,t|t-\tau}$ model. And temporary regression model sets variables selected in the cross-correlation test by significance and expected sign. We assume that defaults of portfolios are independent given the observable (time-lagged) macro variables and the non-observable (contemporary) latent factor at time t. Differentiating time-varing portfolio loss rate cumulative distribution function in section 2.2.3 a with respect to $\ell_{g,t}$ gives the probability densty function by inverse function theorem as

$$f(\ell_{g,t}) = \sqrt{\frac{1-\rho_{g,t}}{\rho_{g,t}}} \cdot exp\left[\frac{1}{2\rho_{g,t}}\left(h_{g,t} - \sqrt{1-\rho_{g,t}}\Phi^{-1}(\ell_{g,t})\right)^2\right] \cdot exp\left[\frac{1}{2}(\Phi^{-1}(\ell_{g,t}))^2\right] \tag{14}$$

The second stage, the constant ρ_g and h_g are estimated using unconditional loss distribution in section 2.2.3.

$$\max_{\boldsymbol{\theta}} \prod_{i=1}^{N} \sqrt{\frac{1-\rho_g}{\rho_g}} \cdot exp\left[\frac{1}{2\rho_g}\left(h_g - \sqrt{1-\rho_g}\Phi^{-1}(\ell_{g,t})\right)^2\right] \cdot exp\left[\frac{1}{2}(\Phi^{-1}(\ell_{g,t}))^2\right] \tag{15}$$

where $\ell_{g,t}$ is a observable charge-off rate at time t for sector g. These will be compared to the time-varying estimates in our final model as static estimates.

The third stage, we calculate initial values of the time-varying loss distribution model in eq(12). The initial $\rho_{g,t-1}$ is calculated by first 5 quaters data. And the initial values of β_0 and β_k^z in time-varing default threshold model, the sensitivity to economic variables, are calculated assuming that asset correlation is fixed all the time from the second stage.

The fourth stage, The estimates of the final time-varying model are obtained based on the maximum likelihood function, including the time-varying asset correlatton $\rho_{g,t|t-s}$ in eq (12) and time-varing default threshold $h_{g,t|t-\tau}$ in eq (11) given earlier,

$$\max_{\boldsymbol{\theta_{final}}} \prod_{t=1}^{N} \sqrt{\frac{1-\rho_{g,t}}{\rho_{g,t}}} \cdot exp\left[\frac{1}{2\rho_{g,t}}\left(h_{g,t|t-\tau} - \sqrt{1-\rho_{g,t}}\Phi^{-1}(\ell_{g,t})\right)^2\right] \cdot exp\left[\frac{1}{2}(\Phi^{-1}(\ell_{g,t}))^2\right] \quad (16)$$

where $\boldsymbol{\theta_{final}} = \left(\rho_{g,t}, h_{g,t|t-\tau}\right)'$ for the time varying asset correlation parameters $\boldsymbol{\alpha_g} = (\alpha_{g,0}, \alpha_{g,1}, \alpha_{g,2})'$ and the estimates of parallax economic variables describing time-varying default threshold are $\boldsymbol{\beta_g}^z = (\beta_0, \beta_1^z, \beta_2^z, \cdots, \beta_k^z)'$. The time-lagged loss distribution $u_{g,t-s}$ in section 2.2.2 estimate by $F^{Vas}(\ell_{g,t-s}|\rho_{g,t-s}, h_{g,t-s})$ as initial value is uesed in the parameters of the third step. And the final models are selected by AIC and SBC include optimal time persistence s as short-term shock in time-varying asset correlation model in section 2.2.2. Through the above four stages, the time-varying $\rho_{g,t}$ and time-varying $h_{g,t|t-\tau}$ can be calculated for each point in time, which means to be able to estimate time-varying loss distribution in section 2.2.3. The final estimates of time-varying are from 1991:Q2 to 2019:Q3, excluding the 5 quarter data used to calculate the initial value in the third stage.

2.3 Risk sources decompositon and contribution

2.3.1 Risk sources decompositon

Using the log-likelihood function in eq(16), the estimation set of the parameters of each risk source model can calculate the estimation of each point-in-time for the source of credit risk, together with the portfolio loss distributions. The expected value of the unconditional loss distribution at each time point is shown as a function of the default threshold of the conditional distribution[9].

$$E[F_{g,t}] = \Phi\left(\frac{\frac{\widehat{h_{g,t}}}{\sqrt{1-\widehat{\rho_{g,t}}}}}{\sqrt{1+(-\frac{\sqrt{\widehat{\rho_{g,t}}}}{\sqrt{1-\widehat{\rho_{g,t}}}})^2}}\right) = \Phi\left(\widehat{h_{g,t}}\right) \quad (17)$$

[9]For $X \sim N(0,1)$, then $E[\Phi(aX+b)] = \Phi(\frac{b}{\sqrt{1+a^2}})$.

Using conditional distribution 5, define point-in-time conditional distributions are

$$p(y) = \Phi(\frac{h_{g,t} - \sqrt{\widehat{\rho_{g,t}}} \cdot \widehat{f_{g,t}}}{\sqrt{1 - \widehat{\rho_{g,t}}}}) \tag{18}$$

.

From inverse conditional distributions can drive systematic common factor at time t

$$f_{g,t} = \frac{\widehat{h_{g,t}} - \sqrt{1 - \widehat{\rho_{g,t}}} \cdot \Phi^{-1}(\ell_{g,t})}{\sqrt{\widehat{\rho_{g,t}}}} \tag{19}$$

$f_{g,t}$ can be defined "Pure frailty effect" for portfolio g at time t that the presence of common latent factors, even when controlling for expected loss given observable macroeconomic condition and for sensitivity loading of systematic factor. Furthermore, this method can calculate another frailty under static assumptions of the risk source using the parameter estimations in 15.

2.3.2 Risk sources contribution

To measure the contribution to each point-in-time loss of each estimated risk source, we propose a conditional copula simulation method for Hoeffding decomposition. The Copula function[10]widely used in financial applications[11] for decoupling a multi-variate joint distribution to maginal distributions and their dependence structure. And Hoeffding decomposition[12] is a frequently method[13] of decomposition for factor contributions in risk management. Determining the different sources in the portfolio and measuring contributions are critical to ensuring stability for the configured assets by managing[14] and controlling[15] the correct sources.

In particular, Rosen and Saunders [2010], Lee and Poon [2014] show that the random variable portfolio loss can be decomposing as the sum of expected loss given all subset of systematic risk factors(macro economic or frailty). They measure the contribution of the loss distribution for each factor using a linear multi-factor model. However, Tasche [2008], Cherny et al. [2010] show that Hoeffding decomposition can be used to decompose the effects of nonlinear factors. The simple case present in these paper for decomposing of two factors F_1 and F_2. We can write the portfolio loss $L = H(F_1, F_2)$ as

[10]See Nelsen [2007], Cherubini et al. [2004]Nelsen [2007], Cherubini et al. [2004] for the full description and mathematical backgrounds.

[11]See McNeil et al. [2015], Patton [2006], Lee and Yang [2019] for various financail filed modelling methods.

[12]See Van der Vaart [2000]for originally developement methodology.

[13]See Rosen and Saunders [2010], Lee and Poon [2014] for application of credit portfolios.

[14]Asset allocation, Risk budgeting

[15]Pricing of derivatives, Hedge

$$
\begin{aligned}
L &= E[L|\cdot] \\
&+(E[L|F_1] - E[L|\cdot]) + (E[L|F_2] - E[L|\cdot]) \\
&+(E[L|F_1, F_2] - (E[L|F_1] - E[L|\cdot]) - (E[L|F_2] - E[L|\cdot]) - E[L|\cdot]).
\end{aligned} \tag{20}
$$

where the first row term $E[L|\cdot]$ is the expected loss without any risk factors(unconditional expected as constant). In the second row term $E[L|F_1]$ and $E[L|F_2]$ denote the risk contributions for portfolio loss L_g from factor F_1 and F_2,respectively. The $(E[L|F_k] - E[L|\cdot])$ operation owing to estimate pure risk contribution from factor F_k. The last row term represents the residual risk contribution even controlling unconditional and individual risk factor contribution. Also this means pure expect loss from comovement in the factors F_1 and F_2.

Our decomposing method extends eq 20 to three risk sources model for within portfolio risk contribution. Among them, we are interesting in the following four terms in order to confirm the pure contribution of each source and basis expected loss at each time.

$E[L_{g,t}|\cdot]$: The basis expected loss in portfolio g at time t.

$(E[L_{g,t}|\Phi\left(\widehat{h_{g,t}}\right)] - E[L_{g,t}|\cdot])$: From risk source for implied PD in macroecnomic covariate.

$(E[L_{g,t}|\widehat{\rho_{g,t}}] - E[L_{g,t}|\cdot])$: From risk source for contagion(asset correlation).

$(E[L_{g,t}|\widehat{f_{g,t}}] - E[L_{g,t}|\cdot])$: From risk source for pure frailty effect controlled other sources.

However, we could not know the joint density of the portfolio loss given estimated risk sources at time t. Thes, we suggest the conditional copula simulation method that estimates the joint distribution by the kernel density based on the empirical dependence structure between portfolio loss and estimated risk sources. For the purpose, we apply Novosyolov [2017] method for conditional distribution for portfolio loss given risk sources at time t. The expected value is calculated for the conditional loss distribution using 1 million Monte Carlo simulation reflecting the Gaussian copula dependence structure from the empirical kernel joint density.

3 Empirical Analysis

3.1 Data

For the empirical study, we use the quarterly aggregated charge-off rates data of U.S. commercial banking system by the loan sector level. These data are collrected from Federal Deposit Insurance Corporation(FDIC) for "Mortgages"[16] , "Business"[17] ,"Rest"[18], "Credit cards"[19], "Individuals"[20], "Lease"[21] during 1984:Q1~2019:Q3. These data include debt information from all companies and individual owners who are affiliated with the FDIC. During the entire data period, our analysis terms are from 1990:Q1 to 2019:Q3 when all data of six research categories(sectors) are represented. The definition of charge-off obligors is the number of the loan as 120 days delinquency[22] referred by Federal regulatory institutions. We calculated charge-off ratio that divide the number of total charge-offs by average outstanding each period by sector. To obtain the annualized rates, we multiplied to each quarterly charge-off rates by factor 4, because banks commonly measure credit risk over the one-year horizon.

Table 1 shows main descriptive statistics for 119 quarterly charge-off rates of across sectors and aggregated portfolio that calculated by the preceding process. The rate of credit cards is on average the highest sector, while mortgages have the lowest. The credit cards charge-off rate is highest on volitility also among the sector. As the left skewness of unconditional loss distribution is high across all sectors, it shows the fat-tail characteristics of the credit assets portfolio as compared to a normal distribution. In addition, this asymmetric distribution shows that the portfolio default data shows clustering since the contagion or spillover effect in crisis periods.

[Table 1 is here.]

Fig. 1 displays the six sector's charge-off rates that were aggregated portfolio. The charge-off rates were time-lagged increasing in almost all industries as behind three economic contraction periods in the U.S. that were been defined by the National Bureau of Economic Research(NBER). These business cycles[23] are CREC(the commercial real estate crisis during 1990:Q3~1991:Q1), DBC(the dotcom bubble crisis during 2001:Q1~2001:Q4), and GFC(the Great Financial Crisis during 2007:Q4~2009:Q2) that pointed to gray area in Fig. 1 During the three times the crisis window, charge-off rates of sectors are increasing stand out. Especially, the credit card sector's increase is greater than in other industries. The peaks of rate

[16] Real Estate Loans Secured by 1-4 Family Residential Properties.

[17] Commercial & Industrial Loans to U.S. Addressees.

[18] All Other Loans.

[19] Credit Cards.

[20] Other Loans to Individuals.

[21] Lease Financing Receivables.

[22] This rule provides a good reason for using lagged macroeconomic variables for market expectation PD when setting up the time-vary default threshold model.

[23] https://www.nber.org/cycles.html

are lagged on some quarterly for crisis periods. Also, The GFC window is having to biggest impact across all sectors. Mortgage sectors have not significantly changed much during other crises except GFCs. Business and credit cards are seen as the most sensitive industries in times of crisis. Looking at the three crises during the study period, each sector reacts sensitively depending on the type of crisis, which means that the sector could be responsible for risks across the financial market.

[Figure 1 is here.]

Many studies[24] of credit risk feilds have highlighted the economic adaptability of credit portfolios. Our model based on ASRF has systematic single credit risk common factor that integrated economic conditions. The following representative economic indicators were used to reflect the effects on the various portfolios credit. Based on privious research for the correlation between macroeconomic variables and credit exposure, we consider for macro factors seasonally adjusted real GDP(GDP), the House Price Index(HPI), the consumer price index(CPI), the unemployment rate(UMEMP), and debt to income ratio(DTI) from Federal Reserve database. The market-based indicators are used the S&P500 index return(S&P500), the 1-year Treasury note rates(T_1Y), the 10-year(T_10Y), the interest rate spread between 10-year and 1-year Treasury note rates(Curvature), the 3-month T-bill rate(TB3MS), the TED Spread rates(TED) and the bank prime loan rates(Prime) in Federal Reserve Economic Data. A total of 12 macroeconomic and market variables were used in the time-varying default threshold(PD) model equation 11 in raw data or differential terms.

3.2 Time-varying risk sources decomposing

Basel recommended the obligor's default probability reflect changes insufficiently long economic cycles include downturns. Similarly, the simultaneous default correlation of borrowers within portfolio expressed as asset correlation provides only conservative criteria under empirical results. However, risk sources within the portfolio are inconsistent and varies with changes in economic conditions. This is because the value of the asset and the default threshold are bound to change by economic conditions. Therefore, this paper estimates the models with the assumptions of the time-varying asset correlation and time-varying default threshold as shown in eq (11) and eq (12).

[24]For the corporate exposures : The Real GDP growth, the S&P500 index return, the 3-month T-bill rate and the interest rate spead between 10-year and 1-year Treasury note rates.(See Koopman et al. [2011], Duffie et al. [2009]Koopman et al. [2011], Duffie et al. [2009] etc.)

For the retail exposures : The GDP growth rate, the unemplyment rate and 3-month real interest rate.(See Jiménez and Mencia [2009], Lee and Poon [2014] etc.)

For the non-performin loans: The GDP growth, the 30-year mortgage rate, the Consumer price index, the Industry production,the Prime loan rate and the Housing price index.(See Betz et al. [2020], Ghosh [2017] etc.)

3.2.1 Time-varying PD model form macro sources

The correlation between 12 macroeconomic variables[25] and 6 sectors charge-off[26] was examined as shown in table 2 before estimating the overall model time-varying default thresholds that changes with economic cycles. We denote the $\tau - th$ sample cross correlations between realized default threshold of sector g and macro variable z_k are $Corr(\Phi^{-1}(\ell_{g,t}), z_{k,t-\tau})$.

[Table 2 is here.]

Each sector has slightly different variables affected by the time difference, but the GDP growth, unemployment growth, the one-year interest rate change, and stock index show leaded cross-correlation across all sectors. The Debt-to-income ratio and the TED Spread are significant explainable time leading variables that the correlation of parallax 4 quaters are strongest for the credit crunch in all areas except mortgages sector. Especially, almost macro variables in sector mortgages and credit cards show the most significant correlation at 4 leading times. In practice, these variables are important because it can be used to forecast increases in the sector's credit risk even considering the timing of the announcement of economic variables and the charge-off data characteristics of the variables 120 days delinquency. In the business and lease sector were found to be correlated with various economic variables more than others due to the various retail loan type.

Table 3 panel A-1 shows the final model estimated of time-varying $h_{g,t}$ by the method described in section 2.2.4. Among the economic variables selected through the above cross correlation analysis, most of the variables used in the final model are those with strong time lags correlation.

[Table 3 is here.]

The DTI ratio (time lag 4) associated with debt repayment capability was chosen as a significant variable in five sectors except for mortgages. Unlike other sectors, the mortgage sector appears to be negatively affected by changes in HPI index. Our results show that the assets of mortgages, business, and personal-related sectors(individuals, credit cards) differ in macro factors to consider depending on the type of the loan.

In this way, the time-varying expected PD can be modeled using exogenous economic variables. Many studies have tried to explain the default of obligor and find predictable economic covariates. However, our model is to separate the sources of portfolio credit risk into three factors and make them manageable rather than the PD's own predictions. In particular, we would like to focus on developing logical and practical indicators that can explain the default clustering. So the estimated detail statistical information for the model's ability to predict the economic variables used in the time-varying default threshold(PD) model is not

[25] Total : 24 macro variables (Raw data and similar differential terms-The GDP and the HPI were used to growth rate scale)

[26] The conditional expectation of default threshold $h_{g,t}$ is approximated by inverse cumulatevie standard nomal distribution $\Phi^{-1}(\ell_{g,t})$

reported separately. However, table 3 panel A-1 last row shows a high correlation between the predict from this model and the actual portfolio loss. It means that predict values are suitable for the expectations of default inherent in macroeconomic variables.

3.2.2 Time-varying asset correlatoin model from contagion

Table 3 panel A-2 shows the final model parameters of time-varying $\rho_{g,t}$. In time-varying asset correlation in eq (12) , the coefficients are very significant across all sectors. The long-term memory $\alpha_{g,1}$ means to the relationship between $\rho_{g,t}$ and $\rho_{g,t-1}$ for capturing a persistence of asset correlations over time. In other words, the positive significance sign of $\alpha_{g,1}$ means that asset correlation accelerates when it begins growing in economic downturns or upturns. The persistence over the previous s lags time for short-term impact $\alpha_{g,2}$ are significant across all sectors. It reflects the volatility in the loss that imply about dynamics for asset correlation within the portfolio and external macro shocks during s times. In summary, this model has combined the long-term trend effect $\alpha_{g,1}$ and the variability effect from recent shocks $\alpha_{g,2}$ for asset correlation within the portfolio.

[Figure (2) is here.]

The credit card sector, where the level of default and its volatility are significant, shows the low level asset correlation as in the static model, although some increasing around GFC and DBC. Compared to other industries, there are relatively large coefficient of $\alpha_{g,2}$ and low-level mean of correlation. These are called retail sectors, and the assets in the portfolio consist of a large number borrowers and small-size exposure with relatively well-diversification. The credit card's PD is high due to a little(minor) delinquency, but the phenomenon of default clustering does not show even in the economic crisis. This sector with the highest default rate and volatility have a smaller significant $\alpha_{credit\,cards,1}$ than other sectors. This means that obligors within sector are more sensitive to short-term shocks than to long-term time-lagged effects. And high volatility during the economic downturn due to the base effect of the usual high default level, not to the increase in the correlation among assets. This retail loan portfolio is a low systematic risk exposure resulting from economic changes rather than idiosyncratic factors.

In the case of mortgages, it is a very important sector since the volatility of asset correlation and their exposure size although the default rate is the lowest. Asset correlation showed rapidly increasing since the 2008 GFC period and had continuously large volatility. And it was increasing sensitively to small economic shocks and reaching a peak in 2012. This means that after the economic crisis, the effects appear at a time lag and the effects remain in the portfolio loss for a considerable period of time. In particular, The uncertainty that cannot be explained by observable macro variables during the economic crisis has appeared, which can have the effect of underestimating the tail risk when evaluated by a static model.

It should be noted that the default clustering that responds to each economic crisis is different for each industry, so asset correlation showed different responses in DBC and GFC. Looking at mortgage and business, in the case of DBC crisis, the business sector has a significant default clustering and there is no reaction in the mortgage sector. Considering the effect of the two sectors, which account for 70% of the total loan market, the default clustering occurring in different sectors during each crisis can be said to be a contagion or spillover effect throughout the entire economic system.

3.3 Contribution of risk sources

3.3.1 Risk factor contribution within sector

It is a very important step for portfolio management or utilization that decomposing the risk sources of the time-varying loss distribution and assessing the contribution of each point in time. Figure 3and 4 show the risk sources for each industry portfolio, and the conditional expectations and relative values calculated by simulation in section (3.3).

[Figure 3and 4 are here.]

The first row is the patial expected value of the portfolio loss given by each risk source. The horizontal line is the sector's average expected loss when not given any risk sources. These levels show the largest mortgage and the smallest credit card, such as the average loss given in table 1 for each sector. The basis default rate of each sector appears to be the largest any risk source, but during the crisis the conditional expected loss given risk sources increases rapidly . But for the retail sector as credit cards and individuals, the risk sources effect not exceed to portfolio default rate basis even if in crisis economic condition. This is a consistent result showing the characteristics of the retail portfolio, which is well diversified and has low asset correlation.

In the mortgage sector, from the beginning of the GFC crisis, the impact of the expected loss from economic variables increases and then decreases. But the actual portfolio loss does not decrease since continuing default clustering, so a fast increase in the effect of asset correlation. The pure risk effect is also increasing in crisis, but its size is smaller than other risk sources. Especially, It should be noted that during the GFC period, the contribution of economic variables is increasing in all sectors, but there is no default clustering in all sectors. Credit markets do not perceive the expected value of losses from observable macroeconomic variables as uncertainty. This means that the credit portfolio expected loss due to economic variations and the increase in default rate beyond that are unexpected uncertainty resulting from asset correlation.

3.3.2 Risk factor contribution across sector

sector spillover effect - empirical results(working)

3.4 Hedging possibility

Each decomposed risk source dynamics can be useful in portfolio management and other fields.(eg.pricing CDO or build derivatives underlying risk sources.) We examine hedge possibility of each risk source for portfolio strategy by tradable market contracts. If it is possible to hedge the time-varying asset correlation by economic conditions and uncertainty, the portfolio risk due to the frailty effect can be further reduced as observably controllable part. We test the hedge possibility using S&P/Case-Shiller Home Price Index[27] for mortgages sector with the largest exposure and time-varying correlation among all sectors. The index can be a very useful hedge tool from a practical perspective as derivatives based on underlying assets are being traded and indexes in major cities are also being produced separately.

[Table (4) is here]

Table (4) shows the correlation between each risk sources and composite index since 2006:Q2[28]. The correlation with the realized PD in mortgage sector show -0.26 higher than other risk sources. However, the correlation with PD expected by lagged value of the macro economic variable also has a not small value. In addition, the correlation for the decomposed time-varying transition effect (asset correlation) is also significantly presented as -0.14. But, the correlation for the estimated pure frailty effect shows a low value. In these results, we find the following insights. Firstly, the manager can make efficient strategies for the portfolio by selectively hedge against risk sources. Secondly, It is possible to hedge some of the macroeconomic variables and asset correlations, which are important sources of the risk of the credit portfolio identified in section (3.2). This is a useful approach since the difficulty of hedge for each macroeconomic variable. Thirdly, the pure A effect hedge effect is not enough, so it can still be regarded as a pure risk that cannot be a hedge.

3.5 Evaluate Basel's criteria

3.5.1 Evaluate Basel's criteria

The function of Basel's required capital is taking two approaches to asset correlation. First, some of the retail sectors do not take into the change over time through the constant of a fixed value. Second, the given asset correlation equation is assuming an inverse relation for PD that limits the upper and lower values. Our previous findings support Basel's criteria and much further research that reverse relationship considering inter-sectoral PD and asset correlation[29].Also, asset correlation based on static models shows that Basel's criteria are sufficiently conservative similarly to the previous studies. However, estimated time-varying asset

[27] The CME introduce that derivatives are comprehensive tools for managing the U.S. housing risk.

[28] When futures trading for this index began.

[29] The high-level charge-off rate in the credit cards sector shows low-level asset correlation either static or time-varying in 3. On the other hand, the mortgages sector shows a higher constant asset correlation despite relatively low charge-off than other sectors.

correlation in section (3.2.2) and (3.3) show that the phenomenon of PD and asset correlation increase rapidly during the economic crisis by default clustering. These show the relation between PD and asset correlation is not the reverse considering the time perspective. Thus, we evaluate whether Basel's proposed criteria are sufficiently conservative in crisis using our time-varying asset correlation estimates[30]. For this purpose, we calculate asymptotic standard error for time-varying estimation obtain from the multivariate delta-method[31]. Using the parameter estimates in the loss distribution Likelihood function and their covariance matrix, The confidence interval of the estimation of time-varying asset correlation was calculated and presented as a gray band in eq (2).

Firstly, look at $\rho_{g,constnat}$ in figure (2) from our static model in eq (15), all sector's constant assset correaltion are much lower than Basel criteria except Rest and Lease sectors. This shows that Basel's standards are conservative enough as previous results. we apply the lower limit of 3% for two sectors, because Because obligors characteristics with both retail and corporate and the smallest exposure. However, the $\rho_{rest,constnat}$ and $\rho_{Lease,constnat}$ also slightly exceeds the lower limit and are sufficiently conservative considering the upper limit of 16%. These Basel standards can be adequate and less procyclical to requirement capital for banking stability. However, it has a weakness of not being able to captue the default risk increasing in the portfolio during economic crisis.

Secondly, It can be seen that the $\rho_{g,t}$ estimated by the time-varying asset correlation model exceeds the constant correlation $\rho_{g,constnat}$ except rest sector. This suggests that asset correlation within the portfolio may be underestimated by the static model. These results mean may not enough regulatory capital for financial institution due to default clustering when uncertainty grows up in crisis. Conversely, It means that each financial institution have far more surplus capital for stability when economic conditions are stable. Thes, there is an excessive amount of regulatory capital being imposed in normal economic conditions. In particular, especially carefully sectors are mortgages and credit cards. In the mortgage sector, the smallest default rate is realized but the static model also has an asset correlation of 10 percent and is close to Basel's constant standard around the GFC period. It implied that Basel's constant value criteria for the mortgage 15% is not sufficiently conservative considering model risk. Moreover, considering mortgage exposure which accounts for half of the loan market, timely monitoring, and management tool for asset correlation is necessary. In contrast, credit cards show the high defult rate, but both relatively low correlation of static and time-varying models. The large volatility in the GFC is due to the base effect of the high-level PD in the sector rather than to the default clustering. Individuals, Rest, and Lease sectors also show

[30] Although the definition of Basel criteria for each sector cannot be exactly the same, we are mapped to reflect sector-specific characteristics.

[31] The multivariate delta method use a deriving from propagation error for function of random variable using information matrix. See Ver Hoef [2012], Doob [1935] for the full description and mathematical backgrounds. And see Duan et al. [2011] for application of credit risk model.

Basel's criteria being exceeded. In particular, the three sectors' asset correlation increased and exceed the lower limit of Basel during the DBC and GFC periods when mortgages and business sectors increased. This can be suspected of the contagion or spillover effect by two sectors that the large size of the loan[32].

As a result of the reassessment by our model, Basel criteria confirmed that could be a gap between static standards and time-varying estimations for asset correlation. Furthermore, it can be seen that some criteria for constant value in times of economic crisis are not sufficient for stability.

[32]The mortgages and business sectors account for about 70 percent of the total exposure.

4 Conclusion

In managing and regulating the credit risk of loan portfolios, measurement and rational estimation of the sources that constitute risks are very important. Our research has presented simple and easy methods for time-varying credit portfolio loss distribution and for their souces decomposing base on the ASRF model under the LHP assumption, which is the basis for the Basel regulations. And, we propose a process for estimating the contribution of risk sources and for managing them when the economic conditions are given at each point in time. Furthermore, the hedge possibility of using tradeable vehicles is compared with portfolio loss and each risk source. In addition, the stability and over-deficiency of regulatory capital in times of crisis is checked by time-varying asset correlation for Basel's criteria.

This paper investigates that the frailty effect, which is considered an unknown factor to explain the default clustering, can be divided into asset correlation and pure frailty. As a result of applying to the U.S aggregated loan portfolio, the time-varying asset correlation is different from the static asset correlation of PD considering only the cross-section cartegories in some sectors. Our methodology could identify not only the cross-sectional diversity(eg. sector, region, credit grade etc.), but also the dynamics of loan assets comovement that comprises the portfolio held by financial institutions. It is a newly practical perspective for default clustering as uncertainty beyond expected portfolio loss by economic covariate.We are examined that the response to the loss distribution of the loan portfolio was different for each type of economic crisis in DBC and GFC. In the dynamics and contribution of each risk source, it shows the loss characteristics of the portfolio itself play a determining role in normal times, but macro variables and asset correlation play an important role in the crisis. In addition, it is found that the hedging of each risk source was more effective than the hedging on portfolio losses. The pure frailty effect is found to be difficult to hedge. Basel's criteria are conservative enough, but estimates of asset correlation using the static model show that there is a possibility of under estimation in the event of an economic downturn.

This model will be a useful tool for portfolio management not only for regulators but also for financial institutions. This method is to better reflect the fat tail of the portfolio loss distribution called default clustering in the economic downturn, thereby realizing the dynamics of the loss distribution.

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Tables

Table 1: Descriptive statistics for charge-off rates by sector in annualized

This table shows the descriptive statistics for charge-off rate by sector in 1990:Q1~2019:Q3. Charge-off historical data in FDIC covers since 1984 :Q1 all sectors except mortgages, but this paper is used sub time horizon(1990:Q1~) data that all sectors are covered. Because the mortgages sector is important that exposure weight is 46% on average all over time.

	Mortgages	Business	Rest	Credit Cards	Individuals	Lease
N	119	119	119	119	119	119
Mean	0.0045	0.0108	0.0059	0.0546	0.0159	0.0059
Std	0.0059	0.0072	0.0056	0.0177	0.0058	0.0036
Skew	1.8708	1.2294	2.6095	2.1781	1.7169	1.2235
Kurt	2.2970	0.7219	7.4004	6.3780	3.1119	1.0525
Min	0.0005	0.0033	0.0016	0.0347	0.0086	0.0017
Q1	0.0012	0.0053	0.0025	0.0437	0.0125	0.0033
Med	0.0017	0.0078	0.0040	0.0504	0.0144	0.0046
Q3	0.0040	0.0150	0.0066	0.0604	0.0178	0.0085
Max	0.0254	0.0332	0.0311	0.1444	0.0362	0.0177
Weight*	0.46	0.23	0.06	0.09	0.13	0.03

* $Weight = average(sector's\, exposure/total\, exposure)$ in time horizon(1990:Q1~2019:Q3)

Table 2: Cross correlation for expected PD model

This table presents the effective pearson correlation of realized dafault threshold and macro variables that satisfied the expected sign and statistical significance level at 5%. The reporting numbers denote that macro variables are the time leading horizon on quarterly. We consider raw data and one quarterly differential(or growth rate) data in time lead 1,2,3,4. The sequence of lag numbers means an order by the absolute value of the pearson correlation. For example, the case of GDP in use modeling of mortgages is a effective time lag correlation coefficients are only in the differential terms. And the sequence of 4 3 2 1 means that an absolute value of 4 quarters lag correlation coefficient is bigger than 3 quarters ones. That means $Corr(\Phi^{-1}(\ell_{g,t}), GDP_{t-4}/GDP_{t-5}) > Corr(\Phi^{-1}(\ell_{g,t}), GDP_{t-3}/GDP_{t-4})$. In addition, we tested not only the time leading correlation but also the backwardness of economic variables. However, we were interested in the time -leading correlation of macro variables, so we did not report any correlation coefficient of time lag that. The significance test of the final chosen correlation coefficient was conducted at a 95% confidence level.

		Mortgages	Business	Rest	Creditcards	Individuals	Lease
GDP	raw						
	diff	4 3 2 1	3 4 1	3 2 4 1	4 3 2 1	3 1 4 2	3 2 4 1
HPI	raw						
	diff	4 3 2 1	2 3 4 1		3 4 2 1		3 2
CPI	raw		1 2 3 4				1 2 3 4
	diff						
Unemloyment	raw						
	diff	4 3 2 1	3 4 2 1	2 3 1 4	4 3 2 1	2 3 1 4	4 3 2 1
DTI	raw		4 3 2 1	4 3 2 1	4 3 2 1	4 3 2 1	4 3 2 1
	diff						
S&P500	raw	4	4 3 1 2	3 4 1 2	4 3	4 1 2 3	4 2 3 1
	diff						
T_1Y	raw		4				4 3
	diff	3 4 2 1	3 4 2 1	3 2 1 4	4 3 2 1	1 4 3 2	4 3 2 1
T_10Y	raw			4			
	diff						
Curvature	raw						
	diff		2 3 4 1	2 3 1 4			4 2 3
TB3MS	raw						
	diff	4 3 2 1					
TED	raw		4 3 2	4 3 2 1	4 3 2	4 3 2 1	4 3 2
	diff		3 1				
Prime	raw		4				4 3
	diff	4 3 2 1	3 2 4 1		4 3 1 2		4 3 2 1

Table 3: Time-varying model prameters

Panel A : This table presents the estimated parameters for time-varying default thresholds $h_{g,t}$ in equation[eq:default threshold in port] and time-varying rho $\rho_{g,t}$in equation[eq:time-varying asset correlation]. In addition, the table contains the summary descriptive statistics for $\rho_{g,t}$ and $h_{g,t}$ that calculated by equation (13)each time. The optimal moving window size s in equation(13) were selected by AIC and SBC statistics. One, two and three asterisks indicate significance at the 5%, 1%, and 0.1% confidence levels, respectively.
Panel B: This table shows the unconditional constant parameters of rho $\rho_g$and h_g in Equation (9)(Static model).

Panel A-1 : Time-varying default threshold(PD) model (by macro)

var_name	Mortgages	Business	Rest	Creditcards	Individuals	Lease
constant	-2.502 ***	-2.507 ***	-3.874 ***	-2.495 ***	-3.023 ***	-4.088 ***
CPI_{t-1}		-0.005 ***				
ΔDTI_{t-4}		0.062 ***	0.107 ***	0.079 ***	0.074 ***	0.140 ***
ΔGDP_{t-3}		-6.518 ***			-2.236 **	-6.337 ***
ΔGDP_{t-4}	-14.635 ***					
ΔHPI_{t-2}		-3.448 ***				
ΔHPI_{t-3}				-1.386 **		
ΔHPI_{t-4}	-18.266 ***					
$\Delta PRIME_{t-3}$		-0.122 **				
$\Delta PRIME_{t-4}$						-0.072 ***
$TREASURY_{t-4}$		-0.025 **				
TED_{t-4}			0.168 ***			
$\Delta UNEMPLOY_{t-2}$			0.268 ***		0.071 *	
$\Delta UNEMPLOY_{t-3}$		0.202 ***				
$\Delta UNEMPLOY_{t-4}$				0.099 ***		0.075 **
$Mean(\widehat{PD})$	0.0042	0.0098	0.0057	0.0537	0.0156	0.0061
$Std(\widehat{PD})$	0.0051	0.0065	0.0048	0.0118	0.0033	0.0035
$corr(E[P\hat{D}], realized[PD])$	0.7681	0.8376	0889	0.8170	0.8279	0.8012

Panel A-2 : Time-varying $\rho_{g,t}$ model

	Mortgages	Business	Rest	Creditcards	Individuals	Lease
α_0	-0.401	-0.536 ***	-0.463 ***	-0.623 ***	-0.631 ***	-0.556 ***
α_1	1.359 ***	2.572 ***	1.056 ***	0.835 ***	4.631 ***	5.167 ***
α_2	0.019 ***	0.045 ***	0.025 **	0.051 ***	0.055 ***	0.021 ***
$Mean(\widehat{\rho})$	0.0412	0.0132	0.0156	0.0057	0.0053	0.0118
$Std(\widehat{\rho})$	0.0218	0.0114	0.0064	0.0120	0.0061	0.0110
Optimal s	2	1	5	3	2	5
Liklihood	572.00	521.30	571.87	395.86	524.59	584.28

Panel B : Constant parameters(Static model)

var name	Mortgages	Business	Rest	Creditcards	Individuals	Lease
$\widehat{PD_{g,}}$	0.0042	0.0104	0.0056	0.0550	0.0160	0.0058
$\widehat{\rho}$	0.1036	0.0493	0.0546	0.0188	0.0163	0.0367
Liklihood	513.07	435.05	496.62	317.69	444.23	509.07

Table 4: Hedge performace of risk sources

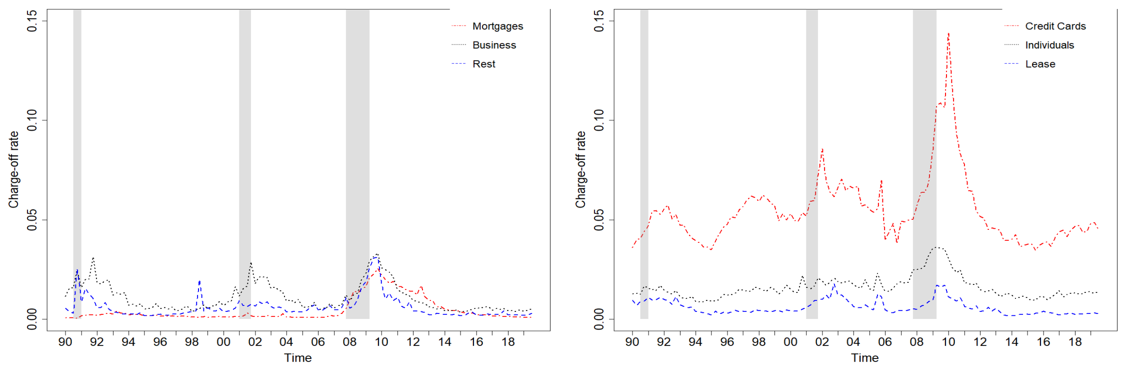
This table shows the hedgerble posibility for risk sources and sector charge-off by tradable market vhichle.

	$realized[PD]$	$E[\widehat{PD}]$	$\widehat{\rho}$	$\widehat{f}$
S&P /Case-Shiller Index	-0.26	-0.21	-0.14	0.05

Figures

Figure 1: Annualized net charge-off rate

These figures show the historical charge-off rate by sector during 1990:1Q~2019:3Q. The gray bars show U.S. business cycle contraction periods: commercial real estate crisis from 1990:Q3 to 1991:Q1, dotcom bubble period from 2001:Q1 to 2001:Q4 and the Great Recession from 2007:Q4 to 2009:Q2 defined by the National Bureau of Economic Research.

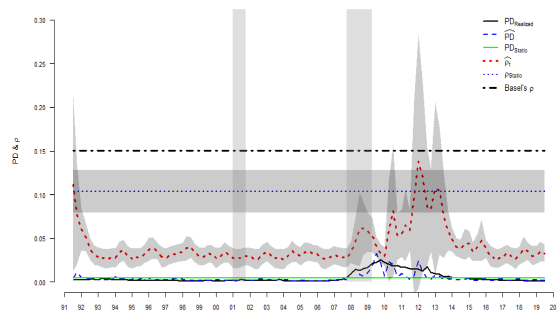


(a) Mortgages, Business and Rest

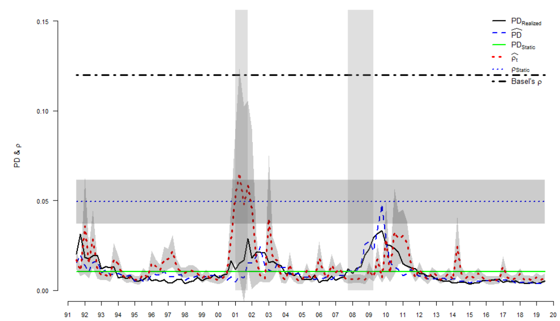
(b) Credit Cards, Individual and Lease

Figure 2: Time-varying risk sources and sector charge-off

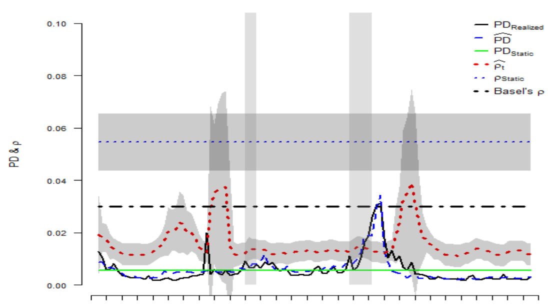
These figures compare the estimated time-varying $\rho_{g,t}$,$PD_{g,t}$ from time-varying model and constant $\rho_{g,constant}$,$PD_{g,constant}$ from static model with historial annaulized charge-off by sector. The axis range is marked differently for each sector to show information more efficiently. (eg. Mortgages:0~0.3, Business and Credit-cards:0~0.15, others:0~0.1). The gray band surrounding the two asset correlation estimates is the 95% confidence interval calculated using the delta method. The gray band around the two asset correlation estimates($\rho_{g,t}$,$\rho_{g,constant}$) is the 95% confidence interval using the delta method. Basel's criterias show constant value or lower bound of mapping exposure class in (5)



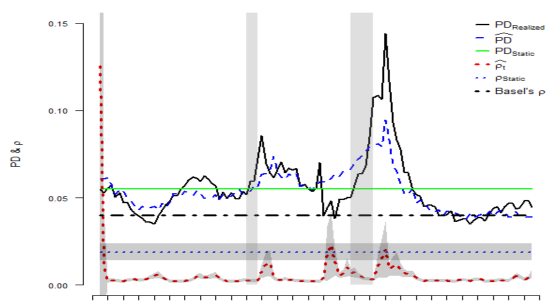
(a) Mortgages



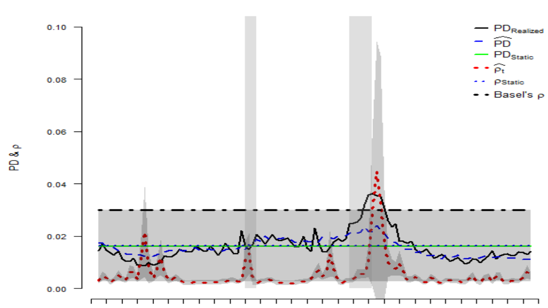
(b) Business



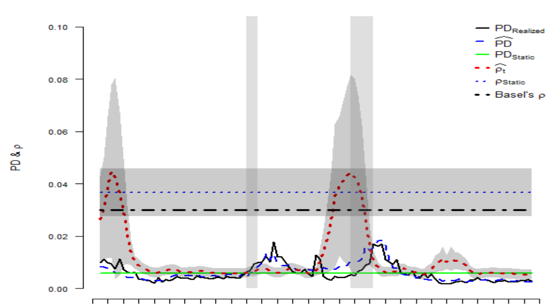
(c) Rest



(d) Credit Cards



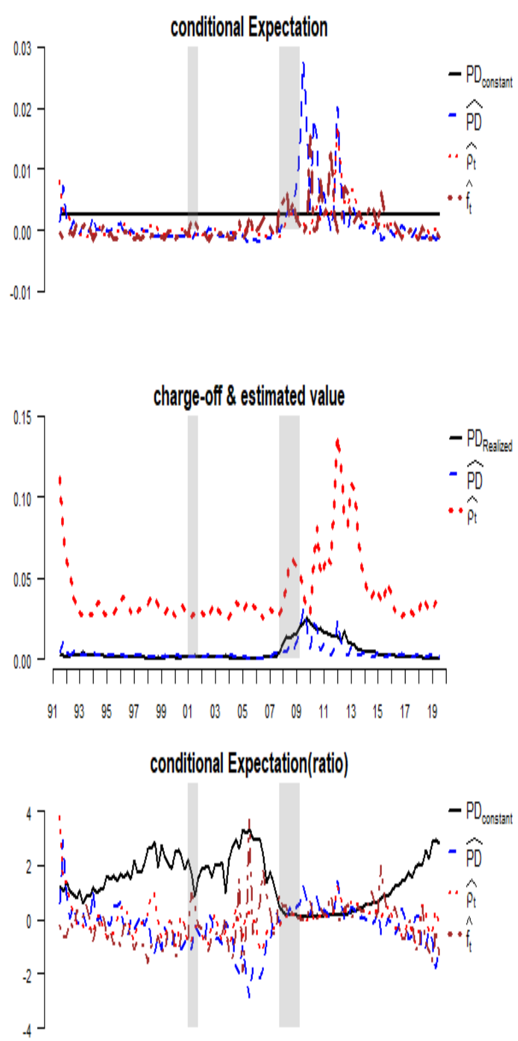
(e) Individuals



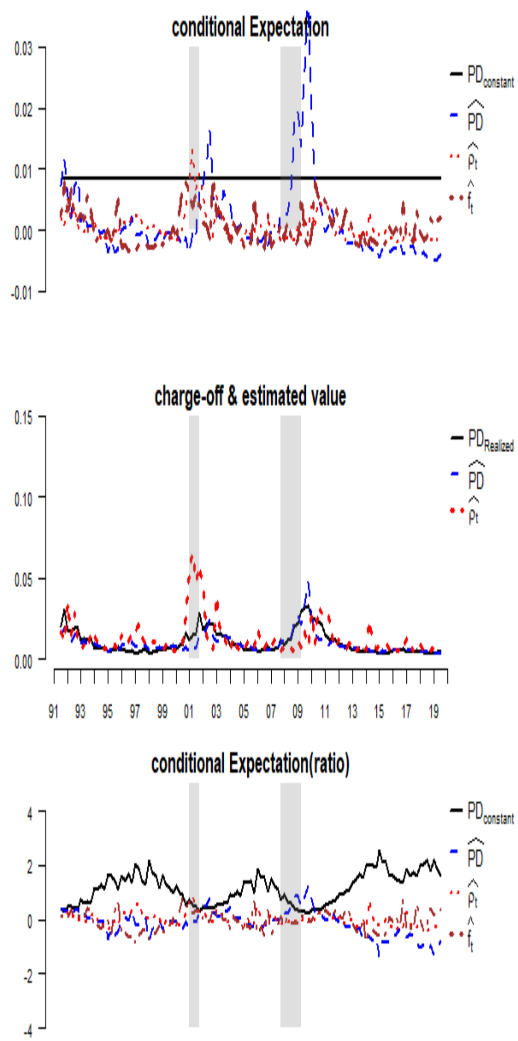
(f) Lease

Figure 3: Risk factor contribution within sector

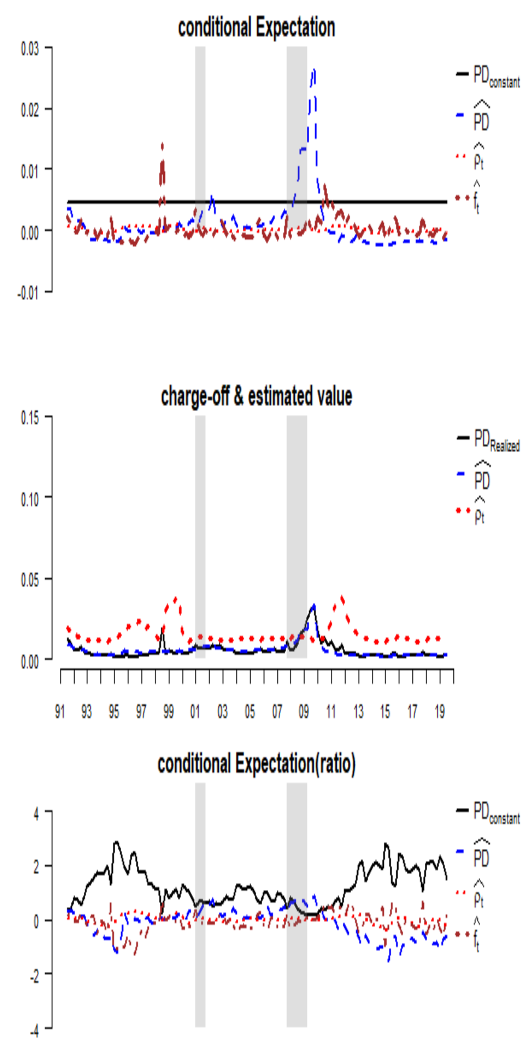
These figures show the contribution of portfolio loss distribution by the estimated risk sources for some sectors(Mortgages, Business and Rest). These are represented by three pictures of each sector. The estimated risk source in the middle line was presented, and the above line with the conditional expected value of theirs for the loss distribution, and the below line with the relative proportion of the conditional expected value by time.



(a) Mortgages



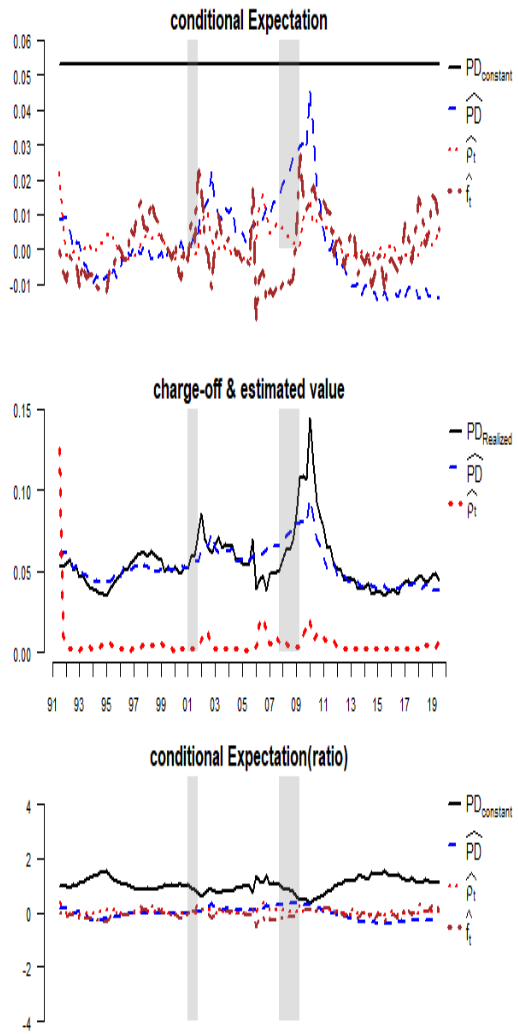
(b) Business



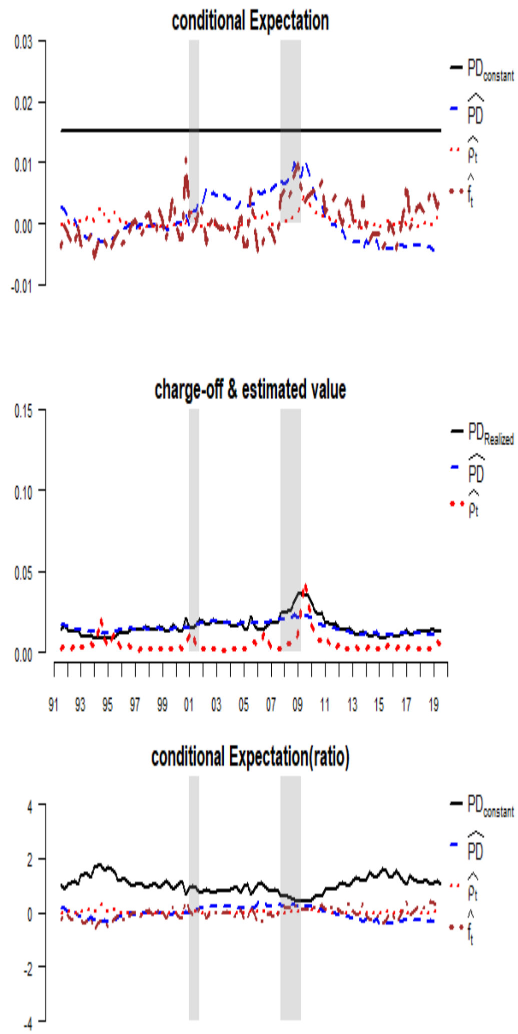
(c) Rest

Figure 4: Risk factor contribution within sector(continue)

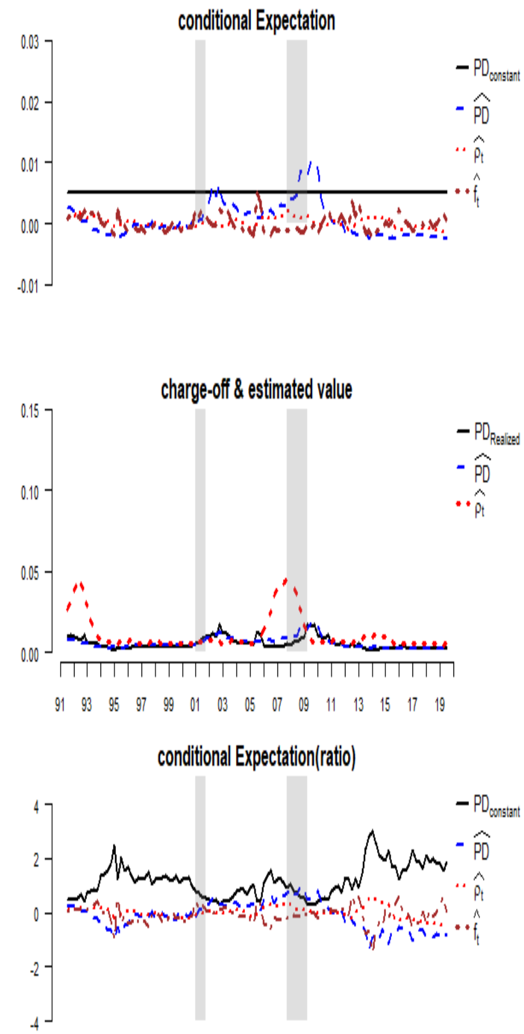
These figures show the contribution of portfolio loss distribution by the estimated risk sources for some sectors(Credit cards, Individual and Lease). These are represented by three pictures of each sector. The estimated risk source in the middle line was presented, and the above line with the conditional expected value of theirs for the loss distribution, and the below line with the relative proportion of the conditional expected value by time. The axis range is marked differently for credit cards sector in conditional expectation value to show information more efficiently. (Creditcards:0~0.06, others:0~0.03).



(a) Credit Cards



(b) Individuals



(c) Lease

Figure 5: Risk factor contribution across sector(working)

These figures show the contribution of portfolio loss distribution from risk sources in other sectors(Mortgages and Business).

(a) Credit Cards (b) Lease (c) Individuals

Appendix

1. Basel's asset correlation criteria

Table 5: Basel's criteria for asset correlation

The table is a asset correlation rules in risk weight function on Basel committee for banking supervision.(see BCBS (2019))

Exposure class	Loan type	Flexiblilty	Range	Rule
Corporate,sovereign and bank	Corporate etc.*	$F(PD)$**	[12%,24%]	$0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.24 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right)$
	FI***	$F(PD)$	[15%,30%]	$1.25 \cdot \left[0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.24 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right)\right]$
	SMEs****	$F(PD)$	[8%,24%]	$0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.24 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right) + 0.04 \cdot \left(1 - \frac{(Size-5)}{45}\right)$
	Specialised lending	$F(PD)$	[12%,30%]	$0.12 \cdot \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})} + 0.30 \cdot \left(1 - \frac{(1-e^{-50 \cdot PD})}{(1-e^{-50})}\right)$
Retail	Residential mortgage	Fixed	15%	15%
	Qualifying revolving	Fixed	4%	4%
	Others	$F(PD)$	[3%,16%]	$0.03 \cdot \frac{(1-e^{-35 \cdot PD})}{(1-e^{-35})} + 0.16 \cdot \left(1 - \frac{(1-e^{-35 \cdot PD})}{(1-e^{-35})}\right)$

* Corporates belonging to groups with total considated revenues exceeding EUR 500 million exception other categories in corporate,sovereign and bank exposure class.

**The asset correlation of categories written F(PD) means function of default probability.

***Financial Institutions.

****Small or Medium-sixed entities