Asymmetric Asset Correlations in Credit Portfolios

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Abstract

This study proposes a new time-varying credit risk model to describe the procyclicality and asymmetry of asset correlations in credit portfolios under the Basel's regulatory framework. Our suggested model is developed based on Patton (2006)'s conditional dependence and Glosten et al. (1993)'s asymmetric conditional volatility models. With the aggregated mortgage and business loan portfolios in the U.S., we show the outperformance of the suggested model over the regulatory one together with strong empirical evidence of procyclical and asymmetric asset correlation. Furthermore, we find that Basel's criteria of asset correlation could be insufficient during economic downturns largely due to the asymmetric asset correlation.

Keywords: Asymmetric asset correlation, Credit portfolio risk, Time-varying risk parameters, Procyclicality, ASRF model, Basel criteria.

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1 Introduction

Asset correlations (AC) and average probability of default (PD) are important risk parameters for determining regulatory capital charges for credit risk in the asymptotic single risk factor approach. The Basel regulation prescribes a conservative asset correlation (AC) criteria, e.g. constant or decreasing function of PD by asset (loan) type (see Basel (2006) and Basel (2019)), as in Table 1, and prohibits financial institutions from using any internal AC to ensure the stability of the financial system against adverse economic cycles. The supervisory AC criteria is suggested based on the cross-sectional studies of the G10 supervisors' data (see Calem et al. (2003), Lopez (2004), Catarineu-Rabell et al. (2005), and Lee et al. (2021)).

[Table 1 is here]

However, many studies find the procyclicality of AC (see Botha and van Vuuren (2010), Lee and Lin (2012), Lee et al. (2011), Siarka (2014), and Stoffberg and van Vuuren (2016)). Lee et al. (2011) even find the asymmetric behavior of AC rising and declining during economic downturns upturns with different magnitude. In this respect, we propose a novel model to reflect the procyclicality and asymmetry of AC under the regulatory framework.

Our suggested model generalizes the regulatory model with static parameters by allowing time-varying PD and AC. For the PD, we adopt the dynamic default threshold by Hamerle et al. (2003), Rösch (2003), and Crook and Bellotti (2010). For the AC, we combine the process of Patton (2006)'s conditional dependence and Glosten et al. (1993)'s asymmetric conditional volatility model. Our model enables us to examine the procyclicality and asymmetric clustering of AC according to the status of the economy.

In the empirical study with the U.S. aggregated loan sector portfolios, our suggested model outperforms the supervisory static model in terms of goodness-of-fit and shows strong evidence of procyclical and asymmetric AC. Furthermore, we find that the Basel's conservative AC criteria could be insufficient during economic crises and it becomes even worse when considering the asymmetric behavior of the AC.

2 Methodology

Static credit risk model

In the structural model by Merton (1974), the default of obligor i occurs when the asset value V_i drops below a default threshold h_i within a risk horizon. Thus the default probability of obligor i is given by

$$\mathbb{P}(V_i = \sqrt{\rho_i}F + \sqrt{1-\rho_i}\varepsilon_i < h_i), \tag{1}$$

where F is the single systematic risk factor, and ε_i is the idiosyncratic factor. The factors F and ε_i are assumed to be independent follow $N(0,1)$ and standard normal, thus, $V_i \sim N(0,1)$. For the large homogeneous portflio (LHP) consisting of enormous number of obligors with the identical h and ρ, the conditional default probability of all obligors given the systematic factor f is

$$\mathbb{P}(V_i < h|F = f) = \Phi\left(\frac{h - \sqrt{\rho}f}{\sqrt{1-\rho}}\right), \tag{2}$$

where $\Phi(\cdot)$ is the cumulative distribution function of $N(0,1)$. The conditional default probability in Eq. (2) is empirically observed as the default rate L, whose unconditional probability distribution is derived as

$$\mathbb{F}(\ell; h, \rho) = \mathbb{P}[L < \ell; h, \rho] = 1 - \Phi\left(\frac{h - \sqrt{1-\rho}\Phi^{-1}(\ell)}{\sqrt{\rho}}\right), \tag{3}$$

where ℓ is a realization of the portfolio default rate L. In Eq. (3), the parameter h determines the average portfolio PD as $E[L] = \Phi(h)$. The parameter ρ is called the asset correlation between obligors as $Corr(V_i, V_j) = \rho$. It determines the extent to which PD distribution spreads (see Gordy (2003) and Lee et al. (2020)). The internal rating-based (IRB) approach adopts Eq. (3) for the regulatory capital. For more details of the static credit risk model, refer to Vasicek (2002).

Time-varying credit risk model

This section suggests a new credit risk model with a time-varying default threshold (TVDT) and time-varying asset correlation (TVAC).

We first define the time-varying default threshold of the LHP at time t as

$$h_t = \beta_0 + \sum_{k=1}^{K} z_{k,t-\tau_k}\beta_k, \tag{4}$$

where $z_{k,t-\tau_k}$ is k-th observable macroeconomic variable at time $t - \tau_k$ for $k = 1, 2, \cdots, K$ with a positive time lag τ_k. Our TVDT in Eq. (4) is similar to Hamerle et al. (2003), Rösch (2003) and Crook and Bellotti (2010) but different in that it employs lagged macroeconomic variables to reflect delinquency periods before defaults.

Second, in order to model asymmetric and procyclicality of AC, we suggest asset correlation at time t as

$$\rho_t = \tilde{\Lambda}\left(\alpha_0 + \alpha_1\rho_{t-1} + \frac{1}{S}\sum_{s=1}^{S}(\alpha_2 + \alpha_3 I_{t-s})\left(\Phi^{-1}\left(u_{t-s}\right)\right)^2\right), \tag{5}$$

where

$$I_{t-s} = \begin{cases} 1 & if \quad \Phi^{-1}\left(u_{t-s}\right) \geq C \\ 0 & if \quad \Phi^{-1}\left(u_{t-s}\right) < C \end{cases},$$

S is a positive integer, $\tilde{\Lambda}(x) = (1 + e^{-kx})^{-1}$, $k > 0$, $-\infty < \alpha_0 < \infty$, $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, $-\infty < \alpha_3 < \infty$, and $-\infty < C < \infty$. The logistic transformation $\tilde{\Lambda}(x)$ restricts the bounds of the TVAC between 0 and 1. The univariate variance term of $\left(\Phi^{-1}\left(u_{t-s}\right)\right)^2$ is calculated based on the portfolio PD distribution at time $t-s$ given h_{t-s} and ρ_{t-s}, i.e. $u_{t-s} = \mathbb{F}(\ell_{t-s}; h_{t-s}, \rho_{t-s})$. Eq. (5) is built on the time-varying conditional dependence model by Patton (2006) to reflect procyclicality of ρ and the asymmetric volatility model by Glosten et al. (1993) to accommodate the asymmetry of ρ. The parameter C indicates the threshold above and below which the TVAC becomes asymmetric. Default shocks above C have greater impacts on conditional asset correlation. Note that our TVAC in Eq. (5) includes symmetric one as a special case with $\alpha_3 = 0$.

Our asymmetric TVAC can model uncertainties, or excess default clustering, leftover after explained by the TVDT in a realized portfolio PD and also examine the asymmetric dynamics of AC in boom and recession economic conditions.

Estimation

Our time-varying credit risk model has the same form of the cumulative unconditional distribution of portfolio PD in Eq. (3) but with the time-varying h_t in Eq. (4) and ρ_t in Eq.(5). The parameters are estimated by maximum-likelihood method.

3 Empirical Analysis

Data

We use the quarterly charge-off rates of the U.S. commercial banking system during 1990:Q1 $\sim$ 2020:Q4 from the Federal Deposit Insurance Corporation (FDIC). In particular, we focus on "Mortgages" and "Business" sectors since they have the largest proportion in the total loan market as causative sectors during the Dotcom Bubble Crisis (DBC) and the Global Financial Crisis (GFC), respectively.[1]

[Table 2 is here]

Table 2 shows descriptive statistics for 124 annualized charge-off rates. The mean and standard deviation of the Mortgages are lower than the Business. The distribution of the charge-off shows strong positive skewness and fat-tail compared to a normal distribution, which are stylized characteristics of credit portfolios mainly due to the concentrated defaults in economic downturns, i.e., the default clustering.

[1] The average exposure of these sectors is about 68% of total loan market (Mortgages : 42.8% and Business: 26.5%) during the entire sample period.

Estimation results

Table 3 shows the estimates of h and ρ, the average PD ($\Phi(h)$), and the log-likelihood of the static model in Eq. (3). The estimated average PD is similar to the average charge-off rate in Table 2. The higher asset correlation estimate of the Mortgage than the Business is due to its more severe defaults during the GFC as shown in higher skewness and kurtosis in Table 2.

[Table 3 is here]

Table 4 presents the estimation results of our time-varying credit risk model. For TVDT in Panel A, we initially consider the lagged values of seasonally adjusted 5 macroeconomic variables[2] and 7 market-based indicators[3] based on previous similar studies. For the Mortgage, ΔGDP_{t-4} and ΔHPI_{t-4} are significantly selected. For the Business, ΔGDP_{t-3}, ΔHPI_{t-2}, CPI_{t-1}, DTI_{t-4}, $\Delta Prime_{t-3}$, T_1Y_{t-4}, and $\Delta Unemploy_{t-3}$ are significantly selected. The averages of the estimated PD, $Mean(\Phi(h_t))$, from symmetric and asymmetric models for the Mortgages and the Business are similar to the average PDs in Table 2. This confirms the validity of the estimated results of our time-varying credit risk model.

[Table 4 is here]

For TVAC in Panel B, we compare two types of TVAC: one is symmetric with $\alpha_3 = 0$ and the other is asymmetric with $\alpha_3 \neq 0$ to emphasize the importance of asymmetric asset correlation. The parameters α_1 and α_2 are strongly significant in the symmetric and asymmetric models. This result highly supports the procyclicality of asset correlation, i.e. large asset correlations tend to be followed by large asset correlations and small asset correlations tend to be followed by small asset correlations. The parameter α_3 is also strongly significant to the Mortgages at 0.1% significance level. The significance of α_3 implies that the TVAC tend to rise during economic downturns and decline during economic upturns but with different magnitude. This asymmetric AC for the Mortgage occurs when an actual default rate is an extreme event with a probability of less than 0.79%, since $\mathbb{P}\left(I_{t-s}=1\right) = \mathbb{P}\left(\Phi^{-1}\left(u_{t-s}\right) \geq C = 2.412\right) = 0.0079$ in

[2]the real GDP (GDP), the house price index (HPI), the consumer price index (CPI), the unemployment rate (Unemploy) debt-to-income ratio (DTI).

[3]the S&P500 index return (S&P500), the 1-year (T_1Y) and the 10-year (T_10Y) treasury note rates, the interest rate spread between the 10-year and 1-year treasury note rates (Curvature), the 3-month T-bill rate (TB3MS), the TED spread rate (TED) and the bank prime loan rate (Prime).

Eq. (5). The procyclical and asymmetric AC also implies that obligors' asset returns become more sensitive to the changes of systematic factor, i.e. greater systematic risk, during economic downturns.

The averages of time-varying asset correlation, $Mean(\rho_t)$, in Panel B are very smaller than those of the static models in Table 3. However, this is consistent with Hamerle et al. (2003) that point out static models' over-estimation for asset correlation.

[Table 5 is here]

Furthermore, we compare the goodness-of-fits of the static and two time-varying credit risk models. Table 5 presents the results of the likelihood ratio tests of three models. The time-varying credit risk models significantly improve the goodness-of-fit since p-values are smaller than 0.0001. For the Mortgages, the asymmetric asset correlation model outperforms the model with the symmetric one. Lastly, the correlation between the estimated conditional PD ($\Phi(h_t)$) and ℓ_t in Panel A of Table 4 is higher for the asymmetric asset correlation than the symmetric one, which also reinforces the validity of the estimated results of our time-varying credit risk model with asymmetric asset correlation.

Fig. 1 shows the observed charge-off rates, the estimated PDs, and asset correlations based on static, symmetric and asymmetric TVAC estimates in Table 3 and 4. The GFC (2007:Q4∼2009:Q2), the DBC (2001:Q1∼2001:Q4), and the COVID-19 pandemic crisis (2020:Q1∼2020:Q2) periods from the National Bureau of Economic Research are indicated as gray bars.

[Figure 1 is here]

The conditionally expected probability of default, i.e., $PD_t = \Phi(h_t)$, given macroeconomic variables makes good predictions during the non-crisis period for the Mortgages and the Business. However, during or after the crisis the predictability of PD_t becomes worse. This unpredictability or uncertainty provokes the asset correlation to significantly increase since the asset correlation determines the variance and tail of default probability distribution. These results clearly show the procyclicality and asymmetric clustering of asset correlation. This is a consistent result with Rosch and Scheule (2004).

The estimates of α_3 for asymmetry is more significant for the Mortgages than the Business, which is due to more fat-tailedness of the Mortgages during the GFC as in Fig. 1 and Table 2. The estimated asymmetric ρ_t in Fig. 1 (a) clearly shows the asymmetric impact of default shocks on the asset correlation.

Evaluation of the Basel criteria

The static asset correlations ρ^{Static} are less than the Basel criteria ρ^{Basel} in Fig. 1 (b) and (d). However, the estimated TVACs in Fig. 1 show the clustering of successive high asset correlations around the crises. The TVACs even exceed the Basel criteria ρ^{Basel} for the Mortgages and the lower bound of SMEs for the Business.[4] These empirical results imply that the Basel criteria could be insufficient to cover extreme default clustering during crises in the presence of the procyclicality and asymmetric dynamics of the AC.

Moreover, the insufficiency of Basel's criteria becomes more intensified when considering the upper bound of 95% confidence interval (gray band) of the estimated TVACs in Fig. 1.

4 Conclusion

We propose a credit risk model with a time-varying default threshold and asset correlation. Our time-varying default threshold is determined by macroeconomic variables. Our time-varying asset correlation is developed based on a GJR-GARCH type volatility model and Patton (2006)'s conditional dependence model in order to reflect procyclicality and asymmetry of asset correlation in a credit portfolio.

For the U.S. loan portfolio, the empirical study strongly supports the validity of the suggested model and finds strong evidence of the procyclicality and asymmetric clustering of asset correlation. Last but not least, we find that the regulatory criteria for asset correlation could be insufficient in the presence of the procyclicality and asymmetry of asset correlation during economic downturns.

[4] In case of the Business, Fig. 1(c) and Fig. 1(d) contain two lower bounds of asset correlation: one $\rho^{Basel}_{Corp.}$ for corporates and the other is ρ^{Basel}_{SMEs} for SMEs since the Business's charge-off data include both of them.

References

Basel (2006). *International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. Bank for International Settlements, Basel Committee on Banking Supervision, Basel.

Basel, I. (2019). Calculation of rwa for credit risk. *Basel committee on banking supervision.*

Botha, M. and van Vuuren, G. (2010). Implied asset correlation in retail loan portfolios. *Journal of Risk Management in Financial Institutions*, 3(2):156–173.

Calem, P. S., Follain, J. R., et al. (2003). The asset-correlation parameter in basel ii for mortgages on single-family residences. *Board of Governors of the Federal Reserve System*, 27.

Catarineu-Rabell, E., Jackson, P., and Tsomocos, D. P. (2005). Procyclicality and the new basel accord-banks choice of loan rating system. *Economic Theory*, 26(3):537–557.

Crook, J. and Bellotti, T. (2010). Time varying and dynamic models for default risk in consumer loans. *Journal of Royal Statistical Society A*, 137(2):283–305.

Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, 48(5):1779–1801.

Gordy, M. B. (2003). A risk-factor model foundation for ratings-based bank capital rules. *Journal of Financial Intermediation*, 12(3):199–232.

Hamerle, A., Liebig, T., and Rösch, D. (2003). Benchmarking asset correlations. *Risk*, 16(11):77–81.

Lee, S.-C. and Lin, C.-T. (2012). Book-to-market equity, operating risk, and asset correlations: Implications for basel capital requirement. *Journal of international financial markets, institutions and money*, 22(4):973–989.

Lee, S.-C., Lin, C.-T., and Yang, C.-K. (2011). The asymmetric behavior and procyclical impact of asset correlations. *Journal of Banking & Finance*, 35(10):2559–2568.

Lee, Y., Rösch, D., and Scheule, H. (2020). Systematic credit risk in securitised mortgage portfolios. *Journal of Banking & Finance*, 122:105996.

Lee, Y., Rösch, D., and Scheule, H. (2021). Systematic credit risk in securitised mortgage portfolios. *Journal of Banking & Finance*, 122:105996.

Lopez, J. A. (2004). The empirical relationship between average asset correlation, firm probability of default and asset size. *Journal of Financial Intermediation*, 13(2):265–283.

Merton, R. C. (1974). On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance*, 7:141–183.

Patton, A. (2006). Modelling asymmetric exchange rate dependence. *International economic review*, 47(2):527–556.

Rösch, D. (2003). Correlations and business cycles of credit risk: Evidence from bankruptcies in germany. *Financial Markets and Portfolio Management*, 17(3):309–331.

Rosch, D. and Scheule, H. (2004). Forecasting retail portfolio credit risk. *Journal of Risk Finance*, 5(2):16–32.

Siarka, P. (2014). Asset correlation of retail loans in the context of the new basel capital accord. *Journal of Credit Risk*, 10(2).

Stoffberg, H. J. and van Vuuren, G. (2016). Asset correlations in single factor credit risk models: an empirical investigation. *Applied Economics*, 48(17):1602–1617.

Vasicek, O. (2002). Loan portfolio value. *Risk*, December:160–162.

Tables

Table 1: Basel's criteria for asset correlation

Exposure class	Loan type	Relation with PD	Lower and uppper bound
Corp, sovereign and bank	Corporate etc.*	Inverse	[12%, 24%]
	Financial institutions	Inverse	[15%, 30%]
	SMEs**	Inverse	[8%, 24%]
	Specialized lending	Inverse	[12%, 30%]
Retail	Residential mortgage	Fixed	15%
	Qualifying revolving	Fixed	4%
	Others	Inverse	[3%, 16%]

* Corporate with total consolidated revenues exceeding EUR 500 million
**Small or Medium-sized entities

Table 2: Descriptive statistics for charge-off rates by sector

	N	Mean	Std	Skew	Kurt	Min	Q1	Med	Q3	Max
Mortgages	124	0.0043	0.0058	1.9525	5.6344	0.0005	0.0011	0.0017	0.0034	0.0254
Business	124	0.0106	0.0071	1.3062	3.9559	0.0033	0.0053	0.0076	0.0146	0.0332

Table 3: Estimates of the static model

Variable name	Mortgages	Business
h	-2.6437	-2.3189
ρ	0.1070***	0.0480***
$PD = \Phi(h)$	0.0041***	0.0102***
Log-likelihood	540.29	457.27

One, two, and three asterisks indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively.

Table 4: Estimates of the time-varying model

Panel A : Estimates of time-varying default threshold h_t

Parameter	Mortgages			Business		
	Symmetric	Asymmetric		Symmetric	Asymmetric	
		C=0	C=2.412		C=0	C=1.645
Constant	-2.511***	-2.705***	-2.667***	-2.617***	-2.622***	-2.598***
ΔGDP_{t-3}				-6.009***	-6.038***	-5.930***
ΔGDP_{t-4}	-14.662***	-6.592***	-8.150***			
ΔHPI_{t-2}				-3.657***	-3.647***	-3.496***
ΔHPI_{t-4}	-17.969***	-12.460***	-13.908***			
CPI_{t-1}				-0.005***	-0.005***	-0.005***
DTI_{t-4}				0.068***	0.068***	0.067***
$\Delta Prime_{t-3}$				-0.105***	-0.104***	-0.117***
$T1Y_{t-4}$				-0.023***	-0.024***	-0.023***
$\Delta Unemploy_{t-3}$				-0.181***	-0.182***	-0.186***
$Mean(\Phi(h_t))$	0.0040	0.0025	0.0027	0.0095	0.0095	0.0095
$Std(\Phi(h_t))$	0.0047	0.0017	0.0021	0.0060	0.0060	0.0060
$Corr(\Phi(h_t), \ell_t)$	0.7624	0.7823	0.7797	0.8417	0.8410	0.8410

Panel B : Estimates of the time-varying asset correlation ρ_t

Parameter	Mortgages			Business		
	Symmetric	Asymmetric		Symmetric	Asymmetric	
		C=0	C=2.412		C=0	C=1.645
α_0	-0.393***	-0.446***	-0.430***	-0.543***	-0.542***	-0.538***
α_1	1.239***	1.018***	1.188***	2.987***	2.919***	3.191***
α_2	0.021***	0.037*	0.024***	0.044***	0.040	0.034*
α_3		0.000	0.017***		0.005	0.014
S	2	1	1	1	1	1
$Mean(\rho_t)$	0.0461	0.0511	0.0473	0.0142	0.0143	0.0141
$Std(\rho_t)$	0.0271	0.0658	0.0552	0.0153	0.0157	0.0149
Log-likelihood	597.22	599.49	600.83	545.57	545.59	546.04

Δ denotes the difference operator.
One, two, and three asterisks indicate statistical significance at the 5%, 1%, and 0.1% levels, respectively.

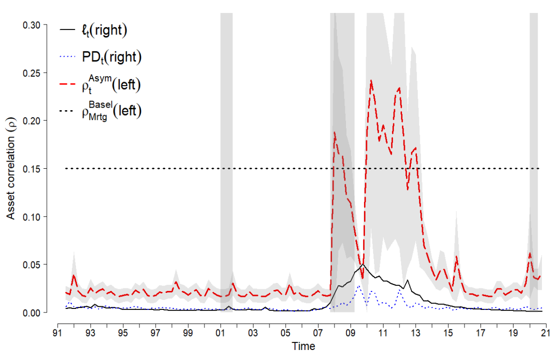
Table 5: Log-likelihood ratio tests

Sector	Mortgages			Business		
Alternative	Null			Null		
	Static	Symmetric ρ_t	Asymmetric ρ_t	Static	Symmetric ρ_t	Asymmetric ρ_t
Static	540.29	-	-	457.27	-	-
Symmetric ρ_t	<0.0001	597.22	-	<0.0001	545.57	-
Asymmetric ρ_t	<0.0001	0.0071	600.83	<0.0001	0.3272	546.04

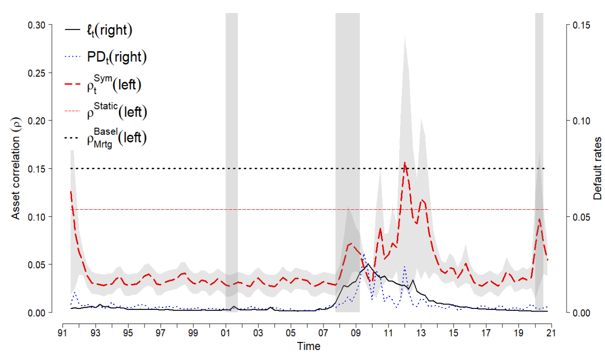
Log-likelihoods are in the diagonals and p-values from the likelihood ratio tests (null model vs alternative model) are in the off-diagonals

Figures

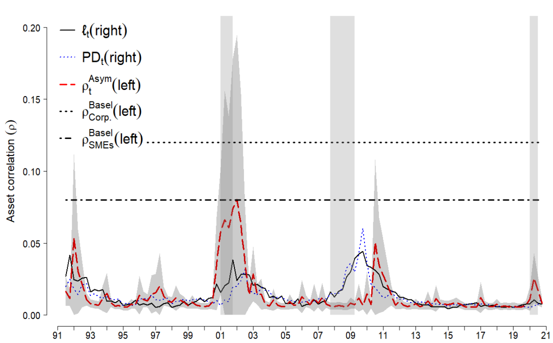
Figure 1: Asset correlations and sector charge-off rate



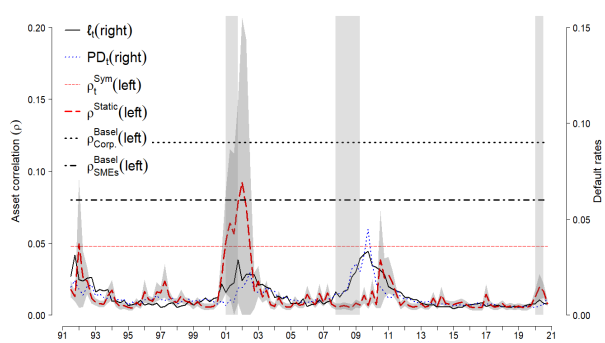
(a) Mortgages (Asymmetric)



(b) Mortgages (Symmetric)



(c) Business (Asymmetric)



(d) Business (Symmetric)