Liquidity Premia and Regime-Switching Transaction Costs and Cash Dividends*

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Abstract

We investigate the effect of the regime-switching transaction costs and dividends on liquidity premium and investor's optimal strategy. With reasonably calibrated parameters, we show that counter-cyclical transaction costs substantially raise liquidity premium while pro-cyclical dividends amplify this effect. More importantly, we observe that cash dividends can no longer play a role as a liquidity provider if the volatility of pro-cyclical dividends increases. Our model provides a universal framework in the liquidity contexts, so that we can examine the effect of regime switching on liquidity premium for all combinations of regimes in transaction costs and dividends.

Keywords: Liquidity Premium; Portfolio Choice; Transaction Cost; Dividend; Regime Switch

JEL classification: D11; G11; C61

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1 Introduction

Market parameters are stochastically changing, and one of the easiest ways to model changing market parameters is to employ a regime switching model reflecting the whole market environment. Many researchers have found interesting results on the frequently changing market liquidity. For example, Jones (2002) shows that when the bid-ask spread is used to measure liquidity, there could be a cycle, and Næs Randi and Arne (2011) find a strong relation between stock market liquidity and business cycle. They also show that investors' portfolio compositions change with the business cycle and that investor participation is related to market liquidity.

The long-lasting goal of studying liquidity is to understand about *liquidity premium* in asset pricing. To explain the effect of market illiquidity on asset's risk premium, investor's portfolio selection models with transaction costs have been widely employed. By exploiting such models, people can explain how liquidity premium arises in asset pricing. First, transaction costs directly lead to investor's wealth reduction. When an investor sets up the initial position she wants and/or whenever she adjusts her latest position, she has to pay transaction costs that cause a decrease in wealth. This leads to the commensurate liquidity premium of illiquid assets in market equilibrium. Second, the optimal investment strategy in the presence of transaction costs inevitably leads to no-trading region in which neither sale nor purchase of the illiquid asset is optimal. The existence of such no-trading region implies that the investor is not able to reach the optimal risk exposure, defined as the optimal investment proportion invested in the illiquid asset, in the liquid market. This suboptimality leads to the investor's utility loss compared to her optimal utility in the absence of transaction costs, necessitating a commensurate liquidity premium.

The portfolio selection problem in the presence of transaction costs has a long history. Constantinides (1986) introduces a portfolio selection model of an individual with transaction costs to calculate the liquidity premium with the reasonably-chosen market parameters. He finds that the ratio of liquidity premium over transaction costs (LPTC) is much smaller than the empirical premium.[1] While Dumas and Luciano (1991) finds an optimal transaction strategy for an investor who wants to maximize her long-term growth utility, they assume that the possible combination of portfolios is constant. Koo (1992) and Liu and Loewenstein (2002) derive optimal investment strategies for general models but they do not analyze liquidity premia. Academics have developed numerous advanced models to fill the gap between theoretical and empirical studies. For instance, Jang *et al.* (2007) employ

[1] Some empirical papers argue that liquidity premia due to transaction costs is insufficient to explain the considerable high ratio of LPTC. See, e.g., Amihud and Mendelson (1986).

a regime-switching market parameters to implement a stochastic investment opportunity that forces investors to trade substantial amounts more frequently. Their model generates a larger wealth reduction because the investor is forced to trade in a large amount anytime the regime changes, and that subsequently incurs a high transaction cost. Lynch and Tan (2011) show that return predictability can be a major factor in the liquidity premium. They set the stock return depending on the external factors and assume the presence of stochastic liquidity and labor to find out the reason for a high liquidity premium. Dai *et al.* (2016) suggest that the market closure can affect the liquidity premium resulting in suboptimal risk exposure. They say that the volatility of the market closure greatly differs from the on-going market, and this difference may cause a high LPTC ratio. Moreover, Chen *et al.* (2020) show that the incomplete information can cause a high LPTC ratio. This study further extends Jang *et al.* (2007), and they provide a solid empirical observation at the high level of the LPTC ratio. However, most of them do not consider the *stochastic changes* in transaction costs and dividends in the financial market.

Jang *et al.* (2007) show that the LPTC ratio in the bear regime is much higher than that in the bull regime assuming constant transaction costs. The assumption of change in transaction costs might give a different result. For instance, the presence of a much higher transaction cost for a bear regime can give us a less LPTC ratio because its denominator is apparently large, which is the opposite of the result of Jang *et al.* (2007). At the same time, the consideration of dividend changes in modeling illiquidity could be interesting because dividends paid in cash or cash equivalents can enhance liquidity of illiquid assets. Therefore, simultaneous consideration of the changes in transaction costs and cash dividends is likely to give new results that cannot be revealed in the existing liquidity studies.

Lower transaction cost can be thought of as higher liquidity, which is the characteristic of a bull regime. However, research on the regime switching of dividends is scarce, despite a slightly different trend of dividend yield during recessions as shown in Figure 2(a) and Figure 2(b).[2] During both Global Financial Crisis of 2008 and early coronavirus disease of 2019 in U.S., the average dividend yield was higher than that of the previous month's. For example, the S&P500 dividend yield in December 2007 was 1.36% whereas it was 1.96% in January 2008.The pattern in the average dividend yield of KOSPI was similar to that of S&P500. During Global Financial Crisis in Korea, the average dividend yield increased. However, in the period from April 2014 to March 2015, the average dividend yield in Korea decreased compared to the yield in March 2014. This means that the dividend yield was counter-cyclical (lower in bull and higher in bear) during the period. Since the regime switching of dividend

[2]The monthly OECD based recession indicators for US and Korea are from Federal Reserve Economic Data (FRED), monthly dividend yield data for US is from Robert Shiller's online data, and monthly dividend yield data for Korea is from Korean Statistical Information Service (KOSIS).

is different depending on a time period and a country, an investor needs to change her investment strategy regarding the regime switching.

We represent the stochastic changes by employing two-state regime-switching transaction cost and dividend parameters in our model. Specifically, to describe the sole effect of the existence of regime switching on liquidity premium, we first identify the two components of the risk premia; one stemming from liquidity risk in the absence of regime switching and the other stemming from regime-switching risk, which is defined as an additional liquidity premium in the presence the regime changes in transaction costs and dividends. Consequently, we show that the premium of the second component at Merton's line (Merton's optimal investment strategy[3]), the optimal portfolio position for the case in the absence of transaction costs, is likely to be higher when transaction costs are pro-cyclical and dividends are counter-cyclical.

In our model with reasonably calibrated parameters, we show that when transaction costs are counter-cyclical, the risk premium is higher in the bear regime. However, we also show that when transaction costs are pro-cyclical, the risk premium is higher at a smaller value of the illiquid asset-to-wealth ratio and lower at its larger value in the bull regime. This could be because of an investor's precautionary behavior against illiquidity. Moreover, we find that the investor's aversion to the risk due to the change in transaction costs can be high. In particular, when transaction costs are high in the bear regime and low in the bull regime (or equivalently, counter-cyclical), liquidity premium increases. This is because when transaction costs are counter-cyclical, the investor's no-trading region in the bear regime expands and the investor's utility score significantly drops. The expected staying time at the bull regime is insufficient to fully exploit the gain from the narrow no-trading region in the presence of the counter-cyclical transaction cost. Therefore, the overall investor's utility score declines, and the investor requires more liquidity premium.

When it comes to dividends, we show that the existence of pro-cyclical dividends paid in liquid asset (e.g. cash) can raise the LPTC ratio. To specify this effect, we calculate and compare the LPTC ratios with different dividend volatilities in each regime. As the volatility of pro-cyclical dividend increases, its role as a liquidity provider weakens, leading to a higher LPTC ratio. We can observe that the counter-cyclical dividends can lower the LPTC ratio, whereas the pro-cyclical dividends can raise it.

We also report some interesting results with the illiquidity measures, such as expected holding time and expected transaction cost, as well as LPTC ratio. The expected transaction cost represents the expectation of the discounted sum of transaction costs paid over the entire investment horizon.

[3]It is derived by solving our problem without considering transaction costs.

Similarly, the expected holding time is the time it takes the investor's position to exit the no-trading region. We find that the counter-cyclical transaction costs can raise the expected transaction costs, which leads to a high liquidity premium. However, the results obtained from the expected holding time are a bit mixed. For instance, in a parameter set with sufficient level of transaction cost mean, the expected holding time is the highest when the transaction costs are pro-cyclical.[4]

Technically, we introduce a modified version of the numerical method stated in Dai and Zhong (2008), a penalty method frequently used for continuous-time portfolio selection problems. We first reduce the portfolio selection problem with two state variables into that with one state variable, the stock-to-unliquidated-wealth ratio, and look for a proper form of linearized penalty term after discretizing the Hamilton-Jacobi-Bellman (HJB) equation. We had to modify the existing penalty method because the dynamics of the liquid asset holdings in our model can be seriously affected by the dynamics of the cash dividends which are proportional to the illiquid stock holdings.

The rest of this paper is organized as follows. Section 2 describes the financial market and the investor's problem and defines liquidity measures that are used in the paper. Section 3 provides the optimal investment strategies, the numerical result of liquidity premia, expected holding time, and expected transaction cost.

2 The Model and Liquidity Measures

2.1 The Model

We assume an infinitely living investor who can access perfectly liquid risk-free asset (call it "cash")[5] and an illiquid risky asset (call it "stock"). The liquid stock is assumed to give the holders cash dividends. Thus, the investor's cash inflow is either from the liquid and illiquid asset returns or cash dividends. The investor should pay transaction costs whenever she trades the illiquid stock. Then, the amount of the investor's cash (x) and stock (y) holdings follow the dynamics:

$$dx_t = \big(rx_t + \delta_{\epsilon_t} y_t - C(t)\big)dt - \big(1 + \lambda_{\epsilon_t}\big)dL_t + \big(1 - \mu_{\epsilon_t}\big)dM_t,$$
$$dy_t = \alpha_{\epsilon_t} y_t dt + \sigma_{\epsilon_t} y_t dB_t + dL_t - dM_t,$$

[4]However for the low level of transaction cost mean (=0.1%), the expected holding time is highest when transaction costs are the same for both regimes. This could be because the expected holding time is not directly connected with rising illiquidity. In other words, the expected holding time can be an ambiguous metric to measure illiquidity compared to the expected transaction cost.

[5]Here, we assume the "cash" is also an asset which can give us a return. In reality, we can consider the "cash" is a risk-free and (almost perfectly) liquid asset such as money market funds.

where B_t is a standard Brownian motion, r is the risk-free rate (or equivalently, the rate of the liquid asset return), δ is the dividend yield, λ (μ) is the purchase (sale, resp.) transaction cost, and dL_t (dM_t) is the amount of liquidity stock bought (sold, respectively) by the investor during an infinitesimal time interval $[t, t + dt)$. We also assume that α is the liquid stock return, σ is its volatility, and $C(t)$ is the investor's consumption rate.

The two-state regime-switching process, $\epsilon_t \in \{B, b\}$, represents the time-t state where the financial market stays, and we assume regime B stands for a *bull* regime and regime b stands for a *bear* regime. Regime i ($i \in \{B, b\}$) switches into the regime j ($j \in \{B, b\}$ and $j \neq i$) at the first jump time of the Poisson process with an intensity η_i.[6] Thus, in our model, the investor can have a stochastically changing investment opportunity because the market parameters can change across the two regimes.

The investor wants to maximize her utility obtained from the intermediate consumption, which implies that the value function V of the investor at time t should be defined as

$$V(x_t, y_t) := \max_{(C_t, L_t, M_t)} \mathbb{E}_t\Big[\int_t^{\infty} e^{-\beta t} \frac{1}{1-\gamma} C_t^{1-\gamma} dt\Big], \tag{1}$$

where $\mathbb{E}_t[\cdot]$ means the time t-conditional expectation, β is the investor's *subjective* discount rate, and γ is her relative risk aversion. For the convenience sake, we let the value function in regime i be V^i. The value function has the homogeneity of $V^i(kx_t, ky_t) = k^{1-\gamma} V^i(x_t, y_t)$ for arbitrary positive constant k, and thus, we can define

$$\psi^i(z_t) := (x_t + y_t)^{\gamma - 1} V^i(1 - z_t, z_t), \text{ and } W^i(z_t) := \frac{\log\big((1-\gamma)\psi^i(z_t)\big)}{1-\gamma},$$

for the stock-to-unliquidated wealth (STUW) ratio, $z_t := \dfrac{y_t}{x_t + y_t}$.

Using the standard asset allocation theory, the *modified* value function W^i and the optimal investment strategy can be obtained by solving the Hamilton–Jacobi–Bellman (HJB) equation in the following theorem.

Theorem 2.1. *The modified value function $W^i(z)$ ($i \in \{B, b\}$) satisfies the following HJB equation:*

$$\max_{(C_t, L_t, M_t)} \{\mathcal{L}_2 W^i(z),\ \mathscr{L}_2 W^i(z),\ \mathscr{M}_2 W^i(z)\} = 0, \tag{2}$$

[6] The regime-switching process is an alternating renewal process with two states. See, Ross (1982) for the details. By the definition, the two Poisson processes that make up the alternating renewal process are independent with each other.

where

$$\mathscr{L}_2 W^i(z) := (1+\lambda_i z)W_z^i(z) - \lambda_i, \text{ and } \mathscr{M}_2 W^i(z) := -(1-\mu_i z)W_z^i(z) - \mu_i,$$

and

$$\begin{aligned}\mathcal{L}_2 W^i(z) := &-\Big(\frac{\beta}{1-\gamma} + \frac{1}{2}\sigma_i^2 z^2 \gamma - (r(1-z)+\delta_i z) - \alpha_i z\Big)\\ &+ \big(-\sigma_i^2 z^2 \gamma(1-z) - z(r(1-z)+\delta_i z) + \alpha_i z(1-z)\big)W_z^i(z)\\ &+ \frac{1}{2}\sigma_i^2 z^2 (1-z)^2\big((1-\gamma)(W_z^i(z))^2 + W_{zz}^i(z)\big)\\ &+ \Big(\frac{1}{1-\gamma} - 1\Big)e^{-\frac{1-\gamma}{\gamma}W^i(z)}(1-zW_z^i(z)) + \frac{1}{1-\gamma}\eta_i\big(e^{(1-\gamma)(W^j(z)-W^i(z))} - 1\big)\big),\end{aligned}$$

for $j \neq i$ ($j \in \{B, b\}$). Here, $W_z^i(z) := \dfrac{\partial W^i}{\partial z}(z)$ and $W_{zz}^i(z) := \dfrac{\partial^2 W^i}{\partial z^2}(z)$.

Proof. See A.1. □

The investor's optimal consumption and portfolio choice problem is converted into a problem to find a solution of partial differential equation with three forms. These forms can build up three regions associated with the optimal investment strategies: the *buy region* where $\mathscr{L}_2 W^i(z) = 0$ holds, the *sell region* where $\mathscr{M}_2 W^i(z) = 0$ holds, and *no-trading region* where $\mathcal{L}_2 W^i(z) = 0$ holds. As seen in Jang *et al.* (2007), if the investor's initial position lies on the buy or sell regions for the both regimes, the optimal investor immediately adjusts her portfolio so that her position lies on the no-trading region. This type of strategy is called *bang–bang.* Notice that the optimal investor's strategy can be in a no-trading region for each regime at the beginning, but it can change with alterations in the market regime. At the time when the market regime change occurs, the optimal investor immediately changes her current position in regime i to be the position on the no-trading region for regime j.[7]

To make the problem feasible, Davis and Norman (1990) have made standard assumptions in their study, and we take the assumption:

Assumption

$$\beta > \max_{i\in\{B,b\}} \Big\{\frac{\gamma}{1-\gamma}\Big(r(1-\gamma) + \frac{(\alpha_i - r)^2}{\sigma_i^2}\Big)\Big\}.$$

[7]These three partial differential equations are solved through the penalty method. More descriptive explanations are included in Appendix A.2.

Furthermore, we assume $\alpha_i > r$ $(i \in \{B, b\})$, which implies the risk premium of the stock to be always positive.[8]

2.2 Liquidity Premium and Liquidity Measures

Before defining liquidity measures, we first explore the STUW ratio process, z_t. As a matter of fact, we can get the evolution equation of the ratio in an explicit form: for $i \in \{B, b\}$,

$$dz_t = \alpha_i(z_t)dt - \sigma_i z(1-z)dB_t,$$

where

$$\alpha_i(z) = z\big(-r(1-z) - \delta_i z + \alpha_i(1-z) + \sigma_i^2 z(1-z) + \big((1-\gamma)\psi(z) - z\psi_z(z)\big)^{-\frac{1}{\gamma}}\big).$$

Now we calculate the steady-state distribution of z_t. For the following theorem, we assume that the no-trading region can be separated for simplicity,[9] $\underline{z}_i$ to be the lower bound of the no-trading region in regime i, and $\bar{z}_i$ to be its upper bound. Notice that the investor's stock purchases occur at $\underline{z}_i$ and her stock sales occur at $\bar{z}_i$. (See Figure 1)

Theorem 2.2. *Suppose $\underline{z}_b < \bar{z}_b < \underline{z}_B < \bar{z}_B$. Then, the steady-state density functions, $\phi_i(z)$ for z_t in regime $i \in \{B, b\}$ are the solutions of*

$$\frac{1}{2}\sigma_B^2 z_B^2(1-z_B)^2\phi_B''(z) - \big(\alpha_B(z_B) - 2\sigma_B^2 z_B(2z_B^2 - 3z_B + 1)\big)\phi_B'(z) - \big(\eta_B - \sigma_B^2(6z_B^2 - 6z_B + 1) + \alpha_B'(z_B)\big)\phi_B(z) = 0,$$

subject to

$$\frac{1}{2}\sigma_B^2 \bar{z}_B^2(1-\bar{z}_B)^2\phi_B'(\bar{z}_B) - \big(\alpha_B(\bar{z}_B) - \sigma_B^2 \bar{z}_B(2\bar{z}_B^2 - 3\bar{z}_B + 1\big)\phi_B(\bar{z}_B) = 0,$$
$$\frac{1}{2}\sigma_B^2 \underline{z}_B^2(1-\underline{z}_B)^2\phi_B'(\underline{z}_B) - \big(\alpha_B(\underline{z}_B) - \sigma_B^2 \underline{z}_B(2\underline{z}_B^2 - 3\underline{z}_B + 1\big)\phi_B(\underline{z}_B) + \frac{\eta_B\eta_b}{\eta_B + \eta_b} = 0,$$

[8] The assumption is obvious if we can take the assumption that the majority of the stock market participants are risk averse. However, this constraint can be lifted if the investor does not make a consumption. See, e.g., Chen *et al.* (2020).

[9] On the premise that the return of the bull regime is much higher than that of the bear regime, we can assume that the sell boundary of the bear regime resides at a lower point than the buy boundary of the bull regime. We assert that the other cases also have similar results and we leave their proofs to the reader. See Jang *et al.* (2007) for the details.

and

$$\frac{1}{2}\sigma_b^2 z_b^2(1-z_b)^2\phi_b^{''}(z)-\big(\alpha_b(z_b)-2\sigma_b^2 z_b(2z_b^2-3z_b+1\big)\phi_b^{'}(z)-\big(\eta_b-\sigma_b^2(6z_b^2-6z_b+1)+\alpha_b^{'}(z_b)\big)\phi_b(z)=0,$$

subject to

$$\begin{aligned}&\frac{1}{2}\sigma_b^2\bar{z}_b^2(1-\bar{z}_b)^2\phi_b^{'}(\bar{z}_b)-\big(\alpha_b(\bar{z}_b)-\sigma_b^2\bar{z}_b(2\bar{z}_b^2-3\bar{z}_b+1\big)\phi_b(\bar{z}_b)-\frac{\eta_B\eta_b}{\eta_B+\eta_b}=0,\\&\frac{1}{2}\sigma_b^2\underline{z}_b^2(1-\underline{z}_b)^2\phi_b^{'}(\underline{z}_b)-\big(\alpha_b(\underline{z}_b)-\sigma_b^2\underline{z}_b(2\underline{z}_b^2-3\underline{z}_b+1\big)\phi_b(\underline{z}_b)=0.\end{aligned}$$

Proof: Follow the steps in Jang *et al.* (2007).

2.2.1 Expected LPTC Ratio

We first move to the definition of liquidity premium. In general, it is defined as the magnitude of the stock return that investors are willing to give up to get rid of transaction costs. Specifically, we define the liquidity premium at a STUW ratio z as follows.

Definition 2.1. *The liquidity premium $\Delta_i(z)$ at z in regime $i\in\{B,b\}$ is defined by*

$$W^{NTC,i}(z;\alpha_i-\Delta_i(z))=W^i(z;\alpha_i),$$

where $W^i(z;\alpha_i)$ is the value function $W^i(z)$ where the expected rate of return of the liquid stock for regime i is α_i, and $W^{NTC,i}(z;\zeta_i)$ is the value function $W^i(z)$ where the transaction costs are zero all the time and the expected rate of return of the liquid stock for regime i is ζ_i.

Notice that when transaction costs are small (large) the corresponding liquidity premium is small (large, resp). To factor out this scale effect, the LPTC ratio is introduced[10] as $\dfrac{\Delta_i(z)}{\lambda_i}$. Many researchers have chosen the STUW ratio z to be the ratio on the *Merton's line* which is an optimal investment strategy drawn on $(x,y)-$plane where there exists no transaction costs in the financial market.

Moreover, following Jang *et al.* (2007), we define the *expected LPTC* ratio over the no-trading region. Using the average LPTC ratio to observe liquidity premium in the regime switching model

[10] This type of liquidity metric has been introduced several papers, e.g., Constantinides (1986) and Jang *et al.* (2007)

seems more reasonable because the Merton's line may go outside the no-trading region.[11]

Definition 2.2. *Suppose the purchase transaction cost and sale transaction cost are the same, i.e.,* $\lambda_i = \mu_i$ *for* $i \in \{B, b\}$*. The expected LPTC ratio in regime* i *is defined as*

$$\mathbb{E}_i\Big[\frac{\Delta_i}{\lambda_i}\Big] := \int_{z\in NT_i} \frac{\Delta_i(z)}{\lambda_i}\phi_i(z)dz \tag{3}$$

where NT_i *stands for the no-trading region in regime* i *and* $\phi_i(z)$ *is the steady-state density function at a STUW ratio* z*, which is defined in Theorem 2.2. Moreover, the expected LPTC ratio for all regimes is defined as*

$$\bar{\Delta} := \mathbb{E}_B\Big[\frac{\Delta_B}{\lambda_B}\Big]\frac{\eta_B}{\eta_B+\eta_b} + \mathbb{E}_b\Big[\frac{\Delta_b}{\lambda_b}\Big]\frac{\eta_b}{\eta_B+\eta_b}.^{12} \tag{4}$$

2.2.2 Expected Transaction Cost

Under the assumption of the separate no-trading regions across regimes, we can calculate the *expected transaction cost* for the whole investment horizon which is defined as the sum of discounted transaction costs over the whole infinite investment period.

Definition 2.3. *The expected transaction cost* $C_i(x, y)$ *at* (x, y) *for regime* $i \in \{B, b\}$ *is defined as*

$$C_i(x,y) = E_0\Big[\int_0^{\tau_i} e^{-\beta t}(\lambda_i dL_t^* + \mu_i dM_t^*) + e^{-\beta\tau_i}C_j(x_{\tau_i}, y_{\tau_i})\Big], \text{ for } j \neq i.$$

Here, τ_i *is the first regime jump time where the initial regime is* i*, and* L_t^* (M_t^*) *is the optimal cumulative purchase (sale, resp.) amount, that is,* L_t $(M_t, resp.)$ *in optimum.*

Theorem 2.3. *The expected transaction costs can have a reduced form of*

$$C_i(x,y) =: (x+y)g_i(\frac{y}{x+y}),$$

[11]For example, Figure 4 in Jang *et al.* (2007) reports that the Merton's line lies on the buy region. This means that the Merton's line cannot be on the no-trading region under some parameter sets.

[12]Notice that the multifliers $\frac{\eta_i}{\eta_B+\eta_b}$ for $i \in \{B.b\}$ are the steady-state distribution of $\epsilon_t = i$.

where for $\underline{z}_i < z < \bar{z}_i$, $g_i(z)$ *solves*

$$\frac{1}{2}\sigma_i^2 z^2 (1-z)^2 g_i''(z) + \big(z(1-z)(\alpha_i - r) - \delta_i z^2 + z e^{-\frac{(1-\gamma)W(z)}{\gamma}} (1 - zW_z(z))^{-\frac{1}{\gamma}}\big) g_i'(z)$$
$$+ \big(\alpha_i z + r(1-z) + \delta_i z - e^{-\frac{(1-\gamma)W(z)}{\gamma}} (1 - zW_z(z))^{-\frac{1}{\gamma}} - \beta - \eta_i\big) g_i(z) + \eta_j g_j(z) = 0,$$

with the boundary conditions of

$$-\lambda_i g_i(\underline{z}_i) + (1 + \lambda_i \underline{z}_i) g_i'(\underline{z}_i) + \lambda_i = 0,$$
$$-\mu_i g_i(\bar{z}_i) + (-1 + \mu_i \bar{z}_i) g_i'(\bar{z}_i) + \mu_i = 0.$$

For the cases where $z > \bar{z}_b$ *and* $z < \underline{z}_B$, $g_i(z)$ *can be derived as*

$$g_b(z) = \frac{1}{-1 + \mu_b \bar{z}_b}\big((-1 + \mu_b z)\, g_b(\bar{z}_b) + \mu_b (\bar{z}_b - z)\big), \quad \text{for } z > \bar{z}_b,$$
$$g_B(z) = \frac{1}{1 + \lambda_B \underline{z}_B}\big((1 + \lambda_B z)\, g_B(\underline{z}_B) + \lambda_B (\underline{z}_B - z)\big), \quad \text{for } z < \underline{z}_B.$$

Proof: See Appendix A.3.

Abusing the definition, we call $g_i(z)$ the expected transaction cost at a STUW ratio z for regime i.

2.2.3 Expected Holding Time

The *expected holding time* defined in this paper is the expected time of the next sale.

Theorem 2.4. *Suppose the current regime is* $i \in \{B, b\}$ *and* $\underline{z}_b < \bar{z}_b < \underline{z}_B < \bar{z}_B$. *We define the next sale time as*

$$\tau^S := inf\{t \geq 0 : z_t = \bar{z}_B \text{ or } z_t = \bar{z}_b\}.$$

Let the expected holding time for regime i *be*

$$T_i(z) := E_0[\tau^S | z_0 = z, \epsilon_0 = i],$$

where z_0 (ϵ_0) represents the initial value of z_t (ϵ_t, resp.). Then, $T_B(z)$ and $T_b(z)$ are the solutions of

$$
\begin{aligned}
&\frac{1}{2}\sigma_B^2 z^2(1-z)^2 T_B^{''}(z) + \alpha_B(z)T_B^{'}(z) - \eta_B T_B(z) + 1 = 0,\\
&\frac{1}{2}\sigma_b^2 z^2(1-z)^2 T_b^{''}(z) + \alpha_b(z)T_b^{'}(z) - \eta_b(T_B(\bar{z}_B) - T_b(z)) + 1 = 0,
\end{aligned}
\tag{5}
$$

subject to

$$T_i(\bar{z}_i) = 0 \text{ and } T_i^{'}(\underline{z}_i) = 0.$$

Proof: See Appendix A.4.

3 Optimal Investment Strategies and Liquidity Premium

In this section, we investigate the effects of regime-switching transaction costs and dividends on the optimal investment strategy and liquidity premia. We have introduced the three liquidity measures: the expected LPTC ratio, the expected transaction cost, and the expected holding time. However, we cannot get their analytic formula. We developed a numerical method based on the penalty method in Dai and Zhong (2010) to solve our problem.[13]

For numerical analyses, the baseline parameter values are set as follows: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, the stock return $\alpha_B = \alpha_b = 0.1394$, the regime-switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$[14] and the discount rate $\beta = 0.051$. We set the mean dividend yield as 4.38%.[15] We subtract 4% from the stock return to compensate for the effect of the cash dividend yield partially. Therefore, the stock return in our model is $\alpha_B = \alpha_b = 0.0994$. We set the stock's return to be the same for both the bull and bear regimes. Moreover, the investor's risk aversion is 7.[16]

In a separate experiment, we compare the premium of a regime switching model with transaction costs and the premium of a single-regime model with transaction costs. Table 1 shows the premium of the former model at Merton's line, i.e., the STUW ratio $z = \frac{\mu - r + \delta(\epsilon(t)))}{\gamma\sigma^2}$. We only examine the case when the mean of transaction costs is 0.5%. Compared with the values in Table 1, the single-regime

[13] See Appendix A.2 for details.

[14] We use the U.S. equity market parameters that are estimated by Ang and Bekaert (2002).

[15] We first extract the S&P500 dividend yield from January 1, 1950 to December 31, 2020 using the data from Quandl. Then, utilizing the recession indicator data from FRED, we calculate the dividend yield for both bull and bear regimes. The mean value is computed by using the regime-switching intensities.

[16] The distribution of relative risk aversion in Kaplow (2005) shows that the relative risk aversion varies from near 0 to 10.

model has a lower premium with the value of 0.0018. This can be interpreted as the premium of the former model consisting of two components: regime-switching risk premium and liquidity premium. In other words, the difference between premia in the two models stems from the regime-switching risk.

3.1 Optimal Investment Strategies

The role of regime switching on transaction costs and dividends in optimal investment strategy is discussed in this section. The portfolio selection problem with transaction costs also provides three distinct regions of investment strategy: buy, sell, and no-trading regions. However, depending on the regime-switching transaction costs and dividends, the position of the three regions can drastically change over time. This can amplify the effect of the regime switching on the liquidity premium.

3.1.1 The Effects of Regime-switching Transaction Costs

Figure 3 shows a graphical representation of the optimal investment strategy where the regions between the two solid, dashed, and dash-dotted lines represent the no-trading region for the cases with counter-cyclical, mean-constant, and pro-cyclical transaction costs, respectively. If transaction costs are counter-cyclical, the no-trading region of the bear regime is the widest. This result is intuitive because the investor will widen the no-trading region to avoid a large wealth reduction. The optimal investment strategy outside the no-trading region is to immediately shift the position to the boundary of the no-trading region. If the no-trading region in the bear regime is narrow, the optimal investor should rebalance her position to hold more stock to reach the boundary of the no-trading region. This rebalancing induces a higher transaction cost, which is why the investor has a large no-trading region in the bear regime when the transaction cost is counter-cyclical.

3.1.2 The Effects of Regime-switching Dividends

The pro-cyclical dividend may encourage the investor to become more involved in the stock market, because the pro-cyclical dividend is similar to increased stock return under reasonably calibrated parameters. As a matter of fact, under such parameter condition, the bull regime seems substantially longer than the bear regime. Figures 4(a) and 4(c) show that when dividends are pro-cyclical, the difference between bull and bear stock holdings is higher, which is obvious since a lower dividend in the bear regime lowers liquidity.

To describe the effect of dividends more precisely, we also analyze the effect of dividend volatility on the LPTC ratio in Figure 5. The standard deviation between bull and bear dividends is referred to as dividend volatility. When the dividend is lower in bull regime and higher in bear regime (or equivantly, counter-cyclical), the LPTC ratio decreases as the volatility increases. However, when the dividend is higher in bull and lower in bear (or equivalently, pro-cyclical), the dividend volatility increases, which leads to increasing LPTC ratio. For the counter-cyclical dividend case, the LPTC ratio decreases as dividend volatility increases. When the volatility is greater, the dividend in the bear regime is higher than that of the bull regime, leading to higher liquidity in bear regime. Therefore, the overall LPTC ratio is lowered. For the case of pro-cyclical dividend case, the LPTC ratio increases as the dividend volatility increases. A higher dividend in bull regime and a comparatively lower dividend in bear regime worsen the liquidity. In other words, the effect of the cash dividend as a liquidity provider diminishes for the case of pro-cyclical dividends.

3.2 Liquidity Premium

3.2.1 Liquidity Premium

Figure 6 shows liquidity premia over different STUW ratios. The counter-cyclical transaction costs show a substantial increase in liquidity premia as shown by the dashed lines of Figure 6. This result is puzzling because it can be assumed that the decrease in the liquidity premium in the bull regime, which lasts for around 80% of the investment horizon, would lower the overall liquidity premium. However, the result is possible if the loss in the utility score from the suboptimal risk exposure in the bear regime is large enough to dominate the gain in the utility score from the lower illiquidity in the bull regime. It shows that the liquidity premium can be mainly induced by the asset with high transaction costs, and this can be explained with an adoption of regime switching.

On the other hand, the expected LPTC ratio is the highest when transaction costs are counter-cyclical, whereas pro-cyclical or mean-constant cases show no substantial differences. As the mean value of transaction costs decreases, this effect becomes more visible. This could be because when transaction costs are counter-cyclical, the overall investor's utility score drops since the investor's no-trading region in the bear regime widens.

The expected LPTC ratio can also be increased by the regime-switching dividends. When the dividends are pro-cyclical, the investor receives a smaller amount of cash in the bear regime. Recalling that the dividend is paid in cash, the smaller amount of dividend payout in the bear regime lowers the provided liquidity, pushing the liquidity premium higher.

Consistent with the result of liquidity premia, the expected transaction costs are the largest when the transaction costs are counter-cyclical. In contrast, pro-cyclical transaction costs show the longest expected holding time. This different implication can be because the widened no-trading region of the long-lasting bull regime is the main reason in increasing the expected holding time.

3.2.2 The Expected Transaction Cost

Figure 7 shows the expected transaction cost as a function of the STUW ratio in each regime. In all cases of regime switching, the expected transaction costs in the no-trading region are U-shaped for both regimes. This can be interpreted as when the position is distant from the boundary of the no-trading region, the optimal investor is less likely to move her portfolio outside the no-trading region. As soon as the stock position goes outside the no-trading region, the optimal investor immediately rebalances her position to remain within the no-trading region, incurring a transaction cost. Therefore, as the stock position is closer to the boundaries of the no-trading region, the expected transaction cost will surge.

A similar logic can be applied to the tilted U-shape of the expected transaction cost graphs. For example, in the bull regime, the right part of the U-shape graph is significantly higher than the left. However in the bear regime, the left part is higher than the right. This is due to regime switching. Under our assumption, there is always a chance that the regime will switch to the bear regime in the bull regime. When the STUW ratio is high and the regime changes to the bear regime, the investor should pay more transaction cost to reach the no-trading region in the bear regime compared to the case in which the STUW ratio is low. Therefore, in the bull regime, the right part of the U-shape graph with a higher STUW ratio has a higher level of the expected transaction cost. Similarly, in the bear regime, there is always a possibility that the regime switches to the bull regime, making the left part of the U-shape graph to have a higher level of the expected transaction cost.

3.2.3 The Expected Holding Time

Figure 8 shows the expected holding time as a function of the STUW ratio in each regime. In all cases of regime-switching transaction costs, the expected holding time decreases as the STUW ratio increases. When the STUW ratio is high, the expected staying time drops. This could also be interpreted as a higher probability of touching boundaries of a no-trading region, and it makes the optimal investor to rebalance her portfolio more frequently, leading to a shorter holding time.

When the dividends are pro-cyclical, the results of the expected holding time are a bit mixed. The decrease in expected holding time is affected by the low transition probability to the bear regime. A decline in the expected holding time is particularly sharp in the bull regime and it can induce a decrease in the total expected holding time.

4 Conclusion

In this paper, we investigate the effect of time-varying transaction costs and dividends. To figure out the effect of regime switching, we identify two components of the risk premium: liquidity risk and regime-switching risk. We find that when transaction costs are pro-cyclical and dividends are counter-cyclical, the regime-switching risk premium at Merton's line is higher. Moreover, we show that the expected LPTC ratio is the highest when transaction costs are counter-cyclical and dividends are pro-cyclical. This result stems from the sharp drop of the investor's utility score due to an expansion of the investor's no-trading region in the bear regime. However, the result of the expected holding time is unexpected. For example, when transaction cost mean is sufficiently high, pro-cyclical transaction costs and pro-cyclical dividends make higher expected holding time. To further describe the effect of regime-switching dividends on liquidity premium, we analyze how different dividend volatilies in each regime changes the expected LPTC ratios. We find out that the counter-cyclical dividends can lower the ratio, and the result is opposite when dividends are pro-cyclical. These analyses could be done by our numerical contribution utilizing the concept of penalty method. Through this method, we could solve the HJB equation with boundaries (constraints).

Future studies could investigate the association between transaction cost and dividend. Our paper examines the case when their correlation is either 1 or -1. Consideration of the correlation could be done by setting two different stochastic processes for transaction cost and dividend instead of regime switching. This would provide more detailed analysis on co-effect of them on liquidity premium. We deal with two types of assets: liquid risk-free asset and illiquid risky asset; however, there are stocks that are easily traded due to the fact that there is a large volume of stock shares traded frequently. Therefore, future research could be conducted in more realistic settings by adopting an additional liquid asset to a model.

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A Proofs

A.1 Proof of Theorem 2.1

Using the definition of equation (1), we can easily derive the following HJB equation:

$$\max\{\mathcal{L}_0 V^i(x,y),\ \mathscr{L}_0 V^i(x,y),\ \mathscr{M}_0 V^i(x,y)\} = 0, \tag{6}$$

where $\mathscr{L}_0 V^i(x,y)$ and $\mathscr{M}_0 V^i(x,y)$ are defined as

$$\mathscr{L}_0 V^i(x,y) := -(1+\lambda_i)V^i_x(x,y) + V^i_y(x,y), \text{ and } \mathscr{M}_0 V^i(x,y) := (1-\mu_i)V^i_x(x,y) - V^i_y(x,y).$$

Also, $\mathcal{L}_0$ is given by

$$\begin{aligned}\mathcal{L}_0 V^i(x,y) = {}& \frac{1}{2}\sigma_i^2 y^2 V^i_{yy}(x,y) + (r_i x + \delta_i y)V^i_x(x,y) + \alpha_i y V^i_y(x,y) - \beta V^i(x,y) \\ & + \Big(\frac{1}{1-\gamma} - 1\Big)(V^i_x(x,y))^{\frac{1-\gamma}{-\gamma}} + \eta_j(V^j(x,y) - V^i(x,y)),\end{aligned}$$

where $i \in \{B, b\}$ indicates the current regime and j indicates the other regime.

Using a change of variable, we reduce the dimension of our problem by one. For the variable $z = \dfrac{y}{x+y}$, we have

$$\mathscr{L}_1 \psi^i(z) = (1+\lambda_i z)\psi_z(z) - \lambda_i \gamma \psi(z), \text{ and } \mathscr{M}_1 \psi^i(z) = -(1-\mu_i z)\psi_z(z) - \mu_i \gamma \psi(z).$$

and

$$\begin{aligned}\mathcal{L}_1 \psi^i(z) = {}& \Big(\frac{1}{2}\sigma_i^2 z^2 \gamma(\gamma-1) + (r(1-z) + \delta_i z)(1-\gamma) + \alpha_i z(1-\gamma) - \beta\Big)\psi^i(z) \\ & + \Big(-\sigma_i^2 z^2 \gamma(1-z) - z(r(1-z) + \delta_i z) + \alpha_i z(1-z)\Big)\psi^i_z(z) + \frac{1}{2}\sigma_i^2 z^2 (1-z)^2 \psi^i_{zz}(z) \\ & + \Big(\frac{1}{1-\gamma} - 1\Big)\{(1-\gamma)\psi^i(z) - z\psi^i_z(z)\}^{\frac{1-\gamma}{-\gamma}} + \eta_i(\psi^j(z) - \psi^i(z)).\end{aligned}$$

Taking an additional transformation, we can say that our problem is converted into the problem of

$$\max\{\mathcal{L}_2 W^i(z),\ \mathscr{L}_2 W^i(z),\ \mathscr{M}_2 W^i(z)\} = 0,$$

with

$$\mathscr{L}_2 W^i(z) = (1+\lambda_i z)W^i_z(z) - \lambda_i, \text{ and } \mathscr{M}_2 W^i(z) = -(1-\mu_i z)W^i_z(z) - \mu_i,$$

and

$$\begin{aligned}\mathcal{L}_2 W^i(z) = &-\Big(\frac{\beta}{1-\gamma} + \frac{1}{2}\sigma_i^2 z^2\gamma - (r_i(1-z)+\delta_i z) - \alpha_i z\Big)\\ &+\Big(-\sigma_i^2 z^2\gamma(1-z) - z(r_i(1-z)+\delta_i z) + \alpha_i z(1-z)\Big)W_z(z)\\ &+\frac{1}{2}\sigma_i^2 z^2(1-z)^2((1-\gamma)(W_z^i(z))^2 + W_{zz}^i(z))\\ &+(\frac{1}{1-\gamma}-1)e^{-\frac{1-\gamma}{\gamma}W^i(z)}(1-zW_z^i(z)) + \frac{1}{1-\gamma}\eta_i(e^{(1-\gamma)(W^j(z)-W^i(z))}-1)).\end{aligned}$$

For the mathematical details, see Dai and Zhong (2010).

A.2 Numerical Methods

When we adopt the penalty method to our problem, we can modify our HJB (2) in Theorem 2.1 into the equation below:

$$\mathcal{L}_2 W^i(z) + P(\mathscr{L}_2 W^i(z))^+) + P(\mathscr{M}_2 W^i(z))^+) = 0,$$

where P equals to the penalty coefficient which is a large number. However, there are several nonlinear terms in the equation, such as the second order derivative, the consumption term, the regime-switching term, and the penalty terms. We linearize all these terms by using second-order central method for the second order derivative, Taylor expansion for the exponential terms, and linearized penalty terms that are stated in Forsyth and Vetzal (2002). Specifically, the penalty terms and the exponential terms are expressed as:

$$P(\mathscr{L}_2 W^i(z))^+ = \begin{cases} Large & \text{if } \mathscr{L}_2 W^i(z)) > 0,\\ 0 & \text{otherwise,}\end{cases}$$

$$P(\mathscr{M}_2 W^i(z))^+ = \begin{cases} Large & \text{if } \mathscr{M}_2 W^i(z)) > 0,\\ 0 & \text{otherwise,}\end{cases}$$

$$\frac{1}{1-\gamma}\eta_i(e^{(1-\gamma)(W^j(z)-W^i(z))}-1) \sim \eta_i(W^j(z)-W^i(z)).$$

Moreover, a first order derivative is discretized as:

$$cW_z \sim \frac{W_{n+1}(z)-W_n(z)}{\Delta z}\max\{c,0\} + \frac{W_n(z)-W_{n-1}(z)}{\Delta z}\min\{c,0\},$$

where c is a coefficient multiplied to the first order derivative of a spatial variable (or equivalently, the wealth variable z) and n is a n-th wealth value of a wealth grid. We then use the matrix form of finite difference method and iteratively calculate it to find out the value vector under a predetermined convergence tolerance.

A.3 Proof of Theorem 2.3

This is easily verified through the relationship between C_i and g_i. Following Jang *et al.* (2007), we can get the following equation for C_i:

$$
\begin{aligned}
&\frac{\partial C_i}{\partial x}(x,y)\left(rx+\delta_i y - C_i(x,y)\right) + \frac{\partial C_i}{\partial x}(x,y)\left(-\left(1+\lambda_i\right)dL_t^* + \left(1-\mu_i\right)dM_t^*\right)\\
&\quad + \frac{\partial C_i}{\partial y}(x,y)\left(\alpha_i y + dL_t^* - dM_t^*\right) + \frac{1}{2}\sigma_i^2 y^2 \frac{\partial^2 C_i}{\partial y^2}(x,y) - \beta C_i(x,y) + \lambda dL_t^* + \mu_i dD_t^*\\
&= \frac{1}{2}\sigma_i^2 z^2 \left(1-z\right)^2 g_i''(z) + \Big(z\left(1-z\right)\left(\alpha_i - r\right) - \delta_i z^2 + z e^{-\frac{(1-\gamma)W(z)}{\gamma}}\left(1 - zW_z(z)\right)^{-\frac{1}{\gamma}}\Big) g_i'(z)\\
&\quad + \Big(\alpha_i z + r\left(1-z\right) + \delta_i z - e^{-\frac{(1-\gamma)W(z)}{\gamma}}\left(1 - zW_z(z)\right)^{-\frac{1}{\gamma}} - \beta - \eta_i\Big) g_i(z) + \eta_j g_j(z)\\
&= 0,
\end{aligned}
$$

where the first equality is driven by dividing $(x+y)$ on the first equation.

A.4 Proof of Theorem 2.4

When the regime shifts from regime B to regime b, it is optimal to sell the stock immediately, leading to

$$
\begin{aligned}
T_B(z) &= E_0[\tau^S | z_0 = z]\\
&= E_0[\tau^{BS} \wedge \tau_B | z_0 = z]\\
&= E_0\Big[\int_0^{\tau^{BS}} e^{-\eta_B t} dt | z_0 = z\Big],
\end{aligned} \tag{7}
$$

where τ^{BS} is the next sale time during regime B.

On the other hand, when the regime shifts from regime b to regime B, we should consider whether (1) the sale time is earlier or (2) the regime jump time is earlier. For the case of (2), we should consider the case where the investor sells the stock in regime B. Therefore, we get

$$
\begin{aligned}
T_b(z) &= E_0[\tau^S | z_0 = z]\\
&= E_0[\tau^{bS} \wedge \tau_b | z_0 = z] + T_B(\bar{z}_B) E_0[1_{\{\tau_b < \tau^{bS}\}} | z_0 = z]\\
&= E_0\Big[\int_0^{\tau^{bS}} (1 + \lambda_b T_B(\bar{z}_B)) e^{-\eta_b t} dt | z_0 = z\Big].
\end{aligned} \tag{8}
$$

where τ^{bS} is the next sale time during regime b. Using the Feynman-Kac theorem, Equation (7) and (8) can be derived from the equations in (5) of the theorem above.

Premium at Merton Line		Transaction Costs (Median Mean 0.5%)		
		Counter	Mean	Pro
Dividend	Counter	0.0039	0.0039	0.0042
	Mean	0.0018	0.0018	0.0019
	Pro	0.0015	0.0015	0.0016

Table 1: Premium of a Regime-switching Model with Transaction Costs. Premium of the regime-switching model with transaction costs is calculated at Merton's line ($z = \frac{\mu - r + \delta(\epsilon(t)))}{\gamma\sigma^2}$). The column shows values according to dividends with different cyclicalities and the row shows values with cyclicalities in transaction costs. For the dividend pair (bull and bear), the higher one is set to be 4.5% and the mean be 4.38%. For a transaction costs pair, lower transaction cost is set to be 0.25% and higher transaction cost is calculated to match the mean. Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$

Expected LPTC Ratio		Transaction Costs								
		High Mean (1%)			Median Mean (0.5%)			Low Mean (0.1%)		
		Counter	Mean	Pro	Counter	Mean	Pro	Counter	Mean	Pro
Dividend	Counter	1.2305	0.3302	0.2570	1.5073	0.3485	0.2682	1.6946	0.3559	0.2688
	Mean	1.2440	0.3326	0.2587	1.5197	0.3507	0.2699	1.7048	0.3580	0.2702
	Pro	1.3442	0.3500	0.2714	1.6045	0.3670	0.2816	1.7802	0.3718	0.2799

(a) Expected LPTC Ratio

Expected Transaction Costs		Transaction Costs								
		High Mean (1%)			Median Mean (0.5%)			Low Mean (0.1%)		
		Counter	Mean	Pro	Counter	Mean	Pro	Counter	Mean	Pro
Dividend	Counter	0.0032	0.0025	0.0020	0.0026	0.0014	0.0011	0.0006	0.0003	0.0002
	Mean	0.0033	0.0025	0.0020	0.0026	0.0014	0.0011	0.0007	0.0003	0.0002
	Pro	0.0037	0.0026	0.0022	0.0028	0.0014	0.0012	0.0007	0.0003	0.0003

(b) Expected Transaction Costs

Expected Holding Time		Transaction Costs								
		High Mean (1%)			Median Mean (0.5%)			Low Mean (0.1%)		
		Counter	Mean	Pro	Counter	Mean	Pro	Counter	Mean	Pro
Dividend	Counter	0.4794	0.7929	0.8324	0.3106	0.4638	0.4776	0.1603	0.1825	0.1782
	Mean	0.4856	0.8034	0.8426	0.3155	0.4648	0.4846	0.1602	0.1860	0.1807
	Pro	0.4919	0.8301	1.0063	0.3181	0.4797	0.4949	0.1559	0.1831	0.1779

(c) Expected Holding Time

Table 2: Robustness Test with Additional Measures. Robustness test of the expected LPTC ratio, transaction costs, and holding time with various transaction costs and dividends. The column shows values according to dividend with different cyclicalities and the row shows values with different mean-levels and cyclicalities in transaction costs. For the dividend, the high level is set to be 4.5% and the mean be 4.38%. Three cases of transaction costs are shown in the table: 0.1%, 0.5%, and 1%. For each case, lower transaction costs are set to be 0.05%, 0.25%, and 0.5% and higher transaction costs are calculated to match the mean. Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$

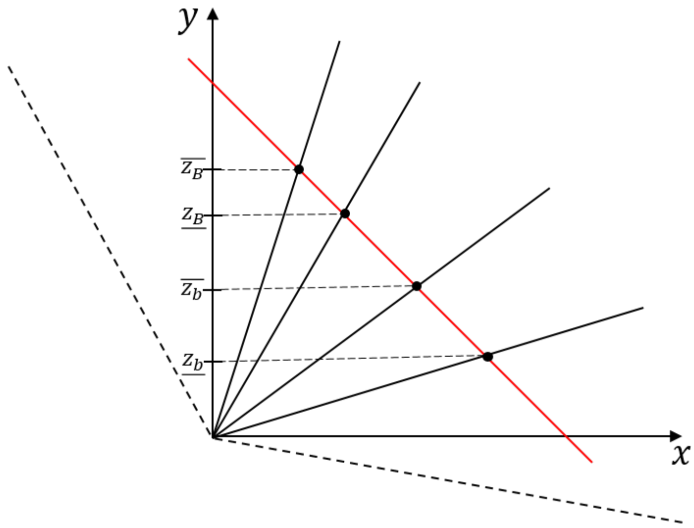
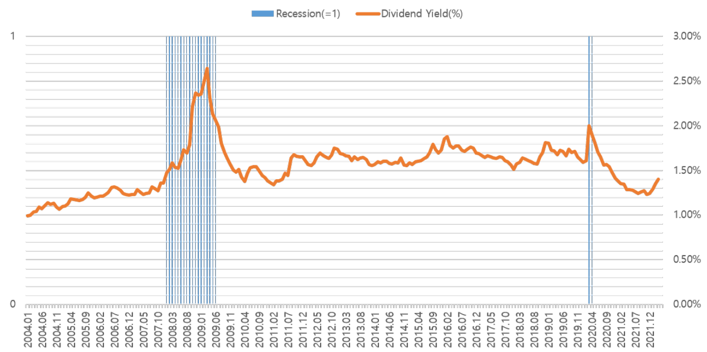
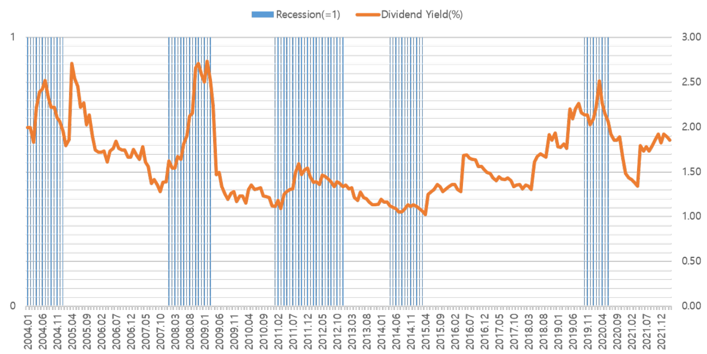


Figure 1: No-trading Regions. x is an amount invested in a liquid asset, y is an amount invested in an illiquid asset, stock sale occurs at $\bar{z}_i$, and stock purchase occurs at $\underline{z}_i$. Dashed lines indicate boundaries of trading regions. The red line is a Merton's line ($z = \frac{y}{x+y} = \frac{\mu - r + \delta(\epsilon(t)))}{\gamma\sigma^2}$).



(a) Dividend Yields of S&P500



(b) Dividend Yields of KOSPI

Figure 2: Dividend Yields of S&P500 and KOSPI. Dividend yield is plotted in an orange line with vertical gray lines indicating recessions.

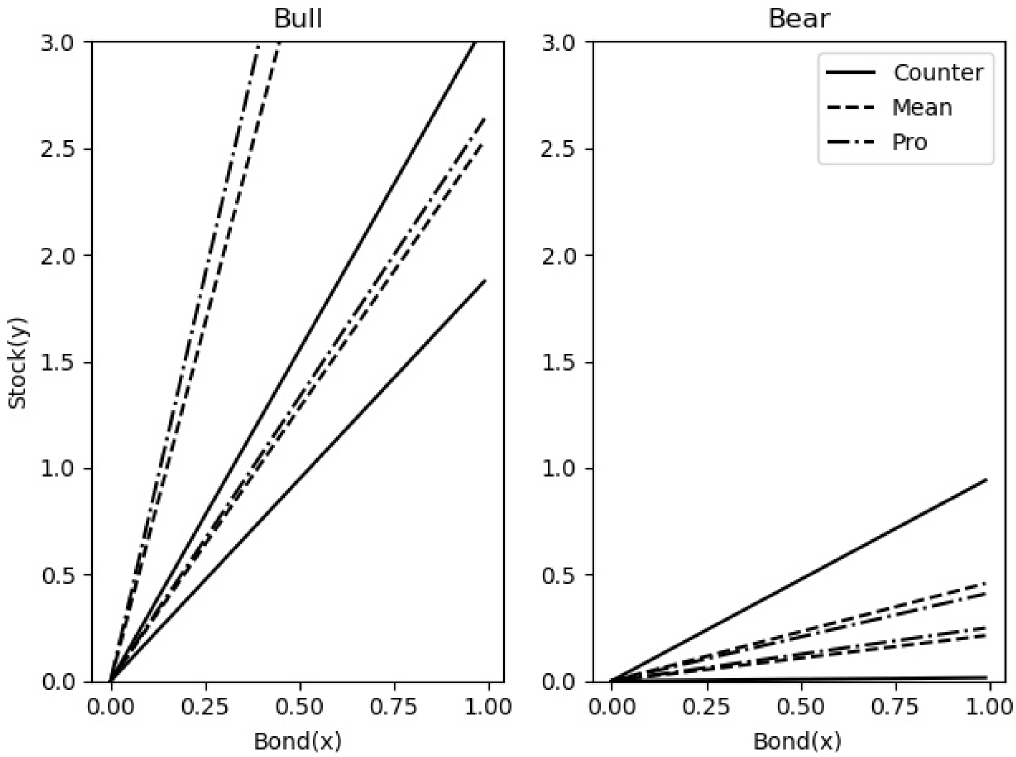
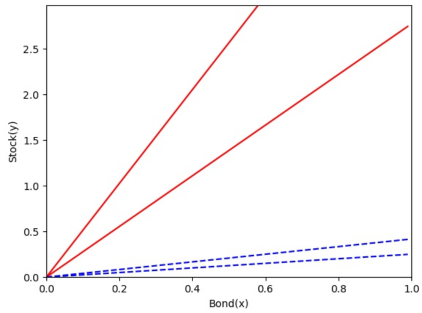
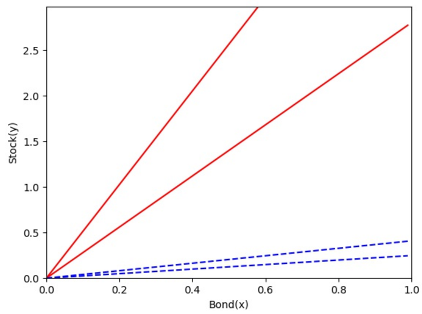


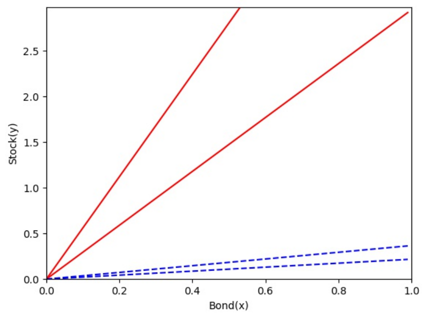
Figure 3: Optimal Investment Strategies with Changing Transaction Costs. Optimal investment strategies of the bull and bear regimes with the counter-cyclical (solid line), mean-constant (dashed line), and pro-cyclical (dash-dotted line) transaction costs. x-axis represents the investor's cash holdings and y-axis represents the investor's stock holdings. A triangular region between two lines is the no-transaction region. The left region of the triangle is the sell-region and the right is the buy-region. Mean transaction costs are set to be 1.0% and the dividends are constant (4.38%). The lower transaction cost is 0.5% and the higher one is calculated with transition probability to match the mean (1.0%). Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$



(a) Counter-cyclical dividend

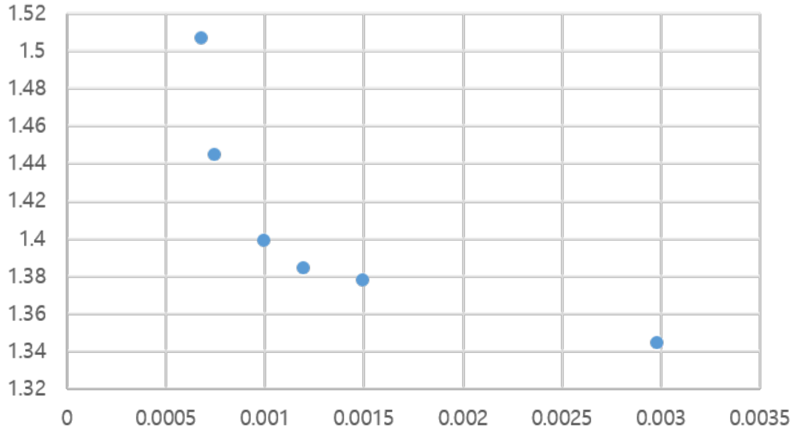


(b) Mean-Constant dividend

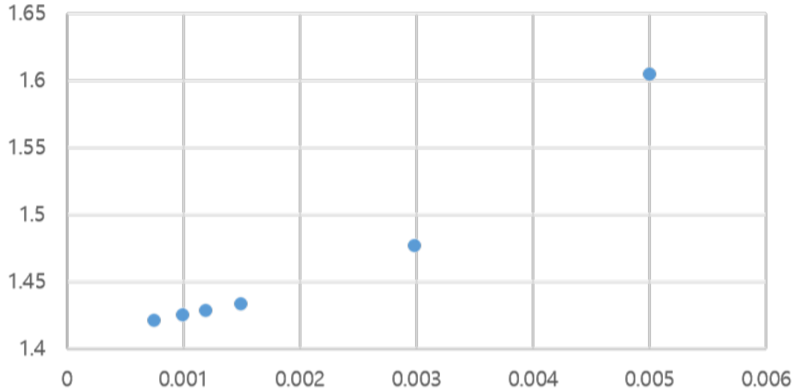


(c) Pro-cyclical dividend

Figure 4: Optimal Investment Strategies with Changing Dividends. Optimal investment strategies of the bull and bear regimes with the counter-cyclical, mean-constant, and pro-cyclical dividends. x-axis represents the investor's cash holdings and y-axis represents the investor's stock holdings. A triangular region between two lines is the no-transaction region. The left region of the triangle is the sell-region and the right is the buy-region. Red means the bull regime and blue means the bear regime. Mean transaction costs are set to be 0.5% and the dividends are constant (4.38%). The pro-cyclical dividend yield is 4.5% and 3.5% for bull and bear regime, respectively. Mean dividend yield is calculated by taking average of two dividends with corresponding Markov switching probability (4.38%), and for counter-cyclical case, bear regime's yield is 4.5%. Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$



(a) Counter-cyclical Dividend



(b) Pro-cyclical Dividend

Figure 5: LPTC Ratios Over Different Dividend Volatilities. LPTC ratios over different volatilities of regime-switching dividends are plotted. x-axis represents the dividend volatility and y-axis represents the LPTC ratio. Transaction costs for both graphs are same with the mean value of 0.5%. The mean value of dividends in all cases is constant (4.38%). Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$

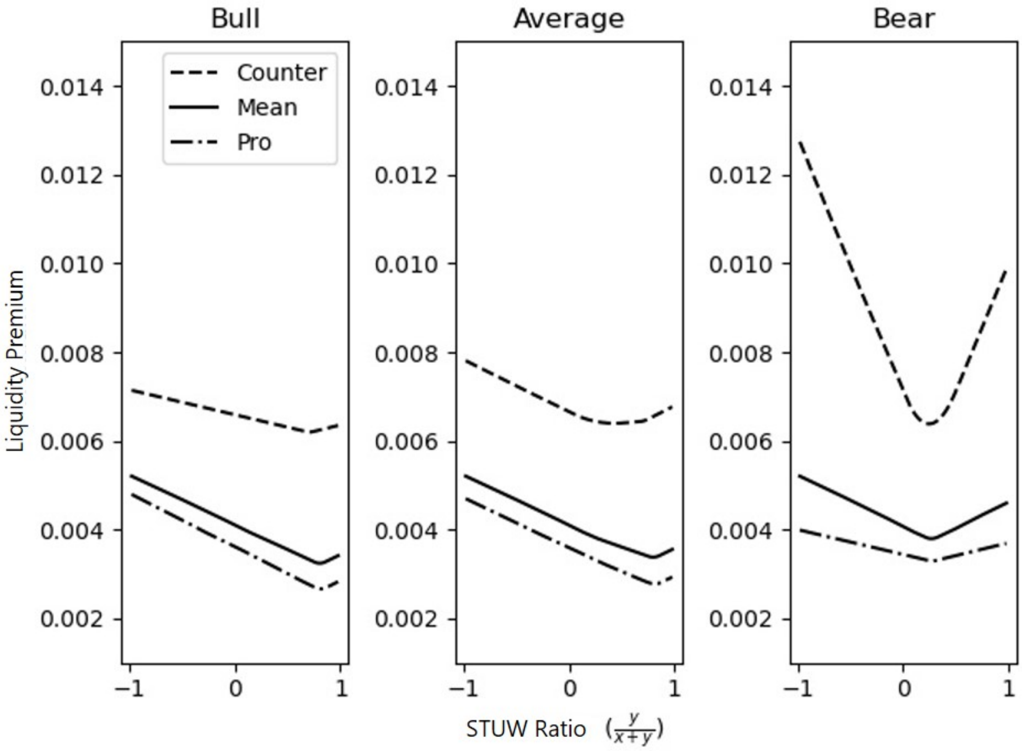
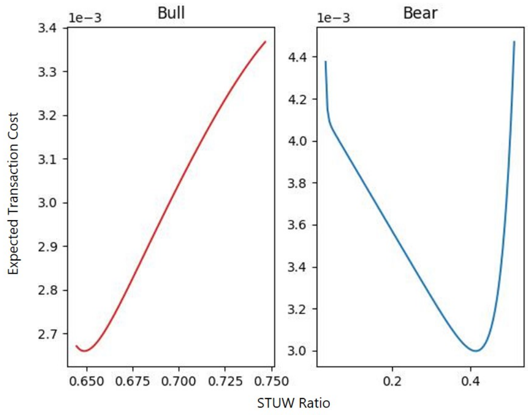
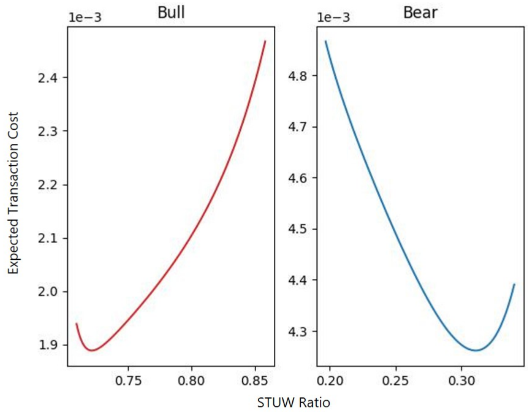


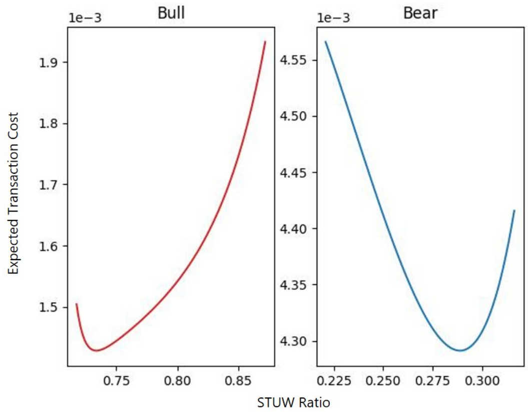
Figure 6: Liquidity Premia. Liquidity premia of counter-cyclical, mean-constant, and pro-cyclical transaction costs. Dashed, dash-dotted, and solid lines represent the cases of counter-cyclical, pro-cyclical and mean transaction costs, respectively. x-axis represents investor's stock-to-wealth ratios. The mean transaction cost is 1.0%. The lower transaction cost is 0.5% and the higher one is calculated with transition probability to match the mean (1.0%). Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$



(a) Counter-cyclical Transaction Costs with 1% Mean

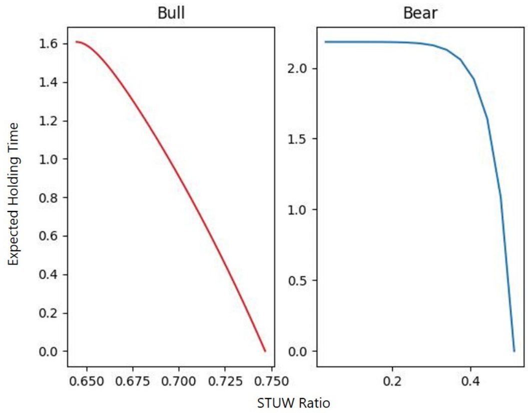


(b) Mean-Constant Transaction Costs with 1% Mean

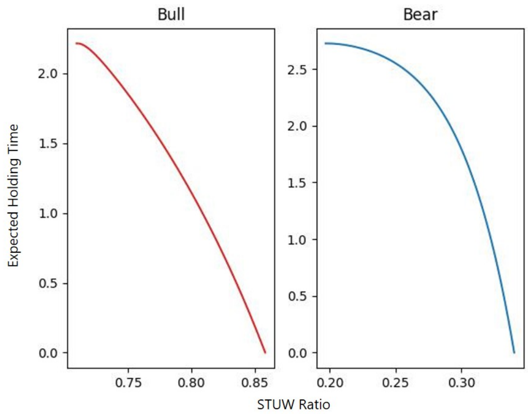


(c) Pro-cyclical Transaction Costs with 1% Mean

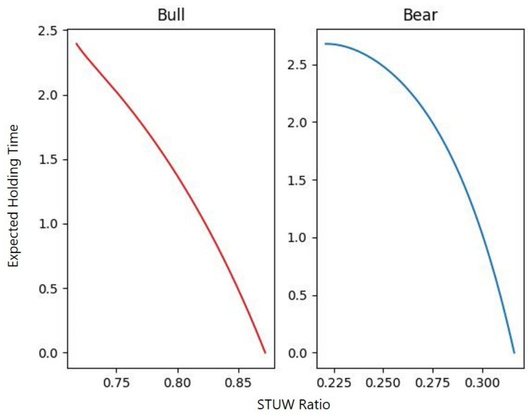
Figure 7: Expected Transaction Costs. The expected transaction costs with different values of regime-switching transaction costs. The red means bull and the blue means bear regime. x-axis represents the investor's stock ratio and y-axis represents the investor's expected transaction cost. Mean transaction costs are set to be 1.0%. The dividend yield is constant (4.38%). The lower transaction cost is half of the mean and the higher one is calculated with transition probability to match the mean. Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$



(a) Counter-cyclical Transaction Costs with 1% Mean



(b) Mean-Constant Transaction Costs with 1% Mean



(c) Pro-cyclical Transaction Costs with 1% Mean

Figure 8: Expected Holding Time. The expected holding time with different values of regime-switching transaction costs. The red means bull and the blue means bear regime. x-axis represents the investor's stock ratio and y-axis represents the investor's expected transaction costs. Mean transaction costs are set to be 1.0%. The dividend yield is constant (4.38%). The lower transaction cost is half of the mean and the higher one is calculated with transition probability to match the mean. Parameters: risk-free rate $r = 0.051$, the stock volatilities $\sigma_B = 0.1306$, $\sigma_b = 0.2438$, and the discount rate $\beta = 0.051$, stock return $\alpha_B = \alpha_b = 0.0994$, regime switching intensities $\eta_B = 0.2353, \eta_b = 1.7391$, risk aversion $\gamma = 7$