Extreme Movements in Aggregate Volatility and Stock Returns

Byungmin Oh*

October 28, 2022

Abstract

Expected extremeness of movements in aggregate volatility, measured by kurtosis of aggregate volatility (KOV), varies significantly through time. The KOV measure is constructed using daily snapshots of VIX option prices and, therefore, conditional and forward-looking. KOV exposure is a significant cross-sectional determinant of stock returns. KOV risk premium is negative and robust to controlling for exposures to alternative measures of economic uncertainty and stock characteristics.

JEL Classifications: G12

Keywords: ICAPM; kurtosis of volatility; VIX; stock returns

*College of Business, Korea Advanced Institute of Science and Technology. byungmin.oh@kaist.ac.kr.

1 Introduction

This study investigates whether exposure to expected extremeness of movements in aggregate volatility is a cross-sectional determinant of stock returns. I measure the expected extremeness by kurtosis of the probability distribution of aggregate volatility. Throughout the manuscript, I refer to the volatility, skewness, and kurtosis of aggregate volatility as VOV (volatility-of-volatility), SOV (skewness-of-volatility), and KOV (kurtosis-of-volatility), respectively. Time variation of these moments characterizes the conditional nature of the distribution of aggregate volatility. A number of recent studies demonstrate that the distribution of aggregate volatility contains useful information about the underlying economic conditions and also has pricing implications.[1] Because KOV and SOV capture the tail properties of the distribution of aggregate volatility, they are appropriate to examine the pricing effect that arises from investors' attitudes towards extreme volatility movements. This is the first attempt, I am aware of, to investigate whether aggregate volatility tail events affect the stock returns.

The moments of aggregate volatility are extracted from VIX option prices. Specifically, I use the model-free methodology of Bakshi et al. (2003) to calculate the risk-neutral volatility, skewness, and kurtosis of VIX.[2] Because the moments are estimated using daily snapshots of VIX option prices, the resulting estimates are conditional and forward-looking. This method also resolves the so-called peso problem that emerges when using data on realized VIX. The peso problem arises when a rare but significant movement in aggregate volatility could have happend but did not happen in the sample. I overcome this problem by estimating the ex ante moments of VIX without using data on realized VIX.

I first perform univariate portfolio sorts to investigate the association between the KOV exposure and the expected return. By sorting all U.S. common stocks from 2008 through

[1] See, for example, Bollerslev et al. (2009), Bali and Zhou (2016), Agarwal et al. (2017), Hollstein and Prokopczuk (2018), Cheng (2019), Huang et al. (2019), and Kostopoulos et al. (2021).

[2] Although the spot VIX is not necessarily equal to the expected discounted VIX, the method developed by Bakshi et al. (2003) can still be applied using the VIX futures price instead of the spot VIX.

2020 into quintile portfolios according to the sensitivities of their returns to KOV innovations, I find that stocks with higher sensitivities to KOV innovations earn lower average returns. There is a significant difference of -0.40% per month in terms of raw returns and -0.38% per month in terms of Fama and French (2018) six-factor alpha between the top KOV beta quintile portfolio and the bottom KOV beta quintile portfolio.

I then investigate whether the negative association between KOV beta and the expected stock return can be explained by risk variables that are plausibly similar to KOV risk. I perform a bivariate portfolio analysis and Fama and MacBeth (1973) regressions to control for CAPM beta, VIX beta, VIX of VIX (VVIX) beta, variance risk premium beta, and several stock characteristics such as size, book-to-market, momentum, and liquidity. The results show that the pricing effect of KOV beta is robust to controlling for these measures. I find that estimates of the price of KOV risk are consistently negative and statistically significant across various econometric specifications and imply an annual premium of approximately -5.53% to -8.46%.

The KOV factor used in the empirical analyses is more sensitive to the "tailedness" of the distribution of aggregate volatility than the existing measures of uncertainty about aggregate volatility. It thus enables investigating whether investors are willing to pay to hedge against fluctuations in the tailedness of the distribution of aggregate volatility, or, fluctuations in the expected extremeness of volatility movements. The negative KOV premium implies that the hedge against these fluctuations is valuable to the investors and the marginal value of wealth is higher in states with higher expected extremeness of volatility movements.

I also investigate the pricing effect of SOV beta and do not find evidence of a significant association between SOV beta and future stock returns. While SOV measures the expected "signed" extremeness of volatility movements, KOV measures the expected "unsigned" extremeness. The results presented in this paper are therefore consistent with the previously documented puzzling findings on the pricing kernel projection onto aggregate volatility. Song and Xiu (2016) show that the projection of the pricing kernel onto the VIX displays a U-shaped pattern, which suggests that both high VIX state claims and low VIX state claims are relatively expensive. Their results indicate that investors have higher marginal value of wealth in both high and low volatility states; that is, both high and low volatility states are unfavorable to investors. The results presented in this paper thus complement the existing evidence by showing that the extreme volatility movements, regardless of the directions of the movements, are unfavorable to investors.

The investigation of whether the higher-order risks of aggregate volatility have the pricing effect is motivated by the existing literature on aggregate stock market anomalies and properties. Part of this literature tries to explain the stylized empirical facts about the stock market using representative agent models, building on the seminal contribution by Bansal and Yaron (2004). Studies in this strand of literature typically model dynamics of aggregate consumption and dividend growth in a way that captures certain characteristics observed in the data and recover the pricing kernel that is consistent with the dynamics (Bansal and Yaron, 2004; Drechsler and Yaron, 2011; Bansal et al., 2012). Notably, Bollerslev et al. (2009) allow richer volatility dynamics by including time-varying aggregate volatility-of-volatility and show that the projection of the pricing kernel onto the volatility-of-volatility in consumption growth should be non-constant to generate the volatility dynamics.[3] In other words, they argue that the pricing kernel that is consistent with time-varying aggregate volatility-of-volatility depends on aggregate volatility-of-volatility.

Although models of this kind are originally developed to explain the characteristics of the aggregate stock market, their implications for the cross section of expected stock returns have also received attention recently. Bali and Zhou (2016) test the pricing implications of Bollerslev et al. (2009) using equity portfolios and find that portfolios that are highly correlated with the variance risk premium earn higher returns on average. Hollstein and Prokopczuk (2018) explore the same implications with an alternative expression of the

[3]Omitting some technical details, this means that the projection of a random variable m, the pricing kernel, onto the space of random variables that are measurable with respect to the the σ-algebra $\mathcal{F}$ generated by a random variable q, aggregate volatility-of-volatility, is a random variable that is not $\mathcal{F}$-a.s. constant.

pricing kernel and find that sensitivity to aggregate volatility-of-volatility carries a negative premium. Because the higher moments of aggregate volatility are also significantly time-varying, I conjecture that the pricing kernel depends on the higher moments of aggregate volatility and exposures to innovations in these moments can explain part of the cross-sectional variation in expected stock returns.

This study contributes to a rapidly growing literature on the role of uncertainty about aggregate volatility in asset pricing. Agarwal et al. (2017) show the importance of VOV risk in explaining hedge fund returns. Huang et al. (2019) demonstrate that the price of VOV risk is negative building on the work of Bakshi and Kapadia (2003) that finds a negative volatility risk premium. Kostopoulos et al. (2021) show that an increase in VOV is associated with an increase in investors' risk aversion and trading activity. By constructing a factor that is more sensitive to the tailedness of the distribution of aggregate volatility than the measures used in the previous studies, I show that investors are willing to pay to hedge against fluctuations in the expected extremeness of volatility movements.

This study also adds to the existing literature on pricing factors extracted from futures and option prices (Ang et al., 2006; Chang et al., 2013; Cremers et al., 2015; Bali and Zhou, 2016; Hollstein and Prokopczuk, 2018; Lu and Murray, 2019). Futures and option prices contain information about investors' perceptions towards future investment opportunitiy set. Because theories of intertemporal asset pricing suggest that exposure to change in future investment opportunity set is priced, their implications should be tested using forward-looking factors. The results presented in the paper show that tails of the distribution of aggregate volatility restored from out-of-the-money calls and puts contain more information about the underlying economic condition than acknowledged by prior research.

This study is also related to studies that investigate the relevance of higher-order risks for asset pricing (Kraus and Litzenberger, 1976; Friend and Westerfield, 1980; Sears and Wei, 1985; Lim, 1989; Harvey and Siddique, 2000; Dittmar, 2002). However, because the higher-order CAPMs are static models, risks in time-varying forward-looking moments are not priced in these models. Notable exceptions include Chabi-Yo (2012) and Chang et al. (2013), who explore the importance of forward-looking higher-order moments of the market return theoretically and empirically, respectively. I add to the literature by showing that KOV, which captures the tail property of aggregate volatility, also plays an important role in explaining the expected stock returns.

2 The Analytical Framework

The empirical model that I use to obtain sensitivities to innovations in the moments of the market return and aggregate volatility is as follows:

$$
\begin{aligned}
R_{i,d} - R_{f,d} = \beta_0 &+ \beta_i^{MKT}(MKT_d - R_{f,d}) \\
&+ \beta_i^{\Delta VOL}\Delta VOL_d + \beta_i^{\Delta SKEW}\Delta SKEW_d + \beta_i^{\Delta KURT}\Delta KURT_d \\
&+ \beta_i^{\Delta VOV}\Delta VOV_d + \beta_i^{\Delta SOV}\Delta SOV_d + \beta_i^{\Delta KOV}\Delta KOV_d + \varepsilon_{i,d},
\end{aligned} \tag{1}
$$

where $R_{i,d}$, MKT_d, and $R_{f,d}$ are the returns of stock i, market portfolio, and risk-free asset over the five-trading day period of day $d-4$ through d. VOL, $SKEW$, $KURT$, VOV, SOV, and KOV over the same period, where VOL, $SKEW$ and $KURT$ are the volatility, skewness, and kurtosis of the market return and VOV, SOV, and KOV are the volatility, skewness, and kurtosis of aggregate volatility. $\Delta VOL_d = VOL_d - E_{d-5}[VOL_d]$ and the five-day innovations for other moments are defined similarly. Section 3.3 describes how the innovations in the moments are measured. I use the five-day returns and innovations to reduce the influence of measurement noises due to the bid-ask spread and nonsynchronous trading in the stock and option markets.[4]

To minimize estimation error, I apply a Bayes shrinkage method to adjust the beta estimates following Fama and French (1997) and Lu and Murray (2019). The following

[4]Several previous studies use similar methods to address such concerns. See, for example, Hou and Moskowitz (2005), Frazzini and Pedersen (2014), and Lu and Murray (2019).

description is for the adjustment of $\beta^{\Delta KOV}$ estimates and I apply the same adjustment process to estimates of $\beta^{\Delta VOL}$, $\beta^{\Delta SKEW}$, $\beta^{\Delta KURT}$, $\beta^{\Delta VOV}$, and $\beta^{\Delta SOV}$. For each stock i and month m, I estimate the model (1) and let $\beta^{\Delta KOV}_{\text{OLS},i,m}$ be the estimated coefficient on ΔKOV_d and $\left(\sigma^{\Delta KOV}_{\text{OLS},i,m}\right)^2$ be the estimated variance of the OLS estimate $\beta^{\Delta KOV}_{\text{OLS},i,m}$. Then, for each month m, I take the prior mean, $\beta^{\Delta KOV}_{\text{Prior},m}$, to be the average $\beta^{\Delta KOV}_{\text{OLS}}$ across all stock-month observations between June 2008 and month m, inclusive; that is,

$$\beta^{\Delta KOV}_{\text{Prior},m} = \frac{\sum_{t\le m}\sum_{j\in S_t}\beta^{\Delta KOV}_{\text{OLS},j,t}}{n_m}, \tag{2}$$

where S_t is the set of stocks with valid values of $\beta^{\Delta KOV}_{\text{OLS},j,t}$ for months t between June 2008 and month m, inclusive, and $n_m \equiv \sum_{t\le m}|S_t|$.[5] I also take the prior variance, $\left(\sigma^{\Delta KOV}_{\text{Prior},m}\right)^2$, to be the sample variance of $\beta^{\Delta KOV}_{\text{OLS},i,m}$ over the same period; that is,

$$\left(\sigma^{\Delta KOV}_{\text{Prior},m}\right)^2 = \frac{\sum_{t\le m}\sum_{j\in S_t}\left(\beta^{\Delta KOV}_{\text{OLS},j,t} - \beta^{\Delta KOV}_{\text{Prior},m}\right)^2}{n_m - 1}. \tag{3}$$

The Bayes-adjusted estimate $\beta^{\Delta KOV}_{i,m}$ is the inverse variance-weighted average of the OLS estimate and the prior mean; that is,

$$\begin{aligned}\beta^{\Delta KOV}_{i,m} =& \frac{\left(\sigma^{\Delta KOV}_{\text{OLS},i,m}\right)^{-2}}{\left(\sigma^{\Delta KOV}_{\text{OLS},i,m}\right)^{-2} + \left(\sigma^{\Delta KOV}_{\text{Prior},m}\right)^{-2}}\beta^{\Delta KOV}_{\text{OLS},i,m} \\ &+ \frac{\left(\sigma^{\Delta KOV}_{\text{Prior},m}\right)^{-2}}{\left(\sigma^{\Delta KOV}_{\text{OLS},i,m}\right)^{-2} + \left(\sigma^{\Delta KOV}_{\text{Prior},m}\right)^{-2}}\beta^{\Delta KOV}_{\text{Prior},m}.\end{aligned} \tag{4}$$

Intuitively, the Bayes-adjusted estimate is the weighted average of the most recent estimate and the prior mean, where more (less) weight is given to the recent estimate if the estimate is accompanied by a smaller (larger) standard error. I use the Bayes-adjusted estimates throught the empirical analyses.

[5] Because the time series of the moments start from January 2007 and the computation of the innovations requires at least a six-month length of the past time series, the first beta estimates are calculated at the end of June 2008.

The first set of empirical results is based on univariate sorts. I sort the available stocks based on the Bayes-adjusted estimate of $\beta^{\Delta KOV}$ into quintile portfolios and examine the cross-sectional variation of returns on the portfolios. I then proceed by performing bivariate sorts. To control for CAPM beta and other plausibly similar risk characteristics, I first sort the available stocks based on each of the control variables into quintile portfolios and then sort them again within each control variable qunitile based on the Bayes-adjusted estimate of $\beta^{\Delta KOV}$. Because the stocks in the same control variable quintile have similar values of the control variable, the cross-sectional variation across the $\beta^{\Delta KOV}$ quintile portfolios within each control variable quintile is unlikely to be explained by the control variable.

The last set of results is based on cross-sectional regressions of individual stock returns on $\beta^{\Delta KOV}$ and other cross-sectional determinants of stock returns. I use coefficient estimates obtained from time series regressions to examine the prices of the risks λ from the cross-sectional relation:

$$E\left[R_i\right] - R_f = \lambda_0 + \lambda_i^{\Delta KOV} \beta_i^{\Delta KOV} + \gamma_i^{\prime} \Lambda, \tag{5}$$

where γ_i is a vector of cross-sectional determinants of expected stock return.

Although existing theories provide little guidance regarding the signs of the premiums of KOV and SOV risks, the intertemporal capital asset pricing model (ICAPM), following Merton (1973), allows forming a prior for the sign of the risk premiums. Risks of the factors are priced if investors are willing to pay to hedge against changes in future investment opportunity set due to changes in the factors. The prices of risks, therefore, depend on whether increases in the factors reflect enhancements or deteriorations in the investors' investment opportunity set. For example, if high value of a given factor today is related to an unfavorable investment opportunity set tomorrow, then an asset whose return is positively related to the innovation in the factor provides a hedge against a deterioration in the investment opportunity set. When investors are risk averse, expected return for such asset is low, that is, the price of risk of the factor is negative, because the hedge provided by this asset is valuable to investors. In the opposite scenario in which high value of the factor today is related to a favorable investment opportunity set tomorrow, the price of risk of the factor is positive.

The logic behind the ICAPM, combined with the recent empirical findings, hints at the sign of the prices of KOV and SOV risks. Song and Xiu (2016) show that the projection of the pricing kernel onto the VIX displays a U-shaped pattern, which suggests that both high VIX state claims and low VIX state claims are relatively expensive. Because their results indicate that investors have higher marginal value of wealth in both high and low volatility states, one would expect that increases in KOV reflect deteriorations in the investment opportunity set and the sign of the price of KOV risk is negative. On the other hand, determining whether increases in SOV are associated with enhancements or deteriorations in the investment opportunity set is less clear.

3 Data Description and Factor Construction

3.1 What Do Higher Moments of Aggregate Volatility Measure?

Kurtosis is the expectation of the standardized values raised to the fourth power. The standardized value raised to the fourth power can be thought of the extremeness of an outcome, because raising a number that is larger than 1 in magnitude to the fourth power results in a positive number that is much larger in magnitude whereas raising a number that is smaller than 1 in magnitude to the fourth power makes it closer to zero. Therefore, KOV is an appropriate measure of the expected extremeness of movements in aggregate volatility. Similarly, SOV can be seen as a measure of the expected "signed" extremeness of movements in aggregate volatility in the sense that a high (low) value of SOV indicates that the extremeness of increases in aggregate volatility is expected to be much larger (smaller) than the extremeness of decreases in aggregate volatility.

3.2 Estimating Higher Moments of Aggregate Volatility

I use the model-free methodology proposed by Bakshi et al. (2003) to estimate the moments of the market return and aggregate volatility from prices of S&P 500 Index options and VIX options. I use options with time-to-maturity of one month to calculate the implied moments over a 30-day horizon. Thus, a high value of KOV today indicates that the innovation in the spot VIX over the next 30 calendar days is expected to be highly extreme. The details of the methodology and implementation are provided in the Appendix. Because VOV (VOL) has a correlation of 0.95 (0.99) with CBOE VVIX (VIX) Index, I use $VVIX$ (VIX) for VOV (VOL) throughout the empirical analyses for maximum comparability with the results documented by previous studies (Ang et al., 2006; Chang et al., 2013; Hollstein and Prokopczuk, 2018).

VIX option data are obtained from OptionMetrics Ivy DB and start in 2006. However, I use the 15-year period from 2007 through 2020 because daily trading volumes of out-of-the-money options were low in 2006. Because estimation of higher moments puts more weight on prices of out-of-the-money options, I exclude the data from 2006.

The daily estimates of $VVIX$, SOV, and KOV are shown in Figure 1. All three time series vary significantly through time. Contrary to the implied skewness of the market return, which is always negative, the sign of the implied skewness of volatility frequently changes. The implied kurtosis of volatility is larger than three for most of the days with a few exceptions.

3.3 Measuring Innovations in Moments of VIX

Follwing previous studies (e.g., Chang et al., 2013), I fit an appropriate ARMA model to the time series for VIX, $SKEW$, $KURT$, $VVIX$, SOV, and KOV to measure the innovations in the moments. The ARMA models are fitted at the end of month m using only past moments and the parameter estimates are used to calculate the forecasts and residuals of the moments in momth $m + 1$. I require at least a six-month length of past time series for the estimation. Figure 2 shows the autocorrelation function (ACF) plots of the original time series, the daily first differences, and the daily ARMA(1,1) residuals of KOV. ACF plots for the other five moments are also available upon request. The plots show that the ARMA(1,1) model is needed to remove the autocorrelation of KOV. The same is also true for $SKEW$, $KURT$, and SOV, whereas taking first differences is enough to remove the autocorrelation for VIX and $VVIX$ as documented by previous studies. I thus use the sum of five daily first differences over the five-day period as the innovation in VIX and $VVIX$. When the ARMA(1,1) models are fitted to the entire time series of $SKEW$, $KURT$, SOV, and $KURT$, the estimates of the AR(1) parameters for all six moments are very close to 1, with values of 0.95 to 0.99 (untabulated), suggesting that one can fit an MA model to the first differences of the moments. I fit the MA(1) model to each time series of daily first differences of $SKEW$, $KURT$, SOV, and KOV:

$$X_d - X_{d-1} = \varepsilon_d + \theta_X \varepsilon_{d-1}, \tag{6}$$

where X represents one of the four moments. It immediately follows that

$$E_{d-5}\left[X_d - X_{d-5}\right] = E_{d-5}\left[\sum_{t=d-4}^{d} (X_t - X_{t-1})\right] = E_{d-5}\left[\sum_{t=d-4}^{d} \varepsilon_t + \theta_X \varepsilon_{t-1}\right] = \theta_X \varepsilon_{d-5}. \tag{7}$$

Therefore, I subtract the day-$(d-5)$ residual multiplied by the MA parameter estimate from the sum of five daily first differences for day $d-4$ through d to calculate the five-day innovation in $SKEW$, $KURT$, SOV, and KOV over the five-day period of day $d-4$ through d; that is,

$$\Delta X_d = \sum_{t=d-4}^{d} (X_t - X_{t-1}) - \theta_X \varepsilon_{d-5}. \tag{8}$$

When the MA(1) model is fitted using the entire time series of daily first differences, the estimates of θ_X are -0.235, -0.096, -0.354, and -0.271 for $SKEW$, $KURT$, SOV, and KOV, respectively.

To check for robustness, I repeat the univariate portfolio analysis using the sum of five daily first differences as the the innovation over the five-day period. Using first differences has an advantage that it does not necessitate a six-month estimation period. The results of the robustness test are similar to the original results and presented in Section 4.4.

3.4 Data on Stock Returns and Characteristics

I obtain stock market information from Center for Research in Securities Prices (CRSP) daily and monthly stock files and financial information from Compustat. The sample used in the empirical tests includes all U.S. common stocks in the CRSP database. The market return, risk-free rate, and the factor mimicking portfolio returns are obtained from Kenneth French's data library[6] and Kewei Hou, Chen Xue, and Lu Zhang's website.[7]

4 Results

4.1 Univariate Portfolio Sorts

To examine the association between $\beta^{\Delta KOV}$ and expected stock return, I start by performing univariate portfolio sorts. At the end of month m, I form quintile portfolios by sorting stocks based on $\beta^{\Delta KOV}$, where $\beta^{\Delta KOV}$ quintile five portfolio comprises stocks with the highest values of $\beta^{\Delta KOV}$ and $\beta^{\Delta KOV}$ quintile one portfolio comprises stocks with the lowest values of $\beta^{\Delta KOV}$. Stocks are weighted equally within each of the $\beta^{\Delta KOV}$ quintile portfolios. Table 1 presents the average month-$(m+1)$ excess returns of the $\beta^{\Delta KOV}$ quintile portfolios and a zero-investment portfolio ($\beta^{\Delta KOV}$ $5-1$) that is long the $\beta^{\Delta KOV}$ quintile five portfolio and short the $\beta^{\Delta KOV}$ quintile one portfolio. The results show that the average excess return is monotonically decreasing in $\beta^{\Delta KOV}$. The $\beta^{\Delta KOV}$ quintile one portfolio and the $\beta^{\Delta KOV}$ quintile five portfolio earn an average excess return of 1.33% per month and 0.93% per month,

[6]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
[7]http://global-q.org/factors.html.

respectively. The $\beta^{\Delta KOV}$ $5-1$ portfolio earns an average return of -0.40% per month, which is economically and statistically significant with a Newey-West t-statistic of -2.13.

To further investigate whether the monotonically decreasing pattern in the excess return of the $\beta^{\Delta KOV}$-sorted portfolios is derived from premiums attached to exposures to other risk factors, I calculate and report the alphas of the quintile portfolios relative to the CAPM, Fama and French (1993) (FF3), Fama and French (1993) and Carhart (1997) (FFC), Fama and French (2015) (FF5), Fama and French (2018) (FF6), Hou et al. (2015) ($q4$), and Hou et al. (2021) ($q5$) factor models. The results indicate that the relation between $\beta^{\Delta KOV}$ and the expected return is not perfectly explained by exposures to previously identified risk factors because the alphas are also monotonically decreasing in $\beta^{\Delta KOV}$. The $\beta^{\Delta KOV}$ $5-1$ portfolio generates monthly alphas of -0.39% to -0.45% with Newey-West t-statistics of -2.01 to -2.35, which are negative and statistically significant.

As emphasized by Ang et al. (2006, page 271), "finding large spreads in the post-formation loadings is a very stringent requirement" in tests of a factor risk-based explanation of a cross-sectional variation in the expected return. The results presented above show the relation between the post-formation returns and the pre-formation $\beta^{\Delta KOV}$. Thus, I also test whether the spread in the post-formation returns is accompanied by the spread in the post-formation KOV beta. The post-formation sensitivities to ΔKOV are obtained by regressing the post-formation five-day excess returns of each portfolio on the contemporaneous ΔKOV and other factors, as in equation (1).

The results presented in Table 1 show that the post-formation sensitivity of $\beta^{\Delta KOV}$ $5-1$ portfolio returns to ΔKOV is 1.36 with a t-statistic of 6.36. By construction, the pre-formation values of $\beta^{\Delta KOV}$ increase from -1.34 for the $\beta^{\Delta KOV}$ quintile one portfolio to 1.37 for $\beta^{\Delta KOV}$ quintile five portfolio. Although the KOV factor loading is an imperfect measure of the true forward-looking KOV factor loading, sorting on the pre-formation exposure generates economically and statistically significant spread in the post-formation exposure to ΔKOV.

4.2 Bivariate Portfolio Sorts

In this subsection, I proceed to investigate the possibility that the association between $\beta^{\Delta KOV}$ and the expected stock return can be explained by risk variables that are plausibly similar to KOV risk. Table 2 reports the sensitivities of returns on the $\beta^{\Delta KOV}$ quintile portfolios to each of the plausibly similar risk variables. I describe each risk variable as I discuss the corresponding results.

To control for the effect of the other risk variables, I perform bivariate portfolio sorts to create portfolios that are similarly exposed to a control variable with a significant $\beta^{\Delta KOV}$ spread. At the end of month m, I form quintile portfolios by sorting stocks based on the control variable. Within each control variable quintile, I form quintile portfolios by sorting stocks based on $\beta^{\Delta KOV}$. Stocks are weighted equally within each of the resulting 25 portfolios. Five portfolios with the same rank of $\beta^{\Delta KOV}$ are weighted equally again and the resulting five portfolios are referred to as the bivariate $\beta^{\Delta KOV}$ quintile portfolios. Because the bivariate $\beta^{\Delta KOV}$ quintile portfolios have similar values of the control variable, any return pattern across the bivariate $\beta^{\Delta KOV}$ quintile portfolios is unlikely to be driven by the control variable. Table 3 reports the average month-$(m+1)$ excess returns of the bivariate $\beta^{\Delta KOV}$ quintile portfolios and a zero-investment portfolio ($\beta^{\Delta KOV}$ $5-1$) that is long the bivariate $\beta^{\Delta KOV}$ quintile five portfolio and short the bivariate $\beta^{\Delta KOV}$ quintile one portfolio. Each column is labeled by the name of the control variable used to construct the bivariate portfolios.

The first risk variable controlled for is CAPM beta (β^{CAPM}). β^{CAPM} is the estimated coefficient on the excess market return from a twelve-month rolling window regression of daily excess stock returns on the contemporaneous excess market returns. Table 2 shows a negative association between $\beta^{\Delta KOV}$ and β^{CAPM}. Because the CAPM predicts that CAPM beta is positively priced, there is a possibility that CAPM beta explain the negative relation between $\beta^{\Delta KOV}$ and average stock returns. However, the column of Table 3 labeled "β^{CAPM}" shows that the bivariate $\beta^{\Delta KOV}$ $5-1$ portfolio that is neutral to β^{CAPM} still earns a highly significant negative return of -0.46% per month with a t-statistic of -2.16. Therefore, the negative association between $\beta^{\Delta KOV}$ and the expected return cannot be explained by the exposure to the market risk. Meanwhile, Frazzini and Pedersen (2014) show that CAPM beta is negatively associated with alphas due to leverage constraints. In constrast to the pattern in average stock returns, the pattern in alphas presented in Table 1 is unlikely to be explained by exposures to the market risk. Table 3 indeed exhibits similar monotonically decreasing patterns in alphas relative to several asset pricing models. The bivariate $\beta^{\Delta KOV}$ $5-1$ portfolio that is neutral to β^{CAPM} earns highly significant negative alphas of -0.45% to -0.50% per month with t-statistics between -2.25 and -2.51.

Although KOV is a measure of "variability" of aggregate volatility that is more sensitive to the tailedness, there is a possibility that the pricing effect of KOV is subsumed by effects of alternative measures of economic uncertainty and uncertainty about aggregate volatility. Thus, I investigate whether exposures to factors that capture the variability of aggregate return and volatility. Ang et al. (2006) find a negative relation between average stock returns and VIX beta ($\beta^{\Delta VIX}$). $\beta^{\Delta VIX}$ is the estimated coefficient on the VIX change from a one-month rolling window regression of daily excess stock returns on the contemporaneous excess market returns and VIX changes. Table 2 shows a negative association between $\beta^{\Delta KOV}$ and $\beta^{\Delta VIX}$, though the association is statistically significant only at the 10% level. Because previous studies document a negative relation between the expected stock return and the exposure to the volatility risk, I expect to observe similar monotonically decreasing patterns in average returns and alphas across bivariate $\beta^{\Delta KOV}$ quintile portfolios that are neutral to $\beta^{\Delta VIX}$. As expected, the column of Table 3 labeled "$\beta^{\Delta VIX}$" shows that the bivariate $\beta^{\Delta KOV}$ $5-1$ portfolio that is neutral to $\beta^{\Delta VIX}$ earns a highly significant negative return of -0.47% with a t-statistic of -2.25 and highly significant negative alphas of -0.47% to -0.56% per month with t-statistics between -2.44 and -3.23.

I then examine two measures that can capture the variability of aggregate volatility. Prior research shows that exposures aggregate volatility-of-volatility ($VVIX$) and the variance risk premium (VRP) affect the expected stock return (Bali and Zhou, 2016; Hollstein and

Prokopczuk, 2018). Because change in the risk-neutral moments of volatility can also lead to change in the VRP, there is a possibility that the observed pattern in returns and alphas of univariate $\beta^{\Delta KOV}$ quintile portfolios are derived from exposures to $VVIX$ and VRP. $\beta^{\Delta VVIX}$ is defined as the estimated coefficient on the $VVIX$ change from a twelve-month rolling window regression of daily excess stock returns on the contemporaneous excess market returns, VIX changes, and $VVIX$ changes. Furthermore, $\beta^{\Delta VRP}$ is defined as the estimated coefficient on the VRP change from a twelve-month rolling window regression of daily excess stock returns on the contemporaneous excess market returns and VRP changes. As presented in Table 2, I do not observe a significant association between $\beta^{\Delta KOV}$ and $\beta^{\Delta VVIX}$, as well as between $\beta^{\Delta KOV}$ and $\beta^{\Delta VRP}$. Therefore, I expect similar monotonically decreasing patterns in average returns and alphas across bivariate $\beta^{\Delta KOV}$ quintiles that are neutral to $\beta^{\Delta VVIX}$ and $\beta^{\Delta VRP}$. The column of Table 3 labeled "$\beta^{\Delta VVIX}$" shows that the bivariate $\beta^{\Delta KOV}$ $5-1$ portfolio that is neutral to $\beta^{\Delta VVIX}$ earns a highly significant negative return of -0.38% with a t-statistic of -2.13 and highly significant negative alphas of -0.39% to -0.49% per month with t-statistics between -2.25 and -2.46. The column labeled "$\beta^{\Delta VRP}$" also shows that the bivariate $\beta^{\Delta KOV}$ $5-1$ portfolio that is neutral to $\beta^{\Delta VRP}$ earns a highly significant negative return of -0.42% with a t-statistic of -2.33 and highly significant negative alphas of -0.42% to -0.47% per month with t-statistics between -2.37 and -3.11.

4.3 Multivariate Analysis

Bivariate portfolio analysis enables controlling for the effect of one variable on the expected stock return. To control for multiple variables simultaneously, I use a Fama and MacBeth (1973) regression analysis. Each month m, I run the following cross-sectional regression:

$$r_{i,m+1} = \lambda_{0,m} + \lambda_m^{\Delta KOV} \beta_{i,m}^{\Delta KOV} + \gamma_{i,m}^{\prime} \Lambda_m + \varepsilon_{i,m}, \tag{9}$$

where $r_{i,m+1}$ is month-$(m+1)$ excess return of stock i, $\beta_{i,m}^{\Delta KOV}$ is month-m value of $\beta^{\Delta KOV}$ of stock i, and $\gamma_{i,m}$ is a vector of month-m values of control variables for stock i. All independent variables are winsorized at the 0.5% and 99.5% levels on a monthly basis. If the effect of $\beta^{\Delta KOV}$ on the expected return is distinct from the effects of the control variables, the coefficient estimates of $\beta^{\Delta KOV}$ should remain negative.

Table 4 presents the time series averages of the monthly cross-sectional regression coefficient estimates. Column (1) reports the results of estimating model (9) with $\beta^{\Delta KOV}$ as the only independent variable. The average coefficient estimate of $\beta^{\Delta KOV}$ is -0.17 with a t-statistic of -2.73, which is negative and statistically significant at the 1% level. This is consistent with the results from the univariate portfolio analysis and indicates a strong negative relation between $\beta^{\Delta KOV}$ and the expected return. Next, I control for exposure to the market risk by including β^{CAPM} as the first control variable and report the results in column (2). The average coefficient on $\beta^{\Delta KOV}$ is even greater in magnitude compared with the univariate specification and statistically significant at the 1% level. Thus, the negative cross-sectional relation between $\beta^{\Delta KOV}$ and future stock returns is not explained by exposure to the market risk. Then, I proceed to include ME (log of the market capitalization), BM (log of the ratio of the book value of equity to the market capitalization), MOM (11-month stock return in months $m-11$ through $m-1$), Y, INV, and $ILLIQ$ as control variables in various forms to control for the effect of size, book-to-market ratio, momentum, profitability, investment, and liquidity.[8] The results presented in columns (3) through (7) show that the average coefficient on $\beta^{\Delta KOV}$ remains negative and statistically significant at the 1% level in each of the specifications with multiple control variables.

I continue by examining whether the results are affected by the inclusion of factor betas. In addition to β^{CAPM}, I include β^{SMB}, β^{HML}, β^{UMD}, β^{RMW}, and β^{CMA}, the estimated coefficients of SMB, HML, UMD, RMW, and CMA from a twelve-month rolling window regression of daily excess stock returns on the contemporaneous returns of the six factors of

[8]The profitability measure Y and the investment measure INV are calculated following Fama and French (2015). The illiquidity measure $ILLIQ$ is calculated following Amihud (2002).

Fama and French (2018). The results are reported in columns (1) through (6) of Table 5. I find that the inclusion of factor betas does not perfectly subsume the effect of $\beta^{\Delta KOV}$ on future returns. The results show that the coefficient estimate of $\beta^{\Delta KOV}$ is between -0.17 and -0.26 with t-statistics ranging from -2.73 and -3.41.

I further examine whether the results are affected by the inclusion of plausibly similar risk variables considered in Section 4.2. In addition to stock characteristics or factor betas, I include $\beta^{\Delta VIX}$, $\beta^{\Delta VVIX}$, and $\beta^{\Delta VRP}$ as the control variables. The results are reported in columns (1) through (8) of Table 6. I find that the inclusion of the risk variables does not perfectly subsume the effect of $\beta^{\Delta KOV}$ on future returns. The results reveal that the coefficient estimate of $\beta^{\Delta KOV}$ is between -0.21 and -0.26 with t-statistics ranging from -2.86 and -3.56.

To assess the economic significance of the results of Fama and MacBeth (1973) regressions, I use information from univariate portfolio sorts. The difference in $\beta^{\Delta KOV}$ between the $\beta^{\Delta KOV}$ quintile five and one portfolios is $1.37 - (-1.34) = 2.71$. Multiplying this difference by the average coefficient estimates of $\beta^{\Delta KOV}$ in Tables 4, 5, and 6 yields estimated annual premiums between -5.53% and -8.46%.

4.4 Robustness Checks

This section discusses the results from a battery of additional robustness checks. Each panel of Table 7 reproduces the univariate portfolio analysis results by employing alternative empirical setups.

The univariate portfolio analysis described in Section 4.1 is performed using an equal-weighting scheme. Thus, there is a possibility that the results are driven by overweighting relatively small stocks. To address this concern, I repeat the univariate portfolio analysis using a value-weighting scheme; that is, stocks in eacn of the univariate quintile portfolios are weighted according to their market capitalization. The value-weighted $\beta^{\Delta KOV}$ $5-1$ portfolio generates FF6 alpha that is much smaller in magnitude compared to the equal-weighted $\beta^{\Delta KOV}$ $5-1$ portfolio; it amounts to -0.18% per month, which is not statistically significant at the 10% level.

To investigate the influence of extremely large stocks on the results, I repeat the analysis excluding the largest stocks. The results presented in Panel A of Table 7 show that, when the top 1% largest stocks in the sample are excluded at the end of each month, the value-weighted $\beta^{\Delta KOV}$ $5-1$ portfolio generates FF6 alpha of -0.28% per month, which is statistically significant at the 10% level. When the largest 5%, 10% and 20% of stocks are excluded, the value-weighted $\beta^{\Delta KOV}$ $5-1$ portfolio generates FF6 alpha of -0.33%, -0.35%, and -0.43% with a t-statistic of -1.91, -2.07, and -2.16, respectively. I conclude that it is the extremely large stocks for which the effect of $\beta^{\Delta KOV}$ is negligible; the significantly negative relation between $\beta^{\Delta KOV}$ and future returns is robust for all other stocks.

I next examine the stability of the relation between $\beta^{\Delta KOV}$ and future returns when alternative lengths of the beta estimation period are chosen. The results from choosing alternative beta estimation period lengths are presented in Panel B of Table 7. When three-month period is used to estimate $\beta^{\Delta KOV}$, the $\beta^{\Delta KOV}$ $5-1$ portfolio generates FF6 alpha of -0.34% per month, which is statistically significant only at the 10% level. However, when six-month and nine-month periods are used, FF6 alphas amount to -0.41% and -0.49% per month with t-statistics of -2.27 and -2.36, respectively. The results indicate that the length of the pre-formation beta estimation window should be long enough to generate a significant post-formation return spread.

I proceed to examine whether the results are affected by applying different data filterings. Panel C of Table 7 reports the results from using alternative data filtering methods. When only stocks listed in NYSE, Amex, and Nasdaq are included in the sample, the $\beta^{\Delta KOV}$ $5-1$ portfolio generates FF6 alpha of -0.46% per month, which is negative and statistically significant at the 5% level. The analysis is repeated again excluding micro-cap stocks, which I define as stocks with a market capitalization below the 10th percentile. When the micro-cap stocks are excluded, the $\beta^{\Delta KOV}$ $5-1$ portfolio generates FF6 alpha of -0.36% per month, which is statistically significant at the 10% level. The results reveal that the effect of $\beta^{\Delta KOV}$ is slightly stronger in relatively small stocks.

Finally, I investigate whether the results are robust to using an alternative time series model to estimate the innovations in the moments. I use the sum of five daily first differences of the moments as the five-day innovations and repeat the univariate portfolio analysis. Panel D of Table 7 reports the results of using an alternative time series model for the calculation of the innovations. The $\beta^{\Delta KOV}$ $5-1$ portfolio generates FF6 alpha of -0.44% per month, which is still negative and statistically significant at the 5% level. The results reveal that the choice of the time series model does not affect the results much.

4.5 Alternative Interpretations of the Results

Option implied moments used in the empirical tests are risk-neutral moments. Changes in risk-neutral moments reflect both changes in the physical moment and changes in the pricing kernel. If the KOV risk premium is a manifestation of the premium attached to risk in time-varying physical kurtosis-of-volatility, then the results indicate that investors are willing to pay to hedge against fluctuations in the "tailedness" of the physical probability distribution of aggregate volatility, or, fluctuations in the expected extremeness of volatility movements. The negative price of risk would also imply that the marginal value of wealth is higher in states with higher likelihood of extreme volatility movements.

However, because the change in risk-neutral KOV also reflect the change in the pricing kernel, the following alternative interpretation cannot be ruled out.[9] The triangular relation among physical probabilities, risk-neutral probabilities, and pricing kernel implies that a more pronounced U-shape of the volatility projection of the pricing kernel is associated with higher risk-neutral KOV. If the negative KOV risk premium can largely be attributed to the premium attached to risk of time-varying U-shapedness of the pricing kernel, then the results presented in this paper indicate that both high VIX state claims and low VIX state

[9]The fact that aggregate volatility is measured by VIX, which is also computed as the square root of the "risk-neutral" expectated average variance of the logged market return, allows even more interpretations.

claims are even more expensive when investors are in the bad state. A detailed analysis of the KOV risk premium is an interesting question for future research.

5 Conclusion

This study shows that the exposure to expected extremeness of movements in aggregate volatility, measured by KOV, is a cross-sectional determinant of stock returns. After a careful examination of the possibility that the negative association between KOV exposure and stock returns can be explained by other risk variables and stock characteristics, I find a robust and significant negative KOV risk premium. The study makes contributions to several strands of literature by demonstrating the importance of the higher-order aggregate volatility risk in explaining stock returns.

References

Agarwal, V., Y. E. Arisoy, and N. Y. Naik (2017). Volatility of aggregate volatility and hedge fund returns. *Journal of Financial Economics 125*(3), 491–510.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial mMarkets 5*(1), 31–56.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *The Journal of Finance 61*(1), 259–299.

Bakshi, G. and N. Kapadia (2003). Delta-hedged gains and the negative market volatility risk premium. *The Review of Financial Studies 16*(2), 527–566.

Bakshi, G., N. Kapadia, and D. Madan (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *The Review of Financial Studies 16*(1), 101–143.

Bakshi, G. and D. Madan (2000). Spanning and derivative-security valuation. *Journal of Financial Economics 55*(2), 205–238.

Bali, T. G. and H. Zhou (2016). Risk, uncertainty, and expected returns. *Journal of Financial and Quantitative Analysis 51*(3), 707–735.

Bansal, R., D. Kiku, and A. Yaron (2012). An empirical evaluation of the long-run risks model for asset prices. *Critical Finance Review 1*(1), 183–221.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance 59*(4), 1481–1509.

Bollerslev, T., G. Tauchen, and H. Zhou (2009). Expected stock returns and variance risk premia. *The Review of Financial Studies 22*(11), 4463–4492.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance 52*(1), 57–82.

Chabi-Yo, F. (2012). Pricing kernels with stochastic skewness and volatility risk. *Management Science 58*(3), 624–640.

Chang, B. Y., P. Christoffersen, and K. Jacobs (2013). Market skewness risk and the cross section of stock returns. *Journal of Financial Economics 107*(1), 46–68.

Cheng, I.-H. (2019). The vix premium. *The Review of Financial Studies 32*(1), 180–227.

Cremers, M., M. Halling, and D. Weinbaum (2015). Aggregate jump and volatility risk in the cross-section of stock returns. *The Journal of Finance 70*(2), 577–614.

Dittmar, R. F. (2002). Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns. *The Journal of Finance 57*(1), 369–403.

Drechsler, I. and A. Yaron (2011). What's vol got to do with it. *The Review of Financial Studies 24*(1), 1–45.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics 33*(1), 3–56.

Fama, E. F. and K. R. French (1997). Industry costs of equity. *Journal of Financial Economics 43*(2), 153–193.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of financial economics 116*(1), 1–22.

Fama, E. F. and K. R. French (2018). Choosing factors. *Journal of financial economics 128*(2), 234–252.

Fama, E. F. and J. D. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy 81*(3), 607–636.

Frazzini, A. and L. H. Pedersen (2014). Betting against beta. *Journal of financial economics 111*(1), 1–25.

Friend, I. and R. Westerfield (1980). Co-skewness and capital asset pricing. *The Journal of Finance 35*(4), 897–913.

Harvey, C. R. and A. Siddique (2000). Conditional skewness in asset pricing tests. *The Journal of Finance 55*(3), 1263–1295.

Hollstein, F. and M. Prokopczuk (2018). How aggregate volatility-of-volatility affects stock returns. *The Review of Asset Pricing Studies 8*(2), 253–292.

Hou, K., H. Mo, C. Xue, and L. Zhang (2021). An augmented q-factor model with expected growth. *Review of Finance 25*(1), 1–41.

Hou, K. and T. J. Moskowitz (2005). Market frictions, price delay, and the cross-section of expected returns. *The Review of Financial Studies 18*(3), 981–1020.

Hou, K., C. Xue, and L. Zhang (2015). Digesting anomalies: An investment approach. *The Review of Financial Studies 28*(3), 650–705.

Huang, D., C. Schlag, I. Shaliastovich, and J. Thimme (2019). Volatility-of-volatility risk. *Journal of Financial and Quantitative Analysis 54*(6), 2423–2452.

Kostopoulos, D., S. Meyer, and C. Uhr (2021). Ambiguity about volatility and investor behavior. *Journal of Financial Economics*.

Kraus, A. and R. H. Litzenberger (1976). Skewness preference and the valuation of risk assets. *The Journal of Finance 31*(4), 1085–1100.

Lim, K.-G. (1989). A new test of the three-moment capital asset pricing model. *Journal of Financial and Quantitative Analysis 24*(2), 205–216.

Lu, Z. and S. Murray (2019). Bear beta. *Journal of Financial Economics 131*(3), 736–760.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica*, 867–887.

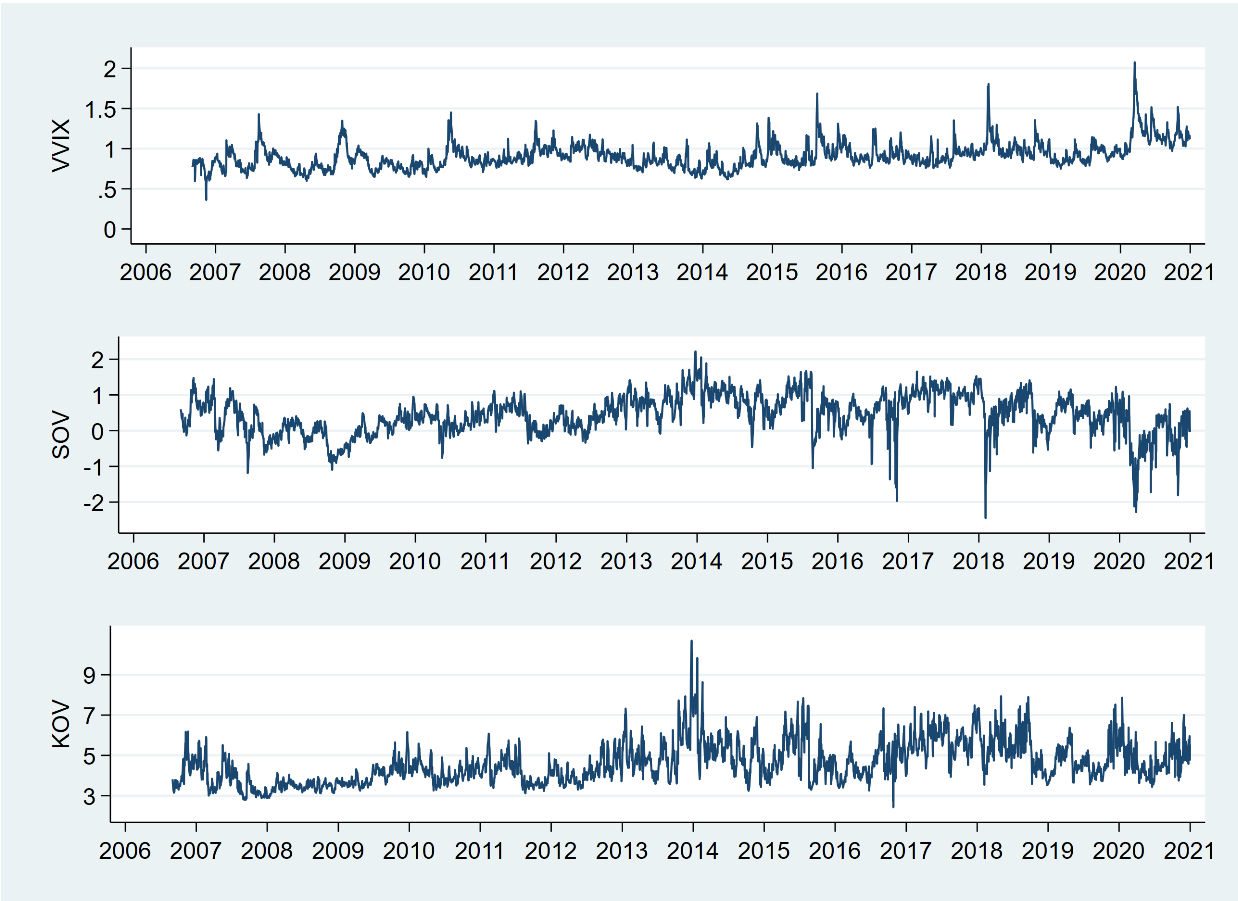
Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–708.

Sears, R. S. and K. J. Wei (1985). Asset pricing, higher moments, and the market risk premium: A note. *The Journal of Finance 40*(4), 1251–1253.

Song, Z. and D. Xiu (2016). A tale of two option markets: Pricing kernels and volatility risk. *Journal of Econometrics 190*(1), 176–196.

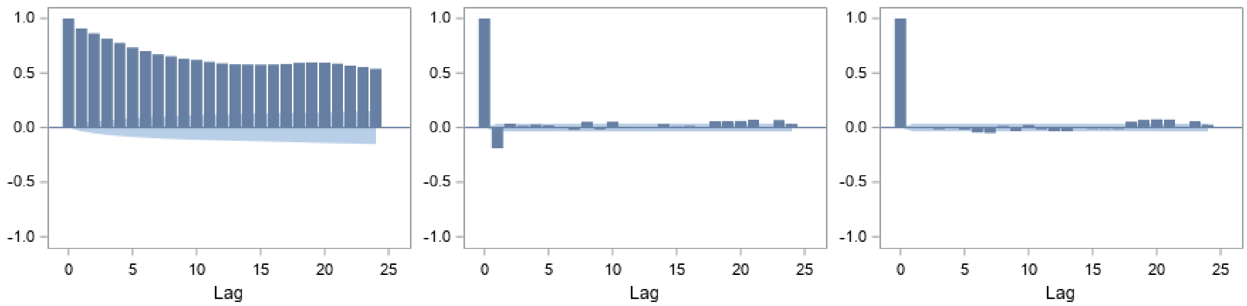
Figures and Tables

Figure 1: Daily Option-Implied Moments of VIX



Notes: This figure plots the daily option-implied volatility, skewness, and kurtosis of VIX. The moments are estimated using daily VIX option prices data. The data come from OptionMetrics Ivy DB.

Figure 2: Autocorrelation functions of KOV



Notes: This figure plots the autucorrelation functions of the original time series, the daily first differences, and the daily ARMA(1,1) residuals of KOV. The colored areas around the horizontal axis indicate 95% confidence intervals.

Table 1: $\beta^{\Delta KOV}$-sorted portfolios returns

Value	1	2	3	4	5	$5-1$
Excess Return	1.33	1.18	1.01	1.01	0.93	-0.40**
	(2.02)	(2.26)	(2.19)	(2.03)	-1.73	(-2.13)
CAPM α	0.06	0.06	-0.03	-0.10	-0.34	-0.40**
	(0.16)	(0.31)	(-0.17)	(-0.58)	(-1.37)	(-2.01)
FF3 α	0.18	0.18	0.10	0.02	-0.21	-0.39**
	(1.03)	(1.19)	(0.95)	(0.23)	(-1.63)	(-2.26)
FFC α	0.18	0.17	0.09	0.02	-0.22	-0.40**
	(1.21)	(1.37)	(0.95)	(0.17)	(-1.88)	(-2.11)
FF5 α	0.28	0.25	0.15	0.10	-0.12	-0.40**
	(1.35)	(1.56)	(1.37)	(1.13)	(-0.81)	(-2.08)
FF6 α	0.25	0.22	0.13	0.09	-0.14	-0.38**
	(1.18)	(2.06)	(1.54)	(1.05)	(-1.17)	(-2.15)
$q4$ α	0.40	0.28	0.16	0.09	-0.05	-0.45**
	(1.92)	(2.32)	(1.97)	(1.09)	(-0.46)	(-2.25)
$q5$ α	0.50	0.35	0.23	0.15	0.08	-0.42**
	(2.56)	(3.65)	(3.08)	(2.23)	(0.78)	(-2.35)
Pre-formation $\beta^{\Delta KOV}$	-1.34	-0.45	0.01	0.47	1.37	
Post-formation $\beta^{\Delta KOV}$	-0.67	-0.23	0.01	0.22	0.69	1.36***
	(-6.20)	(-5.32)	(0.41)	(5.14)	(6.34)	(6.36)

Notes: This table presents the average month-$(m+1)$ excess returns and alphas of the $\beta_{\Delta KOV}$ quintile portfolios. At the end of month m, quintile portfolios are formed by sorting stocks based on $\beta_{\Delta KOV}$, where $\beta_{\Delta KOV}$ quintile five portfolio comprises stocks with the highest values of $\beta_{\Delta KOV}$ and $\beta_{\Delta KOV}$ quintile one portfolio comprises stocks with the lowest values of $\beta_{\Delta KOV}$. Stocks are weighted equally within each of the $\beta_{\Delta KOV}$ quintile portfolios. The column labeled "$5-1$" presents the average excess returns and alphas for a zero-investment portfolio $(5-1)$ that is long the $\beta_{\Delta KOV}$ quintile five portfolio and short the $\beta_{\Delta KOV}$ quintile one portfolio. Alphas (α) are obtained using the capital asset pricing model (CAPM), Fama and French (1993) three-factor model (FF3), Fama and French (1993) and Carhart (1997) four-factor model (FFC), Fama and French (2015) five-factor model (FF5), Fama and French (2018) six-factor model (FF6), Hou et al. (2015) q-factor model ($q4$), and Hou et al. (2021) augmented q-factor model ($q5$). t-statistics adjusted following Newey and West (1987) using 12 lags are reported in parentheses. The row labeled "Pre-formation $\beta^{\Delta KOV}$" shows the average of the values of pre-formation $\beta_{\Delta KOV}$ for each of the portfolios. The row labeled "Post-formation $\beta^{\Delta KOV}$" reports the post-formation $\beta_{\Delta KOV}$, calculated as the estimated coefficient of ΔKOV innovations obtained from a regression of the post-formation five-day excess returns of each portfolio on the contemporaneous ΔKOV and other factors, as in equation (1). t-statistics reported in parentheses for the post-formation sensitivities are adjusted following Newey and West (1987) using 22 lags. ***, **, and * are used to indicate statistical significance of the average excess return and alphas of the $\beta_{\Delta KOV}$ $5-1$ portfolio at the 1%, 5%, and 10% levels, respectively.

Table 2: $\beta^{\Delta KOV}$-sorted portfolios risk characteristics

Value	1	2	3	4	5	$5-1$
Pre-formation β^{MKT}	1.24	1.00	0.91	0.90	1.00	-0.24***
	(25.51)	(42.59)	(57.24)	(35.50)	(18.78)	(-2.71)
Pre-formation $\beta^{\Delta VIX}$	0.07	0.01	-0.02	-0.03	-0.04	-0.11*
	(2.60)	(0.85)	(-1.98)	(-2.55)	(-1.29)	(-1.92)
Pre-formation $\beta^{\Delta VVIX}$	-0.01	-0.00	0.00	0.00	0.01	0.02
	(-1.15)	(-0.31)	(0.79)	(0.26)	(1.17)	(1.21)
Pre-formation $\beta^{\Delta VRP}$	-0.03	-0.02	0.00	0.03	0.03	0.05
	(-1.34)	(-0.98)	(0.38)	(1.19)	(1.03)	(1.43)

Notes: This table presents the risk characteristics of the $\beta_{\Delta KOV}$ quintile portfolios. At the end of month m, quintile portfolios are formed by sorting stocks based on $\beta_{\Delta KOV}$, where $\beta_{\Delta KOV}$ quintile five portfolio comprises stocks with the highest values of $\beta_{\Delta KOV}$ and $\beta_{\Delta KOV}$ quintile one portfolio comprises stocks with the lowest values of $\beta_{\Delta KOV}$. Stocks are weighted equally within each of the $\beta_{\Delta KOV}$ quintile portfolios. The column labeled "$5-1$" presents the risk characteristics for a zero-investment portfolio $(5-1)$ that is long the $\beta_{\Delta KOV}$ quintile five portfolio and short the $\beta_{\Delta KOV}$ quintile one portfolio. Each row presents the average coefficient estimates on each risk variable obtained from rolling window regressions of pre-formation five-day excess returns on the contemporaneous changes in the risk variable and other factors, as described in Section 4.2. t-statistics adjusted following Newey and West (1987) using 12 lags are reported in parentheses. ***, **, and * are used to indicate statistical significance of the average coefficient estimates of the $\beta_{\Delta KOV}$ $5-1$ portfolio at the 1%, 5%, and 10% levels, respectively.

Table 3: Bivariate $\beta^{\Delta KOV}$-sorted portfolios

Portfolio	Value	β^{CAPM}	$\beta^{\Delta VIX}$	$\beta^{\Delta VVIX}$	$\beta^{\Delta VRP}$
$\beta^{\Delta KOV}$ 1	Excess return	1.30	1.34	1.29	1.33
		(1.98)	(2.05)	(1.97)	(1.94)
$\beta^{\Delta KOV}$ 2	Excess return	1.16	1.19	1.18	1.07
		(2.17)	(2.30)	(2.21)	(2.04)
$\beta^{\Delta KOV}$ 3	Excess return	1.10	1.01	1.04	1.03
		(2.15)	(2.10)	(2.24)	(2.04)
$\beta^{\Delta KOV}$ 4	Excess return	0.94	1.06	0.96	0.98
		(1.71)	(2.12)	(1.99)	(1.86)
$\beta^{\Delta KOV}$ 5	Excess return	0.84	0.87	0.91	0.91
		(1.05)	(1.49)	(1.67)	(1.15)
$\beta^{\Delta KOV}$ $5-1$	Excess return	-0.46**	-0.47**	-0.38**	-0.42**
		(-2.16)	(-2.25)	(-2.13)	(-2.33)
$\beta^{\Delta KOV}$ $5-1$	CAPM α	-0.46**	-0.49**	-0.39**	-0.44**
		(-2.51)	(-2.44)	(-2.25)	(-2.37)
$\beta^{\Delta KOV}$ $5-1$	FF3 α	-0.47***	-0.49**	-0.39**	-0.43**
		(-2.64)	(-2.45)	(-2.28)	(-2.42)
$\beta^{\Delta KOV}$ $5-1$	FFC α	-0.47**	-0.49**	-0.40**	-0.42**
		(-2.57)	(-2.49)	(-2.31)	(-2.41)
$\beta^{\Delta KOV}$ $5-1$	FF5 α	-0.45**	-0.51**	-0.42**	-0.42**
		(-2.56)	(-2.52)	(-2.33)	(-2.39)
$\beta^{\Delta KOV}$ $5-1$	FF6 α	0.46**	-0.50**	-0.42**	-0.43**
		(-2.47)	(-2.56)	(-2.35)	(-2.44)
$\beta^{\Delta KOV}$ $5-1$	$q4$ α	-0.48**	-0.53***	-0.47**	-0.45***
		(-2.35)	(-2.98)	(-2.41)	(-3.03)
$\beta^{\Delta KOV}$ $5-1$	$q5$ α	-0.50***	-0.56***	-0.49**	-0.47***
		(-2.65)	(-3.23)	(-2.46)	(-3.11)

Notes: This table presents the average month-$(m+1)$ excess returns and alphas of the bivariate $\beta_{\Delta KOV}$ quintile portfolios. The control variables are β^{MKT}, $\beta^{\Delta VIX}$, $\beta^{\Delta VVIX}$, and $\beta^{\Delta VRP}$. At the end of month m, stocks are sorted into quintile portfolios based on each of the control variables. Within eacn control variable quintile, stocks are sorted again into quintile portfolios based on $\beta^{\Delta KOV}$. Stocks are weighted equally within each of the resulting 25 portfolios. Five portfolios with the same rank of $\beta^{\Delta KOV}$ are weighted equally again and the resulting five portfolios are referred to as the bivariate $\beta^{\Delta KOV}$ quintile portfolios. The column labeled "$5-1$" presents the average excess returns and alphas for each zero-investment portfolio $(5-1)$ that is long the bivariate $\beta_{\Delta KOV}$ quintile five portfolio and short the bivariate $\beta_{\Delta KOV}$ quintile one portfolio. Alphas (α) are obtained using the capital asset pricing model (CAPM), Fama and French (1993) three-factor model (FF3), Fama and French (1993) and Carhart (1997) four-factor model (FFC), Fama and French (2015) five-factor model (FF5), Fama and French (2018) six-factor model (FF6), Hou et al. (2015) q-factor model ($q4$), and Hou et al. (2021) augmented q-factor model ($q5$). t-statistics adjusted following Newey and West (1987) using 12 lags are reported in parentheses. ***, **, and * are used to indicate statistical significance of the average excess returns and alphas of the bivariate $\beta_{\Delta KOV}$ $5-1$ portfolios at the 1%, 5%, and 10% levels, respectively.

Table 4: Price of KOV risk: characteristics controlled

	Future excess returns						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\beta^{\Delta KOV}$	-0.17***	-0.20***	-0.23***	-0.24***	-0.24***	-0.23***	-0.19***
	(-2.73)	(-2.96)	(-3.21)	(-3.22)	(-3.31)	(-3.24)	(-2.83)
β^{MKT}		-0.30	-0.40	-0.35	-0.34	-0.44	-0.44
		(-0.63)	(-0.67)	(-0.69)	(-0.65)	(-0.72)	(-0.76)
ME			0.05***	0.05***	0.04***	0.05***	0.06***
			(3.58)	(3.61)	(3.26)	(3.46)	(3.51)
BM			-0.04***	-0.03***	-0.03***	-0.03***	-0.03***
			(-3.27)	(-3.15)	(-2.95)	(-2.72)	(-2.88)
MOM				-0.06		-0.05	-0.05
				(-1.31)		(-1.44)	(-1.56)
Y					1.15***	1.02***	1.12***
					(4.13)	(3.74)	(3.86)
INV					-0.55***	-0.47***	-0.51***
					(-3.38)	(-3.19)	(-3.26)
$ILLIQ$							0.00
							(4.11)
Intercept	0.51	0.87*	0.95**	1.12**	1.03**	1.08**	0.96**
	(0.93)	(1.81)	(2.12)	(2.23)	(2.25)	(2.31)	(2.18)
Adj. R^2 (percent)	0.6	3.3	5.7	6.8	6.7	7.1	7.4
No. of observations	3891	3891	3891	3891	3891	3891	3891

Notes: This table presents the time series averages of the cross-sectional regressions of month-$(m+1)$ excess stock returns on month-m $\beta^{\Delta KOV}$ and firm characteristics. ME is the log of the market capitalization. BM is the log of the ratio of the book value of equity to the market capitalization. MOM is the 11-month stock return in months $m-11$ through $m-1$. The profitability measure Y and the investment measure INV are calculated following Fama and French (2015). The illiquidity measure $ILLIQ$ is calculated following Amihud (2002). t-statistics adjusted following Newey and West (1987) using 12 lags are presented in parentheses. The average adjusted R-squared (Adj. R^2) and the average number of observations are also reported. ***, **, and * are used to indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 5: Price of KOV risk: betas controlled

	Future excess returns					
	(1)	(2)	(3)	(4)	(5)	(6)
$\beta^{\Delta KOV}$	-0.17***	-0.20***	-0.25***	-0.25***	-0.26***	-0.24***
	(-2.73)	(-2.96)	(-3.36)	(-3.41)	(-3.31)	(-3.13)
β^{MKT}		-0.30	-0.42	-0.35	-0.31	-0.41
		(-0.63)	(-0.69)	(-0.71)	(-0.66)	(-0.70)
β^{SMB}			0.13	0.09	0.08	0.11
			(0.97)	(1.12)	(1.17)	(1.05)
β^{HML}			-0.11	-0.17	-0.17	-0.17
			(-1.31)	(-1.26)	(-1.21)	(-1.33)
β^{UMD}				-0.65		-0.48
				(-1.28)		(-1.13)
β^{RMW}					0.19**	0.18**
					(2.34)	(2.17)
β^{CMA}					-0.14	-0.13
					(-1.39)	(-1.27)
Intercept	0.51	0.87*	0.95**	1.12**	1.03**	1.08**
	(0.93)	(1.81)	(2.12)	(2.23)	(2.25)	(2.31)
Adj. R^2 (percent)	0.6	3.3	4.9	5.5	5.5	5.8
No. of observations	3891	3891	3891	3891	3891	3891

Notes: This table presents the time series averages of the cross-sectional regressions of month-$(m+1)$ excess stock returns on month-m $\beta^{\Delta KOV}$ and factor betas. Included betas are β^{SMB}, β^{HML}, β^{UMD}, β^{RMW}, and β^{CMA}, the estimated coefficients of SMB, HML, UMD, RMW, and CMA obtained from a twelve-month rolling window regression of daily excess stock returns on the contemporaneous returns of the six factors of Fama and French (2018). t-statistics adjusted following Newey and West (1987) using 12 lags are presented in parentheses. The average adjusted R-squared (Adj. R^2) and the average number of observations are also reported. ***, **, and * are used to indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 6: Price of KOV risk: risk characteristics controlled

	Future excess returns							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\beta^{\Delta KOV}$	-0.22***	-0.23***	-0.24***	-0.26***	-0.25***	-0.26***	-0.21***	-0.23***
	(-3.21)	(-3.35)	(-3.42)	(-3.55)	(-3.56)	(-3.31)	(-2.86)	(-3.18)
$\beta^{\Delta VIX}$	-0.27*	-0.31**					-0.23	-0.25
	(-1.74)	(-2.18)					(-1.41)	(-1.49)
$\beta^{\Delta VVIX}$			-0.53**	-0.49**			-0.51**	-0.48*
			(-2.31)	(-2.07)			(-2.17)	(-1.85)
$\beta^{\Delta VRP}$					0.03**	0.04**	0.02*	0.03**
					(2.23)	(2.41)	(1.86)	(2.17)
Controls	Char	Beta	Char	Beta	Char	Beta	Char	Beta
Adj. R^2 (percent)	7.4	5.8	7.5	6.0	7.5	5.9	7.9	7.4
Observations	3891	3891	3891	3891	3891	3891	3891	3891

Notes: This table presents the time series averages of the cross-sectional regressions of month-$(m+1)$ excess stock returns on month-m $\beta^{\Delta KOV}$ and risk variables that are plausibly similar to KOV risk. Included risk variables are $\beta^{\Delta VIX}$, $\beta^{\Delta VVIX}$, and $\beta^{\Delta VRP}$, the estimated coefficients of ΔVIX, $\Delta VVIX$, and ΔVRP obtained from rolling window regressions of daily excess stock returns on the contemporaneous changes in the risk variable and other factors, as described in Section 4.2. t-statistics adjusted following Newey and West (1987) using 12 lags are presented in parentheses. The average adjusted R-squared (Adj. R^2) and the average number of observations are also reported. ***, **, and * are used to indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 7: Robustness Checks

	1	2	3	4	5	5 − 1
Panel A. Value-Weighted Returns						
All	0.07	0.00	-0.11	-0.03	-0.11	-0.18
	(0.48)	(0.03)	(-1.83)	(-0.77)	(-0.86)	(-1.32)
Excl. >=1%	0.01	0.04	-0.02	-0.08	-0.27	-0.28*
	(0.09)	(1.01)	(-0.38)	(-1.37)	(-2.32)	(-1.76)
Excl. >=5%	0.02	0.03	-0.03	-0.11	-0.31	-0.33*
	(0.16)	(0.47)	(-0.58)	(-1.83)	(-2.87)	(-1.91)
Excl. >=10%	0.02	0.05	-0.01	-0.13	-0.33	-0.35**
	(0.12)	(0.76)	(-0.21)	(-2.03)	(-3.05)	(-2.07)
Excl. >=20%	0.03	0.06	-0.03	-0.16	-0.41	-0.43**
	(0.18)	(0.87)	(-0.47)	(-2.13)	(-3.78)	(-2.16)
Panel B. Estimation Period						
Three-month beta	0.05	0.09	0.02	-0.18	-0.29	-0.34*
	(0.38)	(1.04)	(0.34)	(-2.29)	(-2.28)	(-1.71)
Six-month beta	0.07	0.04	0.05	-0.11	-0.34	-0.41**
	(0.49)	(0.32)	(0.53)	(-2.27)	(-3.41)	(-2.27)
Nine-month beta	0.05	0.09	0.01	-0.17	-0.44	-0.49**
	(0.38)	(0.91)	(0.25)	(-2.38)	(-4.37)	(-2.36)
Panel C. Data Filtering						
NYSE/Amex/Nasdaq	0.01	0.06	0.04	-0.13	-0.45	-0.46**
	(0.14)	(0.88)	(0.52)	(-2.31)	(-4.42)	(-2.18)
Excl. micro-cap	0.05	0.11	-0.04	-0.09	-0.32	-0.36*
	(0.73)	(1.13)	(-0.51)	(-1.85)	(-2.81)	(-1.86)
Panel D. Time Series Model						
MA(1) to first differences	0.02	0.06	-0.08	-0.14	-0.42	-0.44**
	(0.38)	(1.05)	(-1.13)	(-2.16)	(-3.97)	(-2.26)

Notes: Each panel of this table reproduces the univariate portfolio analysis results by employing alternative empirical setups. Fama and French (2018) six-factor alphas are reported for each of the portfolios formed by sorting stocks based on $\beta_{\Delta KOV}$ and for the zero-investment portfolio $(5-1)$. Panel A presents the results of using a value-weighting scheme. The results are produced for the full sample (All) and for subsamples that exclude the 1%, 5%, 10%, and 20% largest stocks (as measured by their market capitalization at the end of each month). Panel B presents the results from using rolling three-month, six-month, and nine-month KOV beta estimation periods. Panel C presents the results from using a subsample that includes stocks listed in NYSE, Amex, and Nasdaq only and a subsample that excludes stocks with a market capitalization below the 10th percentile. Panel D presents the results of using an alternative time series model. I use the sum five daily first differences of the moments as the innovation in the moments of the market return and aggregate volatility. ***, **, and * are used to indicate statistical significance of the alphas of the $\beta_{\Delta KOV}$ $5-1$ portfolios at the 1%, 5%, and 10% levels, respectively.

Appendix. Extracting Option-Implied Moments

In the following description, $S(t)$ is either time-t S&P 500 index or time$t$ price of VIX futures that matures at time T multiplied by $e^{-r(T-t)}$ where r is the constant risk-free rate.

Let $R(t,T) = \ln S(T) - \ln S(t)$. When S is the VIX futures price, $S(T)$ is equal to the spot VIX at time T. I use the method of Bakshi et al. (2003) to extract the following moments from option prices:

$$VOL(t,T)\,(\text{or } VOV(t,T)) = \left\{ E_t^Q\left[\left(R(t,T) - E_t^Q\left[R(t,T)\right] \right)^2 \right] \right\}^{1/2}, \tag{10}$$

$$SKEW(t,T)\,(\text{or } SOV(t,T)) = \frac{E_t^Q\left[\left(R(t,T) - E_t^Q\left[R(t,T)\right] \right)^3 \right]}{\left\{ E_t^Q\left[\left(R(t,T) - E_t^Q\left[R(t,T)\right] \right)^2 \right] \right\}^{3/2}}, \tag{11}$$

$$KURT(t,T)\,(\text{or } KOV(t,T)) = \frac{E_t^Q\left[\left(R(t,T) - E_t^Q\left[R(t,T)\right] \right)^4 \right]}{\left\{ E_t^Q\left[\left(R(t,T) - E_t^Q\left[R(t,T)\right] \right)^2 \right] \right\}^{2}}, \tag{12}$$

where $E_t^Q[\cdot]$ is the expected value under the risk-neutral measure. Expanding the powers inside the expectations, these moments are expressed as functions of

$$E_t^Q\left[R(t,T)\right], E_t^Q\left[R^2(t,T)\right], E_t^Q\left[R^3(t,T)\right], E_t^Q\left[R^4(t,T)\right] \tag{13}$$

or

$$E_t^Q\left[e^{-r(T-t)}R(t,T)\right], E_t^Q\left[e^{-r(T-t)}R^2(t,T)\right], E_t^Q\left[e^{-r(T-t)}R^3(t,T)\right], E_t^Q\left[e^{-r(T-t)}R^4(t,T)\right]. \tag{14}$$

The quantities in equation (14) can be interpreted as prices of the contracts whose payoffs,

$H[S]$ ($S = S(T)$ for notational convenience), are

$$H[S] = \begin{cases} R(t,T), \\ R^2(t,T), \\ R^3(t,T), \\ R^4(t,T). \end{cases} \tag{15}$$

Bakshi and Madan (2000) demonstrate that any twice continuously differentiable payoff function, $H[S]$, can be spanned by a portfolio of risk-free bonds, the underlying asset, and out-of-the-money (OTM) calls and puts

$$H[S] = H[\overline{S}] + (S - \overline{S})H_S[\overline{S}] + \int_{\overline{S}}^{\infty} H_{SS}[K](S-K)^+ \, dK + \int_0^{\overline{S}} H_{SS}[K](K-S)^+ \, dK. \tag{16}$$

The prices of these contracts are

$$\begin{aligned} E_t^Q \left\{ e^{-r(T-t)} H[S] \right\} &= (H[\overline{S}] - \overline{S} H_S[\overline{S}]) e^{-r(T-t)} + H_S[\overline{S}] S(t) \\ &\quad + \int_{\overline{S}}^{\infty} H_{SS}[K] C(t, T-t; K) \, dK + \int_0^{\overline{S}} H_{SS}[K] P(t, T-t; K) \, dK, \end{aligned} \tag{17}$$

where $C(t, \tau; K)$ and $P(t, \tau; K)$ are prices of European call and put options with time to maturity τ and strike price K. As a result, the prices of the contracts can be calculated using the prices of a risk-free zero coupon bond, the S&P 500 index (or the discounted VIX futures price), and a series of OTM calls and puts written on the index (or VIX). I use this methodology to first calculate the quantities in equation (14) and then use these quantities to calculate the option-implied moments in equations (10), (11), and (12). I choose $\overline{S} = U(t)$ when I use equation (17) to calculate the quantities in equation (14).

I obtain the data on S&P 500 index options and VIX options from OptionMetrics Ivy DB. I define the price of an option as the average of the bid and ask quotes. I filter out options with zeros bids and those with quotes that do not satisfy standard no-arbitrage conditions. Finally, I eliminate in-the-money options because they are less liquid than out-of-the-money and at-the-money options. Specifically, I eliminate call options with strike prices of less than 97% of the underlying asset price ($K/S < 0.97$) and put options with strike prices of more than 103% of the underlying asset price ($K/S > 1.03$). I estimate only the moments for days that have at least two OTM call prices and two OTM put prices available.

Because I do not have a continuum of strike prices, I estimate the call and put prices for strike prices between adjacent strike prices using cubic splines. Specifically, for each maturity, I interpolate implied volatilities using a cubic spline across moneyness levels (K/S) to obtain a continuum of implied volatilities. For moneyness levels below or above the available moneyness level in the market, I use the implied volatility of the lowest or highest available strike price. I obtain a fine grid of one thousand implied volatilities for moneyness levels between 0.01% and 300% and these implied volatilities are converted again into call and put prices. Integrals in equation (17) are calculated using trapezoidal numerical integration. Linear interpolation of the moments across maturities is used to calculate the moments at a fixed 30-day horizon.