Term structure and risk premiums of commodity futures with linear regressions

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Abstract

We apply the regression-based affine term structure model developed by Adrian et al. (2013) to estimate the term structure of commodity futures. This model has the advantage of a simple and fast algorithm, can accommodate a variety of observable and unspanned factors, and can be applied to daily and even real-time observations. The estimated results show that the model appropriately captures the time-series variation across different maturities and exhibits satisfactory performance in capturing cross-sectional variation for specific months. Furthermore, we investigate the relationship between existing commodity risk factor returns and the risk premiums inferred by the model. Our analysis reveals that different risk factor returns explain spot and term premiums in different ways. Therefore, by using the model's advantages, we can better understand the term structure and risk premiums in commodity futures.

Keywords: Commodity futures, Affine term structure, Commodity risk premiums, Asset prcing
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1. Introduction

Commodity futures are one of the oldest derivatives and are economically significant because their underlying assets are based on raw materials for production. Over the past two decades, the size of commodity futures markets has rapidly increased. The growth of commodity futures markets can be attributed to the influx of financial institutions, known as the financialization of commodities, which has increased trading volume and open interest in commodity futures markets, resulting in significant changes in price volatility. [1] The expansion of institutional investors draws much attention to the importance of risk management and derivatives pricing due to the expansion of commodities included in portfolios and the growth of related derivatives and exchange-traded products(ETPs). [2] This paper proposes a simple and efficient method for estimating the term structure of commodity futures, which is essential for risk management and derivatives valuation.

As demand and interest in commodities continue to grow, the necessity for estimating the term structure for risk management and related derivatives valuation also increases. Neuberger (1999) and Veld-Merkoulova and De Roon (2003) highlight that investors with long-term exposure to commodities may encounter rollover risk when using short-term futures contracts. They propose methodologies using longer-term futures contracts to mitigate such risks by estimating the prices of medium to long-term futures through the estimation of commodity term structures, thereby providing more precise and cost-effective methods. Furthermore, financial institutions issuing various derivatives often hedge by diversifying across multiple maturities rather than relying solely on single-maturity (usually nearby contracts), which can reduce hedging costs and may impact even longer contracts (Henderson

[1] See e.g. Tang and Xiong (2012); Cheng et al. (2015); Henderson et al. (2015); Sockin and Xiong (2015); Basak and Pavlova (2016); Brogaard et al. (2018); Baker (2021); Goldstein and Yang (2022); Ready and Ready (2022); Da et al. (2023); Kang et al. (2023). As reported by Kang et al. (2023), the financialization has begun around 2004. They report that while the proportion of commercial traders has remained relatively stable, the proportion of non-commercial speculators has increased from 18.89% before financialization to 37.71% afterward.

[2] Henderson et al. (2015) analyze commodity-linked notes(CLNs) and suggest that the hedging demand from financial institutions can affect long-term commodity futures prices.

et al., 2015). Several studies report that many firms, particularly in the energy industry, use derivatives such as swaps, options, or collars to hedge against the risks associated with their commodities (Acharya et al., 2013; Mixon et al., 2018). Therefore, accurate term structure estimation is essential for analyzing medium to long-term contracts that can be utilized to reduce hedging costs and provide information necessary for evaluating derivatives like swaps or calendar options. Notably, as the trading volume declines and liquidity diminishes with longer contract maturities in most commodity futures contracts, the price discovery of longer contracts may be problematic. Therefore, estimating the term structure of commodity futures aims to accurately construct prices for unobserved intervals using available futures prices and estimate prices beyond the observed periods.

The domain of term structure models is perhaps most extensively developed in the context of interest rates [3]. Additionally, term structure models have been developed in other asset classes such as equity and foreign exchange [4] Modeling commodity futures term structure has also received substantial attention. [5] Analogous to the assumptions of stochastic movements of multiple state variables (e.g., short rate) for interest rates, various factors such as the spot price, convenience yield, interest rate, and long-term price are considered either individually or in combination to derive commodity futures term structure. Among these, the earliest models assume the stochastic process of spot prices to model the term structure of commodities. The most well-known model of this type, proposed by Brennan and Schwartz (1985), assumes geometric Brownian motion for spot prices to develop an affine term structure model. Furthermore, other studies have derived models by assuming mean-reverting processes of spot price (Schwartz, 1997; Routledge et al., 2000). In addition to assuming stochastic movements of spot prices, some models have incorporated stochastic convenience yield and/or interest rates, resulting in two- or three-factor models. For ex-

[3] See Piazzesi (2010) for a review.

[4] See Lettau and Wachter (2011); Binsbergen et al. (2012); Van Binsbergen et al. (2013); Van Binsbergen and Koijen (2017); Bansal et al. (2021); Ulrich et al. (2022) for the equity and see Backus et al. (2001); Lustig et al. (2011, 2019) for the foreign exchange.

[5] Lautier (2005) surveys term structure models of commodity futures.

ample, Gibson and Schwartz (1990) propose a two-factor model considering stochastic spot prices and convenience yield. Additionally, three-factor models incorporating stochastic interest rates alongside spot prices and convenience yield have also been developed(Schwartz, 1997; Liu and Tang, 2010).

Numerous well-known academic studies exist on exponentially affine term structure models of commodity futures.[6] These conventional methodologies rely on unobservable state variables (such as spot price or convenience yields), thus resorting to parameter estimation through state space modeling with Kalman filter for term structure estimation. These methodologies may typically be time-consuming for estimation and may suffer from reduced efficiency due to assumptions regarding unobservable state variables. Dempster and Tang (2011) argue that estimation via state space form includes measurement errors, which are serially correlated and can influence parameter estimation. In contrast, Adrian et al. (2013) (hereafter ACM) propose a simple and fast ordinary least squares methodology for estimating the term structure of interest rates, claiming computational efficiency and the ability to allow small pricing errors. Moreover, their model can incorporate more factors, including observable and unspanned factors, and can be applied at daily frequency. Consequently, we apply the ACM model to the commodity futures to estimate the individual commodity futures term structure.

Adrian et al. (2013) propose a methodology to estimate the term structure of interest rates. They assume that a pricing kernel is exponentially affine, prices of risk are affine to state variables, and shocks to state variables and log yield observation errors are conditionally normally distributed. They use a regression under these assumptions to provide a simple approach for estimating the term structure of interest rates. They argue that the regression-based term structure model can reduce computing time and consider various factors. Building upon their methodology, we attempt to estimate the term structure of

[6]For further information, please refer to Schwartz (1997); Schwartz and Smith (2000); Geman and Nguyen (2005); Casassus and Collin-Dufresne (2005); Dempster et al. (2008); Liu and Tang (2010); Dempster and Tang (2011).

commodity futures using a setup analogous to that employed by ACM. To achieve this, we define excess returns for commodity futures and extract principal component analysis (PCA) factors using selected four to five maturities of each futures contract within observable maturities to estimate the term structure model of an individual commodity.

It is well established that three PCA factors are widely used for estimating the term structure of interest rates. These three PCA factors for interest rates represent the term structures's level, slope, and curvature (Litterman, 1991). Similarly, we can use PCA factors applied to commodity futures. For instance, some studies suggest that the slope of the futures curve is related to the level of inventories (Kaldor, 1939; Working, 1949; Fama and French, 1987). Additionally, the basis, which is the difference between the spot price and the contemporaneous futures price, represents the slope of the futures term structure and contains information about expected futures excess returns (Fama and French, 1987; Gorton and Rouwenhorst, 2006; De Roon et al., 1998; Gorton et al., 2012). A recent study by Boons and Prado (2019) reports that the basis-momentum shows promising results in predicting commodity spot and term premiums, and suggests that the basis-momentum is related to the slope and curvature of the futures term structure. We confirm through PCA analysis of commodity futures data that the first three PCA factors explain over 98% of the overall price or return movements. Consequently, utilizing the three factors extracted from PCA to estimate the term structure of commodity futures can be appropriate.

We confirm that the regression-based term structure model is successfully applied to commodity futures. The fitted prices estimated from the model are well-defined, demonstrating a good reflection of the term structure characteristics of individual commodities. For validation, we divide the data into in-sample and out-of-sample. The test results indicate that the pricing errors are small and economically acceptable. The time-series regressions explain 99.8% of the variation for the in-sample data and 94.3% for the out-of-sample data at each maturity. Furthermore, we analyze the relationship between observed prices for all maturities on each date and the corresponding fitted prices. Although the pricing errors from the cross-sectional regression are slightly higher than those from the time-series regression, 86.5% of the cross-sectional variation for in-sample and 68.5% for out-of-sample are explained. However, given the sensitivity of regression to outliers and the potential for noise in observed prices of longer-dated contracts, the difference between the two prices is economically insignificant. In particular, the root mean squared errors (RMSEs) for pricing errors vary across commodities, ranging from \$0.008 to \$0.144 on a log price basis. Excluding the natural gas futures contract which has the largest pricing error, the largest error in cross-sectional variation is \$0.088. Therefore, we posit that the regression-based term structure model based on the first three PCA factors effectively and errorlessly estimates the term structure of commodity futures.

Another advantage of the regression-based term structure model is that the model can provide estimates of commodity risk premiums. We examine the relationship between various risk factors, as suggested in existing literature, and the estimated risk premiums from the model. Specifically, we explore the relationships using nine risk factors—momentum, basis, basis-momentum, skewness, inflation beta, volatility, hedging pressure, order imbalance, and value as well as the S&P GSCI index return. The total risk premium is found to be associated with the S&P GSCI return, basis, and hedging pressure factors, the spot premium is related to momentum and volatility factors, and the term premium is associated with the S&P GSCI return, basis, and hedging pressure factors. Additionally, we conduct subsample analysis by dividing the entire sample into pre- and post-financialization periods. The analysis reveals that the risk factors related to each risk premium differ before and after the financialization. For instance, before the financialization, the spot premium is associated with momentum, basis-momentum, inflation beta, and volatility. However, after the financialization, only the volatility factor is significant. Furthermore, while volatility and inflation beta are significant for the term premium before, factors such as the S&P GSCI return and value become significant afterward. These findings suggest that commodity risk premiums consist of spot and term premiums, each reflecting different types of priced risks.

Moreover, the results indicate that as financialization progresses, the risk factors traditionally priced may differ from those significantly reflected in risk premiums afterward.

Our study contributes to the existing literature on the term structure of commodity futures and commodity risk premiums. First, we demonstrate the successful application of a regression-based term structure model to commodities. Unlike stocks or interest rates, observing spot prices of commodities is challenging because spot prices are formed in over-the-counter(OTC) markets. Moreover, although the inflow of institutional investors into commodity futures markets makes price information for longer maturities more observable, futures prices of longer maturities are still limited. Despite these constraints, the regression-based term structure model enables estimation with minimal errors and less computing power even for futures contracts with maturities longer than those typically used for estimation. Next, the model's advantage lies in estimating term premiums for multiple unobservable maturities, allowing for analysis thereof. From the analysis, we show that existing risk factors affect term premiums and spot premiums differently. Therefore, our study presents a significant contribution by demonstrating the simple and fast estimation of commodity term structures for individual commodities. Furthermore, this approach allows for the consideration of various risk factors and the use of granular levels of daily or intraday observations.

The rest of the paper is organized as follows. Section 2 explains the regression-based affine term structure model. Section 3 describes the commodity futures data, conducts principal component analysis, and computes risk factors and long-short portfolio returns. Section 4 presents the empirical results. Section 5 concludes.

2. Regression-based term structure model

In this section, we outline the regression-based term structure model suggested by Adrian et al. (2013) in terms of commodities futures. Assume that the a $K \times 1$ state variable X_t

follows the vector autoregressive process:

$$X_{t+1} = \mu + \Phi X_t + v_{t+1} \tag{1}$$

where the error term v_{t+1} conditionally follows a normal distribution:

$$v_{t+1}|\{X_s\}_{s=0}^{t} \sim N(0, \Sigma) \tag{2}$$

Let S_t denote the spot price of the commodity futures contract and $F_t^{(n)}$ be the futures price for delivery at time $t+n$ at time t. The cost-of-carry model implies that the futures prices equal

$$F_t^{(n)} = S_t e^{y_t^{(n)} \times n} \tag{3}$$

where $y_t^{(n)}$ is the per-period net cost of carry or basis for maturity n. Let $f_t^{(n)}$ as the log futures price and s_t as the log spot price. Given the pricing kernel M_t, we have

$$F_t^{(n)} = E_t \left[M_t F_{t+1}^{(n-1)} \right] \tag{4}$$

and the pricing kernel M_{t+1} is assumed to be exponentially affine:

$$M_{t+1} = \exp\left(-\frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\Sigma^{-1/2}v_{t+1} \right) \tag{5}$$

where λ_t is the market price of risks and has the affine form:

$$\lambda_t = \Sigma^{-1/2}(\lambda_0 + \lambda_1 X_t) \tag{6}$$

Following Gorton et al. (2012); Yang (2013); Sakkas and Tessaromatis (2020), denote $rx_{t+1}^{(n-1)}$ the log excess holding returns on a fully collateralized futures positions:

$$rx_{t+1}^{(n-1)} = f_{t+1}^{(n-1)} - f_t^{(n)} \tag{7}$$

Plugging equations (5) and (7) into equation (4), we have

$$1 = E_t \left[\exp \left(rx_{t+1}^{(n-1)} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\Sigma^{-1/2}v_{t+1} \right) \right] \tag{8}$$

Assume that $\{rx_{t+1}^{(n-1)}, v_{t+1}\}$ are jointly normally distributed. Adrian et al. (2013) find that

$$E_t[rx_{t+1}^{(n-1)}] = \beta_t^{(n-1)'}[\lambda_0 + \lambda_1 X_t] - \frac{1}{2}Var_t[rx_{t+1}^{(n-1)}] \tag{9}$$

where $\beta_t^{(n-1)'} = Cov[rx_{t+1}^{(n-1)}, v_{t+1}']\Sigma^{-1}$. Szymanowska et al. (2014) show that the expected one-period excess futures return can be expressed as the sum of the spot premium and the term premium. The spot risk premium $\pi_{s,t}$ as the expected spot return in excess of the one-period basis

$$E_t[r_{s,t+1}] = E_t[s_{t+1} - s_t] = y_t^{(1)} + \pi_{s,t} \tag{10}$$

Furthermore, a term premium $\pi_{y,t}^{(n)}$ is the expected deviation from the expectation hypothesis of the term structure of the basis satisfying

$$ny_t^{(n)} = y_t^{(1)} + (n-1)E_t[y_{t+1}^{(n-1)}] - \pi_{y,t}^{(n)} \tag{11}$$

Using the cost-of-carry relation in (3), $rx_{t+1}^{(n-1)}$ can be expressed as the sum of spot premium and the term premium

$$E_t[rx_{t+1}^{(n-1)}] = \pi_{s,t} + \pi_{y,t}^{(n)} \tag{12}$$

The unexpected return can be written as a component related to v_{t+1} and another that is conditionally orthogonal. Then, the unexpected return can be written as

$$rx_{t+1}^{(n-1)} - E_t[rx_{t+1}^{(n-1)}] = \beta_t^{(n-1)'}v_{t+1} + e_{t+1}^{(n-1)} \tag{13}$$

where $e_{t+1}^{(n-1)}$ is conditional independently and identically distributed with variance σ^2. Combining equations (9) and (13) and assuming constant β, the return generation process can be written as a vector form:

$$\mathrm{rx} = \beta(\lambda_0 \imath_T' + \lambda_1 X_-) - \frac{1}{2}(B^* \mathrm{vec}(\Sigma) + \sigma^2 \imath_N)\imath_T' + \beta' V + E \tag{14}$$

where rx denotes an $N \times T$ of excess returns, $\beta = [\beta^{(1)}\beta^{(2)} \cdots \beta^{(N)}]$ is a $K \times N$ factor loadings, $X_- = [X_0 X_1 \cdots X_{T-1}]$ is a $K \times T$ lagged pricing factors, $B^* = [\mathrm{vec}(\beta^{(1)}\beta^{(1)\prime}) \cdots \mathrm{vec}(\beta^{(N)}\beta^{(N)\prime})]'$ is an $N \times K^2$ matrix, V is a $K \times T$ innovations, E is an $N \times T$ residuals, and $\imath_N$ and $\imath_T$ are a $N \times 1$ and $T \times 1$ vector of ones.

Based on equation (14), Adrian et al. (2013) propose the three-step regression-based estimator for the parameters. First, estimate the equation (1) to get the innovation vector $\hat{V}$ and the variance-covariacne matrix $\hat{\Sigma} = \hat{V}\hat{V}'/T$. Second, estimate the following regressions to get the variance of pricing errors $\hat{\sigma}^2 = tr(\hat{E}\hat{E}')/NT$ and to construct $\hat{B}^*$:

$$\mathrm{rx} = \mathbf{a}\imath_T' + \beta'\hat{V} + \mathbf{c}X_- + E \tag{15}$$

Finally, based on parameters $\hat{\mathbf{a}}, \hat{\beta}, \mathbf{c}, \hat{\sigma}^2$, and $\hat{\Sigma}$, the coefficients of the market price of risks can cross-sectionally be estimated by

$$\hat{\lambda}_0 = (\hat{\beta}\hat{\beta}')^{-1}\hat{\beta}\left(\hat{\mathbf{a}} + \frac{1}{2}\left(\hat{B}^*\mathrm{vec}(\hat{\Sigma}) + \hat{\sigma}^2\imath_N\right)\right) \tag{16}$$

$$\hat{\lambda}_1 = (\hat{\beta}\hat{\beta}')^{-1}\hat{\beta}\hat{\mathbf{c}} \tag{17}$$

Based on the estimated parameters, the log prices of commodity futures are exponentially affine in the state variables X_t:

$$\ln F_t^{(n)} = A_n + B_n' X_t + u_t^{(n)} \tag{18}$$

Plugging equation (18) into equation (7), we find that

$$rx_{t+1}^{(n-1)} = A_{n-1} + B_{n-1}' X_{t+1} + u_{t+1}^{(n-1)} - A_n - B_n' X_t - u_t^{(n)} \tag{19}$$

Using equation (1) and (9) and equating equation (19) as equation (13), we have the following linear restrictions

$$A_n = A_{n-1} + B'_{n-1}(\mu - \lambda_0) + \frac{1}{2}(B'_{n-1}\Sigma B_{n-1} + \sigma^2) \tag{20}$$

$$B'_n = B'_{n-1}(\Phi - \lambda_1) \tag{21}$$

where A_1 and B_1 can be estimated from equation (18).

3. Data and variables

3.1. Commodity futures data

We collect data on liquid exchange-traded futures contracts from the Datastream. The sample period extends from January 1973 to December 2023. Our sample consists of 27 futures contracts across five major sectors (energy, grains & oilseeds, livestock, softs, and metals) listed on major North American exchanges including the Chicago Mercantile Exchange(CME), the Chicago Board of Trade(CBOT), the New York Mercantile Exchange (NYMEX), the Commodity Exchange(COMEX), the Intercontinental Exchange-US(ICE-US). The Datastream provides daily time series data for each futures contract, ranging from near-term to long-term maturities. We calculate monthly returns based on the final observation of each month for each futures contract. While both closing and settlement prices are commonly employed in the literature, settlement prices yield a greater number of observations, thereby providing more comprehensive price information. Therefore, we use settlement prices for our analysis. It is worth noting that the correlation between the closing and the settlement prices exceeds 95% in our sample. In unreported results, we also employ the closing prices for our analysis and the results remain unchanged. Table 1 provides detailed information about the sample.

PLEASE INSERT TABLE 1 AROUND HERE.

Among the selected 27 contracts, we use only 17 futures contracts to estimate the term structure because some futures contracts do not have enough observations. Furthermore, we employ three factors extracted from the PCA as state variables. Given our intention to use three PCA factors for estimating the term structure, we restrict our analysis to 17 futures contracts with at least six or more maturities from nearby contracts. Thus, the selected 17 futures contracts encompass a minimum of six or more maturities and have continuous monthly returns with at least 200 observations. The selected futures contracts include crude oil(CL), gasoline(RB), heating oil(HO), natural gas(NG) in the energy category, corn(C), soybeans(S), soybean meal(SM), soybean oil(BO) in the grains & oilseeds category, feeder cattle(FC), live cattle(LC), lean hogs(LH) in the livestock category, cocoa(CC), coffee(KC), cotton(CT) in the softs category, gold(GC), silver(SI), copper(HG) in the metal category. These meticulously selected futures contracts are actively traded by many investors globally and exhibit high liquidity.

Table 2 presents a description of futures contracts used for term structure estimation. All observations of selected futures contracts have continuous monthly prices until December 2023. For instance, the "Obs.Mat." of the crude oil futures contract(CL) indicates the availability of continuous price information for up to the ninth maturity over a total of 482 months. Additionally, the "Used mat." for crude oil indicates that we use the third, fifth, seventh, eighth, and ninth maturities to estimate the term structure. The "Max mat." represents the maximum maturity observed for each futures contract during the sample period, ensuring a minimum of 100 or more observations for each maturity. For instance, in the case of feeder cattle(FC), we use maturities up to the sixth maturity for the term structure estimation. But, the seventh or eighth maturity has at least 100 observations available for each maturity although there are some missing values during the sample period. Therefore, we estimate the term structure using maturities up to the "Used Mat." and evaluate the performance of the estimated term structure models using observations up to "Max mat.".

PLEASE INSERT TABLE 2 AROUND HERE.

Next, we compute factor returns based on the initial sample of 27 futures contracts to examine the relationship between the risk premiums estimated from the term structure model and various factors known to explain commodity risk premiums in the existing literature, These factors include 1) momentum, 2) basis, 3) basis-momentum, 4) skewness, 5) inflation beta, 6) volatility, 7) hedging pressure, 8) open interest, and 9) value. The definitions and calculation methods for each risk factor are provided in the appendix. We use the factor returns from December 1986 onwards because the Commitments of Traders (COT) Reports announced by the Commodity Futures Trading Commission(CFTC) have been available since 1986 and computing hedging pressure requires data from the previous 12 months. Therefore, we use the final factor returns from December 1986 to December 2023.

3.2. Commodity factor portfolios

Following Sakkas and Tessaromatis (2020), we construct nine long-short commodity factor returns. To create commodity factor portfolios, we sort commodities based on each characteristic as of the previous month and then construct three portfolios using the next month's commodity returns. We continue rebalancing portfolios based on these characteristics at the end of each month to construct factor portfolios. Based on the three portfolios formed for each characteristic, we construct long-short portfolios for each characteristic defined as the difference in returns between the high and low portfolios.

Table 3 presents descriptive statistics for the constructed factor portfolios. All returns are annualized. The last two columns show the difference in returns between the high and low portfolios and the corresponding t-statistics. The S&P GSCI low returns in the first column represent the monthly returns calculated from the S&P GSCI index. Among the various factor returns, the long-short portfolio returns of the basis, value, basis-momentum, and momentum factors are statistically significant. While there exist some differences in returns between high and low portfolios for other factors, they are not statistically significant. The signs of the long-short portfolio returns are consistent with most of the existing literature, except for the case of the value factor, which has the opposite sign. However, this result is consistent with the finding reported by Asness et al. (2013) that momentum and value factor returns are negatively correlated. Furthermore, these results are in line with various factor tests conducted by Sakkas and Tessaromatis (2020). They find that the returns of multi-factor commodity portfolios combining momentum, basis, basis-momentum, hedging pressure, and value commodity factors perform the best through several statistical tests. Thus, the results are consistent with previous literature suggesting that factors such as momentum, basis, and basis-momentum along with equal-weighted average returns can explain commodity risk premia (Yang, 2013; Szymanowska et al., 2014; Bakshi et al., 2019; Sakkas and Tessaromatis, 2020; Boons and Prado, 2019).

PLEASE INSERT TABLE 3 AROUND HERE.

3.3. Principal component analysis

To estimate the term structure model of commodity futures, we employ three PCA factors as state variables. The use of three PCA factors to estimate the term structure model is common in interest rate model (Piazzesi, 2010). Specifically, the three PCA factors are often labeled as level, slope, and curvature and have been highly effective in estimating the term structure (Litterman, 1991). Analogously, we speculate that three PCA factors are effective in capturing the shape of the commodity futures term structure. Table 4 displays the three PCA factors based on log prices and changes in log prices. The patterns are similar across both methodologies. First, the first factor explains a significant portion of the variance. In the case of PCA based on log prices, the first factor explains between 92.54% and 99.81% of the variance for each futures contract. While the second and third factors exhibit slightly lower explanatory power compared to the first factor, the three PCA factors explain nearly 99% of the variance. A similar pattern can be observed with PCA based on the changes in log prices. Although there is a slight decrease in explanatory power compared to PCA based on log prices, the three factors based on the changes in log prices explain at least 80% of the variance, with the majority demonstrating explanatory power exceeding 95%.

PLEASE INSERT TABLE 4 AROUND HERE.

Figure 1 illustrates the patterns of factor loadings resulting from PCA analysis. The top panel of Figure 1 depicts the factor loadings based on log prices, and the bottom panel represents the factor loadings calculated from changes in log prices. Similar to PCA in interest rates where the first factor represents the level, the second factor represents the slope, and the third factor represents the curvature, an identical pattern emerges in PCA analysis for commodities. First, the first factor reveals a straight-lined pattern indicating an equal-weighted average across all maturities, suggesting an overall representation of the level of commodity prices or returns. The pattern of the second factor reflects differing weights between long and short maturities, effectively capturing the difference between long-term and short-term levels by assigning negative weights to short-term prices or returns and positive weights to long-term levels. Lastly, for the third factor, positive weights are assigned to short and long maturities while negative weights are assigned to middle-matured contracts, indicating the concavity of the overall term structure. In conclusion, the three PCA factors effectively represent the level, slope, and curvature of commodity futures prices or returns, thereby providing a representative description of the entire term structure.

PLEASE INSERT FIGURE 1 AROUND HERE.

Next, we investigate the relationship between the first three PCA components and various commodity pricing factors studied in the existing literature through regression analysis. We regress each PCA component as the dependent variable on the long-short portfolio returns calculated in the previous section as the independent variables across all commodities. We multiply the second PCA factor value by a negative one to represent the slope which is the difference between long and short-term levels. Moreover, we include commodity and year-fixed effects through all regression analyses because individual commodities can have unique characteristics and may be influenced differently over time. All standard errors are clustered by commodity and year.

PLEASE INSERT TABLE 5 AROUND HERE.

Table 5 presents the results of regressions between each PCA component and commodity risk factors. Columns 1-3 display the results for each PCA component based on log prices and Columns 4-6 show the results for the changes in log prices. First, the signs of the coefficients show similar results for both log prices and the changes in log prices. Specifically, the first PCA component exhibits a positive relationship with the S&P GSCI returns. Considering that the first component represents the average level across all maturities, the positive relationship with the S&P GSCI which has the composition of major commodities is quite reasonable. However, in the regression of the second component, the coefficients for the S&P GSCI show statistically significant differences but different signs. This result suggests that the market returns may be related differently to the level and changes in level. Excepting this case, statistically significant coefficients exhibit consistent patterns. The momentum is negatively related to slope, indicating that the effect of momentum is associated with reducing the price difference between short and long-term contracts. Moreover, given that a large basis in our construction results in decreased returns, the negative relationship between the second PCA component and basis aligns with the notion that basis generally represents slope (Yang, 2013). Boons and Prado (2019) argue that basis-momentum reflects the slope and curvature of commodity futures term structure, which is supported by the significantly positive relationship between basis-momentum and the second PCA. Furthermore, the negative relationship between inflation beta and the first and second PCA components contradicts the previous results that commodities sensitive to inflation show positive returns, suggesting the presence of multicollinearity among the factors.

4. Empirical results

4.1. Term structure estimation results

In this section, we analyze the results of term structure estimation. We estimate the term structure using the first three PCA components based on log prices and investigate the pricing errors between the fitted prices from the estimated term structure model and the observed prices. Table 6 presents the time series properties of the pricing errors. In addition to statistics for maturities of 3, 6, 12, 18, and 36 months, the table reports the mean, standard deviation, maximum, and minimum statistics for all maturities in the last four columns.

PLEASE INSERT TABLE 6 AROUND HERE.

Panel A displays the observations for each maturity, Panel B presents the mean of the pricing errors, Panel C shows the standard deviation of the pricing errors, Panel D reports the autocorrelation of order one for the pricing errors, and Panel E indicates the root mean squared error(RMSE) for the pricing errors. Panel A provides the average observed values for the selected maturities. For instance, crude oil (CL), gasoline (RB), heating oil (HO), natural gas (NG), and copper (HG) have more than 100 observations up to a maturity of 36 months. This result implies that energy or agricultural futures generally have sufficient observations for longer-term contracts, while softs or livestock futures contracts are rarely traded for longer maturities.

Panel B displays the average of the pricing errors which is the difference between the observed log prices and the fitted log prices for a given maturity. Although the mean of pricing errors tends to increase slightly for longer maturities, it remains at relatively low levels overall. The commodity with the largest pricing error is lean hogs(LH), showing a maximum error of \$0.187 for the 12th maturity, but averaging \$0.062. Most other commodities exhibit low levels of pricing errors. When seeing the standard deviation in Panel C, these results suggest that pricing errors generally do not deviate significantly from their average. Panel D illustrates the serial correlation of pricing errors. The result shows that the average autocorrelation is approximately 0.69. The autocorrelation appears to increase as the maturity is longer. This result may reflect a higher dependence on previous prices for longer maturities due to low liquidity.

Finally, Panel E presents the average root mean squared error (RMSE) for the pricing errors. While Panel B displays the average values of pricing errors, the average may underestimate the pricing errors as they can take both positive and negative values. Therefore, the RMSE serves as a better indicator of how much the fitted values of the term structure deviate from the actual observations. On average, the RMSE is \$0.041, indicating an error of less than \$1.041 ($e^{0.041} = 1.041$) in terms of prices. Thus, the overall pricing errors are quite small in the estimated regression-based term structure of commodity futures.

PLEASE INSERT FIGURE 2 AROUND HERE.

Figure 2 displays the time series of fitted and observed prices. Due to limited space, we represent exemplary commodities with the smallest and largest RMSE from each commodity category. The left panel of Figure 2 represents commodities with the smallest RMSE in each category, while the right panel shows those with the largest RMSE. Specifically, we select crude oil(CL, RMSE=0.025) and natural gas(NG, RMSE=0.144) in energy, soybean oil(BO, RMSE=0.017) and corn(C, RMSE=0.064) in grains & oilseeds, live cattle(LC, RMSE=0.023) and lean hogs(LH, RMSE=0.088) in livestock, cocoa(CC, RMSE=0.008) and cotton(CT, RMSE = 0.034) in softs, and copper(HG, RMSE=0.016) and gold(GC, RMSE=0.024) for metals. Because we use different maturities for each commodity futures to estimate the term structure, each plot represents the time series of the $m + 1$th futures contract after maturity m which is the maximum maturity used for the term structure estimation of the corresponding futures. For instance, for crude oil (CL), we display the prices of the tenth maturity futures as the term structure is estimated using observations up to the first nine months. The left panel plots reveal negligible differences between observed and fitted prices, consistent with the lowest RMSE reported in Table 6. The right panel shows slightly more errors compared to those on the left. Nevertheless, the overall movement of fitted prices closely tracks the observed price trends over time. Moreover, for commodity corn(C), we can observe the well-fitted prices even for periods with no observed data. Thus, based on the above analysis, we confirm that the term structure model proposed in this paper can effectively estimate the prices of individual commodity futures with small errors.

Next, we perform the goodness-of-fit tests on the estimated values. We conduct regression analyses on each commodity futures contract from two dimensions. The objective is to examine the extent to which the fitted prices explain the observed prices and to compare the R-squares. First, we compute the R-squares from the time-series regression of observed prices on fitted prices for each maturity, assessing how well the time-series variation is captured. Second, we compute the R-squares from the cross-sectional regression of observed prices on the fitted prices for maturities given date, evaluating how well the futures curve for a given date is represented. Moreover, to assess the validity of the model, we separate the regression analysis into two parts: one used for the estimation (In-sample: up to "Obs. mat.") and the other not used for the estimation (Out-of-sample: from "Obs. mat."+1 to "Max. mat.").

PLEASE INSERT TABLE 7 AROUND HERE.

Table 7 presents summary statistics of the R-squares obtained from the regression analyses. Panel A provides a summary of the R-squares obtained from time-series regression, which examines the extent to which the fitted prices explain the observed prices at specific maturity. We perform the time-series regressions for each commodity at various maturities and summarize the results for both in-sample and out-of-sample data. The statistics in the left columns of the panel indicate that in most cases, the in-sample R-squares explain the time-series variation almost perfectly, with an average R-squares of 99.8% across all commodities. The average maximum and minimum R-squares are 99.9% and 99.6%, respectively. Additionally, the standard deviation is extremely low, averaging at 0.001, indicating minimal variability. In the right panel, we present a summary of R-squares for maturities beyond the term structure estimation. Despite a slight decrease compared to the in-sample R-squares, the average R-squares remain high at 94.3%. The finding that the average minimum R-squares is 88.9% suggests that even in the out-of-sample period, the fitted prices successfully capture the time-series variation of observed prices.

PLEASE INSERT FIGURE 3 AROUND HERE.

Figure 3 illustrates the regression parameters and R-squares obtained from the time-series regressions, comparing the results from the in-sample (left panel) and out-of-sample (right panel). For each maturity, the top row depicts the average R-squares, the second row shows the average coefficient (β) for the fitted prices, and the last row illustrates the average intercept (α). First, in the graph representing the R-squares at the top, the solid line indicates the average R-squares and the dashed line represents the value of one. The average R-squares for the in-sample closely approach one, and for the out-of-sample data, they gradually decrease with maturity but still maintain substantial explanatory power. Therefore, the fitted prices from the term-structure model can effectively capture the time-series variation of the observed prices. Second, the second and third rows represent the average α and β of the regressions, respectively, by maturity. If the fitted prices accurately explain the observed prices, the αs should be close to zero, and the βs should be close to one. The βs are consistently close to one, and αs remain nearly at zero for in-sample. Although the results deviate slightly from expectations for out-of-sample, the deviations are not significant. For instance, the βs in the second row show variation around one by maturity, but they remain within the range of 0.96 to 1.02. Furthermore, the αs in the third row mostly move within a range of 0.25, with the majority being near zero.

Next, we examine the cross-sectional variation. The primary objective of this paper is to estimate the term structure model. In this sense, estimating prices for longer maturities or maturities with missing observations in some maturities for a given date may be more critical than investigating the time-series variation. Panel B of Table 7 presents the cross-sectional regression results. The left columns present the cross-sectional regression results using the maturities employed in the term-structure estimation. The right columns represent the results using observed prices and fitted prices for given maturities from the month following those used for estimation up to the maximum observable maturity. Therefore, the in-sample regression uses observations based on a minimum of six to a maximum of 12 maturities for a specific date depending on the futures contract. The results on the right side are based on a minimum of five to a maximum of 24 maturities. Then, the reported R-squares obtained for each date are averaged. Hence, the results may be subject to small-sample bias due to the potential for inaccuracy in conducting regressions with limited observations.

First, the average R-squares for all commodities in the in-sample stand at 86.5%. This result indicates that the cross-sectional results still have a high level of explanatory power although lower than the 99.8% reported in the time series results. On the other hand, the out-of-sample R-squares decrease to an average of 68.5% but this varies across commodities. Nevertheless, the average R-squares for other futures except some futures contracts(RB, NG, C, S) show values above 77%. These results show that the cross-sectional regression results are somewhat lower than the time-series regression results. However, the magnitude of the errors in terms of prices is relatively small. Therefore, we conclude that the regression-based term structure model accurately estimates the time-series variation for each given maturity with minimal error. Nevertheless, minor estimation errors may exist in capturing the price variation of different maturities. This result implies that our term structure estimation can capture the overall time variations of individual futures but may have some errors in the extent of price movements of longer maturities.

PLEASE INSERT FIGURE 4 AROUND HERE.

Figure 4 illustrates the time-series variation of estimated parameters and R-squares from the cross-sectional regression. The left panels depict the time-series plots of R-squares, β, and α for the in-sample, while the right panels display those for the out-of-sample. The in-sample results (left panel) demonstrate that R-squares mostly remain above 80%, βs are close to one, and αs are nearly approaching zero. Thus, the maturities used for term structure estimation exhibit highly accurate estimation results. Conversely, the out-of-sample R-squares show considerable variation, yet consistently demonstrate explanatory power within the stable range of 60–70%. Moreover, both αs and βs fluctuate around the desired values in the out-of-sample. Although the out-of-sample results are volatile, one can overcome this issue by including more observations from longer maturities when estimating the term structure. Moreover, the cross-sectional regression exhibits slightly lower explanatory power than time-series regression, but the statistics still maintain a high level of explanatory power and stable coefficient values. Therefore, we can conclude that the regression-based term structure model adequately reflects the actual term structure.

PLEASE INSERT FIGURE 5 AROUND HERE.

Next, Figure 5 illustrates the relationship between observed and fitted prices for the term structures of individual commodity futures. The selected commodities are the same as those chosen earlier. The selected months are those with the median R-squares for each futures contract in 2023. Most figures exhibit that the fitted prices appear to closely match the shape of the actual term structure except for some futures. Notably, the fitted prices are well estimated for futures with a monotonic term structure shape. However, for Copper(HG) and Gold(GC) in the final row, the fitted prices exhibit a pattern of increase followed by a decrease, whereas the observed prices exhibit an upward trend. Nevertheless, this difference appears to be negligible when considering actual price differences. One possible reason for such errors is that we use the entire observations to estimate coefficients of factor regressions, which may not appropriately reflect the recent movements of the futures prices. In other words, the slope of the term structure may be reversed or the weights on the curvature may be changed during the sample period. One possible solution to address these issues could be using data that reflects recent trends rather than the entire period or including more price observations for longer contracts. Nevertheless, the estimated term structure can capture individual futures characteristics such as seasonality or preferences for specific maturities appearing in certain futures. For example, the fitted prices of natural gas (NS) correctly reflect the seasonality of energy prices although there may be some errors. Moreover, the fitted prices can also capture certain patterns that occur in live cattle(LC) or lean hogs (LH) futures which exhibit higher prices in middle maturities. The model demonstrates excellent performance for the in-sample but has some errors for longer maturities which are not used for the estimation. Nevertheless, the model's ability to produce the smallest errors for the in-sample is a significant advantage. Therefore, we conclude that the regression-based term structure model used in this paper can effectively capture the term structure shape reflecting the overall characteristics of commodity futures.

We have examined the relationship between the fitted and the observed prices in both the time and cross-sectional dimensions. Our findings suggest that the regression-based term structure model appropriately captures the time-series variation across different maturities and demonstrates reasonably good performance in capturing cross-sectional variation for specific months. Basak and Pavlova (2016) posit in their theoretical models that the participation of numerous institutional investors in the commodity futures market may cause an increase in price volatility and trading volume since the early 2000s. Moreover, Henderson et al. (2015) demonstrate that new types of derivatives like CLNs may induce uninformed and predetermined flows into and out of futures contracts. These studies imply that the observed futures prices may deviate temporarily from their fundamental value and the gap between the fitted and actual prices may widen after the financialization. To investigate the effect of the financialization, we first average the RMSEs for all maturities of each futures contract on specific dates and then calculate the average RMSE for all futures contracts for each date.

PLEASE INSERT FIGURE 6 AROUND HERE.

Figure 6 illustrates the time series of the average RMSEs for the 17 commodity futures contracts. This figure illustrates that the RMSE varies over time. While the in-sample RMSEs are quite stable over time, the out-of-sample RMSEs increase after the 2000s, indicating the potential impact of financialization. Hence, one of the possible factors contributing to the observed cross-sectional errors in the out-of-sample might be the increased volatility in actual prices due to the participation of financial institutions in the commodity futures markets. In other words, the increased involvement of financial institutions since the 2000s may lead to greater volatility in actual prices, resulting in errors between model-predicted and observed prices. This finding suggests that using only observations reflecting recent trends over the past 10 or 20 years or including more observations from longer maturities can result in accurate term structure estimates. Adrian et al. (2013) argue that the regression-based term structure model can allow daily or even intraday observations. This argument suggests that one can estimate the model with more frequent observations even with the relatively short sample period.

4.2. Risk premium and factor risks

In this section, we explore how various risk factor returns can explain the estimated risk premium from the model. Equation (9) shows that the total risk premium can be expressed as $\hat{B}'_{n-1}(\hat{\lambda}_0 + \hat{\lambda}_1 X_t) - \frac{1}{2}(\hat{B}'_{n-1}\hat{\Sigma}\hat{B}'_{n-1} + \hat{\sigma}^2)$. As shown in Szymanowska et al. (2014), the expected excess futures return can be represented as the sum of spot premium and term premium. Therefore, we can calculate the total risk premium for each commodity by maturity using parameters estimated from the term structure model. Moreover, assuming that the total premium of the first nearby contract represents the spot premium without the term premium, we can interpret the difference between the total premium of subsequent maturities and the spot premium as the term premium. Our regressions include commodity and year-fixed effects to control for the commodity-specific characteristics and the changes in premiums over time. All standard errors are doubly clustered for commodity and year.

PLEASE INSERT TABLE 8 AROUND HERE.

Table 8 presents the regression results of total risk premium on factor returns. The first column presents the results for the entire period from December 1985 to December 2023. Columns 2 and 3 present the results before and after the financialization, respectively. In Column 1, the coefficient of the S&P GSCI returns is positively and statistically significant. This finding is consistent with previous literature suggesting that the equal-weighted average of commodity futures returns can explain individual commodity returns (Bodie and Rosansky, 1980; Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006). Next, the coefficient for basis is negative and significant. This result is consistent with prior findings suggesting that the smaller basis leads to larger expected commodity returns (Gorton et al., 2012; Yang, 2013; Szymanowska et al., 2014; Bakshi et al., 2019; Boons and Prado, 2019). The coefficient for hedging pressure is also negative and significant. This result indicates that an increase in demand for hedging may result in a decline in the expected returns.

Next, Column 2 presents the results of subsample analysis for the post-financialization period, starting from 2004. During this period, we continue to observe a positive relationship between the return of the S&P GSCI and total risk premiums. This finding is unsurprising given that commodity futures in our sample are predominantly included in the S&P GSCI index and tend to attract greater attention from institutional investors. In addition, the coefficient for the value factor is positive and significant, suggesting a preference for value premiums by investors after the financialization. On the other hand, Column 3 shows the analysis for the period before the financialization, revealing somewhat different patterns compared to the post-financialization results. Specifically, during the pre-financialization period, we observe positive results between inflation beta and risk premiums and negative results between volatility and premiums. These findings suggest that factor returns related to commodity risk premiums may differ before and after the financialization.

PLEASE INSERT TABLE 9 AROUND HERE.

Table 9 presents the regression results between spot premium and factor risk returns. Column 1 indicates that momentum and volatility exhibit the highest relevance to spot premium during the entire sample period and are statistically positive and significant. Column 2 reveals that volatility is associated with spot premium after the financialization. On the other hand, Column 3 shows that momentum, basis-momentum, inflation beta, and volatility are related to spot premium before the financialization. These results suggest that the relationship between factor risk returns and spot premium may differ before and after financialization. Regardless of the period, volatility emerges as the risk factor most closely related to spot premium. However, the relationship with S&P GSCI return, which primarily represents the market return, appears insignificant.

PLEASE INSERT TABLE 10 AROUND HERE.

Next, we conduct regression analyses between term premium and factor returns. Table 10 reports the results. The estimated term premiums range from the second nearby contract to the 36th contract. The total term premiums are divided into four periods and analyzed using the average for each period. Column 1 represents the segment with maturities of up to six months, Column 2 covers maturities from 7 to 12 months, Column 3 encompasses maturities from 13 to 24 months, and Column 4 includes maturities from 25 to 36 months as the dependent variable. Column 1 indicates that only the value factor return is statistically significant. This finding suggests that factor returns may not significantly account for the term premiums of short-term futures contracts. However, the term premiums of longer maturities are positively and statistically associated with the S&P GSCI returns. As the maturity increases, the value of the t-statistics becomes larger, indicating that the term premiums of long-term futures contracts are strongly related to the market returns. An intriguing finding is that the basis factor return becomes increasingly negative and significant with longer maturities. This finding aligns with the definition of the basis as the price difference between long and short-term futures. Moreover, the hedging pressure appears negative and statistically significant, implying that the compensation received from hedging pressure decreases for longer-term futures contracts as hedging pressure increases. A notable distinction arises between factor risk returns associated with spot premium and those associated with term premium. While the spot premium is related to factors such as momentum or volatility representing short-term movements and price risk, the term premiums are associated with broader market movements and the characteristics of futures contracts such as basis. These findings suggest that existing risk factors are differently related to the spot and the term premiums.

PLEASE INSERT TABLE 11 AROUND HERE.

Table 11 presents the results of dividing the results in Table 10 into periods before and after financialization. Columns 1-4 demonstrate the analyses for the period after financialization, while Columns 5-8 present those before financialization. In Columns 1-4, the most significant observation is the positive and significant relationship between S&P GSCI returns and term premiums after financialization. This result implies that the term premium becomes more closely associated with overall market returns due to the participation of institutional investors after the financialization. Moreover, the coefficients for the value factor are statistically positive and significant, implying that the institutional investors' investment behavior responds to market-wide movements and value investments after financialization.

On the other hand, factors such as inflation and volatility are significantly related to the term premiums before financialization. These results suggest that the term premiums on the longer maturities may decline as volatility increases. Moreover, longer commodity futures that are sensitive to inflation exhibit higher expected returns before the financialization, as evidenced by the significant and positive coefficient on the inflation beta. Therefore, the risk factors associated with commodity futures term premiums differ before and after financialization.

Thus far, we have examined the relationship between total, spot, and term premiums with the risk factors defined in the existing literature. The empirical results show that the risk premium can be differently related to various risk factors depending on the period of financialization and the type of premium. However, these results may not imply that other factors are unimportant. As seen in the previous cross-sectional regression, the estimated term structure model might contain some pricing errors. Nevertheless, a significant implication of this analysis is that each futures risk premium is related differently depending on the type of the premium and the financialization period rather than uniformly sharing the same risk factor. Moreover, these results suggest that all futures across different maturities should share the same spot premium but the expected return on longer maturities having additional term premiums can be determined by other risk factors that may be less related to the spot premium.

5. Conclusion

In this paper, we apply a regression-based term structure model to estimate the term structure of commodity futures. The main advantage of this model lies in its computational efficiency and flexibility to incorporate various risk factors, as well as its applicability across various data frequencies (Adrian et al., 2013). The estimated results closely match observed prices with minimal pricing errors. Furthermore, the model enables the estimation of risk premiums, allowing us to analyze their relationships with existing factor risks through regression analysis.

We employ the first three PCA components as state variables to estimate the model. Comparing the fitted with observed prices, we find small pricing errors between the two prices at each maturity. Furthermore, when comparing the fitted and observed prices for all maturities on specific dates, we observe that the pricing errors increase slightly with small amounts compared to time series analysis. Overall, the estimated term structure accurately captures the actual shape of the term structure for each date. Moreover, the estimated term structure can appropriately capture the distinctive characteristics of individual commodity futures. However, the presence of small pricing errors suggests the necessity of incorporating commodity-specific factors in addition to the three PCA factors. One of the advantages of the regression-based term structure model is that we can consider more factors beyond PCA factors. In other words, we can potentially enhance the performance of term structure estimation by accounting for additional characteristics of individual commodities, such as seasonality.

Finally, we investigate whether the estimated risk premium from the model can be explained by existing factor risk returns through regression analysis. The results show that momentum and volatility factor returns are associated with spot premium, while S&P GSCI return, basis factor, and hedging pressure are associated with term premium. Furthermore, factors such as S&P GSCI or value exhibit a closer relationship with term premiums after the financialization period, whereas inflation beta or volatility demonstrate an association with term premiums before financialization. These findings suggest that different risk factors can explain the spot and term premiums in different ways. Moreover, these results imply that different risk factors may also explain each premium in varying ways before and after financialization.

In conclusion, the regression-based term structure model presented in this paper effectively captures the movements of commodity futures prices and the shape of the term structure. The commodity futures markets are segmented into different categories and most trading is concentrated on the nearby contracts although the recent financialization results in increasing trading volume. Nevertheless, we have demonstrated that the regression-based term structure model can successfully estimate the actual term structure even with relatively few observations. Employing the advantage of the regression-based term structure model, researchers can achieve even lower errors in estimating the accurate term structure model by adding commodity-specific risk factors or using daily or intraday-level data to capture the recent trends of individual commodity futures.

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Table 1 Commodity futures data

This table displays 27 commodity futures, the categories they belong (Category), the exchange where they are listed(Exchange), the exchange ticker(Ticker), the code in the Commitment of Traders reports available from the Commodity Futures Trading Commission(CFTC code), the year-month of the first and last observations in our sample(Start, End), the indicator of whether they are used for the term structure estimation(TS Used). The commodity futures contracts are listed and traded on the Chicago Board of Trade(CBT), the Chicago Mercantile Exchange(CME), the New York Commodity Exchange(COMEX), the Intercontinental Exchange U.S.(ICE-US), and the New Tork Mercantile Exchange(NYMEX).

Category	Commodity futures	Exchange	Ticker	CFTC code	Start	End	TS Used
Energy	Crude Oil	NYMEX	CL	067651	1983:03	2023:12	Yes
	Gasoline	NYMEX	RB	111659	2005:10	2023:12	Yes
	Heating Oil	NYMEX	HO	022651	1980:01	2023:12	Yes
	Natural Gas	NYMEX	NG	023651	1990:04	2023:12	Yes
Grains & Oilseeds	Oats	CBT	O	004601, 004603	1973:01	2023:12	
	Rough Rice	CBT	RR	039601, 039781	1981:04	2023:12	
	Corn	CBT	C	002601, 002602	1973:01	2023:12	Yes
	Soybeans	CBT	S	005601, 005602	1976:01	2023:12	Yes
	Soybean Meal	CBT	SM	026603	1976:01	2023:12	Yes
	Soybean Oil	CBT	BO	007601	1996:03	2023:12	Yes
	Wheat	CBT	W	001601, 001602	1996:03	2023:12	
	Red Wheat	CBT	KW	001611, 001612	1980:01	2023:12	
Livestock	Feeder Cattle	CME	FC	061641	1978:07	2023:12	Yes
	Lean Hogs	CME	LH	054641, 054642	1976:01	2023:12	Yes
	Live Cattle	CME	LC	057642	1980:01	2023:12	Yes
Softs	Lumber	CME	LB	058641, 058643	1978:07	2023:02	
	Milk	CME	DK	052642	2000:07	2023:12	
	Cocoa	ICE-US	CC	073732	1973:01	2023:12	Yes
	Coffee	ICE-US	KC	083731	1979:11	2023:12	Yes
	Cotton	ICE-US	CT	033661	1980:01	2023:12	Yes
	Sugar	ICE-US	SB	080732	1973:01	2023:12	
	Orange Juice	ICE-US	OJ	040701	1973:01	2023:12	
Metal	Gold	COMEX	GC	088691	1978:03	2023:12	Yes
	Silver	COMEX	SI	084691	1973:01	2023:12	Yes
	Copper	COMEX	HG	085691, 085692	1988:07	2023:12	Yes
	Palladium	NYMEX	PA	075651	1980:01	2023:12	
	Platinum	NYMEX	PL	076651	1973:01	2023:12	

Table 2 Commodity futures sample for term structure estimation

This table lists 17 commodity futures used for the term structure estimation, and tabulates the categories they belong (Category), the exchange where they are listed(Exchange), the starting year-month for the term structure estimation period (Start), total observed months (#Month), maximum observable maturity without missing values during the sample period (Obs.Mat.), maturities used for the term structure estimation(Used Mat.), maximum maturities with at least 100 observations during the sample period(Max.Mat.). All observations end in December 2023.

Category	Commodity futures	Ticker	Start	#Month	Obs. Mat.	Used Mat.	Max Mat.
Energy	Crude Oil	CL	1983:11	482	9	3,5,7,8,9	36
	Gasoline	RB	2005:10	219	12	3,5,7,9,12	36
	Heating Oil	HO	1985:10	459	10	3,5,6,8,10	36
	Natural Gas	NG	1990:04	405	11	3,5,7,9,11	36
Grains & Oilseed	Corn	C	1978:04	549	6	3,4,5,6	15
	Soybeans	S	1978:10	543	7	3,4,5,7	20
	Soybean Meal	SM	1976:01	576	7	3,4,5,7	23
	Soybean Oil	BO	1996:03	334	9	3,5,7,8,9	24
Livestock	Feeder Cattle	FC	1992:06	379	6	3,4,5,6	8
	Lean Hogs	LH	1976:01	576	7	3,4,5,7	12
	Live Cattle	LC	1983:01	492	6	3,4,5,6	9
Softs	Cocoa	CC	1980:05	524	6	3,4,5,6	10
	Coffee	KC	1979:11	530	6	3,4,5,6	15
	Cotton	CT	1980:01	528	6	3,4,5,6	15
Metal	Gold	GC	1979:10	531	12	3,5,7,9,12	21
	Silver	SI	1978:06	547	12	3,5,7,9,12	19
	Copper	HG	1988:09	424	11	3,5,7,9,11	36

Table 3 Commodity factor portfolios

This table presents the descriptive statistics for the period 1986.12 to 2023.12 of the S&P GSCI and the commodity factor portfolios of the low, medium, high, and long-short portfolios(DIff.) computed based on 27 commodity futures contracts. The last column is the *t*-statistics for the differences in returns between high- and low-portfolios. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Factor	N	Low	Medium	High	Diff.	t-stat
S&P GSCI	446	0.054				
Momentum	446	−0.008	0.010	0.065	0.074**	2.181
Basis	446	0.223	−0.002	−0.145	−0.369***	−11.906
Basis-Mom	446	−0.045	0.012	0.099	0.144***	4.796
Skewness	446	0.028	0.027	0.014	−0.014	−0.516
Inflation Beta	446	0.022	0.018	0.029	0.007	0.213
Volatility	446	0.028	0.027	0.014	−0.014	−0.516
Hedging Pressure	446	0.017	0.029	0.018	0.000	0.014
Open Interest	446	0.018	0.046	0.002	−0.016	−0.651
Value	446	0.149	−0.007	−0.076	−0.225***	−6.737

Table 4 Variance explained from principal component analysis

This table reports the proportion of variance explained by the first three components extracted from principal component analysis (PCA). PCAk indicates the proportion explained by the k-th component, and PCA1-3 represents the cumulative proportion explained by the first three components. The first four columns present statistics for PCA components extracted from log prices and the next four columns report those from changes in log prices.

Commodity	PCA from log price				PCA from changes in log price			
	PCA1	PCA2	PCA3	PCA1-3	PCA1	PCA2	PCA3	PCA1-3
CL	99.81%	0.18%	0.00%	100.00%	97.89%	1.93%	0.15%	99.97%
RB	96.43%	1.69%	1.39%	99.52%	86.52%	4.37%	3.14%	94.03%
HO	99.67%	0.25%	0.06%	99.98%	93.75%	4.01%	1.53%	99.30%
NG	97.05%	1.74%	0.70%	99.50%	73.30%	8.34%	6.59%	88.24%
C	98.97%	0.77%	0.17%	99.90%	91.21%	3.59%	2.85%	97.65%
S	99.30%	0.54%	0.12%	99.96%	94.16%	3.29%	1.40%	98.85%
SM	99.10%	0.76%	0.10%	99.96%	92.76%	4.80%	1.22%	98.79%
BO	99.71%	0.27%	0.02%	99.99%	98.07%	1.42%	0.28%	99.77%
FC	99.66%	0.29%	0.04%	99.98%	90.63%	5.33%	1.90%	97.86%
LH	92.54%	4.68%	1.86%	99.08%	50.66%	18.55%	12.64%	81.84%
LC	98.86%	0.73%	0.27%	99.85%	67.54%	12.00%	9.12%	88.66%
CC	99.74%	0.24%	0.02%	99.99%	98.19%	1.39%	0.29%	99.87%
KC	99.15%	0.81%	0.03%	99.99%	97.84%	1.68%	0.26%	99.78%
CT	96.39%	2.74%	0.47%	99.60%	85.51%	6.80%	3.37%	95.67%
GC	99.93%	0.07%	0.00%	100.00%	99.49%	0.39%	0.07%	99.95%
SI	99.90%	0.10%	0.00%	100.00%	99.21%	0.58%	0.12%	99.91%
HG	99.89%	0.10%	0.01%	100.00%	98.80%	0.97%	0.14%	99.91%

Table 5 Regression analysis of principal components and factor risks

This table presents the regression results of each PCA component on factor risks. The dependent variables in the first three columns are each PCA component extracted from log prices and those in the next three columns are from changes in log prices. All independent variables are defined in the Appendix. All regressions include year and commodity fixed effects. t-statistics based on standard errors double-clustered at the commodity and year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	log price			changes in log price		
	PCA1	PCA2	PCA3	PCA1	PCA2	PCA3
S&P GSCI	0.237***	−0.014**	−0.003	1.624***	0.048*	0.016
	(3.33)	(−2.77)	(−0.98)	(5.87)	(1.81)	(1.42)
Momentum	0.040	−0.010**	0.002	−0.024	−0.040*	−0.003
	(1.16)	(−2.43)	(1.18)	(−0.32)	(−2.12)	(−0.49)
Basis	−0.005	−0.005	0.003	−0.113	−0.037***	0.021
	(−0.14)	(−0.65)	(1.18)	(−0.85)	(−2.93)	(1.12)
Basis-Mom	0.012	0.016***	0.003	−0.096	0.046***	−0.010
	(0.22)	(3.34)	(0.85)	(−0.80)	(2.99)	(−0.78)
Skewness	−0.048	0.003	−0.006	0.100	0.005	0.009
	(−0.98)	(0.60)	(−1.05)	(1.60)	(0.33)	(0.74)
Inflation Beta	−0.076*	−0.009*	0.001	−0.474***	−0.043**	0.007
	(−1.77)	(−2.09)	(0.48)	(−3.11)	(−2.22)	(0.41)
Volatility	−0.008	−0.003	−0.001	0.238**	0.018	0.020**
	(−0.40)	(−1.08)	(−0.23)	(2.37)	(0.88)	(2.19)
Hedging Pressure	−0.022	−0.007	0.008	−0.269*	−0.031	0.015
	(−0.72)	(−1.28)	(1.33)	(−1.84)	(−1.08)	(0.96)
Order Imbalance	0.001	0.011*	−0.001	−0.025	0.006	−0.010
	(0.04)	(2.02)	(−0.15)	(−0.29)	(0.26)	(−0.77)
Value	0.032	−0.002	−0.000	−0.080	0.002	−0.014
	(0.83)	(−0.79)	(−0.01)	(−0.75)	(0.09)	(−1.38)
Constant	0.133***	−0.004***	0.001	−0.108***	−0.017**	0.006
	(41.80)	(−5.99)	(0.95)	(−4.53)	(−2.57)	(1.35)
Commodity Fixed	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed	Yes	Yes	Yes	Yes	Yes	Yes
Observations	7,113	7,113	7,113	7,108	7,108	7,108
Adjusted R^2	0.697	0.090	−0.000	0.171	0.003	−0.005

Table 6 Summary of term structure model estimation

This table summarizes the time series statistics of the pricing errors. The first five columns report the sample statistics for the selected maturities in 3, 6, 12, 18, and 36 months. The last four columns report the summary statistics for maturities from 1 month to 36 months. Panel A reports the number of observations, Panel B reports the average of pricing errors, Panel C reports the standard deviation of pricing errors, Panel D reports the autocorrelation coefficient of order one, and Panel E reports the root mean squared errors. The blank cells imply that there are no observations for the maturity in the month.

Commodity	m= 3	m= 6	m= 12	m= 18	m= 36	Mean	Std.Ev.	Max	Min
Panel A. Number of Observations									
CL	482	482	442	410	215	397	73	482	215
RB	219	219	219	203	203	208	8	219	203
HO	459	459	448	344	170	309	131	459	170
NG	405	405	398	381	304	364	35	405	304
C	549	549	237			382	168	549	100
S	543	543	269	149		359	174	543	115
SM	576	576	289	192		357	180	576	111
BO	334	334	291	202		260	81	334	100
FC	379	379				369	26	379	304
LH	576	576	103			461	180	576	103
LC	492	492				427	115	492	209
CC	524	524				457	113	524	186
KC	530	530	194			357	169	530	113
CT	528	528	194			380	162	528	111
GC	531	531	531	393		458	112	531	159
SI	547	547	547	363		489	85	547	353
HG	424	424	419	373	160	319	118	424	160
Panel B. Average of log pricing errors									
CL	−0.000	0.000	0.000	0.002	0.038	0.004	0.007	0.038	−0.000
RB	−0.007	−0.006	−0.008	−0.007	0.013	−0.002	0.007	0.013	−0.011
HO	−0.004	−0.004	−0.004	0.003	0.048	0.017	0.021	0.048	−0.004
NG	0.004	−0.004	0.002	−0.037	−0.109	−0.048	0.045	0.004	−0.109
C	0.005	−0.005	−0.070			−0.041	0.047	0.007	−0.141
S	−0.001	0.002	−0.031	−0.032		−0.017	0.016	0.002	−0.038
SM	−0.003	−0.000	−0.015	0.035		0.012	0.026	0.067	−0.015
BO	−0.001	−0.002	−0.004	−0.007		−0.003	0.003	0.000	−0.009
FC	−0.005	−0.001				0.001	0.006	0.011	−0.006
LH	0.021	0.008	0.187			0.062	0.070	0.187	−0.001
LC	−0.005	−0.004				−0.003	0.002	0.000	−0.005
CC	−0.003	−0.003				−0.003	0.001	0.000	−0.005
KC	−0.001	−0.001	0.000			−0.000	0.002	0.004	−0.004
CT	−0.003	−0.005	0.001			−0.002	0.006	0.011	−0.009
GC	−0.000	0.003	0.003	0.067		0.022	0.031	0.084	−0.000
SI	−0.014	−0.013	−0.018	0.006		−0.010	0.008	0.009	−0.019
HG	−0.000	−0.000	0.000	−0.000	0.034	0.009	0.013	0.034	−0.002
Panel C. Standard Deviation of log pricing errors									
CL	0.002	0.002	0.007	0.018	0.050	0.024	0.022	0.069	0.001
RB	0.019	0.016	0.027	0.069	0.099	0.060	0.033	0.104	0.015

Table 6 Summary of term structure model estimation - Continued

Commodity	m= 3	m= 6	m= 12	m= 18	m= 36	Mean	Std.Ev.	Max	Min
HO	0.013	0.009	0.038	0.071	0.104	0.057	0.034	0.104	0.006
NG	0.035	0.034	0.073	0.216	0.213	0.132	0.070	0.221	0.023
C	0.017	0.018	0.068			0.045	0.026	0.082	0.009
S	0.009	0.008	0.064	0.047		0.034	0.022	0.066	0.005
SM	0.010	0.008	0.066	0.049		0.038	0.022	0.072	0.006
BO	0.004	0.004	0.021	0.033		0.016	0.011	0.033	0.003
FC	0.006	0.005				0.009	0.006	0.019	0.003
LH	0.032	0.028	0.083			0.055	0.030	0.101	0.025
LC	0.014	0.015				0.023	0.012	0.042	0.008
CC	0.004	0.003				0.007	0.004	0.015	0.001
KC	0.004	0.004	0.024			0.014	0.009	0.030	0.003
CT	0.015	0.013	0.054			0.034	0.021	0.070	0.009
GC	0.001	0.002	0.002	0.029		0.010	0.012	0.036	0.001
SI	0.009	0.008	0.012	0.030		0.013	0.008	0.035	0.000
HG	0.003	0.002	0.004	0.013	0.031	0.011	0.009	0.031	0.001

Panel D. Autocorrelation coefficient of order one

CL	0.663	0.638	0.659	0.798	0.907	0.778	0.119	0.927	0.494
RB	0.163	0.050	0.556	0.844	0.919	0.638	0.349	0.928	−0.110
HO	0.707	0.655	0.848	0.916	0.941	0.856	0.119	0.942	0.537
NG	0.507	0.506	0.696	0.849	0.874	0.763	0.162	0.902	0.402
C	0.762	0.697	0.816			0.752	0.099	0.880	0.531
S	0.594	0.616	0.859	0.859		0.748	0.122	0.899	0.521
SM	0.660	0.547	0.835	0.772		0.723	0.137	0.869	0.356
BO	0.655	0.574	0.571	0.898		0.650	0.142	0.898	0.292
FC	0.322	0.525				0.493	0.208	0.721	0.185
LH	0.417	0.287	0.750			0.574	0.205	0.850	0.268
LC	0.530	0.578				0.627	0.140	0.810	0.397
CC	0.662	0.770				0.581	0.126	0.770	0.372
KC	0.527	0.550	0.754			0.632	0.148	0.838	0.357
CT	0.354	0.714	0.908			0.725	0.198	0.947	0.354
GC	−0.187	0.886	0.896	0.902		0.658	0.418	0.950	−0.381
SI	0.862	0.905	0.955	0.915		0.869	0.117	0.955	0.465
HG	0.580	0.044	0.556	0.848	0.950	0.738	0.207	0.950	0.044

Panel E. Root Mean Squared Errors (RMSE)

CL	0.002	0.002	0.007	0.018	0.062	0.025	0.023	0.069	0.001
RB	0.020	0.017	0.028	0.069	0.100	0.060	0.032	0.104	0.016
HO	0.014	0.010	0.038	0.071	0.114	0.061	0.037	0.114	0.007
NG	0.035	0.034	0.073	0.219	0.239	0.144	0.078	0.239	0.024
C	0.018	0.019	0.098			0.064	0.050	0.162	0.009
S	0.009	0.008	0.071	0.056		0.039	0.026	0.074	0.005
SM	0.010	0.008	0.067	0.060		0.044	0.027	0.083	0.006
BO	0.005	0.004	0.022	0.033		0.017	0.011	0.033	0.003
FC	0.007	0.005				0.010	0.006	0.022	0.003
LH	0.038	0.029	0.204			0.088	0.071	0.204	0.025
LC	0.014	0.015				0.023	0.012	0.042	0.008
CC	0.005	0.005				0.008	0.004	0.015	0.001
KC	0.004	0.004	0.024			0.014	0.009	0.030	0.003

Table 6 Summary of term structure model estimation - Continued

Commodity	m= 3	m= 6	m= 12	m= 18	m= 36	Mean	Std.Ev.	Max	Min
CT	0.015	0.014	0.054			0.034	0.020	0.070	0.009
GC	0.001	0.004	0.004	0.073		0.024	0.033	0.091	0.001
SI	0.017	0.015	0.021	0.030		0.019	0.007	0.036	0.000
HG	0.003	0.002	0.004	0.013	0.046	0.016	0.014	0.046	0.001

Table 7 Summary of R-squares from the regression analysis

This table presents the results of regression analyses between observed prices and fitted prices. Panel A shows the results of time-series regression conducted for each maturity across different commodities. Panel B displays the results of cross-sectional regression performed for all maturities observable on each date across various commodities. The "In-sample" refers to the sample period used for term-structure estimation (up to "Obs.Mat."). The "out-of-sample" denotes the sample period from the next maturity following the one used for the term structure estimation, up to any maturities with at least 100 observations (from "Obs.Mat."+1 to "Max.Mat.").

Commodity	In-sample					Out of sample				
	N	Mean	Std.Ev.	Max	Min	N	Mean	Std.Ev.	Max	Min
Panel A. Time-series regression										
CL	9	1.000	0.000	1.000	1.000	27	0.995	0.008	1.000	0.960
RB	12	0.995	0.003	0.997	0.987	24	0.905	0.046	0.974	0.830
HO	10	1.000	0.000	1.000	1.000	26	0.953	0.032	0.999	0.908
NG	11	0.994	0.002	0.998	0.991	25	0.843	0.061	0.978	0.754
C	6	0.997	0.002	0.999	0.994	9	0.940	0.053	0.989	0.827
S	7	0.999	0.000	1.000	0.999	13	0.948	0.031	0.995	0.887
SM	7	0.999	0.000	1.000	0.999	16	0.902	0.092	0.995	0.750
BO	9	1.000	0.000	1.000	1.000	15	0.994	0.002	1.000	0.991
FC	6	0.999	0.001	1.000	0.998	2	0.998	0.001	0.999	0.997
LH	7	0.988	0.003	0.992	0.983	5	0.731	0.143	0.928	0.619
LC	6	0.997	0.002	0.999	0.994	3	0.977	0.012	0.991	0.967
CC	6	1.000	0.000	1.000	0.999	4	0.999	0.001	1.000	0.998
KC	6	1.000	0.000	1.000	0.999	9	0.989	0.009	0.999	0.976
CT	6	0.995	0.003	0.999	0.989	9	0.859	0.109	0.981	0.700
GC	12	1.000	0.000	1.000	1.000	9	0.997	0.006	1.000	0.981
SI	12	1.000	0.000	1.000	1.000	7	0.999	0.001	1.000	0.997
HG	11	1.000	0.000	1.000	1.000	25	0.995	0.007	1.000	0.976
Total	8.4	0.998	0.001	0.999	0.996	13.4	0.943	0.036	0.990	0.889
Panel B. Cross-sectional regression										
CL	482	0.984	0.067	1.000	0.088	452	0.774	0.289	1.000	0.000
RB	219	0.841	0.128	0.990	0.237	203	0.367	0.214	0.826	0.001
HO	459	0.883	0.161	1.000	0.031	443	0.609	0.332	1.000	0.000
NG	405	0.749	0.184	0.973	0.051	384	0.194	0.220	0.996	0.000
C	549	0.765	0.312	1.000	0.000	374	0.520	0.311	1.000	0.000
S	543	0.884	0.141	1.000	0.012	479	0.550	0.363	1.000	0.000
SM	576	0.890	0.173	0.999	0.001	484	0.678	0.329	1.000	0.000
BO	334	0.931	0.141	0.999	0.078	296	0.615	0.292	1.000	0.000
FC	379	0.793	0.207	0.997	0.001	285	1.000	0.000	1.000	1.000
LH	576	0.749	0.231	0.994	0.000	438	0.744	0.358	1.000	0.000
LC	492	0.687	0.252	0.970	0.000	245	0.652	0.356	1.000	0.000
CC	524	0.918	0.151	1.000	0.000	387	0.891	0.235	1.000	0.000
KC	530	0.969	0.077	1.000	0.366	297	0.930	0.166	1.000	0.001
CT	528	0.792	0.253	0.999	0.000	433	0.705	0.320	1.000	0.000
GC	531	0.997	0.008	1.000	0.839	418	0.632	0.327	1.000	0.001
SI	547	0.916	0.127	0.986	0.215	395	0.950	0.130	1.000	0.009
HG	424	0.959	0.106	1.000	0.006	409	0.842	0.229	1.000	0.000
Total	476.4	0.865	0.160	0.994	0.113	377.8	0.685	0.263	0.990	0.060

Table 8 Regression analysis for the commodity futures risk premium

This table presents the results of the regression analysis of the total risk premium computed from the estimated term structure model on factor risks. The dependent variable is the average of the estimated total risk premium for each maturity. Column 1 presents the results for the entire period, Column 2 represents the results for the period after financialization (post-2004), and Column 3 displays the results for the period before financialization (pre-2004). All independent variables are defined in the Appendix. All regressions include year and commodity fixed effects. t-statistics based on standard errors double-clustered at the commodity and year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Whole	Post 2004	Pre 2004
S&P GSCI	0.019**	0.022***	0.004
	(2.79)	(4.48)	(0.16)
Momentum	−0.006	0.012	−0.012
	(−0.81)	(1.09)	(−0.78)
Basis	−0.017*	−0.020	−0.014
	(−1.75)	(−0.97)	(−0.83)
Basis-Mom	0.001	−0.009	0.009
	(0.07)	(−0.42)	(0.40)
Skewness	−0.001	−0.002	0.003
	(−0.09)	(−0.09)	(0.19)
Inflation Beta	0.001	−0.002	0.022**
	(0.25)	(−0.23)	(2.45)
Volatility	−0.011	0.000	−0.036***
	(−1.19)	(0.02)	(−3.05)
Hedging Pressure	−0.025*	−0.023	−0.024
	(−1.86)	(−1.32)	(−0.63)
Order Imbalance	−0.001	0.015	−0.025
	(−0.09)	(0.78)	(−1.16)
Value	0.008	0.021***	−0.007
	(1.28)	(3.00)	(−0.62)
Constant	0.164***	0.243***	0.058***
	(91.46)	(42.71)	(48.37)
Commodity Fixed	Yes	Yes	Yes
Year Fixed	Yes	Yes	Yes
Observations	7113	3867	3054
Adjusted R^2	0.107	0.120	0.143

Table 9 Regression analysis for the spot premium

This table presents the results of the regression analysis of the spot risk premium computed from the estimated term structure model on factor risks. The spot risk premium represents the risk premium of the first nearby futures contract. Column 1 presents the results for the entire period, Column 2 represents the results for the period after financialization (post-2004), and Column 3 displays the results for the period before financialization (pre-2004). All independent variables are defined in the Appendix. All regressions include year and commodity fixed effects. t-statistics based on standard errors double-clustered at the commodity and year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Whole	Post 2004	Pre 2004
S&P GSCI	0.000	0.000	0.000
	(0.87)	(0.89)	(0.78)
Momentum	0.001**	0.000	0.001*
	(2.57)	(1.21)	(1.89)
Basis	0.000	0.000	−0.000
	(0.03)	(0.19)	(−0.41)
Basis-Mom	−0.001	−0.000	−0.001*
	(−1.46)	(−0.17)	(−1.89)
Skewness	−0.000	0.001	−0.001
	(−0.03)	(1.20)	(−1.44)
Inflation Beta	0.000	0.000	0.000*
	(0.95)	(0.33)	(1.96)
Volatility	0.000**	0.000*	0.001***
	(2.21)	(1.78)	(4.86)
Hedging Pressure	0.000	0.001	−0.000
	(0.51)	(1.09)	(−0.41)
Order Imbalance	0.000	0.001	0.000
	(0.85)	(1.35)	(0.62)
Value	0.000	0.000	0.000
	(0.93)	(1.29)	(0.69)
Constant	−0.002***	−0.006***	0.003***
	(−61.99)	(−74.55)	(48.00)
Commodity Fixed	Yes	Yes	Yes
Year Fixed	Yes	Yes	Yes
Observations	7113	3867	3054
Adjusted R^2	0.242	0.344	0.140

Table 10 Regression analysis for the term premium

This table presents the results of the regression analysis of the term premiums computed from the estimated term structure model on factor risks. For each maturity, the term premium is calculated by subtracting the spot risk premium from the total risk premium. The dependent variable in Column 1 represents the average term premium from the second to the sixth maturity, in Column 2 from the seventh to the 12th maturity, in Column 3 from the 13th to the 24th maturity, and in Column 4 from the 25th to the 36th maturity. All independent variables are defined in the Appendix. All regressions include year and commodity fixed effects. t-statistics based on standard errors double-clustered at the commodity and year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	TP(2-6)	TP(7-12)	TP(13-24)	TP(25-26)
S&P GSCI	0.004	0.003**	0.008**	0.008***
	(1.74)	(2.19)	(2.90)	(3.03)
Momentum	0.000	−0.001	−0.002	−0.002
	(0.45)	(−0.73)	(−0.90)	(−0.73)
Basis	−0.002	−0.003	−0.006*	−0.007*
	(−1.06)	(−1.29)	(−1.85)	(−2.03)
Basis-Mom	−0.004	−0.000	0.001	0.001
	(−1.13)	(−0.04)	(0.11)	(0.18)
Skewness	0.000	−0.001	−0.000	−0.001
	(0.02)	(−0.19)	(−0.02)	(−0.13)
Inflation Beta	0.001	0.000	0.000	0.001
	(0.71)	(0.24)	(0.24)	(0.31)
Volatility	−0.001	−0.002	−0.004	−0.004
	(−0.60)	(−1.24)	(−1.13)	(−1.22)
Hedging Pressure	−0.004	−0.005*	−0.009*	−0.009*
	(−1.50)	(−1.94)	(−1.84)	(−1.80)
Order Imbalance	−0.002	−0.000	−0.000	−0.001
	(−0.51)	(−0.09)	(−0.01)	(−0.12)
Value	0.003*	0.002	0.003	0.003
	(1.85)	(1.22)	(1.26)	(1.27)
Constant	0.021***	0.030***	0.062***	0.067***
	(37.38)	(77.71)	(85.07)	(107.82)
Commodity Fixed	Yes	Yes	Yes	Yes
Year Fixed	Yes	Yes	Yes	Yes
Observations	7113	7113	7113	7113
Adjusted R^2	0.107	0.133	0.097	0.149

Table 11 Subsample regression analysis for the term premium

This table presents the results of the regression analysis of the term premiums computed from the estimated term structure model on factor risks for the entire period, post-financialization, and pre-financialization periods. For each maturity, the term premium is calculated by subtracting the spot risk premium from the total risk premium. The dependent variables in Columns 1 and 5 represent the average term premium from the second to the sixth maturity, in Columns 2 and 6 from the seventh to the 12th maturity, in Columns 3 and 7 from the 13th to the 24th maturity, and in Columns 4 and 8 from the 25th to the 36th maturity. Columns 1-4 represent the results for the post-financialization period and Columns 5-8 display those for the pre-financialization period. All independent variables are defined in the Appendix. All regressions include year and commodity fixed effects. *t*-statistics based on standard errors double-clustered at the commodity and year level are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Post 2004				Pre 2004			
	TP(2-6)	TP(7-12)	TP(13-24)	TP(25-26)	TP(2-6)	TP(7-12)	TP(13-24)	TP(25-26)
S&P GSCI	0.004**	0.003**	0.009***	0.009***	0.001	0.001	0.001	0.001
	(2.15)	(2.51)	(5.46)	(4.94)	(0.31)	(0.23)	(0.12)	(0.14)
Momentum	0.003	0.002	0.004	0.005	0.001	−0.002	−0.005	−0.005
	(1.40)	(0.96)	(1.09)	(1.14)	(0.19)	(−0.82)	(−0.78)	(−0.72)
Basis	−0.003	−0.004	−0.008	−0.007	−0.003	−0.003	−0.005	−0.006
	(−0.65)	(−0.91)	(−1.04)	(−0.93)	(−0.76)	(−0.66)	(−0.79)	(−0.96)
Basis-Mom	−0.004	−0.002	−0.003	−0.003	−0.003	0.001	0.004	0.005
	(−1.37)	(−0.41)	(−0.42)	(−0.36)	(−0.50)	(0.13)	(0.48)	(0.56)
Skewness	0.002	−0.001	0.000	−0.002	−0.001	0.001	0.001	0.002
	(0.48)	(−0.18)	(0.06)	(−0.22)	(−0.39)	(0.25)	(0.11)	(0.26)
Inflation Beta	−0.000	−0.001	−0.001	−0.000	0.005**	0.005*	0.008**	0.008**
	(−0.02)	(−0.37)	(−0.24)	(−0.10)	(2.21)	(2.12)	(2.48)	(2.64)
Volatility	0.001	0.000	0.000	−0.000	−0.005	−0.007***	−0.013***	−0.014***
	(0.43)	(0.11)	(0.06)	(−0.10)	(−1.62)	(−3.11)	(−2.99)	(−2.97)
Hedging Pressure	−0.003	−0.005	−0.008	−0.008	−0.008	−0.006	−0.009	−0.008
	(−0.64)	(−1.52)	(−1.25)	(−1.21)	(−1.00)	(−0.76)	(−0.59)	(−0.56)
Order Imbalance	0.002	0.003	0.006	0.006	−0.005	−0.005	−0.009	−0.010
	(0.63)	(0.74)	(0.84)	(0.78)	(−1.11)	(−1.10)	(−1.07)	(−1.24)
Value	0.004**	0.004**	0.008***	0.008**	0.001	−0.001	−0.003	−0.003
	(2.24)	(2.87)	(3.00)	(2.85)	(0.38)	(−0.32)	(−0.74)	(−0.75)
Constant	0.022***	0.043***	0.091***	0.101***	0.017***	0.012***	0.022***	0.021***
	(16.82)	(36.96)	(41.61)	(49.25)	(27.33)	(29.24)	(61.89)	(50.43)
Commodity Fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3867	3867	3867	3867	3054	3054	3054	3054
Adjusted R^2	0.090	0.161	0.110	0.131	0.160	0.121	0.119	0.276

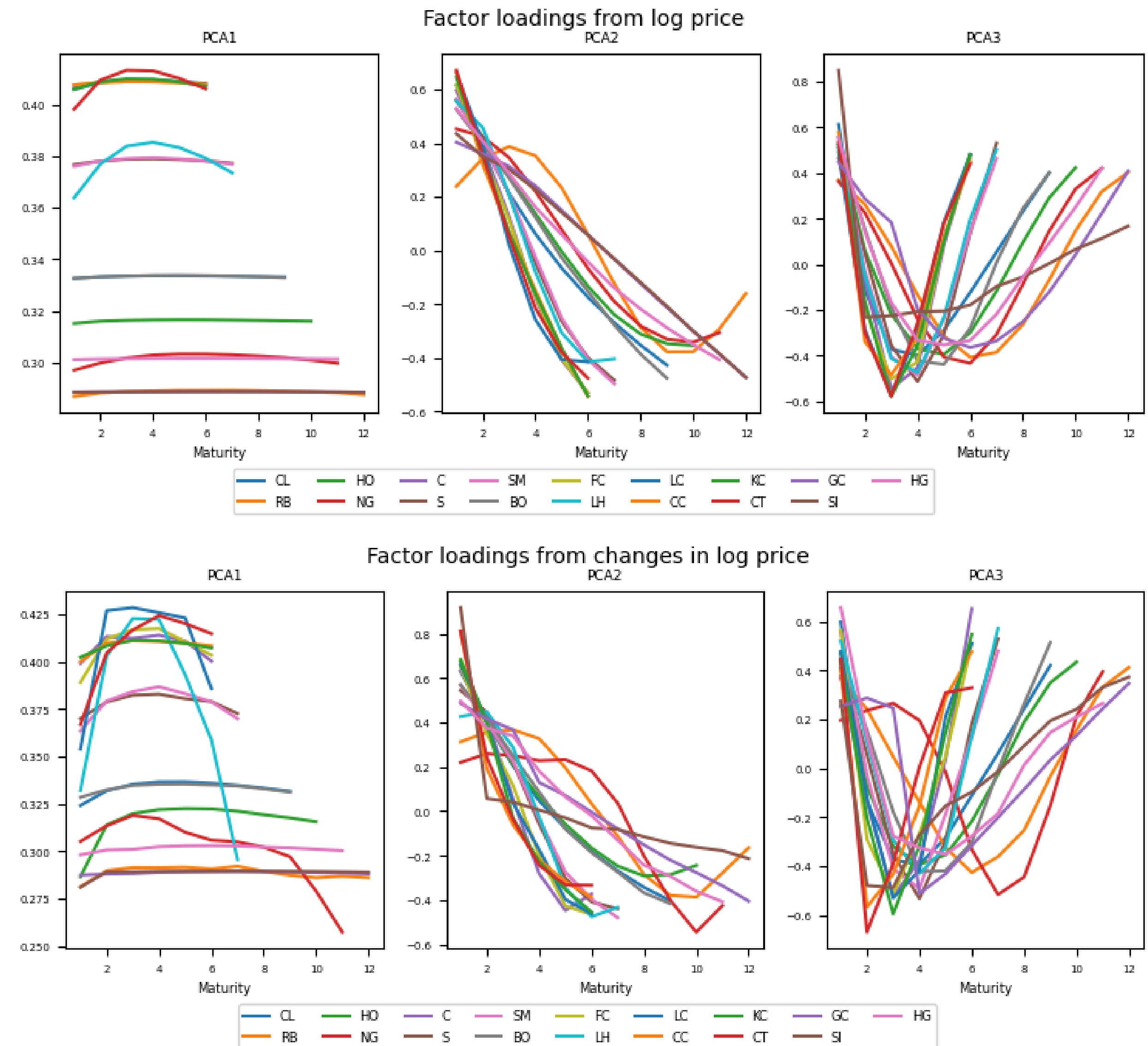

Figure 1 Factor loadings from the principal component analysis. These figures show the factor loadings of PCA for each commodity futures. The upper three panels illustrate the factor loadings of PCA for log prices, while the bottom three panels represent the factor loadings of PCA for changes in log prices. The first column shows the factor loadings for the first factor, the second column for the second factor, and the third column for the third factor.

Figure 2 Time series of fitted and observed prices. These figures illustrate the time series of the observed and the fitted prices estimated by the term structure model for selected commodity futures. For each commodity category, the left column illustrates graphs for the commodities with the lowest RMSE, while the right column shows those with the highest RMSE. The solid lines represent the observed log prices, and the dashed lines correspond to the fitted log prices.

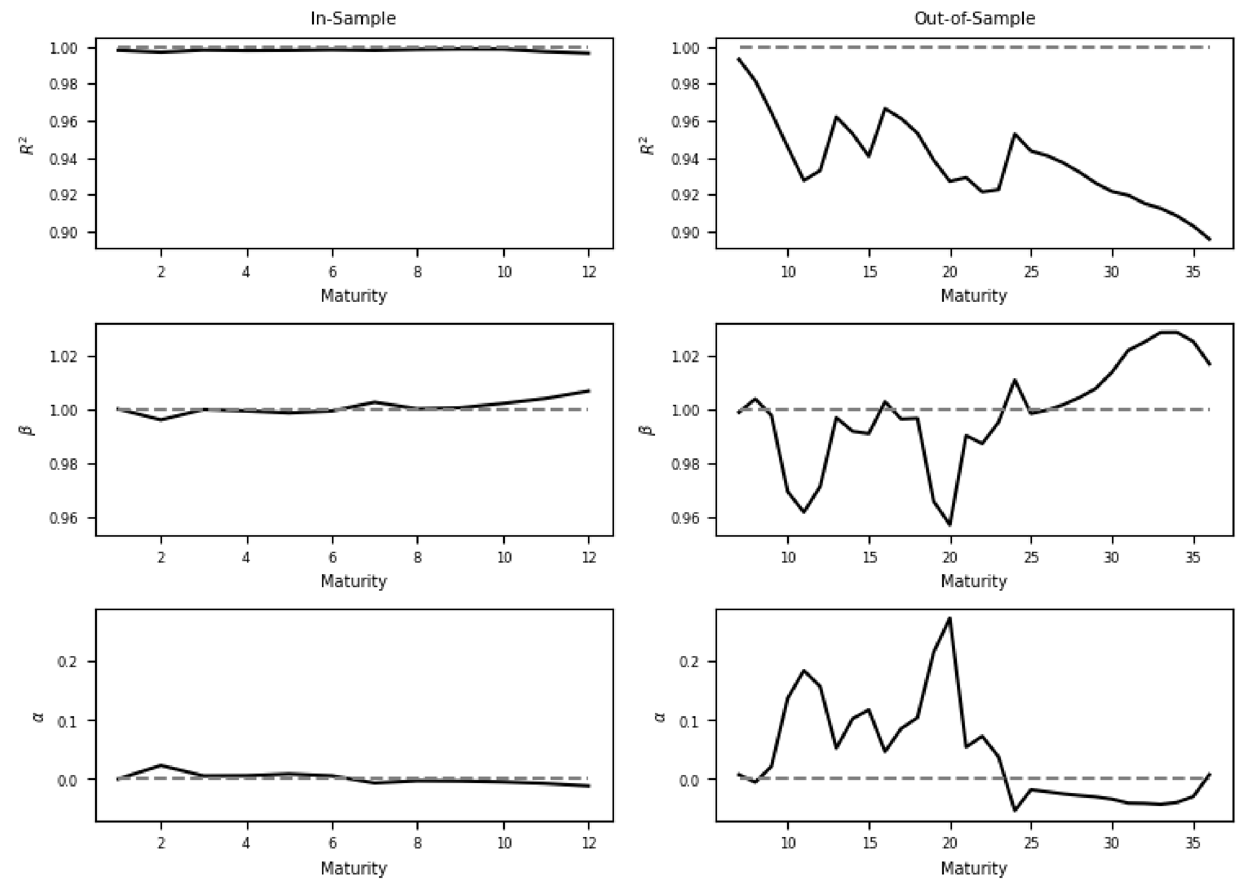

Figure 3 Cross-sectional averages of time series regression parameters. These figures present the average coefficients and R-squares resulting from the time series regression of observed prices on fitted prices. The time-series regression is performed for each commodity futures and maturity. The left panel illustrates the in-sample results, while the right panel depicts the out-of-sample outcomes. The first row displays the averages of R-squares(R^2), the second row indicates the averages of the coefficients (β) for the fitted prices, and the third row presents the averages of the intercepts (α) of the regressions. Solid lines represent the averages of each variable. The dashed line in the first row represents a value of one, in the second row represents one, and in the third row represents zero. The "In-sample" refers to the sample period used for term-structure estimation (up to "Obs.Mat."). The "out-of-sample" denotes the sample period from the next maturity following the one used for the term structure estimation, up to any maturities with at least 100 observations (from "Obs.Mat."+1 to "Max.Mat.").

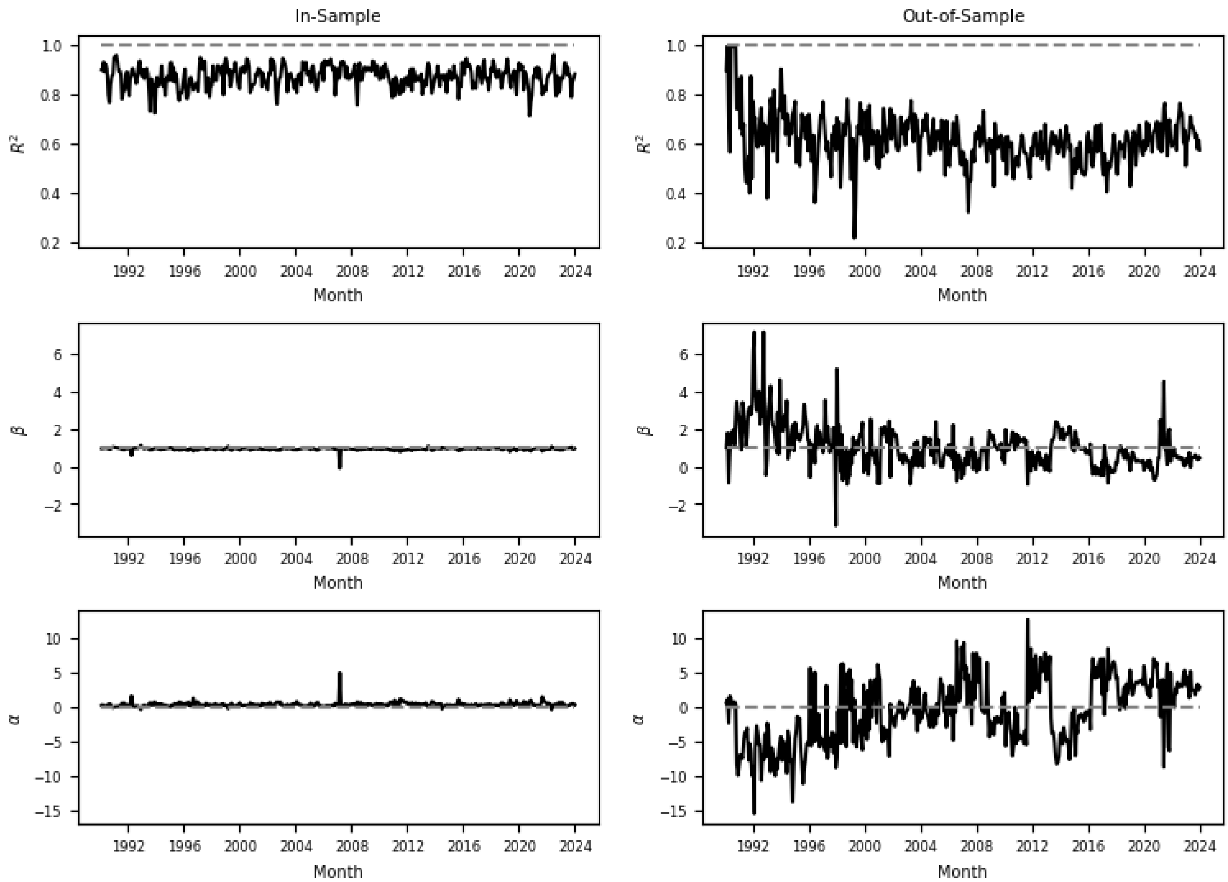

Figure 4 Time series averages of cross-sectional regression parameters. These figures present the average coefficients and R-squares resulting from the cross-sectional regression of observed prices on fitted prices. The cross-sectional regression is performed for each commodity futures and date. The left panel illustrates the in-sample results, while the right panel depicts the out-of-sample outcomes. The first row displays the averages of R-squares(R^2), the second row indicates the averages of the coefficients (β) for the fitted prices, and the third row presents the averages of the intercepts (α) of the regressions. Solid lines represent the averages of each variable. The dashed line in the first row represents a value of one, in the second row represents one, and in the third row represents zero. The "In-sample" refers to the sample period used for term-structure estimation (up to "Obs.Mat."). The "out-of-sample" denotes the sample period from the next maturity following the one used for the term structure estimation, up to any maturities with at least 100 observations (from "Obs.Mat."+1 to "Max.Mat.").

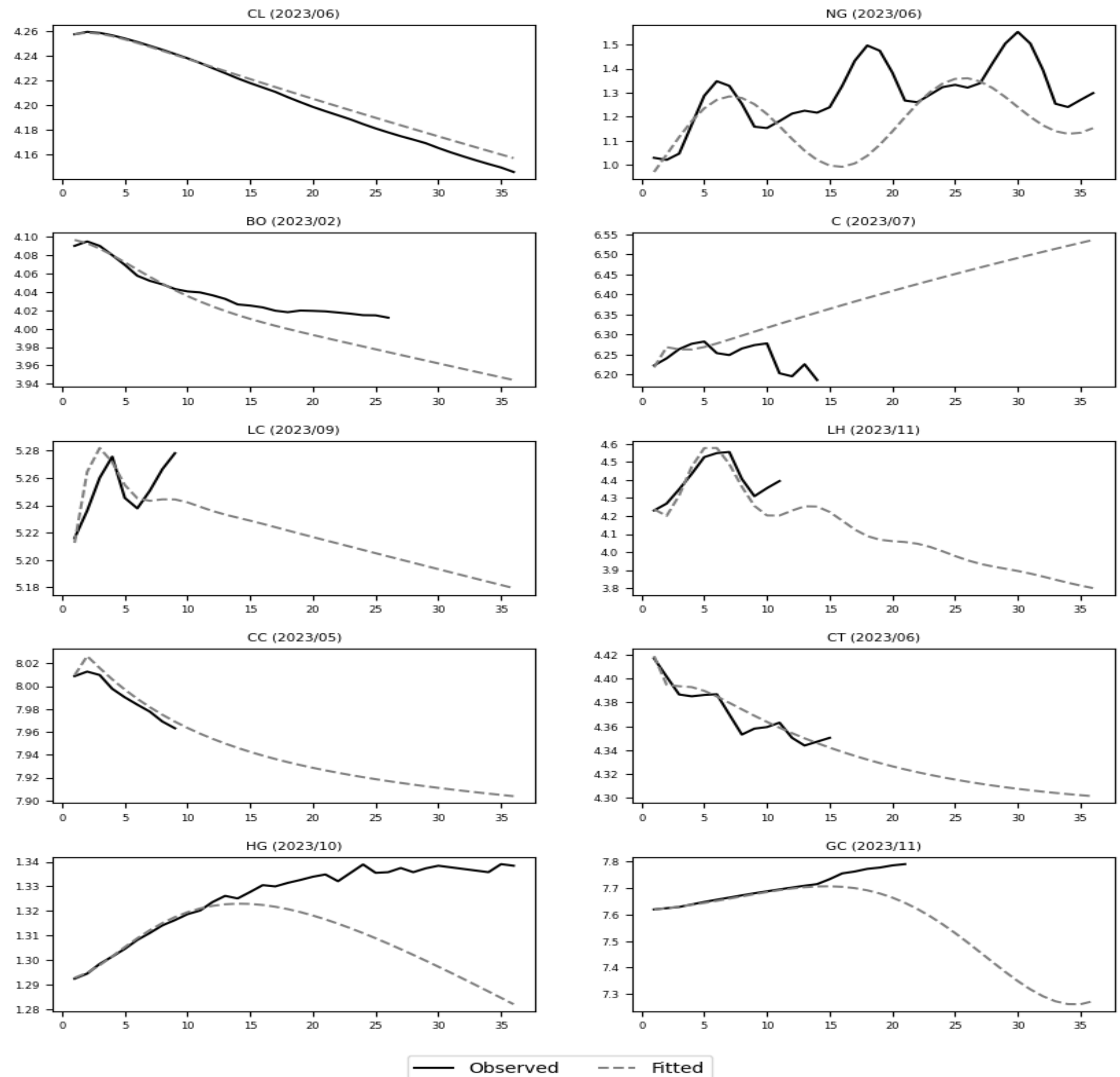

Figure 5 Term structure of fitted and observed prices. These figures depict the fitted and observed term structures for a specific month in 2023. The selected month for each commodity futures corresponds to the month with the median value of R-squares from cross-sectional regressions of each month during 2023. For each commodity category, the left column illustrates graphs for the commodities with the lowest RMSE, while the right column shows those with the highest RMSE. The solid lines represent the observed log prices, and the dashed lines correspond to the fitted log prices.

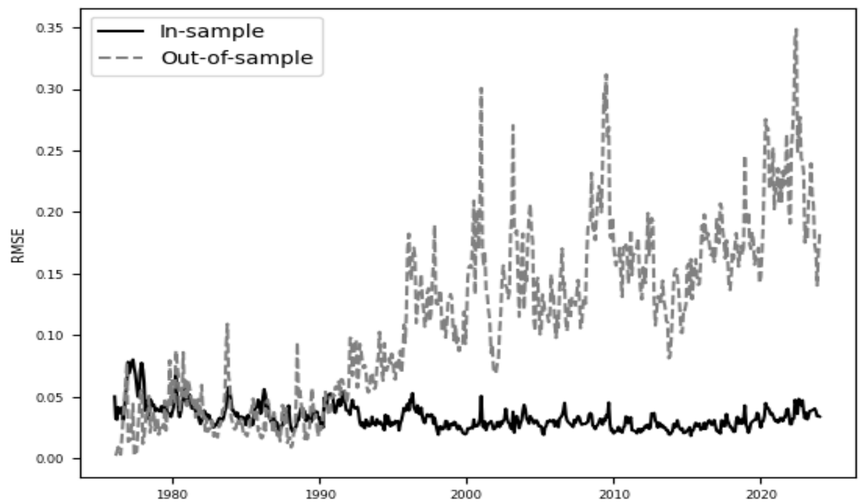

Figure 6 Average root mean squared errors over time. This figure illustrates a time series plot of the average root mean square errors (RMSE) of pricing errors. The RMSE for each futures contract is calculated by using all available maturities with at least 100 observations for each date. The solid lines represent the in-sample RMSE, and the dashed lines correspond to the out-of-sample RMSE.

Appendix A. Commodity risk factors

Let $F_t^{(n)}$ be the futures price for delivery at time $t+n$ at time t. The monthly excess returns at month $t+1$ are defined as $R_{t+1}^{(n-1)} = F_{t+1}^{(n-1)}/F_t^{(n)} - 1$. The definitions of variables follow Sakkas and Tessaromatis (2020).

- Momentum: the cumulative excess returns of nearby futures contracts from the prior 12 months such that $\prod_{s=t-11}^{t}(1+R_s^{(1)}) - 1$.

- Basis: the price difference between the first- and the second-nearby futures such that $F_t^{(2)}/F_t^{(1)} - 1$.

- Basis-Mom: the difference between momentum in a first- and second-nearby futures such that $\prod_{s=t-11}^{t}\left(1+R_s^{(1)}\right) - \prod_{s=t-11}^{t}\left(1+R_s^{(2)}\right)$ (Boons and Prado, 2019).

- Skewness: Pearson's moment coefficient of skewness at t using the daily first-nearby futures returns of the previous 12-months such that $\frac{\frac{1}{D}\sum_{d=1}^{D}(R_d^{(1)}-\mu_t)^3}{\sigma_t^3}$ where D is the number of daily observations, μ_t is the mean of returns, and σ_t is the standard deviation of returns (Fernandez-Perez et al., 2018).

- Inflation Beta: the coefficient of the following regression of first-nearby futures on unexpected inflation changes (ΔCPI) in monthly inflation using the previous 60-month observations (Szymanowska et al., 2014)

$$R_s^{(1)} = \alpha + \beta_t^{CPI}\Delta CPI_s + e_s \qquad s = t-59, \cdots, t$$

- Volatility: the coefficient of variation, i.e., the variance divided by absolute mean first-nearby futures returns during the prior 36 months such that $\sigma_t^2/|\mu_t|$ (Dhume, 2011; Szymanowska et al., 2014).

- Hedging Pressure: the difference between the number of short and number of long open interests divided by the sum of open interests by commercial traders during the last 12 months (Kang et al., 2020) such that

$$\sum_{i=0}^{11}\frac{Short_{t-i} - Long_{t-i}}{Short_{t-i} + Long_{t-i}}$$

- Order Imbalance: the monthly change in total open interest such that $OI_t - OI_{t-1}$ (Hong and Yogo, 2012; Szymanowska et al., 2014).

- Value: average futures price of the first nearby futures contracts from 4.5 to 5.5 years ago divided by the futures price of the first nearby futures contract at time t (Asness et al., 2013) such that $\bar{F}_{t-54,t-66}^{(1)}/F_t^{(1)} - 1$ where $\bar{F}_{t-54,t-66}^{(1)}$ is the average of the the first nearby futures from 4.5 to 5.5 years.