

Tug of War in Corporate Environmental Lobbying*

Byeong-Je An[†] Hyun-Soo Choi[‡] Hugh Hoikwang Kim[§]

Paul Youngwook Kim[¶]

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Abstract

We investigate how firms with opposing environmental stances can engage in political competition to influence policymakers' decisions through lobbying. Our theoretical model predicts that these firms end up spending more on lobbying due to political competition, especially when the climate policy uncertainty is higher. We empirically test the lobbying “tug of war” using U.S. corporate lobbying data. Importantly, excessive lobbying increases the cost of capital and decreases a firm's R&D and capital investment. Overall, our study highlights the hidden costs of intense lobbying competition among firms with polarized environmental interests.

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[†]Fowler College of Business, San Diego State University (e-mail: ban@sdsu.edu)

[‡]Korea Advanced Institute of Science and Technology (e-mail: hschoi19@kaist.ac.kr)

[§]Darla Moore School of Business, University of South Carolina (e-mail: hugh.kim@moore.sc.edu)

[¶]Korea Advanced Institute of Science and Technology (e-mail: ywkim9424@kaist.ac.kr)

1. Introduction

Corporate lobbying serves as an important avenue for companies to influence policy decisions, ensuring they align with their business interests. With a global push for environmentally sustainable business practices, governments introduce environmental policies while firms intensify their efforts to sway policymakers in favor of their business interests. Due to the differing environmental impacts of firms' operations, firms with conflicting environmental interests often engage in political competition to influence policymakers.

The literature on corporate lobbying is extensive. So far, however, there has been little analysis of the strategic aspect of corporate lobbying competition among firms with opposing environmental stances. In this paper, we theoretically and empirically explore how firms can engage in political competition, where success hinges on exerting marginal efforts exceeding those of competitors. With increasing interest in environmental policies and growing political polarization around climate policies, the tug-of-war among firms with opposing environmental stances is likely to become more intense, and firms will incur substantial costs in both direct and indirect ways. Thus, understanding the strategic mechanism and financial outcome of corporate lobbying competition is crucial.

An anecdotal example of political competition occurs between Tesla, Inc. and other automakers in 2023. When the U.S. Environmental Protection Agency (EPA) floated the idea of setting tougher emission guidelines for new vehicle models from 2027 to 2032, firms in the U.S. auto industry reacted in markedly different ways. Tesla, a leading EV manufacturer, welcomed the initiative and further proposed an accelerated adoption of EVs. However, other firms criticized the EPA's initial guideline as a "neither reasonable nor achievable" goal and proposed a more moderate pace of EV adoption (Reuters (2023)). Both sides approached politicians through their lobbying channels to increase the chance that their preferred proposal is adopted.¹

¹The Lobbying Report (LD-2), reported to the House of Representatives and the Senate, show that both sides spent lobbying expenditures on this issue. The final rule implemented in early 2024 is more moderate than the initial proposal, postponing the starting year of emission reduction from 2027 to 2030 (Financial

In this paper, we first build a model of lobbying decisions among firms with varying environmental stances. In the first stage of the model, firms propose a desired level of climate policy to be implemented and spend lobbying expenditures to influence a policymaker’s decision. A key feature of the model is that a policymaker is likely to implement environmental policies favorable to the firm with the greatest lobbying expenditure. However, the outcome of lobbying is not certain because the policymaker does not exactly follow the proposal of the winning firm, and there exists some uncertainty about the stringency of the implemented policy.

In the second stage of the model, firms compete in the product market by Cournot under the climate policy implemented in the first stage. Depending on a firm’s environmental stance, the implemented policy differently affects the firm’s marginal cost of production. Specifically, a firm’s marginal cost of production is lower when the firm’s stance aligns with the implemented policy (e.g., a green firm’s cost of production is lower when a stringent pro-environmental policy is implemented) but higher otherwise. Therefore, a firm needs to consider the cost and benefit of lobbying expenditures; higher expenditures incur higher costs in the first stage but increase the chance that a more favorable policy is implemented, decreasing the cost of production in the second stage.

One key prediction of the model is a U-shaped relation between corporate environmental stances and lobbying expenditures. In other words, firms at the polarized ends of the environmental spectrum—Brown firms and Green firms—spend more on lobbying to counteract the efforts of firms on the opposite side, leading to excessive lobbying expenditures termed the “lobbying tug-of-war” effect.

Another prediction of the model is that the tug-of-war effect becomes more pronounced when the policymakers’ decision-making is more uncertain. A priori, the expected impact of climate policy uncertainty on environmental lobbying is ambiguous due to two opposing dynamics at play. When a firm aligns with policymakers upon winning a lobbying tug-of-war, it

Times (2024)).

enjoys monopolistic advantages from reduced production costs under favorable climate policies. Therefore, during periods of high climate policy uncertainty, firms may find it beneficial to propose more extreme policies and increase lobbying efforts to secure these advantages. Conversely, higher climate policy uncertainty may discourage firms from initiating aggressive lobbying against opponents. This reluctance arises from the increased risk of wasting lobbying resources if the firm’s stance misaligns with policymakers, even after winning the lobbying tug-of-war. The model solution indicates that the first mechanism, emphasizing the pursuit of monopolistic benefits, dominates the second mechanism, suggesting that firms are more inclined to increase lobbying efforts in the face of higher climate policy uncertainty.

Importantly, our model predicts that the tug-of-war in environmental lobbying can elevate a firm’s cost of capital. As policy uncertainty heightens, two forces come into play. First, for a given level of lobbying expenditure, the probability of losing a lobbying tug-of-war increases. While it may seem initially counterintuitive, as Green or Brown firms boost their lobbying efforts during high policy uncertainty, this often triggers a reciprocal response from opponents, intensifying the lobbying competition. Consequently, the likelihood of losing the lobbying tug-of-war actually rises. Furthermore, the potential cost of losing the tug-of-war escalates as the implemented policy proposed by the competitor becomes more unfavorable. These two dynamics combined contribute to an increase in a firm’s cost of capital as policy uncertainty rises. This political competition risk is distinct from other forms of political risk firms encounter, such as termination of procurement contracts or economic policy uncertainty. Termed “lobby-induced political risk,” it emerges specifically due to the competitive lobbying environment shaping policymakers’ decisions.

We empirically test the model’s predictions using mandatory lobbying reports filed by U.S. lobbying entities. The U.S. firms have been filing “Lobbying Report (LD-2)” mandated by the Lobbying Disclosure Act of 1995. This law requires firms to report detailed information about their lobbying activities, including client firms, lobbyists, lobbying issues, and lobbying expenditures. We construct U.S. firms’ lobbying activities related to environmental issues

using subject areas and keywords for environmental issues in the reports from 2003 to 2019.

We also construct a firm’s environmental stance using carbon emissions data from Trucost. We posit that firms with lower carbon emissions (Green-side firms) lobby for strengthening pro-environmental agendas, while those with higher carbon emissions (Brown-side firms) lobby for the opposite. We validate this premise using a subsample of the firms with available information about their lobbying directions. For our empirical analysis, we control for various firm-level characteristics with year and firm fixed effects.

Our identification strategy exploits the varying intensity of the lobbying competition between Green and Brown firms over time. A firm’s operational characteristics and environmental stances are likely stable over time; thus, the varying intensity of competition is primarily driven by the external political environment. The tug-of-war effect is more distinctively identified during periods when contentious environmental bills are proposed, elevating the level of competition. We define periods of high political competition based on the level of climate policy uncertainty, measured by the Climate Policy Uncertainty (CPU) index (Gavriliadis (2021)).² We estimate a regression of a firm’s lobbying activity on the interaction term between a firm’s polarized environmental stance (*Polarized ENV. Interests*) and *High CPU*, an indicator variable for the high CPU period. Our theoretical model predicts that firms in polarized environmental stances are more likely to engage in the lobbying tug-of-war and spend more on lobbying when climate policy uncertainty is high.

The empirical analysis confirms that firms at the polarized ends of the environmental spectrum engage more in the lobbying tug-of-war when the CPU is high. The lobbying expenditure of these firms increases by 7% of one standard deviation of lobbying expenditure when the CPU is high. We also find that the increase in lobbying expenditure and probability of lobbying is not driven solely by Green-side or Brown-side firms; both groups increase their lobbying.

We also confirm the moderating effect of a firm’s product market competition on the

²This approach adopts the textual search on news articles of Baker et al. (2016) for the uncertainty in climate policies.

lobbying tug-of-war, as our model predicts. The tug-of-war effect is more pronounced when the product market competition is more intense.

To address potential endogeneity concerns about the CPU measure, we use the changes in the U.S. Environmental Protection Agency (EPA) administrators affiliated with different political parties as an exogenous shock to climate policy uncertainty. The EPA administrator plays a critical role in determining the direction and stringency of environmental regulations. Given the differing stances of the Republican and Democratic parties on environmental regulations, changes in the EPA administrator from different political parties are likely to increase climate policy uncertainty, particularly during the initial years following such changes, thus meeting the relevance condition. These changes are generally not driven by corporate environmental lobbying but are commonly associated with changes in the U.S. presidency, which tend to impact overall economic policy uncertainty. By controlling for overall economic policy uncertainty in our analysis, we posit that changes in the EPA administrator affiliated with a different party are not directly linked to corporate environmental lobbying, thereby satisfying the exclusion restriction condition. Using this instrumental variable, we reconfirm our earlier finding that firms at the polarized ends of the environmental spectrum are more likely to engage in and spend more on environmental lobbying when climate policy uncertainty is high.

As the strategic aspect of lobbying tug-of-war increases the political risk a firm faces, we evaluate its effect on firms' cost of capital. Using the implied cost of capital based on five different methodologies in the literature (Easton (2004), Gode and Mohanram (2003), Gordon and Gordon (1997), Gebhardt et al. (2001) and Claus and Thomas (2001)), we find that the cost of capital increases for firms engaging in the lobbying tug-of-war during high CPU periods. The magnitude of the effect is estimated at about 3% of the sample average.

The result is robust when controlling for the general political risks a firm faces. While the lobbying tug-of-war may correlate with general political risk associated with the firm's cost of capital (Grotteria (2024)), our model predicts that lobby-induced political risk is a distinct factor elevating the firm's cost of capital. Therefore, we additionally control for firms' general

political risk using standardized firm-level political risk defined by Hassan et al. (2019) and find that the implied cost of capital still increases for firms engaged in the lobbying tug-of-war.

The increase in the implied cost of capital among firms in the lobbying tug-of-war raises the question of whether political competition negatively affects firms' investments. When political competition affects a firm's cost of capital, the real cost can be substantial compared to the dollar amount of lobbying. We find that firms in the tug-of-war competition significantly reduce their R&D, CAPEX, and total investments. Note that we identify the distinct effect of lobby-induced political risk by controlling for general political risk (Grotteria (2024)). These results suggest that the total cost of the tug-of-war in corporate environmental lobbying can be much more substantial than direct lobbying expenditures. The lobbying tug-of-war can negatively affect a firm's long-term viability.

Our research contributes to the literature on the effect of lobbying and political connections on firms' performance. Borisov et al. (2016) find the positive value of corporate lobbying by showing that firm value decreases after an exogenous shock limiting a firm's ability to lobby. Similarly, some prior research find a positive firm performance through political connections (Faccio et al. (2006), Goldman et al. (2009), Cooper et al. (2010), Correia (2014), Lee et al. (2014), Akey (2015), Lambert (2019), Brown and Huang (2020), Heitz et al. (2021)). However, Bertrand et al. (2018) find that political connections can be costly to firms without much benefit. Delmas et al. (2016) find a U-shape pattern in firms' greenhouse gas emissions and lobbying expenditures, highlighting the intensified lobbying competition among firms with opposing interests. Kang (2016) argues that lobbying activities by opposing parties can dampen the effect of firms' lobbying. Grotteria (2024) find that political competition can also increase the cost of capital, reducing firms' R&D investments. Unlike Grotteria (2024) where the outcome of lobbying efforts is independent of competing firms' lobbying expenses, we incorporate the strategic aspect of lobbying into the model—a firm's probability of winning depends on its relative size of lobbying efforts compared to its competitor. This can lead to excessive lobbying expenditures (i.e., lobbying tug-of-war) and an increase in the cost of

capital due to the political uncertainty the lobbying itself brings about, negatively affecting firms' real activities, such as R&D and capital investment.

This paper also contributes to the literature on the real cost of implementing ESG practices. There is an extensive body of literature discussing the effect of ESG on firm value (Hong and Kacperczyk (2009), Di Giuli and Kostovetsky (2014), Ferrell et al. (2016), Lins et al. (2017), Albuquerque et al. (2019)). However, no prior studies explore the cost of engaging in political competition to influence the ESG regulatory environment. We find that the strategic aspect of firms' environmental stances is an important determinant of their lobbying activities. Excessive political competition can negatively affect other business activities, such as long-term investments and R&D, adding to the cost of implementing ESG practices.

The rest of the paper is organized as follows. Section 2 describes the model of tug-of-war in corporate environmental lobbying. Section 3 details our data. Section 4 presents empirical tests of the model's predictions. Section 5 concludes with a discussion of policy implications.

2. Theoretical Model and Predictions

2.1. Model Setup

We develop a two-period model of competitive environmental lobbying. There are three types of firms, Green (G), Neutral (N), and Brown (B), and one policymaker in the model. At $t = 0$, each firm $i \in \{G, N, B\}$ simultaneously chooses lobbying expenditure $s_i \geq 0$ and proposes a target climate policy level $y_i \in \mathbb{R}$. Without loss of generality, we interpret positive y as the sustainable climate policy that benefits Green firms and negative y as the unsustainable climate policy that benefits Brown firms. Upon receiving lobbying s_i from three firms, the policymaker determines the winner with the largest lobby amount, $w = \arg \max_i s_i$, and implements the winner's desired policy y_w with some uncertainty. As the implemented policy affects firms' marginal cost of production at $t = 1$, firms compete through lobbying expenditures s_i to secure their desired policies.

To propose its desired policy, a firm needs to pay an upfront cost at $t = 0$. Building on Hirsch and Shotts (2015), we set an upfront cost as $L(s_i, y_i) = \alpha(s_i + \gamma y_i^2)$, where α is the marginal cost of lobbying and γ measures the marginal cost of proposing a more extreme policy.³ The s_i term captures direct monetary expenses related to lobbying, and the y_i^2 term captures the implicit non-monetary costs for firms when proposing a policy deviating from the neutral point. For example, non-monetary costs include the additional operating costs for Green firms, such as a higher R&D investment and adjustment to the production line for pro-environmental operations, and the litigation risk for Brown firms.

We assume that the winner's policy is not always implemented exactly as the winner desires. Specifically, while the direction of the policy follows the winner's desired policy (y_w), the implemented policy can be more or less extreme than the winner's proposed policy; $\tilde{y} = \tilde{\beta} y_w$, where $\tilde{\beta} = 1 + \sqrt{\Delta}$ with probability 0.5 and $\tilde{\beta} = 1 - \sqrt{\Delta}$ with probability 0.5. We assume that the expectation of $\tilde{\beta}$ is equal to 1, indicating that the policymaker is expected to be unbiased, and the variance of $\tilde{\beta}$ is equal to Δ , which we interpret as the degree of climate policy uncertainty.

At $t = 1$, given the implemented policy \tilde{y} , the three firms engage in Cournot competition to maximize profit. The marginal cost of production c_i is given by

$$c_i = K - x_i \tilde{y},$$

where $x_i = x > 0$, 0, and $-x$ for $i = G, N, B$, respectively, and \tilde{y} is the equilibrium policy level determined at $t = 0$. In this setting, the Green firm benefits from the Green policy (i.e., $\tilde{y} > 0$) because its production cost (c_i) decreases as a positive policy is implemented. Likewise, the Brown firm benefits from a negative policy level. Thus, the model incorporates the political

³Hirsch and Shotts (2015) derive a similar form of upfront cost from the policymaker's utility s as $u - y^2$, where u is the policy quality, and y is the ideology of a policy. For the indifferent utility of the policymaker, policy quality should exponentially increase with the extremeness of the policy. Firms' policy proposals can be summarized as u , which is a function of s and y . In our case, s corresponds to the lobbying amount and y corresponds to the level of proposed policy.

situation where firms' environmental stances conflict.⁴

Let q_i be the quantity of product produced by firm i . The price of the product is determined by quantities produced by the three firms,

$$P = a - \tilde{b} \sum_i q_i, \text{ where } a, \tilde{b} > 0,$$

where \tilde{b} ($\equiv b/\tilde{\theta}$) represents the inverse of the price sensitivity of demands, as a higher \tilde{b} indicates a lesser change in demand with respect to changes in price. This parameter \tilde{b} is jointly determined by the shock to the demand function ($\tilde{\theta}$) and the level of product market competition ($1/b$). A random variable $\tilde{\theta}$ exogenously shifts the price sensitivity of demand, which is realized at $t = 1$ as $\theta_h > 1$ with probability p and $\theta_l < 1$ with probability $1 - p$. We assume that $\tilde{\theta}$ is independent of $\tilde{\beta}$ and has the expectation of one. This implies that the expected price sensitivity of demand is $\mathbb{E}[\tilde{b}^{-1}] = b^{-1}$. Consequently, higher values of b suggest less competitive product marketson average, leading to a reduced price sensitivity in expectation.

Firm i maximizes its operating profit $v_i = (P - c_i)q_i$. To account for risk aversion, the operating profit is discounted by a stochastic discount factor \tilde{M} , defined as

$$\tilde{M} \equiv \frac{\widehat{\text{Pr}}(\tilde{\theta})}{\text{Pr}(\tilde{\theta})},$$

where $\widehat{\text{Pr}}(\tilde{\theta})$ represents the risk-neutral probability of demand shocks. The mean of this stochastic discount factor is set to one, thereby normalizing the risk-free rate to zero. To specify the risk-neutral probability of θ_h , we express it as:

$$\widehat{\text{Pr}}(\tilde{\theta} = \theta_h) = \frac{1 - \eta\theta_l}{\eta(\theta_h - \theta_l)},$$

⁴Grotteria (2024) considers that the goal of lobbying is acquiring more government contracts. In our model, however, firms try to influence government climate policy, which in turn can affect the cost of production.

where $\eta \in [1, 1/\theta_l)$ measures the level of the firm's risk aversion or risk price.⁵ Consequently, the discounted value of a demand shock is calculated as $\mathbb{E}[\tilde{M}\tilde{\theta}] = 1/\eta$.⁶

2.2. Model Solution

We first summarize the results of the second-period Cournot competition given the equilibrium policy level \tilde{y} , as proved in Lemma 1 in the Appendix. Firm i 's optimal quantity and profit are given by

$$q_i^* = \frac{a - K + 4x_i\tilde{y}}{4\tilde{b}} \text{ and } v_i = \frac{(a - K + 4x_i\tilde{y})^2}{16\tilde{b}}. \quad (1)$$

An important observation is that the optimal operating profit of the Neutral firm (v_N) is independent of the equilibrium policy level \tilde{y} since $x_N = 0$. Note that the Neutral firm's profit net of lobbying cost, $v_N - L(s_N, y_N)$ is always dominated by v_N , which is the profit when the Neutral firm does not lobby and proposes a neutral policy level, $s_N = y_N = 0$. In other words, the Neutral firm does not participate in lobbying and does not propose a policy deviating from the neutral point in equilibrium. Therefore, it is sufficient to analyze the competition between Green and Brown firms.

Additionally, it is noteworthy that the operating profit of both Green and Brown firms exhibits convexity with respect to the equilibrium policy level. For instance, the optimal quantity of the Green firm linearly increases with the pro-environmental tilt of the equilibrium policy, owing to its comparative advantage in marginal cost. Consequently, the profit per unit for the Green firm also shows a linear increase with the equilibrium policy level, resulting in convex total profit dynamics.

Based on these observations, we conjecture that Green and Brown firms use a mixed strategy over a common non-empty interval of lobbying amounts $[0, \bar{s}]$. Thus, a cumulative

⁵The risk-neutral probability of θ_h resembles the one utilized in the binomial option pricing model. However, a key distinction lies in the normalization of expected demand shocks to one, allowing η to capture the risk premium associated with exposure to demand shocks.

⁶A set of reasonable assumptions for $\tilde{\theta}$ is in Assumption 1 in the Appendix for the proof of the model.

distribution function (CDF) $F_i(s_i)$ over $[0, \bar{s}]$ and a unique proposed policy for each s_i , $y_i(s_i)$ characterize firms' strategies. Let $\Pi_i(s_i, y_i; y_{-i}, F_{-i})$ denote firm i 's expected profit net of lobbying cost given lobbying amount s_i , proposed policy y_i , and the opponent's lobbying strategy, $\{y_{-i}, F_{-i}\}$. Then, firm i 's expected profit at $t = 0$ is given by

$$\Pi_i(s_i, y_i; y_{-i}, F_{-i}) = -L(s_i, y_i) + F_{-i}(s_i) \cdot \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_i)] + \int_{s_i}^{\infty} \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_{-i}(s_{-i}))] dF_{-i} \quad (2)$$

The first term is the upfront cost of proposing a policy with level y_i and lobbying expenditure s_i . The second term is the risk-adjusted expected payoff when firm i is the winner, where $F_{-i}(s_i)$ is the probability that the opponent's lobbying amount is lower than firm i 's lobbying amount (i.e., the probability of winning), and $\mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_i)]$ is the risk-adjusted expected operating profit when a policy $\tilde{\beta}y_i$ is adopted. The third term is the risk-adjusted expected payoff when firm i loses, integrating the expected operating profit upon losing over the opponent's lobbying strategy from s_i to ∞ . Note that, upon losing, the equilibrium policy is $\tilde{\beta}$ times the one proposed by the opponent, $\tilde{\beta}y_{-i}(s_{-i})$.

Given a fixed lobbying expenditure s_i , the optimal policy level to be proposed y_i^* can be derived⁷ by taking the first-order condition of y_i :

$$0 = \frac{\partial \Pi_i(s_i, y_i^*)}{\partial y_i} = -2\alpha\gamma y_i^* + F_{-i}(s_i) \cdot \mathbb{E}[\tilde{M}\tilde{\beta}v_i'(\tilde{\beta}y_i^*)]. \quad (3)$$

The first term is the marginal cost of proposing a more extreme policy, and the second term is the marginal increase in the expected operating profit conditional on winning the lobby. That is, while a more extreme policy costs more, it increases the expected operating profit on winning the lobby. By substituting $v_i'(\tilde{\beta}y_i^*)$ and solving the expectation, we obtain the optimal

⁷For simplicity, we solve the model as a firm decides y_i for a given s_i . The solution is equivalent to solving the model assuming that a firm decides s_i for a given y_i as there exists a one-to-one correspondence between s_i and y_i .

policy level to be proposed as follows:

$$y_i^*(s_i) = \frac{x_i(a - K)F_{-i}(s_i)}{4\{b\alpha\gamma\eta - (1 + \Delta)x^2F_{-i}(s_i)\}}. \quad (4)$$

In equilibrium, any s_i in the support of the mixed strategy should yield the same utility. That is, the first derivative of Π_i with respect to s_i must equal zero:

$$0 = \frac{\partial \Pi_i(s_i, y_i^*(s_i))}{\partial s_i} = -\alpha + f_{-i}(s_i) \cdot \mathbb{E}[\tilde{M}\{v_i(\tilde{\beta}y_i^*(s_i)) - v_i(\tilde{\beta}y_{-i}^*(s_i))\}], \quad (5)$$

for all $s_i \in [0, \bar{s}]$. The first term is the marginal cost of increasing the lobby amount, and the second term is the marginal benefit of doing so. The marginal benefit is the product of two components: the marginal increase in winning probability $f_{-i}(s_i)$ and the expected increase in operating profit upon winning relative to losing. That is, firms optimize the lobbying amount so that the cost of lobbying equals the benefit of the increased chance of winning and the increased expected operating profit.

In the following theorem, we summarize the equilibrium lobbying strategies of Green and Brown firms.

Theorem 1. *The mixed strategy of Green and Brown firms is symmetric, $F(s) = F_G(s) = F_B(s)$ over $[0, \bar{s}]$. The inverse of unique CDF $F(s)$ is given by*

$$s = F^{-1}(F) = \left(\frac{a - K}{2x(1 + \Delta)} \right)^2 \left(\gamma \ln \frac{b\alpha\gamma\eta}{b\alpha\gamma\eta - (1 + \Delta)x^2F} - \frac{(1 + \Delta)x^2}{b\alpha\eta} F \right), \quad (6)$$

for $F \in [0, 1]$. The maximum lobbying expenditure is given by $\bar{s} = F^{-1}(1)$. Green (Brown) firm's optimal lobbying expenditure is increasing (decreasing) and convex in the proposed policy level.

All formal proofs are provided in the Appendix. Here, we outline the proof steps. In equilibrium, both firms exhibit common support for their mixed strategies $[0, \bar{s}]$. It's important to point out that this outcome is not a result of the model's symmetric structure, but it

emerges from the strategic dynamics in lobbying competition. First, both Green and Brown firms abstaining from any lobbying efforts cannot be an equilibrium since a small deviation with minimal lobbying by any firm would secure a more favorable policy, increasing its utility. Second, if the Green firm allocates some lobbying efforts with a non-zero probability for the mixed strategy, the Brown firm should respond with a similar allocation of lobbying efforts. Otherwise, the mixed strategy of Green firm is not optimal as the firm incurs unnecessary lobbying efforts without increasing the winning probability. These two dynamics indicate the interconnected nature of the mixed strategies adopted by both firms. Consequently, due to the strategic underpinnings of political competition, both firms are motivated to closely mirror each other's lobbying strategies, engaging in a strategic tug-of-war game.

The equilibrium characterization in Theorem 1 also implies that Green firm's optimal policy level to be proposed is always positive, while the Brown firm's optimal policy level is always negative. The firm's lobbying expenditure increases with a proposal containing a more extreme policy. Moreover, the optimal policy level is convex in lobbying expenditure, creating a U-shaped relation between a firm's proposed policy and lobbying expenditure. Figure 1 graphically illustrates the implication of this prediction, showing a clear U-shaped relation between firms' proposed climate policy and their lobbying expenditures. This coincides with the U-shaped pattern in Delmas et al. (2016) regarding firms' greenhouse gas emissions and lobbying expenditure, highlighting the intensified lobbying competition among firms with opposing interests.

2.3. Empirical Predictions

In this section, we discuss the empirical implications generated by the model. These implications address: 1) the variations in the likelihood of lobbying expenditure under different levels of policy uncertainty, 2) the variations in the lobbying expenditure's sensitivity to policy uncertainty under different market conditions, and 3) the impact of political competition on the cost of capital.

Model Prediction 1. *As climate policy uncertainty (Δ) intensifies, firms at the polarized ends of the environmental spectrum increase their lobbying expenditures.*⁸

The model’s first prediction suggests that heightened policy uncertainty (Δ) drives firms to lobby more aggressively. This stems from the convex nature of optimal operating profit $v_i(\tilde{y})$ in equilibrium policy levels for environmentally polarized firms (see Equation (1)). Aligning with policymakers on climate policy can equate to market monopolization, significantly reducing production costs compared to competitors—a substantial advantage. As policy uncertainty rises, the marginal benefit of proposing extreme policies increases while the marginal cost remains constant. Consequently, firms are incentivized to propose more favorable policies, as the potential rewards far outweigh the risks, especially in the face of uncertainty.

Moreover, since both Green and Brown firms escalate lobbying expenditures when advocating for extreme policies (Theorem 1), heightened policy uncertainty leads to increased lobbying spending by either Green or Brown firms, likely reciprocated by the other. This escalating lobbying activity intensifies the tug-of-war dynamics as firms compete for conflicting policy objectives, reflecting their strategic adaptation to evolving regulatory landscapes.

We further explore the heterogeneity of the tug-of-war competition in the equilibrium lobbying strategy by the market conditions and the firm characteristics to derive testable hypotheses.

Model Prediction 2. *The sensitivity of lobbying expenditure to policy uncertainty (Δ) is increasing in the level of competition within the product market ($1/b$).*⁹

Another prediction of the model is that when policy uncertainty is elevated, firms facing higher product market competition ($1/b$) will demonstrate a relatively larger increase in lobbying expenditures compared to firms operating in less competitive environments. This prediction stems from the observation that, in highly competitive product markets, the po-

⁸In the Appendix, we demonstrate that the maximum lobbying amount for both Green and Brown firms increases with Δ , and the lobbying amount is first-order stochastically increasing in Δ .

⁹In the Appendix, we demonstrate that the sensitivity of the maximum lobbying amount and the expected lobbying expenditure to Δ for both Green and Brown firms increase with $1/b$.

tential benefits derived from winning lobbying efforts are magnified due to the increased optimal profit, as shown in Equation (1). Consequently, firms facing heightened product market competition are incentivized to allocate more resources toward lobbying activities to secure favorable outcomes. Building on this prediction, we hypothesize that firms operating in industries marked by high levels of competition are more inclined to increase their lobbying expenditures in response to intensified competition in lobbying, as reflected by elevated policy uncertainty.

The model also provides implications regarding the cost of equity capital due to political competition. We define our measure of the cost of capital as the ratio of expected net profit at $t = 1$ to its risk-adjusted present value at $t = 0$ minus one. The following model prediction presents our hypothesis about the cost of capital:

Model Prediction 3. *The cost of capital for Green and Brown firms is higher than that of Neutral firm. Moreover, the spread in the cost of capital between Green/Brown and Neutral firms increases as climate policy uncertainty rises.*

Relative to Neutral firms, the marginal production costs of Green and Brown firms depend on the equilibrium environmental policy, which is subject to uncertainty even without a lobbying tug-of-war (general political risk). Additionally, lobbying competition among Green and Brown firms exacerbates each firm's exposure to systematic demand shocks (lobby-induced political risk) because there is a risk that an unfavorable climate policy is adopted following the loss of lobbying tug-of-war despite significant lobbying expenditures. Consequently, Green and Brown firms face higher costs of capital than Neutral firm.

The model also predicts that the disparity in the cost of capital between Green/Brown and Neutral firms increases with policy uncertainty. Initially, this may seem counterintuitive because, in times of high policy uncertainty, Green and Brown firms tend to increase their lobbying efforts, potentially reducing the probability of losing the lobbying tug-of-war. However, the strategic nature of lobbying competition complicates this relationship. When one firm increases lobbying expenditures in response to high policy uncertainty, its opponent typically

reciprocates, leading to an intensified lobbying tug-of-war. Consequently, the probability of losing the lobbying tug-of-war for a given level of lobbying expenditures actually *increases*. Moreover, if a firm does end up losing the lobbying tug-of-war, the resulting climate policy is likely to be highly unfavorable. This combination of the heightened probability of losing and adverse policy outcomes upon losing contributes to an increased cost of capital for firms engaged in lobbying tug-of-war. Overall, we hypothesize that firms involved in lobbying competition face higher costs of capital during periods of heightened policy uncertainty.

To distinguish lobby-induced political risk from the general political risk faced by Green and Brown firms, we consider a situation where the policymaker chooses the Green or Brown proposed policy with equal probability as a benchmark in Proposition 1 in the Appendix. This implies that firms' lobbying expenditures do not affect the probability of how a favorable climate policy is implemented, thus eliminating the influence of the lobbying tug-of-war. Despite this, the equilibrium climate policy remains uncertain due to policymakers' uncertain preferences, which is distinct from lobby-induced political risk. We find that the disparity in the cost of capital between Green/Brown and Neutral firms is smaller without the lobbying tug-of-war. This suggests that Green/Brown firms experience a higher cost of capital than Neutral firms as policy uncertainty intensifies, even after accounting for general political risk.

3. Variables and Summary Statistics

The main variable for our analysis is corporate lobbying expenditure. We use the filings of "Lobbying Report (LD-2)" by U.S. firms as mandated by the Lobbying Disclosure Act of 1995.¹⁰ This law requires firms to report detailed information about their lobbying activities, including client firms, registrants/lobbyists, issues for lobbying, and the amount spent on lobbying. In Appendix Table 1, we provide an example of a lobbying report. While all lobbying

¹⁰Contributions from Political Action Committee (PAC) also provide data on monetary contributions related to political stances and are widely used in the literature. However, we use the Lobbying Report (LD-2) to examine firms' lobbying activities since PAC contributions are primarily raised from individuals (Ansolabehere et al. (2003)) and involve less money than lobbying expenditure (Bombardini and Trebbi (2012)).

reports are publicly available at the U.S. Senate Lobbying Disclosure Act Database,¹¹ we use the Lobbyview database (Kim (2018)),¹² which compiles all available information in the lobbying reports into a machine-readable database from 2003 to 2019.

We first identify the lobbying activity related to environmental issues. Each report identifies the General Issue Areas¹³ and the keywords for Specific Lobbying Issues within the General Issue Areas that the registrant devotes his lobbying effort to the client during the reporting period. For environmental lobbying, we use the lobbying reports with General Issue Areas of “Clean Air and Water (CAW),” “Environmental/Superfund (ENV),” and “Waste (WAS),” and also refine our focus to the ones with keywords such as “CAFE (Corporate Average Fuel Economy),” “Cap and Trade,” “Carbon,” “Clean Air,” “Clean Energy,” “Climate, Emission,” “Environment,” “EPA,” “GHG,” “Global Warming,” “Greenhouse,” “Keystone,” “Kyoto,” and “Renewable” in the Specific Lobbying Issues.

We construct our first measure of a firm’s environmental lobbying activity by aggregating all expenditures on environmental lobbying. As a single lobbying report may contain multiple areas of lobbying, including both environmental and non-environmental issues, our measure can overestimate the environmental lobbying amounts if we simply aggregate all amounts in the lobbying reports. Using the information on the list of lobbyists assigned to each lobbying area in a lobbying report, we compute a weight for environmental issues in each report and derive a weighted amount of environmental lobbying.¹⁴ We aggregate expenditures on environmental lobbying by a client firm i in a year t to define *ENV Lobbying Expenses* $_{i,t}$.¹⁵

Note that firms are not required to disclose exact amounts of lobbying expenditure if the amount is less than US\$5,000, but they can still be differentiated from other firms without

¹¹ Available at <https://lobbyingdisclosure.house.gov/>.

¹² Available at <https://www.lobbyview.org/>. We use the most updated version of lobbying reports if there are amendments to the reports.

¹³ We provide the full list of “General Issue Areas” in Appendix Table 2.

¹⁴ We also confirm the robustness of our results using a measure that uses equally divided amounts over different areas of lobbying when there are multiple areas in a lobbying report, as in Ahn et al. (2021).

¹⁵ The Lobbying reports (LD-2) were filed biannually until 2007. However, since the Lobbying Disclosure Act of 1995 was amended by the Honest Leadership and Open Government Act of 2007, the lobbying reports have been disclosed quarterly since 2008.

any lobbying expense through an indicator provided in the data. As the aggregated expenditure amounts would underestimate the lobbying activities with small amounts, we construct our second measure of a firm’s environmental lobbying activity as an indicator variable $I_ENV\ Lobbying_{i,t}$, which is an indicator variable that equals 1 if the firm engages in any environmental lobbying in a year and 0 otherwise.

To examine the tug-of-war competition in corporate environmental lobbying, we build a firm’s environmental stance using carbon emissions data from Trucost. We use a firm’s carbon intensity with Scope 1 emissions, which is the direct emission amount of GHG normalized by the firm’s revenue.¹⁶ We define *Green Side* as firms in the bottom tercile of the average carbon emission intensity in the previous year within a 2-digit SIC industry. *Brown Side* is similarly defined as firms in the top tercile of average carbon emission intensity in the previous year within a 2-digit SIC industry. Note that we are focusing on within-industry variations of firms regarding their environmental stances, in accordance with our model setup of competing in the product market.

The lobbying reports provide firms’ lobbying amounts only and not the direction of the lobbying. We posit that the firms with lower carbon emissions (Green-side firms) lobby for strengthening pro-environmental agendas and those with more carbon emissions (Brown-side firms) lobby for the opposite. We validate our premise by matching our sample to data with firms’ lobbying directions. Despite the limited coverage, Maplight¹⁷ provides firms’ stances on a bill for their lobbying effort. Using the matched sample, we examine how firms’ stance on a bill is related to their carbon emission intensity. As reported in Appendix Table 3, the matched sample includes 17 bills with the political stance of 34 firms. We find that most Green-side firms (83%) support green-oriented bills and the majority of Brown-side firms (62%) support brown-oriented bills, consistent with our premise on the relation between a firm’s environmental stance and their lobbying directions. Interestingly, the firms’ overall

¹⁶Scope 1 emissions measure the direct emissions of a firm, Scope 2 emissions measure the emissions from producing energy consumed by the firm, and Scope 3 emissions measures all other emissions associated with the firm’s operation, such as those from the supply chain.

¹⁷<https://www.maplight.org/data-series/>

environmental stance is evenly split between the green and brown policies (51% vs. 49%), highlighting the tug-of-war nature of the firm’s environmental lobbying.

Our identification strategy is to exploit the varying intensity of the lobbying competition between firms in the polarized environmental spectrum over time. We expect to identify the tug-of-war effect during periods with a higher level of climate policy uncertainty. For the empirical execution, we measure the climate policy uncertainty using the Climate Policy Uncertainty (CPU) index by Gavriilidis (2021),¹⁸ which adopts the textual search on news articles of Baker et al. (2016) for the uncertainty in climate policies.

We use other standard firm-level variables from Compustat, such as total assets, book-to-market ratio, cash holdings, profitability, R&D expenditure, capital expenditure, and leverage ratio. We also use standardized firm-level political risk defined by Hassan et al. (2019) to control for the general political risk of the firm. The detailed definitions of all variables are reported in Table 1.

3.1. Summary Statistics

Table 2 reports the summary statistics for variables used in our analysis. Our data is from 2003 to 2019, and all variables are winsorized at the 2.5-percentile and 97.5-percentile values to minimize the impact of extreme outliers, except the actual lobbying expenses. Panel A reports the summary statistics of variables measuring firms’ lobbying activities constructed from lobbying reports. We find that only 46% of firms in the sample indicate a positive lobbying amount. About 16% of the sample firms have some lobbying activities on environmental issues. The average total lobbying expenditures—be it environmental or not—is \$710,106, but it is highly skewed with a median value of \$0 and the 90th percentile value of \$1,720,000. Regarding environmental lobbying, the average expenditure decreases to \$77,438 with a standard deviation of \$457,378.¹⁹ However, limiting sample to the firms with positive environmental

¹⁸www.policyuncertainty.com

¹⁹Given the large stake of public policies, the relatively small size of lobbying expenditure is a long-standing puzzle in the political science and economics literature, often called “Tullock’s Paradox” (Tullock (1972)). See

lobbying expenses, we find that the average total lobbying expenditure is \$2,578,156 with a median value of \$1,060,000 and the 90th percentile value of \$7,200,000. This is approximately four times larger than the average in the full sample. The average environmental lobbying expenditure of \$505,937 is also much higher than that of the full sample.

In Panel B, we report the summary statistics of variables on firms' climate policy stances constructed from the Trucost database. *Avg. Emission Intensity* has a mean of 102.2 with a standard deviation of 240.8. *Green Side* is an indicator variable that equals 1 when the firms belong in the bottom tercile of *Avg. Emission Intensity* within an industry in a year and 0 otherwise. *Brown Side* is an indicator variable that equals 1 when the firms belong in the top tercile of *Avg. Emission Intensity* within the industry in a year and 0 otherwise. We then define a dummy variable, *Polarized ENV. Interests*, that equals 1 if the firms are either green or brown and 0 otherwise, to indicate the firms who would engage in the lobbying tug-of-war.

Panel C of Table 2 reports the summary statistics of the Climate Policy Uncertainty (CPU) index by Gavriilidis (2021). CPU is available at monthly frequency and we use the average value of CPU in a year. While the CPU index has a mean of 104.7, it shows significant variation across time with the 90th percentile of 199.9 and the 10th percentile of 35.91. We define High CPU as a dummy variable that equals 1 if the CPU level of the year is in the top tercile in our sample period and 0 otherwise.

Panel D reports the summary statistics of variables on the firm-level characteristics. For example, the asset size of the firm (*Total Assets*) is \$9.3 billion on average with a standard deviation of \$17 billion, and the book-to-market ratio (*Book-to-Market*) has a mean of 0.42 with a standard deviation of 0.34. *Cash to TA* is the ratio of cash to the total asset, with a mean of 0.17 and a standard deviation of 0.19. *Profitability* is the ratio of operating income before depreciation to the total asset, with a mean of 0.11 and a standard deviation of 0.14. *R&D to TA* is the ratio of R&D expenses to the total asset, with a mean of 0.04 and a standard deviation of 0.08. *CAPEX to TA* is the ratio of capital expenditure to the total asset, with

Zingales (2017) for more discussion.

a mean of 0.05 and a standard deviation of 0.05. *Leverage* is the debt-to-asset ratio with a mean of 0.25 and a standard deviation of 0.19.

Panel E reports the summary statistics of firms' average implied cost of capital and political risk. For measuring a firm's implied cost of capital, we use five implied cost of capital measures in the literature: Easton (2004), Gode and Mohanram (2003), Gordon and Gordon (1997), Gebhardt et al. (2001), and Gebhardt et al. (2001). *Avg. Implied Cost of Capital* is the average value of the five implied cost of capital measures, with a mean of 0.096 and a standard deviation of 0.034. For the firm-level exposure to political risk, we use *Political Risk* measure from Hassan et al. (2019) that has a mean of 0.858 and a standard deviation of 1.

In Panel F, we report the summary statistics of variables related to the real effects on the firms such as corporate investment. $\Delta R\&D$ is the annual percentage change in the R&D expenses, with a mean of 0.04 and a standard deviation of 0.16. $\Delta CAPEX$ is the annual percentage change in CAPEX, with a mean of 0.12 and a standard deviation of 0.52. $\Delta Total Investment$ is the annual percentage change in total investment, with a mean of 0.08 and a standard deviation of 0.35.

4. Empirical Analysis

4.1. Tug of War in Environmental Lobbying

The main prediction of the model in Section 2 is that firms in the more extreme environmental spectrum lobby more to beat the other firms with opposing environmental stances in order to influence the direction of climate policy. In addition, Model Prediction 1 contends that uncertainty about climate policy induces firms to engage more in the tug of war.

Our identification strategy is based on the premise that the intensity of lobbying tug-of-war between firms with polarized environmental interests varies over time, and the tug-of-war effect would be more distinctively identified when competition is more intense due to a higher level of climate policy uncertainty. That is, the excessive lobbying expenditures by Green-side

and Brown-side firms relative to Neutral firms are likely to be dynamic and these firms are more likely to engage in the lobbying tug-of-war when the CPU is high, which is presented in Model Prediction 1. Our baseline regression specification is as follows.

$$\begin{aligned} \text{Lobbying Expenditure}_{i,t} = & \beta_1 \cdot \text{Polarized ENV. Interests}_{i,t} \times \text{High CPU}_t \\ & + \beta_2 \cdot \text{Polarized ENV. Interests}_{i,t} + \Gamma \cdot X_{i,t-1} + \alpha_i + \alpha_t + \epsilon_{i,t}, \end{aligned} \quad (7)$$

where *Polarized ENV. Interests* is a dummy variable that equals one when a firm is in the polarized environmental spectrum (i.e., *Green Side* or *Brown Side*) and zero otherwise. *High CPU* is a dummy variable that equals one if the CPU level of the year is in the top tercile in our sample period and zero otherwise, and $X_{i,t}$ are the lagged firm-level characteristics such as total assets, book-to-market ratio, cash holdings, profitability, R&D expenditure, capital expenditure, and leverage ratio. We also include firm fixed effects (α_i) and year fixed effects (α_t).

Panel A of Table 3 reports the estimated coefficients of regressions of lobbying activities on *Polarized ENV. Interests* \times *High CPU*. Columns (1)–(2) report the regression results using *LENV Lobbying*, which is a dummy variable of whether the firms engage in lobbying or not, as a dependent variable. Column (1) reports the univariate regression result, and Column (2) reports the regression results controlling for various firm characteristics. We find that the estimated coefficients on the *Polarized ENV. Interests* \times *High CPU* are positive and statistically significant. The result shows that firms with polarized environmental interests are more likely to engage in a lobbying tug-of-war when the climate policy uncertainty is higher. During these periods, they increase the lobbying efforts by 2.8% or 2.7%, depending on the model specification.

Columns (3)–(4) report the regression results using $\text{Ln}(1+\text{ENV Lobbying})$, which is the amount of corporate lobbying, as a dependent variable. Column (3) reports a univariate regression result with firm and year fixed effects; the estimated coefficient on *Polarized ENV. Interests* \times *High CPU* is 0.304 with t -statistic of 2.38. That is, firms with polarized en-

environmental interests increase their lobbying amounts more than other firms with neutral environmental stances during times of high policy uncertainty, indicating intensified tug-of-war competition. The economic significance is substantial—the tug-of-war competition during times with high policy uncertainty increases lobbying expenditure by 7% of one standard deviation of $\ln(1+ENV\text{ Lobbying})$. Column (4) reports a similar regression with various controls of firm characteristics. While the magnitude of the coefficient on *Polarized ENV. Interests* \times *High CPU* slightly reduces to 0.288, we find that it is still positive and statistically significant with t -statistic of 2.30.

We also find that the increased lobbying activities during the high CPU period occur on both Green-side and Brown-side firms. In Panel B of Table 3, we split *Polarized ENV. Interests* into separate dummies of *Green Side* and *Brown Side*, and estimate coefficients of regressions of lobbying activities on *Green Side* \times *High CPU* and *Brown Side* \times *High CPU*. We find that the estimated coefficients on both *Green Side* \times *High CPU* and *Brown Side* \times *High CPU* are positive and statistically significant.

It is noteworthy that our analysis does not explicitly account for the “free-riding” incentive among firms with the same environmental interests. In situations where a peer firm in the same environmental spectrum is actively lobbying to influence climate policy, a firm may be tempted to reduce its own lobbying efforts, potentially diminishing overall lobbying activity among firms with similar environmental stances. This behavior could counteract the expected tug-of-war effect between Green-side and Brown-side firms. As a consequence, our empirical findings may underestimate the true magnitude of the tug-of-war effect between firms with polarized environmental interests.

4.2. Heterogeneity in the Effect of Tug of War

The analysis results above show that all else equal, firms with opposing environmental stances engage in a tug of war by spending more lobbying expenditure on environmental issues when the climate policy uncertainty is high. In this section, we investigate the moder-

ating effect of firm’s product market competition on the lobbying tug-of-war, following Model Prediction 2.

Model Prediction 2 states that firms with higher product market competition respond to tug-of-war competition more sensitively. Using the HHI index within a 2-digit SIC code based on Green/Neutral/Brown firms’ aggregated sales as a measure of the degree of firms’ product market competition, we consider the firms below the median HHI index in our sample as being in more competitive industries. We separately estimate the regression of lobbying expenditure on the *Polarized ENV. Interests* \times *High CPU* in Table 3 for the high and low competition industries.

Table 4 reports the estimated coefficients. Columns (1)–(2) use *L-ENV Lobbying* as the dependent variable. Column (1) reports the regression result using firms facing high product market competition, and Column (2) reports the same regression using firms facing low product market competition. We find that the estimated coefficient on *Polarized ENV. Interests* \times *High CPU* is significantly positive in the industry with high competition, while the coefficients are much weaker with the low competition industry as our model predicts. We find that participation in environmental lobbying is significantly higher in highly competitive industries than in low-competitive industries. Columns (3)–(4) of Table 4 use $\ln(1+ENV\ Lobbying)$ as the dependent variable. We find that the firms in tug-of-war are more likely to spend on environmental lobbying by 63.1% when facing high competition and by 6% with low competition, and the difference is statistically significant.

4.3. Instrumental Variable Analysis

While we find that firms in the polarized environmental spectrum engage in the lobbying tug-of-war when the climate policy uncertainty is high, there remains a concern that the climate policy uncertainty can be endogenous. To address this issue, we use the changes in the U.S. Environmental Protection Agency (EPA) administrators affiliated with different political parties as an exogenous shock to the climate policy uncertainty. EPA develops and

enforces environmental regulations, and the EPA administrator plays an important role in setting the direction and stringency of the regulations. The changes in the EPA administrators with different political parties are likely to increase climate policy uncertainty, satisfying the relevance condition.²⁰ And the changes in the EPA administrators' political party are not likely to be driven by factors related to corporate environmental lobbying. A common reason for these changes is a change in the U.S. president, which tends to impact the overall economic policy uncertainty. By controlling for the overall economic policy uncertainty in our analysis, we posit that changes in the EPA administrator are not directly linked to corporate environmental lobbying, thereby satisfying the exclusion restriction condition.

We define *EPA Admin. Change* as an indicator variable for the first two years of service for new U.S. EPA administrators from different political parties. Table 5 reports the instrumental variable (IV) regression results using *EPA Admin. Change* as an instrument variable for *High CPU*. As our main interest is the interaction term of *Polarized ENV. Interests* \times *High CPU*, we instrument *High CPU* and *Polarized ENV. Interests* \times *High CPU* with *EPA Admin. Change* and *Polarized ENV. Interests* \times *EPA Admin. Change*. Columns (1)–(2) report the first-stage regressions of *High CPU* and *Polarized ENV. Interests* \times *High CPU* on *EPA Admin. Change* and *Polarized ENV. Interests* \times *EPA Admin. Change*. The regression specifications are similar to that for Column (2) of Table 3 with firm fixed effects, but we replace the time fixed effects with macro variables, such as *GDP Growth*, *Consumer Confidence Index*,²¹ *1-Year Treasury Rate* because *EPA Admin. Change* would be absorbed by the time fixed effects. Importantly, we control for the economic policy uncertainty ($\ln(EPU)$),²² which is likely to be correlated with the U.S. presidential change. As for the relevance condition of the IV, we find that *EPA Admin. Change* increases *High CPU* by 0.976 with *t*-statistic of 8.63, and *Polarized*

²⁰In our sample period of 2003-2019, omitting interim administrators, we observe two changes in the affiliated party of EPA administrators. The first transition was in 2009 from Stephen L. Johnson (Republican) to Lisa P. Jackson (Democrat) and the second transition was in 2017 to Scott Pruitt (Republican).

²¹We define *Consumer Confidence Index* as the logarithm of annual Consumer Sentiment Index released by The University of Michigan.

²²We define $\ln(EPU)$ as the logarithm of average monthly economic policy uncertainty from Baker et al. (2016).

ENV. Interests \times *EPA Admin. Change* increases *Polarized ENV. Interests* \times *High CPU* by 0.857 with t -statistic of 9.21. The F -statistic for the joint significance is 41.62, well above the conventional threshold of 10 (Stock and Yogo (2005)).

Columns (3)–(4) of Table 5 report the second-stage regression results. We find that firms in the polarized environmental spectrum are more likely to engage in and spend more on environmental lobbying when facing *High CPU*, consistent with our baseline findings.

4.4. The Real Effect of Tug of War

So far, we show that the firms in the polarized environmental spectrum engage in lobbying tug-of-war when the climate policy uncertainty rises. Political competition can be costly, but it is not very expensive if the cost refers only to lobbying expenditures. The direct loss from lobbying competition will be the amounts used for lobbying, which is much smaller than the firm size, as shown in Table 2. We argue, however, the lobbying tug-of-war introduces a political risk that will increase the cost of capital (Model Prediction 3), potentially leading to a reduction in other investments for a firm’s long-term viability.

We first test whether a firm experiences a higher cost of capital following the lobbying tug-of-war. In Table 6, we use the implied cost of capital as the dependent variable and regress it on *Polarized ENV. Interests* \times *High CPU*, controlling for various firm characteristics with year and firm fixed effects. The implied cost of capital is the average of implied cost of capital computed by five different methodologies in literature: Easton (2004), Gode and Mohanram (2003), Gordon and Gordon (1997), Gebhardt et al. (2001) and Claus and Thomas (2001). In Column (1), we find that the cost of capital of the firms engaging in the lobbying tug-of-war increases during the high CPU periods. The magnitude of the effect is estimated at about 3% of the average value of the implied cost of capital.

While the lobbying tug-of-war is possibly correlated with the general political risk a firm faces that increases the firm’s cost of capital (Grotteria (2024)), our model predicts that the lobby-induced political risk distinctively increases the firm’s cost of capital. In Column (2) of

Table 6, we additionally control for firms' general political risk using standardized firm-level political risk defined by Hassan et al. (2019). We find that the firm's implied cost of capital still increases for the firms engaged in the lobbying tug-of-war even after controlling for general political risk. The magnitude of the effect is similar to the result in Column (1), indicating the relative importance of lobby-induced political competition on a firm's cost of capital.

The increase in the cost of capital among firms in the lobbying tug-of-war leads to a question of whether the real investment of firms can be negatively affected by political competition. To the extent that the lobby-induced political risk increases a firm's cost of capital and subsequently reduces a firm's actual investment, the real cost of the lobbying tug-of-war can be substantial compared to the size of the lobbying expenditures per se. In Table 7, we regress a firm's investment (e.g., R&D expenditure changes, capital expenditure changes, total investment changes) on *Polarized ENV. Interests* \times *High CPU*, controlling various firm characteristics with firm and year fixed effects.

Columns (1)–(3) of Table 7 report the regression result of changes in long-term investment spending (R&D expenditure, capital expenditure, and total investment) on *Polarized ENV. Interests* \times *High CPU*. In Column (1), the estimated coefficient on $\Delta R\&D$ is -0.011 with t -statistic of -2.78, implying that firms in tug-of-war significantly decrease R&D expenditures more than others when the climate policy uncertainty rises. Regarding the economic significance, firms in the tug of war during the high CPU period decrease R&D expenditures by 7% of one standard deviation of $\Delta R\&D$.

In Column (2), the estimated coefficient of $\Delta CAPEX$ is -0.050 with t -statistic of -3.19. That is, firms in tug-of-war during the high CPU period decrease capital expenditure more than others. In terms of economic significance, firms in the tug of war decrease capital expenditures by 10% of one standard deviation of $\Delta CAPEX$. As a result, in Column (3), the estimated coefficient of $\Delta Total Investment$ is -0.030 with t -statistic of -2.93, suggesting that firms in tug-of-war decrease total investments of the firms more than others. The economic significance is large that firms in the tug of war decrease total investments by 9% of one

standard deviation of $\Delta Total Investment$.

5. Conclusion

This paper examines the strategic lobbying decisions of firms with polarized environmental interests. We build a theoretical model in which Green-side and Brown-side firms compete in influencing policymakers' decisions. A key prediction of the model is that firms in the polarized environmental spectrum are competing to spend more lobbying expenditures to keep up with the firms on the other side of the stances, leading to excessive lobbying expenditures called the "lobbying tug-of-war" effect.

Using the lobbying reports filed by U.S. lobbying entities, we empirically test and confirm key predictions of the model; the tug-of-war effect is more pronounced when climate policy is uncertain, and the product market competition is more intense. The lobbying tug-of-war increases the cost of capital for firms, even after controlling for general firm-level political risk, leading to a reduction in real investments. These results highlight that the actual cost of lobbying tug-of-war is much larger than the dollar amount of lobbying expenditure itself.

A potential policy implication is that the government needs to set a clear policy direction regarding environmental policies so firms can avoid a costly lobbying tug-of-war and instead invest in new technologies to transform their businesses conforming with the intended environment policy.

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Figure 1: The Relation of Environmental Lobbying and Proposed Policy Level

The figure plots a U-shaped relation between a firm's proposed policy and lobbying expenditure in the model when Green and Brown firms use mixed strategy at the equilibrium. See Proof of Proposition 2 in the Appendix for the explicit form of lobbying expenditure by proposed policy level.

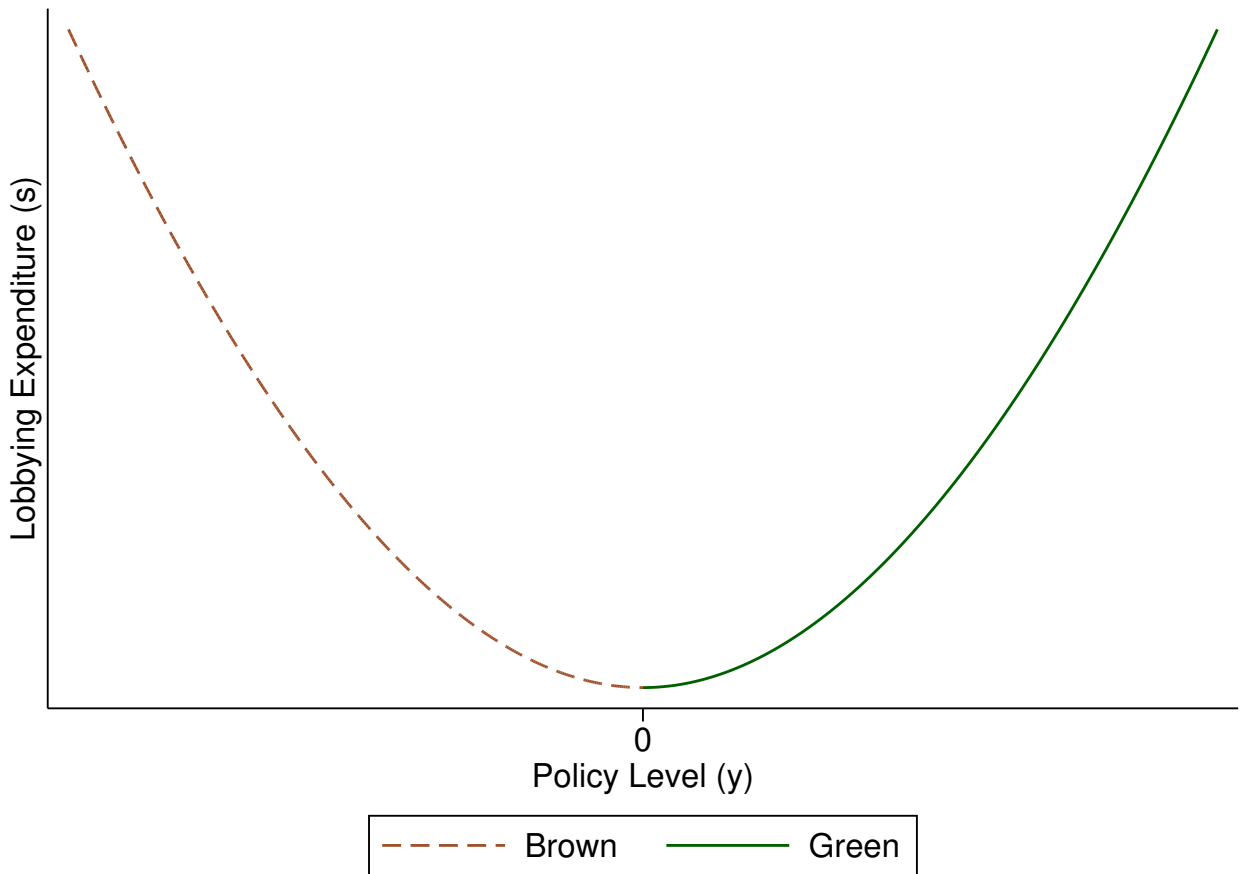


Table 1: Variable Description

We describe all the variables we use in the empirical analysis. The variables on lobbying activities are from Lobbying Reports “Form LD-2” in the U.S. The variables on the environmental stance of firms are from the Trucost database. The variables on firms’ characteristics are from Compustat.

Variable	Description
Lobbying Activity	
<i>L</i> Lobbying	1 if a firm <i>i</i> participates lobbying activity in year <i>t</i> ; 0 otherwise
<i>L</i> ENV Lobbying	1 if a firm <i>i</i> participates lobbying activity related to environment issues in year <i>t</i> ; 0 otherwise
Lobbying Expenses (\$)	Total amount of lobbying expense that a firm <i>i</i> spends in year <i>t</i>
ENV Lobbying Expenses (\$)	Total amount of lobbying expense related to environment issues that a firm <i>i</i> spends in year <i>t</i>
$\ln(1+ENV\ Lobbying)$	The logarithm of one plus <i>ENV Lobbying Expenses</i>
Environmental Stance	
<i>Avg. Emission Intensity</i>	A firm <i>i</i> ’s average carbon emission intensity scope 1 in year <i>t</i> – 2 and <i>t</i> – 1
<i>Green Side</i>	1 if a firm <i>i</i> ’s Average Emission Intensity is in the bottom tercile within 2-digit SIC industry level at the end of year <i>t</i> – 1; 0 otherwise
<i>Brown Side</i>	1 if a firm <i>i</i> ’s Average Emission Intensity is in the top tercile within 2-digit SIC industry level at the end of year <i>t</i> – 1; 0 otherwise
<i>Polarized ENV. Interests</i>	1 if either <i>Green Side</i> , or <i>Brown Side</i> equals 1; 0 otherwise
Climate Policy Uncertainty	
<i>High CPU</i>	1 if the average value of monthly Climate Policy Uncertainty (CPU) index (Gavriilidis (2021)) in year <i>t</i> – 1 is in top tercile; 0 otherwise
Firm Characteristics	
<i>Total Assets (in million USD)</i>	Book value of total assets of a firm <i>i</i> at the end of year <i>t</i>
$\ln(Total\ Assets)$	The logarithm of <i>Total Assets</i>
<i>Book-to-Market</i>	The ratio of book value of total equity to market value of total equity of a firm <i>i</i> at the end of year <i>t</i>
<i>Cash to TA</i>	The ratio of cash and cash equivalents to <i>Total Assets</i> of a firm <i>i</i> in year <i>t</i>
<i>Profitability</i>	The ratio of operating income before depreciation to <i>Total Assets</i> of a firm <i>i</i> in year <i>t</i>
<i>R&D to TA</i>	The ratio of R&D expenses to <i>Total Assets</i> of a firm <i>i</i> in year <i>t</i>
<i>CAPEX to TA</i>	The ratio of capital expenditures to <i>Total Assets</i> of a firm <i>i</i> in year <i>t</i>
<i>Leverage</i>	The ratio of long-term debt plus debt in current liabilities to <i>Total Assets</i> of a firm <i>i</i> in year <i>t</i>
Implied Cost of Capital and Political Risk	
<i>Avg. Implied Cost of Capital</i>	The average value of five implied cost of capital measures following Easton (2004), Gode and Mohanram (2003), Gordon and Gordon (1997), Gebhardt et al. (2001), and Claus and Thomas (2001)
<i>Political Risk</i>	Standardized value of the average firm-level political risk (Hassan et al. (2019)) in year <i>t</i>
Long-Term Real Effect	
$\Delta R\&D$	Percentage changes in R&D expenses of a firm <i>i</i> from year <i>t</i> to year <i>t</i> + 1
$\Delta CAPEX$	Percentage changes in capital expenditures of a firm <i>i</i> from year <i>t</i> to year <i>t</i> + 1
$\Delta Total\ Investment$	Percentage changes in R&D expenses plus capital expenditures of a firm <i>i</i> from year <i>t</i> to year <i>t</i> + 1

Table 2: Summary Statistics

We report the summary statistics of variables in our analysis. Our sample includes U.S. public firms with available information in Trucost and Compustat from 2003 to 2019. Panel A reports the variables on lobbying activities from Lobbying Reports “Form LD-2” in the U.S. Panel B reports the variables on the environmental stance of firms from the Trucost database. Panel C reports the macro-level Climate Policy Uncertainty (CPU) index from Gavriilidis (2021). Panel D reports the variables on firms’ characteristics from Compustat. Panel E reports the variables on the firm’s implied cost of capital, which is the average value of five implied cost of capital measures in literature, and firm-level political risk from Hassan et al. (2019). Panel F reports the variables on firms’ long-term investments from Compustat. The details of the variables are discussed in Table 1. All variables are constructed at the firm-year level. We winsorize all variables at the 2.5% and 97.5% levels except (environmental) lobbying expenses.

	Obs	Mean	Std.Dev.	10th	50th	90th
Panel A: Lobbying Activity						
<i>I_Lobbying (0/1)</i>	14,367	0.46	0.50	0	0	1
<i>I_ENV Lobbying (0/1)</i>	14,367	0.16	0.36	0	0	1
<i>Lobbying Expenses (\$)</i>	14,367	710,106	2,123,434	0	0	1,720,000
<i>ENV Lobbying Expenses (\$)</i>	14,367	77,438	457,378	0	0	80,000
<i>Ln(1+ENV Lobbying)</i>	14,367	1.81	4.30	0	0	11.29
Limiting to Firms with Positive ENV Lobbying						
<i>Lobbying Expenses (\$) if >0</i>	2,199	2,578,156	3,735,667	140,000	1,060,000	7,200,000
<i>ENV Lobbying Expenses (\$) if >0</i>	2,199	505,937	1,072,562	21,000	155,304	1,231,619
Panel B: Carbon Emissions						
<i>Avg. Emission Intensity (tCO₂/\$M)</i>	14,367	102.2	240.8	4.00	19.69	299.6
<i>Green Side</i>	14,367	0.33	0.47	0	0	1
<i>Brown Side</i>	14,367	0.31	0.46	0	0	1
<i>Polarized ENV. Interests</i>	14,367	0.64	0.48	0	1	1
Panel C: Climate Policy Uncertainty						
<i>CPU Index</i>	14,367	104.7	48.74	35.91	100.9	199.9
Panel D: Firm Characteristics						
<i>Total Assets (in million USD)</i>	14,367	9,275	16,737	373.9	3,162	24,867
<i>Ln(Total Assets)</i>	14,367	8.03	1.57	5.92	8.06	10.12
<i>Book-to-Market</i>	14,367	0.42	0.34	0.09	0.34	0.85
<i>Cash to TA</i>	14,367	0.17	0.19	0.01	0.10	0.44
<i>Profitability</i>	14,367	0.11	0.14	0.01	0.13	0.24
<i>R&D to TA</i>	14,367	0.04	0.08	0	0.00	0.12
<i>CAPEX to TA</i>	14,367	0.05	0.05	0.01	0.03	0.11
<i>Leverage</i>	14,367	0.25	0.19	0	0.24	0.52
Panel E: Implied Cost of Capital and Political Risk						
<i>Avg. Implied Cost of Capital</i>	9,641	0.096	0.034	0.066	0.089	0.133
<i>Political Risk</i>	12,887	0.858	1	0.150	0.583	1.817
Panel F: Long-Term Real Effect						
<i>ΔR&D</i>	12,887	0.04	0.16	-0.08	0	0.23
<i>ΔCAPEX</i>	12,887	0.12	0.52	-0.41	0.04	0.70
<i>ΔTotal Investment</i>	12,887	0.08	0.35	-0.32	0.05	0.52

Table 3: Tug of War on Corporate Environmental Lobbying

We report the panel regression results of firms' environmental stances (*Polarized ENV. Interests_t*) on their environmental lobbying activities when facing high Climate Policy Uncertainty (CPU). We use the firm-year observations from 2003 to 2019. Panel A reports our baseline regression results. Our main independent variable is *Polarized ENV. Interests_t*, which is a dummy equals to 1 if either *Green Side_t* or *Brown Side_t* equals to 1. *High CPU* is a dummy equals to 1 if the average value of lagged monthly CPU index is in the top tercile. In Column (1)-(2), the dependent variable is *LENV Lobbying_t*, a dummy equals 1 if a firm *i* conducts lobbying activity related to environmental issues in year *t*. In Column (1), we include firm and year fixed effects. In Column (2), we add other firm-level characteristics. In Column (3)-(4), the dependent variable is *Ln(1+ENV Lobbying_t)*, which is the logarithm of one plus total amount of lobbying expense related to environmental issues that a firm *i* spend in year *t*. In Panel B, we separate Polarized ENV. Interests into two dummy variables of Green Side and Brown Side. The *t*-statistics reported in parentheses are based on standard errors clustered at the firm and year level. ***, **, * denote 1%, 5%, and 10% statistical significance.

Panel A: Baseline Regression	(1)	(2)	(3)	(4)
	<i>LENV Lobbying_t</i>	<i>LENV Lobbying_t</i>	<i>Ln(1+ENV Lobbying_t)</i>	<i>Ln(1+ENV Lobbying_t)</i>
Polarized ENV. Interests _t × High CPU _t	0.028*** (2.95)	0.027** (2.89)	0.304** (2.38)	0.288** (2.30)
Polarized ENV. Interests _t	-0.009 (-1.12)	-0.008 (-1.02)	-0.088 (-0.95)	-0.077 (-0.85)
Ln(Total Assets _{t-1})		0.051*** (4.45)		0.596*** (4.36)
Book-to-Market _{t-1}		-0.022* (-1.94)		-0.211 (-1.41)
Cash to TA _{t-1}		-0.008 (-0.21)		0.49 (0.11)
Profitability _{t-1}		-0.051 (-1.18)		-0.502 (-1.05)
R&D to TA _{t-1}		0.084 (0.71)		1.020 (0.79)
CAPEX to TA _{t-1}		0.029 (0.23)		-0.508 (-0.33)
Leverage _{t-1}		-0.011 (-0.36)		-0.234 (-0.66)
Observations	14,367	14,367	14,367	14,367
Adj. R-squared	0.595	0.597	0.634	0.637
Firm FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES

Table 3 Continues

Panel B: By Environmental Stances	(1)	(2)	(3)	(4)
	<i>I-ENV Lobbying_t</i>	<i>I-ENV Lobbying_t</i>	<i>Ln(1+ENV Lobbying_t)</i>	<i>Ln(1+ENV Lobbying_t)</i>
Green Side _t × High CPU _t	0.033** (2.83)	0.031** (2.74)	0.326** (2.33)	0.301** (2.21)
Brown Side _t × High CPU _t	0.022* (1.86)	0.022* (1.86)	0.282* (1.77)	0.277* (1.75)
Green Side _t	-0.002 (-0.17)	-0.002 (-0.16)	0.009 (0.08)	0.010 (0.09)
Brown Side _t	-0.018 (-1.63)	-0.016 (-1.46)	-0.196 (-1.57)	-0.174 (-1.41)
Ln(Total Assets _{t-1})		0.051*** (4.35)		0.592*** (4.28)
Book-to-Market _{t-1}		-0.022* (-1.93)		-0.209 (-1.40)
Cash to TA _{t-1}		-0.007 (-0.20)		0.053 (0.13)
Profitability _{t-1}		-0.051 (-1.19)		-0.504 (-1.06)
R&D to TA _{t-1}		0.085 (0.72)		1.033 (0.80)
CAPEX to TA _{t-1}		0.026 (0.21)		-0.525 (-0.34)
Leverage _{t-1}		-0.012 (-0.39)		-0.245 (-0.70)
Observations	14,367	14,367	14,367	14,367
Adj. R-squared	0.595	0.597	0.635	0.637
Firm FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES

Table 4: The Effect of Product Market Competition on the Lobbying Tug-of-War

We report the panel regression results of firms' environmental stances (*Polarized ENV. Interests_t*) on their environmental lobbying activities when facing high Climate Policy Uncertainty (CPU), by product market competition. A firm is facing high product market competition if 2-digit SIC level HHI of Green/Neutral/Brown firms' aggregated sales is below median in year $t - 1$. In Column (1)-(2), the dependent variable is $I_ENV Lobbying_t$. Column (1) includes the firms in an industry with high product market competition and Column (2) includes firms in an industry with low product market competition. Regression specification is the same as Column (2) of Panel B in Table 3. The differences in the estimated coefficient of *Polarized ENV. Interests_t* \times *High CPU_t* between groups are at the bottom of the table. In Column (3)-(4), the dependent variable is $Ln(1+ENV Lobbying_t)$. The t -statistics reported in parentheses are based on standard errors clustered at the firm and year level. ***, **, * denote 1%, 5%, and 10% statistical significance.

<i>Product Market Competition:</i>	<i>High</i>	<i>Low</i>	<i>High</i>	<i>Low</i>
	(1)	(2)	(3)	(4)
	<i>I_ENV Lobbying_t</i>		<i>Ln(1+ENV Lobbying_t)</i>	
Polarized ENV. Interests _t \times High CPU _t	0.050*** (3.67)	0.010 (0.67)	0.631*** (4.13)	0.059 (0.35)
Polarized ENV. Interests _t	-0.014 (-1.06)	0.001 (0.07)	-0.196 (-1.29)	0.047 (0.36)
Ln(Total Assets _{t-1})	0.059*** (3.78)	0.058*** (3.43)	0.761*** (4.09)	0.567*** (3.04)
Book-to-Market _{t-1}	-0.010 (-0.56)	-0.032* (-2.05)	-0.179 (-0.80)	-0.212 (-1.07)
Cash to TA _{t-1}	0.054 (0.94)	-0.082 (-1.40)	0.709 (1.08)	-0.767 (-1.19)
Profitability _{t-1}	0.060 (0.92)	-0.209** (-2.74)	0.706 (1.05)	-2.134** (-2.55)
R&D to TA _{t-1}	0.035 (0.28)	0.637 (1.73)	0.584 (0.40)	6.828* (1.87)
CAPEX to TA _{t-1}	-0.010 (-0.06)	0.016 (0.11)	-0.818 (-0.47)	-1.043 (-0.50)
Leverage _{t-1}	0.070 (1.73)	-0.139*** (-2.97)	0.689 (1.63)	-1.584*** (-3.14)
Observations	6,943	6,511	6,943	6,511
Adj. R-squared	0.569	0.645	0.607	0.688
Firm FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES
<i>Difference Between Groups</i>				
Polarized ENV. Interests _t \times High CPU _t	0.040 (1.64)		0.572** (2.40)	

Table 5: Instrumental Variable Analysis

We report the instrumental variables regression results using the changes in the U.S. Environmental Protection Agency (EPA) administrators affiliated with different political parties as an exogenous shock to the level of climate policy uncertainty. $EPA\ Admin.\ Change_t$ is a dummy equals 1 if year t is within the first two years of service for new U.S. EPA administrators from different political parties (i.e., the year 2009, 2010, 2017, and 2018). We use $EPA\ Admin.\ Change_t$ and $Polarized\ ENV.\ Interests_t \times EPA\ Admin.\ Change_t$ as the instrument variables for $High\ CPU_t$ and $Polarized\ ENV.\ Interests_t \times High\ CPU_t$. We replace year fixed effects with macro variables, such as $GDP\ Growth$, $Consumer\ Confidence\ Index$, $1\text{-}Year\ Treasury\ Rate$, and $Ln(EPU)$. We include other variables of firm characteristics and firm fixed effects. The table reports point estimates with t -statistics in parentheses. Standard errors in Columns (1)-(2) are clustered at the firm and year level. Standard errors in Columns (3)-(4) are calculated using the bootstrap method, and clustered at the firm and year level. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels.

	1st Stage		2nd Stage	
	(1)	(2)	(3)	(4)
	$High\ CPU_t$	$Polarized\ ENV.\ Interests_t \times High\ CPU_t$	$L\text{-}ENV\ Lobbying_t$	$Ln(1+ENV\ Lobbying_t)$
EPA Admin. Change _t	0.976*** (8.63)	0.078 (0.99)		
Polarized ENV. Interests _t × EPA Admin. Change _t	-0.003 (-0.58)	0.857*** (9.21)		
$Polarized\ ENV.\ \widehat{Interests}_t \times High\ CPU_t$			0.025** (2.59)	0.278** (2.60)
$\widehat{High\ CPU}_t$			0.010 (1.10)	0.131 (1.33)
Polarized ENV. Interests _t	-0.013 (-1.71)	0.142 (1.59)	-0.005 (-0.70)	-0.030 (-0.42)
Ln(Total Assets _{t-1})	0.043 (1.06)	0.024 (0.97)	0.066*** (11.82)	0.820*** (13.80)
Book-to-Market _{t-1}	-0.140 (-1.70)	-0.087 (-1.68)	-0.021* (-1.65)	-0.181 (-1.28)
Cash to TA _{t-1}	0.076 (1.27)	0.034 (0.87)	0.006 (0.18)	0.249 (0.62)
Profitability _{t-1}	-0.136 (-1.55)	-0.065 (-1.28)	-0.040 (-1.01)	-0.337 (-0.76)
R&D to TA _{t-1}	-0.024 (-0.14)	0.026 (0.21)	0.137 (1.34)	1.851* (1.66)
CAPEX to TA _{t-1}	-0.811 (-1.19)	-0.601 (-1.32)	0.042 (0.41)	-0.222 (-0.19)
Leverage _{t-1}	-0.131 (-1.19)	-0.101 (-1.46)	0.005 (0.27)	0.057 (0.26)
GDP Growth _{t-1}	0.064 (0.67)	0.041 (0.68)	-0.006** (-2.27)	-0.049 (-1.63)
Consumer Confidence Index _{t-1}	-1.081 (-1.09)	-0.698 (-1.09)	-0.157*** (-5.10)	-2.332*** (-6.64)
1-Year Treasury Rate _{t-1}	0.075 (1.17)	0.046 (1.16)	0.014*** (7.21)	0.149*** (6.84)
Ln(EPU _{t-1})	0.101 (0.49)	0.055 (0.43)	-0.008 (-0.61)	-0.149 (-0.96)
Observations	14,367	14,367	14,367	14,367
Adj. R-squared	0.683	0.750	0.595	0.634
Firm FE	YES	YES	YES	YES
F-Statistics	41.62			

Table 6: The Effect of Lobbying Tug-of-War on Cost of Capital

We report the panel regression results of firms' environmental stances (*Polarized ENV. Interests_t*) on their average implied cost of capital (ICC) when facing high Climate Policy Uncertainty (CPU). *Avg. Implied Cost of Capital* is the average value of five ICC measures following Easton (2004), Gode and Mohanram (2003), Gordon and Gordon (1997), Gebhardt et al. (2001), and Claus and Thomas (2001). In Column (1), we include the Polarized ENV. Interests dummy interacted with High CPU controlling for various firm characteristics with firm and year fixed effects. In Column (2), we additionally include firm-level general political risk from Hassan et al. (2019). The *t*-statistics reported in parentheses are based on standard errors clustered at the firm and year level. ***, **, * denote 1%, 5%, and 10% statistical significance.

	(1)	(2)
	<i>Avg. Implied Cost of Capital_{t+1}</i>	
Polarized ENV. Interests _t × High CPU _t	0.003* (1.75)	0.003* (1.76)
Polarized ENV. Interests _t	-0.001 (-0.84)	-0.001 (-0.84)
Ln(Total Assets _t)	-0.005** (-2.88)	-0.005** (-2.88)
Book-to-Market _t	0.026*** (3.86)	0.026*** (3.86)
Cash to TA _t	-0.016*** (-3.55)	-0.016*** (-3.53)
Profitability _t	-0.038*** (-3.20)	-0.038*** (-3.20)
Leverage _t	0.025*** (4.32)	0.025*** (4.32)
Political Risk _t		-0.000 (-0.73)
Observations	9,641	9,641
Adj. R-squared	0.570	0.570
Firm FE	YES	YES
Year FE	YES	YES

Table 7: Real Effect of the Lobbying Tug-of-War

We report the panel regression results of firms' environmental stances (*Polarized ENV. Interests_t*) on firm's long-term investments ($\Delta R\&D_{t+1}$, $\Delta CAPEX_{t+1}$, $\Delta Total Investment_{t+1}$) when facing high Climate Policy Uncertainty (CPU). In Column (1), the dependent variable is $\Delta R\&D_{t+1}$, which is the percentage change of R&D expenses of a firm i from year t to year $t+1$. We include the Polarized ENV. Interests dummy interacted with High CPU controlling for various firm characteristics with firm and year fixed effects. In Column (2), the dependent variable is $\Delta CAPEX_{t+1}$, which is the percentage change of capital expenditures of a firm i from year t to year $t+1$. In Column (3), the dependent variable is $\Delta Total Investment_{t+1}$, which is the percentage change of R&D expenses plus capital expenditures of a firm i from year t to year $t+1$. The t -statistics reported in parentheses are based on standard errors clustered at the firm and year level. ***, **, * denote 1%, 5%, and 10% statistical significance.

	(1)	(2)	(3)
	$\Delta R\&D_{t+1}$	$\Delta CAPEX_{t+1}$	$\Delta Total Investment_{t+1}$
Polarized ENV. Interests _t × High CPU _t	-0.011** (-2.78)	-0.050*** (-3.19)	-0.030*** (-2.93)
Polarized ENV. Interests _t	0.005 (1.65)	0.007 (0.76)	0.008 (1.18)
Ln(Total Assets _t)	-0.009 (-1.53)	-0.031** (-2.70)	-0.036** (-2.83)
Book-to-Market _t	-0.011 (-1.41)	-0.279*** (-8.98)	-0.245*** (-9.51)
Cash to TA _t	0.161*** (4.93)	0.728*** (8.05)	0.524*** (8.18)
Profitability _t	0.141*** (4.20)	0.022 (0.14)	0.199 (1.63)
Leverage _t	-0.021 (-1.62)	-0.171*** (-3.41)	-0.190*** (-5.89)
Political Risk _t	-0.001 (-0.73)	-0.002 (-0.41)	-0.006* (-2.09)
Observations	12,887	12,887	12,887
Adj. R-squared	0.282	0.141	0.169
Firm FE	YES	YES	YES
Year FE	YES	YES	YES

TUG OF WAR IN CORPORATE
ENVIRONMENTAL LOBBYING

INTERNET APPENDIX
FOR ONLINE PUBLICATION

A. Supplemental Tables

Appendix Table 1: An Example of Lobbying Report from “Form LD-2”

Clerk of the House of Representatives Legislative Resource Center 135 Cannon Building Washington, DC 20515 http://lobbyingdisclosure.house.gov	Secretary of the Senate Office of Public Records 232 Hart Building Washington, DC 20510 http://www.senate.gov/lobby
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LOBBYING REPORT

Lobbying Disclosure Act of 1995 (Section 5) - All Filers Are Required to Complete This Page

1. Registrant Name <input checked="" type="checkbox"/> Organization/Lobbying Firm <input type="checkbox"/> Self Employed Individual Bingham McCutchen LLP	
2. Address Address1 2020 K Street, NW Address2 _____ City Washington State DC Zip Code 20006 Country USA	
3. Principal place of business (if different than line 2) City _____ State _____ Zip Code _____ Country _____	
4a. Contact Name Mr. CHARLES H. KNAUSS	b. Telephone Number 2023736000
c. E-mail maurcia.brown@bingham.com	5. Senate ID# 51560-470
7. Client Name <input type="checkbox"/> Self <input type="checkbox"/> Check if client is a state or local government or instrumentality EXXON MOBIL CORPORATION	6. House ID# 349100024

TYPE OF REPORT 8. Year 2011 Q1 (1/1 - 3/31) Q2 (4/1 - 6/30) Q3 (7/1 - 9/30) Q4 (10/1 - 12/31)

9. Check if this filing amends a previously filed version of this report
10. Check if this is a Termination Report Termination Date 06/30/2011 11. No Lobbying Issue Activity

INCOME OR EXPENSES - YOU MUST complete either Line 12 or Line 13	
<p>12. Lobbying</p> <p>INCOME relating to lobbying activities for this reporting period was:</p> <p>Less than \$5,000 <input type="checkbox"/></p> <p>\$5,000 or more <input checked="" type="checkbox"/> \$ <u>70,000.00</u></p> <p>Provide a good faith estimate, rounded to the nearest \$10,000, of all lobbying related income for the client (including all payments to the registrant by any other entity for lobbying activities on behalf of the client).</p>	<p>13. Organizations</p> <p>EXPENSE relating to lobbying activities for this reporting period were:</p> <p>Less than \$5,000 <input type="checkbox"/></p> <p>\$5,000 or more <input type="checkbox"/> \$ _____</p> <p>14. REPORTING Check box to indicate expense accounting method. See instructions for description of options.</p> <p><input type="checkbox"/> Method A. Reporting amounts using LDA definitions only</p> <p><input type="checkbox"/> Method B. Reporting amounts under section 6033(b)(8) of the Internal Revenue Code</p> <p><input type="checkbox"/> Method C. Reporting amounts under section 162(e) of the Internal Revenue Code</p>

Signature Digitally Signed By: Charles H. Knauss, Partner Date 07/22/2011

Appendix Table 1 Continues

LOBBYING ACTIVITY. Select as many codes as necessary to reflect the general issue areas in which the registrant engaged in lobbying on behalf of the client during the reporting period. Using a separate page for each code, provide information as requested. Add additional page(s) as needed.

15. General issue area code CAW

16. Specific lobbying issues

Clean Air Act Regulatory Issues Interior, Environment and Related Agencies Appropriations Bill, FY2012 Draft Clean Air Act Legislation
--

17. House(s) of Congress and Federal agencies Check if None

U.S. HOUSE OF REPRESENTATIVES, U.S. SENATE, Commerce - Dept of (DOC), Executive Office of the President (EOP), Environmental Protection Agency (EPA), Small Business Administration (SBA)

18. Name of each individual who acted as a lobbyist in this issue area

First Name	Last Name	Suffix	Covered Official Position (if applicable)	New
Teresa	Gorman			<input type="checkbox"/>

19. Interest of each foreign entity in the specific issues listed on line 16 above Check if None

--

Appendix Table 2: List of General Issue Area Codes

The table provides the complete list of general issue area codes and their descriptions in “Form LD-2.”

Code	Description	Code	Description
ACC	Accounting	HOM	Homeland Security
ADV	Advertising	HOU	Housing
AER	Aerospace	IMM	Immigration
AGR	Agriculture	IND	Indian/Native American Affairs
ALC	Alcohol & Drug Abuse	INS	Insurance
ANI	Animals	LBR	Labor Issues/Antitrust/Workplace
APP	Apparel/Clothing Industry/Textiles	INT	Intelligence and Surveillance
ART	Arts/Entertainment	LAW	Law Enforcement/Crime/Criminal Justice
AUT	Automotive Industry	MAN	Manufacturing
AVI	Aviation/Aircraft/Airlines	MAR	Marine/Maritime/Boating/Fisheries
BAN	Banking	MED	Medical/Disease Research/Clinical Labs
BNK	Bankruptcy	MIA	Media (Information/Publishing)
BEV	Beverage Industry	MMM	Medicare/Medicaid
BUD	Budget/Appropriations	MON	Minting/Money/Gold Standard
CAW	Clean Air & Water (Quality)	NAT	Natural Resources
CDT	Commodities (Big Ticket)	PHA	Pharmacy
CHM	Chemicals/Chemical Industry	POS	Postal
CIV	Civil Rights/Civil Liberties	RRR	Railroads
COM	Communications/Broadcasting/Radio/TV	RES	Real Estate/Land Use/Conservation
CPI	Computer Industry	REL	Religion
CSP	Consumer Issues/Safety/Protection	RET	Retirement
CON	Constitution	ROD	Roads/Highway
CPT	Copyright/Patent/Trademark	SCI	Science/Technology
DEF	Defense	SMB	Small Business
DOC	District of Columbia	SPO	Sports/Athletics
DIS	Disaster Planning/Emergencies	TAR	Miscellaneous Tariff Bills
ECN	Economics/Economic Development	TAX	Taxation/Internal Revenue Code
EDU	Education	TEC	Telecommunications
ENG	Energy/Nuclear	TOB	Tobacco
ENV	Environmental/Superfund	TOR	Torts
FAM	Family Issues/Abortion/Adoption	TRD	Trade (Domestic & Foreign)
FIR	Firearms/Guns/Ammunition	TRA	Transportation
FIN	Financial Institutions/Investments/Securities	TOU	Travel/Tourism
FOO	Food Industry (Safety, Labeling, etc.)	TRU	Trucking/Shipping
FOR	Foreign Relations	URB	Urban Development/Municipalities
FUE	Fuel/Gas/Oil	UNM	Unemployment
GAM	Gaming/Gambling/Casino	UTI	Utilities
GOV	Government Issues	VET	Veterans
HCR	Health Issues	WAS	Waste (hazardous/solid/interstate/nuclear)
		WEL	Welfare

Appendix Table 3: Firm’s Environmental Stance and Direction of Environmental Lobbying

We tabulate environmental lobbying direction by the firm’s environmental stance (Green/Neutral/Brown). We first extract all the Congress bills mentioned in the lobbying issues of the lobbying report and match the bills to the Congress Roll Call data by the League of Conservation Voters (<https://scorecard.lcv.org/scorecard?year=all>) to identify the environmental direction of the bill. We then match the bills to the MapLight data, which identifies the firm’s position on the bills whether they support or oppose the bill passage in Congress. We identify 59 firm-year-level observations in our sample. We define that a firm has “Green (Brown) Lobbying Direction” if the firm supports pro(anti)-environmental bills, or opposes anti(pro)-environmental bills.

Firm’s Environmental Stance	Lobbying Direction		<i>Total</i>
	<i>Green</i>	<i>Brown</i>	
<i>Green</i>	15 (83%)	3 (17%)	18
<i>Neutral</i>	4 (33%)	8 (67%)	12
<i>Brown</i>	11 (38%)	18 (62%)	29
<i>Total</i>	30 (51%)	29 (49%)	59

B. Proof of the Model

Throughout the proof of the model, we assume the following.

Assumption 1. For θ_l , θ_h , $\tilde{\theta}$ and $\tilde{\beta}$,

- 1) $\mathbb{E}[\tilde{\theta}] = 1$
- 2) $\theta_l < \frac{1}{\eta} < 1 < \theta_h$
- 3) $\tilde{\theta}$ and $\tilde{\beta}$ are independent.

Assumption 2. $m_0 \equiv b\alpha\gamma\eta/\{(1 + \sqrt{\Delta})x^2\}$ satisfies the following condition:

$$m_0 > \max \left\{ 2, \frac{a + 3K}{4K} \right\}$$

Assumption 2 guarantees that the marginal cost of production and the optimal production quantity are always positive for any equilibrium policy level so that the Cournot competition is well-defined. This assumption also ensures that firms' net profit is concave in the proposed policy level.

Equilibrium of the second-period Cournot competition

Lemma 1. The firm i 's optimal quantity and profit are given by

$$q_i^* = \frac{a - K + 4x_i\tilde{y}}{4\tilde{b}} \text{ and } v_i = \frac{(a - K + 4x_i\tilde{y})^2}{16\tilde{b}}.$$

Proof of Lemma 1 By taking the first-order condition of v_i with respect to q_i , the optimal quantities in the equilibrium satisfy

$$2q_G + q_N + q_B = \frac{a - K + x\tilde{y}}{\tilde{b}}, \quad q_G + 2q_N + q_B = \frac{a - K}{\tilde{b}}, \quad q_G + q_N + 2q_B = \frac{a - K - x\tilde{y}}{\tilde{b}}.$$

Solving the linear system yields each firm i 's optimal quantity q_i^* and profit v_i . ■

Proof of Theorem 1

Given that firm i 's net profit is

$$\Pi_i(s_i, y_i; y_{-i}, F_{-i}) = -L(s_i, y_i) + F_{-i}(s_i) \cdot \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_i)] + \int_{s_i}^{\infty} \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_{-i}(s_{-i}))] dF_{-i}$$

where $L(s_i, y_i) = \alpha(s_i + \gamma y_i^2)$, the following lemma holds. (Proofs are analogous to that of Hirsch and Shotts (2015).)

Lemma 2. *The following properties hold.*

1. At any $s_i > 0$ where $-i$ has no atom, proposing $(s_i, y_i^*(s_i))$ with $y_i^*(s_i) = \frac{x_i(a-K)F(s_i)}{4\{b\alpha\gamma\eta - (1+\Delta)x^2F(s_i)\}}$ is strictly better than proposing any other policy.
2. In any equilibrium, $F_k(0) = 0$ for some $k \in \{G, B\}$ and the support of the score CDFs (F_G, F_B) over $s \geq 0$ is common, convex, atomless, and includes 0.

The optimal policy proposed by each firm is directly derived from the first-order condition of y_i . The next step is to derive a mixed strategy of Green (F_G) and Brown firm (F_B) in the equilibrium²³. In the equilibrium, any of $(s_i, y_i^*(s_i))$ in the mixed strategy yields the same utility. Let $[0, \bar{s}]$ be the common support of F_G and F_B . Then, for every $s \in [0, \bar{s}]$, both Green and Brown firms maximize their expected utility when the optimal policies $y_i^*(s)$ is developed. That is, $s_i \in \arg \max_{s_i} \{\Pi_i(s_i, y_i^*(s_i); s_{-i}, y_{-i}, F_{-i})\}$ with probability 1. So, the first-order condition of s_i should be satisfied:

$$0 = \frac{\partial \Pi_i(s_i, y_i^*(s_i))}{\partial s_i} = -\alpha + f_{-i}(s_i) \cdot \mathbb{E}[\tilde{M}\{v_i(\tilde{\beta}y_i^*(s_i)) - v_i(\tilde{\beta}y_{-i}^*(s_i))\}],$$

which implies

$$\begin{aligned} \alpha &= f_B(s) \cdot \frac{x^2(a-K)^2}{8b\eta} \left(\frac{F_B(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)} + \frac{F_G(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)} \right) \\ &\quad \times \left\{ 1 + \frac{(1+\Delta)x^2}{2} \left(\frac{F_B(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)} - \frac{F_G(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)} \right) \right\}, \\ \alpha &= f_G(s) \cdot \frac{x^2(a-K)^2}{8b\eta} \left(\frac{F_B(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)} + \frac{F_G(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)} \right) \\ &\quad \times \left\{ 1 - \frac{(1+\Delta)x^2}{2} \left(\frac{F_B(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)} - \frac{F_G(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)} \right) \right\} \end{aligned}$$

for all $s \in [0, \bar{s}]$. Then,

$$\begin{aligned} &f_B(s) \cdot \left\{ 1 + \frac{(1+\Delta)x^2}{2} \left(\frac{F_B(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)} - \frac{F_G(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)} \right) \right\} \\ &= f_G(s) \cdot \left\{ 1 - \frac{(1+\Delta)x^2}{2} \left(\frac{F_B(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)} - \frac{F_G(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)} \right) \right\}, \end{aligned}$$

²³We follow the restriction in Hirsch and Shotts (2015) that the firms use strategies that are the sum of an absolutely continuous and a discrete distribution for technical convenience.

for all $s \in [0, \bar{s}]$. By rearranging terms,

$$\begin{aligned} f_B(s) - f_G(s) + \frac{b\alpha\gamma\eta(1+\Delta)x^2}{2} \cdot \frac{\{f_B(s) + f_G(s)\}\{F_B(s) - F_G(s)\}}{\{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)\}\{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)\}} &= 0 \\ \implies [\{F_B(s) - F_G(s)\}e^{\int_0^s R(t)dt}]' &= 0 \quad \forall s \in [0, \bar{s}], \end{aligned}$$

where $R(s) := \frac{b\alpha\gamma\eta(1+\Delta)x^2}{2} \cdot \frac{f_B(s)+f_G(s)}{\{b\alpha\gamma\eta - (1+\Delta)x^2F_B(s)\}\{b\alpha\gamma\eta - (1+\Delta)x^2F_G(s)\}}$ is a nonnegative function and integrable almost everywhere by the assumption mentioned in footnote 23. Taking integral from s to \bar{s} yields $\{F_B(s) - F_G(s)\}e^{\int_0^s R(t)dt} = 0 \quad \forall s \in [0, \bar{s}]$, concluding that F_B and F_G should be identical.

Let $F_B = F_G = F$. We obtain a single differential equation $\alpha = f(s) \cdot \frac{x^2(a-K)^2}{4b\eta} \left(\frac{F(s)}{b\alpha\gamma\eta - (1+\Delta)x^2F(s)} \right)$. Substituting $s = F^{-1}(F)$ and applying the chain rule $(\frac{1}{f(F^{-1}(F))}) = \frac{\partial}{\partial F} F^{-1}(F)$, we induce

$$\frac{\partial}{\partial F} F^{-1}(F) = \frac{x^2(a-K)^2}{4b\alpha\eta} \left(\frac{F}{b\alpha\gamma\eta - (1+\Delta)x^2F} \right).$$

By integrating both sides with initial condition $F(0) = 0$, we obtain the inverse of CDF²⁴.

To show monotonicity and convexity of lobbying expenditures with respect to the level of proposed policy, $F(s_i)$ can be rewritten as

$$F(s_i) = \frac{4b\alpha\gamma\eta y_i}{x_i\{a - K + 4x_i(1+\Delta)y_i\}},$$

for $i \in \{G, B\}$. Plugging it into the inverse of CDF formula, the optimal lobbying amount s_i by optimal proposed policy y_i is expressed as

$$s_i = \gamma \left(\frac{a-K}{2x(1+\Delta)} \right)^2 \left\{ \ln \left(1 + \frac{4x_i(1+\Delta)y_i}{a-K} \right) - \frac{4x_i(1+\Delta)y_i}{a-K+4x_i(1+\Delta)y_i} \right\}.$$

Taking first order derivative of s_i by y_i ,

$$\frac{\partial s_i}{\partial y_i} = \frac{4\gamma(a-K)^2 y_i}{\{a-K+4x_i(1+\Delta)y_i\}^2},$$

which is strictly positive (negative) for positive (negative) y_i . It follows that Green (Brown) firm's optimal lobbying expenditure is increasing (decreasing) in proposed policy level y . Lastly, the convexity follows from the second order derivative of s_i by y_i :

$$\frac{\partial^2 s_i}{\partial y_i^2} = \frac{4\gamma(a-K)^2(a-K-4x_i(1+\Delta)y_i)}{\{a-K+4x_i(1+\Delta)y_i\}^3} > 0,$$

²⁴The uniqueness of the solution is assured by Lipschitz continuity.

as $a - K \pm 4x_i(1 + \Delta)y_i > 0$ by Assumption 2. ■

Proof of Model Predictions

Model Prediction 1 We first show that the maximum value of lobbying expenditure in the equilibrium

$\bar{s} = F^{-1}(1) = \left(\frac{a-K}{2x(1+\Delta)}\right)^2 \left(\gamma \ln \frac{b\alpha\gamma\eta}{b\alpha\gamma\eta - (1+\Delta)x^2} - \frac{(1+\Delta)x^2}{b\alpha\eta}\right)$ is increasing in Δ . Taking the derivative of \bar{s} by Δ ,

$$\begin{aligned} \frac{\partial \bar{s}}{\partial \Delta} &= \frac{\gamma(a-K)^2}{4(1+\Delta)^3 x^2} \left(\frac{(1+\Delta)x^2}{b\alpha\gamma\eta - (1+\Delta)x^2} + \frac{(1+\Delta)x^2}{b\alpha\gamma\eta} - 2 \ln \frac{b\alpha\gamma\eta}{b\alpha\gamma\eta - (1+\Delta)x^2} \right) \\ &= \frac{\gamma(a-K)^2}{4(1+\Delta)^3 x^2} \left(\frac{1}{m-1} + \frac{1}{m} - 2 \ln \frac{m}{m-1} \right) \\ &= \frac{\gamma(a-K)^2}{4(1+\Delta)^3 x^2} g(m), \end{aligned}$$

where $m = b\alpha\gamma\eta/\{(1+\Delta)x^2\} \geq m_0 > \max\{2, \frac{a+3K}{4K}\}$ by Assumption 2. Computing $g'(m)$ yields

$$g'(m) = -\frac{1}{\{m(m-1)\}^2} < 0.$$

Observing that $g(2) > 0$ and $\lim_{m \rightarrow \infty} g(m) = 0$, $g(m)$ is strictly positive. To derive stochastic dominance of s by Δ , we use $F^{-1}(F(s; p); p) = s$ for fixed s and any parameter p . Following that $\frac{\partial F}{\partial p} = -\frac{\partial F^{-1}/\partial p}{\partial F^{-1}/\partial F}$, s is first-order stochastically increasing in p if and only if $\frac{\partial F^{-1}}{\partial p}$ is positive. As $F^{-1}(F) = \left(\frac{a-K}{2x(1+\Delta)}\right)^2 \left(\gamma \ln \frac{b\alpha\gamma\eta}{b\alpha\gamma\eta - (1+\Delta)x^2 F} - \frac{(1+\Delta)x^2}{b\alpha\eta} F\right)$ is increasing in Δ , s is first-order stochastically larger as Δ increases. ■

Model Prediction 2 Taking the derivative of $\frac{\partial \bar{s}}{\partial \Delta}$ by b , we obtain

$$\frac{\partial \bar{s}}{\partial \Delta \partial b} = \frac{\partial}{\partial \Delta} \frac{\partial \bar{s}}{\partial b} = \frac{\partial}{\partial \Delta} \left(-\frac{x^2(a-K)^2}{4b^2\alpha\eta\{b\alpha\gamma\eta - (1+\Delta)x^2\}} \right) = -\frac{x^4(a-K)^2}{4b^2\alpha\eta\{b\alpha\gamma\eta - (1+\Delta)x^2\}^2} < 0.$$

Next, we show that the responsiveness of the expected lobbying expenditure to Δ is decreasing in b . The expected value of lobbying expenditure can be expressed as

$$\begin{aligned} \mathbb{E}[s] &= \int_0^{\bar{s}} s dF \\ &= \gamma \left(\frac{a-K}{2x(1+\Delta)} \right)^2 \int_0^{\bar{s}} \left\{ \ln \frac{m}{m-F(s)} - \frac{F(s)}{m} \right\} f(s) ds \\ &= \gamma \left(\frac{a-K}{2x(1+\Delta)} \right)^2 \int_0^1 \left\{ \ln \frac{m}{m-t} - \frac{t}{m} \right\} dt \end{aligned}$$

where $m = b\alpha\gamma\eta/\{(1 + \Delta)x^2\}$. Differentiating it by b , we have

$$\begin{aligned}\frac{\partial \mathbb{E}[s]}{\partial b} &= \frac{\partial \mathbb{E}[s]}{\partial m} \frac{\partial m}{\partial b} \\ &= \gamma \left(\frac{a - K}{2x(1 + \Delta)} \right)^2 \int_0^1 \left\{ \frac{1}{m} - \frac{1}{m - t} + \frac{t}{m^2} \right\} dt \cdot \frac{m}{b} \\ &= \frac{\gamma}{b} \left(\frac{a - K}{2x(1 + \Delta)} \right)^2 \int_0^1 \frac{-t^2}{m(m - t)} dt \\ &= \frac{\gamma}{b} \left(\frac{x(a - K)}{2b\alpha\gamma\eta} \right)^2 \int_0^1 \frac{-mt^2}{m - t} dt.\end{aligned}$$

Then,

$$\begin{aligned}\frac{\partial}{\partial b} \left(\frac{\partial \mathbb{E}[s]}{\partial \Delta} \right) &= \frac{\partial}{\partial m} \left(\frac{\partial \mathbb{E}[s]}{\partial b} \right) \frac{\partial m}{\partial \Delta} \\ &= \frac{\gamma}{b} \left(\frac{x(a - K)}{2b\alpha\gamma\eta} \right)^2 \int_0^1 \left\{ \frac{-t^2}{m - t} + \frac{mt^2}{(m - t)^2} \right\} dt \cdot \frac{-m}{1 + \Delta} \\ &= \frac{\gamma}{b(1 + \Delta)} \left(\frac{x(a - K)}{2b\alpha\gamma\eta} \right)^2 \int_0^1 \frac{-mt^3}{(m - t)^2} dt < 0,\end{aligned}$$

implying that $\frac{\partial \mathbb{E}[s]}{\partial \Delta}$ is decreasing in b . ■

Model Prediction 3 First, we show that firm i 's expected utility is strictly positive. A firm i 's expected utility at the competitive equilibrium is:

$$U_i^* = \Pi_i(\bar{s}, y_i(\bar{s})) = -\alpha(\bar{s} + \gamma y_i(\bar{s})^2) + \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_i(\bar{s}))] = -\alpha\bar{s} + \frac{(a - K)^2}{16b\eta} \left(1 + \frac{x^2}{b\alpha\gamma\eta - (1 + \Delta)x^2} \right)$$

Substituting $\bar{s} = \left(\frac{a - K}{2x(1 + \Delta)} \right)^2 \left(\gamma \ln \frac{b\alpha\gamma\eta}{b\alpha\gamma\eta - (1 + \Delta)x^2} - \frac{(1 + \Delta)x^2}{b\alpha\eta} \right)$ and $m = b\alpha\gamma\eta/\{(1 + \Delta)x^2\}$,

$$\begin{aligned}\Pi_i(\bar{s}, y_i(\bar{s})) &= \frac{(a - K)^2}{4} \left(-\frac{\alpha\gamma}{(1 + \Delta)^2 x^2} \ln \frac{b\alpha\gamma\eta}{b\alpha\gamma\eta - (1 + \Delta)x^2} + \frac{x^2}{4b\eta(b\alpha\gamma\eta - (1 + \Delta)x^2)} \right. \\ &\quad \left. + \frac{1}{b\eta(1 + \Delta)} + \frac{1}{4b\eta} \right) \\ &= \frac{(a - K)^2}{4b\eta(1 + \Delta)} \left(-m \ln \frac{m}{m - 1} + \frac{1}{4(m - 1)} + 1 + \frac{1 + \Delta}{4} \right) \\ &\geq \frac{(a - K)^2}{4b\eta(1 + \Delta)} \left(h(m) + \frac{1}{4} \right),\end{aligned}$$

where $h(m) = -m \ln \frac{m}{m-1} + \frac{1}{4(m-1)} + 1$. By taking first and second-order derivatives of $h(m)$,

$$\begin{aligned} h'(m) &= -\ln \frac{m}{m-1} + \frac{4m-5}{4(m-1)^2} \\ h''(m) &= -\frac{m-2}{2m(m-1)^3}. \end{aligned}$$

Observe that $h(m)$ is strictly concave on $m > 2$. Since $h'(2) > 0$ and $\lim_{m \rightarrow \infty} h'(m) = 0$, $h(m)$ is strictly increasing on $m > 2$. From $h(2) \in (-\frac{1}{4}, 0)$, U_i^* is strictly positive on $m > 2$.

Next, we compute the firm i 's cost of capital, which is the expected rate of return. The expected rate of return r_i can be expressed as the ratio of expected profit (P_{1i}) at $t = 1$, which is a function of s , to its discounted value at $t = 0$, which is the firm i 's expected utility U_i^* , minus one. For Neutral firm, the expected profit at $t = 1$, P_{1N} , is $\frac{(a-K)^2}{16b}$, and the expected utility U_N^* is $\frac{(a-K)^2}{16b\eta}$. So, the expected rate of return of Neutral firm r_N is equal to $\eta - 1$. For Green and Brown firms, recall that the firm i 's expected utility U_i^* is constant over $s \in [0, \bar{s}]$. Define $L_s = \alpha(s + \gamma y_i(s)^2)$. Observing that

$$\begin{aligned} \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y)] &= \mathbb{E}\left[\tilde{M} \cdot \frac{\tilde{\theta}}{16b}(a-K+4x_i\tilde{\beta}y)^2\right] \\ &= \mathbb{E}[\tilde{M}\tilde{\theta}] \cdot \mathbb{E}\left[\frac{(a-K+4x_i\tilde{\beta}y)^2}{16b}\right] \\ &= \frac{1}{\eta} \mathbb{E}\left[\frac{(a-K+4x_i\tilde{\beta}y)^2}{16b}\right] \\ &= \frac{1}{\eta} \mathbb{E}[v_i(\tilde{\beta}y)] \end{aligned}$$

for any policy y , we derive P_{1i} in terms of U_i^* , L_s , and η :

$$\begin{aligned} U_i^* &= -L_s + F(s) \cdot \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_i)] + \int_s^\infty \mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_{-i}(s_{-i}))] dF \\ &= -L_s + F(s) \cdot \frac{1}{\eta} \mathbb{E}[v_i(\tilde{\beta}y_i)] + \frac{1}{\eta} \cdot \int_s^\infty \mathbb{E}[v_i(\tilde{\beta}y_{-i}(s_{-i}))] dF \\ &= \frac{1}{\eta} P_{1i} - \frac{\eta-1}{\eta} L_s \\ &\implies P_{1i} = \eta U_i^* + (\eta-1)L_s. \end{aligned}$$

It follows that the Green (Brown) firm's expected rate of return r_i is

$$r_i = \frac{P_{1i}}{U_i^*} - 1 = (\eta-1) \left(1 + \frac{L_s}{U_i^*}\right) > \eta-1 \quad \forall s \in (0, \bar{s})$$

since $U_i^* > 0$ and $L_s > 0$ for any positive s .

To show the unconditional cost of capital $\mathbb{E}[r_i]$ is increasing in Δ , it is sufficient to show that the expected lobbying expenditure $\mathbb{E}[L_s] = \alpha\mathbb{E}[s + \gamma y_i(s)^2]$ is increasing, and the expected utility U_i^* is decreasing in Δ when $m > 3$. To show $\mathbb{E}[L_s]$ is increasing in Δ , it is sufficient to prove that $|y_i(s)| = |y_i(\Delta; s)|$ is increasing in Δ for all fixed s and firm i as s is first-order stochastically increasing in Δ . Recall the optimal lobbying amount s_i (which is fixed) expressed by optimal proposed policy y_i :

$$s_i = \gamma \left(\frac{a - K}{2x(1 + \Delta)} \right)^2 \left\{ \ln \left(1 + \frac{4x_i(1 + \Delta)y_i}{a - K} \right) - \frac{4x_i(1 + \Delta)y_i}{a - K + 4x_i(1 + \Delta)y_i} \right\}.$$

Differentiating both side by Δ yields,

$$\ln \left(1 + \frac{4x_i(1 + \Delta)y_i}{a - K} \right) - \frac{4x_i(1 + \Delta)y_i}{a - K + 4x_i(1 + \Delta)y_i} - \frac{8x_i^2(1 + \Delta)^2 y_i^2 + 8x_i^2(1 + \Delta)^3 y_i \frac{\partial y_i}{\partial \Delta}}{\{a - K + 4x_i(1 + \Delta)y_i\}^2} = 0$$

Let $t = \frac{4x_i(1 + \Delta)y_i}{a - K}$. Then,

$$\ln \left(1 + \frac{4x_i(1 + \Delta)y_i}{a - K} \right) - \frac{4x_i(1 + \Delta)y_i}{a - K + 4x_i(1 + \Delta)y_i} - \frac{8x_i^2(1 + \Delta)^2 y_i^2}{\{a - K + 4x_i(1 + \Delta)y_i\}^2} = \ln(1 + t) - \frac{t}{1 + t} - \frac{t^2}{2(1 + t)^2}.$$

This is positive at $t > 0$ as

$$\begin{aligned} \left\{ \ln(1 + t) - \frac{t}{1 + t} - \frac{t^2}{2(1 + t)^2} \right\}' &= \frac{1}{1 + t} - \frac{1}{(1 + t)^2} - \frac{t}{(1 + t)^2} + \frac{t^2}{(1 + t)^3} \\ &= \frac{t^2}{(1 + t)^3} > 0. \end{aligned}$$

It follows that $y_i \frac{\partial y_i}{\partial \Delta} > 0$. At the Green firm's side ($x_i, y_i > 0$), the optimal proposed policy is positive and increasing in Δ . On the other hand, Brown firm's optimal proposed policy is negative and decreasing in Δ . Therefore, $|y_i(s)|$ is increasing in Δ for both firms.

By differentiating U_i^* by Δ , it shows that

$$\begin{aligned} \frac{\partial U_i^*}{\partial \Delta} &= -\alpha \frac{\partial \bar{s}}{\partial \Delta} + \frac{(a - K)^2}{16b\eta} \frac{x^4}{\{b\alpha\gamma\eta - (1 + \Delta)x^2\}^2} \\ &= -\frac{\alpha\gamma(a - K)^2}{4(1 + \Delta)^3 x^2} \left\{ \frac{1}{m - 1} + \frac{1}{m} - 2 \ln \frac{m}{m - 1} - \frac{1}{4m(m - 1)^2} \right\} > 0 \end{aligned}$$

as $\lim_{m \rightarrow \infty} \left\{ \frac{1}{m - 1} + \frac{1}{m} - 2 \ln \frac{m}{m - 1} - \frac{1}{4m(m - 1)^2} \right\} = 0$ and $\left\{ \frac{1}{m - 1} + \frac{1}{m} - 2 \ln \frac{m}{m - 1} - \frac{1}{4m(m - 1)^2} \right\}' = -\frac{m - 3}{4m^2(m - 1)^3} < 0$ for $m > 3$. ■

Proposition 1. *Suppose that the policymaker chooses Green or Brown's proposed policy with equal probability.*

The difference in a cost of capital between Green/Brown and Neutral firm is smaller than the one in the presence of lobbying tug-of-war.

Proof of Proposition 1 Suppose that the winner is randomly determined by 50% regardless of how much Green and Brown firm lobby. Then, both firms would maximize their expected utility:

$$\Pi_i(s_i, y_i) = -\alpha(s + \gamma y_i^2) + \frac{1}{2}\mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_i)] + \frac{1}{2}\mathbb{E}[\tilde{M}v_i(\tilde{\beta}y_{-i})]$$

Since the lobby amount would not change their future payoffs, both firms would choose only policy level and not lobby at all, i.e. there is no tug of war. From the first-order condition, the optimal policy level of Green firm, y_w , and its expected utility are:

$$y_w = \frac{x(a - K)}{4(2b\alpha\gamma\eta - (1 + \Delta)x^2)} = \frac{a - K}{4x(1 + \Delta)(2m - 1)} < y(\bar{s})$$

$$\Pi_G^{NG} = \frac{(a - K)^2}{16b\eta} \left\{ 1 - \frac{(b\alpha\gamma\eta - (1 + \Delta)x^2)x^2}{(2b\alpha\gamma\eta - (1 + \Delta)x^2)^2} \right\} = \frac{(a - K)^2}{16b\eta(1 + \Delta)} \left\{ 1 + \Delta - \frac{m - 1}{(2m - 1)^2} \right\}$$

where $m = b\alpha\gamma\eta/\{(1 + \Delta)x^2\}$. Then, the cost of capital r_{NG} when there is no tug of war, is

$$r_{NG} = (\eta - 1) \left(1 + \frac{\alpha\gamma y_w^2}{\Pi_G^{NG}} \right)$$

The difference between Green (or Brown) firm's cost of capital when tug of war occurs, $\mathbb{E}[r_i]$, and that when there is no tug of war, r_{NG} , measures the extra risk induced by tug of war:

$$\mathbb{E}[r_i] - r_{NG} = (\eta - 1) \left(\frac{\mathbb{E}[L_s]}{\Pi_G} - \frac{\alpha\gamma y_w^2}{\Pi_G^{NG}} \right)$$

The expected upfront cost of Green (Brown) firm facing the tug of war $\mathbb{E}[L_s]$ is

$$\begin{aligned} \mathbb{E}[L_s] &= \int_0^{\bar{s}} \alpha\{s + \gamma y_i(s)^2\} dF \\ &= \frac{\alpha\gamma(a - K)^2}{4x^2(1 + \Delta)^2} \int_0^{\bar{s}} \left[\ln \frac{m}{m - F(s)} - \frac{F(s)}{m} + \frac{F(s)^2}{4\{m - F(s)\}^2} \right] f(s) ds \\ &= \frac{(a - K)^2}{4b\eta(1 + \Delta)} m \int_0^1 \ln \frac{m}{m - t} - \frac{t}{m} + \frac{t^2}{4(m - t)^2} dt \\ &= \frac{(a - K)^2}{4b\eta(1 + \Delta)} m \left(\frac{3}{2}m - 1 \right) \left\{ \ln \frac{m - 1}{m} + \frac{1}{2} \left(\frac{1}{m - 1} + \frac{1}{m} \right) \right\}. \end{aligned}$$

The last formula is induced by the following step:

$$\begin{aligned}
\int_0^1 \ln \frac{m}{m-t} - \frac{t}{m} + \frac{t^2}{4(m-t)^2} dt &= \int_{m-1}^m \ln \frac{m}{u} - \frac{m-u}{m} + \frac{(m-u)^2}{4u^2} du \\
&= \int_{m-1}^m \ln m - \ln u - 1 + \frac{u}{m} + \frac{m^2}{4u^2} - \frac{m}{2u} + \frac{1}{4} du \\
&= \ln m - \frac{3}{4} + \left[-u \ln u + u + \frac{u^2}{2m} - \frac{m^2}{4u} - \frac{m}{2} \ln u \right] \Big|_{m-1}^m \\
&= \left(\frac{3}{2}m - 1 \right) \ln \frac{m-1}{m} + \frac{1}{4} + \frac{m}{4} - \frac{(m-1)^2}{2m} + \frac{m^2}{4(m-1)} \\
&= \left(\frac{3}{2}m - 1 \right) \ln \frac{m-1}{m} + \frac{(3m-2)(2m-1)}{4m(m-1)} \\
&= \left(\frac{3}{2}m - 1 \right) \left\{ \ln \frac{m-1}{m} + \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\}
\end{aligned}$$

From the two terms:

$$\frac{\mathbb{E}[L_s]}{\Pi_G} = \frac{m \left(\frac{3}{2}m - 1 \right) \left\{ \ln \frac{m-1}{m} + \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\}}{-m \ln \frac{m}{m-1} + \frac{1}{4(m-1)} + 1 + \frac{1+\Delta}{4}}$$

and

$$\frac{\alpha \gamma y_w^2}{\Pi_G^{NG}} = \frac{\alpha \gamma \frac{(a-K)^2}{16x^2(1+\Delta)^2(2m-1)^2}}{\frac{(a-K)^2}{16b\eta(1+\Delta)} \left\{ 1 + \Delta - \frac{m-1}{(2m-1)^2} \right\}} = \frac{m}{(2m-1)^2 \left\{ (1+\Delta) - \frac{m-1}{(2m-1)^2} \right\}},$$

we need to show that

$$\begin{aligned}
Q(m) &:= \left(\frac{3}{2}m - 1 \right) \left\{ \ln \frac{m-1}{m} + \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\} \cdot \left[(2m-1)^2 \left\{ (1+\Delta) - \frac{m-1}{(2m-1)^2} \right\} \right] \\
&\quad - \left(-m \ln \frac{m}{m-1} + \frac{1}{4(m-1)} + 1 + \frac{1+\Delta}{4} \right) \\
&= (1+\Delta) \left[(2m-1)^2 \left(\frac{3}{2}m - 1 \right) \left\{ \ln \frac{m-1}{m} + \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\} - \frac{1}{4} \right] \\
&\quad - \left[(m-1) \left(\frac{3}{2}m - 1 \right) \left\{ \ln \frac{m-1}{m} + \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\} \right] - \left(-m \ln \frac{m}{m-1} + \frac{1}{4(m-1)} + 1 \right) > 0
\end{aligned}$$

in $m > 2$. Define $h_0(m)$, $h_1(m)$, and $h_2(m)$ as

$$\begin{aligned} h_0(m) &= \left(\frac{3}{2}m - 1\right) \left\{ \ln \frac{m-1}{m} + \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\} \\ &= \int_0^1 \ln \frac{m}{m-t} - \frac{t}{m} + \frac{t^2}{4(m-t)^2} dt \\ h_1(m) &= mh_0(m) \\ h_2(m) &= m^2 h_0(m). \end{aligned}$$

By differentiating $h_0(m)$, $h_1(m)$, and $h_2(m)$, we have

$$\begin{aligned} h'_0(m) &= \int_0^1 -\frac{t^2}{m^2(m-t)} - \frac{t^2}{2(m-t)^3} dt \\ h'_1(m) &= \int_0^1 \ln \frac{m}{m-t} - \frac{t}{m-t} - \frac{t^2(m+t)}{4(m-t)^3} dt \\ h'_2(m) &= \int_0^1 2m \ln \frac{m}{m-t} - \frac{mt}{m-t} - t - \frac{mt^3}{2(m-t)^3} dt. \end{aligned}$$

It follows that $h'_0(m)$ and $h'_1(m)$ are negative in $m > 2$ since $\ln \frac{m}{m-t} = \ln \left(1 + \frac{t}{m-t}\right) < \frac{t}{m-t}$. For $h'_2(m)$, the function inside the integral is 0 at $t = 0$, and decreasing in t since

$$\frac{\partial}{\partial t} \left\{ 2m \ln \frac{m}{m-t} - \frac{mt}{m-t} - t - \frac{mt^3}{2(m-t)^3} \right\} = -\frac{t^2}{(m-t)^2} - \frac{3mt^2}{2(m-t)^3} - \frac{3mt^3}{2(m-t)^4} < 0.$$

Thus, $h'_2(m)$ is also negative in $m > 2$. Taken together, the first term of (the latter expression of) $Q(m)$ can be expressed as $1 + \Delta$ times the difference of two positive and decreasing functions: $4h_2(m) + h_0(m) - (4h_1(m) + \frac{1}{4})$.

From the dominated convergence theorem,

$$\begin{aligned} \lim_{m \rightarrow \infty} h_0(m) &= \int_0^1 \lim_{m \rightarrow \infty} \left\{ \ln \frac{m}{m-t} - \frac{t}{m} + \frac{t^2}{4(m-t)^2} \right\} dt = 0 \\ \lim_{m \rightarrow \infty} h_1(m) &= \int_0^1 \lim_{m \rightarrow \infty} \left\{ m \ln \frac{m}{m-t} - t + \frac{mt^2}{4(m-t)^2} \right\} dt = 0 \\ \lim_{m \rightarrow \infty} h_2(m) &= \int_0^1 \lim_{m \rightarrow \infty} \left\{ m^2 \ln \frac{m}{m-t} - mt + \frac{m^2 t^2}{4(m-t)^2} \right\} dt \\ &= \int_0^1 \lim_{u \rightarrow 0} \left[\frac{1}{u^2} \left\{ \ln \frac{1}{1-ut} - ut \right\} + \frac{t^2}{4(1-ut)^2} \right] dt \\ &= \int_0^1 \frac{3t^2}{4} dt = \frac{1}{4}, \end{aligned}$$

implying that $4h_2(m) + h_0(m) > 1$ for $m > 2$. If $m > 2.5$,

$$4h_2(m) + h_0(m) - (4h_1(m) + \frac{1}{4}) > 1 - (4h_1(2.5) + \frac{1}{4}) > 0.$$

If $2 < m \leq 2.5$,

$$4h_2(m) + h_0(m) - (4h_1(m) + \frac{1}{4}) > 4h_2(2.5) + h_0(2.5) - (4h_1(2) + \frac{1}{4}) > 0.$$

So the first term of (the latter expression of) $Q(m)$ is positive in $m > 2$. For the other terms,

$$\begin{aligned} & - \left[(m-1) \left(\frac{3}{2}m - 1 \right) \left\{ \ln \frac{m-1}{m} + \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\} \right] - \left(-m \ln \frac{m}{m-1} + \frac{1}{4(m-1)} + 1 \right) \\ & = \left(\frac{3}{2}m^2 - \frac{3}{2}m + 1 \right) \ln \frac{m}{m-1} - \frac{1}{2}(m-1) \left(\frac{3}{2}m - 1 \right) \left(\frac{1}{m-1} + \frac{1}{m} \right) - \frac{1}{4(m-1)} - 1 \\ & = \left(\frac{3}{2}m^2 - \frac{3}{2}m + 1 \right) \ln \frac{m}{m-1} - \frac{1}{4m(m-1)} (6m^3 - 9m^2 + 6m - 2) \\ & = \left(\frac{3}{2}m^2 - \frac{3}{2}m + 1 \right) \ln \frac{m}{m-1} - \frac{1}{4m(m-1)} (3m^2 - 3m + 2)(2m-1) + \frac{1}{4(m-1)} \\ & = \left(\frac{3}{2}m^2 - \frac{3}{2}m + 1 \right) \left\{ \ln \frac{m}{m-1} - \frac{1}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) \right\} + \frac{1}{4(m-1)} \end{aligned}$$

Let the above expression $T(m)$. Observe that $T(2) > 0$ and $\lim_{m \rightarrow \infty} T(m) = 0$. If we show $T(m)$ is decreasing in $m > 2$, we are done. Taking the derivatives of $T(m)$,

$$\begin{aligned} T'(m) &= \frac{3}{2}(2m-1) \ln \left(\frac{m}{m-1} \right) - \frac{1}{m-1} + \frac{1}{m} + \frac{1}{4(m-1)^2} + \frac{1}{2m^2} - 3 \\ T''(m) &= 3 \ln \left(\frac{m}{m-1} \right) - \frac{3}{2} \left(\frac{1}{m-1} + \frac{1}{m} \right) + \frac{1}{(m-1)^2} - \frac{1}{m^2} - \frac{1}{2(m-1)^3} - \frac{1}{m^3} \\ T'''(m) &= 3 \left(-\frac{1}{m-1} + \frac{1}{m} \right) + \frac{3}{2} \left\{ \frac{1}{(m-1)^2} + \frac{1}{m^2} \right\} - \frac{2}{(m-1)^3} + \frac{2}{m^3} + \frac{3}{2(m-1)^4} + \frac{3}{m^4} \\ &= \frac{-6m^3 + 23m^2 - 20m + 6}{2m^4(m-1)^4} \end{aligned}$$

Since the numerator of $T'''(m)$ has the local maximum 10 at $m = 2$, $T'''(m) > 0$ in $2 < m < a_0$ and $T'''(m) < 0$ in $m > a_0$ for some $a_0 > 2$. That is, $T''(m)$ is increasing (decreasing) when $m(> 2)$ is smaller (greater) than a_0 . Observing that $T''(2) < 0$ and $\lim_{m \rightarrow \infty} T''(m) = 0$, $T''(a_0)$ should be positive, and there exists the unique $a_1 \in (2, a_0)$ such that $T''(m) < 0$ in $2 < m < a_1$ and $T''(m) > 0$ in $m > a_1$. Finally, $T'(2) < 0$ and $\lim_{m \rightarrow \infty} T'(m) = 0$ lead to $T'(m) < 0$ for $m > 2$. ■