Option Pricing under Extended Normal Distribution

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July 5, 2004

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Abstract

This paper proposes a closed pricing formula for European options when the dynamics of an underlying asset price does not follow the typical geometric Brownian motion as in a Black-Scholes framework. Instead the return distribution of the underlying asset is supposed to have any different degrees of skewness and kurtosis relative to the normal distribution by introducing the extended normal distribution. We suggest the moment restriction so that the pricing model under any arbitrary distribution for an underlying asset must satisfy the arbitrage-free condition.

Numerical experiments and comparison of empirical performance of the proposed model with the Black-Scholes, ad hoc Black-Scholes, and Gram-Chalier distribution model are carried out. In particular, we make an estimation of implied parameters such as volatility, skewness and kurtosis of the return on the underlying asset from the market prices of the KOSPI 200 index options, and perform in-sample and out-of-sample test. These results support the previous finding that the actual density of the underlying asset shows skewness to the left and high peaks.

Key Words: option pricing, extended normal distribution, Gram-Chalier series expansion, martingale restriction, moment restriction, skewness, kurtosis, KOSPI 200 index option

1 Introduction

One of the assumptions previously made in earlier works on the valuation of an option is that the underlying stock price follows a geometric Brownian motion through time which produces a log-normal distribution for the stock price between any two points in time. The diffusion processes provide a nice framework to analyze lots of financial derivatives mathematically and simplify the analysis with a relatively-less complicated Ito stochastic calculus. The option pricing models of Black-Scholes (1973) and Merton (1973) are derived in this framework.

However, there are several deficiences reported in the Black-Scholes framework. First, the market prices of options are frequently different from those derived using Black-Scholes formula especially in deep-in-the-money and deep-out-of-the money options. These mis-pricings arise since the true return distribution of the underlying asset shows asymmetric leptokurtic features such as a high peak or heavier tails with skewed left or right. These deficiencies are addressed in empirical studies by Black (1975), Macbeth and Merville (1980), Rubinstein (1985), and Whaley (1982). Another drawback is the so-called volatility smile: the implied volatility of an option as a function of its strike price resembles smile curve. However it should be constant in a framework of Black-Scholes model.

A number of studies have been made to overcome these deficiencies, and the approaches are categorized into three groups. The first group extends the Black-Scholes framework by incorporating stochastic jumps or stochastic volatility. This group includes Merton (1973), Hull and White (1987), Heston (1993), Bates (1996), and Bakshi, Cao, and Chen (1997). Since the models of this group explain the stock movement with multiple stochastic factors, the market is no longer complete, and so there may exist many equivalent martingale measures, which complicates dealing with the analysis.

The second group uses other distributions for the rate of return on the underlying asset rather than the normal distribution. Jarrow and Rudd (1982), Corrado and Su (1996), Rubinstein (1998) and Li (2000) develop option pricing models by using distribution with third and fourth moments for the underlying assets. In particular, Jarrow and Rudd (1982) use Edgeworth series expansion to the log-normal distribution function, and Corrado and Su (1996) apply the Gram-Chalier series expansion to the normal distribution function. Ritchey (1990) proposes an option pricing formula under an assumption that the underlying asset's return is distributed as k-component independent normal mixtures. Janicki et al.

(1997) and Hurst et al. consider the problem of pricing options when the process of the underlying asset is assumed to be driven by an α-stable Levy motion. Bibby and Sørensen (1997) propose a pricing model under the assumption that the underlying asset price follows a hyperbolic diffusion process. Despite their usefulness in explaining the stock movement with skewness and kurtosis, their stochastic models does not satisfy the condition of Levy process, and so it is not easy to transform the models in any discrete framework such as binomial tree model.

Finally, third group estimates the stochastic density function of the underlying asset directly from the market option prices. This group includes Derman and Kani (1994), Dupire (1994), and Rubinstein (1994). Also a study of Aït-Sahalia and Lo (1998) estimates the stochastic density function with a non-parametric method.

As pointed out by Harrison and Kreps (1979), and Harrison and Pliska (1981, 1983), the value of a European style option is a discounted expected value of the option payoff at the maturity under risk-neutral measure. In a complete market, only one risk-neutral measure exists, and the discounted asset price should satisfy the martingale property. This condition is called *martingale restriction* by Longstaff (1995). In this paper, we propose an equivalent condition to the martingale restriction, namely, *moment restriction*, using the property of moment generating function so that the option pricing model satisfy the risk-neutral condition or arbitrage-free condition. Using this restriction we correct the pricing formula of Corrado-Su (1996) based on the Gram-Chalier series expansion, which however is not satisfying the martingale restriction.

The main part of our research is to suggest another pricing formula for European options when the return of the underlying asset follows the extended normal distribution, which allows different degrees of skewness and kurtosis of the return on the underlying asset by taking a linear combination of two normal distributions with Gram-Chalier series expansion.

By imposing the moment restriction on the expected value of the natural logarithm of the asset price, we derive another risk neutral option pricing model. The form of extended normal distribution suggested in this paper has been never addressed in previous researches, as we know, and the associated pricing model outperforms previous models such as Black-Scholes (1973), Corrado-Su (1996), and Dumas, Flemming, and Whaley (1998), as a result of in-sample test, and out-of-sample test.

The paper is organized in the following way. Section 2 describes a general model for the valuation of European options when the underlying asset follows a general probability distribution under a risk-neutral measure. We begin Section 3 with the derivation of a pricing formula when the underlying asset follows an extended normal distribution. This section is the main part of this paper. Section 4 represents numerical examples and compares the empirical performance of the proposed model with previous pricing models. For this purpose, we estimate the values of the parameters such as volatility, skewness and kurtosis from the market prices of the KOSPI 200 index options. Finally, Section 5 concludes with suggestions for future research.

2 Risk-neutral Valuation Model

We assume throughout this paper that (1) there are no friction such as asymmetric taxes, transaction costs, bid-ask spreads, and trading takes place continuously in the financial markets, (2) there are two tradable assets in the market, a risky asset and a risk-free asset, (3) the risk-free interest rate and the volatility of the rate of return on the risky asset are known and constant through time. We also assume the market is complete so that there exists a unique equivalent martingale measure for this market. Let S_t denote the risky asset price at time t. Under a risk-neutral measure, $\mathbb{P}$, where it exists, the asset price should satisfy the following condition, which is called *martingale restriction* by Longstaff (1995):

Martingale restriction: $\mathbb{E}\{S_T\} = S_0 e^{rT}$,

where $\mathbb{E}$ denotes the expectation operator under the risk-neutral measure $\mathbb{P}$, and S_0 is an initial price of underlying asset. The moment generating function, $M(\theta)$, of an arbitrary distribution, under which a random variable Y follows, is given by

$$M(\theta) = \mathbb{E}\{e^{\theta Y}\}. \tag{1}$$

In particular, the moment generating function for $\ln S_T$ with $\theta = 1$ is given by

$$M(1) = \mathbb{E}\{e^{\ln S_T}\} = \mathbb{E}\{S_T\}. \tag{2}$$

Thus martingale restriction can be rewritten as follows, which we call *moment restriction* for the risk-neutral valuation, and it imposes the restriction for the mean of futures stock prices under the risk-neutral measure.

Moment restriction: $\quad M(1) = S_0 e^{rT}.$

Unless the mean of the future stock price after T satisfies the moment generating function with $\theta = 1$, the valuation formula from this framework is not a risk-neutral valuation scheme, and so it is not no arbitrage option pricing model.

Now consider the option pricing model with moment restriction. When the density function of logarithmic asset price is $h(y)$, the price of a European call option C, with exercise price X, and time to maturity of T, is given by

$$\begin{aligned} C &= e^{-rT}\mathbb{E}\{(S_T - X)^+\} \\ &= e^{-rT}\int_X^\infty (S_T - X)d\mathbb{P}(S_T) \\ &= e^{-rT}\int_{\ln X}^\infty \{\exp(y) - X\}\, h(y)dy. \end{aligned} \tag{3}$$

We apply the moment restriction to the two distributions for logarithmic stock prices at maturity; normal distribution, and Gram-Chalier distribution, and derive the corresponding option pricing formula under each different distribution in the following subsection.

2.1 Normal distribution

Black-Scholes (1973) assume that the natural logarithm of the asset price follows the normal distribution with mean m and standard deviation of $\sigma\sqrt{T}$. Since the moment generating function of the normal distribution is given by $M(\theta) = \exp\{m\theta + \sigma^2\theta^2 T/2\}$, it follows from the moment restriction that:

$$M(1) = \exp\{m + \sigma^2 T/2\} = S_0 e^{rT}. \tag{4}$$

Thus the mean value m is given by

$$\begin{aligned} m &= \ln(S_0 e^{rT}) - \frac{1}{2}\sigma^2 T \\ &= \ln(S_0) + (r - \frac{1}{2}\sigma^2)T. \end{aligned} \tag{5}$$

If we let $h_{m,s}$ denote the density of normal with mean m and standard deviation $s = \sigma\sqrt{T}$, the price of a European call C with exercise price X, and time to maturity T, is follows from the Equation (3) that

$$\begin{aligned}
C &= e^{-rT} \int_{\ln X}^{\infty} \{\exp(y) - X\} \, h_{m,s}(y) dy \\
&= e^{-rT} \int_{(\ln X - m)/s}^{\infty} \{\exp(sy + m) - X\} \, h_{0,1}(y) dy \\
&= e^{-rT} \exp\left(m + \frac{1}{2}s^2\right) \int_{(\ln X - m)/s}^{\infty} h_{s,1}(y) dy - e^{-rT} X \int_{(\ln X - m)/s}^{\infty} h_{0,1}(y) dy \\
&= S_0 \int_{(\ln X - m)/s}^{\infty} h_{s,1}(y) dy - e^{-rT} X \int_{(\ln X - m)/s}^{\infty} h_{0,1}(y) dy.
\end{aligned}$$

By letting $\Phi(x) = \int_{-\infty}^{x} h_{0,1}(x) dx$, the same formula as Black-Scholes(1973) can be derived by

$$\begin{aligned}
C &= S_0 \Phi\left(\frac{m + s^2 - \ln X}{s}\right) - X e^{-rT} \Phi\left(\frac{m - \ln X}{s}\right) \\
&= S_0 \Phi(d_1) - X e^{-rT} \Phi(d_2),
\end{aligned} \tag{6}$$

where

$$d_1 = \frac{\ln(S_0/X) + \left(r + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}}, \;\; d_2 = d_1 - \sigma\sqrt{T}.$$

2.2 Gram-Chalier distribution

In this subsection we derive a pricing formula of a European call when the risk neutral distribution of the future asset price follows a different distribution from the normal with non-zero skewness and kurtosis greater than three.

Now consider a Gram-Chalier series expansion, $H_{\xi,k}(x)$, as follows

$$\begin{aligned}
H_{\xi,k}(x) &= h(x) - \frac{1}{6}\xi \frac{d^3 h(x)}{dx^3} + \frac{1}{24}(k-3) \frac{d^4 h(x)}{dx^4} \\
&= h(x) \left\{ 1 + \frac{1}{6}\xi(x^3 - 3x) + \frac{1}{24}(k-3)(x^4 - 6x^2 + 3) \right\},
\end{aligned} \tag{7}$$

where h denotes the density function of standard normal. Next, define a new probability density function.

Definition 1. If the density function of a random variable Y is given by

$$f_{m,\sigma,\xi,k}(y) = \frac{1}{\sigma} \, H_{\xi,k}\left(\frac{y - m}{\sigma}\right), \tag{8}$$

then the random variable is called to follow *Gram-Chalier distibution*, $GC(m, \sigma, \xi, k)$, with mean m, standard deviation σ, skewness ξ, and kurtosis k.

Figure 1 illustrates the density function of normal and Gram-Chalier distribution, with skewness(ξ) -0.3 and kurtosis(k) 3.4. The moment generating function of the Gram-Chalier distribution is given by

$$M(\theta) = \exp(m\theta) \int_{-\infty}^{\infty} \exp(\sigma\theta z) H_{\xi,k}(z) dz. \tag{9}$$

Now we will derive the pricing formula for a European call when the logarithmic price of the asset follows a Gram-Chalier distribution. We suppose the natural logarithm of the asset price at time T, $\ln S_T$, follows a Gram-Chalier distribution, $GC(m, \sigma\sqrt{T}, \xi, k)$. The mean of $\ln S_T$, m, should satisfy the moment restriction, and so m is given using the moment generating function as follows:

$$m = \ln \left[\frac{S_0 \exp(rT)}{\int_{-\infty}^{\infty} \exp(sz) H_{\xi,k}(z) dz} \right]. \tag{10}$$

Since

$$\int_{-\infty}^{\infty} \exp(sz) \frac{d^i h(z)}{dz^i} dz = (-s)^i \int_{-\infty}^{\infty} \exp(sz) h(z) dz = (-s)^i \exp\left(\frac{1}{2}s^2\right),$$

the denumerator inside of the logarithmic function of Equation (10) is evaluated as:

$$\int_{-\infty}^{\infty} \exp(sz) H_{\xi,k}(z) dz = \exp\left(\frac{1}{2}s^2\right) \left\{1 + \frac{1}{6}\xi s^3 + \frac{1}{24}(k-3)s^4\right\}.$$

Therefore the mean m is obtained as follows:

$$m = \ln S_0 + (r - \frac{1}{2}\sigma^2)T - \ln\left\{1 + \frac{1}{6}\xi s^3 + \frac{1}{24}(k-3)s^4\right\}. \tag{11}$$

In the Black-Scholes formula, the third term of Equation (11) disappears since the skewness(ξ) is 0, and the kurtosis(k) is 3. Thus the Black-Scholes formula can be regarded as a special case of this formula.

Remark 1. *In the formula of Corrado-Su (1996), they assume the natural logarithm of the asset price at time T, $\ln S_T$ follows the Gram-Chalier distribution, $GC(m', \sigma\sqrt{T}, \xi, k)$, where*

$$m' = \ln S_0 + (r - \frac{1}{2}\sigma^2)T,$$

which is the same as in the Black-Scholes model, although the underlying distribution changes. As a result, the expected value of the asset price at the maturity under the risk-neutral measure is given by

$$\mathbb{E}\{S_T\} = S_0 e^{rT}\left\{1 + \frac{\xi}{6}s^3 + \frac{k-3}{24}s^4\right\} \neq S_0 e^{rT},$$

which shows that the pricing model of Corrado-Su(1996) does not satisfy risk-neutral valuation scheme.

Once the distribution for the asset price at the maturity is provided, we can evaluate the price of a European call as follows.

Proposition 1. *If the natural logarithm of the asset price follows the Gram-Chalier distribution, $GC(m, \sigma\sqrt{T}, \xi, k)$ under the risk-neutral measure, where the mean value m is given by Equation (11), then the arbitrage-free price of a European call, C, with exercise price X and time to maturity T is given by*

$$\begin{aligned} C &= S_0\Phi(D_1) - Xe^{-rT}\Phi(D_2) \\ &\quad + Xe^{-rT}\left\{\sigma\sqrt{T}E_1\Phi'(D_2) + E_1\Phi''(D_2) + E_2\Phi'''(D_2)\right\}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} D_1 &= \frac{\ln(S_0/X) + \left(r + \frac{1}{2}\sigma^2\right)T - \ln\left\{1 + \sigma^2 T E_1\right\}}{\sigma\sqrt{T}}, \\ D_2 &= D_1 - \sigma\sqrt{T}, \\ E_1 &= \frac{1}{6}\xi\sigma\sqrt{T} + \frac{1}{24}(k-3)\sigma^2 T, \\ E_2 &= \frac{1}{24}(k-3)\sigma\sqrt{T}, \end{aligned}$$

and where $\Phi, \Phi', \Phi'', \Phi'''$ denote the normal distribution function, its first, second, and third derivative function, respectively.

Proof. See the Appendix. □

If the skewness is 0, and kurtosis is 3, i.e., $\xi = 0$, $k = 3$, then both E_1 and E_2 equal to zero, and so the pricing formula is the same as what Black-Scholes (1973) derived.

Remark 2. *In their independent research, Jurczenko et al. (2004) also corrected the pricing formula of Corrado-Su (1996), and proposed another formula by imposing the restriction of risk-neutral condition. Their corrected formula is the same as Equation (12) we derived.*

3 Option Pricing under Extended Normal Distribution

We begin this section with introducing a new distribution for the asset price at the maturity. Let $h_{m,\sigma}$ denote the density function of normal distribution with mean m and standard deviation σ. Also define p, α, and β for $k \geq 0$ as follows:

$$\begin{aligned} p &= 1 - \frac{9}{k^2}, \\ \alpha^2 &= 1 - \frac{1}{p}\sqrt{p(1-p)\left(\frac{k}{3}-1\right)}, \\ \beta^2 &= 1 + \frac{1}{1-p}\sqrt{p(1-p)\left(\frac{k}{3}-1\right)}. \end{aligned} \tag{13}$$

This allows us to adjust the levels of kurtosis to any non-negative real numbers in a distribution. Now consider the following transformations which allow the distribution to have any flexible levels of skewness. First we define a linear combination of two normal density functions with each having different standard deviation α, and β, respectively, $l_k(x) = p\ h_{0,\alpha}(x) + (1-p)\ h_{0,\beta}(x)$. Next, define a new function, $J_{\xi,k}$ for real ξ,

$$\begin{aligned} J_{\xi,k}(x) &= l_k(x) - \frac{\xi}{6} l_k'''(x) \\ &= p\left\{1 + \frac{\xi}{6\alpha^6}\left(x^3 - 3\alpha^2 x\right)\right\} h_{0,\alpha}(x) + (1-p)\left\{1 + \frac{\xi}{6\beta^6}\left(x^3 - 3\beta^2 x\right)\right\} h_{0,\beta}(x). \end{aligned} \tag{14}$$

Thus the new function $J_{\xi,k}(x)$ has the flexible levels of kurtosis and skewness, after two transformations. In order to make the function a probability density, we finally normalize it as follows.

Definition 2. If the density function of a random variable Y is given by

$$f_{m,\sigma,\xi,k}(y) = \frac{1}{\sigma} J_{\xi,k}\left(\frac{y-m}{\sigma}\right), \tag{15}$$

then the random variable Y is called to follow *extended normal distribution* $EN(m, \sigma, \xi, k)$ with mean m, standard deviation σ, skewness ξ, and kurtosis k.

The moment generating function of extended normal distribution, $EN(m, \sigma, \xi, k)$, is given by:

$$M(\theta) = \exp(m\theta) \int_{-\infty}^{\infty} \exp(\sigma\theta z)\ J_{\xi,k}(z) dz \tag{16}$$

Now suppose that the logarithmic asset price at time T follows the extended normal distribution, $EN(m, \sigma\sqrt{T}, \xi, k)$. The mean of $\ln S_T$ under the risk-neutral measure is given by

$$m = \ln\left[\frac{S_0 \exp(rT)}{\int_{-\infty}^{\infty} \exp(sz)\ J_{\xi,k}(z)dz}\right], \tag{17}$$

where $s = \sigma\sqrt{T}$. Since

$$\begin{aligned}\int_{-\infty}^{\infty} \exp(sz) J_{\xi,k}(z)dz &= \left(1 + \frac{1}{6}\xi s^3\right)\int_{-\infty}^{\infty} \exp(sz) l_k(z)dz \\ &= \left(1 + \frac{1}{6}\xi s^3\right)\left\{pe^{\frac{1}{2}\alpha^2 s^2} + (1-p)e^{\frac{1}{2}\beta^2 s^2}\right\},\end{aligned}$$

we can obtain the mean value:

$$m = \ln S_0 + rT - \ln(A+B) - \ln(1 + \frac{1}{6}\xi s^3), \tag{18}$$

where

$$A = pe^{\frac{1}{2}\alpha^2 s^2},\ B = (1-p)e^{\frac{1}{2}\beta^2 s^2}.$$

Now consider the price of a European call option.

Proposition 2. *If the natural logarithm of the asset price follows the Gram-Chalier distribution, $EN(m, \sigma\sqrt{T}, \xi, k)$ under the risk-neutral measure, where the mean value m is given by Equation (17), then the arbitrage-free price of a European call, C, with exercise price X and time to maturity T is given by*

$$\begin{aligned}C = S_0 &\left[\frac{A}{A+B}\Phi\left(\frac{D}{\alpha} + \alpha s\right) + \frac{B}{A+B}\Phi\left(\frac{D}{\beta} + \beta s\right)\right] \\ &- Xe^{-rT}\left[p\Phi\left(\frac{D}{\alpha}\right) + (1-p)\Phi\left(\frac{D}{\beta}\right)\right] \\ &+ \frac{\xi X s}{6}e^{-rT}\left[\frac{p}{\alpha}\left(s - \frac{D}{\alpha^2}\right)\Phi'\left(\frac{D}{\alpha}\right) + \frac{1-p}{\beta}\left(s - \frac{D}{\beta^2}\right)\Phi'\left(\frac{D}{\beta}\right)\right],\end{aligned} \tag{19}$$

where

$$D = \frac{\ln(S_0/X) + rT - \ln(A+B) - \ln\left(1 + \frac{1}{6}\xi s^3\right)}{\sigma\sqrt{T}}$$

Proof. See the Appendix. □

If the skewness is 0, and the kurtosis is 3, i.e., $\xi = 0$, and $k = 3$, then the above pricing formula coincides to the Black-Scholes formula.

Now consider an underlying asset that pays a dividend yield at a rate q per annum. If we let $F_0 = S_0 e^{(r-q)T}$, then the price of a European call is given by

$$\begin{aligned} C = {} & F_0 e^{-rT} \left[\frac{A}{A+B} \Phi\left(\frac{D}{\alpha} + \alpha s\right) + \frac{B}{A+B} \Phi\left(\frac{D}{\beta} + \beta s\right) \right] \\ & - X e^{-rT} \left[p \Phi\left(\frac{D}{\alpha}\right) + (1-p) \Phi\left(\frac{D}{\beta}\right) \right] \\ & + X e^{-rT} \frac{\xi s}{6} \left[\frac{p}{\alpha} \left(s - \frac{D}{\alpha^2}\right) \Phi'\left(\frac{D}{\alpha}\right) + \frac{1-p}{\beta} \left(s - \frac{D}{\beta^2}\right) \Phi'\left(\frac{D}{\beta}\right) \right], \end{aligned} \tag{20}$$

where

$$D = \frac{\ln(F_0/X) - \ln(A+B) - \ln\left(1 + \frac{1}{6}\xi s^3\right)}{\sigma\sqrt{T}}.$$

In fitting the implied parameters of the proposed model, and conducting empirical tests to follow, we will use the above pricing formula in Equation (20) instead of using that in Equation (19).

4 Empirical Performance

This section gives some numerical examples and compares the empirical performance of the proposed model with the three previous models with respect to in-sample and out-of-sample pricing errors.

4.1 Data Description

We use, for our empirical work, KOSPI 200 (Korea Stock Price Index 200) option prices provided officially by the KSE (Korean Stock Exchanges). These options are traded at KSE with expiration dates in the three near-term months along with the following one month from the March expiration cycle (March, June, September, December). The sample period extends from Dec 13, 2002 through Dec 11, 2003, which is from the next day of the maturity date of options that expire December 2002 through the day right before the maturity date of options that expire December 2003. We use as the risk-free interest rate the CD 91 days interest rates announced daily by Bank of Korea. For the purpose of parameter estimation for several models to follow, we use the price quotes at 2:45 PM on each trading day.

There are several possible approaches to determining the expected future rate of dividend payments by the stocks that compose the index until the expiration of an option. For example, Bakshi, Cao and Chen (1997) estimate the daily dividends directly from the market, and Poteshman (2001) extracts the quantity Se^{-qT}, where q denotes the dividend yield per annum, from the transactions data from the index futures markets via spot-futures parity. The approach of Poteshman (2001) is simple to use, but is not applicable at least in Korea, since the KOSPI 200 index futures contracts expire only in March, June, September, and December. We instead determine the implied futures price F_0 from the call and put prices by using the put-call parity:

$$C - P = e^{-rT}(F_0 - X), \tag{21}$$

where C and P are the call and put option prices on the KOSPI 200 index respectively, and X is the strike price. Finally we eliminate the deep in-the-money options and options with less than six-days to expiration since they may incur liquidity-related biases.

4.2 Numerical Illustration

As reported in Table 1 and 2, and illustrated in Figure 2 and 3, we compute the European call option prices using the extended normal model we propose here whose pricing formula is expressed in Equation (20), with varying the kurtosis but fixing skewness zero, and with varying skewness but fixing kurtosis 3, respectively. In particular, Table 1 presents call option prices at kurtosis of 3.0, 4.0, 5.0, and 6.0, respectively for the range of exercise prices from 80.0 to 115.0 with 2.5 interval. Also Table 2 shows the call prices at skewness of -0.4, -0.2, 0.0, 0.2 and 0.4, respectively for the same range of exercise prices. If skewness is 0 and kurtosis is 3.0, it turns out that the prices are equivalent to those computed using the Black-Scholes formula. Figure 2 and 3 illustrate the difference of option prices derived by subtracting the Black-Scholes prices from the prices using the extended normal model, with varying the kurtosis or the skewness. In computing the option prices, the futures price F_0 is 97.5, the risk-free interest rate r is 5%, the volatility of the underlying asset σ is 30%, and the time to maturity is 0.1 year.

As shown in Figure 2, the ITM (in-the-money) and OTM (out-of-the-money) option prices derived using the extended normal model become larger, and ATM (at-the-money) option prices become smaller if the kurtosis increases. Also as shown in Figure 3, the ITM option prices derived using the extended normal model become smaller, but OTM option prices become larger if the skewness increases. These results imply that if the actual kurtosis of the distribution for the underlying asset is greater than three, the Black-Scholes model systematically overprices ATM calls while it underprices deep ITM and deep OTM calls. They also imply that if the actual skewness of the distribution for the underlying asset is less than zero, the Black-Scholes model systematically overprices OTM calls while it underprices ITM calls.

4.3 Parameter Estimation and In-Sample Performance

For the empirical comparison to follow, we concentrate on the four model: the Black-Scholes model (hereafter BS), the ad hoc Black-Scholes model (hereafter ahBS) based on the deterministic volatility function proposed by Dumas, Flemming and Whaley (1998), the corrected version of Corrado-Su (1996, 1997) model (hereafter CS), and the extended normal model we propose here (hereafter EN).

In estimating the parameters of the deterministic volatility function in ad hoc Black-Scholes model, we approximate the volatility with the quadratic polynomial function:

$$\sigma(X) = \alpha(X/F - \beta)^2 + \gamma, \tag{22}$$

where F is the futures price, X is the exercise price, and the parameters α, β, γ denote the degree of smile, the center axis of the smile, and the minimum level of the volatility, respectively. As we point out previously, the Corrado-Su model does not satisfy the arbitrage-free condition and it has been corrected by Jurczenko et al. (2004). We call as CS here the corrected version of Corrado-Su model.

In making an estimation of the implied values of parameters for each model, we apply the Levenberg-Marquart method. First let the function $V(X, \sigma, \xi, k)$ denote the model-based option price computed from the proposed pricing formula when the exercise price is X, volatility is σ, skewness is ξ, and kurtosis is k, respectively. Secondly V_i is denoted by the market price of a KOSPI 200 index option with exercise price X_i for all $i = 1, 2, \ldots, N$. Then one can obtain the estimated values of parameters (σ, ξ and k) by minimizing the following term:

$$\sum_{i=1}^{N} \{V(X_i, \sigma, \xi, k) - V_i\}^2 . \tag{23}$$

Since the ITM call option data has biases due to a low level of liquidity, we transform the OTM put option prices into the equivalent call prices by applying the put-call parity, and use them in fitting the parameters, as in Aït-Sahalia and Lo (1998).

Table 3 shows the estimated values of the associated parameters for each model; implied volatility for BS, α, β, γ for ahBS, implied volatility, implied skewness and implied kurtosis for CS and EN model. The values of the implied volatility for each model is nearly the same in that the maximal difference between them is at most 0.01 as seen in Table 3. The degree of skewness turns out to be negative values and the kurtosis are greater than three for the CS and EN model, as expected. The difference of the values are greater than that of implied volatility, but is not larger than 0.05 for skewness and 0.35 for kurtosis, respectively. This result implies that the actual density of the underlying stock index shows skewness to the left and high peaks as pointed out by previous researches.

Table 4 reports the monthly average MSE (mean squared errors) between the market price and the model determined price for each KOSPI 200 index option collected at 2:45 PM on every trading day, with respect to moneyness, F/X, where F is the implied futures price and X is the strike price. As one expects, the model determined prices with more parameters better fit the market prices. The result that ahBS, CS, and EN models with three structured parameters have less pricing errors than the BS with one parameter is not so surprising. Therefore it is a natural step to examine each model's out-of-sample pricing performance to follow.

4.4 Out-of-Sample Pricing Performance

To measure the out-of-sample pricing performance for each model, we first estimate the parameter values from the previous day's market option prices and then use them as input to compute the current day's model determined option prices. This procedure is repeated for every option and each day in the sample to evaluate the pricing errors in three different ways: MSE (mean squared errors), MAE (mean absolute errors), and MPE (mean percentage errors). Table 5, 6 and 7 summarize the average pricing errors between the market price and the model price with different measures with respect to moneyness, F/X, where F is the implied futures price and X is the strike price. The out-of sample pricing performance of the model EN is better than the BS model and not less than the ahBS and CS models on all three measures, MSE, MAE and MPE.

5 Conclusion

This research proposes a closed pricing formula for European options when the natural logarithm of the underlying asset price is extended normally distributed, which allows us to manipulate the skewness and kurtosis for the distribution on the underlying return. This study also suggests the moment restriction so that the pricing model under any arbitrary distribution for an underlying asset must satisfy the arbitrage-free condition or risk-neutral valuation scheme. Those two works constitute the main contribution of this paper.

Numerical experiments and comparison of empirical performance of the proposed model with the Black-Scholes, ad hoc Black-Scholes, and Gram-Chalier distribution model are carried out to show that the new proposed formula fits the real market prices better than the classical Black-Scholes formula, and competes with the ad hoc Black-Scholes and Gram-Chalier model. In particular, we make an estimation of implied volatility, skewness and kurtosis of the return on the underlying assets from the market prices of the KOSPI 200 index options, and perform in-sample and out-of-sample test. These results support the previous finding that the actual density of the underlying stock index shows skewness to the left and high peaks.

Future research includes in-sample and out-of-sample empirical test to verify the empirical performance of this proposed model using foreign option market data such as S&P 500 index option in the U.S.

Appendix

A Proof of Proposition 1

Proof. Consider the expected value of the option payoff at the maturity under the risk-neutral measure:

$$\begin{aligned}
\mathbb{E}\{(S_T - X)^+\} &= \int_{\ln X}^{\infty} \{\exp(y) - X\} f_{m,s,\xi,k}(y) dy \\
&= \int_{(\ln X - m)/s}^{\infty} \{\exp(sy + m) - X\} H_{\xi,k}(y) dy \\
&= \int_{(\ln X - m)/s}^{\infty} \{\exp(sy + m) - X\} h(y) dy \\
&- \frac{1}{6}\xi \int_{(\ln X - m)/s}^{\infty} \{\exp(sy + m) - X\} \frac{d^3 h(y)}{dy^3} dy \\
&+ \frac{1}{24}(k-3) \int_{(\ln X - m)/s}^{\infty} \{\exp(sy + m) - X\} \frac{d^4 h(y)}{dy^4} dy,
\end{aligned}$$

where $s = \sigma\sqrt{T}$. Using the partial integration allows the evaluation of the integrand as follows.

$$\begin{aligned}
\int_{(\ln X - m)/s}^{\infty} \{\exp(sy + m) - X\} \frac{d^i h(y)}{dy^i} dy \quad &= \quad -X \sum_{j=0}^{i-2} (-s)^{i-j-1} \frac{d^j h}{dy^j} \left(\frac{\ln X - m}{s} \right) \\
&\quad\; +(-s)^i \int_{(\ln X - m)/s}^{\infty} \exp(sy + m) h(y) dy.
\end{aligned}$$

Thus the expected payoff is given by:

$$\begin{aligned}
\mathbb{E}\{(S_T - X)^+\} \quad = \quad & \left\{ 1 + \frac{1}{6}\xi s^3 + \frac{1}{24}(k-3)s^4 \right\} \int_{(\ln X - m)/s}^{\infty} \exp(sy + m) h(y) dy \\
& -X \int_{(\ln X - m)/s}^{\infty} h(y) dy + X \left\{ \frac{1}{6}\xi s^2 + \frac{1}{24}(k-3)s^3 \right\} h\left(\frac{\ln X - m}{s} \right) \\
& -X \left\{ \frac{1}{6}\xi s + \frac{1}{24}(k-3)s^2 \right\} h'\left(\frac{\ln X - m}{s} \right) \\
& +X \left\{ \frac{1}{24}(k-3)s \right\} h''\left(\frac{\ln X - m}{s} \right)
\end{aligned}$$

Using the normal distribution function, $\Phi(x) = \int_{-\infty}^{x} h(y)dy$, we can simplify the above equation as follows:

$$\begin{aligned}
\mathbb{E}\{(S_T - X)^+\} \quad = \quad & \left\{1 + \frac{1}{6}\xi s^3 + \frac{1}{24}(k-3)s^4\right\} \exp\left(m + \frac{1}{2}s^2\right) \Phi\left(\frac{m + s^2 - \ln X}{s}\right) \\
& -X\Phi\left(\frac{m - \ln X}{s}\right) + X\left\{\frac{1}{6}\xi s^2 + \frac{1}{24}(k-3)s^3\right\} \Phi'\left(\frac{\ln X - m}{s}\right) \\
& -X\left\{\frac{1}{6}\xi s + \frac{1}{24}(k-3)s^2\right\} \Phi''\left(\frac{\ln X - m}{s}\right) \\
& +X\left\{\frac{1}{24}(k-3)s\right\} \Phi'''\left(\frac{\ln X - m}{s}\right)
\end{aligned}$$

Therefore the price of a call option with time to maturity of T, $C = e^{-rT}\mathbb{E}\{(S_T - X)^+\}$, the discounted expected payoff under the risk-neutral measure can be derived as asserted. $\square$

B Proof of Proposition 2

Proof. Consider the expected value of the option payoff at the maturity under the risk-neutral measure:

$$\begin{aligned}
E\{(S_T - X)^+\} &= \int_{(\ln X - m)/s}^{\infty} (\exp(sy + m) - X)\, J_{\xi,k}(y)dy \\
&= \int_{(\ln X - m)/s}^{\infty} (\exp(sy + m) - X)\, l_k(y)dy - \frac{\xi}{6}\left\{sXl_k'\left(\frac{\ln X - m}{s}\right)\right. \\
&\quad \left. -s^2Xl_k\left(\frac{\ln X - m}{s}\right) - s^3\int_{(\ln X - m)/s}^{\infty} \exp(sy + m)l_k(y)dy\right\} \\
&= \left(1 + \frac{1}{6}\xi s^3\right)\int_{(\ln X - m)/s}^{\infty} \exp(sy + m)l_k(y)dy - X\int_{(\ln X - m)/s}^{\infty} l_k(y)dy \\
&\quad + \frac{\xi}{6}\left\{sXl_k'\left(\frac{m - \ln X}{s}\right) + s^2Xl_k\left(\frac{m - \ln X}{s}\right)\right\} \\
&= \frac{S_0e^{rT}}{A + B}\left[p\int_{(\ln X - m)/s}^{\infty} \exp(sy)h_{0,\alpha}(y)dy + (1 - p)\int_{(\ln X - m)/s}^{\infty} \exp(sy)h_{0,\beta}(y)dy\right] \\
&\quad - X\left[p\int_{(\ln X - m)/s}^{\infty} h_{0,\alpha}(y)dy + (1 - p)\int_{(\ln X - m)/s}^{\infty} h_{0,\beta}(y)dy\right] \\
&\quad + \frac{\xi}{6}\left[sX\left\{ph_{0,\alpha}'\left(\frac{m - \ln X}{s}\right) + (1 - p)h_{0,\beta}'\left(\frac{m - \ln X}{s}\right)\right\}\right. \\
&\quad \left. +s^2X\left\{ph_{0,\alpha}\left(\frac{m - \ln X}{s}\right) + (1 - p)h_{0,\beta}\left(\frac{m - \ln X}{s}\right)\right\}\right].
\end{aligned}$$

Rearranging the above equation yields:

$$\begin{aligned}
E\{(S_T-X)^+\} = S_0 e^{rT} & \left[\frac{A}{A+B}\int_{(\ln X-m)/s}^{\infty} h_{\alpha^2 s,\alpha}(y)dy + \frac{B}{A+B}\int_{(\ln X-m)/s}^{\infty} h_{\beta^2 s,\beta}(y)dy\right] \\
& - X\left[p\int_{(\ln X-m)/s}^{\infty} h_{0,\alpha}(y)dy + (1-p)\int_{(\ln X-m)/s}^{\infty} h_{0,\beta}(y)dy\right] \\
& - \frac{\xi X}{6}\left[(m-\ln X)\left\{\frac{p}{\alpha^2}h_{0,\alpha}\left(\frac{m-\ln X}{s}\right) + \frac{1-p}{\beta^2}h_{0,\beta}\left(\frac{m-\ln X}{s}\right)\right\}\right. \\
& \left. -s^2\left\{p h_{0,\alpha}\left(\frac{m-\ln X}{s}\right) + (1-p)h_{0,\beta}\left(\frac{m-\ln X}{s}\right)\right\}\right]
\end{aligned}$$

Transforming $h_{0,\alpha}$ into $h_{0,1}$ by normalizing gives:

$$\begin{aligned}
E\{(S_T-X)^+\} = S_0 e^{rT} & \left[\frac{A}{A+B}\int_{(\ln X-\alpha^2 s^2-m)/\alpha s}^{\infty} h_{0,1}(y)dy + \frac{B}{A+B}\int_{(\ln X-\beta^2 s^2-m)/\beta s}^{\infty} h_{0,1}(y)dy\right] \\
& - X\left[p\int_{(\ln X-m)/\alpha s}^{\infty} h_{0,1}(y)dy + (1-p)\int_{(\ln X-m)/\beta s}^{\infty} h_{0,1}(y)dy\right] \\
& + \frac{\xi X s}{6}\left[\frac{p}{\alpha}\left(s-\frac{m-\ln X}{\alpha^2 s}\right)h_{0,1}\left(\frac{m-\ln X}{\alpha s}\right)\right. \\
& \left. +\frac{1-p}{\beta}\left(s-\frac{m-\ln X}{\beta^2 s}\right)h_{0,1}\left(\frac{m-\ln X}{\beta s}\right)\right]
\end{aligned}$$

It follows by the normal distribution function, $\Phi(x)=\int_{-\infty}^{x} h_{0,1}(y)dy$, that:

$$\begin{aligned}
E\{(S_T-X)^+\} &= S_0 e^{rT}\left[\frac{A}{A+B}\Phi\left(\frac{m+\alpha^2 s^2-\ln X}{\alpha s}\right) + \frac{B}{A+B}\Phi\left(\frac{m+\beta^2 s^2-\ln X}{\beta s}\right)\right] \\
& -X\left[p\Phi\left(\frac{m-\ln X}{\alpha s}\right) + (1-p)\Phi\left(\frac{m-\ln X}{\beta s}\right)\right] \\
& +\frac{\xi X s}{6}\left[\frac{p}{\alpha}\left(s-\frac{m-\ln X}{\alpha^2 s}\right)\Phi'\left(\frac{m-\ln X}{\alpha s}\right)\right. \\
& \left. +\frac{1-p}{\beta}\left(s-\frac{m-\ln X}{\beta^2 s}\right)\Phi'\left(\frac{m-\ln X}{\beta s}\right)\right]
\end{aligned}$$

By letting $D=\frac{m-\ln X}{s}$, we obtain:

$$\begin{aligned}
E\{(S_T-X)^+\} &= S_0 e^{rT}\left[\frac{A}{A+B}\Phi\left(\frac{D}{\alpha}+\alpha s\right) + \frac{B}{A+B}\Phi\left(\frac{D}{\beta}+\beta s\right)\right] \\
& -X\left[p\Phi\left(\frac{D}{\alpha}\right) + (1-p)\Phi\left(\frac{D}{\beta}\right)\right] \\
& +\frac{\xi X s}{6}\left[\frac{p}{\alpha}\left(s-\frac{D}{\alpha^2}\right)\Phi'\left(\frac{D}{\alpha}\right) + \frac{1-p}{\beta}\left(s-\frac{D}{\beta^2}\right)\Phi'\left(\frac{D}{\beta}\right)\right].
\end{aligned}$$

Therefore the price of a European call is given by:

$$
\begin{aligned}
C = S_0 &\left[\frac{A}{A+B} \Phi\left(\frac{D}{\alpha} + \alpha s\right) + \frac{B}{A+B} \Phi\left(\frac{D}{\beta} + \beta s\right)\right] \\
&- X e^{-rT} \left[p\Phi\left(\frac{D}{\alpha}\right) + (1-p)\Phi\left(\frac{D}{\beta}\right)\right] \\
&+ \frac{\xi X s}{6} e^{-rT} \left[\frac{p}{\alpha}\left(s - \frac{D}{\alpha^2}\right)\Phi'\left(\frac{D}{\alpha}\right) + \frac{1-p}{\beta}\left(s - \frac{D}{\beta^2}\right)\Phi'\left(\frac{D}{\beta}\right)\right],
\end{aligned}
\tag{24}
$$

where

$$
D = \frac{\ln(S_0/X) + rT - \ln(A+B) - \ln\left(1 + \frac{1}{6}\xi s^3\right)}{\sigma\sqrt{T}}
$$

□

C Figures and Tables

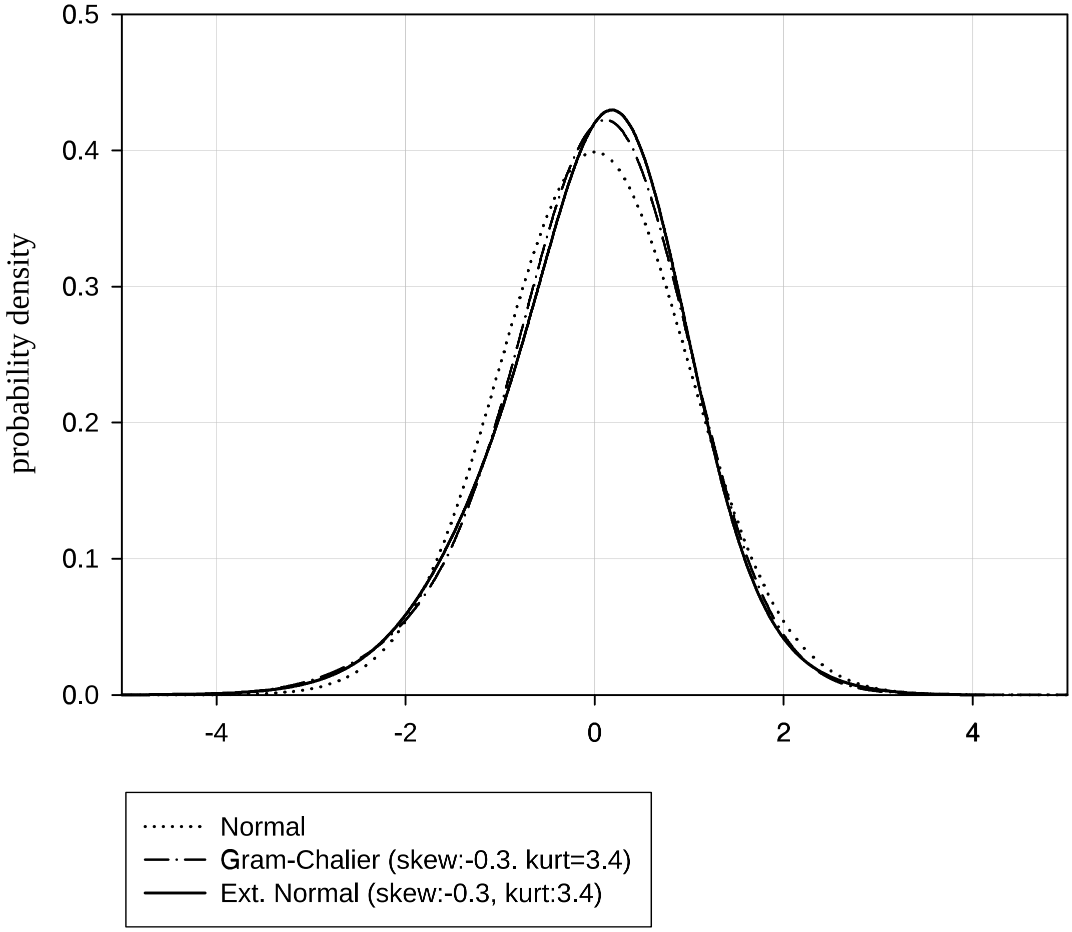

Figure 1: **Comparison of Density for Normal, Gram-Chalier and Extended Normal**

This figure compares the density function of standard normal, Gram-Chalier, and extended normal distributions. In the Gram-Chalier, and extended normal distribution, the skewness is -0.3 and kurtosis is 3.4.

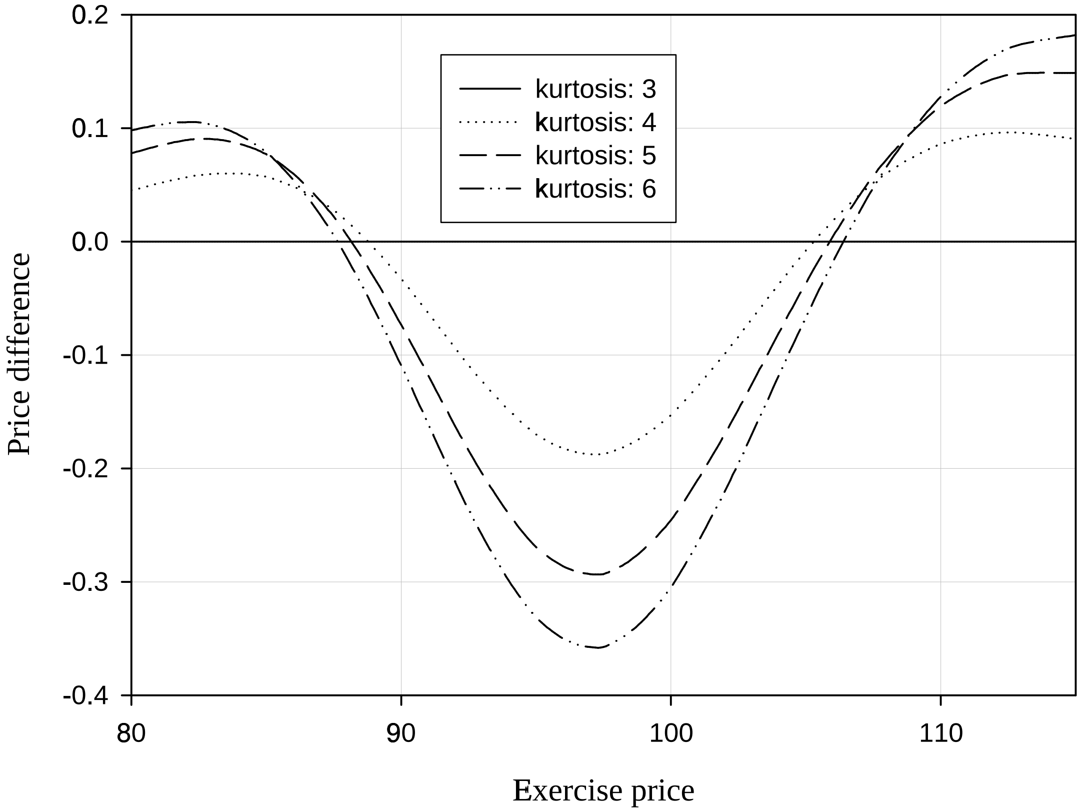

Figure 2: **Call Option Prices with Varying Kurtosis**

This figure compares the differences of European call option prices computed using the extended normal model proposed here, with different kurtosis 3, 4, 5, 6. The current futures price is 97.5, the range of exercise prices is from 80 to 115, risk-free interest rate is 5%, the volatility of underlying is 30%, and the time to maturity is 0.1 year.

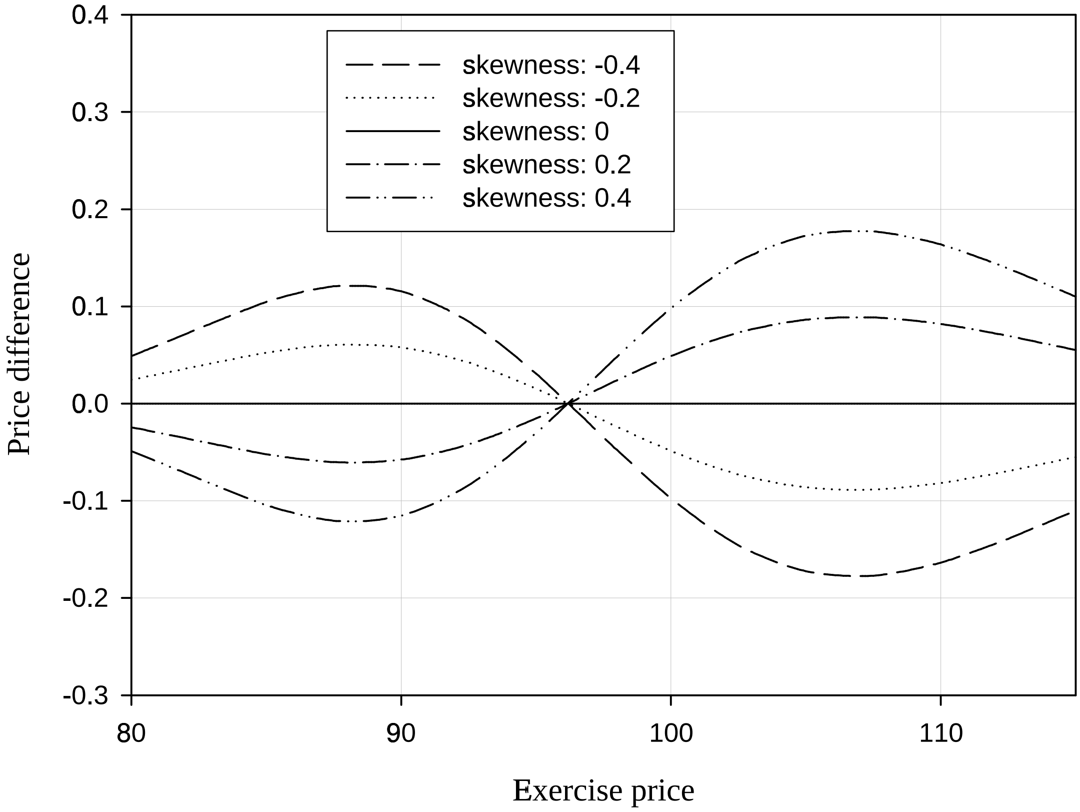

Figure 3: **Call Option Prices with Varying Skewness**

This figure compares the differences of European call option prices computed using the extended normal model proposed here, with different skewness -0.4, -0.2, 0, 0.2, 0.4. The current futures price is 97.5, the range of exercise prices is from 80 to 115, risk-free interest rate is 5%, the volatility of underlying is 30%, and the time to maturity is 0.1 year.

Table 1: **Comparison of Call Option Prices with Varying Kurtosis**

The reported option prices are computed using the formula in Equation (20) based on the extended normal with varying kurtosis but with fixed skewness zero. The current futures price is 97.5, the range of exercise prices is from 80 to 115, risk-free interest rate is 5%, the volatility of underlying is 30%, and the time to maturity is 0.1 year. The reported price difference is computed by subtracting the associated Black-Scholes price from each option price of panel A.

	Strike Price														
Kurtosis	80.0	82.5	85.0	87.5	90.0	92.5	95.0	97.5	100.0	102.5	105.0	107.5	110.0	112.5	115.0
Panel A: Option Prices															
3.0	17.469	15.058	12.722	10.501	8.445	6.600	5.002	3.670	2.605	1.788	1.186	0.761	0.472	0.284	0.166
4.0	17.514	15.117	12.779	10.529	8.413	6.491	4.832	3.483	2.452	1.704	1.178	0.813	0.558	0.380	0.256
5.0	17.547	15.149	12.799	10.524	8.372	6.416	4.732	3.377	2.359	1.639	1.149	0.819	0.592	0.431	0.314
6.0	17.567	15.164	12.801	10.507	8.337	6.364	4.670	3.313	2.300	1.592	1.119	0.808	0.600	0.454	0.348
Panel B: Price Difference from B-S															
3.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0.045	0.059	0.057	0.028	-0.033	-0.109	-0.170	-0.187	-0.153	-0.084	-0.008	0.052	0.086	0.096	0.090
5.0	0.078	0.091	0.077	0.022	-0.073	-0.184	-0.270	-0.293	-0.246	-0.148	-0.037	0.058	0.119	0.147	0.149
6.0	0.098	0.105	0.079	0.006	-0.109	-0.236	-0.332	-0.358	-0.305	-0.196	-0.067	0.047	0.128	0.170	0.182

Table 2: **Comparison of Call Option Prices with Varying Skewness**

The reported option prices are computed using the formula in Equation (20) based on the extended normal with varying skewness but with fixed skewness 3. The current futures price is 97.5, the range of exercise prices is from 80 to 115, risk-free interest rate is 5%, the volatility of underlying is 30%, and the time to maturity is 0.1 year. The reported price difference is computed by subtracting the associated Black-Scholes price from each option price of panel A.

	Strike Price														
Skew	80.0	82.5	85.0	87.5	90.0	92.5	95.0	97.5	100.0	102.5	105.0	107.5	110.0	112.5	115.0
Panel A: Option Prices															
-0.4	17.518	15.136	12.826	10.622	8.561	6.684	5.033	3.636	2.507	1.642	1.013	0.584	0.309	0.145	0.055
-0.2	17.493	15.097	12.774	10.562	8.503	6.642	5.017	3.653	2.556	1.715	1.100	0.672	0.391	0.214	0.111
0.0	17.469	15.058	12.722	10.501	8.445	6.600	5.002	3.670	2.605	1.788	1.186	0.761	0.472	0.284	0.166
0.2	17.444	15.020	12.669	10.441	8.388	6.558	4.987	3.688	2.654	1.861	1.272	0.849	0.554	0.353	0.221
0.4	17.420	14.981	12.617	10.381	8.330	6.516	4.971	3.705	2.703	1.934	1.359	0.938	0.636	0.423	0.275
Panel B: Price Difference from B-S															
-0.4	0.049	0.077	0.105	0.121	0.115	0.084	0.031	-0.035	-0.098	-0.146	-0.173	-0.177	-0.164	-0.139	-0.110
-0.2	0.024	0.039	0.052	0.060	0.058	0.042	0.015	-0.017	-0.049	-0.073	-0.086	-0.089	-0.082	-0.070	-0.055
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.2	-0.024	-0.039	-0.052	-0.060	-0.058	-0.042	-0.015	0.017	0.049	0.073	0.086	0.089	0.082	0.069	0.055
0.4	-0.049	-0.077	-0.105	-0.121	-0.115	-0.084	-0.030	0.035	0.098	0.146	0.173	0.177	0.164	0.139	0.110

Table 3: **Implied Parameters**

Each day in the sample, the parameters of each model are estimated by minimizing the sum of squared pricing errors between the market price and the model determined price for each KOSPI 200 index option. The reported parameter values are the average of the estimated parameters for each month of the year 2003. BS, ahBS, CS, EN stand for the Black-Scholes, the ad hoc Black-Scholes, based on the deterministic volatility function by Dumas, Flemming and Whaley, corrected Corrado-Su, and extended normal model, respectively. The deterministic volatility function in the ahBS is defined in Equation (22), and α, β and γ in ahBS are the associated parameters.

Month	BS	ahBS			CS			EN		
	volatility	α	β	γ	volatility	skewness	kurtosis	volatility	skewness	kurtosis
2003/01	0.32	1.06	1.02	0.32	0.33	-0.05	3.45	0.32	-0.05	3.26
2003/02	0.37	1.29	1.10	0.35	0.37	-0.28	3.50	0.37	-0.25	3.21
2003/03	0.36	1.50	1.09	0.35	0.37	-0.29	3.56	0.37	-0.26	3.25
2003/04	0.41	1.32	1.12	0.38	0.42	-0.36	3.50	0.41	-0.32	3.20
2003/05	0.34	0.81	1.11	0.33	0.34	-0.21	3.33	0.34	-0.19	3.16
2003/06	0.33	0.28	1.43	0.29	0.34	-0.30	3.25	0.33	-0.28	3.11
2003/07	0.27	1.33	1.14	0.26	0.28	-0.29	3.41	0.28	-0.26	3.19
2003/08	0.23	3.08	1.06	0.22	0.24	-0.32	3.74	0.24	-0.27	3.39
2003/09	0.21	2.83	1.05	0.20	0.22	-0.29	3.60	0.21	-0.26	3.27
2003/10	0.22	3.63	1.03	0.21	0.22	-0.25	3.84	0.22	-0.21	3.50
2003/11	0.25	1.67	1.09	0.24	0.26	-0.36	3.55	0.26	-0.33	3.23
2003/12	0.27	1.83	1.10	0.25	0.27	-0.30	3.47	0.27	-0.27	3.22

Table 4: **Pricing Errors in Parameter Fitting**

This table reports the average MSE(mean squared errors) between the market price and the model determined price for each KOSPI 200 index option with respect to moneyness, F/X, where F is the futures price and X is the strike price. BS, ahBS, CS, EN stand for the Black-Scholes, the ad hoc Black-Scholes, based on the deterministic volatility function by Dumas, Flemming and Whaley, corrected Corrado-Su, and extended normal model, respectively.

Month	moneyness<0.95				0.95≤moneyness≤1.05				1.05<moneyness			
	BS	ahBS	CS	EN	BS	ahBS	CS	EN	BS	ahBS	CS	EN
2003/01	0.00149	0.00002	0.00002	0.00004	0.00069	0.00009	0.00005	0.00006	0.00093	0.00005	0.00004	0.00005
2003/02	0.00601	0.00003	0.00005	0.00024	0.00162	0.00006	0.00006	0.00010	0.00241	0.00004	0.00004	0.00005
2003/03	0.00610	0.00004	0.00006	0.00023	0.00136	0.00004	0.00004	0.00010	0.00185	0.00003	0.00003	0.00003
2003/04	0.00972	0.00003	0.00003	0.00023	0.00189	0.00003	0.00003	0.00010	0.00363	0.00002	0.00003	0.00004
2003/05	0.00284	0.00003	0.00003	0.00005	0.00083	0.00003	0.00003	0.00004	0.00125	0.00002	0.00001	0.00002
2003/06	0.00545	0.00007	0.00003	0.00007	0.00162	0.00006	0.00003	0.00004	0.00319	0.00011	0.00002	0.00002
2003/07	0.00415	0.00005	0.00007	0.00020	0.00143	0.00004	0.00003	0.00005	0.00157	0.00004	0.00002	0.00003
2003/08	0.00584	0.00002	0.00017	0.00069	0.00187	0.00002	0.00003	0.00011	0.00070	0.00003	0.00003	0.00009
2003/09	0.00237	0.00005	0.00011	0.00037	0.00126	0.00003	0.00003	0.00007	0.00040	0.00004	0.00004	0.00009
2003/10	0.00304	0.00003	0.00013	0.00045	0.00121	0.00004	0.00004	0.00011	0.00036	0.00004	0.00004	0.00013
2003/11	0.00851	0.00002	0.00003	0.00035	0.00305	0.00003	0.00004	0.00011	0.00280	0.00004	0.00003	0.00004
2003/12	0.00572	0.00002	0.00002	0.00019	0.00214	0.00003	0.00002	0.00007	0.00283	0.00003	0.00002	0.00002
Average	0.00510	0.00003	0.00006	0.00026	0.00158	0.00004	0.00004	0.00008	0.00183	0.00004	0.00003	0.00005

Table 5: **Out-of-Sample Pricing Errors**

We compute the price of each option using the previous day's implied parameters for each model. This table summarizes the average pricing errors between the market price and the model-based price for each KOSIPI 200 index option of the year 2003, with respect to moneyness, F/X, where F is the implied futures price and X is the strike price. The reported pricing errors are evaluated using three different ways: MSE(mean squared errors), MAE(mean absolute errors), and MPE(mean percentage errors). BS, ahBS, CS, EN stand for the Black-Scholes, the ad hoc Black-Scholes, based on the deterministic volatility function by Dumas, Flemming and Whaley, corrected Corrado-Su, and extended normal model, respectively.

Month	moneyness<0.95				0.95≤moneyness≤1.05				1.05<moneyness			
	BS	ahBS	CS	EN	BS	ahBS	CS	EN	BS	ahBS	CS	EN
Panel A: Mean Squared Errors												
2003/01	0.00320	0.00175	0.00156	0.00155	0.00910	0.00956	0.00951	0.00955	0.00430	0.00374	0.00366	0.00363
2003/02	0.00648	0.00086	0.00066	0.00087	0.00325	0.00227	0.00216	0.00215	0.00398	0.00213	0.00221	0.00231
2003/03	0.00520	0.00104	0.00098	0.00115	0.00311	0.00208	0.00207	0.00228	0.00234	0.00079	0.00080	0.00079
2003/04	0.00962	0.00489	0.00498	0.00509	0.01950	0.01667	0.01673	0.01738	0.01242	0.00523	0.00520	0.00514
2003/05	0.00255	0.00252	0.00245	0.00248	0.01244	0.01154	0.01143	0.01138	0.00453	0.00273	0.00276	0.00276
2003/06	0.00446	0.00080	0.00078	0.00081	0.00386	0.00222	0.00231	0.00234	0.00401	0.00087	0.00086	0.00088
2003/07	0.00258	0.00117	0.00103	0.00120	0.00584	0.00434	0.00434	0.00442	0.00180	0.00110	0.00123	0.00131
2003/08	0.00530	0.00056	0.00068	0.00142	0.00539	0.00305	0.00310	0.00331	0.00119	0.00071	0.00066	0.00074
2003/09	0.00220	0.00031	0.00038	0.00047	0.00361	0.00283	0.00272	0.00287	0.00147	0.00024	0.00027	0.00031
2003/10	0.00318	0.00209	0.00240	0.00226	0.01487	0.01355	0.01328	0.01371	0.00261	0.00135	0.00138	0.00150
2003/11	0.00815	0.00118	0.00118	0.00136	0.00844	0.00569	0.00564	0.00585	0.00425	0.00143	0.00141	0.00139
2003/12	0.00437	0.00308	0.00306	0.00289	0.01440	0.01206	0.01204	0.01233	0.00621	0.00346	0.00335	0.00330

Table 6: **Continued**

Month	moneyness<0.95				0.95≤moneyness≤1.05				1.05<moneyness			
	BS	ahBS	CS	EN	BS	ahBS	CS	EN	BS	ahBS	CS	EN
Panel B: Mean Absolute Errors												
2003/01	0.05025	0.03327	0.03027	0.03029	0.08170	0.08301	0.08272	0.08281	0.05898	0.04954	0.04908	0.04899
2003/02	0.07810	0.02309	0.02048	0.02624	0.04664	0.03570	0.03464	0.03476	0.04633	0.03074	0.03163	0.03254
2003/03	0.06935	0.02898	0.02767	0.03117	0.04792	0.04034	0.04018	0.04189	0.04020	0.02136	0.02129	0.02117
2003/04	0.07991	0.05801	0.05925	0.05671	0.11879	0.11623	0.11678	0.11950	0.09198	0.06172	0.06152	0.06125
2003/05	0.03739	0.04099	0.04012	0.04087	0.09854	0.09649	0.09602	0.09586	0.05437	0.04576	0.04596	0.04632
2003/06	0.06275	0.02492	0.02238	0.02456	0.05125	0.04009	0.04163	0.04113	0.05578	0.02539	0.02508	0.02534
2003/07	0.04639	0.02802	0.02735	0.02914	0.06623	0.06007	0.06012	0.06061	0.03387	0.02611	0.02898	0.03040
2003/08	0.06717	0.01977	0.02207	0.03149	0.06142	0.04824	0.04850	0.04994	0.02651	0.02087	0.02055	0.02154
2003/09	0.04445	0.01443	0.01603	0.01955	0.04980	0.04384	0.04287	0.04360	0.02569	0.01265	0.01254	0.01413
2003/10	0.04628	0.03736	0.04075	0.03827	0.10509	0.09710	0.09658	0.09829	0.03940	0.03003	0.03056	0.03213
2003/11	0.08739	0.02596	0.02717	0.02957	0.07544	0.05954	0.05950	0.06127	0.05422	0.02731	0.02673	0.02785
2003/12	0.05466	0.04793	0.04795	0.04487	0.10626	0.10106	0.10109	0.10235	0.06837	0.05180	0.05067	0.05112

Table 7: **Continued**

Month	moneyness<0.95				0.95≤moneyness≤1.05				1.05<moneyness			
	BS	ahBS	CS	EN	BS	ahBS	CS	EN	BS	ahBS	CS	EN
Panel C: Mean Percentage Errors												
2003/01	-0.0073	-0.0041	-0.0035	-0.0036	-0.0207	-0.0273	-0.0268	-0.0267	-0.2208	-0.1333	-0.1194	-0.1400
2003/02	-0.0112	-0.0015	-0.0008	-0.0018	0.0074	-0.0022	-0.0021	-0.0024	0.0716	-0.0131	-0.0091	-0.0257
2003/03	-0.0103	0.0000	0.0007	-0.0007	0.0104	-0.0010	-0.0007	-0.0016	0.0396	-0.0277	-0.0215	-0.0302
2003/04	-0.0110	0.0011	0.0017	0.0004	0.0267	0.0143	0.0145	0.0140	0.1766	0.0303	0.0174	0.0054
2003/05	-0.0044	0.0024	0.0029	0.0024	0.0221	0.0131	0.0131	0.0124	0.0887	-0.0047	0.0024	0.0008
2003/06	-0.0089	-0.0004	0.0008	-0.0001	0.0109	0.0055	0.0043	0.0040	0.1374	-0.0423	-0.0105	-0.0122
2003/07	-0.0067	0.0009	0.0013	0.0002	0.0316	0.0184	0.0189	0.0186	0.0882	-0.0641	-0.0374	-0.1552
2003/08	-0.0092	-0.0005	-0.0002	-0.0025	0.0249	0.0076	0.0083	0.0090	-0.0988	-0.0597	-0.0292	-0.0785
2003/09	-0.0055	-0.0002	0.0001	-0.0014	0.0387	0.0174	0.0181	0.0194	-0.4223	-0.3245	-0.2586	-0.2765
2003/10	-0.0066	0.0006	0.0009	-0.0011	0.0207	-0.0021	-0.0010	0.0001	-0.4024	-0.1731	-0.1035	-0.2344
2003/11	-0.0117	-0.0010	-0.0003	-0.0024	0.0212	0.0005	0.0005	-0.0004	0.1030	-0.0884	-0.0661	-0.0860
2003/12	-0.0072	0.0006	0.0012	-0.0005	0.0190	0.0041	0.0046	0.0056	0.0548	-0.0333	-0.0117	-0.0150

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