Implied Volatility with Transaction Costs and the Market Efficiency of the KOSPI 200 Option Market*

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ABSTRACT

The "smile" in Black-Scholes implied volatilities is observed in the KOSPI 200 index option market. In addition, the Black-Scholes implied volatility is not an unbiased estimator of the future realized volatility over the remaining life of the option.

This paper examines the possibility that measurement errors in variables and transaction costs cause these anomalies. Simulation results show that measurement errors and transaction costs may explain the volatility smile observed in the real world. However, unlike the result of Christensen and Prabhala (1998), empirical results show that measurement errors and transaction costs are not enough to fully explain the bias of the implied volatilities, especially for ATM options, in forecasts of the future realized volatility. We also document evidence that trading strategies that exploit these anomalies can be profitable. We interpret the empirical evidence documented in this paper as suggesting evidence that the KOSPI200 options market may not be efficient.

1. Introduction

In an efficient market, the prices of traded assets reflect all the information available to investors. Specifically, option prices should convey the market information regarding the future volatility of the underlying asset returns. Under the assumptions of Black and Scholes (1973), the so-called Black-Scholes implied volatility should reflect complete information about the future volatility expected to be realized over the remaining life of the option. The Black-Scholes implied volatility is determined uniquely across those options with different exercise prices and the same expiration date. Therefore, the implied volatilities across the moneyness should be flat.

If the market is efficient and no errors in the variables exist, then the observed option prices reflect the rational expectation of investors concerning the future volatilities of the underlying assets. Thus, the BS implied volatility should be an unbiased estimator of the realized volatility. Day and Lewis (1992), using the data from 1983 through 1989, found that the implied volatilities of OEX options have sufficient information regarding one-week volatility.

However, Canina and Figlewski (1993) examined whether the implied volatility was an unbiased estimator of future volatility in the OEX option market from 1983 through 1987, and found that implied volatility had virtually no correlation with future return volatility. Moreover, they found that the historical volatility has better forecasting power than the implied volatility. In addition, Lamoureux and Lastrapes

(1993) documented that the forecasting power of the historical volatility is better than the implied volatility, using the individual stock options data from 1982 to 1984. The foregoing empirical evidence calls into question the joint hypothesis that the Black-Scholes model is correct and the market is efficient. On the other hand, Jorion (1995) showed that the forecasting power of the implied volatilities of the CME exchange options on futures is better than the estimators using historical data, GARCH(1,1) or MA(20).

Christensen and Prabhala (1998) suggested that the result of Canina and Figlewski was caused by problems with the sample period, usage of overlapping data and measurement errors. Christensen and Prabhala reexamined the relationship between implied volatility and the subsequent realized volatility for the OEX options market using the non-overlapping ATM option data for a long time. Using the 2SLS method to diminish the errors in variables, they showed that the implied volatility is an unbiased forecast of future volatility.

In reality, Implied volatilities across the moneyness are not flat. In many markets, a “smile” or “smirk” is observed. This phenomenon can be observed when the underlying asset’s return process is not a geometric Brownian motion or when transaction costs or measurement errors exist, or both.

Many authors showed that the Black-Scholes implied volatility would display a smile pattern when the underlying asset’s return follows a jump-diffusion process or a diffusion process with stochastic volatility. In addition, Kim et al. (1994) and Hentschel (2003) showed that either transaction costs or the measurement errors in variables could cause smiles to occur.

The first goal of this paper is to examine whether the Black-Scholes implied volatility is an unbiased forecast of the realized return volatility and if it is flat across the moneyness in the KOSPI 200 index option market. If the implied volatility is an unbiased estimator of the future volatility that the market expects, then the following equation holds:

$$IV = E_{MKT}(\sigma),$$

where IV is the implied volatility, and $E_{MKT}(\sigma)$ denotes the market's expectation for σ. This relation leads to a regression test for the rationality of a forecast as shown below.

$$\sigma = \alpha + \beta F(\Phi) + u,$$

where σ is the realized volatility of the underlying asset's return, $F(\Phi)$ is the forecast of σ based on the information set Φ, and u is the regression residual. If the forecast is the expected value of σ conditional on Φ, the regression estimates for α and β should be 0 and 1, respectively. Canina and Figlewski considered the implied volatility and the historical volatility as $F_1(\Phi_1)$ and $F_2(\Phi_2)$. They tried to verify that the information contents of implied volatility (Φ_1) include that of historical volatility (Φ_2) by "encompassing regression":

$$\sigma = \alpha + \beta_1 F(\Phi_1) + \beta_2 F(\Phi_2) + u.$$

The full informed forecast's coefficient, β_1 should still be 1 and the less informed forecast should be

$\beta_2 = 0$. We can test the joint hypothesis of the market efficiency and the Black-Sholes economy using the above regression. In this paper, we show that the joint hypothesis does not hold true in KOSPI 200 option market.

The second goal of this paper is to analyze why the hypothesis examined is rejected. First, this paper examines the relation between the Black-Scholes implied volatility and the realized volatility by using a simulation of the conditions under which transaction costs exist. The transaction costs and the measurement errors in variables seem to be able to explain the “smile” but cannot explain the bias of implied volatilities, especially for the ATM options, in forecasts of the realized volatility.

Also, we test the market efficiency by means of the trading strategy that uses the forecast of the future volatility. The strategy is to sell the overvalued options which have implied volatilities that are greater than the forecast and to buy the undervalued options which have implied volatilities that are lower than the forecast. This strategy leads to profits even after considering the transaction costs. Therefore, the market efficiency of the KOSPI 200 option market is doubtful. Additional evidence supporting market inefficiency is given.

We begin the analysis in section 2 with a data description of the KOSPI 200 option market and a simple hypothesis test. The data show a volatility smile, and the hypothesis that the implied volatilities are unbiased estimators of the future realized volatility is rejected for all moneyness. Section 3 examines the effect of the transaction costs and the measurement errors in variables. The simulation procedure is described and the effect is analyzed. In addition, test results obtained by using the 2SLS estimation to diminish the errors in variables are shown. Section 4 suggests a trading strategy using the forecast of the future volatility. Also, in section 4, the profits obtained by the proposed strategy are reported. Our conclusions are presented in section 5.

2. Data Description and the Hypothesis

2.1 Sampling procedure

We use the closing prices for the KOSPI 200 index options traded on the KSE (Korea Stock Exchange). The KOSPI 200 index option contracts are by far the most actively traded index options in terms of the number of contracts traded. In 2002, the number of contracts traded in the KOSPI200 options market was about 1.93 billion, which is larger than the sum of all the options contracts traded in CME, CBOT and CBOE. Trading of KOSPI 200 index options began in July 1997 and our sample covers the period from October 1999 to March 2003. The option expires on the second Thursday of each month, and 43 expiration dates are available within the sample period.

To obtain the historical volatilities and the realized volatilities, the KOSPI 200 index data from August 1999 through April 2003 are used. We eliminated the put options and those options with fewer than 3 or more than 30 days to expiration. In addition, the options that violate the upper and lower bounds for option prices are eliminated to calculate the Black-Scholes implied volatilities. The upper and lower bounds are

$$S_t - D_t - Xe^{-r(T-t)} < C_t < S_t - D_t,$$

where S_t is stock index, D_t is the present value of the dividends over the remaining life of the option, X is the strike price, r is the riskless interest rate, and T is the expiration date. The possibility of an early exercise need not be considered, since the KOSPI 200 index option is European. This elimination procedure results in a remainder of 10371 observations and 785 trading days.

The dividends are obtained from the average of past dividend amounts for the year. We used the 90-day CD rate converted into continuous compounding as a riskless interest rate for each day.

At time t, the historical volatility is calculated using the KOSPI 200 index for the past 50 dates (from t-50 to t-1). The ex-post realized volatility is calculated over each option's life (from t to T). These are computed as the sample standard deviation of the daily index returns. They are calculated from

$$HV_t(\text{or } RV_t) = \sqrt{\frac{252}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2},$$

where $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ and $\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$. The Black-Scholes implied volatility is obtained by inverting the Black-Scholes formula numerically.

2.2 Descriptive Statistics and the Smile

Table 1 reports the descriptive statistics for the variables considered. Descriptive statistics for each year are reported respectively. Panel A shows the properties of the time series of the KOSPI200 index's daily log return. The average of the 1-day return is almost zero and the volatility varies from 0.313 to 0.478. The distribution of the return is negatively skewed and leptokurtic. This implies that the return process does not follow the log-normal distribution and an assumption of the Black-Scholes model is violated.

Panel B reports the statistics for the ex-post realized volatility. The average of the realized volatility and the volatility of return in panel A are approximately the same value. The standard deviation of the realized volatility varies from 0.076 to 0.167.

Panel C presents the statistics for the Black-Scholes implied volatility classified by the moneyness, defined by X/S. We regard an option with a moneyness of less than 0.95 as in-the-money, an option with a moneyness greater than 0.95 but less than 1.05 as at-the-money, and an option with a moneyness greater than 1.05 as out-of-the-money. The mean value of the implied volatilities of at-the-money (ATM) options is much closer to both the 1-day return volatility and the ex-post realized volatility than that of the implied volatilities of in-the-money (ITM) or out-of-the-money (OTM) options. This shows that the implied volatilities of ITM or OTM options may be overestimated under the Black-Scholes economy where the implied volatility should be an unbiased estimator of the realized volatility.

The data also reveal the volatility smile. The implied volatility of ITM or OTM options is greater than that of ATM options and thus the volatility smile is observed. The distribution of the implied volatility is thicker at the ATM than it is at the ITM or the OTM. This phenomenon is illustrated in Figure 1.

Figure 1 shows the relationship between moneyness and the implied volatility. The implied volatility curve is U-shaped and the standard deviation of the implied volatilities of ATM options is much smaller than that of ITM or OTM options as shown in this figure. This fact is confirmed by the standard deviation reported in panel C of table 1.

Roughly speaking, the implied volatilities of the ATM options seem to have the most accurate and efficient information about the realized volatility.

2.3 The Main Hypothesis and Its Test

Under the condition that the market is efficient and that the Black-Scholes assumptions hold, the implied volatility of an option should be an unbiased estimator of the future realized volatility. As in the previous works such as Canina and Figlewski (1993) or Christensen and Prabhala (1998), we can test the joint hypothesis that the market is efficient and the Black-Scholes assumptions hold using the following regression equations:

$$RV_t = \alpha + \beta_i IV_t + \varepsilon_t \quad (1)$$

$$RV_t = \alpha + \beta_h HV_t + \varepsilon_t \quad (2)$$

$$RV_t = \alpha + \beta_i IV_t + \beta_h HV_t + \varepsilon_t \quad (3)$$

where RV_t is the ex-post realized volatility of the KOSPI 200 index returns from the time t through the option's expiration date, HV_t is the historical volatility over the past 50 days, and IV_t is the Black-Scholes implied volatility of the option.

If the implied volatility is an unbiased estimator of the realized volatility, β_i should not be different from 1 statistically in eq. (1). If the market is efficient so that the information contained in the implied volatility contains that of the historical volatility, β_i and β_h should be 1 and 0, respectively in eq. (3).

The results from the OLS regressions are reported in Table 2. We estimate the regression equations using the options with 10, 15, 20, 25, and 30 days to expiration. Because we used the overlapping data, a serial correlation problem occurs. We calculate the standard errors following Hansen (1982)[1] to solve this problem. The coefficients of the implied volatility and the historical volatility, the standard errors, R-square coefficient, and the number of observation are reported for each of the subsamples defined by moneyness.

This table shows that the β_i's are statistically significantly different from 1. The joint hypothesis that the market is efficient and that the Black-Scholes assumptions hold is rejected for every subsample. The coefficients of implied volatility for eq. (1) vary from 0.016 to 0.330 and the coefficient in the ITM or the

OTM is greater than that of the ATM. In comparison with the estimates of eq. (2), the coefficients of the implied volatility are less than those of the historical volatility. Thus, the implied volatility is more severely biased than the historical volatility as a forecast of the future realized volatility.

The coefficients for eq. (3) confirm that the forecasting power of the historical volatility exceeds that of the implied volatility in general. However, we find that the implied volatilities of ATM options have more information than those of the ITM or OTM options, and the slope coefficients of ATM options are roughly the same value as the slope coefficient of the historical volatility. This result is consistent with table 1 and the figure 1. The implied volatilities calculated from the ITM or OTM options are overestimated, and a volatility smile occurs. This means the implied volatility is overvalued and is more severely biased for the ITM or OTM options.

This test shows that the Black-Scholes assumptions are false or that the market is inefficient, or both. Many researchers have tried to explain this anomaly, that is, the volatility smile and the bias of the implied volatility for predicting the future realized volatility. These phenomena might occur because the return does not follow the geometric Brownian motion process. If the return process follows a jump process or, if volatility varies stochastically, the Black-Scholes implied volatility will display a smile and will contain less information about the future volatility.[2]

Another explanation is that there exist transaction costs, market frictions, and measurement errors in variables in the real world. Errors can arise from bid-ask spreads, transaction costs, nonsyncronous trading problem, and finite quotations of the observed prices. Errors caused by transaction costs or by market frictions exist in the option price, which makes it difficult to compute the real implied volatility. These errors make the implied volatility vary proportionally to the vega, causing a smile to occur.

Alternatively, the market is inefficient and a trading strategy that generates profit opportunities exists. There might be overreaction or underreaction in the option market that generates profitable strategies.

In the following sections, we consider the effect of transaction costs and the measurement errors in variables using a simulation. In addition, we test whether trading strategies using forecasts of the future realized volatility and a volatility smile generate profits exceeding transaction costs.

3. Simulation with Transaction Costs and the Test

3.1 Simulation method

For the simulation, we assume the stock index follows a geometric Brownian motion and there exist errors in the stock index and the call option prices. We can generate the stock index series that follows the geometric Brownian motion process using the initial value of the stock index, its expected rate of return, its volatility, and time interval.

$$S_i(t+\Delta t) = S_i(t)\exp\left[\left(\mu - \frac{\sigma_i^{\,2}}{2}\right)\Delta t + \sigma_i \varepsilon_t \sqrt{\Delta t}\right],$$

where we assume that the initial vales of $S(0) = 100$, $\mu = 0.1$, and $\Delta \mathrm{t} = 0.0025$. The disturbance term has a standard normal distribution, $\varepsilon_{\mathrm{t}} \sim N(0,1)$. We assume that the remaining life of the options is 0.125 year and that the period for calculating the historical volatilities is 0.125 year. Therefore, we need a stock index series for 0.25 year and we need to generate 0.25/0.0025=100 stock index prices.

When generating the stock indexes, σ_i is given by a random number between 0.2 and 0.6 to make the regression results reliable. The volatility of the ith series, σ_i, is given by $\sigma_i = 0.4\eta_i + 0.2$, where η_i is uniformly distributed between 0 and 1.

Strike prices are assumed to be 92.5, 95, 97.5, 100, 102.5, 105, and 107.5; 7 different strike prices, at intervals of 2.5. We calculate each call option price using the Black-Scholes formula with the true volatility. This price is regarded as the true option price that reflects the exact implied volatility. However, the option prices we observe are contaminated by errors in the stock index and in the call option price itself. The errors can arise from transaction costs, tick size restriction, or non-synchronicity between the index price and the option price. We generate the errors following Hentschel (2003). The observed stock index, $\widetilde{S}$, and call option prices, $\widetilde{C}$, are calculated by adding the error term to the true price as follows:

$$\widetilde{S}_t = S_t + e_S \text{ , where } e_S \sim N(0, 0.25^2) \text{ and}$$

$$\widetilde{C}_t = C_t + e_C \text{, where } e_C \sim N(0, {\sigma_C}^2)$$

where σ_C is given by $\sigma_C = \begin{cases} 0.03 & (X/S > 1.1) \\ 0.07 & (1.1 > X/S > 1.0) \\ 0.11 & (1.0 > X/S > 0.9) \\ 0.15 & (X/S < 0.9) \end{cases}$ with the moneyness.[3]

$\widetilde{S}$ and $\widetilde{C}$ are the observed stock index and the call price, respectively; S and C are true values; and e_S and e_C are error terms that have independent Gaussian distributions. The prices of the index and the call option prices are rounded off to three decimal places to consider finite quotation.

Figure 2 shows the effect of measurement errors in the stock index and the call option price described earlier. The variance of $\widetilde{C}$ is calculated by

$$Var(\widetilde{C}) = Var(e_C) + \left(\frac{\partial C}{\partial S}\right)' Var(e_S)\left(\frac{\partial C}{\partial S}\right),$$

where $\dfrac{\partial C}{\partial S} = \Phi(d_1)$, $d_1 = \dfrac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{t}}$, and $\Phi(\cdot)$ is the standard normal distribution function.

Figure 2 shows the Black-Scholes option prices and its 95% confidence interval when the measurement errors in the stock index and in the option price are as above. We assume that the strike price is 100, the true implied volatility is 0.3, the time to expiration is 1 month, and the riskless interest rate is 0.05 for this figure. In the upper figure of figure 2, dotted lines indicate the upper and lower confidence intervals and the lower line is the no-arbitrage lower boundary. In the lower figure, the standard deviations of the call option price are represented.

In this figure we can find that the ITM or the OTM option prices can violate the lower no-arbitrage boundary easily. This means that a truncation problem exists in estimating the implied volatility, and that the ITM and the OTM options lying on the no-arbitrage region are overvalued, resulting in a volatility smile. Kim et al. (1994) and Hentschel (2003) showed that the errors in variable may make a smile. This truncation problem is severe in the deep ITM options since the standard deviation of the call option increases as moneyness decreases and the deep OTM option prices observed can never be negative in the real world.

Figure 3 shows that a smile can be caused by errors in variables. We calculate the theoretical price, using the Black-Scholes formula and the real data of the KOSPI 200 index, strike prices, riskless rates, time to expiration, and ex-post realized volatility. Errors are added to the theoretic prices as in the simulation method. Implied volatilities are obtained from these theoretical Black-Scholes option prices with errors. Figure 3 shows that the errors in the option prices can make the volatility smile similar to what is observed in reality, as shown in figure 1.

The next section examines whether measurement errors in variables can make an implied volatility biased, thus rejecting the joint hypothesis that the market is efficient and that the Black-Scholes economy holds.

3.2 Simulation Results

To consider transaction costs and market frictions, a simulation is conducted as described in the previous section. We first generate 3000 series of the stock index and call option prices. The 51st stock index and option prices are regarded as the time-0 prices. The historical volatility is calculated from the 1st through the 51st prices, and realized volatility is calculated from the 51st through the 101st prices. Since we assume that there are 7 different strike prices, we can get 21000 pairs of implied volatility, historical volatility, and realized volatility. In this simulation, there is no overlapping between the observations and the number of observations is large enough. The regression formulas from eq. (1) to eq. (3) are estimated by OLS for each subsample divided by moneyness.

Table 3 presents the estimation results. In panel A, we use the data without errors in variables for a benchmark to compare with the case that considers transaction costs or errors. In eq. (1), constant terms are nearly 0 and the slope coefficients of the implied volatility are nearly 1. Implied volatility is an unbiased forecast of the realized volatility. The result from eq. (2) shows that the historical volatility has some forecasting power. Examining eq. (3), the coefficients of the implied volatility are nearly 1 as before. However, the coefficients of the historical volatility drop to as low as 0, and the sum of the coefficients of the implied volatility and the historical volatility is nearly 1. This result indicates that the forecasting power of the implied volatility far exceeds that of the historical volatility and that the information reflected in the implied volatility contains that of the historical volatility. This result holds regardless of moneyness.

In panel B, the data contain errors in stock index and option prices. In eq. (1), the slope coefficients fall a little over all moneyness and the slope coefficient of deep ITM falls sharply to 0.683. ITM options are more influenced by errors rather than are ATM or OTM options, which is expected. The estimates of eq. (2) are approximately the same as those in panel A.

In eq. (3), the coefficients of the implied volatility and the historical volatility remain at nearly 1 and 0, respectively, for the ATM options. However, the coefficient of the implied volatility is 0.246, less than the historical volatility, 0.619, for the deep ITM options. In addition, the R-square coefficient is 0.689 for the deep ITM, which is dropped from 0.847 in panel A. The coefficients of the implied volatility and the historical volatility for the deep OTM are 0.822 and 0.131, respectively. In the case that there exist errors in the variables, the sum of the coefficients of the implied volatility and the historical volatility is nearly 1.

This table 3 shows that the error can make an implied volatility seem to be biased for deep ITM options. It cannot fully explain, however, the result that the implied volatility is severely biased as shown in table 2.

As Christensen and Prabhala (1998) point out, this result can be caused by the sampling procedure. In table 2, we use overlapping data under the condition that the number of observations is small. Table 4 reports the estimation result, using the overlapping data with a small number of observations. Only 80 series are considered, and the implied volatilities are computed from 51st, 61st, 71st, 81st, and 91st stock index and option prices. Moreover, the historical volatility and realized volatility are obtained from the past 50 pairs of observations and from the next observations over the remaining lifetime, respectively. The total number of observations is 2800.

The regression results that are analogous to table 3 are reported in table 4 using the overlapping data. In table 4, the standard errors are calculated following the Hansen method since we are using overlapping data.

In panel A, the results are similar to those in panel A of table 3. The standard errors of coefficients increase, and the coefficient of the implied volatility deviates from 1 more severely. In eq. (3), however, the coefficients of the implied volatility and the historical volatility are still close to 1 and 0, respectively, and their sum is nearly 1.

In panel B, the results change considerably. The coefficient of the implied volatility in eq. (1) is nearly 0 for deep ITM options. In addition, the value for deep OTM options drops to 0.157. The explanatory powers of the implied volatilities for deep ITM or deep OTM options are less than 10%. In eq. (3), the coefficients of the implied volatility for deep ITM and OTM options are nearly 0 and the historical volatility dominates the implied volatility. These results are consistent with table 2.

However, the coefficients of the implied volatility for ATM and OTM options are still nearly 1 for both eq. (1) and eq. (3). In addition, the R-square coefficients are more than 70%.

To summarize, the measurement errors caused by the transaction costs or market frictions and the problem with the sampling procedure cannot fully explain the anomaly that the implied volatility is biased.

3.3 2SLS estimation

Christensen and Prabhala (1998) used the 2SLS method to reduce the effect of errors in variable. They used a historical volatility and an implied volatility of 1 month ago as instruments to diminish errors in today's implied volatility. They considered past implied volatility to be correlated with true implied volatility but, quite plausibly, uncorrelated to the measurement error associated with present implied volatility. We conduct the 2SLS estimation following Christensen and Prabhala (1998).

We sample the implied volatility of options with 24 days to expiration and 21 days to expiration and examine the 3 week forecasting power of the implied volatility. For the first stage regression, the implied volatilities of options with 21 days to expiration are regressed on a constant, the implied volatility of options with 24 days to expiration, and the historical volatility of the option. The first stage regress equation is,

$$IV_t = \alpha + \beta_1 IV_{t-3} + \beta_2 HV_t + \varepsilon_t, \tag{4}$$

where IV_{t-3} is the implied volatility of the option with 24 days to expiration.

For the second stage regression, the realized volatilities are regressed on a constant, the implied volatility estimated from the first stage regression, and the historical volatility. The second stage regression equations are,

$$RV_t = \alpha + \beta_i \hat{IV_t} + \varepsilon_t, \tag{5}$$

$$RV_t = \alpha + \beta_i \hat{IV_t} + \beta_h HV_t + \varepsilon_t, \tag{6}$$

where $\hat{IV_t}$ is the estimate from the first stage regression.

Table 5 reports the estimation results. Panel A and panel B show the estimates of the first stage regression and those of the second stage regressions. The result of eq. (5) in panel B demonstrates that the coefficients of implied volatilities improved greatly in comparison with the values in table 2. The coefficient values of the implied volatility range from 0.523 to 0.765 for the ATM and OTM options. The coefficient of the implied volatility increases, while that of the historical volatility in eq. (6) decreases, especially for the ATM and the OTM options. The coefficient values of the implied volatility range from 0.403 to 0.709 except for those of the deep ITM options.

The coefficients of the implied volatility obtained by the simulation following the method described in the previous section are nearly 1 except for the deep ITM, though not presented in this paper. The 2SLS method diminishes most of the errors that are mutually independent.

This result indicates that the measurement errors exist and such errors make the implied volatility more severely biased. However, the coefficients of the implied volatility are still far from one. The errors-in-variables problem partially accounts for the bias of the implied volatility, but they cannot fully account for the results of table 2.

4. Forecast of the Future Volatility and Trading Strategy

In the previous section, we examined the effect of the transaction costs and the errors in variables. These errors can result in a volatility smile observed and an implied volatility biased from the actual one. Nevertheless, those errors alone cannot fully account for the empirical anomaly of the KOSPI 200 index option market.

There are two possibilities that may explain the anomaly documented so far. First, there is the possibility that the Black-Scholes model is incorrect. For example, Bakshi, Cao and Chen (1997), Bates (1996) and many other researchers document that the Black-Scholes model is rejected. The failure of the Black-Scholes model might explain the anomaly under discussion. The other possibility is that the market is inefficient. For example, Stein (1989) and Poteshman (2001) discussed this possibility.

In this section, we explore investment strategies that may generate profit opportunities using the forecast of the future volatility and the Black-Scholes formula. If the market is efficient, abnormally profitable strategies cannot exist. The market efficiency is, however, doubtful if there is an abnormally profitable strategy.

4.1 The Forecasting Power of Various Volatility Estimators

For the trading strategy, the forecast of the future volatility is needed. If the BS assumptions are correct and the prices can be observed without errors, we can obtain the unique implied volatility across the moneyness at time t. In the real world, we observe that implied volatilities are different from each other across the moneyness. The observed implied volatility vector, denoted $\tilde{\sigma} = [\sigma_1, \sigma_2, ..., \sigma_n]'$, can be observed from n options with different strike prices. To obtain a forecast of future volatility from these observed implied volatilities, we consider the following statistics using cross-sectional individual implied volatilities:

1. Mean
2. Median
3. The implied volatility of the option whose moneyness is closest to 1
4. Vega-weighted average
5. Hentschel's estimator
6. Hentschel's estimator considering a volatility smile with quadratic specification

The mean and the median of $\tilde{\sigma} = [\sigma_1, \sigma_2, ..., \sigma_n]'$ can be a forecast of the future volatility if the volatility can be regarded as constant over time. The implied volatility of the ATM option is considered because it contains the most accurate information about the future volatility, as shown in a previous section. The vega-weighted average is $\hat{\sigma} = \sum_{i=1}^{n} w_i \sigma_i \Big/ \sum_{i=1}^{n} w_i$, where $w_i = \partial C / \partial \sigma$.

Hentschel's estimator is the cross-sectional estimator of the implied volatilities with measurement errors using the FGLS method. Hentschel (2003) suggested this measure and showed that this is an efficient estimator of the implied volatility.

There exists a weighted average of the observed implied volatilities that has the minimum variance among all such unbiased weighted averages. This efficient estimator is the GLS weighted average obtained by regressing implied volatilities on a constant,

$$\tilde{\sigma} = \iota\sigma + \varepsilon_{\sigma}$$

where $\tilde{\sigma}$ is the observed implied volatility, ι is an n-vector of ones, σ is a true volatility, and ε_{σ} is an error in the observed implied volatility. The efficient estimator of σ is obtained by the GLS estimation as below.

$$\hat{\sigma} = \omega'\tilde{\sigma} = (\iota'\Sigma^{-1}\iota)^{-1}\iota'\Sigma^{-1}\tilde{\sigma},$$

where Σ is the covariance matrix for implied volatilities from a cross-section of options. With independent measurement errors in prices, the covariance matrix for the implied volatilities is obtained as below.

$$\Sigma = Var(\varepsilon_{\sigma}) = E\left[\varepsilon_{\sigma}\varepsilon_{\sigma}'\right] = \frac{\partial\sigma}{\partial x'}\Lambda\frac{\partial\sigma}{\partial x},$$

where $\Lambda = E\left[\varepsilon_{x}\varepsilon_{x}'\right] = diag\left[V\left(\varepsilon_{C_1}\right),\Lambda\ ,V\left(\varepsilon_{C_n}\right),V\left(\varepsilon_{S}\right),V\left(\varepsilon_{r}\right),V\left(\varepsilon_{t}\right)\right]$ is the covariance matrix of the underlying errors and the source of the errors is $x = \left(C_1,...,C_n,S,r,t\right)$. $\frac{\partial\sigma}{\partial x'} = \left(\frac{\partial\sigma_i}{\partial C_i}\right)\left(\frac{\partial C_i}{\partial x_j}\right)$ is the Jacobian matrix of implied volatility derivatives. To implement the FGLS method, the Jacobian matrix can be calculated from partial derivatives of the BS formula.

And the estimator, which considers a volatility smile with a quadratic specification, is obtained by the following regression:

$$\tilde{\sigma} = \iota\beta_0 + (K/S-1)\beta_1 + (K/S-1)^2\beta_2 + \varepsilon_\sigma,$$

$$= X\beta + \varepsilon_\sigma$$

where $K = (K_1, \Lambda, K_n)$ is the column vector of strike prices, $\beta = (\beta_0, \beta_1, \beta_2)$, and $X = [\iota, (K/S-1), (K/S-1)^2]$. The coefficients can be estimated by the FGLS method.

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}\tilde{\sigma}.$$

We take β_0, the value at $K/S = 1$, as a forecast.

For the covariance matrix of the underlying errors, we assume that $V(\varepsilon_C)$ and $V(\varepsilon_S)$ are equal to the variance presumed in the previous section, and $V(\varepsilon_r) = 0.001^2$ and $V(\varepsilon_t) = 0.0001^2$. (0.0001 year is translated to about 50 minutes)

We can obtain the time series of 785 cross-sectional estimators. These are the forecast of the realized volatility over the remaining lifetime of the options with the same expiration date. The forecast estimators are regressed onto the ex-post realized volatilities using eq. (1) and eq. (3). The data of every trading day with less than 30 days to expiration are used.

The results are presented in table 6 and in figure 4. Table 6 reports the regression estimation results and figure 4 shows the relation between the cross-sectional forecasts and the ex-post realized volatilities.

The first column of table 6 presents the result when the historical volatility over the prior 50 trading days is used as a forecast of the future volatility. The second through the seventh columns report the forecasting power of the various cross-sectional forecasts mentioned above. Because we use the data from every trading day, the use of overlapping data is inevitable. Thus, the standard errors are calculated by the Hansen method.

The estimation result of eq. (1) shows that the coefficient of each forecast ranges from 0.065 to 0.224. The coefficients of the historical volatility, implied volatility of ATM option, and Hentschel's estimators are greater than the others. These coefficients have values of approximately 0.2. The R-square coefficient of the regressions is greatest when Hentschel's estimators are used.

In eq. (3), the coefficients of the cross-sectional estimators range from 0.041 to 0.152, and the coefficients of the historical volatility range from 0.112 to 0.204. The sum of the coefficients of the cross-sectional forecast and the historical volatility is about 0.25. In eq. (3) as well as in eq. (1), the coefficients of the implied volatility from the ATM option and those from Hentschel's estimators are greater than the others.

In this table, we can find that the forecasting power of the mean, the median, and the Vega-weighted average of cross-sectional observations is lower than that of Hentschel's estimators or of the implied volatility from the ATM option. However, the forecasting power of the cross-sectional estimators is very low in general.

Figure 4 shows the relationship between the cross-sectional estimators and the ex-post realized volatility. The pairs of the cross-sectional estimators mentioned above and the ex-post realized volatilities are dotted.

The regression line is the solid line. The dashed line shows the points where the value of the estimator equals the ex-post realized volatility.

Positive correlations between the estimates and the ex-post realized volatilities are observed in every figure. However, the deviation from the dashed line is positive for the lower values of the estimators and negative for the higher values of the estimators, thus the slopes of the regression lines are less than 1. This shows that the estimators are somewhat biased.

In the next section, we explore investment strategies using the Black-Scholes formula and the forecasts of the future volatility. The implied volatility of the option whose moneyness is closest to 1 and Hentschel's estimator are used as the forecasts.

4.2 The trading strategy and its profit

We test the market efficiency by way of a trading strategy that uses the forecast of the future volatility. The strategy is "Buy the undervalued and Sell the overvalued" relative to a theoretical price calculated by the Black-Scholes formula and the volatility forecast. The trading strategy is as follows:

1. For each day t, find the theoretical prices for the options with different strike prices using the Black-Scholes formula. We use the forecasts described in the previous section for the future volatility of

KOSPI 200 index.

2. Identify the overvalued and the undervalued by comparison with the theoretical prices computed in the previous step.

3. Compute the hedge ratio for delta-hedging. By the Black-Scholes formula, the hedge ratio is $-\partial C/\partial S = -N(d_1)$. Delta-hedging reduces the risk of the shift of call option prices caused by the change of the underlying asset's price.

4. Buy the undervalued and sell the overvalued with delta-hedging. Set the position in the options to be proportional to the ratio of the difference between the theoretical price and the observed price. The number of call options to be bought or sold, x_t, is calculated by the following equation:

$$x_t = 10000\left(\frac{C_t - C_t^{BS}}{C_t}\right).$$

where C_t is the observed call option price at time t and C_t^{BS} is the theoretical price computed by the Black-Scholes formula. A positive value for x_t indicates a long position in the call option and a short position in the index, and a negative value for x_t indicates a short position in the call option and a long position in the index.

5. We assume the holding period is 1 day. Therefore, the profit generated during a day is calculated by,

$$profit_t = x_t(C_{t+1} - C_t) - x_t \cdot \Delta_t(S_{t+1} - S_t),$$

where S_t is the KOSPI 200 index at time t, and $\Delta = N(d_1)$. The portfolio is rebalanced every day.

The trading profit is shown in table 7. In this table, we assume that we can buy or sell at the observed price without any transaction costs. The data that breach the arbitrage bounds and so were eliminated in the previous analysis are included in this analysis.[4]

Panel A used the ex-post realized volatility as a forecast. Since the ex-post realized volatility is not available in advance, the results of panel A are somewhat imaginary. The profits using the ex-post realized volatility are considered as the maximum value of profits. The estimates from regression equation eq. (3) obtained from the Hentschel's estimator or the ATM estimator are used, respectively, for panels B and C. In panel D, the Hentschel's estimator is used as a forecast.

Each panel reports the number of observations, the total profit, and the total profit over total investment. The values of total profit over total investment can be negative if the net investment is negative. We report the mean of daily profits and their t-values under the null hypothesis, "the mean of daily profits equals zero."

In panel A, the profits are positive in all the subsamples and are statistically significant for almost all the subsamples. In panels B, C, and D, the profits are similar to one another. In general those profits are much smaller, relative to the profits from panel A. This shows that the forecast of the future volatility is not a perfect estimator. The overall average of daily profit changes from 783 to 220 and the t-values also become smaller. One subsample, the OTM option in 2001, produces negative profits. The negative profits in 2001 arise from the 9.11 shock in Korean financial market.

Though nearly the same amounts of profits are shown in panel A regardless of the moneyness, the size of the profits is very different across the moneyness in panels B, C, and D. The largest decline is shown in the OTM options and the smallest decline is shown in the ITM options. Trading the ITM options seems the most profitable.

These results show that the volatility smile is not caused entirely by the misspecification of the underlying asset process assumed by the Black-Scholes model. The Black-Scholes formula seems to identify the overvalued options and the undervalued options very well, and the trading profit is positive in most cases. If the underlying process does not follow a log-normal process but follows another process, such as a jump-diffusion or a process with stochastic volatility, and the market is efficient, then the strategies that uses a volatility smile should generate zero profits on average.

The results of table 7 do not show that the market is inefficient. Due to the transaction costs, investors may not be able to make money with this proposed strategy. Next, we consider the transaction costs caused by bid-ask spreads.[5] We assume that the observed prices are the midpoints of bid and ask prices. Investors are assumed to pay the observed price plus half of the spread when they buy a security, and then receive the observed price minus half of the spread when they sell a security.

We assume that the bid-ask spreads of call options are as follows:

$$\text{bid - ask spreads of the call option} = \begin{cases} 0.01 & C < 3 \\ 0.05 & 3 \le C < 5 \\ 0.10 & 5 \le C < 10 \\ 0.25 & 10 \le C \end{cases}.$$

We also assume that the bid-ask spreads of the index are 0.2 and fixed. These bid-ask spreads of the call option and the index are estimated from the transaction data of call options and futures on the KOSPI 200 index, with the maturities of which are less than one month.

Transaction costs, at time t, caused by bid-ask spreads are computed by the following equation, assuming a one-time rebalance.

$$\text{Transaction costs} = \left| x_t - x_{t-1} \right| \cdot \frac{s_C}{2} + \left| \Delta_t x_t - \Delta_{t-1} x_{t-1} \right| \cdot \frac{s_S}{2},$$

where s_C and s_S are bid-ask spreads of a call option and the KOSPI 200 index, respectively.

The trading profits are shown in table 8. The format of this table is equivalent to that of table 7. The profits reported in panel A are all positive and statistically significant. In comparison with table 7, the daily profits decrease on average to 84% of the daily profits of the case without transaction costs. The ratios range from 52% to 94% for each subsample.

Panels B, C, and D show that the profits are similar in each case. Positive profits are observed for all the ITM subsamples, and most of the other subsamples. However, some negative profits are reported and their t-values are less than those in table 7. In comparison with table 7, the daily profits decrease on average to about 70% of the daily profits of the case without transaction costs. Profits for the ITM subsamples are largest and the profits generated from the ATM or the OTM options are much smaller relative to the profits in panel A.

However, the investment strategy still seems profitable after transaction costs, and the profits from ITM options are still statistically significant. Though not reported in this paper, if trading is permitted only when the option price's deviation from the theoretical price is greater than the bid-ask spreads, the profit of the strategy is greater than the results presented in table 8 and is statistically significant.

These results show that the profits, even after deducting transaction costs, are positive for the trading strategy using the forecast of the future volatility and the Black-Scholes formula. This fact is not consistent with the hypothesis that the KOSPI200 index option market is efficient. However, we can see that the trading profits tend to become smaller with time in tables 7 and 8. This indicates that the profit opportunities have decreased and the market has become more efficient.

4.3 Additional Evidence

We examine the serial-correlation of the errors in the implied volatility. If the volatility smile is caused by transaction costs or measurement errors in variables, then the time-series of the errors in the implied volatility should be independent as Hentschel assumes. We calculate the errors in the implied volatility by subtracting the ex-post realized volatility from the estimated implied volatility as follows:

$$error_t = IV_t - RV_t^{ex-post}$$

Because we restricted the data to only those options within 30 days to expiration, we do not have enough data to analyze the autocorrelation structure of the error. For the test of the serial correlation, a runs-test[6] is conducted. For a run-test, we construct a time-series of indicators that is 1 for the overvalued and -1 for the undervalued. The options that breach the lower no-arbitrage boundary are also considered to be undervalued.

$$indicator_t = Sign(error_t)$$

In addition, the first-order autocorrelation coefficients of the indicators' time-series are examined.

The first-order autocorrelation coefficients and the z-statistics of the runs-tests are calculated for each option's time-series. The distribution of these values is shown in the Figure 5. The left figure shows the histogram for the first-order autocorrelation of the series and the right figure shows the histogram for the z-statistics obtained by the runs-test. Both figures show the positive autocorrelation of the errors in the implied volatilities. This result cannot be accounted for by the transaction costs or by the measurement errors unless they are serially correlated. This evidence is consistent with the results in section 4.2 and implies that market inefficiency may be caused by overreaction or underreaction in the options market.

5. Conclusion

We found that the volatility smile is observed in KOSPI 200 index option market. In addition, we document that the Black-Scholes implied volatility is not an unbiased estimator of the future realized volatility over the remaining life of the option.

These anomalies cannot be accounted for in the pure Black-Scholes economy. This paper examines the possibility that measurement errors in variables and transaction costs can explain the volatility smile and the bias of implied volatility as a forecast of the future volatility. From a simulation with the errors in variables, we can observe a volatility smile similar to those observed in the real world. The volatility smile seems to be explained by the errors in variables and transaction costs, but the bias cannot be fully accounted for, especially with regard to the ATM options. Though we consider the overlapping feature of small samples, the errors in variables cannot explain the coefficients of the implied volatility of the ATM or the OTM options. 2SLS estimation results also suggest that the errors in variables cannot fully explain the bias. The volatility smile and the bias of the implied volatilities in the KOSPI 200 options are explained only partially by transaction costs and measurement errors.

If the anomalies are caused by market inefficiency, some trading strategies using this anomaly should generate profit opportunities. We examined whether a strategy of selling the overvalued and buying the undervalued, using the Black-Scholes formula, brings profits to investors. We document that this strategy with delta-neutral hedging generates substantial trading profits even after taking transaction costs equivalent to the bid-ask spreads into consideration. These results show that the volatility smile is not caused entirely by the misspecification of the underlying process as the Black-Scholes log-normal process. We also find that the profit opportunities tend to disappear over time.

Other evidence supporting the market inefficiency is presented in this paper. The series of the values that indicate the overvalued or the undervalued shows positive serial-correlations. This cannot be explained by the errors in variable or transaction costs. We also found that the option price's deviation from the Black-Scholes theoretical price is too large to be accounted for by measurement errors in variables and errors caused by model specification.

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Footnotes

[1] To settle the serial correlation problem caused by the overlapping data, the Hansen method is used for estimating the covariance matrix for the coefficients.

$\hat{\Psi} = \frac{1}{N}\sum_{k=1}^{N}\sum_{j=1}^{N} Q(k,j)\hat{e}_k\hat{e}_j(X_k{}'X_j)$ where $\hat{e}_k$ and $\hat{e}_j$ are the residual for observations k and j from OLS regression and X_n is the row vector of the independent variable matrix for observation n. In addition, $Q(k,j)$ is an indicator function taking the value 1 if there is an overlap between the observation k and j, and 0 otherwise. The covariance matrix for the coefficients is $\hat{\Omega} = (X'X)^{-1}\hat{\Psi}(X'X)^{-1}$.

[2] Das and Sundaram (1999) and Tompkins (2001) pointed out that a jump or a stochastic volatility cannot fully explain the smile.

[3] The standard deviation of the errors in variable is assumed following Hentschel (2003). Hentschel assumes the standard deviation is one quarter of the bid-ask spread. Vijh (1990) reports that the errors broadly have a normal distribution with a zero mean and a standard deviation of one quarter of the bid-ask spread.

[4] The trading profits using the data that satisfy no arbitrage boundary are a little smaller than the profits using the whole data. However, the positive profits are still obtained with this strategy, and the main results remain qualitatively unchanged.

[5] We ignore the brokerage commissions and other costs because the transaction occurred once a day and these costs are insignificant to the institutional investors.

[6] Assume that the probability of 'H' and 'L' is 0.5 respectively and the events are independent. Expectation of

a number of runs is approximately $E(R) = n/2 + 1$ and the standard error is $SE(R) = \sqrt{n-1}/2$, where n is the number of events. The test statistics is $z = \dfrac{r - E(R)}{SE(R)}$.

Table 1. Descriptive Statistics

		Statistics	total	2003	2002	2001	2000	1999
Panel A: Time series of stock index return								
daily log return of index		mean	-0.0003	0.0005	-0.0005	0.0012	-0.0031	0.0011
		volatility	0.3857	0.3129	0.3339	0.3538	0.4776	0.4017
		skewness	-0.2972	0.1888	-0.1083	-0.6202	-0.2226	-0.0739
		kurtosis	4.8266	3.0405	3.7055	7.5338	3.9560	3.1498
Panel B: Ex-post realized volatility								
realized volatility		mean	0.3701	0.2881	0.3261	0.3671	0.4369	0.3663
		standard deviation	0.1395	0.0804	0.1056	0.1670	0.1426	0.0764
		skewness	0.9754	0.3921	0.4050	2.1377	-0.3608	0.5366
		kurtosis	6.4338	2.1607	4.5105	10.8717	2.5664	3.8336
Panel C: Black-Scholes implied volatility								
implied volatility	ITM (X/S<0.95)	mean	0.6396	0.4230	0.6723	0.6556	0.5800	0.6865
		standard deviation	0.4801	0.1955	0.5809	0.4695	0.3473	0.3116
		skewness	3.1351	3.0849	2.8846	2.5343	4.2115	1.7136
		kurtosis	16.8581	16.6508	13.9978	11.3416	27.5662	7.0241
	ATM (0.95<X/S <1.05)	mean	0.3945	0.3397	0.3618	0.3586	0.4464	0.4683
		standard deviation	0.0977	0.0557	0.0619	0.1075	0.1004	0.0611
		skewness	0.9333	0.4784	1.5563	1.0328	0.9578	0.9909
		kurtosis	4.9266	3.8982	11.6260	4.5442	4.7384	4.8548
	OTM (X/S>1.05)	mean	0.4721	0.4500	0.4295	0.4083	0.5383	0.4983
		standard deviation	0.1538	0.1815	0.1307	0.1155	0.1605	0.0807
		skewness	2.2009	2.7897	3.2879	2.3203	1.8420	2.1060
		kurtosis	10.7048	12.6166	18.0624	12.9686	8.8278	10.1316
	total	mean	0.4890	0.4227	0.4793	0.4602	0.5222	0.5413
		standard deviation	0.2705	0.1703	0.3413	0.2874	0.1943	0.1989
		skewness	5.2240	3.0937	5.3403	4.4782	4.6304	3.4451
		kurtosis	47.5305	15.7518	42.7454	31.7947	48.5311	19.6912

Panel A reports the descriptive statistics for daily time series of log return of KOSPI 200 index from August 1999 through April 2003. Mean, volatility which is the sample standard deviation of returns, skewness, and kurtosis are reported. Panel B reports the statistics for ex-post realized volatility over the remaining life of an oprtion. Panel C shows the statistics for the Black-Scholes implied volatility classified by moneyness. Sample period for panel B and C is from October 1999 to March 2003 and the options whose time to expirations are 3~30 days are used.

Table 2. The Regression Results

	moneyness	X/S<0.85	0.85<X/S <0.95	0.95<X/S <1.05	1.05<X/S <1.15	1.15<X/S	Total
Eq. (1)							
	constant	0.3400	0.2965	0.2731	0.2396	0.2831	0.3455
	se	(0.002)	(0.002)	(0.004)	(0.004)	(0.002)	(0.001)
	IV	0.0155	0.1595	0.2590	0.3304	0.2131	0.0655
	se	(0.003)	(0.004)	(0.009)	(0.008)	(0.003)	(0.001)
	R2	0.0046	0.0360	0.0334	0.0444	0.0526	0.0109
	N	136	309	528	478	625	2076
Eq. (2)							
	constant	0.1383	0.2495	0.2628	0.2437	0.1551	0.2095
	se	(0.014)	(0.006)	(0.005)	(0.005)	(0.004)	(0.002)
	HV	0.6154	0.3033	0.2939	0.3409	0.5765	0.4310
	se	(0.042)	(0.016)	(0.013)	(0.012)	(0.009)	(0.005)
	R2	0.0888	0.0360	0.0314	0.0445	0.1734	0.0791
	N	136	309	528	478	625	2076
Eq.(3)							
	constant	0.1248	0.2230	0.2413	0.2145	0.1562	0.2044
	se	(0.014)	(0.006)	(0.005)	(0.005)	(0.004)	(0.002)
	IV	0.0157	0.1232	0.1694	0.1990	-0.0069	0.0228
	se	(0.003)	(0.004)	(0.008)	(0.008)	(0.004)	(0.001)
	HV	0.6158	0.2342	0.1758	0.2065	0.5823	0.4171
	se	(0.041)	(0.016)	(0.013)	(0.014)	(0.010)	(0.005)
	R2	0.0935	0.0556	0.0406	0.0537	0.1734	0.0804
	N	136	309	528	478	625	2076

This table shows the regression results using the implied volatility of the call options from October 1999 through March 2003. 2076 observations of the options with 10, 15, 20, 25, or 30 days to expiration are used. The regressions are estimated by using the data classified by the moneyness. The standard errors of the slope coefficients are estimated following Hansen (1982) for considering the serial correlation caused by using the overlapping data. Each panel shows the coefficients, the standard errors (se) in parenthesis, R-square coefficient (R2), and the number of observations (N).

Table 3. The Regression Results using the Non-overlapping Data by Simulation

moneyness	X/S<0.85	0.85<X/S <0.95	0.95<X/S <1.05	1.05<X/S <1.15	1.15<X/S	Total
Panel A. No measurement error						
Eq. (1)						
constant	-0.0050	-0.0038	-0.0005	-0.0004	0.0077	-0.0006
se	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
IV	1.0095	1.0062	0.9945	0.9945	0.9736	0.9956
se	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R2	0.8469	0.8898	0.8971	0.8855	0.8232	0.8845
N	2874	4813	5744	4090	3479	21000
Eq. (2)						
constant	0.0815	0.0400	0.0408	0.0466	0.0756	0.0481
se	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
HV	0.8183	0.9006	0.8896	0.8793	0.8304	0.8806
se	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)
R2	0.7028	0.7873	0.7945	0.7792	0.7019	0.7796
N	2874	4813	5744	4090	3479	21000
Eq.(3)						
constant	-0.0049	-0.0038	-0.0005	-0.0005	0.0075	-0.0006
se	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
IV	0.9952	1.0087	1.0321	1.0249	0.9063	0.9940
se	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
HV	0.0140	-0.0025	-0.0378	-0.0304	0.0682	0.0016
se	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
R2	0.8470	0.8898	0.8972	0.8856	0.8240	0.8845
N	2874	4813	5744	4090	3479	21000
Panel B. measurement error in prices						
Eq. (1)						
constant	0.1353	0.0270	0.0170	0.0146	0.0272	0.0331
se	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
IV	0.6827	0.9428	0.9688	0.9774	0.9469	0.9241
se	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R2	0.4666	0.8291	0.8852	0.8768	0.7935	0.8231
N	2440	4678	5744	4090	3384	20336
Eq. (2)						
constant	0.1019	0.0456	0.0429	0.0489	0.0854	0.0523
se	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
HV	0.7798	0.8904	0.8887	0.8794	0.8161	0.8749
se	(0.001)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)
R2	0.6566	0.7662	0.7835	0.7666	0.6758	0.7656
N	2440	4678	5744	4090	3384	20336
Eq.(3)						
constant	0.0583	0.0179	0.0164	0.0146	0.0243	0.0232
se	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
IV	0.2456	0.6821	0.9368	0.9743	0.8216	0.6487
se	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
HV	0.6190	0.2822	0.0334	0.0032	0.1311	0.2989
se	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
R2	0.6891	0.8427	0.8854	0.8768	0.7971	0.8393
N	2440	4678	5744	4090	3384	20336

This table shows the simulation results by OLS. This table shows information contents in an implied volatilitiy when underlying asset price follows the geometric Brownian motion process. We use a non-overlapping data. 3000 series of index and option prices that have 7 different strikes prices are used. The regressions are estimated by using the data classified by the moneyness. In panel A, we assume that there is no measurement error in stock index and option price. In Panel B, the measurement errors that follow independent normal distribution exist in the stock index and option price. The data that violate a no-arbitrage boundary are eliminated. Each panel shows the coefficients, the standard errors (se) in parenthesis, R-square coefficient (R2), and the number of observations (N).

Table 4. The Regression Results using the Overlapping Data by Simulation

moneyness	X/S<0.85	0.85<X/S <0.95	0.95<X/S <1.05	1.05<X/S <1.15	1.15<X/S	Total
Panel A. No measurement error						
Eq. (1)						
constant	-0.0052	0.0168	-0.0067	-0.0148	0.0764	0.0155
se	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
IV	1.0427	0.9587	1.0136	1.0302	0.8225	0.9661
se	(0.004)	(0.002)	(0.002)	(0.003)	(0.003)	(0.001)
R2	0.7258	0.7980	0.8369	0.7863	0.6723	0.7709
N	544	618	644	385	609	2800
Eq. (2)						
constant	0.0750	0.0775	0.0572	0.0571	0.1191	0.0728
se	(0.002)	(0.001)	(0.001)	(0.002)	(0.002)	(0.000)
HV	0.8308	0.7937	0.8561	0.8848	0.7718	0.8378
se	(0.005)	(0.003)	(0.002)	(0.004)	(0.005)	(0.001)
R2	0.6063	0.6591	0.7280	0.6689	0.5463	0.6512
N	544	618	644	385	609	2800
Eq.(3)						
constant	-0.0051	0.0130	-0.0096	-0.0147	0.0792	0.0156
se	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
IV	1.0373	1.2756	1.1639	1.0161	0.9218	1.0164
se	(0.012)	(0.006)	(0.006)	(0.011)	(0.008)	(0.003)
HV	0.0052	-0.3052	-0.1435	0.0143	-0.1122	-0.0513
se	(0.011)	(0.006)	(0.006)	(0.010)	(0.007)	(0.002)
R2	0.7258	0.8083	0.8390	0.7863	0.6740	0.7713
N	544	618	644	385	609	2800
Panel B. measurement error in prices						
Eq. (1)						
constant	0.3968	0.0901	0.0181	-0.0004	0.3850	0.2377
se	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
IV	0.0652	0.7450	0.9691	1.0120	0.1570	0.4083
se	(0.004)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)
R2	0.0128	0.5471	0.7741	0.7397	0.0931	0.2987
N	314	541	643	375	444	2317
Eq. (2)						
constant	0.1016	0.0895	0.0630	0.0709	0.1499	0.0825
se	(0.003)	(0.001)	(0.001)	(0.002)	(0.002)	(0.000)
HV	0.7589	0.7593	0.8485	0.8602	0.7283	0.8174
se	(0.006)	(0.003)	(0.003)	(0.005)	(0.005)	(0.001)
R2	0.5379	0.6039	0.6774	0.6073	0.4973	0.6083
N	314	541	643	375	444	2317
Eq.(3)						
constant	0.1142	0.0577	0.0181	-0.0002	0.1463	0.0720
se	(0.003)	(0.001)	(0.001)	(0.001)	(0.002)	(0.000)
IV	-0.0322	0.3307	0.9688	1.0324	0.0174	0.0954
se	(0.002)	(0.004)	(0.006)	(0.010)	(0.004)	(0.001)
HV	0.7720	0.5029	0.0003	-0.0210	0.7146	0.7377
se	(0.006)	(0.004)	(0.006)	(0.009)	(0.006)	(0.001)
R2	0.5408	0.6429	0.7741	0.7398	0.4983	0.6188
N	314	541	643	375	444	2317

This table shows the simulation result by OLS. This table shows information contents in an implied volatilitiy when underlying asset price follows the geometric Brownian motion process. We use an overlapping data just as table 2. 80 series of index and option prices which have 7 different strikes prices are used. 51th, 61th, 71th, 81th, and 91th data are mutually overlapped. The regressions are estimated by using the data classified by the moneyness. In panel A, we assume that there is no measurement error in stock index and option price. In Panel B, the measurement errors that follow independent normal distribution exist in the stock index and option price. The data that violate a no-arbitrage boundary are eliminated. Each panel shows the coefficients, the standard errors (se) in parenthesis, R-square coefficient (R2), and the number of observations (N). The standard errors of the slope coefficients are estimated following Hansen (1982) for considering the serial correlation caused by using the overlapping data.

Table 5. 2SLS Estimation with KOSPI 200 Option Data

moneyness	X/S<0.85	0.85<X/S <0.95	0.95<X/S <1.05	1.05<X/S <1.15	1.15<X/S	Total
Panel A. first stage regression						
constant	0.7046	0.2691	0.0446	0.0295	-0.0206	0.0661
se	(0.056)	(0.009)	(0.001)	(0.001)	(0.001)	(0.001)
past IV	0.2699	0.3553	0.6155	0.7300	0.8494	0.6341
se	(0.019)	(0.017)	(0.003)	(0.003)	(0.003)	(0.002)
HV	-0.5705	-0.0149	0.2650	0.1864	0.2071	0.2206
se	(0.147)	(0.024)	(0.003)	(0.003)	(0.003)	(0.003)
R2	0.3144	0.0886	0.5966	0.7156	0.8055	0.5406
N	16	75	139	121	141	492
Panel B. second stage regression						
Eq. (5)						
constant	0.4860	0.0925	0.1407	0.1624	0.0659	0.1619
se	(0.012)	(0.016)	(0.008)	(0.008)	(0.005)	(0.003)
IV_hat	-0.0955	0.6654	0.5963	0.5225	0.7648	0.5239
se	(0.017)	(0.040)	(0.020)	(0.020)	(0.011)	(0.008)
R2	0.0140	0.0441	0.0835	0.0731	0.2900	0.1162
N	16	75	139	121	141	492
Eq.(6)						
constant	-0.3897	0.0960	0.1426	0.1629	0.0302	0.1247
se	(0.005)	(0.016)	(0.008)	(0.008)	(0.005)	(0.003)
IV_hat	0.2412	0.7091	0.6753	0.6447	0.4034	0.3762
se	(0.004)	(0.044)	(0.031)	(0.033)	(0.017)	(0.009)
HV	1.8193	-0.0550	-0.0845	-0.1268	0.4841	0.2541
se	(0.010)	(0.022)	(0.024)	(0.027)	(0.017)	(0.009)
R2	0.7958	0.0451	0.0846	0.0757	0.3332	0.1335
N	16	75	139	121	141	492

This table shows the 2SLS estimation result. 2SLS method is used for reducing errors in the implied volatility. The forecasting power of the options with 21 days to expiration are shown. As instruments, the historical volatility and the implied volatility of the option with 24 days to expiration are used. For the first stage regression, implied volatility with 21 days to expiration is regressed on a constant and the instruments. And the estimated implied volatility from the first stage regression is used for second stage regression. The first stage regression results are presented in panel A, and the second stage results are presented in Panel B. The data that violate a no-arbitrage boundary are eliminated. Each panel shows the coefficients, the standard errors(se) in parenthesis, R-square coefficient (R2), and the number of observations (N).

Table 6. Forecasting Power of the Various Cross-Sectional Estimators

	historical volatility	mean	median	closest to ATM	Vega-weighted average	Hentschel's estimator 1	Hentschel's estimator 2
Eq. (1)							
constant	0.2817	0.3362	0.3170	0.2982	0.3225	0.2833	0.2964
se	(0.004)	(0.001)	(0.002)	(0.003)	(0.002)	(0.003)	(0.003)
forecast	0.2241	0.0652	0.1160	0.1763	0.1023	0.2107	0.1710
se	(0.011)	(0.002)	(0.003)	(0.007)	(0.004)	(0.006)	(0.006)
R2	0.0148	0.0053	0.0097	0.0141	0.0090	0.0185	0.0174
N	785	785	785	785	785	785	785
Eq.(3)							
constant		0.2701	0.2673	0.2670	0.2672	0.2640	0.2635
se		(0.004)	(0.004)	(0.005)	(0.005)	(0.004)	(0.005)
forecast		0.0411	0.0715	0.1110	0.0661	0.1524	0.1235
se		(0.002)	(0.003)	(0.006)	(0.003)	(0.006)	(0.005)
HV		0.2036	0.1810	0.1489	0.1865	0.1116	0.1380
se		(0.011)	(0.011)	(0.011)	(0.011)	(0.013)	(0.011)
R2		0.0167	0.0179	0.0187	0.0181	0.0207	0.0217
N		785	785	785	785	785	785

This table reports the regression estimation results using the cross-sectional estimators for the future volatility. For the cross-sectional estimators, we consider the mean, median, the implied volatility of the option whose moneyness is closest to 1, Vega-weighted average, Hentschel's estimator, and Hentschel's estimator considering a smile with quadratic specification. The data cover from October 1999 through March 2003, and cross-sectional estimators of every trading day are used for the regressions. The first column of the table 6 presents the result when the historical volatility over prior 50 trading days is used as a forecast of a future volatility. And the result from the second through the seventh column reports the forecasting power of the various cross-sectional forecast mentioned above. The standard errors are calculated by Hansen (1982) method. Each shows the coefficients, the standard errors(se) in parenthesis, R-square coefficient (R2), and the number of observations (N).

Table 7. Trading Profits with No Transaction Costs

moneyness		Pre-transaction costs trading profit					
		1999	2000	2001	2002	2003	total
panel A. Using ex-post realized volatility							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1673	304	3773
	total profit	246565	683037	259853	438592	51494	1679542
	total profit/total investment	-0.0328	0.0761	0.0176	0.0463	0.0100	0.0544
	mean of daily profit	1385.20	1110.63	259.08	262.16	169.39	445.15
	t-value	3.95	4.47	5.27	6.46	5.09	8.99
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	42764	626463	254937	194571	55621	1174356
	total profit/total investment	-0.0020	0.0267	0.0090	-0.0109	0.0550	0.0877
	mean of daily profit	245.77	862.90	453.62	258.74	283.78	487.28
	t-value	2.92	7.24	3.67	6.17	4.31	9.99
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	62351	3722679	774169	1043023	118042	5720264
	total profit/total investment	-0.0067	0.0206	0.0027	0.0152	0.0149	0.0106
	mean of daily profit	283.41	2069.30	914.01	752.54	227.88	1199.22
	t-value	3.49	7.44	1.42	2.65	6.26	6.80
total	number of observations	572	3140	2412	3811	1018	10953
	total profit	351681	5032180	1288959	1676186	225156	8574162
	total profit/total investment	-0.0092	0.0236	0.0039	0.0278	0.0160	0.0159
	mean of daily profit	614.83	1602.61	534.39	439.83	221.17	782.81
	t-value	5.21	9.47	2.33	4.18	9.01	9.85
panel B. Using Hentschel's estimator and Historical volatility with (eq.3)							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1674	304	3774
	total profit	243982	649146	243578	445432	48073	1630210
	total profit/total investment	-0.0242	-0.0338	0.1377	-0.0652	0.0393	-0.0492
	mean of daily profit	1370.68	1055.52	242.85	266.09	158.13	431.96
	t-value	3.93	4.46	5.01	6.61	6.16	9.05
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	33836	166696	4441	122818	12548	340339
	total profit/total investment	-0.0011	-0.0018	-0.0001	-0.0015	-0.0009	-0.0013
	mean of daily profit	194.46	229.61	7.90	163.32	64.02	141.22
	t-value	1.57	2.78	0.17	4.01	1.29	4.49
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	79563	237953	-34904	131851	23940	438402
	total profit/total investment	-0.0089	-0.0058	0.0022	-0.0041	-0.0040	-0.0042
	mean of daily profit	361.65	132.27	-41.21	95.13	46.22	91.91
	t-value	2.95	3.50	-0.91	3.04	1.84	4.64
total	number of observations	572	3140	2412	3812	1018	10954
	total profit	357380	1053794	213115	700101	84560	2408951
	total profit/total investment	-0.0072	-0.0069	-0.0042	-0.0057	-0.0044	-0.0232
	mean of daily profit	624.79	335.60	88.36	183.66	83.07	219.92
	t-value	4.96	6.11	3.16	8.15	4.68	11.07

moneyness		Pre-transaction costs trading profit					
		1999	2000	2001	2002	2003	total
panel C. Using the implied volatility of ATM option and Historical volatility with (eq.3)							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1674	304	3774
	total profit	243196	649060	243065	442882	47792	1625996
	total profit/total investment	-0.0270	-0.0409	0.0696	-0.1574	0.0223	-0.0737
	mean of daily profit	1366.27	1055.38	242.34	264.57	157.21	430.84
	t-value	3.92	4.46	5.02	6.59	6.17	9.03
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	31623	144036	11528	106566	10349	304102
	total profit/total investment	-0.0012	-0.0018	-0.0004	-0.0016	-0.0010	-0.0015
	mean of daily profit	181.74	198.40	20.51	141.71	52.80	126.18
	t-value	1.70	2.83	0.56	4.34	1.41	4.79
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	78615	160733	-23383	118160	14143	348267
	total profit/total investment	-0.0084	-0.0038	0.0015	-0.0035	-0.0024	-0.0033
	mean of daily profit	357.34	89.35	-27.61	85.25	27.30	73.01
	t-value	3.26	2.56	-0.71	3.33	1.48	4.16
total	number of observations	572	3140	2412	3812	1018	10954
	total profit	353434	953829	231210	667608	72285	2278366
	total profit/total investment	-0.0078	-0.0069	-0.0059	-0.0065	-0.0051	-0.0214
	mean of daily profit	617.89	303.77	95.86	175.13	71.01	207.99
	t-value	5.05	5.69	3.70	8.34	5.01	10.90
panel D. Using Hentschel's estimator							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1674	304	3774
	total profit	258179	669291	244359	439581	46584	1657993
	total profit/total investment	-0.1027	0.1531	0.0194	0.0272	0.0096	0.0467
	mean of daily profit	1450.44	1088.28	243.63	262.59	153.24	439.32
	t-value	3.97	4.48	5.07	6.53	5.27	8.99
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	18717	222687	26565	68789	16328	353086
	total profit/total investment	-0.0064	-0.0825	0.0031	0.0241	0.0061	0.0417
	mean of daily profit	107.57	306.73	47.27	91.47	83.31	146.51
	t-value	1.98	7.10	2.59	6.17	3.25	9.60
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	5382	124271	-2280	24030	3275	154679
	total profit/total investment	-0.0159	-0.1579	0.0018	-0.0051	-0.0460	-0.0217
	mean of daily profit	24.46	69.08	-2.69	17.34	6.32	32.43
	t-value	1.21	6.43	-0.25	2.47	1.57	6.44
total	number of observations	572	3140	2412	3812	1018	10954
	total profit	282278	1016248	268644	532400	66188	2165759
	total profit/total investment	-0.0488	1.1477	0.0135	0.0371	0.0088	-0.3041
	mean of daily profit	493.49	323.65	111.38	139.66	65.02	197.71
	t-value	4.18	6.54	5.33	7.68	6.26	11.37

This table shows trading profits with no transaction costs. The sample period is from Oct. 1999 to Mar. 2003. The options with 5~30 days to expiration and the KOSPI 200 index are traded and the Delta-neutral portfolio is composed. The results are classified by the moneyness and the traded year. In the panel A , the ex-post realized volatilities are used as a forecast. And in the panel B and the panel C, the forecasts by the "Hentschel's estimator + historical volatility" and "the implied volatility of the ATM option + historical volatility" with the (eq.3) are used respectively. In panel D, the Hentschel's estimator is used as a forecast.

Each reports the number of observations, the total profit, the total profit over the total investment, the mean of daily profit, and the t-value under null hypothesis "mean=0".

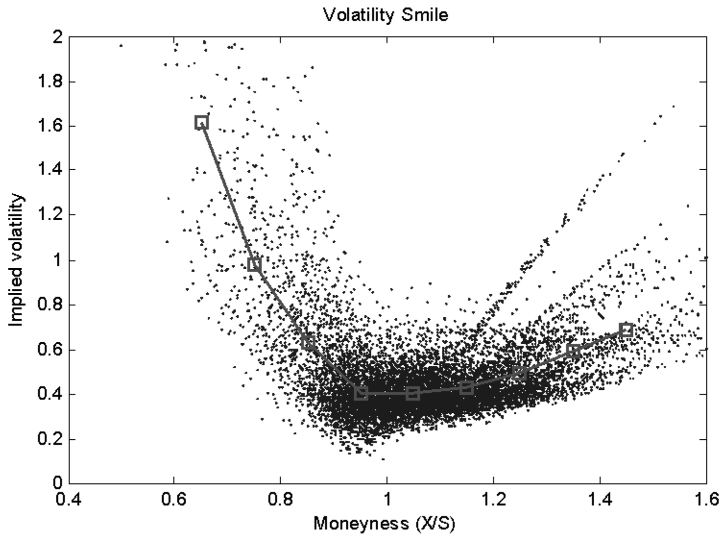
Table 8. Trading Profits after Deducting Transaction Costs

moneyness		Post-transaction costs trading profit					
		1999	2000	2001	2002	2003	total
panel A. Using ex-post realized volatility							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1674	304	3774
	total profit	210785	566430	170404	304015	26822	1278456
	total profit/total investment	-0.0281	0.0631	0.0115	0.0321	0.0052	0.0414
	mean of daily profit	1184.19	921.02	169.89	181.61	88.23	338.75
	t-value	3.44	3.83	3.56	4.62	2.78	7.07
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	27034	527639	182153	131990	41180	909996
	total profit/total investment	-0.0013	0.0225	0.0064	-0.0074	0.0407	0.0680
	mean of daily profit	155.37	726.78	324.12	175.52	210.10	377.59
	t-value	1.86	6.14	2.63	4.29	3.23	7.80
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	56349	3516470	472812	882091	101526	5029248
	total profit/total investment	-0.0060	0.0194	0.0016	0.0128	0.0128	0.0093
	mean of daily profit	256.13	1954.68	558.22	636.43	196.00	1054.35
	t-value	3.17	7.10	0.85	2.58	5.48	6.12
total	number of observations	572	3140	2412	3812	1018	10954
	total profit	294168	4610539	825369	1318096	169528	7217700
	total profit/total investment	-0.0077	0.0216	0.0025	0.0219	0.0120	0.0134
	mean of daily profit	514.28	1468.32	342.19	345.78	166.53	658.91
	t-value	4.45	8.78	1.47	3.77	6.92	8.50
panel B. Using Hentschel's estimator and Historical volatility with (eq.3)							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1674	304	3774
	total profit	207140	532091	159975	313882	23408	1236495
	total profit/total investment	-0.0205	-0.0277	0.0904	-0.0460	0.0191	-0.0373
	mean of daily profit	1163.71	865.19	159.50	187.50	77.00	327.64
	t-value	3.39	3.79	3.40	4.81	3.19	7.10
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	15976	86553	-31677	70472	-240	141084
	total profit/total investment	-0.0005	-0.0009	0.0009	-0.0008	0.0000	-0.0005
	mean of daily profit	91.82	119.22	-56.36	93.71	-1.23	58.54
	t-value	0.75	1.46	-1.23	2.31	-0.02	1.87
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	74766	210016	-47718	112718	17585	367366
	total profit/total investment	-0.0084	-0.0052	0.0031	-0.0035	-0.0030	-0.0035
	mean of daily profit	339.84	116.74	-56.34	81.33	33.95	77.02
	t-value	2.79	3.09	-1.24	2.60	1.35	3.89
total	number of observations	572	3140	2412	3812	1018	10954
	total profit	297882	828659	80580	497072	40752	1744945
	total profit/total investment	-0.0060	-0.0054	-0.0016	-0.0041	-0.0021	-0.0168
	mean of daily profit	520.77	263.90	33.41	130.40	40.03	159.30
	t-value	4.21	4.94	1.22	5.91	2.28	8.22

moneyness		Post-transaction costs trading profit					
		1999	2000	2001	2002	2003	total
panel C. Using the implied volatility of ATM option and Historical volatility with (eq.3)							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1674	304	3774
	total profit	207108	535294	161075	313637	23880	1240993
	total profit/total investment	-0.0230	-0.0337	0.0461	-0.1115	0.0111	-0.0563
	mean of daily profit	1163.53	870.40	160.59	187.36	78.55	328.83
	t-value	3.40	3.81	3.44	4.82	3.29	7.13
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	14959	71884	-20319	62797	-167	129154
	total profit/total investment	-0.0006	-0.0009	0.0007	-0.0010	0.0000	-0.0006
	mean of daily profit	85.97	99.01	-36.15	83.51	-0.85	53.59
	t-value	0.81	1.42	-0.97	2.57	-0.02	2.05
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	73476	131597	-36831	97930	7338	273510
	total profit/total investment	-0.0078	-0.0032	0.0024	-0.0029	-0.0012	-0.0026
	mean of daily profit	333.98	73.15	-43.48	70.66	14.17	57.34
	t-value	3.06	2.09	-1.11	2.76	0.76	3.27
total	number of observations	572	3140	2412	3812	1018	10954
	total profit	295542	738775	103925	474363	31051	1643657
	total profit/total investment	-0.0065	-0.0054	-0.0026	-0.0047	-0.0022	-0.0155
	mean of daily profit	516.68	235.28	43.09	124.44	30.50	150.05
	t-value	4.30	4.54	1.70	6.07	2.20	8.08
panel D. Using Hentschel's estimator							
X/S<0.95 (ITM options)	number of observations	178	615	1003	1674	304	3774
	total profit	221004	560220	164663	309198	23061	1278146
	total profit/total investment	-0.0879	0.1281	0.0131	0.0191	0.0047	0.0360
	mean of daily profit	1241.60	910.93	164.17	184.71	75.86	338.67
	t-value	3.47	3.88	3.53	4.74	2.77	7.16
0.95<X/S <1.05 (ATM options)	number of observations	174	726	562	752	196	2410
	total profit	4082	164822	909	28883	6537	205233
	total profit/total investment	-0.0014	-0.0610	0.0001	0.0101	0.0024	0.0242
	mean of daily profit	23.46	227.03	1.62	38.41	33.35	85.16
	t-value	0.45	5.42	0.09	2.66	1.34	5.75
X/S>1.05 (OTM options)	number of observations	220	1799	847	1386	518	4770
	total profit	-1341	83593	-18045	-3507	-5017	55683
	total profit/total investment	0.0040	-0.1062	0.0145	0.0007	0.0704	-0.0078
	mean of daily profit	-6.10	46.47	-21.30	-2.53	-9.69	11.67
	t-value	-0.30	4.40	-1.95	-0.36	-2.37	2.34
total	number of observations	572	3140	2412	3812	1018	10954
	total profit	223745	808635	147528	334574	24581	1539062
	total profit/total investment	-0.0386	0.9132	0.0074	0.0233	0.0033	-0.2161
	mean of daily profit	391.16	257.53	61.16	87.77	24.15	140.50
	t-value	3.40	5.39	3.02	4.99	2.47	8.36

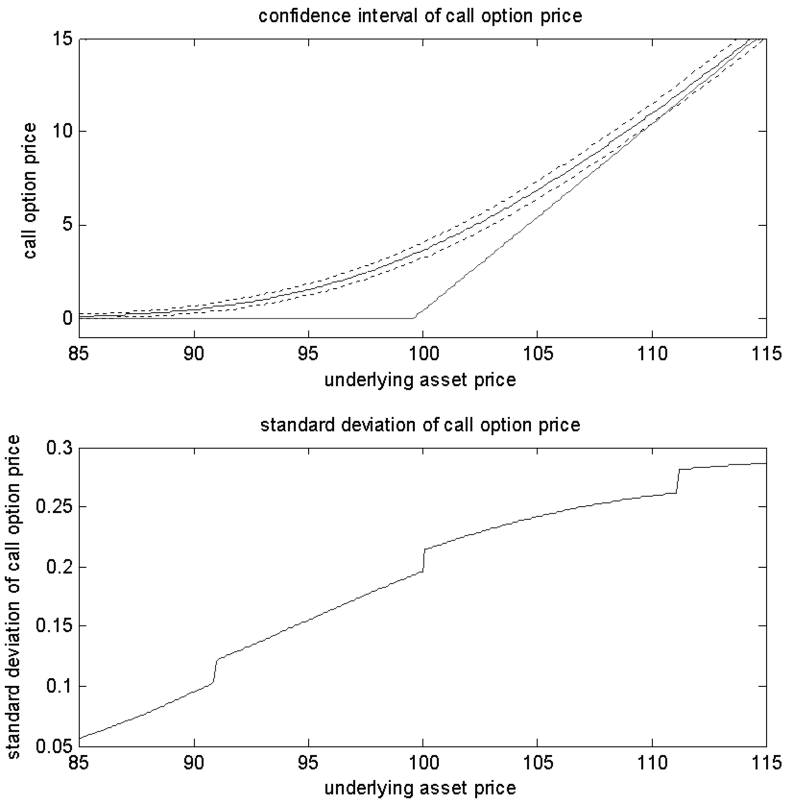
This table shows the trading profits after deducting the transaction costs. The sample period is from Oct. 1999 to Mar. 2003. The options with 5~30 days to expiration and the KOSPI 200 index are traded and the Delta-neutral portfolio is composed. The results are classified by the moneyness and the traded year. Transaction costs are assumed as follows. 0.005 (C<3), 0.025 (3<C<5), 0.05 (5<C<10), and 0.13 (10<C) for a contract of one call option and 0.1 for index. In the panel A , the ex-post realized volatilities are used as a forecast. And in the panel B and the panel C, the forecasts by the "Hentschel's estimator + historical volatility" and "the implied volatility of the ATM option + historical volatility" with the (eq.3) are used respectively. In panel D, the Hentschel's estimator is used as a forecast. Each reports the number of observations, the total profit, the total profit over the total investment, the mean of daily profit, and the t-value under null hypothesis "mean=0".

Figure 1. Volatility smile of KOSPI 200 index option



This figure shows the relation between the moneyness and Black-Scholes implied volatility. 10371 pairs of moneyness and implied volatility are dotted and the mean of the implied volatilities classified by moneyness is marked with square.

Figure 2. The confidence interval of call option price with errors

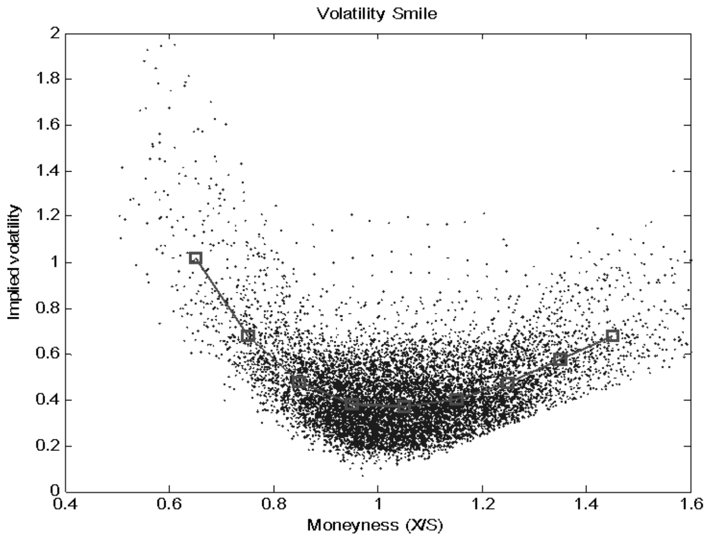


This figure shows the Black-Scholes option prices and their 95% confidential interval when the measurement errors in stock index and option price follow normal distributions and are independent. In the upper figure, dotted lines indicate the upper and lower confidential interval and lower line is a no-arbitrage boundary. In the lower figure, standard deviations of call option price are represented. The standard deviation is calculated by following equation.

$$Var(C) = Var(e_C) + \left(\frac{\partial C}{\partial S}\right)' Var(e_S)\left(\frac{\partial C}{\partial S}\right)$$

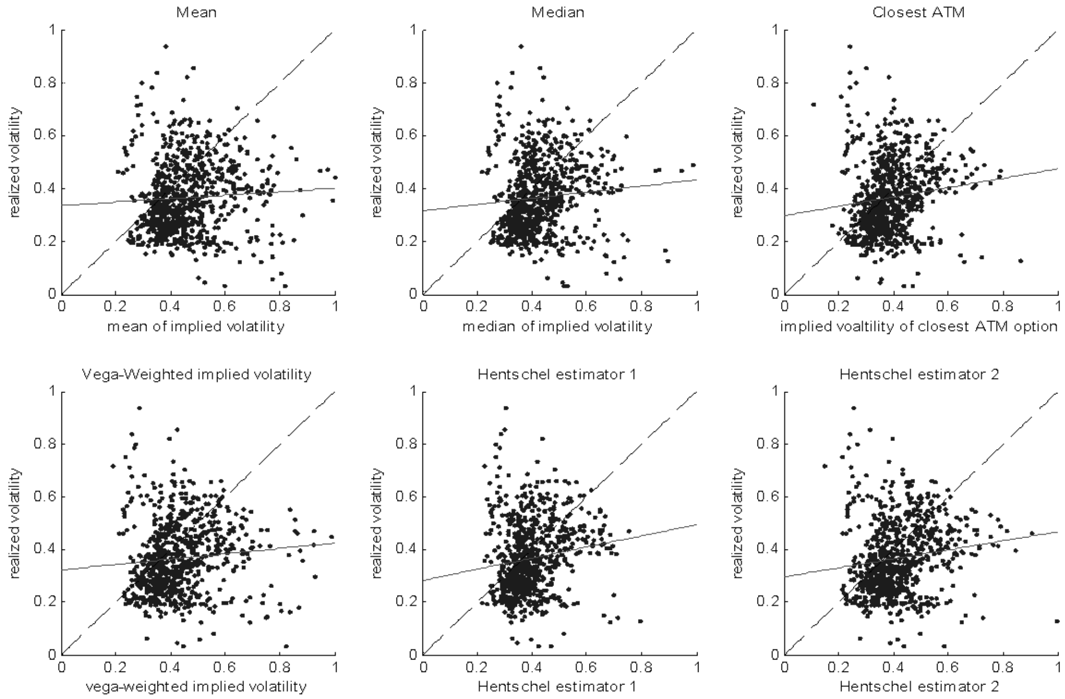
We assume that the strike price of the option is 100, its volatility is 0.3, its time to expiration is 1 month, and riskless interest rate is 0.05.

Figure 3. Volatility smile of the simulation



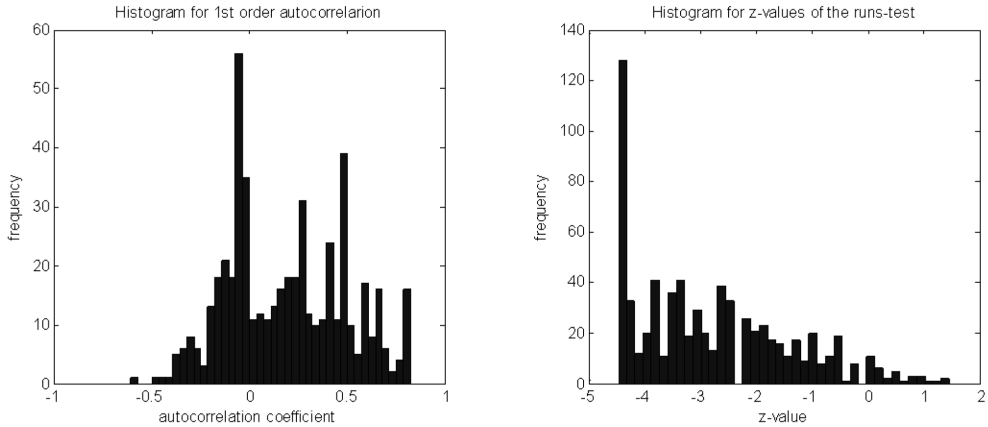
This figure shows the relation between the moneyness and the Black-Scholes implied volatility made by simulation. The theoretical option prices are computed by the Black-Scholes formula with real data and ex-post realized volatilities. Errors are added to the option prices and implied volatilities are obtained from the option prices with errors. Implied volatilities are dotted and the mean of the implied volatilities classified by moneyness is marked with square.

Figure 4. The forecast of a realized volatility using various cross-sectional estimators



These figures show the relation between the cross-sectional estimators and the ex-post realized volatility. For the cross-sectional estimators, we consider the mean, median, the implied volatility of the option whose moneyness is closest to 1, Vega-weighted average, Hentschel's estimator, and Hentschel's estimator considering a smile with quadratic specification. The data cover from October 1999 through March 2003, and cross-sectional estimators of every trading day are dotted. The pairs of the cross-sectional estimators mentioned above and the ex-post realized volatilities are dotted. And the regression line is drawn by a solid line. The dashed line shows the points where the value of estimator equals to the ex-post realized volatility.

Figure 5. The serial-correlation property of the errors in option prices



These figures show the serial-correlation property of the time-series of indicators which is 1 for an overvalued or -1 for an undervalued. The overvalued is the option whose implied volatility is greater than the ex-post realized volatility and the undervalued is the option whose implied volatility is less than the ex-post realized volatility. The left figure shows the histogram for the first-order autocorrelation of the series and the right figure shows the histogram for the z-statistics obtained by run-test.