The Days-of-the-Week Effect and LME Metal Market
~Nonlinear and Random Shuffling Approach~

Takaya MIYANO
Department of Micro System Technology,
College of Science and Engineering, Ritsumeikan University,
Noji-higashi 1-1-1, Kusatsu, Shiga 525-8577, Japan
E-mail : tmiyano@se.ritsumei.ac.jp

Ken-ichi TATSUMI
Faculty of Economics, Gakushuin University,
Mejiro 1-5-1, Toshima-ku, Tokyo 171-8588, Japan
E-mail : Kenichi.Tatsumi@gakushuin.ac.jp

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Abstract

This paper applies nonlinear nonparametric time series analytic tool by Wayland *et al.* (1993) and also proposes a test by random shuffling to detect existence of periodic pattern in a time series data and analyzes the days-of-the-week effect on London Metal Exchange listed non-ferrous metal returns. Although the nonlinear time series techniques are an improved and simpler measure of chaotic complexity, the proposed technique makes it possible to carry out hypotheses testing which has not been executed. The empirical analysis investigated the interpolated daily spot & futures price indexes of LME aluminum and copper since 1989. The results indicate that there is an evidence of the days-of-the-week effect and also that speculative behavior rather than hedging has been eminent since then.

Keywords: anomaly, high frequency data, hypothesis testing, interpolation, rank, random shuffle

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1. Introduction

Modeling and analyzing high-frequency data has become important in finance. Financial time series data exhibits significant nonlinearity, with this nonlinearity predominantly associated with a weekly pattern and also a seasonal pattern.

Since chaos study made clear that nonstochastic factors cause seemingly stochastic dynamic behavior, various methods of nonlinear time series analysis such as Wayland-Bromley-Pickett-Passamante (1993) and Bandt-Pompe (2002) have been presented. The nonlinear time series analysis begins with embeddings, which could naturally be applicable to periodicity analysis.

Another key element is noise. Most methods break down as soon as noise is added to the time series. For these respects, both the nonlinear time series analysis proposed by Wayland *et al.* (1993) and permutation entropy method proposed by Bandt-Pompe (2002) are promising for economic and financial time series data.

Abnormal stock returns, for example, have been globally documented on specific days of the week and in specific months (see Tong (2000)) and called as anomalies. These phenomena require both appropriate treatment of data and appropriate tools of analysis because they are nonlinear and noisy. Our method of nonlinear time series analysis is applied to the days-of-the-week effect on London Metal Exchange listed non-ferrous metal returns. Do metal prices make peculiar fluctuations on Monday, Friday or other days?

This paper applies nonlinear nonparametric time series analytic tool by Wayland *et al.* (1993) and also proposes a test by random shuffling of weekly rank to detect existence of periodic pattern in a time series data. Although the nonlinear time series techniques are an improved and simpler measure of chaotic complexity, the proposed technique makes it possible to carry out hypotheses testing which has not been executed. The empirical analysis investigated the interpolated daily spot & futures price indexes of LME aluminum and copper since 1989.

We will document that there is an evidence of the days-of-the-week effect and also that speculative behavior rather than hedging has been eminent since then.

2. Preceding and Related Researches

There are not many related researches, rigorously speaking, but notionally similar fields and their relationship with ours have to be briefly noted.

2-1. Metal Study

(1) Seasonality of Metal Return

It is well known that metal storage costs are low relative to value. It is also known that the metals are not subject to seasonals in supply or demand. Accordingly the metal futures prices showed less seasonality. This is a regression analytic conjecture of Fama-French (1997), studying monthly data of Comex and New York Mercantile Exchange (NYMEX) for January 1967-May 1984, by comparison with such commodity futures as meat and agriculture, not rigorous verification.

What they did is to detect the monthly effect, not the weekly effect. They documented there was not the monthly effect [1]. We will verify the phenomenon of the weekly effect with statistically more satisfying tools.

(2) Volatility of Metal Return

Several authors have examined the impact of the pricing regime on price variability with reference to the non-ferrous metals industry. Although theoretical arguments are ambiguous, but they suggest that the extent of monopoly power is more important than the pricing regime as a determinant of variability.

In copper market producers with market power had undertook price smoothing. Since the producer pricing system came to an end by the late 1970s, purchasers of copper had incentive to engage in hedging activities through futures contracts. It is also a common knowledge in the aluminum market that the price has behaved differently due to the development of derivatives.

Slade (1991) , with LME listed non-ferrous metals' monthly data from 1970 to 1986, documented that metal-price volatility in the 1980s relative to the 1970s is explained by increased reliance on commodity exchanges, not by declines in the market-structure and concentration variables. This was associated with a move from administered producer pricing to exchange pricing. However Figuerola-Ferretti-Gilbert (2001) extended Slade's sample to the recent years and showed that any early differences between the variability of producer and exchange prices have vanished.

2-2. Methodological Argument and Stock Return Anomaly Study

(1) Methodology

On stock market anomaly study, dummy variables for the days-of-the-week or the months have been employed extensively in a linear OLS regression analysis. There is, however, a problem of multicolinearity among dummy variables in this approach. Chien-Lee-Wang (2002) noted, however, the impact of stock price volatility throughout the week or the year on the application of dummy variable regression model and showed that it yields misleading results.

As for the analytical tool of comovement, the Copula analysis or Kendall’s tau has been well known these days. Although Copula analysis or Kendall’s tau has been utilized as a nonlinear devise, it carries out pair wise matching of two variables. This paper, however, analyzes the degree of coincidence in long run periodic movement of only one single variable.

(2) The Days of the Week Effect on Stock Return

Nonlinear time series analyses explained below have been applied to Japanese financial time series data. After a framework of analysis is well designed, Miyano-Tatsumi (2004) applied the Wayland test and others to the daily stock price index data of Nikkei 225 and Nikkei JASDAQ Average from January 4, 1989 to August 29, 2003 to detect the days of the week effect on the stock index returns and documented the existence of Monday and Friday effect.

3. Data

3-1. Data and Processing

(1) Data Analyzed

Since we will apply the method of the nonlinear time series analysis to London Metal Exchange listed non-ferrous metal returns, daily spot (ca, or cash) & 3 month futures (3m, or 3 month) price indexes of LME aluminum (Al) and copper (Cu) from January 4, 1989 to August 29, 2003 are chosen. This is only for liquidity reason. The trading volume of cash or 3 month futures compared to other long term futures is so large enough in LME that there is no need to correct thin trading. The same thing can be said to the aluminum or the copper compared to rare metals.

The daily price index is based on the settlement price. There is not any price limit in LME and therefore price jump has not often been observed.

The expiry date of the 3 month futures contract in LME is daily expired (the contract is settled daily on every business day) and there is no expiration effects observed often in the same commodities of longer expiration date and also other listed products.

(2) Interpolation

Monday return without Saturday and Sunday interpolation is the rate of change from Friday settlement price through Monday settlement price. Although this return calculates the rate of 3 days price change, the returns on the other days of the week calculate exactly 1 day change. If we combine these data into a series, data with different time intervals are mixed. Time series analytic tools require equidistant on the other hand. This is the reason why an interpolation method will be used extensively in the following.

The sample does not exist naturally on holidays and weekends. Also the data of the Bank holiday and the first day in January are not measured, since these days are the holidays. The nonexistent or missing data of metal prices are linearly interpolated in the following study. Monday return with the interpolation is therefore the rate of change from the estimated Sunday settlement price through Monday settlement price. Filling in the missing values with the estimates, which comprise 5,352 observations, these are then calculated to yield daily returns and then the weekday returns are considered.

The method of the interpolation replaces the missing values by the values interpolated by two days just before and just after when there exist data. If there are n consecutive data missing, the coefficient of interpolation for the i-th value will be ((n-i+1)/ (n+1), i/ (n+1)). Suppose there are no data on six consecutive days. Then the coefficients of interpolation will be (6/7, 1/7), (5/7, 2/7), (4/7, 3/7), (3/7, 4/7), (2/7, 5/7), and (1/7, 6/7).

(3) Return Calculation

The return is the rate of change from the last day's settlement price to the today's settlement price. Annualized percentage daily returns are calculated as 36000 times of them. Fundamental statistics of daily returns are calculated in **Table 1**.

Aluminum return has lower standard deviation than that of copper. There is no evidence of excess kurtosis in both series, although the distributions are more kurtosis than the normal distribution. Skewness of metal returns is more eminent than the normal distribution. Furthermore the interpolated daily returns of metals are distributed more closely to the normal distribution than those of stock returns (see Miyano-Tatsumi (2004)).

Table 1. Statistics of Daily Return

	Aluminum				Copper			
	Average	SD	Skew	Kurtosis	Average	SD	Skew	Kurtosis
Cash	-2.3885	350.4704	0.3225	7.7314	-2.2272	432.9417	0.5571	14.141
3m	-2.5728	306.8892	0.3692	7.8034	-2.1857	362.5833	0.0840	7.2275

3-2. Interaction Effect

For the interpolated daily returns, correlation coefficients are calculated in **Table 2**. The correlation coefficients of about 0.5 between the aluminum return and the copper return has to be said high, but not extremely high.

Table 2. Correlation Coefficients of Daily Returns

		Aluminum		Copper	
		Cash	3m	Cash	3m
Aluminum	Cash	1	0.9656	0.4879	0.5016
	3m		1	0.4999	0.5216
Copper	Cash			1	0.9336
	3m				1

(1) Autocorrelation Coefficients

Other correlation coefficients are calculated also, but not shown in tables.

Autocorrelation coefficients of the aluminum return do not deteriorate, but there is evidence of autocorrelation in the copper return series.

We would expect that some commodity-specific property might affect both spot and futures price behavior. The aluminum futures trading with higher liquidity might yield lower standard deviation of its return in comparison with that of copper.

(2) Correlation Coefficients between Commodities

Correlation coefficients of the aluminum return and the copper returns in different time period does not deteriorate. We would expect that some arbitrage among commodities might affect both spot and futures price behaviors.

4. Nonlinear and Nonparametric Analysis

4-1. Wayland algorithm-the Degree of Visible Determinism

The nonlinear time series analysis by Wayland *et al.* (1993), based on the parallelness of neighboring trajectories in phase space, is an improved and simpler variant of the Kaplan-Glass algorithm (1993).

(1) Embedding and Time Translation

Given a time series { u (t)}, D-dimensional phase space is constructed at t_0 by embedding, as $\mathbf{u}(t_0) = \{u(t_0), u(t_0 - \triangle t), u(t_0 - 2\triangle t), \cdots, u(t_0 - (D-1)\triangle t)\}$, where D is the embedding dimension and $\triangle t$ is an appropriate time lag.

The central point of the Wayland algorithm is as follows. K nearest neighbors of $\mathbf{u}(t_0)$, denoted as $\mathbf{u}(t_i)$, $i = 0,1,2,\cdots,K$, are found then. The vector $\mathbf{u}(t_i + T\triangle t)$ is called the image of $\mathbf{u}(t_i)$ because each $\mathbf{u}(t_i)$ becomes $\mathbf{u}(t_i + T\triangle t)$ as a time of $T\triangle t$ passes.

The image is generated by time translation. Therefore the change in time series process as times go can be described approximately by translation vector $\boldsymbol{v}(t_i) = \mathbf{u}(t_i + T\triangle t) - \mathbf{u}(t_i)$.

(2) Translation Error and Properties of Wayland test

The K translation vectors should point in similar directions if determinism is visible, i.e., the time series process is deterministic. The similarity in direction is gauged in terms of a measure referred to as translation error E_{trans}.

$$E_{trans} = \frac{1}{K+1} \sum_{i=0}^{K} \frac{\| \boldsymbol{v}(t_i) - \hat{\boldsymbol{v}} \|}{\| \hat{\boldsymbol{v}} \|},$$

where

$$\hat{\boldsymbol{v}} = \frac{1}{K+1} \sum_{i=0}^{K} \boldsymbol{v}(t_i).$$

The translation error measures the diversity of directions of nearby trajectories, therefore the degree of visible determinism of the time series data. The more visible the determinism is, the smaller E_{trans} will be.

In Wayland test the E_{trans} estimator is dependent on the embedding dimension D. If $E_{trans} \to 0$, the original time series process is considered to be deterministic. If the original time series process is white-noise, then the translation vector $\boldsymbol{v}(t_i)$ becomes uniformly distributed and the E_{trans} estimate will be close to 1.

If D is less than the intrinsic dimension of the original time series process, the E_{trans} estimate is higher. Even if D is larger than the intrinsic dimension, the E_{trans} estimate may be higher because of the redundancy of the embedding space. The detail is not well known for the intermediate range of D (Miyano (1996)).

4-2. Presentation of Results

We will try 1-week translation for weekly returns, 7-day and 90-day translation for daily returns. **Figures 1 to 3** are for the aluminum and **Figures 4 to 6** for the copper.

Figure1 Wayland test of aluminum cash return (1 week translation)

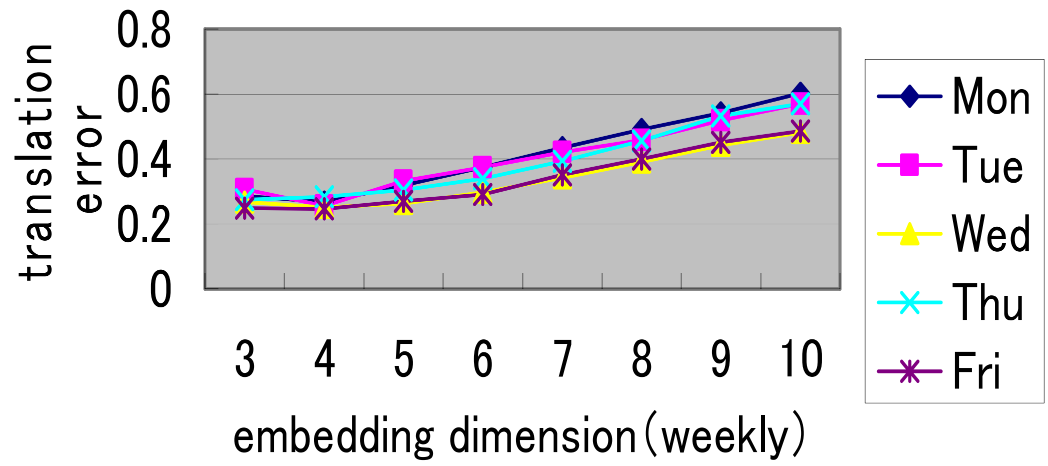
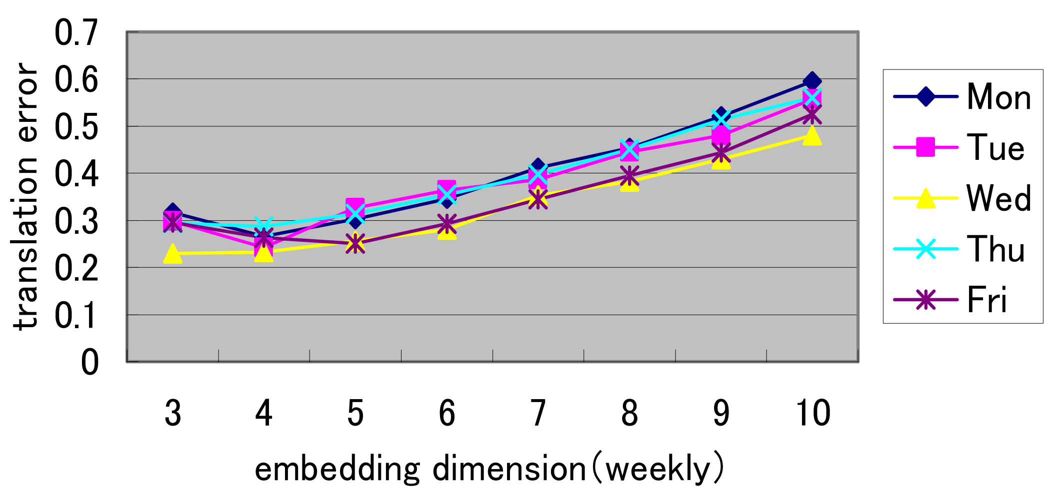


Figure2 Wayland test of aluminum futures return (1 week translation)



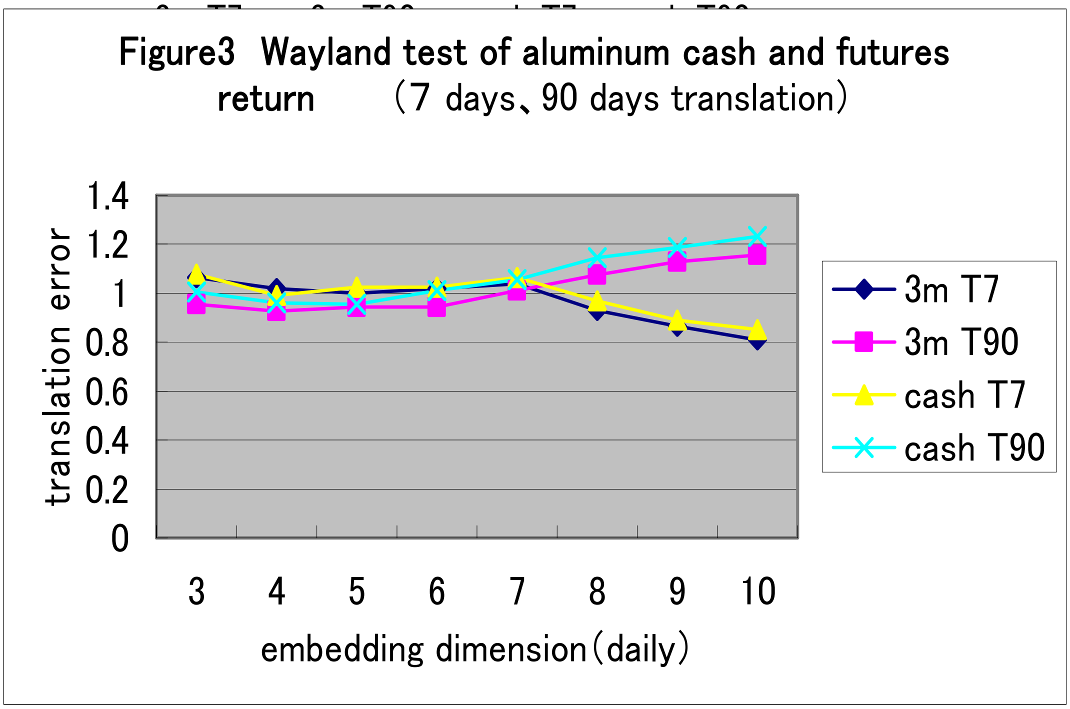


Figure3 Wayland test of aluminum cash and futures return (7 days、90 days translation)
translation error
1.4
1.2
1
0.8
0.6
0.4
0.2
0
3
4
5
6
7
8
9
10
embedding dimension(daily)
3m T7
3m T90
cash T7
cash T90

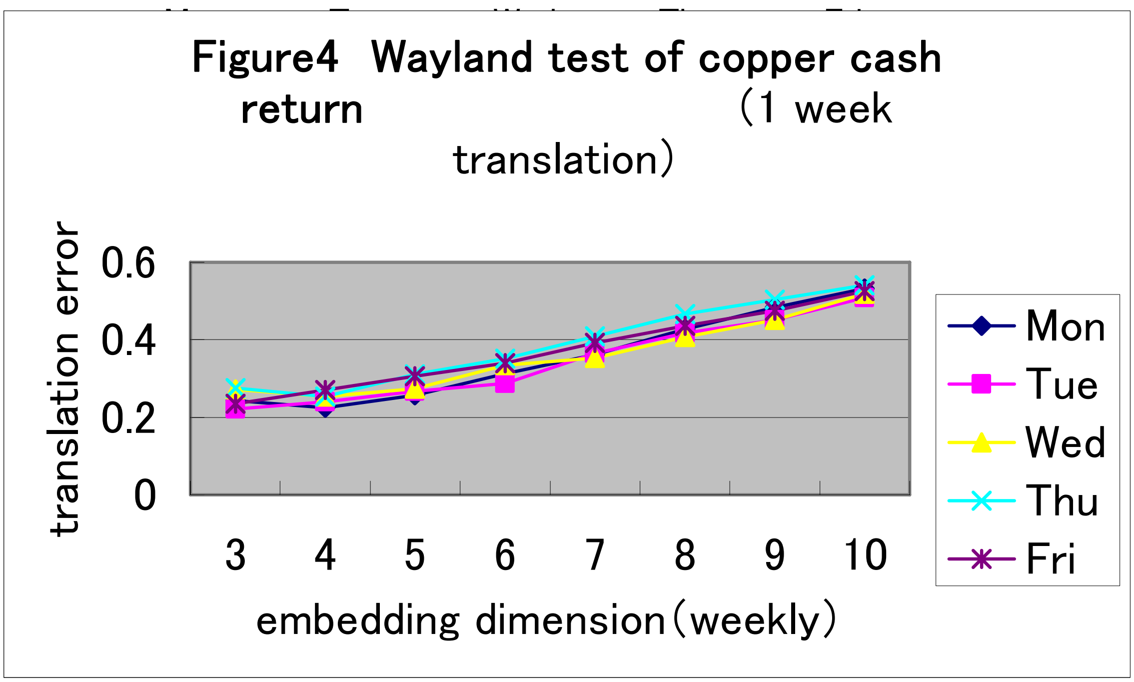


Figure4 Wayland test of copper cash return (1 week translation)
translation error
0.6
0.4
0.2
0
3
4
5
6
7
8
9
10
embedding dimension(weekly)
Mon
Tue
Wed
Thu
Fri

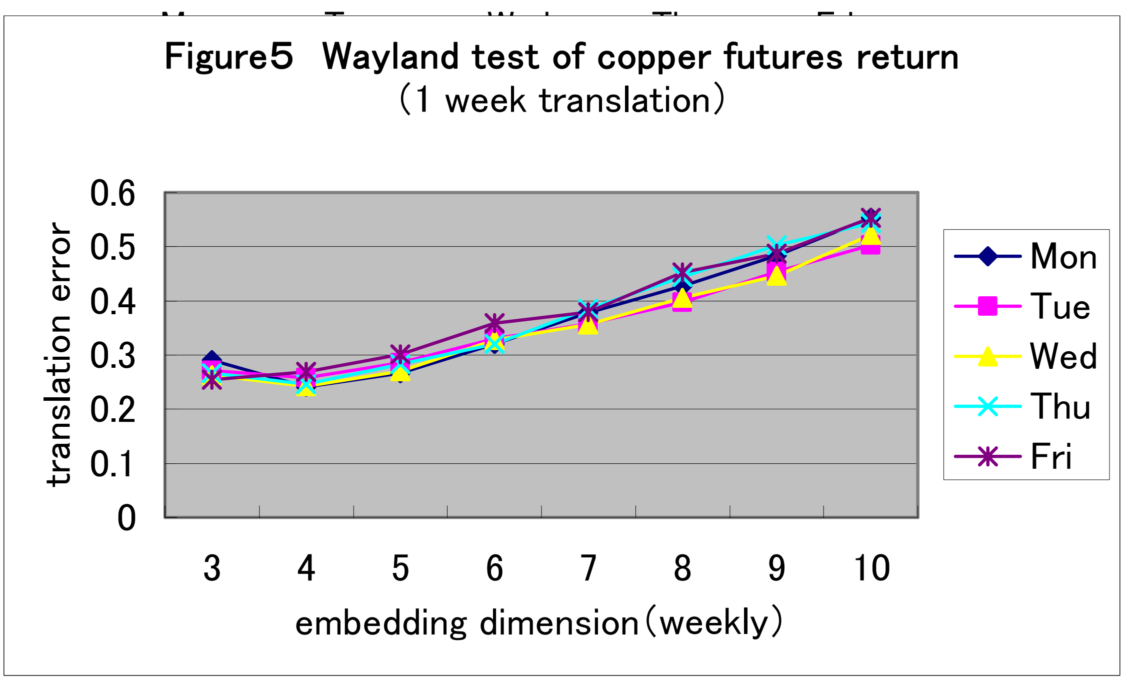


Figure5 Wayland test of copper futures return
(1 week translation)
translation error
0.6
0.5
0.4
0.3
0.2
0.1
0
3
4
5
6
7
8
9
10
embedding dimension(weekly)
Mon
Tue
Wed
Thu
Fri

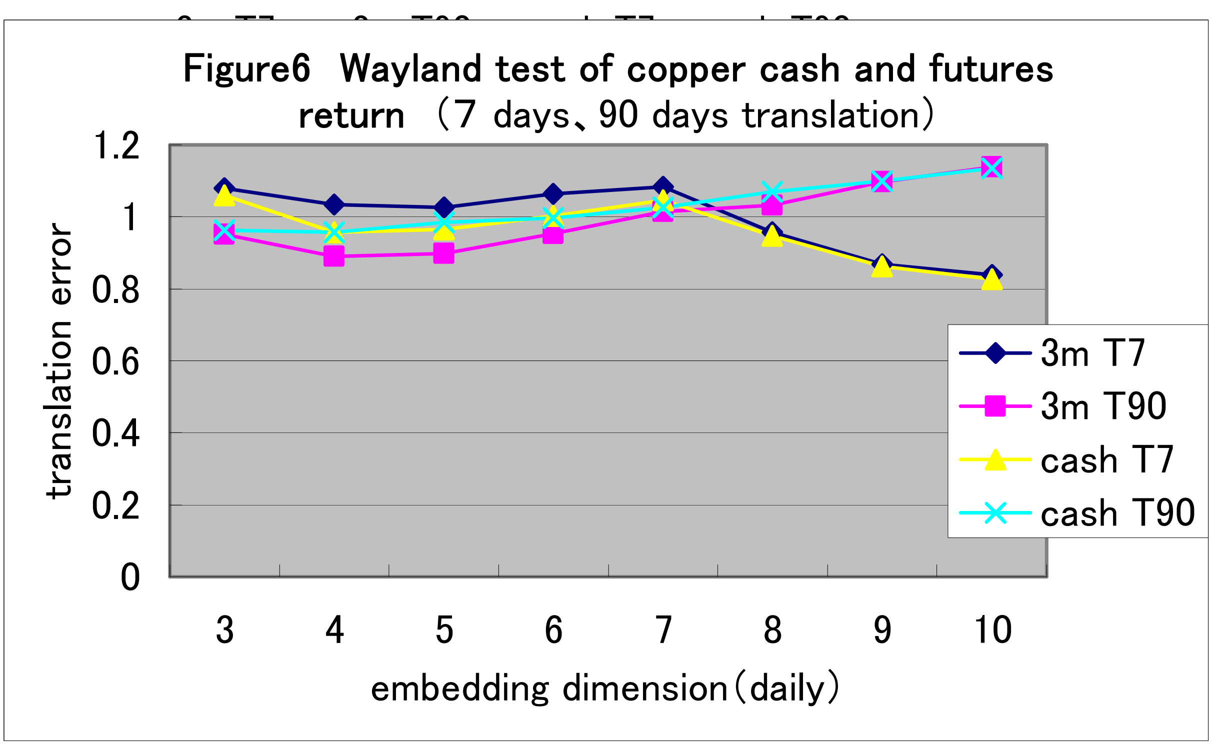


Figure6 Wayland test of copper cash and futures return (7 days、90 days translation)
translation error
1.2
1
0.8
0.6
0.4
0.2
0
3
4
5
6
7
8
9
10
embedding dimension(daily)
3m T7
3m T90
cash T7
cash T90

From **Figures 1-6** we do not see any difference between the aluminum and the copper. The same thing can be said between cash and futures, and also among the days of the week.

In 7-day and 90-day translation for daily cash and futures returns the translation error is one and therefore it turns out to be random regardless of the embedding dimensions (see **Figure 3** and **Figure 6**). It has to be noted that in the case of 7-day translation one week ahead is the same day of the week.

(2) Metal Return Dynamics

By Wayland test we could know several dynamic behaviors of the metal returns.

First of all, since the translation error of one week ahead is relatively small for 3 to 5 week patterns of weekly returns (see **Figures 1, 2** and **Figures 4, 5**), determinism is visible for the patterns.

Secondly, since the translation error is minimized at the embedding dimension of 4 weeks (see **Figures 1, 2** and **Figures 4, 5**), there is a property of 4-week-periodicity for weekly returns. This suggests monthly periodicity.

Thirdly since the translation errors of 7- or 90-day ahead are very close to unity for 3 to 10 day patterns of daily returns (see **Figure 3** and **Figure 6**), they are uncorrelated random noises.

(3) Drawbacks of Wayland test

There are several drawbacks in the Wayland algorithm. First of all, there is no clear threshold of E_{trans} by which the underling dynamics is classified into either a deterministic process or a stochastic process.

Secondly, we have no definite criterion instead of trial-and-error to determine the appropriate value of the time translation T. We introduce financial economics rationale in here.

Thirdly it is difficult, though not impossible, to estimate the reliable interval for estimates of E_{trans}, which prohibits carrying out hypothesis testing. How can we judge when the E_{trans} fluctuates drastically? Wayland test cannot generally give simple and clear conclusions.

We next propose a much simpler procedure in the following.

5. Periodicity Analysis by Rank

5-1. Analytical Framework

(1) Setting

Let a time series $\{u(t)\}$, $t=1,2,\cdots$, N, be given, consisting of N consecutive data points of variable u observed equidistant in time. Suppose we would like to detect whether m consecutive samples in the time series have any periodicity. For examples, m is 5 for weekly pattern of daily data and 12 for yearly pattern of monthly data. The latter is exactly the seasonality problem.

For simplicity of exposition without loss of generality, let N be μ times of m. The whole sample is then divided to μ groups by m consecutive samples. In terms of vectors, $\{u(t)\}$ = $\{(u(1), u(2), \cdots, u(m)), (u(m+1), u(m+2), \cdots, u(2m)), (u(2m+1), u(2m+2), \cdots, u(3m)), \cdots\cdots, (u((\mu-1)m+1), u((\mu-1)m+2), \cdots, u(\mu m))\}$ = $\{\mathbf{u}(m), \mathbf{u}(2m), \mathbf{u}(3m), \cdots\cdots, \mathbf{u}(\mu m)\}$.

We then compose vectors by rank, ranking among the m values.

$\mathbf{y}(im) = (x((i-1)m+1), x((i-1)m+2), \cdots, x(im)), i=1,2,\cdots, \mu,$

where x is the positive integer up to m. The number of combination of the rank becomes m! =M [(2]. We will call these vectors as the original data and consider the first to the m-th columns in the original vectors separately.

(2) Hypothesis Testing in General

It may be likely that we would like to know whether the rank of a specific column is on average higher than that of other columns. Here the average of the specific column is taken over the μ values. More specifically it is interesting to know whether the rank of a specific column has a tendency to be higher than the overall average.

The procedure to test the hypothesis follows. The ranks of the N columns in the original vectors are randomly shuffled 40 times in order to know how often the ranking would appear.

For each column, 40 ranks are used to calculate its average and standard deviation. The derived distribution of the ranks can be used to test a hypothesis whether the realized original rank is significantly larger or smaller than the overall average rank.

If we assume Gaussian process for x, this test statistics might become that of the familiar student's t-test. Tatsumi-Miyano (2004) has rejected a null hypothesis that Monday stock index returns are smaller than their averages in Japan.

5-2. An Application to Daily Metal Return Anomaly

(1) Random Shuffling Analysis by Rank

Let the metal returns on Monday through Friday be R_1, R_2, R_3, R_4, and R_5, and then calculate ranking among them. The highest return gets the number 1 and the lowest is 5. A weekly rank vector will be denoted as $y(5) = (x_1, x_2, x_3, x_4, x_5)$. There will be 5!=120 rank vectors.

The reason why we shuffle data is twofold. It is because they might be noisy, which is also the main reason to consider the rank instead of the absolute value. Second is to know the random process of the rank, since the random shuffling generates the random process.

(2) Method of Hypothesis Testing

The procedure of the hypothesis testing is as follows. We shuffle randomly the daily metal prices within week, that is, from Monday through Friday within the same week, 40 times.

They are called as 40 surrogate data, getting 41 datasets including the original data.

For data with the interpolation, we then count their ranking within week. The highest return gets the number 1 and the lowest is 5.

For each day of the week we calculate both average of the original return ranking and the surrogate return ranking. For each weekday we then calculate average and standard deviation of the 40 average surrogate return ranking.

The difference between the average of the original return ranking and the average of the 40 average surrogate return ranking divided by the standard deviation of the 40 average surrogate return ranking for each day of the week would be considered the student's t distributed.

This t statistics has the meaning under a null hypothesis that the metal return for each day of the week is random and mutually independent [(3]. The null hypothesis should be rejected if the t statistics satisfies the condition $|t| > 2.02$, because the degree of freedom is 40. We will call this null hypothesis as random process hypothesis.

(3) Presentation of Results

The hypothesis tests executed are presented in **Table 3**. Some null hypotheses are not rejected since the t statistics are lower.

Table 3. Student's t-Statistical Test using Surrogate Returns for the Case of Random Shuffle of Daily Metal Returns within Week

	Aluminum		Copper	
	cash	3 month futures	cash	3 month futures
Monday	2.5450*	1.9605	1.5755	1.5166
Tuesday	2.8008*	2.2393*	3.1318*	2.2175*
Wednesday	-0.8435	-0.4308	-0.6174	-0.6736
Thursday	-5.1103*	-4.2514*	-4.2610*	-3.3023*
Friday	-0.4987	-0.5560	-1.0240	-0.7576

Note: * indicates significance at the 95% confidence level for the both-sided test.
number of surrogates = 40.

From **Table 3** we find that Monday and Friday are not, generally speaking, significant. Returns would be random on Wednesday and Friday for both metals. At least it might be concluded that Tuesday and Thursday returns are not random since Tuesday and Thursday returns for both cash and 3 month futures are significant. Positive return on Tuesday means lower ranking than the average, whereas negative return on Friday higher ranking.

5-3. The Days of the Week Effect ~ A Summary of Findings

(1) The random process hypothesis

Since the random process hypothesis is rejected for Tuesday and Thursday, the cash and futures returns of both copper and aluminum can be said to have the days-of-the-week effect. It is important to note that both cash and futures returns have shown non-random behavior in Tuesday and Thursday.

Because of Bank holidays in UK, Monday effect which is very familiar in stock market all over the world might move to Tuesday in the case of LME metal returns. However financial economic reasoning on Friday and Thursday is required, which will become our future work.

(2) Wayland test

The translation errors of the aluminum cash and futures returns on Tuesday and Thursday might be higher at first sight. Since we do not have tools to measure whether the difference among the days of the week is significant, this is just a conjecture. We should say that they might be so or might not be so.

It seems to be sure that the days-of-the-week return on Japanese stock indexes as shown in Miyano-Tatsumi (2004) move more divergently than those on LME metals. We should not say that this is because LME is a global market.

(3) The Effect of Interpolation

Whether or not we have to interpolate and furthermore how we could interpolate the Saturday and Sunday prices may affect Monday return and therefore weekly return ranking, leading naturally to a drastic change in the result [(4].

In the above experiment shown in **Table 3** Monday aluminum return is significant. This might have something to go with the interpolation, which in turn is due to Bank holidays in UK.

6. Discussions and Concluding Remarks

6-1. Hedging or Speculation

We have found that by rejecting the random process hypothesis the metal returns have not behaved randomly for both spot and futures in Tuesday and Thursday. Hedging function of futures contract has to be recalled with this phenomenon.

The result of hedging by 3 month futures is made clear in 3 months after future spot price is determined. The term of 3 month futures contract is approximately 90 days. 90 are not multiples of 7 (one week). Therefore the settlement date of the futures contract might not be the same day of the week as the day of the week when the futures is traded.

If it is true that today's futures price contains information about market participants' expectations about the future [5], we will observe that the same day-of-the-week effect between the settlement date of the futures contract and the date when the futures is traded. Since we have found the contrary, it implies that hedging has not been successful. This might be either because market participants' expectations have been wrong or because hedging has been dominated by speculative behavior.

6-2. Methodological Improvement

Although our random shuffling technique is simple and easily programmed, we are sure that an evidence of its power has been shown. The present paper makes it possible to carry out hypotheses testing. It is not rejected that with the weekends-and-holidays interpolation metal returns are not random on Tuesday and Thursday. There remain several remarks, however, on the methodology.

The nonlinear time series analysis could have begun with the embeddings [6]. Given a time series $\{u(t)\}$, we construct m-dimensional phase space at t_0 with delayed vectors consisting of lagged sequences of data points as,

$$\mathbf{u}(t_0) = \{u(t_0), u(t_0-1), u(t_0-2), \cdots, u(t_0-(m-1))\},$$

where m is the embedding dimension and also periodic time. Then we have 5 times more samples, which might be good for the case of small size sample.

What kind of periodicity is this study detecting? The answer is average return. One might be interested in periodicity of volatility (standard deviation) or higher moments, which could have been executed similarly.

Final remark on perspective of nonlinear analysis is that because tools for nonlinear time series analysis are still developing, we have to watch their progress and judge which to use for nonlinear time series analytical tools.

FOOTNOTES

*) All remaining errors are our own. The latter author would like to thank Japan Commodity Futures Industry Association for financial support.

1) Fama-French (1997) also documented that metal futures prices showed weak forecast power of future spot price and expected premium.

2) Are there any tendencies in the frequency if we calculate its frequency f_j from the μ rank vector data? One extreme is the equal occurrence which leads to uniform distribution of f_j, $j=1, 2, \cdots, M$. The other extreme is the concentration at a periodic pattern, i.e., $f_j = 1$ for some j and 0 for other j's. It will be convenient to invent a measure to show how often a specific pattern is observed. The following quantity has the desirable properties.

$$\sum_{j=1}^{M} \frac{M}{1-M}\left(f_j - \frac{1}{M}\right)^2$$

The minimum is zero when f_j distributes uniformly. The maximum 1 is attained when the frequency concentrates at a periodic pattern. This is a rough measure of persistence of periodic pattern.

3) It might be helpful for understanding to explore the case of the frequencies of the rank vectors. The frequencies ought to be randomly shuffled 40 times. For each rank vector of 120, 40 frequencies are then used to calculate its average and standard deviation. The derived distribution of the frequencies can be utilized to test hypothesis whether the realized frequency is significantly larger or smaller than such a specific value as zero,1/M, or others.

4) In order to eliminate this problem, we take Monday returns out and execute the ranking test in the same way as above. Table 3 in Tatsumi-Miyano (2004) shows the result. Returns are random on Tuesday to Thursday for Nikkei 225, on Wednesday and Thursday for JASDAQ, getting the same results as the interpolated case. These results might suggest that we have to interpolate the Saturday and Sunday stock prices, otherwise it leads to misleading results on Monday return.

5) This is the market efficiency hypothesis. We will test the market efficiency hypothesis by the nonlinear nonparametric time series analysis that futures prices are good predictors of future spot prices.

6) Although many dynamical systems are subject to multiple independent variables to determine their time evolution, there are often cases where only a single variable can be observed. It has been claimed that the embedology is proved to guarantee to reproduce the whole characteristics of the underlying dynamics from time series data about a single variable despite a Q-dimensional multivariate system. However, our technique does not depend on the theorem.

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