A Class of Quadratic Options for Exchange Rate Stabilization

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Abstract

We propose the use of a new option which we call "quadratic," and that central banks could use to smooth exchange rate volatility through the hedging strategies of the issuers. We derive analytic pricing and hedging formulas. We suggest a criterion to derive the optimal (for the Central Bank) option parameters. Finally, we perform several simulation exercises which show the effectiveness of using this option, with or without conventional spot interventions.

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1 Introduction

Even under free-floating exchange rate regimes, there is strong evidence that most central banks intervene in foreign exchange markets in order to affect the level or dynamics of the exchange rate. With respect to the reasons for these practice, according to Neely's (2001) survey, 89.5 percent of monetary authorities intervene to resist short-run trends in exchange rates and 66.7 percent intervene to correct medium-term misalignment of the exchange rate with respect to its fundamental value.

Interventions in foreign exchange markets are believed to be mostly conducted in the spot market where the delivery takes place in two days or less. Forwards and/or currency swaps are rarely used.[1] However, foreign exchange interventions result in several types of costs. First, foreign exchange intervention leads to the unintended change of the monetary base, impairing the independence of monetary policy. *Sterilized* intervention, in which intervention in the foreign exchange market is coupled with the reverse operation in the domestic open market, does not change the monetary base, but then the portfolio in authorities' balance sheet is unintentionally changed. Additionally, the possibility that other market participants find out that the Central Bank is intervening, can lead to spikes in volume with increased volatility. Because of the aforementioned, and other reasons, the attainment of foreign exchange objectives in the most effective possible way is a fundamental problem for central banks.

An alternative approach, which we consider in this paper, is the use of financial options. The idea of using options as central banks' policy tools is not new. Breuer (1999) reports that the Bank of Mexico sold U.S. dollar put/peso call options to build up foreign reserves in appreciation changes of the Mexican peso. The Bank of Spain reportedly sold put options on the peseta to resist devaluation pressures during 1993, though the bank denied it (Neely, 2001). Options can be used by monetary authorities in other context than foreign exchange intervention. Sundaresan and Wang (2004) studied the effects of liquidity provision through "Y2K options" by the Federal Reserve in the wake of the millennium date change event.

Options have several advantages, compared to traditional foreign exchange intervention instruments: First, the leverage of options, defined by the ratio of the hedge portfolio (which is the effective intervention mechanism) to the option premium, is usually large; therefore, monetary authorities can attain the same effect with a lower intervention size. Second, appropriately designed options (as the one we discuss in this paper) play a built-in stabilizer role by inducing option hedge portfolio changes which counter short-term exchange rate

[1]Neely (2001) reports that the Bank of Thailand used forwards in the spring of 1997 and the Reserve Bank of Australia used the spot and swaps markets to mimic a forward market intervention. Mandeng (2003) discusses the use of options by the Central Bank of Colombia.

movements automatically. Third, as we argued before, when interventions are necessary, very often market participants will find profitable to take positions against the monetary authorities, thus making their objectives more costly; options are expected to align the interests of some market participants with those of the Central Bank, lowering the cost of the intervention.

The use of standard options has been considered in the literature (see Breuer 1999 and Zapatero and Reverter 2003). However, despite some advantages with respect to spot market interventions, standard call and put options appear inappropriate when the objective of the Central Bank is to reduce volatility: Long positions in puts will exacerbate exchange rate movements (see Breuer 1999 for a description of the mechanism) while short positions in calls can potentially lead to unbounded losses. In this paper, we propose a new type of option which we call "quadratic option," and that will help the Central Bank to pursue its objective with limited potential losses. The idea is that the Central Bank will buy this option and, as in the case of standard options, its objectives are attained through the hedging strategy of the option issuer. However, the option is designed so as to accomplish those objectives in the most efficient way. For the particular option we propose, we derive an analytic pricing formula. Similarly, we compute analytically the hedging ratio. We find numerically the optimal (for the Central Bank) parameter values of the quadratic option. Additionally, we study numerically several competing intervention strategies: spot only, option only, spot and option, and no-intervention. In our exercises, the use of the quadratic option, especially combined with spot intervention, proves to be efficient in terms of fulfilling the authorities' policy objectives. Additionally, it reduces the amount of spot intervention with only a small investment in the option premium.

This paper is organized as follows: In the next section we discuss the general mechanism through which options can help the Central Bank, and we show the shortcomings of using standard call and put options. In section 3 we introduce and price the quadratic option. In section 4 we derive the optimal parameter values of the quadratic option. In section 5 we present simulation results for several strategies and compare their efficacy. In section 6 we discuss other possible options. We close the paper with some conclusions.

2 Economic Foundations

The basic argument to use options as a foreign exchange policy is as follows. Options are derivative securities whose prices can be approximated by the underlying security and a bank account. It is at the core of option trading to hedge the position in the option, so that the financial intermediary who is buying or selling the option does not have any risk exposure. Both long and short positions in options can lead to losses and must be hedged in order to eliminate all risk exposure. The difference between the risk of a long and a short position is that in a long position the possible total loss is limited to the price paid for the option, while in a short position potential (theoretical) losses are practically unbounded.[2] The intervention mechanism is the following: the Central Bank will take positions in options (accepting the risk associated with the position), but the investment bank that takes the counterpart will hedge its position; for a properly chosen position of the Central Bank, the hedge of the investment bank will induce a trade in the underlying (the currency) in the direction desired by the Central Bank. Intervening through options (instead of directly in the spot currency market) has several advantages, already sketched in the introduction, like the fact that it minimizes the effects on the monetary base, aligns the interests of the investment bank with those of the Central Bank, plus benefits from the automatic stabilization features connatural to the hedging of some options: rebalancing of the hedge portfolio might involve trades that help stabilizing the currency.

Although Central Banks could in theory pursue exchange rate objectives by taking short positions in options, they will be reluctant to do so because of the potential unlimited loss associated with a short position. Also, they would be off-balance entries, which would reduce the transparency of the Central Bank.[3] We will focus then on long positions, for which the maximum possible loss is the price of the option. Breuer (1999) shows that a long put, however, will have destabilizing effects. Here we will focus on an option that we suggest here and will have stabilizing effects through the hedge.

The Central Bank buys this option from an "investment bank," which will hedge, in order to avoid possible losses, by using the money raised from the sale of the option to replicate a long position in the option. In order to replicate that long position, the investment bank has to take a position in the underlying, which in this case will be the foreign currency, and in the "bank" or money market fund. As the price of the underlying changes, the price of the option changes, and so does the composition of the hedging portfolio: the hedging portfolio is self-financing, so only rebalancing is required, and not adding new funds.

To be specific, we call $D(S_t, T-t)$ the option price at time t, with underlying asset price S_t, and time-to-maturity $T-t$; additionally, we denote by M_t the value of money market fund at time t, and by p_t and q_t the quantities to be held at time t of the underlying asset and money market fund, respectively. Therefore, the hedge portfolio (p_t, q_t) will satisfy the

[2] In a short put the maximum possible loss is the strike price, but this is typically so much larger than the price of the put that we can consider it unbounded for practical purposes.

[3] Blejer and Schumacher (2000) analyze in detail the use of derivatives by Central Banks.

following condition:

$$D(S_t, T-t) = p_t \cdot S_t + q_t \cdot M_t, \tag{1}$$

where M_t is assumed to grow continuously at a constant rate r. To derive a specific hedge portfolio, we consider the version of the Black and Scholes (1973) model developed by Garman and Kohlhagen (1983). In their model, r and r_f are the constant domestic and foreign interest rate, respectively. The exchange rate satisfies the following dynamics:

$$\frac{dS_t}{S_t} = (r - r_f)\, dt + \sigma d\widehat{W}_t, \tag{2}$$

where, as usual, S_t is the number of units of the domestic currency needed to buy one unit of the foreign currency, and $d\widehat{W}_t$ is the increment of a standard Wiener process under the risk-neutral measure $\widehat{\mathbb{P}}$ in the probability space $\left(\Omega, \mathcal{F}, \mathbb{F}, \widehat{\mathbb{P}}\right)$. Imposing the self-financing requirement upon (1), applying Itô's formula into $D(S_t, T-t)$, and equating them, we obtain the hedge portfolio:

$$\begin{aligned} p_t &= D_S(S_t, T-t), \\ q_t &= (D(S_t, T-t) - p_t \cdot S_t) / M_t, \end{aligned} \tag{3}$$

where D_S, the derivative of the price of the option with respect to the underlying is called the "delta" or "hedge ratio" of the option and tells us how many units of the underlying are needed to replicate the option. In the Garman and Kohlhagen (1983) model, the hedge ratio of standard options has an analytic expression.

Our objective is to design an option such that long positions by the Central Bank will help to achieve foreign exchange objectives. In addition, we want an option for which we can derive an explicit formula both for its price and its hedge ratio. The reason is that we need to study the effect of using this option in foreign exchange policy, and this analysis becomes practically unfeasible if we cannot compute the hedge ratio. We discuss this option in the next section.

3 Quadratic Option

We propose a new option designed specifically to attain the stabilization objectives of the Central Bank. Additionally, we can find an analytic expression for the price and hedge ratio of this option. This is a European-style option. We denote by S the exchange rate process, defined in the previous section. The payoff of the quadratic option at maturity is,

$$\begin{aligned} D(S_T, 0) &= \max\left(a + bS_T + cS_T^2, 0\right) \\ &= \max\left\{c(S_T - \alpha)(S_T - \beta), 0\right\}, \end{aligned} \tag{4}$$

where a, b, c, α and β are constant parameters such that $c < 0, \alpha < \beta, b = -c(\alpha + \beta), a = c\alpha\beta$. These parameters are to be found using some optimization criterium (that we introduce in the next section). For an exchange rate process that satisfies (2) , the price of the quadratic option is given by

$$D(S_0, T; a, b, c) \equiv I_1 + I_2 + I_3, \tag{5}$$

where

$$\begin{aligned} I_1 &= e^{-rT} a \left\{\Phi(d_1(\beta)) - \Phi(d_1(\alpha))\right\}, \\ I_2 &= e^{-r_f T} b S_0 \left\{\Phi(d_2(\beta)) - \Phi(d_2(\alpha))\right\}, \\ I_3 &= e^{-rT} c S_0^2 \exp\left[\left\{2(r - r_f) + \sigma^2\right\} T\right] \cdot \\ &\quad \left\{\Phi(d_3(\beta)) - \Phi(d_3(\alpha))\right\}, \\ d_1(x) &= \frac{\ln(x/S_0) - (r - r_f - \sigma^2/2)T}{\sigma\sqrt{T}}, \\ d_2(x) &= \frac{\ln(x/S_0) - (r - r_f + \sigma^2/2)T}{\sigma\sqrt{T}}, \\ d_3(x) &= \frac{\ln(x/S_0) - (r - r_f + 3\sigma^2/2)T}{\sigma\sqrt{T}}. \end{aligned}$$

Here, $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution.

Furthermore, the hedge ratio D_S can also be analytically expressed as

$$D_S(S_0, T; a, b, c) \equiv I_1' + I_2' + I_3', \tag{6}$$

where

$$
\begin{aligned}
I_1' &= e^{-rT} a \left\{ \phi\left(d_1(\beta)\right) d_1'(\beta) - \phi\left(d_1(\alpha)\right) d_1'(\alpha) \right\}, \\
I_2' &= e^{-r_f T} b \left\{ \Phi\left(d_2(\beta)\right) - \Phi\left(d_2(\alpha)\right) \right\} + \\
&\quad e^{-r_f T} b S_0 \left\{ \phi\left(d_2(\beta)\right) d_2'(\beta) - \phi\left(d_2(\alpha)\right) d_2'(\alpha) \right\}, \\
I_3' &= 2 e^{-rT} c S_0 \exp\left[\left\{ 2\left(r - r_f\right) + \sigma^2 \right\} T \right] \cdot \\
&\quad \left\{ \Phi\left(d_3(\beta)\right) - \Phi\left(d_3(\alpha)\right) \right\} + \\
&\quad e^{-rT} c S_0^2 \exp\left[\left\{ 2\left(r - r_f\right) + \sigma^2 \right\} T \right] \cdot \\
&\quad \left\{ \phi\left(d_3(\beta)\right) d_3'(\beta) - \phi\left(d_3(\alpha)\right) d_3'(\alpha) \right\}, \\
d_1'(x) &= d_2'(x) = d_3'(x) = -\frac{1}{\sigma\sqrt{T} S_0}.
\end{aligned}
$$

Here, $\phi(\cdot)$ is the p.d.f. of the standard normal distribution.

In figure 1 we plot the price of a quadratic option at maturity and six months before maturity, for the model described above and the following parameter values: $r = .04$, $r_f = .03$, $\sigma = .1$, $c = -.1$, $[\alpha, \beta] = [.9, 1.1]$. Given the "quadratic" nature of the option, payoff and price are symmetric around the band delimited by α and β. The delta of this option, with maturity in six months, is represented in figure 2. Within the band delimited by α and β (region I in the graph) the delta is negatively correlated with the exchange rate, consistent with the objective of the Central Bank: When the exchange rate appreciates (decreases) the issuer of the option will have to buy the underlying (the foreign currency) in order to rebalance its hedge, which will push the exchange rate upwards.

The hedging strategy of the option issuer is aligned with the Central Bank objectives not only within the band range, but also over time. In figure 3 we plot the price of the option for the band $[0.9, 1.1]$ with the parameter values of figures 1 and 2, and for three maturities, 0.5, 0.25 and 0.1 years, respectively. From this graph we observe that, as maturity approaches, in general, the delta of the option increases in absolute value for a given exchange rate. That is, the stabilization pressure increases over time.

From figure 2 we can also measure the leverage effect resulting from using options. For example, for the parameters listed above, the Central Bank can buy the option at a price of 0.006 when $S_t = 1$. If the exchange rate appreciates to $S_t = 0.9$ the hedge demand for foreign currency will be 0.048, which at the exchange rate of 0.9 represents an investment of 0.0522, which is 8.5 times the value of the investment in the option. The drawback of this strategy is that if the exchange rate gets out of the band delimited by α and β the hedging demand reverses and goes in the direction opposite to the objective of the Central Bank. Therefore, it is very important to choose an appropriate band.

The trade-off resulting from the choice of band is clear from figures 4 and 5. In figure 4 we plot the price of the option described above, with a band $[0.9, 1.1]$ and the price of an option with the same parameter values for the exchange rate and the same maturity, but for a band $[0.8, 1.2]$. In figure 5 we plot the deltas of these two options. As we can see, both the price and delta of the option with the broader band are larger than the price and delta of the other option. However, the leverage resulting from the option with broader band is lower: If the Central Bank buys the option with band $[0.8, 1.2]$ at an exchange rate of 1, the price of the option is 0.0343. If the exchange rate appreciates to $S_t = 0.9$ the hedge demand for foreign currency will be 0.1625, which at the exchange rate of 0.9 represents an investment of 0.1795, which is 5.2 times the value of the investment in the option, compared with a leverage of 8.5 for the option with a band $[0.9, 1.1]$. The broader the band, the larger the region over which the beneficial effects of the hedge spread. However, the resulting price pressure is lower. Therefore, it is essential to choose the right band. We later introduce a criterium to pick up the limits of the band.

4 Model for Interventions

The purpose of this paper is to compare the efficiency of quadratic options as an intervention mechanism with traditional intervention in the spot market. However, in order to make any comparisons, we need a model of the effect of the interventions on the exchange rate. The model we present here is based on Zapatero and Reverter (2003). We abstract from effects on interest rates, which would force us to specify monetary policy objectives and would make the model too complicated. We also ignore possible margin requirements, since the quadratic option would be over-the-counter.

The dynamics of the exchange rate, if no intervention takes place, are given by (2). Suppose now that the Central Bank intervenes at moment t in the spot market by buying or selling an amount I_t (buying means $I > 0$ and selling means $I < 0$) of the foreign currency, which will require an investment $S_t I_t$, in units of the domestic currency. We assume that the exchange rate after the intervention will be

$$S_{t+} = S_t e^{I_t}. \tag{7}$$

More explicitly, let us suppose that the Central Bank has chosen to intervene in the spot market when the exchange rate moves out of an implicit target range given by $[\underline{S}, \overline{S}], \underline{S} < \overline{S}$. We assume that the Central Bank intervenes to move the exchange rate to $\underline{S} + \varepsilon$ or $\overline{S} - \varepsilon$ ($\varepsilon > 0$) when the exchange rate exceeds the lower or upper bound. The required amount of intervention of this case is

$$I_t = \begin{cases} \log\left(\frac{\overline{S}-\varepsilon}{S_t}\right), & \text{if } S_t > \overline{S} \\ \log\left(\frac{\underline{S}+\varepsilon}{S_t}\right), & \text{if } S_t < \underline{S}. \end{cases} \tag{8}$$

Therefore, at time t, the exchange rate is given by

$$S_t = S_0 \exp\{\left(r - r_f - \sigma^2/2\right) t + \sigma W_t + \sum_{i=1}^{n} I_{s_i}\} \tag{9}$$

with $0 \leq s_1 < s_2 \ldots s_n < t$.

Similarly, if the Central Bank buys at t one quadratic option with a delta $D_S(S_t)$, the exchange rate after the intervention is,

$$S_{t+} = S_t e^{D_S(S_t)}. \tag{10}$$

Clearly, in order to hedge the position in the option, the issuer of the option has to buy (or sell, if negative) $D_S(S_t)$ units of the underlying, that is, the foreign currency, at the exchange rate S_t. Recall, furthermore, that the option issuer has to rebalance the hedge dynamically, so that $D_S(S_\tau)$, for $\tau > t$, represents the number of units of the foreign currency the issuer of the option holds (or has shortened, if negative) as a result of the updated hedge up to τ. Then, if the Central Bank has not intervened in the spot market, but has bought quadratic options, we have the equivalent of (9),

$$S_t = S_0 \exp\{\left(r - r_f - \sigma^2/2\right) t + \sigma W_t + D_S(S_t)\}. \tag{11}$$

Obviously, in our model of the effects of Central Bank intervention (whether in the spot market or by buying quadratic options) we are ignoring that the intervention of the Central Bank might affect the parameter values of the exchange rate dynamics. For example, the volatility σ might change if the market finds out that the Central Bank is intervening. We assume that the Central Bank will intervene in periods of mild un-stability and the effect on parameter values will not be significant.

In order to study the effect of the different types of Central Bank intervention and compare them with the case of no-intervention, we will simulate the dynamics of the exchange rate and will keep track of the total value of the investment required for intervention purposes. To simplify the notation, we make $\mu = r - r_f$. In order to simulate the exchange rate in the case of no-intervention, we use the standard discretization,

$$\triangle S_t = S_{t+\triangle t} - S_t = S_t \left(\mu \triangle t + \sigma \omega_t\right), \tag{12}$$

where $\omega \sim N\left(0, \sqrt{\triangle t}\right)$ and $\triangle t$ is set to be one business day (=1/252 year) here.

In the case of intervention by the Central Bank, we have to take into account its effect on the exchange rate dynamics according to (7)-(11). We denote the dynamics of the exchange rate in case of intervention as S^i, where $i = s$ if intervention in the spot market, $i = o$ if intervention by buying quadratic options, and $i = so$ if intervention both in the spot market and with quadratic options. In the case of intervention in the spot market, we simulate the dynamics of the exchange rate as

$$S^s_{t+\triangle t} = \begin{cases} S^s_t + S^s_t \left(\mu \cdot \triangle t + \sigma \omega_t\right), & \text{if } S^s_{t+\triangle t} \in \left[\underline{S},\ \overline{S}\right] \\ \overline{S} - \varepsilon, & \text{if } S^s_{t+\triangle t} > \overline{S} \\ \underline{S} + \varepsilon, & \text{if } S^s_{t+\triangle t} < \underline{S} \end{cases} \tag{13}$$

In the case of option intervention, we do not specify for now the size of the intervention. If the Central Bank buys quadratic options at some point, from (11), the exchange rate path is simulated as

$$S^o_{t+\triangle t} = \left[S^o_t + S^o_t \left(\mu \cdot \triangle t + \sigma \omega_t\right)\right] \exp\left(D_S\left(S^o_t\right)\right) \tag{14}$$

where $D_S\left(S^o_t\right)$ denotes the hedge position of the option issuer.

Finally, in the case of intervention both in the spot market and with options, with intervention amount in the spot market given by (8), the exchange rate path is adjusted as follows:

$$S^{so}_{t+\triangle t} = \begin{cases} \left[S^{so}_t + S^{so}_t \left(\mu \cdot \triangle t + \sigma \omega_t\right)\right] \exp\left(D_S\left(S^{so}_t\right)\right), & \text{if } S^{so}_{t+\triangle t} \in \left[\underline{S},\ \overline{S}\right] \\ \overline{S} - \varepsilon, & \text{if } S^{so}_{t+\triangle t} > \overline{S} \\ \underline{S} + \varepsilon, & \text{if } S^{so}_{t+\triangle t} < \underline{S} \end{cases} \tag{15}$$

In the next section, we use this model and the discretization we just described to compare the different types of intervention.

5 Numerical Results

5.1 Optimal Band

Before we perform numerical exercises to compare different types of intervention, we have to address a preliminary issue. As we discussed in section 3, the performance of quadratic options depends on the band. The Central Bank will then have to choose the band so as to optimize its policy objectives. Therefore, we need a criterium for that purpose. In particular, we assume that the Central Bank has two objectives, related, but somehow complementary. We assume that the two main foreign exchange policy objectives of the Central Bank are: low volatility, and an exchange rate that does not deviate too much from a given reference exchange rate. We can formulate the problem as

$$\min E\left[\int_0^T \left\{we^{-\delta t}\left(S_t - S_0\right)^2 + (1-w)\left(dS_t\right)^2\right\} dt\right], \tag{16}$$

where δ is some discount rate and $w \in [0,1]$ measures the weight given to each objective. The first term in the integral formalizes the goal to have the exchange rate as close as possible to some reference exchange rate S_0. Without loss of generality, we assume that the reference exchange rate is the current exchange rate. The second term in the integral provides a measure of overtime volatility. Obviously, both objectives are related, but they are not identical: if the exchange rate deviates from the reference exchange rate, there will be tension between the target of pushing the exchange rate as close as possible to the reference rate, but on the other hand keeping volatility low.

Given a band $[\alpha, \beta]$ for the quadratic option, if the Central Bank takes a long position in it, the resulting hedging demand will be a complicated function of α and β. Therefore, the possibility of deriving optimal α and β analytically seems out of the question, and it will be necessary to try to approximate them analytically.

We use the criterion formalized in (16) to choose the optimal band parameters. We do this numerically: We simulate 1,000 exchange rate paths according to (14), and we calculate the costs given by (16) for different bands $[\alpha, \beta]$. As we change the band parameters, the cost of buying the option also changes, which affects the effectiveness of the option. Since we will compare results across many possible bands, we fix the value of the initial investment and we adjust the number of options to be bought, as we change the band $[\alpha, \beta]$. Parameter values for the benchmark case are $\sigma = .1$, $w = .5$, $r = .04$, $r_f = .03$, $T = .5$, $S_0 = 1$. Table 1 reports the values of the objective function of the Central Bank for different option bands. The optimal band, which yields the lowest objective value, is calculated as $[.92, 1.08]$ for the benchmark case (panel A). For higher volatility, $\sigma = .2$, we have a thinner optimal band, $[.96, 1.04]$, as illustrated in panel B. If we change the weight w from .5 to .1, so that stability becomes substantially more important than deviation from the target, the optimal band changes, as shown in panel C. In particular, the optimal band is not symmetric around the target level anymore.

5.2 Comparison of Interventions

We now analyze the effectiveness of Central Bank intervention with quadratic options with respect to conventional instruments in foreign exchange intervention. We consider the following possible strategies: (i) no-intervention, (ii) intervention only in the spot market, (iii) intervention in both spot and option markets, and (iv) intervention only in the option market. Intervention in the spot market means that the Central Bank will use foreign reserves when the exchange rate hits the implicit target range. Option intervention means that the Central Bank will buy one quadratic option with the optimal band parameters, as described in the previous subsection, and, additionally, will intervene in the spot market when the limits of the implicit target range are reached. For our simulations, we use the discretizations given by (12)-(15).

Table 2 compares the performances of the four cases in terms of the costs defined as (16) and intervention amount. The results are obtained as averages over 1,000 simulations. The benchmark case is set with parameters $T = .5$, $r = .04$, $S_0 = 1$, $\varepsilon = .02$, $\sigma = .1$, $c = -.1$, $r_f = .03$, $w = .5$, $\left[\underline{S}, \overline{S}\right] = [.9, 1.1]$. In the case of intervention with options, the band is computed as described above, that is, so as to minimize (16). It is clear that, in our setting, intervention has a substantial effect in the objective of the Central Bank, given by (16). However, the use of options increases that effect dramatically. This is clear from the case in which only options are used and from the case in which options and spot intervention are combined. In the latter case, the cost in reserves is substantially lower than in the case in which options are not used. Obviously, we are ignoring the net cost of the option (the difference between the initial price and the payoff at maturity). Arguably, on average, the option issuer should make some money, or it would not have incentives to trade the option. However, for a well diversified investment bank, the cost would not have to be that high. One advantage of this type of option for the investment bank is the fact that the potential payoff is bounded, unlike in standard options. As we observe in panel B of table 2, the beneficial effects of using options are magnified when volatility is higher. Arguably, the net cost of using options would also increase. In panel C, we observe that the benefits of both spot intervention and the use of options for intervention, are relatively smaller (but still substantial) when the main objective of the Central Bank is to lower volatility.

6 Alternative Options

We have shown the performance and potential usefulness of the *quadratic* option for foreign exchange policy. Obviously, the *quadratic* option is just one possible type of option which can be considered for those purposes. We discuss other types of options in this section.

A straddle, which is a combination of short currency call and put where the strike of the call is higher than that of the put, offers downward-sloping option deltas, so that it can serve to reduce exchange rate volatility; however, Central Banks are exposed to the risk of possible unbounded losses as option-writers. Another possible strategy consists in a short currency call and a long currency (discussed in Zapatero and Reverter 2003) has features similar to the straddle.

We may create more complicated, and yet flexible options than the *quadratic* option, for the same purposes. For example, consider the following: Define a European option whose payoff at maturity is

$$D\left(S_T,0;X,\alpha,\beta\right)=\begin{cases}0, & S_T\in(0,X)\\ S_T-X, & S_T\in[X,X+\alpha)\\ \alpha, & S_T\in[X+\alpha,X+\alpha+\beta)\\ Y-S_T, & S_T\in[X+\alpha+\beta,Y)\\ 0, & S_T\in[Y,\infty)\end{cases}, \tag{17}$$

where $Y\equiv X+2\alpha+\beta$. Let us call it *range* option. Under the same conditions assumed for the quadratic option, the price of the range option can be obtained analytically. That is, the option price at time 0 is given by

$$D\left(S_0,T;X,\alpha,\beta\right)\equiv I_1+I_2+I_3. \tag{18}$$

We define the following functions

$$\begin{aligned} g\left(a,b\right) &= e^{-rT}\left\{\Phi\left(d_1\left(b\right)\right)-\Phi\left(d_1\left(a\right)\right)\right\}, \\ d_1\left(x\right) &= \frac{\ln\left(x/S_0\right)-\left(r-r_f-\sigma^2/2\right)T}{\sigma\sqrt{T}}, \end{aligned}$$

and

$$\begin{aligned} h\left(a,b\right) &= e^{-\delta T}S_0\left\{\Phi\left(d_2\left(\beta\right)\right)-\Phi\left(d_2\left(\alpha\right)\right)\right\}, \\ d_2\left(x\right) &= \frac{\ln\left(x/S_0\right)-\left(r-r_f+\sigma^2/2\right)T}{\sigma\sqrt{T}}. \end{aligned}$$

We now denote

$$
\begin{aligned}
I_1 &= h(X, X+\alpha) - X \cdot g(X, X+\alpha), \\
I_2 &= \alpha g(X+\alpha, X+\alpha+\beta), \\
I_3 &= Y \cdot g(X+\alpha+\beta, Y) - h(X+\alpha+\beta, Y).
\end{aligned}
$$

By defining the first derivatives of the auxiliary functions,

$$
\begin{aligned}
g'(a,b) &= e^{-rT}\left\{\phi(d_1(\beta))\, d_1'(\beta) - \phi(d_1(\alpha))\, d_1'(\alpha)\right\}, \\
d_1'(x) &= -\frac{1}{\sigma\sqrt{T} S_0},
\end{aligned}
$$

and

$$
\begin{aligned}
h'(a,b) &= e^{-r_f T}\left\{\Phi(d_2(\beta)) - \Phi(d_2(\alpha))\right\} + \\
&\quad e^{-r_f T} S_0 \left\{\phi(d_2(\beta))\, d_2'(\beta) - \phi(d_2(\alpha))\, d_2'(\alpha)\right\}, \\
d_2'(x) &= -\frac{1}{\sigma\sqrt{T} S_0} = d_1'(x),
\end{aligned}
$$

we can write

$$
\begin{aligned}
D_S(S_0, T; a, b, c) &\equiv I_1' + I_2' + I_3', \qquad (19) \\
I_1' &= h'(X, X+\alpha) - X \cdot g'(X, X+\alpha), \\
I_2' &= \alpha g'(X+\alpha, X+\alpha+\beta), \\
I_3' &= Y \cdot g'(X+\alpha+\beta, Y) - h'(X+\alpha+\beta, Y).
\end{aligned}
$$

We expect the characteristics of the range option to be similar to those of the quadratic option, so we do not perform any numerical exercises.

7 Conclusions

In this paper, we discuss the possible use of options for foreign exchange intervention and we compare this strategy to standard spot intervention. In particular, we introduce one particular type of option (which can be considered representative of a larger class) that we call *quadratic* option. We show that it can have significant effects on the objectives of the Central Bank.

Of course, for tractability reasons, in our analysis we overlook many aspects of the problem that would have to be taken into account for actual implementation. For example, the "signalling" implications of using options. In general, equilibrium considerations might be necessary for further insight on the possible use of the quadratic option (or a similar option) for foreign exchange policy.

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Table 1: Values of Objective Function and Optimal Option Band

Values of objective functions are defined as (16) and are multiplied by 10^{-3}. The optimal option band is meant to yield the minimum values of objective functions. By adjusting the number of options, we keep investment in options fixed and equal to the option premium for the benchmark case of a band of $[.9, 1.1]$ and $c = .1$.

Panel A. Benchmark Case: $\sigma = .1, w = .5; r = .04, r_f = .03, T = .5, S_0 = 1$

$\alpha \setminus \beta$	.84	.88	.92	.96	1.00	1.04	**1.08**	1.12	1.16	1.20
.80	14.4	8.99	5.53	3.57	2.46	1.56	.896	.450	.207	.138
.84	-	6.55	4.10	2.49	1.59	.913	.448	.182	.096	.167
.88	-	-	2.92	1.72	.906	.434	.158	.063	.135	.357
.92	-	-	-	1.07	.473	.141	**.042**	.116	.353	.743
.96	-	-	-	-	.250	.070	.113	.360	.773	1.35
1.00	-	-	-	-	-	.244	.436	.809	1.412	2.19
1.04	-	-	-	-	-	-	1.04	1.63	2.31	3.26
1.08	-	-	-	-	-	-	-	2.83	3.97	5.20
1.12	-	-	-	-	-	-	-	-	6.32	8.80
1.16	-	-	-	-	-	-	-	-	-	15.1

Panel B. $\sigma = .2$

$\alpha \setminus \beta$	.84	.88	.92	.96	1.00	**1.04**	1.08	1.12	1.16	1.20
.80	5.91	4.50	3.37	2.50	1.84	1.38	1.09	.950	.925	.989
.84	-	3.44	2.52	1.83	1.34	1.01	.837	.786	.838	.973
.88	-	-	1.86	1.32	.963	.756	.679	.715	.847	1.06
.92	-	-	-	.972	.719	.613	.630	.753	.970	1.27
.96	-	-	-	-	.615	**.595**	.699	.913	1.23	1.62
1.00	-	-	-	-	-	.710	.899	1.21	1.62	2.14
1.04	-	-	-	-	-	-	1.24	1.65	2.18	2.82
1.08	-	-	-	-	-	-	-	2.26	2.91	3.69
1.12	-	-	-	-	-	-	-	-	3.83	4.78
1.16	-	-	-	-	-	-	-	-	-	6.12

Panel C. $\sigma = .2, w = .1$

$\alpha \setminus \beta$	.84	.88	.92	.96	1.00	**1.04**	1.08	1.12	1.16	1.20
.80	1.33	1.00	.733	.556	.426	.335	.279	.252	.248	.262
.84	-	.806	.584	.427	.328	.264	.229	.221	.232	.261
.88	-	-	.477	.338	.256	.215	.200	.208	.236	.280
.92	-	-	-	.293	.219	**.189**	.193	.218	.263	.324
.96	-	-	-	-	.221	.198	.211	.253	.317	.397
1.00	-	-	-	-	-	.246	.266	.317	.400	.503
1.04	-	-	-	-	-	-	.364	.428	.517	.645
1.08	-	-	-	-	-	-	-	.585	.698	.829
1.12	-	-	-	-	-	-	-	-	.925	1.10
1.16	-	-	-	-	-	-	-	-	-	1.43

Table 2: Performance of Intervention Methods

Super script s means spot intervention, so spot and option intervention, and o option intervention, and no super script denotes no-intervention. "Cost" is defined as (16). "Intv" is the required spot intervention amount to meet the target exchange rates as illustrated in (8). Values for costs are multiplied by 10^{-3}. Options are bought with the optimal bands as obtained in table 1.

Panel A: Benchmark Case ($T = .5$, $r = .04$, $S_0 = 1$, $\sigma = .1$, $c = -.1$, $r_f = .03$, $w = .5$),

$[\underline{S}, \overline{S}] = [.9, 1.1]$

	cost	cost^s	cost^{so}	cost^o	Intv^s	Intv^{so}
Mean	.626	.452	.042	.042	.012	.000
Median	.357	.354	.040	.040	.000	.000
S.D.	.695	.322	.011	.011	.024	.000
Min	.032	.032	.022	.022	.000	.000
Max	6.24	1.58	.108	.108	.142	.000

Panel B: $\sigma = .2$

	cost	cost^s	cost^{so}	cost^o	Intv^s	Intv^{so}
Mean	2.55	.697	.382	.595	.077	.018
Median	1.42	.695	.365	.391	.063	.000
S.D.	2.97	.243	.151	.956	.070	.029
Min	.132	.132	.109	.109	.000	.000
Max	29.2	1.34	.885	19.5	.378	.213

Panel C: $\sigma = .2$, $w = .1$

	cost	cost^s	cost^{so}	cost^o	Intv^s	Intv^{so}
Mean	.574	.200	.147	.189	.079	.019
Median	.349	.200	.145	.151	.063	.000
S.D.	.601	.049	.032	.190	.070	.029
Min	.073	.073	.077	.077	.000	.000
Max	5.95	.333	.265	3.82	.378	.200

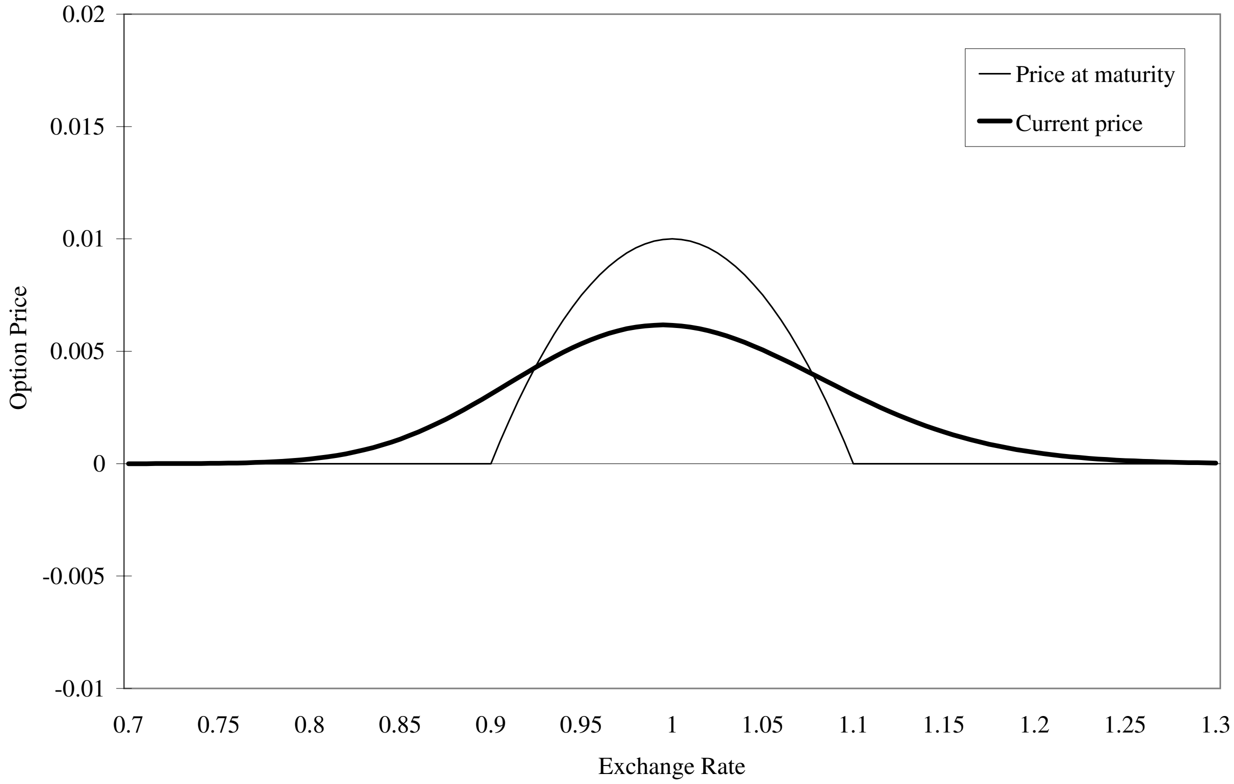

Figure 1: The plot shows the price of a quadratic option at maturity and six months before maturity. Parameter values are: $r = .04$, $r_f = .03$, $\sigma = .1$, $c = -.1$, $[\alpha, \beta] = [.9, 1.1]$.

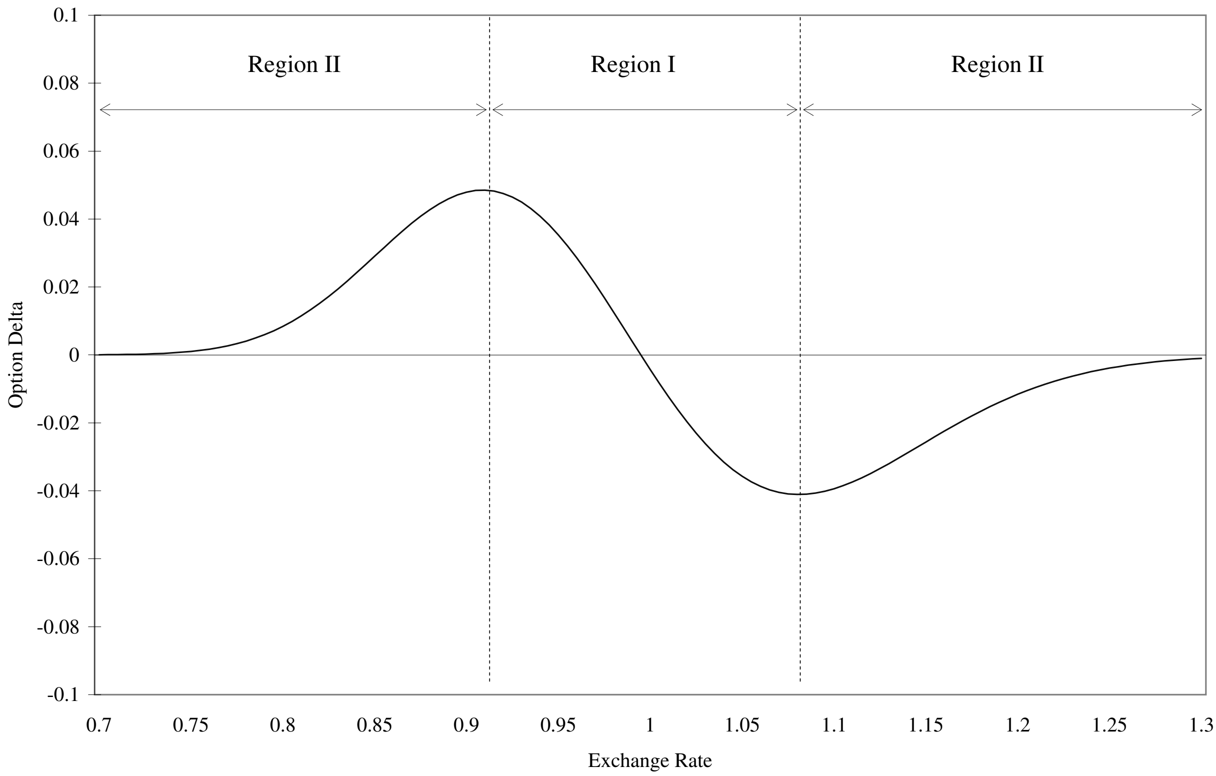

Figure 2: The plot shows the delta of a quadratic option with six months to maturity. Parameter values are: $r = .04$, $r_f = .03$, $\sigma = .1$, $c = -.1$, $[\alpha, \beta] = [.9, 1.1]$.

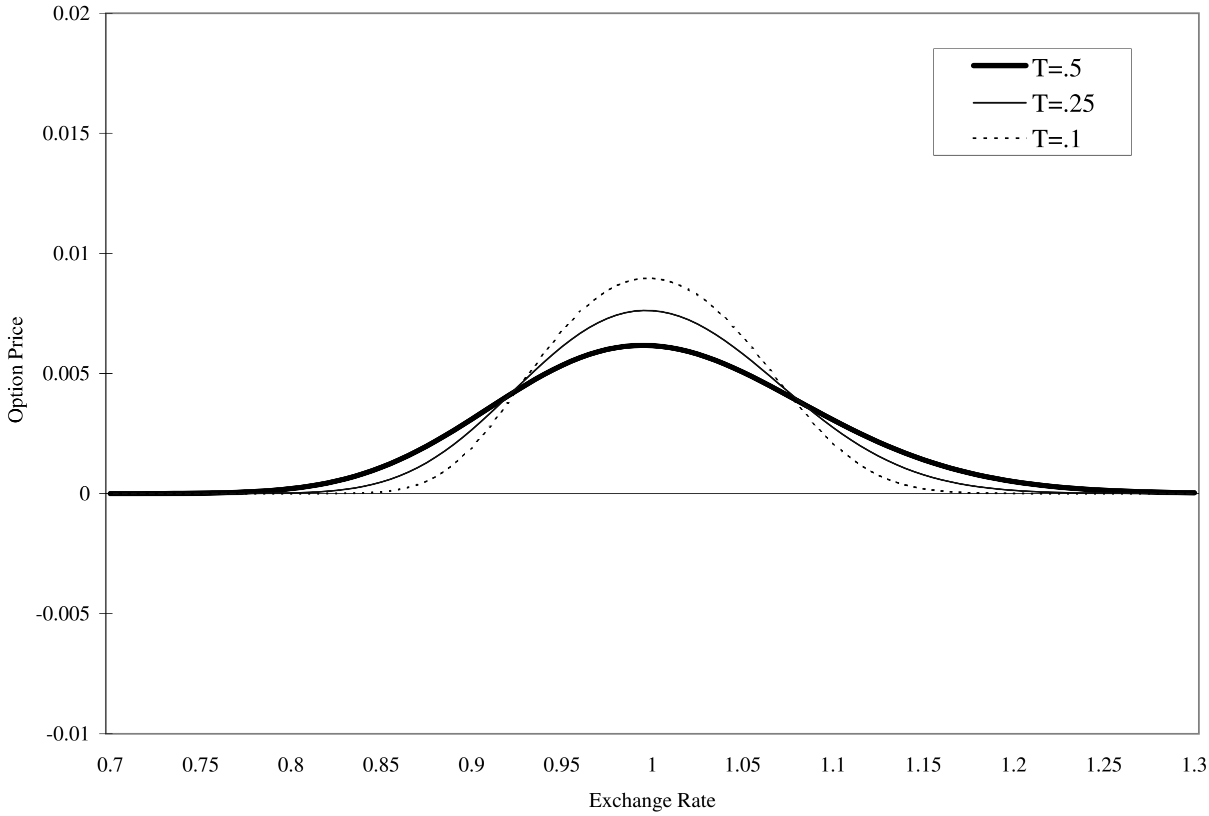

Figure 3: The plot shows the price of a quadratic option with 0.5, 0.25 and 0.1 years to maturity, for parameter values: $r = .04$, $r_f = .03$, $\sigma = .1$, $c = -.1$, $[\alpha, \beta] = [.9, 1.1]$.

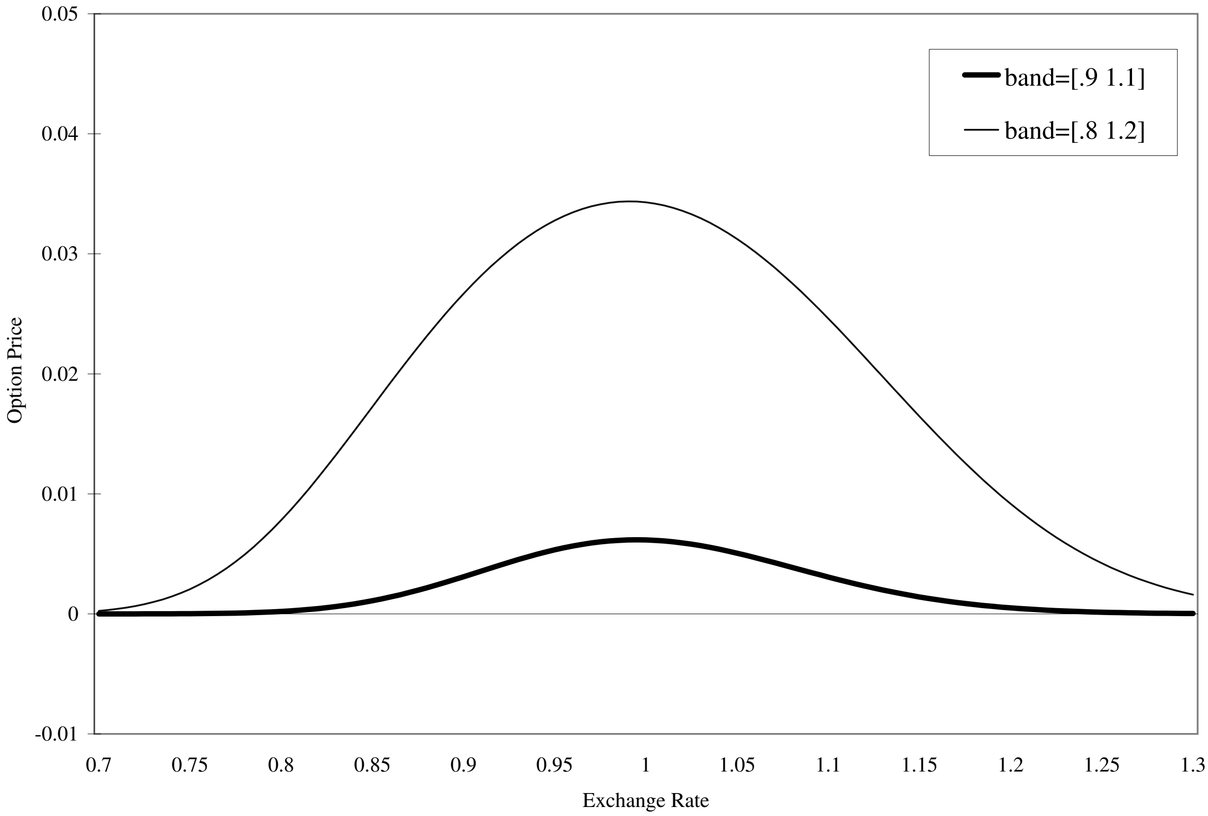

Figure 4: The plot shows the price of two different quadratic options with bands $[0.8, 1.2]$ and $[0.9, 1.1]$, respectively. Other parameter values are: $r = .04$, $r_f = .03$, $T = .5$, $\sigma = .1$, $c = -.1$.

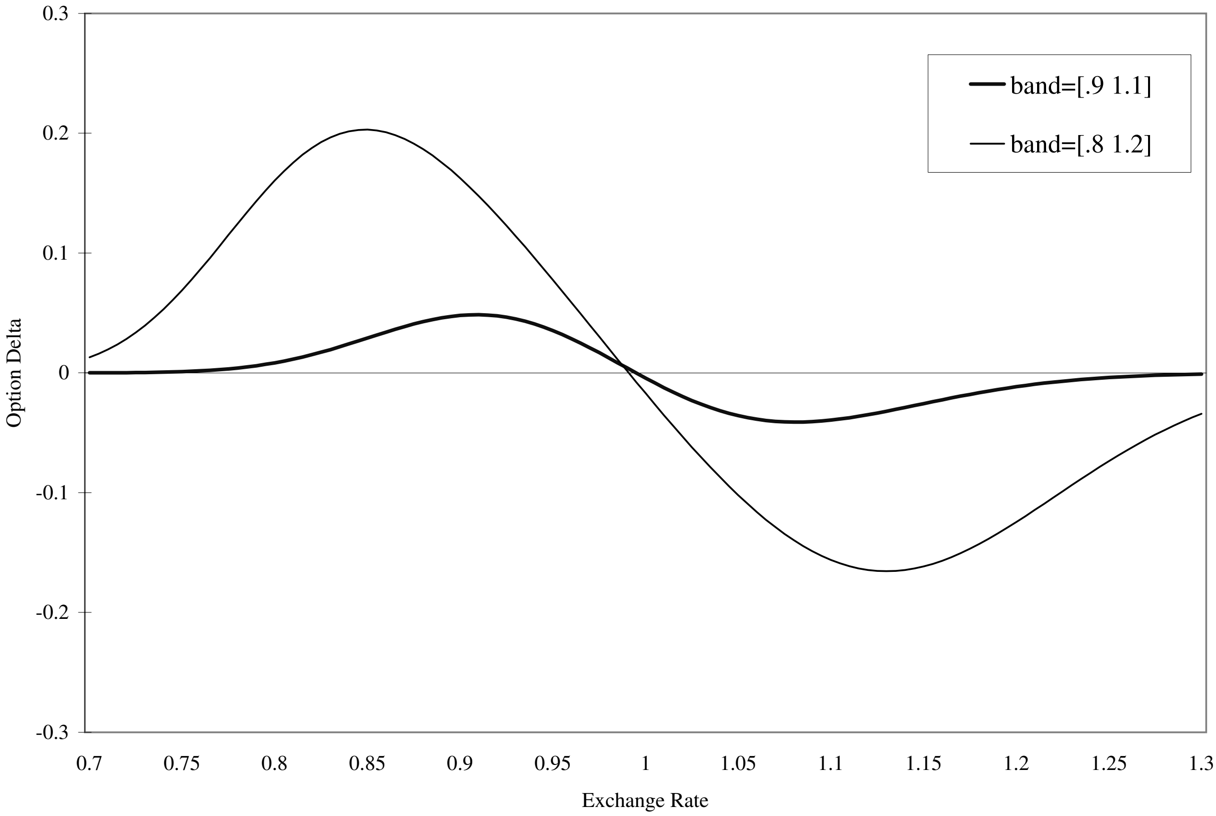

Figure 5: The plot shows the delta of two different quadratic options with bands $[0.8, 1.2]$ and $[0.9, 1.1]$, respectively. Other parameter values are: $r = .04$, $r_f = .03$, $T = .5$, $\sigma = .1$, $c = -.1$.