Option-based Dynamic Hedging of Convertible Bond with Credit Risk

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Abstract

The dynamic hedging strategy has evolved as a favored approach in managing convertible bond portfolios. Moreover, more recent bond valuation studies have clearly demonstrated that credit risk is an important factor related to the profitability of a convertible bond portfolio. The objective of this study is to formulate an option-based dynamic hedging model which accounts for credit risk for practitioners in managing convertible bond portfolio. This paper adopts four option-based dynamic delta-hedging models to account for the transaction costs and credit risk for convertible bond portfolio management. Departing from the traditional dynamic hedging strategy, this study incorporates the KD technical index to formulate a selective hedging strategy to account for asymmetric behavior of investors under bull and bear market conditions. Empirical investigations of five TSE-listed convertible bonds are provided to validate our proposed method. Consistent with the hedging literature, the valuation model with minimum tracking errors outperforms the others. In line with our expectation, transaction cost is an important issue. Moreover, the model takes into account the credit risk which generates the highest profitability. Finally, an incorporation of the KD index as threshold hedging scenario considerably improves the profitability of the underlying CB portfolio.

Key words: Option-based dynamic hedging, KD index, Credit risk, CB portfolio

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I. Introduction

Convertible bonds (CBs) are sophisticated financial instruments and widely traded in the Taiwan's capital market. The static hedging of CB was a standard trading strategy of more sophisticated Taiwanese investors in the past few years. However, the profitability of such a strategy has been eroded resulting from price change in underlying stock due to recent regulatory change on the conversion practice of CBs in

2003. In contrast to the static hedging strategy which is subject to the risk exposure of price change in the underlying stocks, the dynamic hedging strategy could eliminate the risk of price changes in the underlying stocks. Thus, the latter is evolving as a preferred trading strategy in Taiwan's capital market.

Yet, valuation literature presents a variety of model specifications for CBs with no conclusive finding on the best model. Moreover, the traditional approach failed to account for default risks of CB issuers. More recent studies of bonds have clearly demonstrated that credit risk indeed affects the profitability of a convertible bond portfolio (Tong, 1995; Krueger, 1999; Hung *et al*., 2002; Meyer, 2003; Beltratti, 2004). The objectives of this study are two-fold: first, to investigate which model is more appropriate for convertible bond valuation; second, to formulate a dynamic hedging strategy for convertible bond practitioners. This paper adopts four option-based dynamic delta-hedging models that account for the transaction costs and credit risk for convertible bond portfolio management. Departing from the traditional dynamic hedging strategy, this study extends the previous study by incorporating the KD technical index to formulate a selective hedging strategy to account for asymmetric behavior of investors under bull and bear market conditions. Empirical investigations of five TSE-listed convertible bonds are provided to demonstrate the feasibility of our proposed method. Consistent with the hedging literature, the valuation model with minimum tracking errors outperforms the others. In line with our expectation, transaction cost is an important issue. Moreover, the model takes into account the credit risk which generates the most attractive profitability. Finally, application of the KD index enhances the profitability of the underlying portfolio.

The remainder of this paper is organized as follows. Section 2 presents a review of the relevant literature on bond valuations. Section 3 provides a discussion of the experiment design in the study, which is followed in section 4 by an evaluation of the model via numerical examples. The paper concludes with a summary analysis of the findings in section 5.

II. Brief review of the literature

The value and hedge of a CB is sophisticated because of the nature of having many embed options. Traditional methods of pricing a convertible bond decompose value of a CB into the value of a straight bond and the conversion value. The optimal value of a convertible bond at any time before its maturity can be obtained by the discounted value of the straight bond and the conversion value that is higher. It can be formulated as:

$$\text{Value of CB} = \frac{\max\left(\text{straight value, conversion value}\right)}{\text{discount rate}}$$

The optimal time for the holder to exercise the conversion option is when the conversion value exceeds its market value. This method has been widely used by Poensgen (1965), Baumol *et al*. (1966), Weil *et al*. (1968) Walter *et al*. (1973) and Jennings (1974). The traditional method has serious shortcomings and tends to under-estimate the intrinsic value of a CB (Ingersoll, 1977; Cheung and Nelken, 1994).

There are two other distinctive approaches to value a CB. The first type is the contingent-claim approach. This more sophisticated approach values the convertible bond as a sum of a straight bond and a call option on the underlying stock. The pioneer of this approach can be traced back to the work of Black and Scholes (1973). They established the price of a European call option through a well known formula, which is the solution to a second order partial differential equation. This closed analytical solution conferred elegance to the proposed formulation and multiplied in extensive and complementary studies. However, Ingersoll (1977) indicated that the embed option of a convertible bond is of the American type. Thus, the risk discount rate can not be determined easily. Ingersoll (1977), who assumes the specific stochastic process of interest rate and underlying equity price and then applied Ito's lemma to derive a partial differential? equation and priced the CB with closed-form solution. Among the others, Brennan and Schwartz (1977, 1980) extend the previous work by incorporating arbitrage-free argument and exploiting the appropriate boundary conditions. Then, they price CB by solving the partial differential equation. Since the work by Ingersoll (1977) and Brennan *et al*. (1977), the contingent-claims approach to pricing CBs is the norm. However, the presence of senior debts and multiple classes of common stocks in a capital structure of a firm makes this approach difficult to capture the value of a CB (Brrone-Adesi *et al*. 2003). In addition, this method is not exact, since the exercise price on the equity option is not fixed. Recently, Gong *et al*. (2006) adopted the finite difference method to solve the Black and Scholes equation through a multi-stage compound-option model and provide evidence to support the assertion that finite difference method generates higher accuracy and efficiency.

The second type is the traditional binomial (or tree) approach. To price a CB under this approach, the first step is to determine the payoff at the terminal nodes of the stock price tree, and subsequently roll back to the initial node to obtain the price of the underlying CB. This approach has been widely used in practice (Hung 2002, Hull 2003, Jaimungal and Wang 2006).

The recent CB valuation literature has focused on the price effect of a default risk or, more specifically, the potential price change resulting from default of an issuing firm (Tong, 1995; Krueger, 1999; Hung *et al*., 2002; Meyer, 2003; Ayache *et al*., 2003; Beltratti, 2004). The valuation of CB with default risk falls into two approaches, the structural approach and the reduced-form approach. The structural approach treats default as an endogenous event and values the lower boundary on firm value that triggers reorganization of a firm (Merton, 1974; Leland, 1994). The reduced-form approach characterizes default event exogenously by the jump process, and focuses on modeling the likelihood of default (Jarrow and Turnbull, 1995). Most research utilizes the structure form approach to pricing CBs with default risk (Takahashi *et al*. 2001). For details, it can be referred to the works of Nyborg (1996) and Overbeck *et al*. (2005).

Credit risk is typically incorporated in a CB valuation model by adding a constant option-adjusted spread or market-observed spread (Takahashi *et al*., 2001; Hung 2002; Barone-Adesi *et al*., 2003; Overbeck *et al*., 2005). However, Barone-Adesi *et al*. (2003) suggest that the later is a preferred approach because the former unnecessarily penalizes the credit riskless stock upside of the CB.

In sum, the literature on CB valuation clearly indicates the important of incorporating credit risk for pricing CBs.

III. Experimental Design

Our empirical implementation proceeded in two stages. First of all, four option-based models are formulated to estimate the Delta value. Second, appropriate hedge ratios are calculated to dynamically rebalance bond portfolio to circumvent the risk of price change in the underlying securities.

We first convert the stock prices to their return series, and then calculate their volatilities by four kinds of moving averages as follows:

(1) Simple moving average

$$\sigma = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(R_t - \overline{R})^2}\ , \qquad (1)$$

where N is the sample size, R_t is the return at time t, $\overline{R}$ is the average of R_t.

(2) Exponential weighted moving average

$$\sigma = \sqrt{(1-\lambda)\sum_{t=1}^{N}\lambda^{t-1}(R_t - \overline{R})^2}\ , \qquad (2)$$

λ is a parameter to determine the weighting over time.

(3) Implied volatility

The implied volatility is inferred from the Black-Scholes Model (1974).

Black and Scholes (1973) assumed that the underlying stock price follows a geometric Brownian motion with constant volatility,

$$\frac{dS}{S} = \mu dt + \sigma dz \,, \tag{3}$$

where μ is the expected return, σ is the volatility, and z is the Brownian motion.

(4) GARCH-model implied Volatility:

Vtzal (1997) suggests that implied volatility derived from observed option prices utilizing a constant volatility models tends to understate historical volatilities. Moreover, an extensive literature has documented that the use of a single factor model should incorporate time varying volatility in the valuation framework (Hull and White, 1990, 1993; Longstaff and Schwartz, 1992; Vetzal, 1997; Beltratti, 2004; Bollerslev and Zhou, 2006). Following the pioneer work of Bollerslev (1986, 1992), the GARCH class has become a superior model in assessing the stochastic volatility of financial instruments (Gerlach *et al*., 2006). The literature on the valuation of CB has documented that the exponential GARCH models have some advantages over the GARCH class of models (Nelson (1991), Vetzal (1997)). For simplicity, we adopt standard GARCH(1,1) model to capture the stochastic return volatility of the underlying assets. The GARCH model can be formulated as follows:

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t) \\ \sigma_t^2 &= \alpha + \beta_1 \sigma_{t-1}^2 + \beta_2 r_{t-1}^2 \end{aligned} \tag{4}$$

Furthermore, the proxy of credit risk needs to be addressed before we enter the main body of our proposed modeling. Following Tsiveriotis *et al*. (1998), we use market-observed credit spread of straight bond in the valuation of CBs. We use the one-year deposit rate as the riskless interest rate. The risk premium is determined from TCRI (Taiwan Corporate Risk Index). TCRI ranks each individual enterprise by nine risk levels. We assigned every level according to its risk by a premium. These premiums are listed below:

Table1. Market-observed credit spread in Taiwan

TCRI	Risk Premium	TWRI	Risk Premium
1	0.25%	twAAA+	0.125%
2	0.50%	twAAA	0.250%
3	0.75%	twAAA-	0.375%
4	1.00%	twAA+	0.625%
5	1.50%	twAA	0.875%
6	2.00%	twAA-	1.125%
7	3.00%	twA+	1.500%
8	4.00%	twA	1.875%
9	5.00%	twA-	2.250%
		twBBB+	2.750%
		twBBB	3.250%
		twBBB-	3.750%

Then, we compare the performance of four pricing models for the CBs, and choose the best models to perform the dynamic hedging strategy. In the first stage, four pricing models are compared according to their tracking errors. The four models in our study are as follows:

(1) Model 1:

Although the pioneered model of contingent claim approach, Black and Scholes (1973) option pricing formula, is subject to several weaknesses in the valuation of CBs, the model is widely used by practitioners and its proponents because of the advantage on the ease of implementation and it entails low computational cost (Ammann, 2003; Gong *et al*., 2006;). Following Black and Scholes (1973), we take value of a CB as the straight bond plus a call option, or

$$CB = K + \max(\gamma V_T - K, 0)\,, \tag{6}$$

where K is the straight bond value, and a simple European call option is specified to calculate the option value.

$$Call = S \bullet N(d_1) - K(1+r)^{-T} N(d_2)$$
$$d_1 = \frac{In\frac{S}{K} + \left(r + 0.5\sigma^2\right)T}{\sigma\sqrt{T}} \tag{7}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

where S is the price of underlying security, K is the strike price, r is the risk free interest rate or discount rate, N is the distribution function of a normal random variable, and T represents the maturity.

(2) Model 2:

The most important drawback of the model, B-S model, is that the model refers to as European-style option where the CB can only be converted into a common stock at maturity. Nevertheless, most of the CBs can be exercised prior to maturity. The Binomial-tree approach is often utilized to circumvent this drawback (Ammann, 2003). Thus, we also adopt the standard binomial-tree model in the study. In practice, a binomial tree is employed to model the possibility of early exercise accounting for the feature of an American style option. The tree model can be formulated as follows:

$$Call = \frac{1}{a^n}\sum_{j=m}^{n}\binom{n}{j}P^j(1-P)^{n-j}\left(u^j d^{n-j} S - K\right) \tag{8}$$

$$m > \frac{InK - InS - nInd}{Inu - Ind}$$

$$a = e^{r\Delta t}$$

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = e^{-\sigma\sqrt{\Delta t}}$$

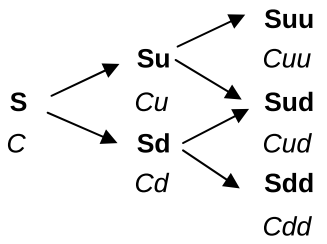
$$p = \frac{a-d}{u-d}$$

where Δt is the time interval used in the tree model.

(3) Model 3

To further account for the important state variable, credit risk, we apply the one factor multiple tree model of Connolly (1998) to valuate the underlying CBs. To fully reflect the nature of differential discount rates in payoff of a CB, we revised the model of Connolly in that the cash flow is discount at a risk free rate, r_{fa}, at period t whenever the value of its underlying stock exceeds the value of the straight bond at time t+1 (if the bonds are converted, the holder of equity is not subject to default risk as cited in Tsiveriotis, 1998). Otherwise, a risky discount rate, r_a, should be introduced.

Although the maximum number of trees in our empirical implementation is 50, we take a two-period discrete time model, for example, to elaborate the process for simplicity. We assume the conversion ratio is 1:1; in other words, one CB can be converted to one share of the stock.



(9)

$\mathrm{Su} = \mathrm{S} \times \mathrm{u}$, $\mathrm{Sd} = \mathrm{S} \times d$,

$\mathrm{Suu} = \mathrm{S} \times \mathrm{u} \times \mathrm{u}$, $\mathrm{Sud} = \mathrm{S} \times \mathrm{u} \times d$, $\mathrm{Sdd} = \mathrm{S} \times d \times d$,

$\mathrm{Cuu} = Max(\mathrm{Suu}, \mathrm{B})$, $\mathrm{Cud} = Max(\mathrm{Sud}, \mathrm{B})$, $\mathrm{Cdd} = Max(\mathrm{Sdd}, \mathrm{B})$

$$\mathrm{Cu} = \begin{cases} \dfrac{Cuu \times p}{(1+r_f)} + \dfrac{Cud \times q}{(1+r_f)} & \text{if Suu} > Cuu \text{ and Sud} > \text{Cud} \\ \dfrac{Cuu \times p}{(1+r_f)} + \dfrac{Cud \times q}{(1+r)} & \text{if Suu} > Cuu \text{ and Sud} \le \text{Cud} \\ \dfrac{Cuu \times p}{(1+r)} + \dfrac{Cud \times q}{(1+r)} & \text{if Suu} \le Cuu \text{ and Sud} \le \text{Cud} \end{cases} \quad (10)$$

$$\mathrm{Cd} = \begin{cases} \dfrac{Cud \times p}{(1+r_f)} + \dfrac{Cdd \times q}{(1+r_f)} & \text{if Sud} > Cud \text{ and Sdd} > \text{Cdd} \\ \dfrac{Cud \times p}{(1+r_f)} + \dfrac{Cdd \times q}{(1+r)} & \text{if Sud} > Cud \text{ and Sdd} \le \text{Cdd} \\ \dfrac{Cud \times p}{(1+r)} + \dfrac{Cdd \times q}{(1+r)} & \text{if Sud} \le Cud \text{ and Sdd} \le \text{Cdd} \end{cases}$$

$$\mathrm{C} = \begin{cases} \dfrac{Cu \times p}{(1+r_f)} + \dfrac{Cd \times q}{(1+r_f)} & \text{if} \quad \text{Su} > Cu \text{ and Sd} > \text{Cd} \\ \dfrac{Cu \times p}{(1+r_f)} + \dfrac{Cd \times q}{(1+r)} & \text{if} \quad \text{Su} > Cu \text{ and Sd} \le \text{Cd} \\ \dfrac{Cu \times p}{(1+r)} + \dfrac{Cd \times q}{(1+r)} & \text{if} \quad \text{Su} \le Cu \text{ and Sd} \le \text{Cd} \end{cases} \quad (11)$$

where the hedge ratio (or delta) is

$$H = \frac{Cu - Cd}{Su - Sd} \quad , \quad (12)$$

and other parameters are listed below:

S: stock price , C: CB price , B: bond value at maturity , σ: volatility ,

r_{fa} : risk free rate , $r_a = r_{fa} + \text{credit spread}$,

$$r_f = (1+r_{fa})^{\frac{1}{n}} - 1, \quad r = (1+r_a)^{\frac{1}{n}} - 1,$$

$$u = e^{\sigma\sqrt{\frac{1}{n}}}, \quad d = \frac{1}{u}, \quad p = \frac{1-d}{u-d}, \quad q = 1-p.$$

(4) Model 4

For simplicity, the riskless yield and risky yield are often assumed to be constant. When rolling back through the tree, two optimal conditions on each node of the tree must be checked. The optimal time for the holder to exercise the conversion option is when the conversion value exceeds its market value. On the other hand, the optimal time for the issuer to call back the convertible bond is when the convertible bond's market value exceeds its call price. When the issuer exercises the call option, it often forces the holders to convert earlier.

Model 4 adopts the most popular option pricing model with credit risk by Tsiveriotis & Fernandes(1998). Tsiveriotis and Fernandes suggest that the value of a CB has components involving different default risks. A new hypothetical security, the "cash-only part of the CB" or the "COCB" is defined; the holder of the COCB is entitled to all cash flows, but no equity flows. A single-factor model is built in which the CB is viewed as an equity-only derivative. A numerical solution is given for the model. The model is described using two parabolic partial differential equations. The value of CB, , follows the Black-Schole's equation

$$\frac{\partial u}{\partial t} + r_g s \frac{\partial u}{\partial S} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 u}{\partial S^2} - (r + r_c)u + f(u, S, t) = 0 \tag{13}$$

where

u : the value of CB

S : the price of the underlying stock

r_c : the observable credit spread implied by the non - convertible bond of the same issure for matrities similar to that of the CB

r_g : the growth of the stock

r : the risk - free interest rate

COCB is also a derivative security with the same stock described by

$$\frac{\partial u}{\partial t} + r_g s \frac{\partial u}{\partial S} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 u}{\partial S^2} + r(u - v) - (r + r_c)v + f(t) = 0 \tag{14}$$

where

v : the value of the COCB

These two equations differ only in the discounting terms, which reveal the different credit treatment of cash payments and equity upside. For these two equations, the authors discuss in detail the final, boundary and other conditions that the solutions have to satisfy due to conversion, callability, and puttability. The boundary conditions are as follows:

Maturity condition：

$$u(S,T)=\begin{cases}\alpha S & \mathrm{S} \geq \mathrm{B}/\alpha \\ \mathrm{B} & \text{elsewhere}\end{cases} \tag{15}$$

$$v(S,T)=\begin{cases}0 & \mathrm{S} \geq \mathrm{B}/\alpha \\ \mathrm{B} & \text{elsewhere}\end{cases} \tag{16}$$

B Put price at maturity

α : conversion ratio

Upside constraints due to conversion

$$u \geq \alpha S \quad \text{for } t \in [0,T] \tag{17}$$

$$v = 0 \quad \text{if } \mathrm{u} \leq \alpha \mathrm{S} \text{ for } t \in [0,T] \tag{18}$$

Callability constraints：

$$u < \max(B_c, \alpha S) \quad \text{for } t \in [T_c, T] \tag{19}$$

$$v = 0 \text{ if } \mathrm{u} \geq \mathrm{B_c} \quad \text{for } t \in [T_c, T] \tag{20}$$

Putability constraint：

$$u \geq B_p \quad \text{for } t \in \lfloor T_p, T \rfloor \quad \text{putability unoccur} \tag{21}$$

$$v = B_p \quad \text{if } \mathrm{B} < \mathrm{B_P} \quad \text{for } t \in \lfloor T_p, T \rfloor \quad \text{putability occur} \tag{22}$$

To compare the effectiveness of these four models, we utilize the approach of Takahashi et al. (2001) using absolute error ratio (AER) as the proxy of tracking errors. The absolute error ratio (AER) is defined as follows:

$$AER = \frac{|\text{Market - Model price}|}{\text{Model price}} \tag{23}$$

For static or dynamic hedges, we calculate the Delta coefficients of these models.

We construct a portfolio by long one share of CB and short Delta unit of its stock, namely, the value of the portfolio is obtained as

$$CB_0 - \Delta_0 CR_0 S_0 . \qquad (24)$$

For every day trading, we adjust the position of the stock according to the Delta if its price change exceeds some upper or lower limits. The trading strategy can be formulated as follows:

$$\begin{array}{ll} CB_0 - \Delta_0 CR_0 S_0 & \text{if } \mathrm{S}_1 < Su_0 \text{ and } \mathrm{S}_1 > \mathrm{S}d_0 \\ CB_0 - \Delta_0 CR_0 S_0 - (\Delta_1 CR_1 - \Delta_0 CR_0) S_1 , & \text{otherwise.} \end{array} \qquad (25)$$

where the upper and lower limits are set as below:

$Su_0 = S_0(1+\mathrm{m})$ upper limit,

$Sd_0 = S_0(1\text{-}\mathrm{m})$ lower limit.

The payoff of the portfolio at every time interval without taking account of the transaction cost is as follows:

$$P_0 = (\max(parity , CB_0) - CB_0) + (\Delta_0 CR_0 S_0 - \Delta_0 CR_0 S_0)$$

$$\begin{aligned} P_1 &= (\max(parity , CB_1) - CB_0) + \Delta_0 CR_0 S_0 + (\Delta_1 CR_1 - \Delta_0 CR_0) S_1 - \Delta_1 CR_1 S_1 \\ &= (\max(parity , CB_1)_1 - CB_0) + \Delta_0 CR_0 (S_0 - S_1) \end{aligned}$$

$$\begin{aligned} P_2 &= (\max(parity, CB_2) - CB_0) + \Delta_0 CR_0 S_0 + (\Delta_1 CR_1 - \Delta_0 CR_0) S_1 + ((\Delta_2 CR_2 - \Delta_1 CR_1) S_2 - \Delta_2 CR_2 S_2 \\ &= (\max(parity, CB_2) - CB_0) + \Delta_0 CR_0 (S_0 - S_1) + \Delta_1 CR_1 (S_1 - S_2) \end{aligned}$$

$$\begin{aligned} P_n &= (\max(parity, CB_n) - CB_0) + \Delta_0 CR_0 (S_0 - S_1) + \Delta_1 CR_1 (S_1 - S_2) + ... + \Delta_{n-1} CR_{n-1} (S_{n-1} - S_n) \\ &= (\max(parity, CB_n) - CB_0) + \sum_{i=0}^{n-1} \Delta_i CR_i (S_i - S_{i+1}) \end{aligned} \qquad (26)$$

where

P_n : the payoff at the n-th period

S_n : stock price at the n-th period

CB_n : CB market value at the n-th period

CR_n : the conversion ration at the n-th period

$$Parity = \frac{\text{spot price}}{\text{conversion price}}$$

Δ_n : the Delta at the n-th period

On the other hand, when the transaction cost is included, the payoff of the portfolio is

$$P_n = (\max(parity , 0.999\, CB_n - 1.001\, CB_0) + \sum_{i=0}^{n-1} 0.994775\ \Delta_i CR_i (S_i - S_{i+1}) - 0.00665\ \Delta_n CR_n S_n . \qquad (27)$$

Unlike the findings of the early literature on the equity market anomalies, numerous researchers suggest that the technical trading strategy may be profitable in many Asian equity markets in general, and Taiwan's equity market in particular (Ratner *et al.*,1999; Ito, 1999; Lai *et al*., 2006). These findings lend support to our final empirical design. The study employs the most popular technical trading rule, 5 day KD index, on short-term market movement for practitioners in the equity market in Taiwan as trigger scenario in hedging the arbitrage of CBs. The KD index is calculated as follow:

$$K(5)_t = \frac{2}{3}K(5)_{t-1} + \frac{1}{3}RSV \quad (28)$$

$$D(5)_t = \frac{2}{3}D(5)_{t-1} + \frac{1}{3}K(5)_t$$

(29)

$$RSV = \frac{C - L5}{H5 - L5} \times 100$$

(30)

Where C, L5 and H5 are the closing price of the previous transaction day, the lowest price and highest price in the latest 5 transaction days, respectively. The value of K and D assumes to be 50 If there is no K and D in the previous transaction day.

The hedge ratio of our strategy at time t, $\Delta_{adjusted\_t}$, is defined as follows：

$$\Delta_{adjusted\_t} = \Delta_{t-1} + \alpha(\Delta_t - \Delta_{t-1}) \text{ if } \Delta_t \geq \Delta_{t-1}$$

$$\Delta_{adjusted\_t} = \Delta_{t-1} + \beta(\Delta_t - \Delta_{t-1}) \text{ if } \Delta_t < \Delta_{t-1}$$

$$\alpha = 0.5 \text{ if } K(5)_{t-1} < D(5)_{t-1} \text{ and } K(5)_t > D(5)_t$$
$$\alpha = 1.5 \text{ if } K(5)_{t-1} > D(5)_{t-1} \text{ and } K(5)_t < D(5)_t$$
$$\alpha = 1 \quad \text{otherwise}$$
$$\beta = 1.5 \text{ if } K(5)_{t-1} < D(5)_{t-1} \text{ and } K(5)_t > D(5)_t$$
$$\beta = 0.5 \text{ if } K(5)_{t-1} > D(5)_{t-1} \text{ and } K(5)_t < D(5)_t \quad (32)$$
$$\alpha = 1 \quad \text{otherwise,}$$

where α and β are stated adjustment multipliers.

IV. Numerical examples

Due to the low liquidity of convertible bonds in Taiwan's equity market, we selected those bonds with larger trading volumes. The data-sampling period is from

April 1, 2002 to June 30, 2004. We chose five convertible bonds: Yuanta 2nd CB (60042), Compeq 2nd CB (23132), OPTP CB(23401), Shin Kong CB 1A(28881), Yang Ming CB 2A(26092). These data are sampled from TEJ (*Taiwan Economic Journal*).

In the first stage, we compare the tracking errors of these four models. The model with minimal tracking error will be used in the second stage to perform dynamic hedge and arbitrage. In the second stage, we also compare the hedging performance based on the frequency of rebalance and the influence of transaction costs.

Figures 1 to 5 and table 1 present the empirical results of the first stage process, respectively. For Yuanta 2nd CB (60042), Compeq 2nd CB (23132), OPTP CB (23401), Shin Kong CB 1A (28881), Yang Ming CB 2A (26092), the predicted prices and actual prices of the observed CBs are plotted in Figures 1 to 5. Table 1 indicates tracking errors of all observed CBs. Complying with the literature in the valuation of CBs, all four models are consistently overstating the actual prices of CBs. Nevertheless, Model 4, where the credit risks of CBs are accounted for, has the smaller tracking error, and hence is the most effective underlying model.

Figure 1. Predicted prices and actual prices of Yuanta 2nd CB

Figure 2. Predicted prices and actual prices of Compeq 2nd CB

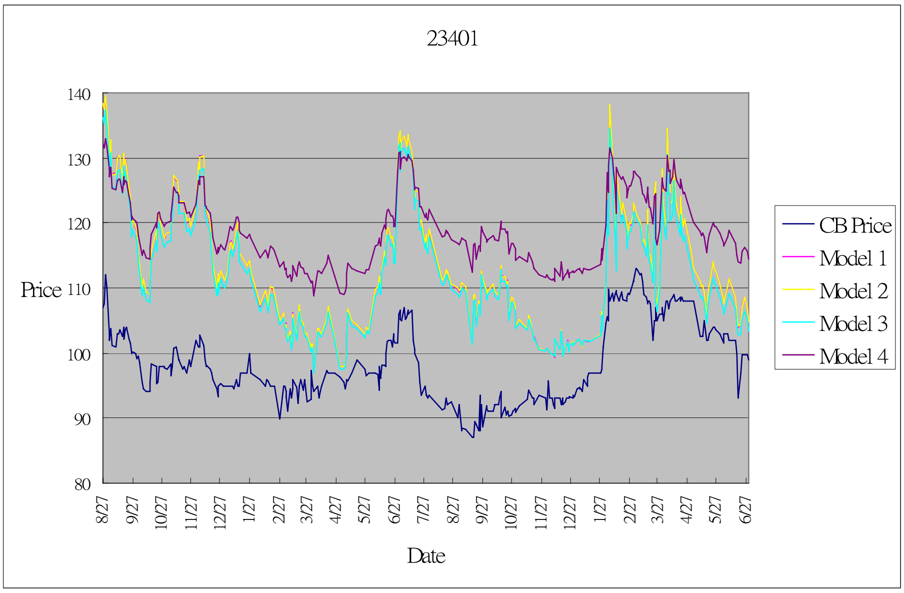


Figure 3. Predicted prices and actual prices of OPTP CB

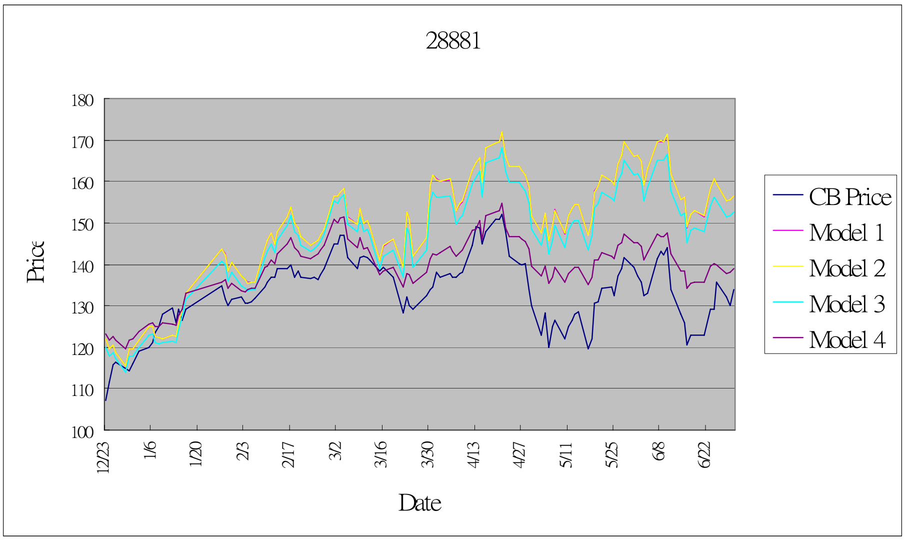


Figure 4. Predicted prices and actual prices of Shin Kong CB 1A

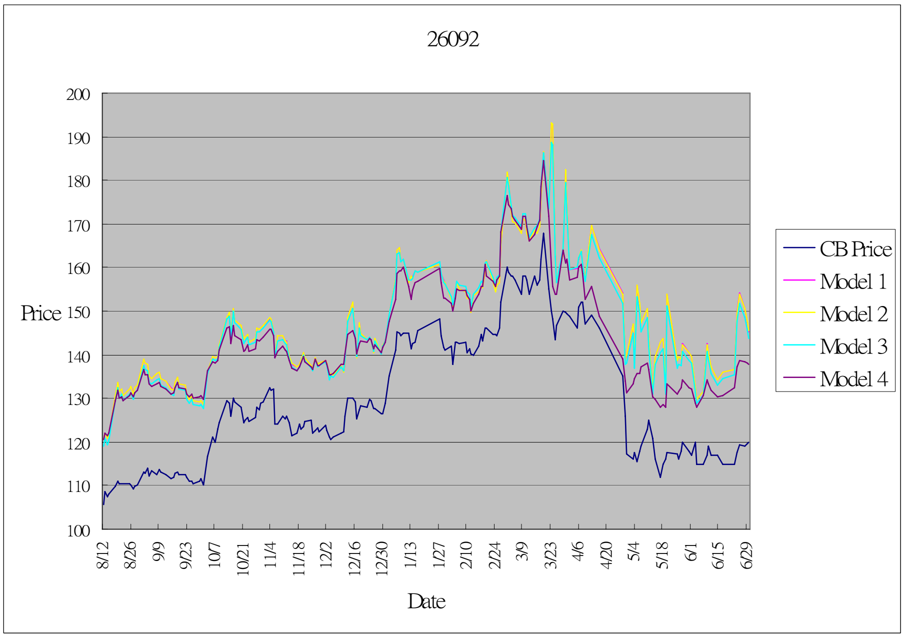


Figure 5. Predicted prices and actual prices of Yang Ming CB 2A

CB	Tracing Error			
	Model 1	Model 2	Model 3	Model 4
60012	0.058826	0.058898	0.053393	0.050456
23132	0.147159	0.147302	0.138204	0.087045
23401	0.131965	0.13204	0.123235	0.17274
28881	0.108604	0.108596	0.09226	0.045087
26092	0.124061	0.124107	0.12035	0.104998
Average	0.116284	0.116353	0.108175	0.106717

To make the comparison more thoughtful, we consider the arbitrage performance of each model with or without transaction costs, and with and without the constraints on margin trading including long or short. The performances of the dynamic hedging and arbitrages are reported in table 2 to 6.

Interestingly, all of our four models produce positive profits. Under the constraint of no long margin trade, model 1 outperforms the others. Under the constraint of no short margin trading, model 4 out performs the others. However, on average, model 4 has the highest return.

Table 2. The performance of dynamic arbitrage of Yuanta 2nd CB

CB		Performance	Average			
			Model 1	Model 2	Model 3	Model 4
60042	Without consider constraint of short sale	With Transaction costs	13.954%	13.992%	12.371%	14.575%
		Without Transaction costs	14.513%	14.551%	13.109%	15.559%
		Ratio of Transaction costs	4.053%	4.040%	6.144%	7.105%
	Consider constraint of short sale	With Transaction costs	13.693%	13.730%	12.253%	15.022%
		Without Transaction costs	15.009%	15.046%	13.741%	16.792%
		Ratio of Transaction costs	9.664%	9.635%	12.358%	12.182%
	Consider constraint of Short sale and adjusted hedge ratio	With Transaction costs	13.864%	13.900%	12.495%	15.762%

Table 3. The performance of dynamic Arbitrages of Compeq 2nd CB

CB		Performance	Average			
			Model 1	Model 2	Model 3	Model 4
23132	Without consider constraint of short sale	With Transaction costs	20.553%	20.550%	20.430%	14.581%
		Without Transaction costs	21.374%	21.366%	21.224%	15.665%
		Ratio of Transaction costs	4.016%	3.990%	3.914%	7.690%
	Consider constraint of short sale	With Transaction costs	5.139%	5.136%	5.247%	4.279%
		Without Transaction costs	6.354%	6.346%	6.438%	5.609%
		Ratio of Transaction costs	24.529%	24.471%	23.508%	34.598%
	Consider constraint of Short sale and adjusted hedge ratio	With Transaction costs	5.205%	5.202%	5.248%	4.998%

Table 4. The performance of dynamic Arbitrages of OPTP CB

CB		Performance	Average			
			Model 1	Model 2	Model 3	Model 4
23401	Without consider constraint of short sale	With Transaction costs	14.819%	14.750%	14.480%	9.165%
		Without Transaction costs	15.347%	15.274%	15.003%	9.544%
		Ratio of Transaction costs	3.522%	3.517%	3.580%	4.132%
	Consider constraint of short sale	With Transaction costs	16.646%	16.585%	16.266%	10.399%
		Without Transaction costs	17.416%	17.352%	17.028%	10.953%
		Ratio of Transaction costs	4.606%	4.605%	4.670%	5.326%
	Consider constraint of Short sale and adjusted hedge ratio	With Transaction costs	16.692%	16.614%	16.357%	10.426%

Table 5. The performance of dynamic Arbitrages of Shin Kong CB 1A

CB		Performance	Average			
			Model 1	Model 2	Model 3	Model 4
28881	Without consider constraint of short sale	With Transaction costs	10.835%	10.835%	10.140%	18.239%
		Without Transaction costs	11.621%	11.621%	10.917%	19.247%
		Ratio of Transaction costs	7.261%	7.261%	7.702%	5.546%
	Consider constraint of short sale	With Transaction costs	8.931%	8.931%	8.259%	17.012%
		Without Transaction costs	10.417%	10.417%	9.733%	18.532%
		Ratio of Transaction costs	16.659%	16.659%	17.971%	8.986%
	Consider constraint of Short sale and adjusted hedge ratio	With Transaction costs	10.335%	10.335%	9.730%	18.890%

Table 6. The performance of dynamic Arbitrages of Yang Ming CB 2A

CB		Performance	Average			
			Model 1	Model 2	Model 3	Model 4
26092	Without consider constraint of short sale	With Transaction costs	6.172%	6.172%	4.979%	9.063%
		Without Transaction costs	6.703%	6.703%	5.641%	10.020%
		Ratio of Transaction costs	8.688%	8.688%	14.766%	11.441%
	Consider constraint of short sale	With Transaction costs	3.547%	3.547%	2.329%	6.996%
		Without Transaction costs	4.860%	4.860%	3.761%	8.592%
		Ratio of Transaction costs	37.322%	37.322%	76.327%	24.177%
	Consider constraint of Short sale and adjusted hedge ratio	With Transaction costs	3.711%	3.711%	2.425%	7.060%

Table 7. Mean performance of arbitrage

CB		Performance	Mean performance			
			Model 1	Model 2	Model 3	Model 4
All CBs	Without consider constraint of short sale	With Transaction costs	13.267%	13.260%	12.480%	13.124%
		Without Transaction costs	13.912%	13.903%	13.179%	14.007%
		Ratio of Transaction costs	5.508%	5.499%	7.221%	7.183%
	Consider constraint of short sale	With Transaction costs	9.591%	9.586%	8.871%	10.742%
		Without Transaction costs	10.811%	10.804%	10.140%	12.095%
		Ratio of Transaction costs	18.556%	18.538%	26.967%	17.054%
	Consider constraint of Short sale and adjusted hedge ratio	With Transaction costs	9.961%	9.952%	9.251%	11.427%

This section examines the arbitrage performance of sample CBs with variation in their rebalance frequencies. The results are listed in Tables 8 and 9. Since the dynamic rebalance strategy entails transaction cost to the portfolio, too frequent rebalance maybe eroding the profit. Consequently, we compare the arbitrage performances of these four models under different rebalance scenarios. The higher degree of rebalances, indeed, increases the cost of transaction and consumes the profit of underlying portfolio to some extent. However, the hedging frequency somewhat between 8-10% seems to generate a better return. On average, model 4 outperforms the others with and without considering the constraint on short trading.

Table 8. The arbitrage performance under different portfolio adjustment frequencies (Without consider constraint on short sale)

	With transaction costs				Without transaction costs			
Hedge frequency	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
0%	13.079%	13.064%	11.768%	12.543%	14.009%	13.990%	12.801%	13.984%
1%	13.098%	13.092%	11.770%	12.815%	13.986%	13.976%	12.755%	14.214%
2%	13.393%	13.369%	12.190%	11.805%	14.190%	14.160%	13.064%	12.996%
3%	13.336%	13.316%	12.385%	11.585%	14.061%	14.040%	13.168%	12.538%
4%	13.092%	13.105%	12.172%	12.280%	13.723%	13.736%	12.866%	13.203%
5%	12.839%	12.818%	12.269%	13.061%	13.423%	13.400%	12.896%	13.808%
6%	13.063%	13.044%	12.345%	13.294%	13.607%	13.586%	12.910%	13.933%
7%	13.384%	13.402%	12.982%	12.977%	13.919%	13.935%	13.557%	13.610%
8%	13.738%	13.724%	13.148%	14.491%	14.236%	14.222%	13.676%	15.120%
9%	13.547%	13.563%	12.910%	14.001%	14.030%	14.047%	13.426%	14.576%
10%	13.363%	13.360%	13.341%	15.516%	13.844%	13.840%	13.847%	16.098%

Table 9. The arbitrage performance under different portfolio adjustment frequencies (consider the constraint on short sale)

	With transaction costs				Without transaction costs			
Hedge frequency	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
0%	9.418%	9.407%	8.212%	10.507%	10.918%	10.904%	9.810%	12.411%
1%	9.435%	9.429%	8.208%	10.744%	10.894%	10.884%	9.757%	12.605%
2%	9.679%	9.654%	8.598%	9.711%	11.050%	11.021%	10.043%	11.367%
3%	9.669%	9.652%	8.751%	9.326%	10.971%	10.953%	10.105%	10.753%
4%	9.410%	9.425%	8.541%	9.981%	10.616%	10.630%	9.807%	11.372%
5%	9.154%	9.137%	8.635%	10.562%	10.316%	10.296%	9.836%	11.782%
6%	9.344%	9.327%	8.645%	10.668%	10.460%	10.440%	9.778%	11.787%
7%	9.698%	9.721%	9.352%	10.525%	10.809%	10.829%	10.503%	11.641%
8%	10.059%	10.046%	9.541%	12.024%	11.131%	11.118%	10.639%	13.138%
9%	9.941%	9.956%	9.358%	11.347%	11.003%	11.019%	10.444%	12.388%
10%	9.694%	9.691%	9.734%	12.761%	10.754%	10.751%	10.819%	13.806%

Table 10 presents empirical results of these models using KD index as hedging trigger variable. The results suggest that the application of KD index, on average, significantly improves the performances of our CB hedging portfolio.

Tabel 10. Comparison of performances with and without KD threshold

CB		Average			
		Model 1	Model 2	Model 3	Model 4
60042	unadjusted hedge ratio	13.693%	13.730%	12.253%	15.022%
	Adjusted hedge ratio	13.864%	13.900%	12.495%	15.762%
23132	unadjusted hedge ratio	5.139%	5.136%	5.247%	4.279%
	Adjusted hedge ratio	5.205%	5.202%	5.248%	4.998%
23401	unadjusted hedge ratio	16.646%	16.585%	16.266%	10.399%
	Adjusted hedge ratio	16.692%	16.614%	16.357%	10.426%
28881	unadjusted hedge ratio	8.931%	8.931%	8.259%	17.012%
	Adjusted hedge ratio	10.335%	10.335%	9.730%	18.890%
26092	unadjusted hedge ratio	3.547%	3.547%	2.329%	6.996%
	Adjusted hedge ratio	3.711%	3.711%	2.425%	7.060%
Rate of Returns	unadjusted hedge ratio	9.591%	9.586%	8.871%	10.742%
	Adjusted hedge ratio	9.961%	9.952%	9.251%	11.427%
	% of excess return	3.860%	3.822%	4.290%	6.383%

V. Conclusions

The dynamic hedging strategy has evolved as a favored approach in managing convertible bond portfolios. Moreover, more recent bond valuation studies have clearly demonstrated that credit risk is an important factor related to the profitability of a convertible bond portfolio. The objective of this study is to formulate an option-based dynamic hedging model which accounts for credit risk for practitioners in managing convertible bond portfolio. This paper adopts four option-based dynamic delta-hedging models that account for the transaction costs and credit risk for convertible bond portfolio management. Departing from the traditional dynamic hedging strategy, this study incorporates the KD technical index to formulate a selective hedging strategy to account for asymmetric behavior of investors under bull and bear market conditions. Our empirical implementation proceeded in two-stages. First of all, four option-based models are established to estimate the Delta value. Second, appropriate hedge ratios are calculated to rebalance bond portfolio to circumvent the risk of price change in the underlying securities.

Empirical investigations of five TSE-listed convertible bonds are provided to demonstrate the feasibility of our proposed method. Consistent with the literature in dynamic hedging, the valuation model with minimum tracking errors outperforms the others, and dynamic delta hedging in our CB portfolio produces significant positive return (Krishnan *et al.*, 2002; Gondzio *et al*., 2003; Meyer, 2003; Beltratti *et al*., 2004) . In line with our expectation, transaction cost is an important issue as documented by Gondzio et al. (2003). Moreover, the model takes into account credit risk, which generates the most attractive profitability. In comparison to the dynamic traditional hedging strategy, our scenario modeling for selective hedging with incorporation of KD as threshold variable could considerably improve performance of CBs. Overall, our results further shed light on the application technical trading strategies for practitioners in CB investment management.

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