Order Imbalance and the Dynamics of Index and Futures Prices

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Abstract

This study uses transaction records of index futures and the index stocks, with bid/ask price quotes, to examine the impact of stock market order imbalance on the dynamic behavior of index futures and cash index prices. Spurious correlation in the index is purged by using an estimate of the "true" index with highly synchronous and active quotes of individual stocks. A smooth transition autoregressive error-correction model (STAECM) is used to describe the nonlinear dynamics of the index and futures prices. Order imbalance in the cash stock market is found to significantly affect the error-correction dynamics of index and futures prices. Order imbalance impedes error-correction particularly when the market impact of order imbalance works against the error-correction force of the cash index, explaining why real potential arbitrage opportunity may persist over some time. Incorporating order imbalance in the framework significantly improves its explanatory power. The findings indicate that a stock market microstructure that allows a quick resolution of order imbalance promotes dynamic arbitrage efficiency between futures and the underlying stocks.

1. Introduction

Index arbitrageurs endeavor to capture any price discrepancy between index futures and the underlying index. Many authors document evidence of persistent and apparently exploitable arbitrage opportunities.

Grossman (1988) conjectures that arbitrage opportunities compensate arbitrageurs for providing liquidity in futures when trading is skewed toward one side of the market. Roll, Schwartz, and Subrahmanyam (2005) find that the level of arbitrage basis of the NYSE composite index futures is negatively related to market liquidity. They use quotes and effective spreads as proxies for market liquidity, finding a significant bi-directional causality relationship between the liquidity proxies and the level of the basis.

Fung (2004) uses order imbalance as a measure of both the direction and the extent of market liquidity; on average, positive order imbalance is associated with positive arbitrage basis, and negative order imbalance is associated with negative arbitrage basis. If arbitrage opportunities are related to liquidity of the stock market, the same market force could be impeding error-correction mechanisms that are supposed to prevent and eliminate such opportunities. This effect may help explain the persistence of arbitrage opportunities.

We examine how and to what extent stock market order imbalance affects error-correction dynamics in index and futures prices, using as an example the Hang Seng Index (HSI) and futures. HSI futures are among the most liquid contracts in the world; HSI represents over 75% of the total market capitalization of stocks listed in Hong Kong.

We use a smooth transition autoregressive error-correction model (STAECM) to capture the nonlinear error-correction dynamics of the index and futures prices. We avoid spurious correlation of the cash index due to infrequent trading and the bid-ask bounce by adopting a mid-quote index that is based on synchronous active quotes of all index stocks.

We examine the robustness of the empirical results by comparing findings before and during the 1997 financial market crisis in Hong Kong. The results show strong contemporaneous relationships between order imbalance and index and futures returns. Moreover, incorporating the market impact effect of order imbalance significantly improves explanatory power of the error-correction model. The benchmark framework, which does not consider the market impact of order imbalance, provides inconsistent inferences as to the error-correction dynamics of the two prices during the crisis period. The cash index become much more responsive to the arbitrage basis during the crisis, indicating that arbitrage-related trades increase when the market becomes more volatile. Order imbalance dictates price movements of both index and futures when the market impact of order imbalance is opposite to the force of error-correction, which helps explain the persistence of index arbitrage opportunities.

2. Literature review

In a frictionless market, arbitrage mechanisms should keep the futures price F_t close to its fair (or theoretical) value, F_t^*. Following Klemkosky and Lee (1991), we can write the fair futures price as $F_t^* = S_t(1 + r - d)^{T-t}$.[1] S_t is the index value at a particular time on day *t*, and *r* and *d* represent, respectively, the riskless rate of interest and the dividend yield of the index portfolio appropriate for the period before the contract matures on day *T*. Hence, *T* - *t* is the time to maturity of the contract; *T* and *t* are measured in fractions of a year.

It follows that the pricing error or the arbitrage basis z_t, defined as the difference between the natural logarithm of the actual and the fair futures prices (i.e., $z_t = \ln F_t - \ln F_t^*$), should always be close to zero.

Early research uses an Engle and Granger (1987) linear error-correction framework to model the conditional price dynamics of index and futures. Ignoring the lagged returns, a typical linear error-correction framework is as follows:

$$\Delta f_t = \omega_1 z_{t-1} + \pi_{1t} \quad (1)$$

$$\Delta s_t = \omega_2 z_{t-1} + \pi_{2t} \quad (2)$$

where $\Delta f_t = \ln F_t - \ln F_{t-1}$ is the futures return between *t* - 1 and *t* conditional on an observed pricing error z_{t-1} at time *t* - 1. Similarly, $\Delta s_t = \ln S_t - \ln S_{t-1}$ is the conditional index returns. ω_1 and ω_2 are the error-correction coefficients for the futures and index returns, respectively. π_{1t} and π_{2t} are the error terms for the two equations.

If z_{t-1} is positive and the futures is overpriced, long-stock short-futures arbitrage should cause the futures to drop and the index to rise; if z_{t-1} is negative and the futures is underpriced, the converse occurs. Hence the conditional futures return is expected to be opposite in sign to the pricing error, and ω_1 is expected to be negative. The conditional index return should have the same sign as the pricing error, and ω_2 is expected to be positive.

The expected error-correction adjustments in index and futures are confirmed by many empirical studies. Studies of the U.S. markets include Garbade and Silber (1983) and Stoll and Whaley (1990). Fung and Jiang (1999) document similar results for the Hong Kong market.

In reality, arbitrage involves substantial transaction costs in trading stocks and futures. Therefore, the futures price may fluctuate randomly when the arbitrage basis does not trigger arbitrage (Kawalla, 1987). It follows that the arbitrage basis reverts toward zero only when the deviation of the futures price is great enough to attract arbitrage. To capture the nonlinear pattern of the error correction dynamics, Yadav, Pope, and Paudyal (1994), Dwyer, Locke, and Yu (1996), and Martens, Kofman, and Vorst (1998) use a version of the threshold autoregressive error-correction (STAECM) process. Following Martens, Kofman, and Vorst (1998) and focusing on the error-correction term, a typical TAEC framework is as follows:

$$z_t = \rho_1 z_{t-1} + e_t^1 \qquad \ln F_{t-1} \le \ln F_{t-1}^* - c_1 \tag{3}$$

$$z_t = \rho_2 z_{t-1} + e_t^2 \qquad \ln F_{t-1}^* - c_1 < \ln F_{t-1} \le \ln F_{t-1}^* + c_2 \tag{4}$$

[1] See Cornell and French (1983) and Modest and Sundaresan (1983) for formal proofs of the relationship.

$$z_t = \rho_3 z_{t-1} + e_t^3 \qquad \ln F_{t-1} > \ln F_{t-1}^* + c_2 \tag{5}$$

where c_1 and c_2 are the costs or required compensation (in percentage of the fair futures price) associated with long-futures short-stock and long-stock short-futures arbitrage, respectively. $\ln F_{t-1}^* - c_1$ is the so-called lower no-arbitrage bound for the (logarithm of) the futures price, and $\ln F_{t-1}^* + c_2$ the upper no-arbitrage bound.

To trigger arbitrage, the futures price has to be either below $\ln F_{t-1}^* - c_1$ (i.e., in regime 1) or above $\ln F_{t-1}^* + c_2$ (i.e., in regime 3). Hence, the arbitrage basis is mean-reverting and the AR(1) coefficients in regimes 1 and 3 (i.e., ρ_1 and ρ_3) are expected to be significantly less than unity. If the futures price is within the no-arbitrage bounds, arbitrage does not take place, and the futures price may move randomly and the AR(1) coefficient in regime 2 (i.e., ρ_2) is expected to be close to unity.

This specification admits non-trivial transactions cost, and allows for asymmetries in the error-correction process of the index and futures prices in response to positive and negative pricing errors. Asymmetries may arise if there are significant institutional restrictions or cost and risk associated with short-selling of equity stock.[2] These effects dampen error-correction more when the futures is underpriced than when it is overpriced. If the constraints, costs, and risks against short-selling have a significant impact on the arbitrage relationship, c_1 will be higher than c_2.

[2] See Draper and Fung (2003) for a detailed discussion of the cost and risk associated with conducting short-stock long-futures arbitrage in the Hong Kong market.

Chan (1992) argues that quasi-arbitrage engaged in by institutional investors with sizable equity portfolios may reduce the impact of the constraints and costs on short-selling. That is, when the futures is underpriced (meaning that the cash stocks are relatively overpriced), institutions may sell part of their stock portfolio and substitute by going long the underpriced futures.

Kempf (1998) shows that the constraints against short-selling impede arbitrage. For the Hong Kong market, Fung and Draper (1999) show that both the extent and the frequency of underpricing is reduced after the Stock Exchange of Hong Kong (today's HKEx) lifted its restriction against stock short-selling.

Moreover, Jiang, Fung, and Cheng (2001) find that the contemporaneous relationship between index and futures strengthens when short-selling is allowed. The result is particularly strong in falling market situations and when the index is overpriced. Hence, the impact of the costs and constraints against short-selling on index and futures dynamics is an empirical issue.

Dwyer, Locke, and Yu (1996) examine the nonlinear dynamics between the S&P 500 futures price and the spot index. Their results show that the model better explains the price dynamics than the linear error-correction model. Martens, Kofman, and Vorst (1998) apply a similar framework to estimate a band around the theoretical S&P 500 futures price where arbitrage is not profitable for most arbitrageurs. Their results show that the arbitrage thresholds are different, given positive and negative pricing errors.

The STAECM model assumes implicitly that the arbitrage triggers or cost thresholds are common for all market participants. Differential trading costs imply arbitrage activities for various levels of mispricings but with different intensities. Traders may also undertake a risky dynamic arbitrage strategy that does not require that the mispricing be great enough to cover total transaction cost if they expect to be able to capture additional profit by unwinding their positions when a large basis reversal occurs before the contract expires.

This potential trading strategy provides an arbitraguer an early-unwinding option (see MacKinlay and Ramaswamy, 1988, and Brennan and Schwartz, 1990). Hence, an arbitrage portfolio can be established whenever an arbitrageur believes that the value of the early-unwinding option is enough to compensate for the difference between transaction cost and the mispricing.

Sofianos (1993) finds, on 2,659 S&P 500 actual index-arbitrage trades that transaction costs outweigh the average mispricing of an arbitrage portfolio; moreover, 18% of positions were created with a mispricing between zero and less than half of the estimated transaction cost. Neal (1996) reports that the number of arbitrage trades (i.e., arbitrage intensity) is positively related to the absolute level of mispricing. He also finds that arbitrage positions are established over a wide spectrum of mispricing; and a majority of arbitrage positions are executed when mispricing amounts to only one index point.

Institutional investors who have large and diversified stock holdings may avoid high equity trading costs and the constraints, cost, and risk of short-selling. To capture a positive pricing error, they may short futures and hedge the position with their equity portfolio to lock in a high riskless return. If the futures is underpriced, they may arbitrage by selling part of their equity portfolio and avoiding the problems and costs of short-selling in long-futures arbitrage (Chan, 1992).

Neal finds that 28% of long-futures arbitrage involves the direct selling of stocks. Hence, a dynamic arbitrage strategy together with heterogeneous classes of arbitrageurs could make arbitrage activities a continuous function of the arbitrage basis.

Kawalla (1991) also show that at any price, the futures can be used to advantage by potential users. Hence, dynamic arbitrage strategy and arbitrageurs with heterogeneous trading costs could make arbitrage activities a continuous function of the arbitrage basis.

To model the potential arbitrage-induced price dynamic for all levels of arbitrage basis, Taylor et al. (2000) and Tse (2001) apply a form of a smooth transition error-correction model (STECM);

$$\Delta f_t = \alpha_1 z_{t-1} F(z_{t-1};\ \gamma_1) + \theta_{1t} \tag{6}$$

$$\Delta s_t = \alpha_2 z_{t-1} F(z_{t-1};\ \gamma_2) + \theta_{2t} \tag{7}$$

The α_i are the error-correction coefficients; and the θ_i are the error terms of the two equations. $F(z_{t-1};\ \gamma_i)$ is the transition function with the form $1-\exp(-\gamma_i z_{t-1}^2)$, and γ_i measures its slope, which indicates how quickly traders in market *i* react to a mispricing. The value of the function $F(z_{t-1};\ \gamma_i)$ increases monotonically over the pricing error, with values bounded between 0 and 1. If the pricing error is small, arbitrage activities are expected to be low, and the value of the transition function is then close to zero; as a result, error-correction adjustments of both futures and index are small, and vice versa.

The error-correction coefficients (i.e., α_i) thus represent the “maximum” adjustment speed in a particular market.

Taylor et al. (2000) adopt the framework to examine how the introduction of SETS, an electronic trading system, affects the dynamic arbitrage efficiency between the FTSE-100 and the underlying cash index. They find that greater adjustments in the spot market, in absolute terms, than adjustments in the futures market during the post-SETS period.

Tse (2001) applies the framework to study the dynamics of the Dow Jones Industrial Average (DJIA) futures price and the underlying cash index. His results show that investors respond more rapidly when the futures is underpriced than when it is overpriced.

Order imbalance has also been found to have a significant impact on stock returns. Executed order imbalance is defined as the difference between the dollar volume crossed at ask prices and that crossed at bid prices. Trades executed at ask prices (i.e., ask trades) represent buyer-initiated trades and those executed at bid prices (i..e, bid trades) represent seller-initiated trades. A positive order imbalance indicates that buying is more active than selling, while a negative order imbalance indicates that selling is more active than buying. Blume, MacKinlay, and Terker (1989) find correlations between the aggregate order imbalance and the concurrent 15-minute market returns of 0.81 and 0.86,on October 19 and 20 significant at individual stock level. Chordia, Roll, and Subrahmanyam (2002) find that order imbalance reduces market liquidity and increases bid and ask spreads.

Easley, O'Hara, and Srinivas (1998) indicate that order imbalance in CBOE options provides information on price movement of the underlying stock. Chan, Chung, and Fong (2002) show that stock order imbalance, not options order imbalance, helps predict quote revision patterns in both stock and options. This shows that order imbalance in a particular market can be associated with price movements in related securities.

Order imbalance in the cash market may also impact the futures market for two reasons. First, there is a liquidity effect; institutions may short index futures to substitute for selling off equities when a large negative order imbalance makes it costly or impossible to unload sizable stock positions. Institutions may be willing to short futures at a discount to induce greater supply of liquidity from the arbitrageurs. This widens the negative basis by pushing down the futures. Similarly, the basis strengthens when institutional buying spills over to the futures market when there is a large positive order imbalance in the cash market.

Second, there is a signaling effect; positive order imbalance signals a rise in the cash market, when traders may buy futures ahead of the impending stock price movement. Similarly, a negative order imbalance signals a potential drop in the market, and traders will short futures ahead of the cash market decline. In this respect, the information effect reinforces the liquidity effect.

Locke and Sayers (1993) have examined the relation between order imbalance and the stock market volatility. Chan and Fong (2000) show that order imbalance could explain the volume-volatility relation. They find that, on a daily basis, the order imbalance is highly correlated with the total number of trades in both NYSE and Nasdaq stocks; the volume-volatility relation is weaker after capturing the impact of order imbalance on the intraday stock return.

Several authors examine how market conditions affect the dynamics between index and futures. In a study of the U.K. FTSE-100 index futures, Yadav and Pope (1994) fail to find a significant relationship between market returns and the arbitrage basis. Fung and Jiang (1999) and Jiang, Fung, and Cheng (2001) report that the futures lead over the cash index strengthens in falling markets and when the futures is underpriced. These results indicate that the hurdle against short-selling impedes the short-stock long-futures arbitrage process when the futures is underpriced.

Harris (1989), Kleidon (1992), and Kleidon and Whaley (1992) have examined the large negative basis between the S&P 500 index and futures during the U.S. market crash on October 19, 1987. Harris shows that the large basis cannot be entirely explained by non-synchronous trading in the stock market. Kleidon and Whaley (1992) argue that the delinkage between the stock and futures markets could be caused by the NYSE's inefficient order routing system at the time.

Blume, MacKinlay, and Terker (1989) however, find a significant positive relationship between order imbalance and the concurrent 15-minute market returns on October 19 and 20, 1987. These results seem to suggest that the behavior of the S&P 500 index-futures basis on October 19 could be associated with the pattern of order imbalance. Shleifer and Vishny (1997) show theoretically that widening of the arbitrage basis under extreme market conditions could paralyze arbitrage if arbitrage capital is exhausted.

Draper and Fung's (2003) examination of the behavior of the arbitrage basis during the Hong Kong financial crisis indicates that the index and futures prices remained closely aligned until the Hong Kong government intervened in both the stock and index derivatives markets. Harris, Sofianos, and Shapiro (1994) find that program trading activities are positively related to market volatility. Hence, it is expected that arbitrage-related trading should intensify during the crisis period, and traders should respond faster to mispricing signals.

Breen, Hodrick, and Korajczyk (2002) find that firm-specific characteristics affect the (positive) relationship between order imbalance and stock returns. Roll, Schwarz, and Subrahmanyam (2005) find a significant bi-directional causality relationship between market liquidity and the NYSE composite index-future basis. Liquidity and basis have a strong contemporaneous relationship, and the time for an error to revert to zero is positively related to market liquidity.

Fung (2004) examines the Hang Seng Index futures and the underlying cash index and finds that the arbitrage basis is positively related to order imbalance; that is, large positive (negative) order imbalance is associated with large positive (negative) arbitrage basis. He also finds an asymmetric relationship between order imbalance and arbitrage basis; a negative order imbalance has a stronger impact on the basis than a positive order imbalance. The time for a negative basis to converge to zero is negatively related to order imbalance, which means that positive (negative) order imbalance speeds up (delays) error correction when the futures is underpriced. The time for a positive basis to converge to zero is not significantly related to order imbalance, however. All these results show that order imbalance in the stock market can significantly affect the error-correction dynamics of index and futures prices. We expand upon this line of research, and examine the dynamic relationships between order imbalance and index and futures returns. We also test how and to what extent order imbalance in the stock market affects the error-correction dynamics of index and futures prices.

3. Model and hypotheses

As a benchmark for measuring the significance of the impact of order imbalance on the error-correction dynamics, following Taylor et al. (2000), we adopt a smooth transition autoregressive (STAR) error-correction model as a benchmark:

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)} \Delta s_{t-1} + (\alpha_{11} + \alpha_{12} D) z_{t-1} \right] F(z_{t-1};\, \gamma) + \eta_{1t} \tag{8}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)} \Delta s_{t-1} + (\alpha_{21} + \alpha_{22} D) z_{t-1} \right] F(z_{t-1};\, \gamma) + \eta_{2t} \tag{9}$$

where D = 1 when $z_{t-1} < 0$, and $= 0$ otherwise.

The error-correction coefficient for futures returns α_{11} is expected to be negative, and the coefficient for α_{12} should be positive if the error-correction process is weakened when the futures is underpriced due to the constraints against short-selling stocks. The error-correction coefficient for index returns α_{21} is expected to be positive, and the coefficient for α_{22} should be negative, if constraints against shorting stocks significantly impede arbitrage. The α_{12} and α_{22} coefficients should be smaller than the corresponding error-correction coefficients α_{11} and α_{21} to preserve convergence between the index and futures prices. The a_{ij} are the auto- and cross-correlation coefficients.

$F(z_{t-1};\, \gamma_t)$ is the transition function, which takes the form $1-\exp\left(-\gamma_i z_{t-1}^2 \big/ \sigma_{z_{t-1}}^2\right)$. Its value increases monotonically with the amount of the pricing error, with values bounded between 0 and 1. If pricing error in the previous period is low, arbitrage is expected to be low and the value of the transition function is then close to zero. As a result, error-correction adjustments in both futures and index prices are small. γ_i measures how quickly investors in market *i* respond to the mispricings. z_{t-1}^2 represents the squared pricing error in the previous period and $\sigma_{z_{t-1}}^2$ the variance of the pricing error. Following Dwyer, Locke, and Yu (1996) and Taylor et al. (2000), z_{t-1}^2 is normalized by $\sigma_{z_{t-1}}^2$ to make the γ_i scale-free measures.

3.1 Impacts of order imbalance on the error-correction dynamics

The market impact of order imbalance may enhance or impede the error-correction process. There are four possible scenarios:

Case 1: Both order imbalance and error are positive ($z_{t-1} > 0$ *and* $OI_t > 0$ *)*

If both order imbalance and error are positive, order imbalance has a positive market impact on both index and futures returns. Positive pricing error triggers short-futures long-stock arbitrage that exerts downward pressure on the futures and upward pressure on the index. The error-correction dynamics of the index would be enhanced by the market impact of order imbalance, and the conditional return to the index is positive. The error-correction force of the futures is countervailed by the opposite market force of order imbalance.

The conditional futures return is ambiguous, and depends on the relative dominance of the two forces. If the market impact of order imbalance is stronger than the error-correction force in futures, then the conditional futures return is positive, and vice versa.

Exhibit 1

Case 1: Positive order imbalance and positive pricing error ($OI_t > 0$ *and* $z_{t-1} > 0$ *)*

Variable	Δf_t	Δs_t
$OI_t > 0$ Market Impact	Positive	Positive
$z_{t-1} > 0$ Error Adjustment	Negative	Positive
Overall Direction	Ambiguous	Positive

Case 2: Order imbalance is negative and pricing error is positive ($OI_t < 0$ *and* $z_{t-1} > 0$ *).*

If order imbalance is negative and pricing error is positive, the conditional return to futures is negative because the market impact of order imbalance and error-correction dynamics affects futures in the same direction. The conditional return to the index is positive only if the error-correction force dominates the market impact of negative order imbalance. The conditional index return can be negative if the market impact force of order imbalance overwhelms the effect of error-correction.

Exhibit 2

Case 2: Negative order imbalance and positive pricing error ($OI_t < 0$ and $z_{t-1} > 0$)

Variable	Δf_t	Δs_t
$OI_t < 0$ Market Impact	Negative	Negative
$z_{t-1} > 0$ Error-Correction	Negative	Positive
Overall Direction	Negative	Ambiguous

Case 3: Order imbalance is positive and pricing error is negative ($OI_t > 0$ and $z_{t-1} < 0$).

If order imbalance is positive and pricing error is negative, the conditional futures return is positive because the error-correction adjustment for the futures price is enhanced by positive order imbalance. The conditional index return could be positive if the market impact of positive order imbalance exceeds the error-correction force.

Exhibit 3

Case 3: Positive order imbalance and negative pricing error ($OI_t > 0$ and $z_{t-1} < 0$)

Variable	Δf_t	Δs_t
$OI_t > 0$ Market Impact	Positive	Positive
$z_{t-1} < 0$ Error-Correction	Positive	Negative
Overall Direction	Positive	Ambiguous

Case 4: Order imbalance is negative and pricing error is negative ($OI_t < 0$ and $z_{t-1} < 0$).

If order imbalance is negative and pricing error is negative, the conditional index return is expected to be negative because the market impact of order imbalance enhances the error-correction mechanism for the index. The conditional futures return could become negative if the market impact of a negative order imbalance exceeds the error-correction force.

Exhibit 4

Case 4: Negative order imbalance and negative pricing error ($OI_t < 0$ *and* $z_{t-1} < 0$ *)*

Variable	Δf_t	Δs_t
$OI_t < 0$ Market Impact	Negative	Negative
$z_{t-1} < 0$ Error-Correction	Positive	Negative
Overall Direction	Ambiguous	Negative

3.2 Modeling the impact of order imbalance - Four-regime STAR model

To test whether order imbalance significantly impedes order correction dynamics, we extend the Taylor et al. (2000) model as follows:

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)} \Delta s_{t-1} + \Bigg[\sum_{n=1}^{p} b_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)} \Delta s_{t-1} + (\beta_{11} D_1 + \beta_{12} D_2 + \beta_{13} D_3 + \beta_{14} D_4) z_{t-1} \Bigg] F(z_{t-1};\, \gamma_1) + \eta_{1t} \tag{10}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)} \Delta s_{t-1} + \Bigg[\sum_{n=1}^{p} b_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)} \Delta s_{t-1} + (\beta_{21} D_1 + \beta_{22} D_2 + \beta_{23} D_3 + \beta_{24} D_4) z_{t-1} \Bigg] F(z_{t-1};\, \gamma_2) + \eta_{2t} \tag{11}$$

$$D_1 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \text{ and } OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_2 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \text{ and } OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_3 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \text{ and } OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_4 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \text{ and } OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

The coefficient for the first dummy variable (i.e., β_1) depicts the conditional response of the futures and the cash index in case 1, and so on for the other cases.

For the futures equation, β_{12} and β_{13} are expected to be (unambiguously) negative, since the market impact of order imbalance enhances error-correction in both cases. β_{11} and β_{14} allow us to test whether order imbalance significantly impede the error-correction mechanism for futures when the two forces drive the futures price in opposite directions. If the market impact of order imbalance dominates, then β_{21} and β_{24} are positive, and vice versa.

For the index equation, β_{11} and β_{14} are expected to be unambiguously positive since the market impact of order imbalance and error-correction affects the index price in the same direction. If β_{21} and β_{22} are negative, then the result will show that the market impact of order imbalance dominates the error-correction force, and vice versa.

4. Data

We obtain time stamped bid/offer quotes for the 33 constituent stocks of the Hang Seng Index (HSI) and transaction records for the stocks and the Hang Seng Index futures from the Hong Kong Exchange from May 1996 through December 1998. The stocks were traded electronically in a screen-based Automatic Matching System (AMS) system. The futures were traded via open outcry.[3] The spot month futures contract is the most liquid of the four concurrently traded maturity months, except on its expiration day. We substitute the next-month contract for the spot month contract on the contract expiration day of each month.

[3] Electronic futures trading began on June 5, 2000, via the Hong Kong Automated Trading System (HKATS).

The data cover the period surrounding the Asian financial crisis of 1997. We separate the sample into two time periods: May 1996 - April 1997 (before the speculative attack on the Thai baht in May) represents the period before the Asian financial crisis, while May 1997 - December 1998 represents the financial crisis period. This second period includes extreme market conditions during the crisis and when there were wide fluctuations in stock and futures prices and trading volumes.

We eliminate data in the month of August 1998 to avoid distortion of the analytical results when unusual trading activities occurred upon the direct intervention of the Hong Kong government in both the index futures and the stock markets. To reduce the influence of extreme observations on the test results, we eliminate observations of arbitrage basis and order imbalance that are more than five standard deviations from their means.

After application of these procedures, the mean and standard deviation of the arbitrage basis (order imbalance) are -0.4059% and 0.5480% (0.001 and 16.676), respectively for the pre-crisis period. For the crisis period, the mean and standard deviation of the arbitrage basis (order imbalance) are -0.6622% and 0.944% (-0.831 and 21.688), respectively. All are wide variations of arbitrage basis and order imbalance in the crisis period.

Dividend information including the ex-date, the payment day, and the actual amount of dividend for the constituent stocks is also obtained from the Exchange. To construct the market value weight for each index stock, we obtain market capitalization information and closing index quotes from Hang Seng Index Services Limited. Hong

Kong Inter-Bank Offer Rates (HIBORs) for maturities of one-day to one-month come from Datastream.

4.1 Construction of the mid-quote index

Studies of index-futures relationships have been plagued by measurement problems in the index caused by infrequent trading and the bid/ask bounce. Miller, Muthuswamy, and Whaley (1994) show that part of the negative correlation in the basis can be explained by the effect of infrequent trading, which delays adjustment of the cash index. The problems are especially pronounced in highly volatile periods when simultaneous selling and buying occurs, causing a large bid and ask bounce in the observed index (Harris, 1989; Harris, Sofianos, and Shapiro, 1994). Yet only during stressful market situations are there the large variations in the basis that provide a meaningful test of its dynamic behavior.

Following Blume, MacKinlay, and Terker (1989), we negate the effects of infrequent trading by adopting a reconstructed time series of the index based on the mid-quote synchronous active bid/offer prices of the index's constituent stocks. Chan, Chung, and Johnson (1993) also indicate that the use of mid-quotes reduces the impact of the discreteness in the tick size on the responsiveness of the traded price. Such an approach also controls for the bias in index returns directly due to order imbalance (See Lease, Masulis, and Page, 1991). The bias could also be induced by arbitrage itself, as the index could be moved to either side of the spread as a result of index arbitrage (Harris, Sofiano, and Shapiro, 1994).

The Hang Seng Index (HSI) is a value-weighted index. The current index value is the ratio of the current total market value of the index stocks divided by the total market capitalization at the previous day's close, multiplied by the value of the index at the previous day's close. Following the index construction method, the mid-quote index at time τ on day t is equal to:

$$S_{t_\tau}^{m} = \sum^{33} W_{it} \left(P_{it_\tau}^{a} + P_{it_\tau}^{b} \right) / 2 \tag{14}$$

where $S_{t_\tau}^{m}$ is the mid-quote index at time τ on day *t*, and W_{it} is the market value weight for security *I* on day *t*. $P_{it_\tau}^{a}$ and $P_{it_\tau}^{b}$ are, respectively, the ask and bid price for stock *i* at time τ on day *t*.

As the quotes are refreshed every 30 seconds, a mid-quote index is obtained for a 30-second interval when all 33 pairs of bid/offer quotes are available. We use the minute-by-minute sample data and the mid-quote index to calculate index returns and the fair futures price. Returns for the overnight non-trading hours and the lunch break are excluded from the analysis. [4]

4.2 Construction of the fair futures price series

To filter out discrete interday changes in the index-futures relationship due to uneven dividend payments to the index, the actual (ex-post) dividend payments accruing to the

[4] Draper and Fung (2003) provide details of the methodology for construction of the quote-based index prices.

index during the remaining life of the contract are factored into the cost-of-carry framework. Let $F_{t_\tau}^*$ be the fair (or theoretical) futures price:

$$F_{t_\tau}^* = S_{t_\tau}^m (1+r)^{T-t} - \sum_{j=t}^{T-t-1} W_{it} D_{ij} (1+r_j)^{T-j} \tag{15}$$

where t and T (as fractions of a year) denote the initiation and the expiration date of the contract, respectively; r_j is the overnight interest rate; r is the riskless rate for the holding period between day t and T; and D_{ij} is the per share cash dividend for stock i at time j.

We measure the degree of pricing error or arbitrage basis in percentage of the fair futures value - i.e., $z_t = \ln F_t - \ln F_t^*$.

4.3 Measurement of order imbalance

Following Blume, MacKinlay, and Terker (1989), we take the order imbalance of an individual stock as equal to its dollar volume crossed at the asked price minus the dollar volume crossed at the bid price within a particular interval. We generally follow Lee and Ready's (1991) approach to identify whether a trade is executed at bid or at ask. A trade is identified as a bid (an ask) trade if the traded price is below (above) the middle of the nearest previous bid and ask quotes. If that fails to identify a trade, the nearest quotes following the trade are used.

The reason is that HKEx retrieves the quote by taking snapshots of the limit order book every 30 seconds. Hence, the quotes following the trade could have been the quotes at which that trade was executed.

When the traded price falls exactly at the middle of the quotes both preceding and after the trade, a tick test is used. If the current traded price is above (below) the previous traded price, the trade is an up tick (down tick), and if is classified as an ask (a bid) trade. If the current traded price is equal to the previous traded price, the trade is classified according to the trade before the previous one. A zero-up tick (i.e., the previous trade is traded at an up tick) is classified as an ask trade, and a zero-down tick (i.e., the previous trade is traded at a down tick) is classified as a bid trade. The process stops when there are no changes in the traded price in the last two transactions, and the trade will not be included in the analysis. The maximum time difference between the current trade and the oldest transaction or quote used for the purpose of identification is restricted to five minutes.

Aggregate order imbalance for the index within a particular time interval is obtained by summing the individual order imbalance of the constituent stock of the index within the same time interval; that is, $O_t = \sum_{i=1}^{33} O_{i\tau}$, where $O_{i\tau}$ denotes the order imbalance of stock *i* measured for the τ^{th} interval.

To make the order imbalance measure free of the level of the market and comparable with the volume of the HSI futures contracts, we convert the aggregate dollar order imbalance into an equivalent number of index futures contracts.

To accomplish this, we divide the aggregate 30-second dollar order imbalance by the mid-quote index prevailing at the end of the interval and by HK$50 (the contract multiplier). Hence:

$$O_\tau = \sum_{i=1}^{33} \varpi_{i\tau} \Big/ (S_t 50). \tag{16}$$

To calculate the one-minute order imbalance, we simply add the two consecutive 30-second order imbalances within the particular non-overlapping one-minute interval. This procedure is followed to calculate order imbalance for other time intervals.

To focus on the information revealed through the trades executed within the AMS system, we discard all non-AMS transactions. A trade is classified according to a matching quote that occurs nearest to the time of the trade. This criterion causes some trades to be classified according to quotes after it. This is possible since a trade could have been executed against a quote that was being revised within a 30-second interval, and the revised quotes are reported only after the trade occurs.

5. Empirical results and interpretation

We first apply a Granger causality test to the relationship between the futures (or cash index) returns and the order imbalance. According to Fung (2004), cash index returns and order imbalance are expected to have a strong contemporaneous relationship. Moreover, since futures returns usually lead cash index returns, futures returns should lead order imbalance. Following Fung and Jiang (1999) and Jiang, Fung, and Cheng (2001), we pre-whiten all series with AR processes.

Table 1 shows the results for the lead-lag relationship between futures returns and order imbalance. The pre-crisis results show that the two series lead and lag one an other, but the coefficient for the one-period lead term in order imbalance is the most significant, with a t-value of 13.72. That is, there is a significant lead of futures returns over order imbalance by one period. The two series have a strong positive contemporaneous relationship.

The contemporaneous and lead-lag relationship between futures returns and order imbalance strengthened during the crisis period. The one-period lead over order imbalance is again most significant. The two-period lead of futures over order imbalance and the contemporaneous relationship also strengthen. The result is consistent with the Blume et al. (1989) conjecture that an order imbalance leads to a price change, and a price change leads in turn to further order imbalance, and so on.

Table 2 shows the lead-lag results between index futures and order imbalance. Results in both periods show that index returns and order imbalance have a very strong contemporaneous relationship. The t-values for the two periods are 38.71 and 51.60. The results also show that order imbalance generally leads index returns by one period, although index returns lead more during the crisis period. Order imbalance has a substantially greater impact on index returns than on futures returns.

5.1 Asymmetrical error-correction mechanism in response to - positive and negative errors benchmark case

To account for the non-constant error variance in the two equations, we adopt a GARCH (1, 1) process to capture stochastic variance: $\sigma_{it}^2 = \varpi_i + A_i a_{it-1}^2 + B_i \sigma_{it-1}^2$; i = 1, 2 where a_{it-1}^2 is the lag 1 squared residuals and σ_{it-1}^2 the lag 1 residual variance of Δf_t and

Δs_t. Equations (1) and (2) are estimated simultaneously using the full information maximum likelihood (FIML) approach.

Table 3 shows the results of the smooth transition autoregressive process for the futures and index returns over the two sample periods. The R-square of the index returns equation increased from 16.63% in the pre-crisis period to 32.24% during the crisis period, while the R-square of the futures returns equation also increases from 4.35% to 8.23%. The error variance can be fitted according to the GARCH (1, 1) for the pre-crisis period, but the error variance is more chaotic during the crisis period and GARCH (1, 1) cannot capture its dynamics. Hence, we use a highly robust heteroscedastic consistent covariance matrix estimation (HCCME) to obtain consistent estimates of the parameters for the crisis period sample.

For the pre-crisis period, the signs of the error-correction coefficients of the futures and index returns are both consistent with hypotheses. The error-correction coefficient for the futures return α_{11} is negative but not significant; while the error correction coefficient α_{21} for cash stock is positive and significant at the 10% level. The signs of the coefficients for the dummy variables are the opposite of the error-correction coefficients. These results are consistent with the proposition that restrictions on shorting stock impede arbitrage and reduce the speed of error-correction when the futures is underpriced (although the coefficients are not significant).

For the crisis period, all the above coefficients are not significant, and the error-correction coefficient and the coefficient for the dummy of the stock equation are of the wrong signs. These results indicate that the benchmark framework cannot describe the error-correction dynamics of index and futures prices, particularly during crisis.

According to Harris, Sofianos, and Shapiro (1994), arbitrageurs should be more responsive to pricing errors during crisis periods. Yet we find reduced response coefficients γ_i for both index and futures during our crisis period. These results are contrary to earlier findings in the U.S. market and may indicate that the benchmark framework is inadequate in capturing the conditional price dynamics of the two prices under volatile market condition.

5.2 Impact of order imbalance on error-correction process – Four-regime case

Table 4 shows the results of the STAR estimations. Four dummy variables are used to denote four different regimes. β_{11} shows the net adjustment coefficient for futures returns when there is positive pricing error when order imbalance is positive. The β_{11} result is positive, which shows that the market impact of order imbalance dominates the effect of error-correction. Because the two effects are offsetting, the coefficient is relatively low (0.00759).

β_{12} shows the net adjustment coefficient when there is positive pricing error when order imbalance is negative. In this case, error-correction is enhanced by the market impact of order imbalance. β_{12} is negative (-0.01376) as predicted.

β_{13} shows the net adjustment coefficient when there is negative pricing error when order imbalance is positive. Again, error-correction should push the futures price up along with the order imbalance. Consistent with expectations, β_{13} is positive (0.00378) (but lower than β_{12} because of impediments against short-stock arbitrage).

β_{14} shows the net adjustment coefficient for futures returns when there is negative pricing error when order imbalance is negative. The force of error-correction should push up the futures price, but negative order imbalance works against this. β_{14} is positive (0.00334), which shows that the market impact of order imbalance exceeds the effect of error-correction.

Results in the crisis period are generally consistent with those in the pre-crisis period, except that all coefficients are higher. Hence the results show that order imbalance can actually impede error-correction mechanisms conditions such as regime 1 (positive order imbalance and positive pricing errors) and regime 4 (negative order imbalance and negative pricing error). In both cases, the error-correction in futures prices is completely offset by the opposite market impact effect of order imbalance. The effect is particularly strong in regime 4, where error-correction is further impeded by restrictions on the short selling of stock.

In the cash results, β_{21} shows the net adjustment coefficient for index returns when there is positive pricing error when order imbalance is positive. In this case, the market impact of order imbalance should push the index up as does error-correction. β_{21} is positive (0.01005), consistent with expectation.

β_{22} shows the net adjustment coefficient when there is positive pricing error and negative order imbalance. In this case, error-correction should push the index up but against the negative market impact of order imbalance. β_{22} is negative (-0.00698), which means that the market impact of order imbalance exceeds the force of error-correction.

β_{23} shows the net adjustment coefficient when there is negative pricing error and order imbalance is positive. Error-correction should push the index down, but positive order imbalance works to push it up. β_{23} is negative (-0.01044). Again, the force of order imbalance exceeds that of error-correction when they are offsetting effects. β_{23} is higher than β_{22}, however, which again shows that the error-correction mechanism is particularly distorted by order imbalance when error-correction is impeded by restrictions on short-stock arbitrage when the futures is underpriced.

β_{24} shows the net adjustment coefficient for futures returns when there is negative pricing error and when order imbalance is negative. In this case, both forces should push the index price down. The coefficient is negative (0.01334), consistent with expectation.

Results in the crisis period are generally consistent with those in the pre-crisis period. Hence, incorporating the market impact effect of order imbalance provides a consistent explanation of the price dynamics of index and futures prices, even during stresses in the market. Moreover, the results of the index equation also confirm those of the futures equation. That is, order imbalance may reverse the error-correction effect when the two forces are offsetting.

In regime 2 (negative order imbalance and positive pricing errors) and regime 3 (positive order imbalance and negative pricing error), the error-correction in index returns is offset by the opposite market impact effect of order imbalance. The effect is particularly strong in regime 3, where error-correction is further impeded by restrictions on the short selling of stock.

Moreover, consistent with expectations, the reaction coefficients γ_i show that both futures and index prices are more responsive to pricing errors during the crisis period. The γ_i for the cash index is about five times higher than that of the pre-crisis period. These results show that incorporating the market impact of order imbalance lets us capture the price dynamics of the index and futures especially under stressful market conditions.[5]

5.3 Test of relative explanatory power

We apply a likelihood ratio test to examine whether incorporating order imbalance improves the overall goodness-of-fit of the framework. Table 5 shows the results for both periods. The high F-statistics suggest rejection of the null hypothesis that the benchmark model and the order imbalance models perform the same in both sample periods. That is, the four-regime model outperforms the benchmark framework.[6]

5.4 Impact of order imbalance on convergence of index and futures prices

Tables 1 and 2 have shown that order imbalance has a greater impact on stock returns than on futures returns. Results of a smooth transition error-correction model

[5] We also test a three-regime framework by combining the two cases of offsetting market impact of order imbalance and direction of error-correction. This allows a test of the overall dominance of the force of order imbalance. The result again shows that order imbalance dominates error-correction when the two forces are of opposite direction (results available upon request).
[6] The three-regime case also outperforms the benchmark framework.

estimation also show that order imbalance dominates the effect of error-correction when the two forces are opposite one another. Hence, order imbalance should strengthen the error-correction process when the imbalance is of the same sign as the arbitrage basis. Order imbalance will work against the error-correction mechanism if the signs of order imbalance and the arbitrage basis are opposite.

Table 6 shows the impacts on index and futures returns at various levels of pricing error under the four different scenarios describing directions of order imbalance and arbitrage basis. For the first two cases, when order imbalance is of the same sign as the pricing error, the error-correction mechanism is strengthened. The results are similar in both periods. In the third and fourth cases, when the pricing error is of the opposite sign as order imbalance, the impact of order imbalance on the index returns impedes the error-correction mechanism.

6. Conclusion

The direction of order imbalance has an effect on the dynamics of index and index futures prices. Order imbalance leads the cash index returns by three minutes, with the lead reduced to one minute during our crisis period. Order imbalance and futures returns both lead and lag each other, but the lead of futures over order imbalance strengthens during the crisis period.

The benchmark model cannot consistently explain the error-correction dynamics especially of the cash index under the stressful market conditions of the crisis period. Our results show that incorporating the market impact of order imbalance provides a consistent explanation of the dynamic error-correction process, particularly under volatile market conditions and when arbitrage and trading activities are intense. Factoring in the potential impact of order imbalance significantly improves the explanatory power of the framework.

Finally, the results show that order imbalance impedes the error-correction process when the market impact of order imbalance is the opposite of the error correction force on the cash index.

References

Blume, M.E., MacKinlay, A.C. & Terker, B. (1989). Order Imbalances and Stock Price Movements on October 19 and 20, 1987. *Journal of Finance*, XLIV(4), 827-848.

Breen, W.J., Hodrick, L.S., & Korajczyk, R.A. (2002). Predicting Equity Liquidity. *Management Science*, 48(4), 470-483.

Brennan, M. & Schwartz, E., (1990). Arbitrage in Stock Index Futures. *Journal Business*, 63: 7-31.

Chan, K. (1992). A Further Analysis of the Lead Lag Relation between the Cash Market and Stock Index Futures Market. *Review of Financial Studies*, 5, 123-152.

Chan, K., Chung, Y.P., & Fong, W.M. (2002). The Informational Role of Stock and Option Volume. *The Review of Financial Studies*, 15(4) 1049-1075.

Chan, K., Chung, Y.P., & Johnson H. (1993). Why Options Prices Lag Stock Prices: A Trading-based Explanation. *Journal of Finance*, 48(5), 1957-1967.

Chan, K. & Fong, W.M. (2000). Trade Size, Order Imbalance, and the Volatility – Volume Relation. *Journal of Financial Economics*, 57, 247-273.

Chordia, T., Roll, R., & Subrahmanyam, A. (2002). Order Imbalance, Liquidity, and Market Returns. *Journal of Financial Economics*, 65(2), 111-130.

Cornell, B., & French, K. (1983). Taxes and the Pricing of Stock Index Futures. *Journal of Finance*, 38(2), 675-694.

Draper, P., & Fung, J.K.W. (2003). Discretionary Government Intervention and the Mispricing of Index Futures. *Journal of Futures Markets*, 23(2), 1159-1189.

Dwyer, G.P., Locke, P., & Yu, W. (1996). Index Arbitrage and Nonlinear Dynamics between the S&P 500 Futures and Cash. *Review of Financial Studies*, 9(1), 301-332.

Easley, D., O'Hara, M., & Srinivas, P.S. (1998). Option Volume and Stock Prices: Evidence on Where Informed Traders Trade. *Journal of Finance*, LIII(2), 431-465.

Engle, R.F., & Granger, C.W.J. (1987). Cointegration and Error Correction: Representation, Estimation, and Testing. *Econometrica*, 55, 251-276.

Fung, J. K.W. (2004). Order Imbalance and the Pricing of Index Futures. Working Paper, Hong Kong Baptist University.

Fung, J.K.W. & Draper, P. (1999). Index Arbitrage Opportunities and Short Sales Constraints. *Journal of Futures Markets*, 10(2), 695-715.

Fung, J.K.W., & Jiang, L. (1999). Restrictions on Short-selling and Spot-Futures Dynamics. *Journal of Business, Finance and Accounting*, 26(1)&(2), 227-248.

Garbade, K.D., & Silber, W.L. (1983). Price Movements and Price Discovery in Futures and Cash Markets. *Review of Economics and Statistics*, 65, 289-297.

Grossman, S.J. (1988). An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies. *Journal of Business*, 61(3), 275-298.

Harris, L. (1989). The October 1987 S&P 500 Stock-Futures Basis. *Journal of Finance*, XLIV(1), 77-99.

Harris, L., Sofianos, G. & Shapiro, J.E. (1994). Program Trading and Intraday Volatility. The *Review of Financial Studies*, 7(4), 653-685.

Jiang, L., Fung, J.K.W., & Cheng, L.T.W. (2001). Lead-lag Relationship between Spot and Futures Markets under Different Short-Selling Regimes. *Financial Review*, 38, 63-88.
Kawalla, I.G. (1987). A note: Debunking the Myth of the Risk-Free Return. *Journal of Futures Markets*, 7, 327-331.
Kawalla, I.G. (1991). Determining the Relevant Fair Value(s) of S&P 500 Futures: A Case Study Approach. *Journal of Futures Markets*, 11(4), 453-460.
Kempf, A. (1998). Short Selling, Unwinding, and Mispricing. *Journal of Futures Markets*, 18(8), 903-923.
Kleidon, A.W. (1992). Arbitrage, Nontrading, and Stale Prices: October 1987. *Journal of Business*, 65(4), 483-507.
Kleidon, A.W., & Whaley, R.E. (1992). One market? Stocks, Futures, and Options During October 1987. *Journal of Finance*, XLVII(3), 851-877.
Klemkosky, R.C. & Lee, J.H. (1991). The Intraday Ex Post and Ex Ante Profitability of Index Arbitrage. *Journal of Futures Markets*, 11(3), 291-311.
Lease, R.C., Masulis, R.W., & Page, J.R. (1991). An Investigation of Market Microstructure Impacts on Event Study Returns. *Journal of Finance*, XLVI(4), 1523-1536.
Lee, C.M.C., & Ready, M.J. (1991). Inferring Trade Direction from Intraday Data. *Journal of Finance*, 46(2), 733-746.
Locke, P.R. & Sayers, C.L. (1993). Intra-Day Futures Price Volatility: Information Effects and Variance Persistence. *Journal of Applied Econometrics*, V(8), 15-30.
Mackinlay, A.C., & Ramaswamy, K. (1988). Index-Futures Arbitrage and the Behavior of Stock Index Futures Prices. *Review of Financial Studies*, 1, 137-158.
Martens, M., Kofman, P., & Vorst, T.C.F. (1998). A Threshold Error-Correction Model for Intraday Futures and Index Returns. *Journal of Applied Econometrics*, 13, 245-263.
Miller, M.H., Muthuswamy, J., & Whaley, R.E. (1994). Mean Reversion of Standard & Poor's 500 Index Basis Changes: Arbitrage-induced or Statistical Illusion? *Journal of Finance*, XLIX(2), 479-513.
Modest, D., & Sundaresan, M. (1983). The Relationship Between Spot and Futures Prices in Stock Index Futures Markets: Some Preliminary Evidence. *Journal of Futures Markets*, 3(1), 15-41.
Neal, R. (1996). Direct Tests of Index Arbitrage Models. *Journal of Financial and Quantitative Analysis*, 31(4), 541-562.
Roll, R., Schwartz, E., & Subrahmanyam, A. (2005). Liquidity and the Law of One Price: The Case of the Futures/Cash Basis.
Shleifer, A., & Vishny, R.W. (1997). The Limits of Arbitrage. The Journal of Finance, LII(1), 35-55.
Sofianos, G. (1993). Index Arbitrage Profitability. *Journal of Derivatives*, 1, 6-20.
Stoll, H.R., & Whaley, R.E. (1990). The Dynamics of Stock Index and Stock Index Futures Returns. *Journal of Financial and Quantitative Analysis*, 25(4), 441-468.
Taylor, N., Dijk, D.V., Frances, P.H., & Lucas, A. (2000). SETS, Arbitrage Activity and Stock Price Dynamics. *Journal of Banking and Finance*, 24, 1289-1306.

Tse, Y. (2001). Index Arbitrage with Heterogeneous Investors: A Smooth Transition Error-Correction Analysis. *Journal of Banking and Finance*, 25, 1829-1855.

Yadav, P.K. & Pope, P.F. (1994). Stock Index Futures Mispricing: Profit Opportunities or Risk Premia? *Journal of Banking & Finance*, 18, 921-953.

Yadav, P.K., & Pope, P.F., and Paudyal, K. (1994). Threshold Autoregressive Modeling in Finance: The Price Differences of Equivalent Assets. *Mathematical Finance*, 4, 205-221.

Table 1
Lead-lag relationship between order imbalance and futures returns

Model: $Fres_t = a_1 + \sum_{i=-5}^{5} b_{1i} OIres_{t+i} + e_{1t}$

Following Stoll and Whaley (1990), lead-lag regressions are applied to pre-whiten futures return residuals ($Fres_t$) and order imbalance residuals ($OIres_{t+i}$).

	Pre-crisis Period		Crisis Period	
	Parameter Estimate	t-value (p-value)	Parameter Estimate	t-value (p-value)
Intercept	– 0.00027	– 0.57 (0.5693)	– 0.00013	– 0.13 (0.8935)
$OIres_{-5}$	– 0.00004	– 1.37 (0.1710)	0.00007	1.46 (0.1445)
$OIres_{-4}$	0.00005	1.71 (0.0867)	0.00001	0.14 (0.8884)
$OIres_{-3}$	0.00010	3.26 (0.0011)	0.00005	0.97 (0.3315)
$OIres_{-2}$	0.00018	6.09 (<.0001)	0.00005	1.10 (0.2713)
$OIres_{-1}$	0.00025	8.52 (<.0001)	0.00048	9.80 (<.0001)
OIres	0.00029	9.59 (<.0001)	0.00099	20.44 (<.0001)
$OIres_{1}$	0.00041	13.72 (<.0001)	0.00128	26.29 (<.0001)
$OIres_{2}$	0.00029	9.76 (<.0001)	0.00073	14.96 (<.0001)
$OIres_{3}$	0.00024	7.86 (<.0001)	0.00035	7.19 (<.0001)
$OIres_{4}$	0.00014	4.60 (<.0001)	0.00026	5.35 (<.0001)
$OIres_{5}$	0.00008	2.74 (0.0061)	0.00014	2.92 (0.0035)
R^2	0.0612		0.1014	
No. of obs.	9223		14160	
F-value (p-value)	54.61 (<0.0001)		145.11 (<0.0001)	

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Table 2
Lead-lag relationship between order imbalance and index returns

Model: $Sres_t = a_1 + \sum_{i=-5}^{5} b_{1i} OIres_{t+i} + e_{1t}$

Following Stoll and Whaley (1990), lead-lag regressions are applied to pre-whiten index return residuals ($Sres_t$) and order imbalance residuals ($OIres_{t+i}$).

	Pre-crisis Period		Crisis Period	
	Parameter Estimate	t-value (p-value)	Parameter Estimate	t-value (p-value)
Intercept	– 0.00014	– 0.45 (0.6519)	– 0.00023	– 0.42 (0.6721)
$OIres_{-5}$	– 0.00001	– 0.42 (0.6768)	– 0.00001	– 0.23 (0.8157)
$OIres_{-4}$	0.000004	0.19 (0.8493)	– 0.00006	– 2.44 (0.0146)
$OIres_{-3}$	0.00007	3.64 (<.0001)	– 0.00005	– 1.79 (0.0740)
$OIres_{-2}$	0.00011	5.65 (<.0001)	– 0.00001	– 0.33 (0.7421)
$OIres_{-1}$	0.00032	16.58 (<.0001)	0.00039	15.10 (<.0001)
OIres	0.00075	38.71 (<.0001)	0.00133	51.60 (<.0001)
$OIres_{1}$	0.000003	0.17 (0.8672)	0.00016	6.30 (<.0001)
$OIres_{2}$	0.00009	4.67 (<.0001)	0.00027	10.54 (<.0001)
$OIres_{3}$	0.00006	3.05 (0.0023)	0.00021	8.06 (<.0001)
$OIres_{4}$	0.00006	3.11 (0.0019)	0.00011	4.39 (<.0001)
$OIres_{5}$	0.00001	0.52 (0.6049)	0.0001	4.03 (<.0001)
R^2	0.1698		0.1855	
No. of obs.	9223		14160	
F-value (p-value)	171.34 (<0.0001)		292.98 (<0.0001)	

Table 3
Summary of the estimation results of the benchmark STAR model

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)} \Delta s_{t-1} + (\alpha_{11} + \alpha_{12} D) z_{t-1} \right] * F(z_{t-1}; \gamma) + \eta_{1t}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)} \Delta s_{t-1} + (\alpha_{21} + \alpha_{22} D) z_{t-1} \right] * F(z_{t-1}; \gamma) + \eta_{2t}$$

D = 1 when $z_{t-1} < 0$ and $= 0$ otherwise. $F(z_{t-1}; \gamma_t) = 1 - \exp\left(-\gamma_i z_{t-1}^2 / \sigma_{z_{t-1}}^2\right)$.

	Pre-crisis Period (11/96 to 4/97)		Crisis period (8/97 to 1/98)	
	Futures	Cash	Futures	Cash
R^2	0.0435	0.1663	0.0823	0.3224
α_{i1}	-0.0011 (-0.39)	0.00269 (1.41)*	-0.02043 (-1.12)	-0.00769 (-0.74)
α_{i2}	0.0003 (0.09)	-0.00171 (-0.73)	0.01535 (0.82)	0.0064 (0.60)
A_i	0.10174 (11.71)***	0.08957 (7.67)***	–	–
B_i	0.86729 (63.83)***	0.83339 (30.89)***	–	–
γ_i	1.01188 (3.18)***	0.95657 (3.05)***	0.17435 (1.91)**	0.13906 (1.71)**
p-values of Ljung-Box statistics				
u_t	0.6112	0.9039	0.0505	0.2478
u_t^2	0.0776	0.3143	–	–

Note: the following GARCH (1,1) process is adopted to account for the non-constant error variance in the index and futures equations: $\sigma_{it}^2 = \varpi_i + A_i a_{it-1}^2 + B_i \sigma_{it-1}^2$; i=1,2. a_{it-1}^2 is the lag 1 squared residuals and σ_{it-1}^2 the lag 1 residual variance of Δf_t and Δs_t. The system is estimated with full information maximum likelihood method. For all periods, outliers with absolute values of either one of df, ds, lagz and boi exceeding 7 standard deviation. are removed. Numbers corresponding to ut and ut2 are the p-values of Ljung-Box Q(24)-statistics residual diagnosis on the null hypothesis that the residuals are white noise. The Ljung-Box Q(24) statistics show that GARCH (1,1) is sufficient to capture the stochastic error variance during the pre-crisis sample period. However, the same process is found insufficient to fit the volatility structure. We adopt the robust Heteroscedastic Consistent Covariance Matrix Estimation (HCCME) to provide consistent estimates of the model parameters. We use 15 lag terms for each estimation. *Significant at the 10% level, **Significant at the 5% level, and ***Significant at the 1% level.

Table 4
Summary of estimation results of benchmark STAR model with four order imbalance regimes

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)} \Delta s_{t-1} + \left(\beta_{11} D_1 + \beta_{12} D_2 + \beta_{13} D_3 + \beta_{14} D_4 \right) z_{t-1} \right] * F\left(z_{t-1}; \gamma_1\right) + \eta_{1t}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)} \Delta s_{t-1} + \left(\beta_{21} D_1 + \beta_{22} D_2 + \beta_{23} D_3 + \beta_{24} D_4 \right) z_{t-1} \right] * F\left(z_{t-1}; \gamma_2\right) + \eta_{2t}$$

$$D_1 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_2 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_3 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_4 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$F\left(z_{t-1}; \gamma_t\right) = 1 - \exp\left(-\gamma_i z_{t-1}^2 / \sigma_{z_{t-1}}^2\right).$$

	Pre-crisis Period (11/96 to 4/97)		Crisis period (8/97 to 1/98)	
	Futures	Cash	Futures	Cash
R^2	0.0454	0.1882	0.0863	0.3421
β_{i1}	0.00759 (1.84)**	0.01005 (5.39)***	0.00499 (0.64)	0.01542 (5.92)***
β_{i2}	-0.01376 (-2.43)***	-0.00698 (-3.35)***	-0.02598 (-2.55)***	-0.0175 (-8.01)***
β_{i3}	-0.00378 (-1.98)**	-0.01044 (-7.79)***	-0.01841 (-5.43)***	-0.01401 (-10.06)***
β_{i4}	0.00334 (1.64)*	0.01334 (9.36)***	0.00633 (2.06)**	0.00975 (6.37)***
A_i	0.1118 (10.96)***	0.11444 (8.20)***	–	–
B_i	0.85268 (51.77)***	0.78098 (22.71)***	–	–
γ_i	0.89791 (1.81)**	2.20297 (3.30)***	0.94174 (2.53)***	11.88921 (3.23)***
p-values of Ljung-Box statistics				
u_t	0.5934	0.7815	0.0813	0.1060
u_t^2	0.0992	0.5507	–	–

Note: refer to the footnote in Table 6.

Table 5
Comparison of explanatory power of benchmark model and four-regime framework

					H0: Benchmark model H1: 4-regime model	
Pre-crisis period		N	7332			
		R-sq	k	q	F	p-value
Benchmark model	futures	0.0435	67			
	cash	0.1663	67			
4-regime model	futures	0.0454	69	2	7.2280	0.0007
	cash	0.1882	69	2	97.9673	1.04E-42
Crisis period		N	14354			
		R-sq	k	q	F	p-value
Benchmark model	futures	0.0823	64			
	cash	0.3224	64			
4-regime model	futures	0.0863	66	2	31.2750	2.799E-14
	cash	0.3421	66	2	213.9182	2.886E-92

High F values allow rejection of the null hypothesis that the four-regime model has the same explanatory power as the benchmark model. The four-regime model outperforms the benchmark model in describing the conditional returns of index and futures, especially in the crisis period.

Table 6
Conditional futures and index returns in four market conditions

	Pre-Crisis				**Crisis**			
	$z_{t-1}>0$ OI>0	$z_{t-1}<0$ OI<0	$z_{t-1}>0$ OI<0	$z_{t-1}<0$ OI>0	$z_{t-1}>0$ OI>0	$z_{t-1}<0$ OI<0	$z_{t-1}>0$ OI<0	$z_{t-1}<0$ OI>0
Impact on futures return	0.0044965 **0.0073789**	-0.0019786 **-0.0032470**	-0.0815389 **-0.1338087**	0.0022399 **0.0036758**	0.0030429 **0.0048727**	-0.0038622 **-0.0061846**	-0.0158492 **-0.0253793**	0.0112311 **0.0179843**
Impact on index return	0.0089433 **0.0100525**	-0.1186360 **-0.1333501**	-0.0062089 **-0.0069790**	0.0092866 **0.0104384**	0.0154149 **0.0154150**	-0.0097489 **-0.0097490**	-0.0174999 **-0.0175000**	0.0140099 **0.0140100**

Note: the upper number denotes the impact with one standard deviation of arbitrage basis, the lower number shows the impact with two standard deviations of arbitrage basis. Note that for one standard deviation in the basis, the last two cases for the crisis period in particular, the opposite impact of order imbalance on index returns disrupt the convergence of the basis. With two standard deviations in the basis, the impact of order imbalance in the index returns impede the convergence process.