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Abstract

This paper examines the estimation and forecasting performance of range-based variance estimators for a set of stocks. The range of liquidity and drift of these stocks affords the examination of their effect on these estimators in a market with a low bid-ask spread. Two-scales realized volatility based on high-frequency data is used as the benchmark for its high efficiency and unbiasedness. There is evidence that the range-based estimators provide an efficient and low-bias alternative to the return-based estimators. These are not negatively biased in the presence of negative autocorrelation and low liquidity as generally suspected. We find the drift to be a major cause of the documented poor performance of Parkinson's estimator. Generally, the estimators that specifically adjust for drift perform better. Forecasting for the volatility up to one month with the daily range-based estimators is about as efficient as forecasting with the benchmark directly, but is more biased. This, more or less, applies to the range of methods generally used for forecasting. The realized range-based volatility estimators (Christensen & Podolskij, 2006) perform only marginally better than the range-based estimators on bias, and are about as efficient. However, on account of simplicity and data requirement, the range-based estimators appear to be more desirable, particularly for forecasting.

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1. Introduction

The measurement and forecasting of volatility of financial assets are important for many financial economics applications. It is extensively documented that volatility is both time-varying and predictable to a certain extent. A number of methods are suggested for its estimation and prediction.

For measuring ex-post volatility for a day, numerous methods based on daily return and price range are available. The methods based on range are more efficient than the return-based methods (refer to Parkinson, 1980; Garman & Klass, 1980; Rogers & Satchell, 1991; Alizadeh, Brandt & Diebold, 2002, among others). Of late, considerable efforts have gone into the use of intraday data for variance estimation. Several estimation methods based on intraday returns are proposed (Andersen, Bollerslev, Diebold & Ebens, 2001; Hansen & Lunde, 2004, among others). Alternatively, the methods that apply the concept of price-range to intraday intervals are also proposed (Martens & Dijk, 2006; Christensen & Podolskij, 2006). Intraday data helps to capture the time-varying volatility more closely. Moreover, its use limits the influence of expected returns that is otherwise difficult to eliminate. These methods improve the efficiency of variance estimation significantly.

But there are limitations to the use of intraday data in the financial markets. First, there are markets where high-frequency price data are simply not available. Even when these are available, their frequency may be very low. For instance there may be less than one price observation for each five minute time interval. Such a low frequency may not allow significant efficiency gains. In such cases, the daily range-based methods offer a good alternative. Second, the observed prices are often contaminated by an array of microstructure errors. This makes the high-frequency data based estimators biased and inconsistent. This phenomenon is more pronounced for the range-based estimators using high-frequency data. The available approaches mitigate the influence of these errors only if certain specific assumptions are true (Zhang, Mykland and Aït-Sahalia, 2005, among others). Third, daily and intraday range-based methods are found to be very sensitive to their strict theoretical assumptions. These problems cast doubts on the relative efficiency of various estimators in an empirical context.

Much of the progress in the volatility research, especially the use of high-frequency data, is limited to volatility estimation. Forecasting ex-ante volatility from their ex-post values is still dominated by methods such as GARCH (Hansen & Lunde, 2005). These methods use squared daily returns as the ex-post measure of volatility. The limited evidence available on the volatility forecasting based on high-frequency data and range is encouraging. Andersen, Bollerslev, Diebold and Labys (2003) found that GARCH, its variants, and the stochastic volatility models, are inferior to the realized volatility based time series methods in the forex market. Similarly, Vipul and Jacob (2006) found that time series methods with the past range-based volatility estimates outperform GARCH for an equity index. These findings can be attributed to the more precise estimates of ex-post volatility used for forecasting. This calls for an investigation of the forecasting performance of high-frequency and range-based estimates for relatively less-liquid assets such as individual stocks.

Against this backdrop the study attempts the following. First, it examines the relative estimation and forecasting performance of a number of daily range-based and high-frequency based volatility estimators. A set of stocks, which distinctly differ on characteristics like liquidity, drift and volatility are used. The use of these stocks allows examining the change in the performance of various estimators and forecasting methods with these characteristics. This would bring more clarity on the choice of optimal estimators and forecasting methods in varying contexts. The available studies on estimation and forecasting are mostly limited to highly liquid assets like indexes and foreign exchange. The forecasts are examined for empirically relevant daily, weekly and monthly periods. The comparison of forecasts based on high-frequency estimators and daily range would give more insights about the gain from high-frequency data. Second, it empirically examines the documented poor performance of one of the important range-based estimators: the Parkinson's estimator. This examination is done by using Parkinson's estimator with price ranges for varying time periods. An insight into the performance of Parkinson's estimator is crucial for understanding the behavior of range-based estimators which use the same basic approach (for e.g. Alizadeh et al, 2002, Chou, 2005, Christensen & Podolskij, 2006).

The study uses, the Two-Scales Realized Volatility of Zhang, Mykland and Aït-Sahalia (2005) as the benchmark to ensure unbiasedness and high efficiency. The price quotations of National Stock Exchange of India (NSE) used in the study provide an opportunity to examine the data that is not significantly contaminated by the bid-ask spread. The transactions at NSE are based on anonymous order matching process. The available studies are conducted in the markets where dealers offer two-way quotes leading to significant bid-ask spread effects.

The remainder of this article is organized as follows. Section 2 briefly discusses the range-based estimators used in this study. Section 3 provides the methodology for estimating and forecasting volatility. Section 4 discusses the results and Section 5 gives the conclusions from the study.

2. Range-based Volatility Estimation

When the expected return during a time interval is zero, the squared return during that interval provides an estimate of the variance of the return process. The range-based estimators are developed on the intuition that the price-range during any interval would more effectively capture volatility than the squared returns. Therefore, a properly scaled squared log-range would give a more efficient estimate of the variance. The scaling factor comes from the expected relationship between the squared log-range and the variance of a process following Brownian motion[1]. Generally, these estimators assume that stock prices follow a geometric Brownian motion with two parameters: the drift and

[1] This factor was developed by Parkinson (1980). It uses the asymptotic distribution of the range of cumulative sums of independent random variables developed by Feller (1951). The moment generating function of the range is developed by Parkinson after applying a suitable truncation to the infinite series involved in the distribution.

the volatility. Parkinson (1980) proposed the first such estimator, assuming a driftless price process. Its variance is claimed to be only about one-fifth of that of the squared returns. This estimator (PK estimator) is

$$\frac{(H_t - L_t)^2}{4ln2} \tag{1}$$

where, H_t and L_t are the log transformed highest and lowest prices observed during the trading day *t*. The following concerns remain with the PK estimator. First, in obtaining the relation between the range and the variance, Parkinson assumed that infinite steps of random walk are observed. With discrete trading, the observed high (low) may underestimate (overestimate) the true value. This is expected to downwardly bias the PK estimator for less liquid assets. Second, the presence of a non-zero drift leaves the estimator upwardly biased by overestimating (H_t - L_t). Garman and Klass (1980) attempted to further improve the efficiency of range-based estimation, using the opening and closing prices in addition to the price-range. They made the same assumptions as the PK estimator. Their estimator is claimed to have a lower variance than the PK estimator. Their estimator (GK estimator) is

$$0.511(H_t - L_t)^2 - .019\{(C_t - O_t)(H_t + L_t - 2O_t) - 2(H_t - O_t)(L_t - O_t)\} - 0.383(C_t - O_t)^2 \tag{2}$$

where, O_t and C_t are the log transformed opening and closing prices for the trading day *t*. The middle term in this expression is relatively very small. Therefore, practically, this estimator is a weighted average of PK estimator and squared 'open-to-close' returns. The squared 'open-to-close' return, a fraction of which is effectively subtracted from the squared low to high return, acts as a proxy to the drift. This drift adjustment helps the GK estimator to be less biased and more efficient in the presence of drift, though it assumes a driftless price process. The issue of drift is more formally addressed by Rogers and Satchell (1991) by offering an estimator which is independent of drift. Their estimator (RS estimator) is

$$(H_t - C_t)(H_t - O_t) + (L_t - C_t)(L_t - O_t). \tag{3}$$

This is claimed to be more efficient than the PK and GK estimators, when the security price has a drift. Later, Kunitomo (1992) and Yang and Zhang (2000) also suggested alternative estimators that take care of the drift, using transformed price process. The estimator of Yang and Zhang uses open, high, low and close prices of multiple periods to obtain a more efficient estimate. Therefore, it does not give the estimate of volatility for a single period. The Kunitomo's estimator uses a Brownian bridge price process, which requires tick-level data. The need for tick-data limits the practical utility of this estimator. The efficiency of the above estimators is confirmed by various studies (Bali & Weinbaum, 2005; Shu & Zhang, 2006, among others).

Motivated by the higher efficiency afforded by range-based estimation, a number of estimators have also been proposed recently. Alizadeh, Brandt and Diebold (2002) developed a stochastic volatility estimator using daily range. Chou (2005), and Brandt and Jones (2006) proposed range-based GARCH type estimators. Quite recently, Martens and Dijk (2006), and Christensen and Podolskij (2006) concurrently proposed a realized range estimator. This, in fact, is a straightforward application of the PK estimator to intraday intervals, primarily motivated by two factors. First, the influence of drift in variance estimation, particularly in PK estimator, can be mitigated by using range over small intraday intervals. Second, the stochastic nature of volatility is captured more closely with the use of such small intervals. This estimator (RRV) takes the following form:

$$\frac{1}{4\ln 2}\sum_{i=1}^{N}\left(H_{t,i}-L_{t,i}\right)^2 \tag{4}$$

where, $H_{t,i}$ and $L_{t,i}$ are the log transformed high and low prices during the i-th intraday period on day t, and N is the total number of intraday periods. RRV is found to be more biased due to the influence of microstructure errors, compared to the estimators based on squared intraday returns. Martens and Dijk (2006) report that RRV estimators are upwardly biased as the observed high-low range is pushed upwards by the bid-ask spread[2]. The daily-range based bias correction of RRV, suggested by Martens and Dijk, may not be optimal as daily range itself may be biased.

Unfortunately, the choice of the best estimator is not so straightforward. Whereas, the daily range-based estimators are predominantly free from the influence of microstructure error, the RRV is not[3]. On the other hand, the RRV may more closely approximate the variance than its daily counterparts. Therefore, the empirical efficiency of these estimators would depend on the extent of microstructure error in the observed prices, liquidity level of the assets and the presence of drift in the price process.

3. Methodology

The performance of range-based volatility estimators is sensitive to the liquidity and the drift of price process and also the autocorrelation of underlying returns. Low liquidity and negative autocorrelation are predicted to lead to an underestimation. The presence of drift is likely to lead to an overestimation, particularly with the PK estimator. Therefore, stocks are selected such that they differ distinctly on these characteristics. Among the various daily range-based estimators, Yang and Zhang (2000) and Kunitomo (1991) are not included in the study[4].

The characteristics of the selected stocks, ACC, Infosys, Reliance and Zeetele[5], are given in Table I. Among the four selected stocks, Reliance has the highest average liquidity (about two transactions per second) followed by Infosys, Zeetele and ACC (about one

[2] In a market where prices alternate between bid and ask, the high price is likely to be an ask price and the low price a bid price; leading to overestimation of each range. In comparison, the price observations involved in squared intraday returns will be more or less equally divided between bid and ask.

[3] A simulation conducted by Alizadeh et al (2002) confirms the relative robustness of PK estimator to one of the significant microstructure errors: the bid-ask bounce.

[4] The estimator of Yang and Zhang is ignored as it does not provide the volatility estimate for a day. Kunitomo's estimator has little practical value as it uses tick-level data.

[5] These four stocks are constituents of the leading Indian equity indexes, SENSEX and NIFTY. Derivatives are also traded on these stocks. ACC (Associated Cement Company) is a leading cement manufacturer, Infosys (Infosys Technologies) is a major software services provider, Reliance (Reliance Industries) is a diversified firm with major interests in petrochemicals, and Zeetele (Zee Telefilms) is a leading entertainment provider.

transaction every two seconds). Zeetele is the most volatile of the four. Average Daily Absolute Return is used as a proxy for the average daily 'open-to-close' drift present in stock prices. Both positive and negative drift increase the estimate of the range-based volatility. Zeetele has the highest daily drift (2.66%) followed by Infosys (1.96%), ACC (1.80%) and Reliance (1.56%). All the stocks have significant negative first-order autocorrelation between intraday returns (in a close range between – 0.31 and – 0.35)[6]. The autocorrelations beyond one lag are insignificant. The partial autocorrelations over different lags decline exponentially. This pattern is similar to a first-order moving average process. It is widely documented that this type of correlation in high-frequency returns is induced by microstructure errors (Campbell, Lo & MacKinlay, 1997). The wide range of liquidity, volatility and daily drift of these stocks provides a good opportunity of testing the effectiveness of different methods for estimation and forecasting of range-based volatility.

The availability of a large number of price observations and the presence of autocorrelation in returns make it undesirable to use the simple realized volatility (RV) as the benchmark. The RV for a day, which is the sum of the squared intraday returns, assumes that returns are uncorrelated. Ideally, the sampling intervals should be as short as possible, inducing the use of all the available data. But, the presence of microstructure noise makes RV inconsistent and biased for the data at very high frequencies (refer to Andersen, Bollerslev, & Meddahi, 2005; Aït-Sahalia, Mykland, & Zhang, 2005a for a detailed discussion). Practically, the RV is estimated using the prices sampled at periods ranging from five minutes to 30 minutes. But, even for a small sampling period like five minutes, a lot of price observations are still ignored.

Due to these considerations, a more appropriate benchmark is the Two-scales Realized Volatility (TSRV) developed by Zhang, Mykland and Aït-Sahalia (2005). It attempts to correct the bias induced by microstructure error and allows the use of all the available price observations. TSRV assumes that the price process is independent of noise, and the returns have a first-order negative autocorrelation. Essentially, the TSRV approach combines the variance at a relatively low and a relatively high frequency to eliminate the influence of microstructure noise. This is based on the result that the realized variance estimated with $N+1$ price observations includes $2N$ times the variance of the noise. Moreover, the variance at the low frequency ($\overline{s}^{2}_{sub}$) is estimated by averaging the variance over different sub-samples[7]. This facilitates the use of complete price data. As the bias correction depends on the scaled difference between the variances estimated for these two frequencies, they should be substantially different. On the other hand, there should be sufficient number of price observations in each sub-sample at the low frequency. The two frequencies are chosen in the light of these considerations. The variance at the low frequency is estimated by sub-sampling prices at every five minutes. The variance at the high frequency (s^{2}_{high}) is estimated using the data sampled at frequencies varying from one

[6] Based on the autocorrelations between one-second returns for each day, averaged over 1256 days

[7] Suppose there are 19,800 equidistant price observations during a 5½ hours trading day. A sub-sample of 66 observations can be formed at a low frequency of '5-minutes' starting with the first observation and then systematically selecting observations, skipping 300 observations at a time. Another sub-sample can be obtained starting with the second observation. Similarly, 298 more sub-samples can be obtained by systematically sampling the rest of the observations.

to four seconds, depending on the liquidity of the stock. The variance for the trading day *t*, estimated using the TSRV approach, is as follows:

$$s_t^2 = \frac{N}{N-\overline{n}}\left(\overline{s}^2_{sub,t} - \frac{\overline{n}}{N} s^2_{high,t}\right) \tag{5}$$

where, $\overline{n}$ is the average number of returns across all the sub-samples at the low frequency and N is the total number of returns at the high frequency[8]. Aït-Sahalia, Mykland and Zhang (2005b) confirmed that the bias correction of TSRV is largely robust even to a serially correlated microstructure noise. The TSRV estimator has been recently used in several studies (Martens & Dijk, 2006, among others).

This study uses the high-frequency data for a period of five years from January 1, 2001 to December 30, 2005 (1256 trading days). The data of the four stocks has been sourced from National Stock Exchange, Mumbai, India (NSE). Price quotations for each day cover about 5 ½ hours time period which is the normal daily trading time for NSE. The raw price data is filtered for outliers and transactions beyond the official closing time of the exchange are discarded. The data for five days having transactions for less than two hours are also removed from the sample. This leaves a total of 1256 trading days for all the stocks. The high-frequency data are sampled based on the transaction time (calendar time sampling). This is induced by the logic that the violation of the assumption of equidistant prices is likely to make the estimation inconsistent for tick-data. Since the NSE stock data are time-stamped to the nearest second, the data are filtered at one-second interval. This leaves an average of 9,669 price observations for Reliance (maximum) and 4,670 for ACC (minimum). The TSRV volatility is estimated using this one-second price data. Wherever the price corresponding to a particular time is not available the nearest previous tick is used to obtain the returns for such cases.

The daily volatility based on the PK, GK, and RS estimators is calculated using the daily open, high, low and closing prices. The volatility based on RRV estimators is reported for 5, 10, 15, and 30 minute periods. Sixty and 120 minute RRV's are also used but their estimations tend to converge to those of the PK estimator as expected. The results for these are not reported for brevity.

The estimate of variance based on TSRV and the other competing estimators is only for the 'open-to-close' period of the market (about 5 ½ hours) each day. However, in the stock market, the participants are exposed to variance over the entire 24-hour day. Therefore, the variance for an entire day is obtained by scaling up the 'open-to-close' variance. The scaling factor estimates the true daily variance using the 'open-to-close' variance and the noisy overnight variance. This is similar to the approaches of Martens (2002) and Koopman, Jungbacker and Hol (2005). The scaling factor *?* is estimated with the following formula,

$$g = \sum_{t=1}^{T} r_t^2 \Big/ \sum_{t=1}^{T} r_{ot}^2 \tag{6}$$

[8] The sub-sampling procedure in TSRV estimator leaves a small number of observations in the beginning or end in each sub-sample. Some researchers apply an area correction for this. But, all the sub-samples together encompass almost all the data for a day and the average variance from the sub-samples captures it. Therefore, the effect of the left out data would be negligible and is ignored.

where, r_{ot}^2 is the squared 'open-to-close' return and r_t^2 is the sum of the squared 'overnight' and 'open-to-close' returns for t-th day. The overnight return is estimated after adjusting for bonus issues and stock splits. A constant scaling factor, based on the data for five years, is applied to the entire period[9]. This factor ranges from 1.299 for ACC to 1.172 for Zeetele. Its magnitude is comparable to that of the factor used by Martens (2002) in the US market[10]. The scaled daily variances are added over five days and 21 days to estimate the weekly and monthly variances respectively. Alternative estimates based on weekly and monthly ranges are very poor compared to the estimates obtained through the above approach. This can be attributed to the following. First, the weekly and monthly ranges may have the presence of considerable drift. Second, the assumption of constant volatility during such a long period contradicts the well-documented stochastic nature of volatility (these results are not reported).

Here, the attempt is to directly forecast volatility from its own past values using a set of time series methods. The range of forecasting methods used to assess the extent of gain from high-frequency data are given below:

MA (q):
$$\hat{s}_{t+1,j} = \frac{1}{q}\sum_{i=1}^{q} s_{t-i+1,j} \tag{7}$$

AR (p):
$$\hat{s}_{t+1,j} = w_j + \sum_{i=1}^{p} a_{ij} s_{t-i+1,j} \qquad a_{ij} \geq 0 \tag{8}$$

EWMA:
$$\hat{s}_{t+1,j} = a_j s_{t,j} + (1 - a_j)\hat{s}_{t,j} \qquad 0 \leq a_j \leq 1 \tag{9}$$

where, $\hat{s}_{t+1,j}$ is the forecast for the period $(t+1)$ using the estimator j; $s_{t,j}$ is the estimate of volatility for the period t using the estimator j; w_j, a_j, and a_{ij} are coefficients.

The following ARMA model is also used to account for the volatility clustering. This method is motivated by GARCH and the recently proposed Conditional Autoregressive Range model of Chou (2005).

ARMA (1, 1):
$$\hat{s}^2_{t+1,j} = d_j + g_j s_{t,j}^{\ 2} + b_j \hat{s}^2_{t,j}; \quad 0 \leq d_j, g_j, b_j \leq 1 \tag{10}$$

where, d_j, $?_j$, and b_j are coefficients. The forecasts are carried out using the rolling fixed-window approach by re-estimating the coefficients for each period. This widely used

[9] Alternative scaling factors based on one, two and three years' rolling windows are also examined. But, the performance of all the estimators is poorer than that of the constant scaling factor based on five years' data. In fact, the use of one scaling factor or the other does not affect the relative performance of the estimators as all of them are equally affected.

[10] Our estimate of overnight variance for the selected stocks is in line with the findings of Hansen and Lunde (2005b) that for the equities included in the DIJA, 20% of the volatility occurs during the inactive 'overnight' period.

practice would help to capture the time variation in volatility (e.g. Figlewski, 1997). As the forecasts are also sensitive to the amount of data used for estimating coefficients, forecasts are carried out using different estimation sets.

The estimators and forecasting methods are evaluated on efficiency and bias. The efficiency is measured by the root mean squared error (RMSE), and bias by the mean bias (Mean Bias). Both RMSE and Mean Bias are estimated using standard deviation as the measure of volatility because variance as a measure would involve fourth moments. Using RMSE rather than mean squared error helps in comparing the magnitude of error with bias. These loss functions are given below:

$$\text{RMSE} = \sqrt{E(\hat{s}_t - s_t)^2} \tag{11}$$

$$\text{Mean Bias} = E(\hat{s}_t - s_t) \tag{12}$$

4. Results

4.1 Estimation

Based on TSRV, the variance induced by microstructure error is only about 10^{-3} times the average realized variance at five-minute frequency for the selected stocks. This implies that the influence of microstructure error is relatively low in variance estimation for the data sampled at five minutes. The estimates of volatility from RRV estimators, particularly at 5 and 10 minutes do not deviate significantly from the TSRV volatility. Therefore, the 'discrete price' correction procedures[11] for the downward bias of range-based estimators are not used. Such bias is perhaps getting offset by the upward bias of microstructure error. This is in line with the findings of Alizadeh et al (2002). At the same time, it contrasts with the finding of Martens and Dijk (2006) that RRV estimator is significantly upward biased due to microstructure error. The relatively insignificant influence of microstructure noise in RRV estimation can be explained by its low order of magnitude. This, probably, is due to the absence of a significant bid-ask spread in the price data sourced from NSE. NSE is an order matching market where the specialists are absent. Therefore, the traders carry out their transactions at the prices quoted by the other traders and not at the bid/ask prices quoted by the specialists.

The mean and standard deviation of volatility estimates for the daily, weekly and monthly time periods are reported in Table II. The mean volatility is the highest for PK estimator, followed by the GK estimator. The RS estimator reports the lowest mean volatility. The PK estimator tends to overestimate the true volatility (measured by TSRV benchmark) for all the stocks and time periods. In contrast, the RS estimator tends to underestimate the volatility for daily estimation.

RRV(5), based on PK estimator with five-minute intervals, invariably estimates the volatility lower than the PK estimator that uses full-day time intervals. RRV(10), RRV(15) and RRV(30) also have a similar pattern compared to PK estimator. But the magnitude of their difference from the PK estimator keeps reducing as the time interval increases from 5

[11] Bias correction procedures are suggested by Rogers and Satchell (1991), and recently by Christensen and Podolskij (2006).

to 30 minutes. The same pattern is shown by the RRV estimator with higher time intervals (not reported for brevity). The difference between the RRV estimators and the PK estimator can mostly be explained by the presence of drift in the returns process. The more the drift (whether positive or negative), the more does it increase the range and therefore, the estimate of volatility. If the time period of estimation is small (say five minutes or 10 minutes), then the drift would have a lower adverse effect on the range. This is why RRV's based on smaller intervals reduce the estimation bias with the same PK estimator that otherwise overestimates the volatility. This contention is further corroborated by the fact that the difference between the PK estimator and RRV(5) is much less for those stocks which have a lower Average Daily Absolute Return (reflecting a lower drift). The difference between RRV(5) and PK estimator is lower for ACC and Reliance (having lower drift) as compared to that for Infosys and Zeetele. The lower mean volatility estimated by the GK and RS estimators, compared to PK estimator, also point towards the significant influence of drift in volatility estimation for stocks. The RRV estimators appear to effectively mitigate the influence of drift. However, RRV based on very small intervals, such as five minutes or 10 minutes, may not capture the price changes spanning over adjacent time intervals. If the price remains the same during an interval, and then suddenly changes and remains at that level during the subsequent interval, the RRV would not be able to capture this volatility. We observed a few such instances for some stocks.

The estimation performance of various estimators is presented in Table III. The Root Mean Square Error (RMSE) and Mean Bias (M.BIAS) for an estimator represent its efficiency and bias respectively. Both the inefficiency and bias increase as we increase the time interval of estimation of volatility from a day to a month. This is due to the fact that the weekly and monthly volatility estimates are derived by cumulating daily volatility over five days and 21 days respectively. Expectedly, the bias, being additive in the same direction, increases much more than the inefficiency, that gets mutually cancelled across different days.

Among PK, GK and RS estimators, the GK estimator generally has the highest efficiency, but RS estimator has the lowest bias. However, the gain on efficiency of the GK estimator generally is much more than its loss on the bias, except for the weekly and monthly estimation for Reliance, where the RS estimator appears to be better. This is because the bias becomes more important in cumulated daily variances, if the magnitude of error in volatility estimation is small (as for Reliance due to lower drift). Therefore, the RS estimator with its inherently low bias performs better. When the magnitude of error is larger (as for the other three stocks due to higher drift), the lower bias of RS estimator is unable to overcome the higher efficiency of the GK estimator.

RRV estimators outperform all the daily range-based estimators on bias and efficiency, except for the weekly and monthly estimation for Reliance. The ratio of RMSE of RRV(5) to PK estimator is higher for Zeetele and Infosys as compared to that for ACC and Reliance. This is possibly due to the relatively higher drift in the former two as compared to the latter two. Moreover, the RMSE for the GK, RS and PK estimators have more or less the same order of rankings as their Average Daily Absolute Return (proxy for the drift). These observations further strengthen our contention that the drift present in the return process primarily causes the underperformance of daily range-based estimators.

The liquidity and volatility do not appear to affect the estimation process significantly. Most of the range-based estimators for our stocks have a positive rather than a negative bias. This is contrary to the apprehension that range-based estimators would be negatively biased due to discreteness of price data. Further, these estimators, for even the least liquid ACC stock, do not show a significant negative bias. On the other hand, the most liquid Reliance stock does not show a significantly lower negative bias compared to ACC. These observations are contrary to the expected behavior of the less liquid and more liquid stocks. The ranking order of RMSE for different stocks also does not follow the ranking order of their volatility levels. It indicates the insensitivity of estimators to the volatility levels of individual stocks. Since the range of return autocorrelation is not wide enough, it is difficult to draw specific inferences about its relative influence on the volatility estimation. However, its magnitude present in the selected stocks is not able to negatively bias the estimation.

Though RRV's perform better than the daily range-based estimators, their use in the volatility estimation by practitioners has two problems. First, its empirical performance is very sensitive to the presence of microstructure error and discreteness of the price series. Bias correction methods proposed by Martens and Dijk (2006) and Christensen, Podolskij and Vetter (2006) are difficult to apply as they vary for each intraday interval. Second, sudden changes in prices in short time periods may distort the estimation significantly. Therefore, if the high-frequency data is available it is advisable to use returns-based estimations like TSRV. However, in many markets the high-frequency data is not readily available.

4.2 Forecasting

The forecasting performance of various estimators using different methods of forecasting the daily, weekly and monthly volatility is reported in Table IV, Table V and Table VI. The methods of forecasting used with PK, GK, RS, RRV and TSRV estimators are simple Moving Average (MA), Exponentially Weighted Moving Average (EWMA), Autoregression (AR), and Autoregressive Moving Average (ARMA). Results for MA for 10 days [MA(10)], EWMA, AR for two-day lags [AR(2)] and ARMA (1,1) are reported in the tables. In addition, MA using lags ranging from two days to 10 days, and AR for lags up to eight days are also used for forecasting. But these are not reported for brevity as their performance is inferior. The results in Table IV to Table VI are for estimation sets of 240 periods for daily forecasting, 40 periods for weekly forecasting and 30 periods for monthly forecasting. This applies to all the methods other than MA. These are the most optimal estimation sets with the available data for the selected stocks.

For each of the forecasting methods for the daily volatility, the efficiency of daily range-based GK and RS estimators is comparable to that of the TSRV benchmark. However, on bias, TSRV performs better as expected. The performance of RRV's lies in between the TSRV and the daily range-based GK and RS estimators. Considering the requirement of high-frequency data and attendant computation for TSRV and RRV's, it appears that there is no significant loss of information in using daily range-based estimators for forecasting. Overall, considering RMSE and Mean Bias together, the GK estimator appears to be the most desirable among the daily range-based estimators, closely followed by the RS estimator. Among the forecasting methods, EWMA appears to be better than the others. ARMA is included among the forecasting methods to more closely account for the volatility clustering. GARCH is not included because its forecasting efficiency is generally found to be poor. However, the forecasting efficiency of ARMA is inferior to EWMA and its bias is significantly more than the other methods as is generally observed for GARCH also (refer to Vipul & Jacob, 2006).

For the weekly forecasting also, the pattern of efficiency and bias of different estimators and methods is similar. Here, the only difference is that the RS estimator performs marginally better than the GK estimator for ACC, Infosys and Reliance. The GK estimator outperforms the others for Zeetele. AR(2) method of forecasting closely follows EWMA and performs better in certain cases. For monthly forecasting, the pattern is similar to the weekly forecasting with GK estimator performing better for Infosys and Zeetele, and RS estimator performing better for ACC and Reliance. Here again, the GK estimator performs relatively better for high-drift stocks, whereas the RS estimator performs better for the low-drift stocks. This is due to the lower bias of RS estimator which becomes the deciding factor if the magnitude of error in the volatility is small as for the first three stocks (due to lower drift). When the magnitude of error in the volatility is large (as in Zeetele due to higher drift), the lower bias of RS estimator is not able to overcome the higher efficiency of the GK estimator. If the magnitude of drift is very small, as is the case with RRV estimators, then even the PK estimator, without any drift correction, gives reasonably good results.

5. Conclusions

In this study the estimation and forecasting performance of various range-based volatility estimators is examined for four different stocks in the Indian market. The range of their characteristics like liquidity, drift and volatility brings about their effect on volatility estimation and forecasting quite clearly. The drift in the stock prices has a major influence on the efficiency of estimation and forecasting of the stock price volatility. The more the drift, the less is the efficiency of estimation and forecasting. The GK estimator, which indirectly adjusts for the drift, performs better for high-drift stocks, whereas the RS estimator performs better for the low-drift stocks. This is explained by the higher efficiency and bias of the GK estimator as compared to the RS estimator. The PK estimator also performs well, if the drift is insignificant. Daily range-based estimators appear to be competitive to high-frequency data based estimators for volatility forecasting up to one month. Among the forecasting methods that use daily range-based methods, EWMA appears to be the most efficient, closely followed by AR(2). The level of liquidity and volatility do not have a significant effect on the estimation and forecasting efficiency of range-based estimators. These estimators, particularly the estimators suggested by Rogers and Satchell, and Garman and Klass give very promising results for forecasting. Their forecasts are as efficient as those with the benchmark Two-scales Realized Volatility. However, they have a higher bias as compared to TSRV. This result is true across all the estimation time-periods and forecasting methods.

In view of these findings, these estimators have a strong case against the best available estimator (TSRV) and the next best alternative (RRV). The latter two estimators require high-frequency data and involve computations that make them difficult to implement. On the other hand, the daily open, close, high and low price data, required for the range-based estimators, is readily available in most of the stock markets. Our results also indicate an absence of negative bias in range-based estimators. Interestingly, this is despite the low liquidity and high negative first-order autocorrelation present in our stocks. The price observations per day for all our stocks are much less than 20,000 – the minimum number theoretically required to ensure unbiased estimation. Moreover, most of our stocks had a high negative first order autocorrelation in their returns (ranging between – 0.31 and – 0.35). The negative bias of the range-based estimators for stocks, reported by Beckers (1983) and Wiggins (1991), was probably due to their comparison with the daily squared returns.

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Table I: Basic Characteristics of the Stocks

Stock	Average One-second Price Observations Per Day*	Average Daily Volatility	Average Daily Absolute Return	First Order Auto-correlation in Returns
ACC	4670	0.0198	0.0180	-0.3095
Infosys	9199	0.0186	0.0196	-0.3147
Zeetele	7116	0.0292	0.0266	-0.3472
Reliance	9669	0.0165	0.0156	-0.3206

* The average daily tick-data observations for ACC, Infosys, Zeetele and Reliance are 12035, 29183, 23200 and 39552 respectively. However, the figures reported in the table are based on one-second calendar time sampling for each of these stocks as these are the observations used for the estimation of volatility.

Table II: Descriptive Statistics of Volatility Estimates

Stock	Statistic	TSRV	PK	GK	RS	RRV(5)	RRV(10)	RRV(15)	RRV(30)
				Panel A: Daily Volatility (open-to-close)					
ACC	Mean	1.948	1.986	1.958	1.904	1.947	1.982	1.987	1.991
	Std. Dev.	(1.061)	(1.247)	(1.139)	(1.170)	(1.015)	(1.075)	(1.092)	(1.119)
Infosys	Mean	1.856	2.017	1.918	1.828	1.915	1.951	1.955	1.962
	Std. Dev.	(1.165)	(1.497)	(1.331)	(1.377)	(1.155)	(1.198)	(1.226)	(1.269)
Zeetele	Mean	2.925	3.015	2.973	2.904	2.924	2.971	2.967	2.969
	Std. Dev.	(1.656)	(1.881)	(1.820)	(1.921)	(1.465)	(1.544)	(1.556)	(1.602)
Reliance	Mean	1.646	1.705	1.665	1.614	1.648	1.681	1.685	1.689
	Std. Dev.	(0.825)	(1.071)	(0.954)	(0.976)	(0.839)	(0.865)	(0.879)	(0.916)
				Panel B: Weekly Volatility					
ACC	Mean	5.105	5.371	5.248	5.173	5.105	5.212	5.230	5.259
	Std. Dev.	(2.405)	(2.604)	(2.380)	(2.353)	(2.268)	(2.398)	(2.434)	(2.475)
Infosys	Mean	4.936	5.601	5.264	5.125	5.104	5.208	5.231	5.272
	Std. Dev.	(2.726)	(3.226)	(2.891)	(2.899)	(2.654)	(2.750)	(2.806)	(2.890)
Zeetele	Mean	7.317	7.800	7.662	7.620	7.263	7.399	7.405	7.431
	Std. Dev.	(3.571)	(3.639)	(3.548)	(3.617)	(3.164)	(3.320)	(3.325)	(3.399)
Reliance	Mean	4.264	4.587	4.428	4.345	4.281	4.372	4.390	4.415
	Std. Dev.	(1.793)	(2.135)	(1.909)	(1.896)	(1.805)	(1.856)	(1.880)	(1.955)
				Panel C: Monthly Volatility					
ACC	Mean	10.696	11.340	11.063	10.942	10.712	10.952	10.992	11.068
	Std. Dev.	(4.553)	(4.673)	(4.268)	(4.164)	(4.173)	(4.426)	(4.493)	(4.543)
Infosys	Mean	10.517	12.082	11.280	11.018	10.880	11.117	11.186	11.301
	Std. Dev.	(4.940)	(5.616)	(5.089)	(5.083)	(4.703)	(4.868)	(4.952)	(5.076)
Zeetele	Mean	15.422	16.618	16.275	16.235	15.252	15.569	15.579	15.672
	Std. Dev.	(6.612)	(6.187)	(6.137)	(6.198)	(5.813)	(6.068)	(6.087)	(6.157)
Reliance	Mean	8.999	9.853	9.419	9.233	9.084	9.284	9.331	9.401
	Std. Dev.	(3.090)	(3.365)	(3.121)	(3.142)	(2.975)	(3.049)	(3.079)	(3.215)

Notes: TSRV is the Two-scales Realized Volatility. PK, GK and RS are the volatilities based on the estimators of Parkinson, Garman and Klass, and Rogers and Satchell respectively. RRV(5), RRV(10), RRV(15), and RRV(30) are the Realized Range-based volatilities based on Parkinson's estimator for various time intervals (5, 10, 15, and 30 minutes). The mean values and standard deviations (in parentheses) of the estimates of volatility are given in percentages.

Table III: Estimation Performance of Various Estimators

Stock	Loss Function	PK	GK	RS	RRV(5)	RRV(10)	RRV(15)	RRV(30)
		Panel A: Daily Volatility (open-to-close)						
ACC	RMSE	0.6315	0.5789	0.7845	0.3728	0.3436	0.3431	0.3494
	M.BIAS	0.0378	0.0098	-0.0437	-0.0009	0.0341	0.0389	0.0435
Infosys	RMSE	0.8329	0.6412	0.8590	0.4873	0.4870	0.4850	0.4925
	M.BIAS	0.1611	0.0616	-0.0281	0.0585	0.0945	0.0984	0.1060
Zeetele	RMSE	0.9215	0.8473	1.1360	0.4672	0.4279	0.4386	0.4528
	M.BIAS	0.0901	0.0480	-0.0207	-0.0011	0.0458	0.0425	0.0435
Reliance	RMSE	0.6688	0.5096	0.5866	0.4644	0.4543	0.4543	0.4582
	M.BIAS	0.0592	0.0198	-0.0312	0.0023	0.0351	0.0394	0.0435
		Panel B: Weekly Volatility						
ACC	RMSE	1.0222	0.8923	1.0994	0.7246	0.6675	0.6671	0.6757
	M.BIAS	0.2657	0.1422	0.0673	-0.0004	0.1070	0.1248	0.1535
Infosys	RMSE	1.4788	1.1041	1.3500	0.9571	0.9763	0.9732	0.9935
	M.BIAS	0.6654	0.3277	0.1889	0.1679	0.2719	0.2956	0.3361
Zeetele	RMSE	1.2708	1.1916	1.4799	0.5828	0.4814	0.5100	0.5543
	M.BIAS	0.4831	0.3450	0.3027	-0.0540	0.0821	0.0876	0.1136
Reliance	RMSE	1.1046	0.7530	0.7491	0.8753	0.8551	0.8550	0.8588
	M.BIAS	0.3233	0.1646	0.0812	0.0179	0.1085	0.1268	0.1512
		Panel C: Monthly Volatility						
ACC	RMSE	1.4982	1.2303	1.4181	1.0926	0.9772	0.9898	1.0142
	M.BIAS	0.6440	0.3666	0.2463	0.0159	0.2556	0.2954	0.3723
Infosys	RMSE	2.4852	1.6377	1.9196	1.4508	1.4793	1.5012	1.5405
	M.BIAS	1.5647	0.7630	0.5007	0.3624	0.5992	0.6682	0.7840
Zeetele	RMSE	1.8184	1.6914	1.8934	0.9541	0.7324	0.7322	0.8376
	M.BIAS	1.1965	0.8537	0.8128	-0.1698	0.1476	0.1568	0.2501
Reliance	RMSE	1.6960	1.0135	0.8716	1.3623	1.3294	1.3219	1.3254
	M.BIAS	0.8546	0.4207	0.2346	0.0851	0.2850	0.3321	0.4021

Notes: TSRV is the Two-scales Realized Volatility. PK, GK and RS are the volatilities based on the estimators of Parkinson, Garman and Klass, and Rogers and Satchell respectively. RRV(5), RRV(10), RRV(15), and RRV(30) are the Realized Range-based volatilities based on Parkinson's estimator for various time intervals (5, 10, 15, and 30 minutes). RMSE and M.BIAS are the root mean squared error and mean bias in percentages respectively. TSRV is used as the benchmark.

Table IV: Forecasting Performance – Daily (open-to-close)

				Panel A : RMSE					
Stock	Method	TSRV	PK	GK	RS	RRV(5)	RRV(10)	RRV(15)	RRV(30)
ACC	MA(10)	1.3748	1.3614	1.3325	1.3323	1.3314	1.3328	1.3368	1.3526
	EWMA	0.7199	0.6772	0.6700	0.6962	0.6863	0.6941	0.6944	0.7002
	AR(2)	0.7362	0.7208	0.7019	0.7236	0.7018	0.7090	0.7114	0.7381
	ARMA	0.7580	0.8239	0.7471	0.7482	0.7536	0.7705	0.7756	0.8094
Infosys	MA(10)	0.9367	0.9789	0.9589	0.9650	0.9429	0.9508	0.9550	0.9590
	EWMA	0.7976	0.8357	0.8036	0.8055	0.8280	0.8422	0.8499	0.8552
	AR(2)	0.7627	0.8152	0.8013	0.8134	0.9214	0.8976	0.9227	0.8670
	ARMA	0.8490	0.9443	0.9261	1.0328	0.9508	0.9456	0.9757	0.9594
Zeetele	MA(10)	1.2225	1.2230	1.2718	1.3428	1.2336	1.2385	1.2349	1.2449
	EWMA	0.9006	0.9455	0.9444	0.9579	0.9086	0.9140	0.9140	0.9208
	AR(2)	0.8909	0.9487	0.9434	0.9513	0.8867	0.8963	0.8981	0.9028
	ARMA	0.8945	1.0619	1.0287	1.0415	0.8868	0.9082	0.9069	0.9144
Reliance	MA(10)	0.7586	0.7631	0.7544	0.7652	0.7555	0.7576	0.7571	0.7587
	EWMA	0.6933	0.7046	0.7028	0.7203	0.6903	0.6953	0.6947	0.6984
	AR(2)	0.7270	0.6887	0.6898	0.7117	0.6773	0.6780	0.6770	0.6759
	ARMA	0.6867	0.8019	0.7273	0.7228	0.7439	0.7516	0.7483	0.7478
				Panel B: Mean Bias					
ACC	MA(10)	0.0043	0.0320	-0.0042	-0.0634	-0.0269	0.0083	0.0127	0.0354
	EWMA	0.0442	0.0572	0.0188	-0.0323	0.0165	0.0420	0.0408	0.0471
	AR(2)	0.1093	0.1164	0.0705	0.0172	0.0396	0.0700	0.0717	0.1159
	ARMA	0.1364	0.3095	0.2188	0.1843	0.1533	0.1893	0.1922	0.2246
Infosys	MA(10)	0.0074	0.1691	0.0690	-0.0210	0.0660	0.1023	0.1059	0.1136
	EWMA	0.0105	0.1901	0.0823	0.0347	0.1014	0.1280	0.1329	0.1355
	AR(2)	0.0689	0.2461	0.1492	0.1008	0.1252	0.1559	0.1629	0.1707
	ARMA	0.1379	0.4617	0.3251	0.3556	0.2871	0.3269	0.3513	0.3629
Zeetele	MA(10)	0.0066	0.0960	0.0524	-0.0175	0.0047	0.0516	0.0483	0.0494
	EWMA	0.0136	0.1437	0.1234	0.1116	0.0596	0.0894	0.0823	0.0823
	AR(2)	0.0844	0.2519	0.2163	0.1790	0.1148	0.1522	0.1485	0.1544
	ARMA	0.2312	0.5476	0.4744	0.4835	0.2177	0.2739	0.2777	0.2945
Reliance	MA(10)	0.0004	0.0598	0.0209	-0.0298	0.0031	0.0358	0.0399	0.0440
	EWMA	0.0064	0.0580	0.0253	-0.0078	0.0160	0.0419	0.0430	0.0433
	AR(2)	0.0424	0.0896	0.0565	0.0300	0.0419	0.0698	0.0726	0.0722
	ARMA	0.1225	0.3570	0.2332	0.2004	0.1958	0.2147	0.2408	0.2327

Notes: Panels A and B provide the performance of different forecasting methods (given in the second column) with various volatility estimators. RMSE and Mean Bias are given in percentages. Daily volatility is estimated for the 'open-to-close' period. TSRV is the Two-scales Realized Volatility. PK, GK and RS are the volatilities based on the estimators of Parkinson, Garman and Klass, and Rogers and Satchell respectively. RRV(5), RRV(10), RRV(15), and RRV(30) are the Realized Range-based volatilities based on Parkinson's estimator for various time intervals (5, 10, 15, and 30 minutes). The estimate of volatility given by TSRV is used as the target forecast.

Table V: Forecasting Performance - Weekly

Panel A : RMSE

Stock	Method	TSRV	PK	GK	RS	RRV(5)	RRV(10)	RRV(15)	RRV(30)
ACC	MA(10)	3.2349	3.2221	3.1651	3.1507	3.1437	3.1482	3.1530	3.1919
	EWMA	1.8295	1.7886	1.6915	1.6463	1.6652	1.6824	1.6860	1.8260
	AR(2)	1.7246	1.7547	1.6743	1.6724	1.6196	1.6414	1.6460	1.7304
	ARMA	2.2030	2.1877	1.9111	1.8043	1.5862	1.6697	1.6447	1.9948
Infosys	MA(10)	2.1235	2.3480	2.1921	2.1538	2.1341	2.1654	2.1756	2.1888
	EWMA	1.8320	1.9513	1.8364	1.8695	1.9285	1.9685	1.9235	1.8824
	AR(2)	1.9022	2.0506	1.8892	1.8944	1.8996	1.9238	1.9193	1.9340
	ARMA	1.9268	2.3289	2.0726	2.2271	2.0792	2.0737	2.0686	2.1805
Zeetele	MA(10)	2.3865	2.4650	2.4328	2.4338	2.3134	2.3391	2.3435	2.3435
	EWMA	1.8199	2.0152	2.0237	2.0790	1.7440	1.8211	1.8292	1.8426
	AR(2)	1.7693	1.9889	1.9362	1.9756	1.7266	1.7843	1.7672	1.8057
	ARMA	1.9153	2.1481	2.2142	2.2597	1.9329	1.9746	1.8896	1.9085
Reliance	MA(10)	1.7519	1.8024	1.7683	1.7730	1.7426	1.7502	1.7510	1.7584
	EWMA	1.5448	1.6390	1.5973	1.6087	1.5763	1.5805	1.5814	1.5736
	AR(2)	1.5919	1.7310	1.6571	1.6490	1.6027	1.6308	1.6351	1.6728
	ARMA	1.6330	1.8314	1.6934	1.8220	1.7330	1.7176	1.6498	1.7650

Panel B: Mean Bias

Stock	Method	TSRV	PK	GK	RS	RRV(5)	RRV(10)	RRV(15)	RRV(30)
ACC	MA(10)	0.1128	0.3139	0.1411	0.0371	-0.0527	0.0550	0.0716	0.1895
	EWMA	0.3633	0.3013	-0.0756	-0.0012	0.0282	0.2141	0.1489	0.3677
	AR(2)	0.4188	0.5446	0.3639	0.2397	0.2060	0.2938	0.2984	0.4411
	ARMA	0.6230	0.8621	0.4815	0.2808	0.3153	0.3699	0.3874	0.6239
Infosys	MA(10)	0.1406	0.8179	0.4766	0.3376	0.3091	0.4169	0.4409	0.4826
	EWMA	0.0481	0.6952	0.4128	0.3508	0.3167	0.4030	0.4253	0.4686
	AR(2)	0.1440	0.8557	0.5174	0.4445	0.4095	0.5030	0.5337	0.5729
	ARMA	0.2517	0.9215	0.5076	0.4556	0.5308	0.5759	0.6327	0.6548
Zeetele	MA(10)	0.2409	0.7240	0.5869	0.5470	0.1852	0.3225	0.3288	0.3550
	EWMA	0.1690	0.7387	0.7072	0.7830	0.1196	0.2813	0.2638	0.3098
	AR(2)	0.2813	0.8600	0.7748	0.7772	0.2857	0.4111	0.4028	0.4395
	ARMA	0.3274	0.9480	0.8463	0.9527	0.3054	0.3869	0.4366	0.5168
Reliance	MA(10)	0.0502	0.3870	0.2249	0.1377	0.0700	0.1621	0.1802	0.2065
	EWMA	0.0413	0.3941	0.2811	0.2136	0.0919	0.1715	0.1884	0.1992
	AR(2)	0.1057	0.4300	0.2911	0.2100	0.1464	0.2252	0.2418	0.2722
	ARMA	0.2115	0.6042	0.4739	0.4598	0.2611	0.3594	0.3866	0.4284

Notes: Panels A and B provide the performance of different forecasting methods (given in the second column) with various volatility estimators. RMSE and Mean Bias are given in percentages. The weekly period represents five trading days. TSRV is the Two-scales Realized Volatility. PK, GK and RS are the volatilities based on the estimators of Parkinson, Garman and Klass, and Rogers and Satchell respectively. RRV(5), RRV(10), RRV(15), and RRV(30) are the Realized Range-based volatilities based on Parkinson's estimator for various time intervals (5, 10, 15, and 30 minutes). The estimate of volatility given by TSRV is used as the target forecast.

Table VI: Forecasting Performance - Monthly

Stock	Method	TSRV	PK	GK	RS	RRV(5)	RRV(10)	RRV(15)	RRV(30)
Panel A : RMSE									
ACC	MA(10)	4.0745	3.9792	3.6840	3.5659	3.3033	3.3971	3.4239	3.8427
	EWMA	3.2349	3.2224	3.1812	3.1777	3.2003	3.2415	3.2797	3.3129
	AR(2)	3.6242	3.4067	3.4448	3.4978	3.4472	3.4651	3.4646	3.4885
	ARMA	4.9674	4.2014	4.5251	4.4053	4.0751	4.3315	4.3496	4.7450
Infosys	MA(10)	3.4209	4.2358	3.7319	3.6559	3.5469	3.6616	3.7027	3.7800
	EWMA	3.5355	3.6971	3.5883	3.7820	3.6130	3.7062	3.7607	3.8106
	AR(2)	3.5256	4.0067	3.5730	3.5675	3.5125	3.6240	3.6776	3.7333
	ARMA	3.5578	4.4232	4.1355	4.2164	3.9719	4.0718	4.1353	3.9195
Zeetele	MA(10)	3.5861	4.1724	3.9594	3.9402	3.3757	3.5687	3.5756	3.6611
	EWMA	2.5748	3.1075	3.0649	3.1084	2.5301	2.5691	2.5591	2.5852
	AR(2)	2.6874	3.6389	3.4396	3.3988	2.7328	2.8433	2.8681	2.8996
	ARMA	3.0215	3.3879	3.5075	3.4566	3.0463	3.0487	3.0439	3.1491
Reliance	MA(10)	2.7026	3.0354	2.8652	2.8544	2.6726	2.7374	2.7515	2.8051
	EWMA	2.8522	3.1299	2.9634	2.9404	2.8922	2.9349	2.9340	3.4719
	AR(2)	3.1338	3.1929	3.0491	3.0289	3.0782	3.1365	3.1276	3.2283
	ARMA	3.3575	3.9447	3.7936	3.6060	3.8951	3.9108	3.8880	4.0926
Panel B: Mean Bias									
ACC	MA(10)	1.2863	1.7566	1.2991	1.0627	0.8072	1.0248	1.0633	1.4077
	EWMA	0.5156	0.5472	0.2548	0.2132	0.0681	0.2429	0.2978	0.3736
	AR(2)	0.4252	0.6795	0.4351	0.3124	0.1542	0.3701	0.3955	0.4489
	ARMA	-0.0348	0.4211	0.4283	0.3343	0.3177	0.3970	0.2964	0.1846
Infosys	MA(10)	1.0252	2.6254	1.8210	1.5813	1.4731	1.7014	1.7801	1.8963
	EWMA	0.5306	1.8907	1.2586	1.3248	1.0199	1.2103	1.3323	1.3652
	AR(2)	0.9899	2.4429	1.7544	1.6908	1.5402	1.7312	1.8427	1.8904
	ARMA	0.8403	2.7039	1.8690	2.2626	1.7069	1.8022	2.0622	1.9478
Zeetele	MA(10)	1.3526	2.6261	2.3165	2.2949	1.2852	1.5767	1.5909	1.6837
	EWMA	0.0777	1.5410	1.3761	1.4147	0.2014	0.4111	0.4078	0.4652
	AR(2)	0.8503	2.4223	2.2118	2.1931	0.9279	1.1659	1.1833	1.2468
	ARMA	-0.2104	1.3617	1.3336	1.6238	0.2680	0.3044	0.2466	0.3915
Reliance	MA(10)	0.5416	1.4204	0.9686	0.7803	0.6435	0.8450	0.8850	0.9521
	EWMA	0.2957	1.3066	0.7706	0.5413	0.6724	0.8375	0.8644	1.0784
	AR(2)	0.0946	1.0702	0.5610	0.2734	0.5023	0.6595	0.6567	0.7353
	ARMA	-0.1396	1.0049	0.3680	0.0140	0.8709	0.7334	0.7574	0.8863

Notes: Panels A and B provide the performance of different forecasting methods (given in the second column) with various volatility estimators. RMSE and Mean Bias are given in percentages. The monthly period represents 21 trading days. TSRV is the Two-scales Realized Volatility. PK, GK and RS are the volatilities based on the estimators of Parkinson, Garman and Klass, and Rogers and Satchell respectively. RRV(5), RRV(10), RRV(15), and RRV(30) are the Realized Range-based volatilities based on Parkinson's estimator for various time intervals (5, 10, 15, and 30 minutes). The estimate of volatility given by TSRV is used as the target forecast.