Deposit Insurance, Bank Competition and Risk Taking *

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December 2006

Abstract

We analyse risk-taking behaviour of banks in the context of a model based on spatial competition. Banks mobilise deposits by offering deposit rates. We show that when the market concentration is low, banks invest in the gambling asset. On the other hand, for sufficiently high levels of market concentration, all banks choose the prudent asset to invest in, and some depositors may even be left out of the market. Our results suggest a discontinuous relation between market concentration and social welfare. We also show that, in a regime of high deposit insurance, banks are more likely to gamble.

JEL Classification numbers: G21, L11, L13.
Keywords: Deposit insurance, Market concentration, Risk-taking.

*An earlier version of this paper has been circulated under the title: *Does Market Concentration Preclude Risk Taking in Banking?*. The authors owe thanks to Jordi Caballé, Gabriel González König, David Pérez-Castrillo, Inés Macho-Stadler, József Sákovics, and especially to Javier Suárez for numerous valuable suggestions. The current and earlier versions of the paper have benefited from helpful comments of the seminar participants at Universitat Autònoma de Barcelona (Barcelona, 2003), Edinburgh School of Economics (Edinburgh, 2003), Escuela de Economía (Guanajuato, 2004) and the MBAA Annual Meeting (Chicago, 2004). The authors gratefully acknowledge financial supports from the projects MEIF-CT-2004-501150 of the European Commission and POA 2003 of the Universidad de Guanajuato. The usual disclaimer applies, however.

1 Introduction

When banks are able to raise deposits to invest in assets with uncertain returns, excessive deposit might induce banks to take more risk. Involvement in high risk activities is viewed as one of the principal causes of several instances of banking crises that the world economy has witnessed during the last few decades.[1]

The main goal of this paper is to analyse the role of market concentration in determining the risk-taking behaviour of banks. The banking sector described here consists of a finite number of banks. Banks are identical with respect to their equity capital. They raise deposits by offering deposit rates. Banks choose between a *prudent asset* and a *gambling asset* to invest their total fund (equity plus deposit). The gambling asset on average yields lower return than the prudent asset, but if the gamble is successful it gives higher return. There is a continuum of depositors. The depositors have one unit of monetary fund apiece, which they may place in a bank to earn the deposit rate offered. They cannot observe banks' portfolio choice and are *not* insured in case the gamble fails.

We analyse a model of bank competition in the context of a circular city model *à la* Salop [8]. Both the banks and the depositors are located uniformly on a unit circle. The depositors incur a per unit transport cost to travel to a bank. In this model banks compete in deposit rates and each bank enjoys certain market power in the deposit market. Market power stems from transport cost. This should not literally be interpreted as the cost (or, time) a depositor spends traveling to a bank. Banks could be differentiated because of differences in ATM facilities, availability in various regions, internet banking services, etc.[2]

In general, our findings do not depend particularly on the source of market power. What is more important is that the amount of deposit in each bank is a continuous function of the deposit rates offered by it and its rivals. Our objective is to analyse the effects of market concentration, and not to model its sources. Ours is possibly one of the simplest formulations that can capture the essence of a monopolistically competitive banking sector.

We analyse two types of symmetric equilibria. A prudent equilibrium, where all banks invest in the prudent asset, and a gambling equilibrium, where all banks invest in the gambling asset. We use the unit transport cost relative to the number of banks as a measure of market concentration. We characterise the equilibrium of the banking sector. We show that when

[1] Recall the great S&L debacle in the United States. In 1985, S&L failures in Maryland caused loss to state deposit insurance fund and Maryland taxpayers of $185 million. The country-wide crisis had a cost of about $600 billion in bailouts.

[2] Sometimes banks gift frying pans and wine glasses to their clients.

concentration is low, banks compete aggressively in order to capture greater market share by offering higher deposit rates. In this case, all the depositors participate, and a *covered* market is said to arise. Here, for very low concentration, all banks invest in the Gambling asset (i.e., a Covered Gambling Equilibrium exists). As concentration increases, all banks investing in the prudent asset can *also* be supported in equilibrium (i.e., a Covered Prudent Equilibrium co-exists along with a Covered Gambling Equilibrium).

For high levels of market concentration, banks never invest in the gambling asset. This is because the banks are interested in preserving the higher rent, which stems from increased market power, by behaving prudently. Hence, one can observe only a Covered Prudent Equilibrium. Finally, when concentration is very high, a *uncovered* market emerges where deposit rate is so low that some depositors find it unprofitable to place their funds in banks and prefer to stay out of the deposit market.

Previous contributions to the literature on risk-taking in banking have mostly focused on the effectiveness of prudential regulations in inhibiting banks from investing in risky projects. Two popular instruments are minimum capital requirements and deposit rate ceilings which are often used to curb fierce competition among banks. Hellmann, Murdock and Stiglitz [3] show, using a dynamic model of prudential regulation, that freely determined deposit rates are inconsistent with Pareto efficiency, and that an optimal regulation policy combines minimum capital requirements and deposit rate ceiling.

Among others, there are some theoretical and empirical works which are particularly close to ours. Keeley [4] estimates a simple static model by using data from the 150 largest bank holding companies in the US, and finds that increased competition may lead to higher risk-taking in banking. Repullo [7] analyses a theoretical model of banking sector, and corroborates this finding. He uses a dynamic model of banking based on spatial competition with full deposit insurance to show that for very low level of market concentration, low intermediation margins reduce banks' franchise value and induce banks invest only in the gambling asset.

In contrast with the above mentioned works, our paper assumes away any sort of deposit insurance. As a result, the depositors in our model become *active* players, since the total volume of deposit with a bank determines whether the bank is going to invest in the prudent asset or in the gambling asset. In Repullo [7], the depositors are fully insured, and hence their decision to place their funds does not influence the portfolio choice of the banks. The depositors simply react to the deposit rates offered by the banks. On the contrary, in our case depositors care about bank's portfolio choice and know that banks who offer relatively high rates tend to gamble. Therefore, in our analysis we must impose simple *consistency* conditions on banks' behavior, which are embedded in the *No Gambling Condition*, which says that in a prudent equilibrium, *given the deposit rates offered and total volume of deposit raised*, banks' profits from investing in the prudent asset must exceed those from the gambling asset.

In this sense, the current setup retains similarity with the work by Matutes and Vives [5], who consider a model of bank competition with no deposit insurance and where depositors have beliefs about the probability of failure of the banks. In their model, banks can choose to invest in various risky assets where the riskiness depends on the market share enjoyed by each bank. It is the presence of depositors' beliefs what generates consistency requirements that should be fulfilled in any equilibrium. The authors show that when there are economies of scale, depositors' expectations provoke vertical differentiation and give rise to multiple equilibria.

Similar to theirs, our model also imposes consistency requirements on the equilibria. But since we avoid the complexity added by the existence of beliefs, these requirements boil down to the No Gambling Condition requiring that if a bank makes its clients believe that it is going to invest in the prudent asset, at equilibrium it indeed does so. In our model, banks' risk-taking behaviour is a fallout of the moral hazard generated through the provision of limited liability (i.e., in case gamble fails, the banks do not have to pay back their depositors which makes the gamble socially undesirable since in case of failure it imposes a loss on the depositors). An important difference between our model and that of Matutes and Vives [5] is that these authors analyse banks who are *not subject to limited liability.* In their model a banking crisis might occur because the depositors have different beliefs about failure (success) probabilities. In our model, under limited liability, increased market concentration ceases away the possibility that the banks invest in the gambling asset. When the banks are not subject to limited liability, as in Matutes and Vives [5], high concentration may always not guarantee prudent behaviour. These authors assert:

> *The multiplicity of equilibria is due to the coordination problem arising from the fact that alternative sets of expectations become self-fulfilling .... The problem is not competition; a monopoly bank could suffer from instability.* (Matutes and Vives [5], page 186).

These equilibrium outcomes have significant implications for social welfare. For a fixed level of equity capital and a given level of concentration, social welfare under a prudent equilibrium is higher than that under a gambling equilibrium (when both co-exist for these levels). In the characterisation of the equilibria we show that beyond a certain level of market concentration a gambling equilibrium ceases to exist. This generates a discontinuous leap in social welfare. Thus higher market concentration becomes efficiency enhancing as competition softens, at equilibrium, and banks no longer invest in the gambling asset. However, for the whole range of market concentration, welfare decreases as market concentration increases.

In the final Section of the paper we consider the possibility of (partial) deposit insurance. The effect of this regulatory measure on social welfare remains ambiguous. Diamond and Dybvig [2] find that deposit insurance system prevents sunspot runs and therefore financial collapse.[3] On the other hand, Demirgüç-Kunt and Detragiache [1] find empirical evidence that explicit deposit insurance has provoked bank failure. Deposit insurance induces fierce competition and thus reduces bank's incentives to behave prudently by increasing the moral hazard at the bank level when they are protected by a limited liability clause. We show that in our model, under high levels of deposit insurance, further increase in it makes a gambling equilibrium more likely to occur.

The reminder of the paper is organised as follows. In Section 2, we lay out the basic model. In the next section, we analyse prudent and gambling equilibria. In Section 4, we discuss the relation between market concentration and social welfare. In Section 5, we present the case when the depositors are (at least partially) insured. We conclude in Section 6. The proofs are relegated to the Appendix.

2 The Model

Consider a banking sector with n risk neutral banks distributed uniformly on a unit circle. A bank i has a fixed amount of equity capital k.[4] Banks compete in deposit rates in order to mobilise deposit. Let $\mathbf{r} = (r_1, ..., r_i, ..., r_n)$ be the deposit rates offered by the banks. Bank i's demand for deposit is given by $D(r_i, r_{-i})$, where r_{-i} is the vector of rates offered by the other banks.

There is a continuum of depositors, also uniformly distributed on the unit circle, with a unit of fund apiece. A depositor can deposit her fund in a bank which pays off a deposit rate in the next period. She incurs a per unit transport cost t in order to travel to a bank.

Banks can choose between a prudent asset and a gambling asset to invest their total fund. The prudent asset yields a constant return α, and the gambling asset gives a return γ with

[3] Capturing the phenomenon of bank run, which is associated with withdrawal of funds by the depositors, is beyond the scope of our model which is a static one.

[4] We do not explicitly model the sources of bank's capital. It might be considered as the total of a bank's issued shares. We assume this to be given to the bank before it enters the deposit market.

probability θ and zero with probability $1-\theta$. The prudent asset has higher expected return ($\alpha > \theta\gamma$), but if the gamble pays off it yields higher private return ($\gamma > \alpha$). The banks are protected by limited liability, i.e., in case the project fails their depositors are not paid back. We also assume that the depositors are not insured in the event of failure. The bank invests both the equity capital and the deposit raised in one of the assets.[5]

If bank i chooses to invest in the prudent asset and the gambling asset, its expected profits, respectively are:

$$\begin{aligned} \pi^P(r_i, r_{-i}, k) &= \alpha k + (\alpha - r_i)D(r_i, r_{-i}), \\ \pi^G(r_i, r_{-i}, k) &= \theta\gamma k + \theta(\gamma - r_i)D(r_i, r_{-i}). \end{aligned}$$

The timing of the game is as follows. Banks simultaneously offer deposit rates. Depositors then choose the bank in which to deposit their funds. The deposit mobilisation is followed by the portfolio choice by the banks. Finally, project outputs are realised and the depositors are paid off.

3 Equilibrium

In this section, we characterise the equilibrium of the economy where banks compete in the deposit market by offering deposit rates and choose a prudent asset or a gambling asset to invest in, and the depositors choose banks to place their funds. We will focus on two types of symmetric equilibria. A *prudent* equilibrium, where all banks choose to invest in the prudent asset, and a *gambling* equilibrium, where all banks invest in the gambling asset. The natural solution concept used here is *Subgame Perfect Equilibrium.*

We solve the stage game by backward induction. Recall the timing of events. In the third stage of the game, the banks decide on their portfolio choice. A bank will choose to invest in the prudent asset if the expected profits from doing so exceeds the expected profits from gambling asset ($\pi^P \geq \pi^G$), i.e., if the deposit of a bank satisfies the following *No Gambling Condition.*

$$D_i \leq \frac{(\alpha - \theta\gamma)k}{(1-\theta)r_i - (\alpha - \theta\gamma)}\ , \tag{NGC}$$

We assume that $(1-\theta) - m > 0$ with $m \equiv \alpha - \theta\gamma$, in order that the term in the *right hand side* of the above inequality is positive. Also whenever the above condition is satisfied

[5] A bank might invest a fraction of total capital in each asset. It is easy to show that optimality would imply that banks choose only one asset to invest in.

with equality, a bank invests in the prudent asset. If the above inequality is reversed, i.e., a *Gambling Condition* (call that (GC)) holds, then a bank will invest in the gambling asset.

Taking the above into account, in the previous stage, a depositor takes the decision whether to place her fund in a bank. Consider a bank i. A depositor at a distance x from this bank will deposit her unit fund if $r_i - 1 \geq tx$ in case this depositor anticipates that the bank will choose the prudent asset in the third stage. In case of the choice of a gambling asset, the above condition turns out to be $\theta r_i - 1 \geq tx$ (since, the expected gross return to the depositor is θr_i). These two conditions guarantee that this depositor participates in the deposit market. Call the above two conditions the *Participation Conditions* and denote them by (PC) and (PC$'$), respectively. If these conditions are reversed, refer to them as (NPC) and (NPC$'$), respectively. Now consider two consecutive banks i and j on the circle, and a depositor between them. Suppose that neither of the banks offers a deposit rate high enough so that, say the (PC) satisfied for this depositor.[6] Then this depositor does not deposit neither in bank i nor in bank j. In such case we say that an *Uncovered Market* arises at equilibrium. On the other hand, if there is no depositor that stays out of the deposit market (i.e., the participation conditions are satisfied for all depositors), a *Covered Market* is said to arise.

Since, the depositors have to incur a per unit transport cost t to travel to a bank, the transport cost relative to the number of banks in the economy (t/n) can be used as an appropriate measure of market concentration. This is because, if the transport cost increases relative to the number of banks, given the total number of depositors, each bank can lower the deposit rate due to sufficiently high market power it gains.

In the first stage of the game, each bank would then set the deposit rate in order to maximise its expected profits. In course of doing that, they must take into account the possible outcomes of the subgame that follows (stages 2 and 3). Hence, the aforesaid restrictions are imposed as constraints on bank's profit maximisation problem. For example, when the banks maximises subject to (NGC) and (PC), then a Covered Prudent Equilibrium (CPE) is said to arise. In fact, there might arise four possible equilibria, which we analyse in the following subsection.

It is worth noting that the above condition (NGC or GC) determines banks' portfolio choice which follows the decision taken by the depositors. If there is a small number of depositors who place their funds in a particular bank, then this bank is more likely to invest in the prudent asset (since, (NGC) is more likely to be satisfied). Hence, the conditions (NGC)

[6] There is one (PC) for each bank.

and (GC) are endogenous, rather than being exogenous constraints. In a regime where the depositors are not insured in case the project fails, this fact bears important consequences on the market equilibria. Had the depositors been insured, they would not rather care about the banks' investment decision.

On the other hand, a market structure (covered or uncovered) is determined by the participation or the non-participation conditions, hence endogenous too. When market concentration is relatively low, there is no depositor who finds it profitable not to place her fund in any of the banks. In this case, all the depositors are served and a *covered* market is said to arise. On the other hand, when the transport cost relative to the number of banks is very high, there might be some depositors who, in equilibrium, would not find it profitable to travel to a bank to place their fund (since, deposit rate would not be high enough in order to compensate for the increased transport cost). In that case, we say that an *uncovered* market emerges. Prior to characterising the equilibrium, we first analyse the necessary conditions for existence of prudent and gambling equilibria under both types of market structures.

3.1 A Covered Market

A covered market is a situation where all the depositors place their funds in any of the n banks rather than keeping their funds idle. We will assume that $\theta\gamma + 2 < 3\theta r_i$.[7]

First, we consider a Covered Prudent Equilibrium (CPE).[8] We compute the demand for deposit issued by bank i when it offers r_i and all the other banks offer r. If the depositors anticipate that all banks are going to choose the prudent asset, then the demand for deposit of bank i is given by:

$$D(r_i,\, r) = \frac{r_i - r}{t} + \frac{1}{n}\ .$$

As we have noted earlier that first, all the banks must comply with the *No Gambling Condition* in order that the market structure arises at equilibrium is indeed a prudent equilibrium. Second, there is no depositor who has an incentive to keep her fund idle, i.e., for any depositor and for any bank the *Participation Condition* (PC) must hold good. Thus bank i's

[7]This condition can be rewritten as $\frac{\theta r_i - 1}{\theta\gamma - 1} > \frac{1}{3}$. This implies that the proportion of net return to the depositor at a distance 0 from bank i to the net return from investing a unit fund must be high enough in order to attract this depositor.

[8]The detailed calculations and proofs of the results in Section 3 are in the appendix.

shareholders will solve the following problem:

$$\max_{r_i} \left\{ \alpha k + (\alpha - r_i) \left(\frac{r_i - r}{t} + \frac{1}{n} \right) \right\}$$
$$\text{subject to} \quad (NGC) \quad \text{and} \quad (PC).$$

Let $r_i = r = r^{CP}$ be the candidate optima for the above maximisation problem, which are summarised below.

$$r^{CP} = \begin{cases} \bar{r} & \text{if } \frac{t}{n} \leq \alpha - \bar{r} \\ \alpha - \frac{t}{n} & \text{if } \alpha - \bar{r} \leq \frac{t}{n} \leq \frac{2(\alpha-1)}{3} \\ 1 + \frac{t}{2n} & \text{if } \frac{2(\alpha-1)}{3} \leq \frac{t}{n} \leq 2(\bar{r} - 1), \end{cases}$$

where $\bar{r} \equiv \frac{m(1+nk)}{1-\theta}$ is the deposit rate which makes the (NGC) bind with equal deposit for all banks. Notice that $\bar{r}$ is an increasing function of a bank's equity capital. If all banks have higher amount of k, (NGC) is more likely to be satisfied for each bank, and hence they are more likely to behave prudent. Also, it is clear from the (NGC) that for very low levels of k, this condition is less likely to be satisfied. So, k can be interpreted as a minimum capital standard imposed by the central bank. And a suitable combination of r_i and k can guarantee that the banks invest in the prudent asset.[9]

In order to interprete the above, first consider the corner solution $\bar{r}$. This occurs when the (NGC) binds. This deposit rate must satisfy (PC), which implies $\frac{t}{n} \leq 2(\bar{r} - 1)$. Also at $\bar{r}$, the profit function must have a non-negative slope which implies $\frac{t}{n} \leq \alpha - \bar{r}$. These two together imply that $\bar{r}$ is a candidate optimum only if $\frac{t}{n} \leq min\{\alpha - \bar{r}, 2(\bar{r} - 1)\}$. The fact that the profit function has non-negative slope at $\bar{r}$ implies that $\bar{r} \geq \alpha - \frac{t}{n}$. Notice that the banks, while offering this deposit rate, are indifferent between choosing the prudent asset and the gambling asset, and hence this rate is the highest one among the rates offered in a CPE.

Next consider the interior solution $\alpha - \frac{t}{n}$. This must satisfy both (NGC) and (PC), which implies $\alpha - \bar{r} \leq \frac{t}{n} \leq \frac{2(\alpha-1)}{3}$.

Finally, consider the other corner solution $\left(1 + \frac{t}{2n}\right)$ which must satisfy (NGC) and at this point the profit function must have a negative slope. These two together implies that $\frac{2(\alpha-1)}{3} \leq \frac{t}{n} \leq 2(\bar{r} - 1)$. Notice that this rate is the one that makes the (PC) binds (the threshold rate between a CPE and a UPE), and hence, the lowest deposit rate offered in a covered market.

Next, we analyse a Covered Gambling Equilibrium (CGE). We compute the demand for

[9] See also Proposition 2 in Hellmann, Murdock and Stiglitz [3] for a detailed discussion.

deposit issued by bank i when it offers a deposit rate r_i and all other banks offer r. Note that if a bank i promises a deposit rate r_i, a depositor in this bank gets (in expected terms) θr_i. If the depositors anticipate that all banks are going to choose the gambling asset (i.e., for all banks the (GC) holds good), the deposit of bank i is given by:

$$D(r_i,\, r) = \frac{\theta(r_i - r)}{t} + \frac{1}{n}\;.$$

Here, one should take two restrictions into account. First, all the banks must comply with the *Gambling Condition* in order that the market structure arises at equilibrium is indeed a gambling equilibrium (stage 3 of the game). Second, there is no depositor who has incentive to keep her fund idle, i.e., the *Participation Condition*, (PC′), must hold good. Hence, bank i's shareholders will solve the following problem:

$$\max_{r_i} \left\{ \theta\gamma k + \theta(\gamma - r_i)\left(\frac{\theta(r_i - r)}{t} + \frac{1}{n}\right)\right\}$$
$$\text{subject to} \quad (GC) \quad \text{and} \quad (PC').$$

Let $r_i = r = r^{CG}$ be the candidate optima for the above maximisation problem. These are summarised below.

$$r^{CG} = \begin{cases} \gamma - \frac{t}{\theta n} & \text{if} \;\; \frac{t}{n} \leq \theta(\gamma - \bar{r}) \\ \bar{r} & \text{if} \;\; \theta(\gamma - \bar{r}) \leq \frac{t}{n} \leq 2(\theta\bar{r} - 1) \\ \frac{1}{\theta}\left(1 + \frac{t}{2n}\right) & \text{if} \;\; \frac{t}{n} \geq 2(\theta\bar{r} - 1). \end{cases}$$

First consider the interior solution $\gamma - \frac{t}{\theta n}$. This must satisfy both (GC) and (PC′), which together imply the upper bound to the level of concentration for which this deposit rate is a candidate solution. Next, consider the corner solution $\bar{r}$. In this case the bounds to the level of concentration emerge from the fact that this deposit rate must satisfy (PC′), and at this deposit rate the profit function must have a negative slope. Finally, consider the other corner solution $\frac{1}{\theta}\left(1 + \frac{t}{2n}\right)$ which must satisfy (GC) and at this point the profit function must have a negative slope, the above two give rise to the lower bound to t/n. It is easy to see that, this is the lowest possible rate offered by a gambling bank in a covered market.

3.2 An Uncovered Market

In an uncovered market between any two consecutive banks on the circle there is a non-empty subset of depositors who do not place their funds in either of the banks. In this section, we focus on two possible equilibria: an Uncovered Prudent Equilibrium (UPE) and an Uncovered Gambling Equilibrium (UGE). First important thing to note is that, a market being covered or uncovered solely depends on the Participation Conditions described in the earlier sections. If the deposit rate posted by each of all the n banks satisfies this condition, then the market is said to be covered. Since this restriction is taken into account in Stage 2 of the game, a covered or an uncovered market structure emerges endogenously at equilibrium.

In an uncovered market, there exist at least two neighbouring banks such that there is a depositor between them who does not deposit in either of the banks. In this section, an individual bank will solve profit maximisation problems similar to those in the previous section. Here, one important restriction is that the deposit rate of a bank does not satisfy the Participation Condition (i.e., (PC) and (PC′) are reversed). Whenever these restrictions are satisfied with equality, a bank chooses a corner solution to the maximisation problem same as what we have obtained in the covered market. Hence, in this section we will ignore this type of solution, and we will refer to this situation as a covered market.

First consider an Uncovered Prudent Equilibrium. If a bank i offers deposit rate r_i, a depositor at distance x from i will prefer to stay home if $r_i - 1 < tx$. Hence, bank i will get a maximum deposit of $x \leq \frac{r_i - 1}{t}$ from either side and it will have the following amount of deposit:

$$D(r_i) = \frac{2(r_i - 1)}{t} \ .$$

In such equilibrium, banks maximise profits subject to the *No Gambling Condition*, and the *No Participation Condition.* Notice that, in an uncovered market, it is sufficient to check that the depositor at $x = \frac{1}{2n}$ does not deposit in either of the banks. Hence, the *No Participation Constraint* boils down to

$$r_i \leq 1 + \frac{t}{2n} \ . \tag{NPC}$$

Therefore, bank i's shareholders will solve the following problem:

$$\max_{r_i} \left\{ \alpha k + (\alpha - r_i) \left(\frac{2(r_i - 1)}{t} \right) \right\}$$
$$\text{subject to} \quad (NGC) \quad \text{and} \quad (NPC).$$

Let $r_i = r = r^{UP}$ be the candidate optima for the above programme, which are summarised below.

$$r^{UP} = \begin{cases} \tilde{r} & \text{if} \quad 2(\bar{r} - 1) \leq \frac{t}{n} \leq \phi^{UP}, \\ \frac{\alpha+1}{2} & \text{if} \quad \frac{t}{n} \geq \phi^{UP}, \end{cases}$$

where $\phi^{UP} \equiv \frac{(\alpha-1)(2\theta\gamma-\alpha(1+\theta)+(1-\theta))}{2((1-\theta)\bar{r}-m)}$, and $\tilde{r}$ is defined by:[10]

$$\frac{2(\tilde{r}-1)}{t} = \frac{mk}{(1-\theta)\tilde{r}-m}.$$

In the rest of this section, we analyse the Uncovered Gambling Equilibrium (UGE). In this case, bank i operates on the part of the demand curve $\frac{2(\theta r_i - 1)}{t}$ above $\frac{mk}{(1-\theta)r_i - m}$ (i.e., it complies with the *Gambling Condition*). Also, in this case, the *No Participation Condition* is slightly different from the case of a prudent bank.

$$r_i \leq \frac{1}{\theta}\left(1 + \frac{t}{2n}\right). \qquad \text{(NPC}'\text{)}$$

Hence, bank i's shareholders will solve the following problem:

$$\max_{r_i} \left\{ \theta\gamma k + \theta(\gamma - r_i)\left(\frac{2(\theta r_i - 1)}{t}\right) \right\}$$
$$\text{subject to} \quad (GC) \quad \text{and} \quad (NPC').$$

Solving the necessary conditions for the above maximisation problem, we get several candidate deposit rates. In the following Proposition we show that none of these candidates constitutes an equilibrium strategy for the banks.

PROPOSITION **1** *An Uncovered Gambling Equilibrium never exists.*

PROOF See Appendix A. □

The above proposition says that when market concentration is too high, for the banks it is not necessary to compete fiercely, and hence they do not offer very high deposit rates which would induce them to choose the gambling asset.

Till now we have provided the necessary conditions under which different deposit rates are candidates for a Nash Equilibrium, and showed that an UGE does not exist. But the above candidates are not necessarily immune to possible credible deviations by a bank. In the following section, we also provide the conditions under which some of the candidates indeed constitute part of a Subgame Perfect Nash Equilibrium strategy profiles.

[10]While interpreting the above necessary conditions, notice that $\tilde{r}$ is a function of t/n. After several steps of tedious algebra one can show that $t/n \geq 2(\tilde{r}-1)$ if and only if $t/n \geq 2(\bar{r}-1)$. Notice that $2(\tilde{r}-1)$ as a function of t/n is increasing and concave, and there exists a fixed point of this function. At this fixed point, $\tilde{r} = \bar{r}$.

3.3 Characterisation of Equilibrium

In the following proposition, we characterise the equilibrium. Recall that the term $\frac{t}{n}$ is used as a measure of market concentration.

Proposition **2** *For a given level equity capital of each bank, k,*

(a) there exists a threshold $\widetilde{\phi}$ such that if $\frac{t}{n} \leq \tilde{\phi}$ (low market concentration), a Covered Gambling Equilibrium exists, with the banks offering deposit rate $\gamma - \frac{t}{\theta n}$;
(b) if $\frac{t}{n} \in \left[\widetilde{\phi}, \phi^G\right]$ (intermediate levels of market concentration), both a Covered Gambling Equilibrium and a Covered Prudent Equilibrium exist, with banks offering $\gamma - \frac{t}{\theta n}$, and $\bar{r}$ or $\alpha - \frac{t}{n}$, respectively ;
(c) if $\frac{t}{n} \in \left[\phi^G, \phi^P\right]$ (high levels of concentration), only a Covered Prudent Equilibrium exists, with banks offering $\alpha - \frac{t}{n}$ or $1 + \frac{t}{2n}$;
(d) if $\frac{t}{n} \geq \phi^P$ (very high concentration), only an Uncovered Prudent Equilibrium exists, with banks offering $\frac{\alpha+1}{2}$ or $\tilde{r}$.

Proof See Appendix B. □

The intuition behind the above proposition is not difficult to understand. When the market concentration is very low, competition erodes banks' profit, thus leaving little incentive for them to invest in prudent asset.

On the other hand, for very high degree of concentration, banks gain monopoly rent, and hence they have incentive to choose prudent asset in order to preserve that. For, even a higher values of $\frac{t}{n}$, the market becomes uncovered, i.e., banks offer even lower deposit rate which is not conducive to attract the depositors located at a longer distance. The above proposition is summarised in the following figure.

[Insert Figure 1 about here]

Also, for intermediate levels of concentration, banks might invest in the prudent asset by offering a lower deposit rate or in the gambling asset offering a higher rate which compensates for the expected loss for the depositors due to a possible failure in gambling.

4 Social Welfare

In this section, we discuss the connection between market concentration and welfare. In the current set up, social welfare is simply the total consumer's surplus, since the deposit rate is a transfer from the banks to the depositors.

For covered markets welfare is independent of deposit rates

$$\begin{aligned} W^{CPE}(t) &= \alpha kn + \alpha D^T - 2nt \int_0^{\frac{1}{2n}} x dx; \\ &= \alpha(nk+1) - \frac{t}{4n}. \\ W^{CGE}(t) &= \theta\gamma(nk+1) - \frac{t}{4n}. \end{aligned}$$

Hence, in a covered market social welfare is always higher under the prudent equilibrium since

$$W^{CPE} - W^{CGE} = m(nk+1) > 0.$$

For a UPE, take the case where the equilibrium deposit rate is $r^{UP} = \widetilde{r}$. Let x^I be the last depositor who goes to the bank. Hence, x^I must satisfy the following:

$$\begin{aligned} \widetilde{r} - tx^I &= 1 \\ x^I &= \frac{\widetilde{r}-1}{t}. \end{aligned}$$

And the total deposit is given by $D^T = \frac{2n(\widetilde{r}-1)}{t}$, and the social welfare is given by

$$\begin{aligned} W^{UP}(t) &= \alpha kn + \alpha D^T - 2nt \int_0^{x^I} x dx \\ &= \alpha kn + 2\alpha n \frac{\widetilde{r}-1}{t} - 2nt \int_0^{\frac{\widetilde{r}-1}{t}} x dx \\ &= n[\alpha k + \frac{\widetilde{r}-1}{t}(2\alpha - \widetilde{r} + 1)]. \end{aligned}$$

Given that the lower bound on concentration beyond we have a UPE is $\frac{t}{n} = 2(\overline{r}-1)$, one can show that

$$\begin{aligned} W^{UP}(n\phi^P) &= \alpha nk + \frac{\widetilde{r}-1}{2(\overline{r}-1)}(2\alpha - \widetilde{r} + 1), \\ W^{CPE}(n\phi^P) &= \alpha nk + \frac{2\alpha - \overline{r} + 1}{2}, \end{aligned}$$

implying that $W^{UP}(n\phi^P) = W^{CPE}(n\phi^P)$ because at this level of market concentration it turns out that $\widetilde{r} = \overline{r}$ (recall that $\widetilde{r}$ is a function of $\frac{t}{n}$).

The above is summarised in Figure 2. The line labeled W^G is the social welfare as a function of market concentration under a gambling equilibrium, and that labeled W^P is the welfare under a prudent equilibrium. Social welfare decreases with the level of market concentration under both types of equilibria. Second, for a given level of market concentration where both the CPE and CGE exist (for $\frac{t}{n} \in [\widetilde{\phi}, \phi^G]$), social welfare is higher in case all banks behaving prudently. Next, at the level of concentration beyond which no bank invests in the gambling asset (at $\frac{t}{n} = \phi^G$), welfare takes a discontinuous leap.

[Insert Figure 2 about here]

The above findings are summarised in the following proposition. There we also show that social welfare is maximised for the level of market concentration $\frac{t}{n} = \widetilde{\phi}$ if all the banks invest in the prudent asset. This is done by showing that social welfare is higher at $\frac{t}{n} = \phi^G$ than at $\frac{t}{n} = 0$ (point A is higher than point A' in Figure 2). Note that for $\frac{t}{n} \in [\widetilde{\phi}, \phi^G]$, social welfare is higher under prudent equilibrium, since for these levels both gambling and prudent equilibria co-exist, and beyond $\frac{t}{n} = \phi^G$ the gambling equilibrium ceases to exist.

PROPOSITION **3** *Social welfare decreases with market concentration both under gambling and prudent equilibrium. The levels of market concentration for which both equilibria co-exist, social welfare is always higher under prudent equilibrium. Moreover, at the level of concentration beyond which a Covered Gambling Equilibrium ceases to exist, social welfare is higher than at zero market concentration.*

PROOF See Appendix C. □

The above proposition should be interpreted carefully. Social welfare is higher at $\frac{t}{n} = \phi^G$ than at $\frac{t}{n} = 0$, and hence higher at $\frac{t}{n} = \widetilde{\phi}$ than at $\frac{t}{n} = 0$ since welfare decreases with market concentration. But between $\widetilde{\phi}$ and ϕ^G, both equilibria co-exist. This implies that, at $\frac{t}{n} = \widetilde{\phi}$ social welfare is maximised only if we are in a CPE. One can no way guarantee that this should always be the case. If, for these levels of market concentration, all banks decide to invest in the gambling asset, we will end up in the welfare curve labeled W^G. Hence, a coordination among the banks is necessary in order to achieve the maximum social welfare. In any case, it is obvious that a CPE Pareto dominates a CGE.

5 Deposit Insurance

In the previous sections we developed a model that assumes away any sort of deposit insurance. As a result, the depositors in our model become *active* players, since the total volume of deposit with a bank determines whether the bank is going to invest in a prudent asset or in a gambling asset. When the depositors are fully insured, their decision to place their funds does not influence the portfolio choice of the banks. Matutes and Vives [5] consider a model of bank competition with no deposit insurance and where depositors have beliefs about the probability of failure of the banks. Repullo citerepullo02 also considers a model with full deposit insurance to show the effect of market concentration on the risk-taking behaviour of banks.

In this section we consider the possibility of (partial) deposit insurance. The effect of this regulatory measure remains ambiguous for low deposit insurance. Deposit insurance may prevent systemic confidence crises, [11], and increase deposit by compensating for transport cost. On the other hand, deposit insurance induces banks to compete and thus reduces bank's incentives to behave prudently by increasing the moral hazard at the bank level since they are protected by a limited liability clause.

We consider a scenario when the depositors are insured by the central banking authority. In a system with deposit insurance (denoted by $\delta \in (\theta,\, 1]$), even if a bank i fails, its depositors are paid back δ fraction of the promised deposit rate r_i. A full insurance scheme corresponds to $\delta = 1$. Whenever $\delta < 1$, the depositors are insured partially. The limiting case, where $\delta = \theta$, corresponds to no insurance. When there is deposit insurance, depositors are paid back (at least partially) regardless of the success of the investment in the risky asset. Hence, we relax the consistency conditions we require on depositors' and banks' behaviour and the demand for deposit under CGE is

$$D(r_i,\, r) = \delta\left(\frac{r_i - r}{t}\right) + \frac{1}{n}.$$

Then, under CGE, the shareholders of bank i will solve the following problem:

$$\max_{r_i} \left\{\theta\gamma k + \theta(\gamma - r_i)\left(\delta\left(\frac{r_i - r}{t}\right) + \frac{1}{n}\right)\right\}$$
$$\text{subject to } \; (GC) \; \text{ and } \; (PC).$$

It is easy to show (similar to the proof of Proposition 2) that only the interior solution $\gamma - \frac{t}{\delta n}$

[11] See Diamond and Dybvig [2], and Matutes and Vives [5].

survives as an equilibrium deposit rate. And this exists only if

$$\frac{t}{n} \leq \delta(\gamma - \bar{r}).$$

In the following proposition we show that when the deposit insurance is sufficiently high, then a CGE exists over a higher range of the levels of market concentration compared to the case of no insurance. In other words, under a regime of (partial, but high) deposit insurance, banks are more likely to gamble.

PROPOSITION **4** *There exists a threshold level of deposit insurance $\bar{\delta} < 1$ such that whenever $\delta \geq \bar{\delta}$, the upper-bound to a Covered Gambling Equilibrium is increasing in δ.*

PROOF See Appendix D. □

Although the effect of deposit insurance on risk taking is not totally unambiguous (for $\delta \leq \bar{\delta}$), the fact that a high deposit insurance might have adverse effects with respect to risk taking is fairly intuitive. In general, since the banks are protected by limited liability in case the gamble fails, a high insurance induces them to gamble. In this case, as the banks do not have to pay back their depositors the underlying moral hazard has more bite on the risk-taking behaviour of the banks. Notice that, under a deposit insurance scheme δ, a bank's objective function under gambling changes (since it shifts out the demand for deposits); whereas that under prudent behaviour remains unchanged. This makes the gambling asset more attractive for the banks. Consequently, deposit insurance induces fiercer competition, and leads to a situation where a gambling equilibrium is more likely to occur.

6 Conclusions

In this paper, we use a model of baking sector based on spatial competition, and analyse the role of market concentration in influencing the risk-taking behaviour of banks. Using a static model we show that, for a very low level of market concentration, banks invest in the gambling asset, since high competition erodes banks' profits, and thus leaves little incentives for them to behave prudently. On the other hand, when the market concentration increases, banks invest only in the prudent asset. For even higher concentration the deposit market becomes uncovered. We assert that, more market concentration works as a device to refrain banks from being involved in high risk activities. The two types of market structure (covered and uncovered) arise endogenously at equilibrium, and these should no way be interpreted as exogenous constraints imposed on bank's optimisation problems. Thus our results have similar essence to Matutes and Vives [5], and Repullo [7].

Furthermore, we present a discontinuous relationship between market concentration and social welfare. This is because of the fact that for a non-empty range of parameter values there might exist two types of equilibria (CGE and CPE).

The current model is distinct from most of the works mentioned in Section 1 in a very important way. In our basic formulation of the model we assume away any sort of deposit insurance. This does not render the depositors indifferent between their bank investing in the prudent asset and the gambling asset, and influences in an important way the so called *No Gambling Condition*. We also show that high deposit insurance makes gambling more likely for a bank. The fact that increasing deposit insurance having an unambiguous effect only for high levels would explain why the empirical evidence has so far not been conclusive.

The main body of the paper shows that increased market concentration ensures that all banks invest in the prudent asset. Much of the debates on bank mergers pose the view that mergers are able to enhance efficiency in case of speculative lending by providing the banks with more market power. One important thing to note is that our results would differ significantly if we have allowed the number of banks to vary. One appropriate way to do this is accommodating entry of new banks in a dynamic context. But, in our understanding, market concentration is essentially a short run concept since, with free entry, the market power each bank enjoys is merely temporary. Perotti and Suárez [6] show that, in the presence of last bank standing effects, appropriate mergers and regulatory policies can enhance efficiency by giving banks incentives to behave prudently.

One important feature of the current paper is that the model presented is essentially a static one. In our opinion, this static nature is more relevant when one analyses the role of market concentration. With similar specifications but in a dynamic setup (without outside intervention), market power of a bank does not prevail permanently due to free entry. Hence, several relevant questions in this context may not be answered with the tools used in the current paper. Zendejas-Castillo [9] analyses a model with entry under a regime of full deposit insurance, and shows that the government can resort to specific entry policy by asking the entrants to pay an entry fee. This makes the market concentration high enough so that all banks invest in the prudent asset (a Covered Prudent Equilibrium) in the long run equilibrium (which is characterised by a zero-profit condition). It is worth noting that, in Zendejas-Castillo [9], the (minimum) level of market concentration that ensures a prudent equilibrium is, in fact, the level for which the social welfare is maximum in our model when the depositors are fully insured.

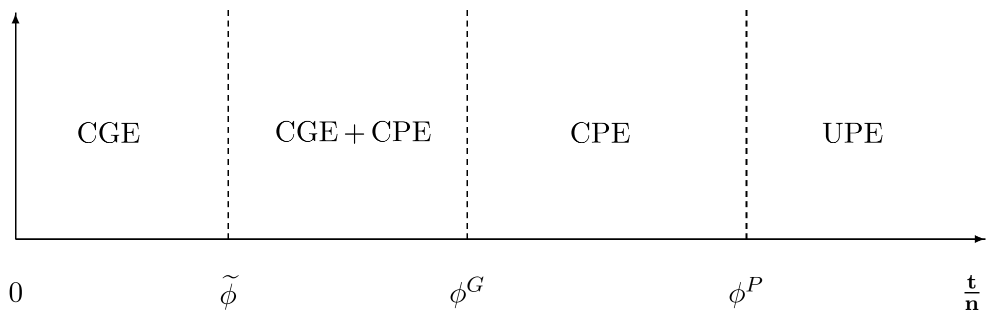

Figure 1: Characterisation of Equilibrium

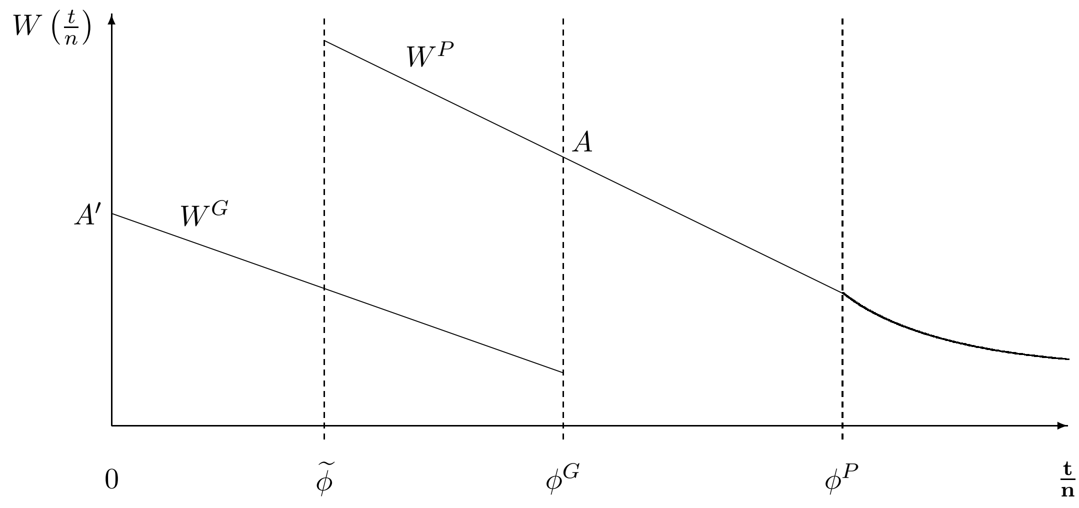

Figure 2: Market Concentration and Social Welfare

Appendix

A Proof of Proposition 1

Let r^{UG} be the candidate maxima of banks' profit maximisation problem in case of a UGE. These candidates (ignoring the corner solution $r^{UG} = \frac{1}{\theta}\left(1 + \frac{t}{2n}\right)$) are below:

$$r^{UG} = max\left\{\frac{\theta\gamma + 1}{2\theta}, \hat{r}\right\}, \quad \text{when} \quad \frac{t}{n} \geq max\{\theta\gamma - 1, \, 2(\theta\hat{r} - 1)\},$$

where $\hat{r}$ is defined by:

$$\frac{2(\theta\hat{r} - 1)}{t} = \frac{mk}{(1-\theta)\hat{r} - m} \, .$$

The restrictions on t/n come from the Kuhn-Tucker (necessary) conditions for the profit maximisation problem.

First, consider the corner solution $\hat{r}$. At this deposit rate, the (NGC) is satisfied with equality. Hence, a bank invests in the prudent asset.

Next, consider the interior solution $\frac{\theta\gamma+1}{2\theta}$. This occurs as an optimum only when $\theta\gamma - 1 \geq 2(\theta\hat{r} - 1)$. In this case, this candidate generates profits equal to

$$\pi^{UG}\left(\frac{\theta\gamma + 1}{2\theta}\right) = \theta\gamma k + \frac{(\theta\gamma - 1)^2}{2t},$$

and that they are lower than the profits under the rival candidate $1 + \frac{t}{2n}$ for UPE since

$$\pi^{UP}\left(1 + \frac{t}{2n}\right) = \alpha k + \left(\alpha - 1 - \frac{t}{2n}\right)\frac{1}{n} \, .$$

The above holds whenever $\frac{t}{n} \geq \theta\gamma - 1$. Hence, $\frac{\theta\gamma+1}{2\theta}$ can be ruled out as part of an equilibrium.

B Proof of Proposition 2

Take a symmetric CGE with deposit rate r and suppose that a bank deviates to a deposit rate that will induce it to behave prudently. The profit function after deviation is:

$$\pi^{G \to P} = \alpha k + (\alpha - r^*)\left(\frac{r^* - \theta r}{t} + \frac{1}{n}\right) \, .$$

This deviation r^* must be credible. So we may have to compute also the deposit rate, r', that will leave it indifferent between investing in the prudent asset and the gambling asset. That is,

$$\frac{r'-\theta r}{t}+\frac{1}{n} = \frac{mk}{(1-\theta)r'-m} \tag{1}$$

$$r'-\theta r = \left(\frac{\bar{r}-r'}{r'-\frac{m}{1-\theta}}\right)\frac{t}{n}\ . \tag{2}$$

Note that the LHS is increasing in r' and the RHS is decreasing. Now take the three candidates for CGE. We will see that deviations arise easily. Nevertheless, for sufficiently low levels of market concentration a CGE exists.

First, consider $r^{CG}=\gamma-\frac{t}{\theta n}$. Suppose first that the bank deviates to a rate r^* that generates a deposit greater than $\frac{1}{n}$. This occurs when

$$\frac{r^*-\theta\gamma}{t}+\frac{2}{n}>\frac{1}{n}\Leftrightarrow r^*>\theta\gamma-\frac{t}{n}\,.$$

In this case, it cannot be that $r^*\geq\overline{r}$ because then the (NGC) is not satisfied and the deviation is not credible. This imposes the restriction that $\theta\gamma-\frac{t}{n}<r^*<\overline{r}$. Hence, there can be no such deviation whenever $\frac{t}{n}<\theta\gamma-\overline{r}$. It is easy to see that if $\frac{t}{n}\geq\theta\gamma-\overline{r}$, then a bank can deviate by choosing $\theta\gamma-\frac{t}{n}$ which is a credible deviation since it generates the same deposit as before. Hence, this candidate for CGE can be ruled out for the interval $[\theta\gamma-\overline{r},\theta(\gamma-\overline{r})]$.

Now, suppose that $\theta\gamma-\overline{r}>\frac{t}{n}$. Notice that

$$\left.\frac{\partial\pi^{G\to P}}{\partial r^*}\right|_{r^*=\theta\gamma-\frac{t}{n}}=\frac{m}{t}>0,$$

so the deviator's profit is increasing in r^* for deviations $r^*\leq\theta\gamma-\frac{t}{n}$. Since this deviation must be credible, the best the deviating bank can do is to set the maximum deposit rate consistent with prudent behavior. After rewriting condition (1), this rate is defined by the expression

$$r'-\theta\gamma+\frac{t}{n} = \left(\frac{\bar{r}-r'}{r'-\frac{m}{1-\theta}}\right)\frac{t}{n}$$

$$r'-\theta\gamma = \left(\frac{\bar{r}-2r'+\frac{m}{1-\theta}}{r'-\frac{m}{1-\theta}}\right)\frac{t}{n}\,.$$

Notice that this condition is not the same as the (NGC) of the maximisation problem while finding a CPE.

Now we want to check when profits after deviation are still below those under the CPE. Profits after and before deviation, respectively, are:

$$\begin{aligned}\pi' &= \alpha k + (\alpha - r')\left(\frac{r' - \theta\gamma}{t} + \frac{2}{n}\right) = \theta\gamma k + \theta(\gamma - r')\left(\frac{r' - \theta\gamma}{t} + \frac{2}{n}\right); \\ \pi^G &= \theta\gamma k + \frac{t}{n^2}.\end{aligned}$$

Tedious algebra shows that $\pi' \leq \pi^G$ if and only if:[12]

$$\frac{t}{n} \leq \theta(\gamma - r') - \sqrt{\theta(\gamma - r')}\sqrt{r'(1-\theta))} < \theta\gamma - \overline{r}, \tag{3}$$

where the last inequality holds good when $\theta\gamma - \overline{r} > \frac{t}{n}$.[13]

Thus we have found an upper bound for this CGE candidate. Note that this bound might be negative. This is the case whenever $\theta\gamma < r'$. However, one can show that if $\frac{t}{n} \leq \theta\gamma - \bar{r}$ then $r' < \theta\gamma - \frac{t}{n}$ so the upper bound is positive. Let us write

$$\varphi\left(\frac{t}{n}\right) = \theta(\gamma - r') - \sqrt{\theta(\gamma - r')}\sqrt{r'(1-\theta))}.$$

The fact that φ is a function of $\frac{t}{n}$ makes it impossible to know *a priori* if condition (3) holds in the region $[0, \theta\gamma - \overline{r}]$. We need then to find a fixed point of $\varphi\left(\frac{t}{n}\right)$ in order to ensure the existence of an interval where this candidate cannot be beaten.

First, it is easy to see that φ is decreasing in r', and by the Implicit Function Theorem, that $\frac{\partial r'}{\partial(\frac{t}{n})} < 0$. Hence, φ is increasing in $\frac{t}{n}$. Moreover, one can show that $r'(0) = \theta\gamma$ and that $r'(\theta\gamma - \bar{r}) = \bar{r}$. Hence, $\varphi(0) = 0$, and

$$\varphi(\theta\gamma - \bar{r}) = \theta(\gamma - \bar{r}) - \sqrt{\theta(\gamma - \bar{r})}\sqrt{\bar{r}(1-\theta))} < \theta\gamma - \bar{r}.$$

Also $\varphi\left(\frac{t}{n}\right)$ is concave. The above ensure the existence of a fixed point, denoted by $\overline{\varphi}$.

Next, consider the corner solution $\bar{r}$. This generates profits equal to $\pi^{CG}(\bar{r}) = \theta\gamma k + \theta(\gamma - \bar{r})\frac{1}{n} = \alpha k + (\alpha - \bar{r})\frac{1}{n}$. If a bank deviates by choosing a deposit rate $r' = \theta\bar{r}$ and the

[12]There is another condition: $\frac{t}{n} \geq \theta(\gamma - r') + \sqrt{\theta(\gamma - r')}\sqrt{r'}(1-\theta)$. But if $\theta\gamma - \overline{r} > \frac{t}{n}$ is assumed then it turns out that $r' < \overline{r}$, and hence this condition never holds in the relevant region.

[13]Note that $\theta(\gamma - r') - \sqrt{\theta(\gamma - r')}\sqrt{r'(1-\theta))} = \theta\gamma - r' + \sqrt{r'(1-\theta)}(\sqrt{r'(1-\theta)} - \sqrt{\theta(\gamma - r')})$. When $\theta\gamma - \overline{r} > \frac{t}{n}$ both $\theta\gamma > r' > \overline{r}$ hold good.

prudent asset, then it gets $\pi' = \alpha k + (\alpha - \theta\bar{r})\frac{1}{n}$, which is higher than that before deviation. Also $r' < \bar{r}$ and the deviation is credible (i.e., the bank indeed wants to be prudent).

Finally, consider the other corner solution $\frac{1}{\theta}\left(1 + \frac{t}{2n}\right)$. It is easy to see that a bank can profitably deviate by posting a deposit rate $1 + \frac{t}{2n}$ and choosing the prudent asset to invest in.

Hence a CGE exists if and only if

$$\frac{t}{n} \leq min\left\{\overline{\varphi}, \frac{2(\theta\gamma - 1)}{3}\right\} \equiv \phi^G.$$

Now consider a candidate for symmetric CPE with deposit rate r and suppose that a bank deviates to a deposit rate that will make it gamble. Following is the profits from deviation.

$$\pi^{P\to G} = \theta\gamma k + \theta(\gamma - r^*)\left(\frac{\theta r^* - r}{t} + \frac{1}{n}\right).$$

Again one should consider as well the limit deposit rate ($\underline{r}$) for the bank to credibly gamble after the deviation. This rate is now defined by the following equality

$$\frac{\theta\underline{r} - r}{t} + \frac{1}{n} = \frac{mk}{(1-\theta)\underline{r} - m}.$$

First, consider the corner solution $r^{CP} = \bar{r}$. The deviation deposit rate is

$$r^* = \frac{\bar{r} + \theta\gamma}{2\theta} - \frac{t}{2\theta n},$$

Notice that for the profits after deviation to be greater than before, r^* must satisfy the following inequality:

$$(\gamma - r^*)(\theta r^* - \overline{r}) > \frac{t}{n}(r^* - \overline{r}).$$

If with the deviation the bank gets a smaller deposit. i.e., if $r^* < \frac{\overline{r}}{\theta}$, it cannot be the case that $r^* < \overline{r}$, because otherwise the banks would want to gamble. And if $\overline{r} \leq r^* \leq \frac{\overline{r}}{\theta}$, it is easy to see that there is no r^* satisfying the condition above. Hence any deviation must be such that $r^* > \frac{\overline{r}}{\theta}$.

Now, let use check what the deviation will be. The following derivative

$$\left.\frac{\partial \pi^{P\to G}}{\partial r^*}\right|_{r^*=\frac{\overline{r}}{\theta}} = \frac{\theta}{t}(-\frac{t}{n} + \theta\gamma - \overline{r})$$

implies that if $\frac{t}{n} \geq \theta\gamma - \overline{r}$, the bank maximises profits by deviating with the minimal r^* possible, that is $\frac{\overline{r}}{\theta}$; but we know that in that case the bank is not better off by deviating. Therefore, $\frac{t}{n}$ must be greater than $\theta\gamma - \overline{r}$. The indirect profit function with this deviation is

$$\pi^* = \theta\gamma k + \frac{1}{t}\left(\frac{\theta\gamma - \overline{r}}{2} + \frac{t}{2n}\right)^2.$$

Recall that the profits previous to the deviation were

$$\pi^P = \alpha k + (\alpha - \overline{r})\frac{1}{n} = \theta\gamma k + \theta(\gamma - \overline{r})\frac{1}{n}.$$

Hence, the deviation is profitable if and only if

$$\frac{(\theta\gamma - \overline{r})^2}{4} - \left(\frac{\theta\gamma + (1-2\theta)\overline{r}}{2}\right)\frac{t}{n} + \left(\frac{t}{2n}\right)^2 > 0.$$

This above condition boils down to:[14]

$$\frac{t}{n} < \theta\gamma + \overline{r}(1-2\theta) - 2\sqrt{(1-\theta)\overline{r}\theta(\gamma - \overline{r})}.$$

We also need to show that this deviation is credible. In fact one can show that $r^* > \overline{r}$ since this holds true whenever $\frac{t}{n} < \theta\gamma + \overline{r}(1-2\theta)$. This together with the fact that by assumption this deviation generates a deposit $D^* > \frac{1}{n}$ implies the consistency of this interior deviation (recall that $\frac{t}{n} < \theta\gamma - \overline{r}$). Hence for this range, this candidate for the CPE can be ruled out; it can only survive in the range

$$\theta\gamma + \overline{r}(1-2\theta) + 2\sqrt{(1-\theta)\overline{r}\theta(\gamma - \overline{r})} > \alpha - \overline{r} = \widetilde{\phi} \leq \frac{t}{n} \leq \alpha - \overline{r}.$$

Now consider the interior solution $r^{CP} = \alpha - \frac{t}{n}$. There are two candidates for the best reply. First one is the interior best response deviation $r^* = \frac{\alpha+\theta\gamma}{2\theta} - \frac{t}{\theta n}$, and the other is the limit deposit rate that is consistent with gambling, denoted by $\underline{r}$.

The limit deposit rate $\underline{r}$ is implicitly defined by:

$$\begin{aligned} \frac{\theta\underline{r} - \alpha}{t} + \frac{2}{n} &= \frac{mk}{(1-\theta)\underline{r} - m}; \\ \theta\underline{r} - \alpha &= \left(\frac{\overline{r} - 2\underline{r} + \frac{m}{1-\theta}}{\underline{r} - \frac{m}{1-\theta}}\right)\frac{t}{n}. \end{aligned} \tag{4}$$

[14]The other one is $\frac{t}{n} \geq \theta\gamma + \overline{r}(1-2\theta) + 2\sqrt{(1-\theta)\overline{r}\theta(\gamma - \overline{r})} > \alpha - \overline{r}$ so it has no bite in this region.

In that case

$$\begin{aligned}
\underline{\pi} &= \theta\gamma k + \theta(\gamma - \underline{r})\left(\frac{\theta\underline{r} - \alpha}{t} + \frac{2}{n}\right) = \alpha k + (\alpha - \underline{r})\left(\frac{\theta\underline{r} - \alpha}{t} + \frac{2}{n}\right) \\
\pi^P &= \alpha k + \frac{t}{n^2}; \\
\pi^P &\geq \pi^* \Leftrightarrow \frac{t}{n^2} \geq (\alpha - \underline{r})\left(\frac{\theta\underline{r} - \alpha}{t} + \frac{2}{n}\right).
\end{aligned}$$

Algebra shows that the last inequality has no solution and that $\pi^P \geq \underline{\pi}$ always holds. Hence, we must focus on the case where the bank deviates with $r^* = \frac{\alpha+\theta\gamma}{2\theta} - \frac{t}{\theta n}$. One can show that this cannot be the case. The deposit generated with this deviation is $\frac{2}{n} - \frac{m}{t}$ which is positive if and only if $\frac{t}{n} > \frac{m}{2}$. We also have

$$\begin{aligned}
\pi^* &= \theta\gamma k + \frac{1}{t}\left(\frac{t}{n} - \frac{m}{2}\right)^2 \\
\pi^P &= \alpha k + \frac{t}{n^2}; \\
\pi^P &< \pi^* \Leftrightarrow \frac{t}{n} < \frac{m^2}{4(1-\theta)\bar{r}}.
\end{aligned}$$

It is clear that $\frac{m^2}{4(1-\theta)\bar{r}} < \frac{m}{2}$. So if under the deviation deposit is positive, the profits it generates are smaller than under our candidate and therefore it survives as a CPE.

Finally, consider the other corner solution $r = 1 + \frac{t}{2n}$. It is clear that the bank will not deviate to an uncovered gambling deposit rate. It will not get a deposit greater than $\frac{1}{n}$ and it will have to pay higher deposit rates. Then, the only alternative is to deviate to a covered gambling deposit rate. The best response deposit rate is

$$r^* = \frac{\theta\gamma + 1}{2\theta} - \frac{t}{4\theta n}.$$

But it is easy to check that with this deposit rate the market is still uncovered. For the consumer at a distance $\frac{1}{2n}$ it holds good that

$$\theta r^* - \frac{t}{2n} = \frac{\theta\gamma + 1}{2} - \frac{3t}{4n} < 1,$$

where the last inequality holds because in this case, $\frac{t}{n} \geq \frac{2(\alpha-1)}{3}$.

Summarising the above, one can say a symmetric CPE exists if and only if

$$\widetilde{\phi} \leq \frac{t}{n} \leq \phi^P, \text{ where } \phi^P \equiv 2(\bar{r} - 1).$$

Notice that the deposit rate set by a bank in case of a UPE does not depend on the rates posted by the other banks. Also, following Proposition 1, it is easy to show that from the interior and the corner solution $\tilde{r}$ there is no profitable deviation.

C Proof of Proposition 3

The facts that social welfare decreases with market concentration, and that it is always higher under prudent equilibrium are obvious from the discussion in the text. We only prove the last part. We want to show that $W^{CP}(n\phi^G) \geq W^{CG}(0)$. First, we have

$$\begin{aligned} W^{CP}(n\phi^G) &= \alpha(nk+1) - \frac{\phi^G}{4}, \\ W^{CG}(0) &= \theta\gamma(nk+1). \end{aligned}$$

Notice that

$$\phi^G \equiv \min\{\overline{\varphi}, \frac{2(\theta\gamma-1)}{3}\} \leq \frac{2(\theta\gamma-1)}{3}.$$

Therefore,

$$W^{CP}(n\phi^G) \geq \alpha(nk+1) - \frac{(\theta\gamma-1)}{6}.$$

On the other hand,

$$\begin{aligned} &\alpha(nk+1) - \frac{(\theta\gamma-1)}{6} \geq \theta\gamma(nk+1), \\ &\Leftrightarrow m(nk+1) = (1-\theta)\bar{r} \geq \frac{(\theta\gamma-1)}{6}, \\ &\Leftrightarrow \bar{r} \geq \frac{(\theta\gamma-1)}{6(1-\theta)}. \end{aligned}$$

We know that $\bar{r} \geq 1$. Now we show that $\frac{(\theta\gamma-1)}{6(1-\theta)} \leq 1$. For this to happen, note that we need

$$\gamma \leq \frac{6(1-\theta)+1}{\theta}.$$

Consider the assumption $\theta\gamma + 2 < 3\theta r_i$. This to be meaningful, we need $r_i \geq 1$. Therefore, the above assumption is equivalent to $\gamma < \frac{3\theta-2}{\theta}$. Hence, it only remains to check that

$$\begin{aligned} &\frac{3\theta-2}{\theta} \leq \frac{6(1-\theta)+1}{\theta}, \\ &\Leftrightarrow \theta \leq 1. \end{aligned}$$

D Proof of Proposition 4

Consider the candidate deposit rate $\gamma - \frac{t}{\delta n}$. It is easy to check that for $\frac{t}{n} \geq \delta\gamma - \bar{r}$, a bank can profitably deviate by choosing the prudent asset and a deposit rate $\delta\gamma - \frac{t}{n}$. So we will focus on the complementary region. A bank can deviate to the prudent asset by choosing a deposit rate given below:

$$r^* = \frac{\delta\gamma + \alpha}{2} - \frac{t}{n},$$

and the deposit is given by

$$D^* = \frac{\alpha - \delta\gamma}{2t} + \frac{1}{n}.$$

This deposit is too high so that if a bank posts r^*, it still wants to gamble. Note that we should require $\frac{t}{n} > \frac{\delta\gamma - \alpha}{2}$ in order to ensure non-negative profits. Let us look at the deposit generated with this deviation. Consistency requires

$$\frac{\alpha - \delta\gamma}{2t} + \frac{1}{n} \leq \frac{mk}{(1-\theta)(\frac{\delta\gamma+\alpha}{2} - \frac{t}{n}) - m}.$$

The above implies that this is the case if and only if

$$\begin{aligned}(\delta - \frac{\alpha}{\gamma})\left(\delta + \frac{1}{\gamma}\left(\alpha - \frac{2m}{1-\theta}\right)\right) &\geq 0,\\ (\delta - \bar{\delta})(\delta - \underline{\delta}) &\geq 0,\end{aligned}$$

where $\bar{\delta} = \alpha/\gamma > \underline{\delta}$. Consider the case when $\delta \geq \bar{\delta}$.

Here the deviation is credible and we must check when profits after deviating to r^* are not higher, that is when

$$\pi^* \leq \pi^G \Leftrightarrow \alpha k + \left(\frac{\alpha - \delta\gamma}{2} + \frac{t}{n}\right)^2 \frac{1}{t} \leq \theta\gamma k + \frac{\theta t}{\delta n^2}.$$

The above requires

$$\delta(\bar{r} - \gamma q)\frac{t}{n} + \delta\frac{(\alpha - \delta\gamma)^2}{4(1-\theta)} + q\left(\frac{t}{n}\right)^2 < 0,$$

where $q = \frac{\delta - \theta}{1-\theta}$. The above expression yields two following roots of t/n.

$$\begin{aligned} z^+ &= \frac{\delta(\gamma q - \bar{r})}{2q} + \frac{1}{2q}\sqrt{\delta^2(\gamma q - \bar{r})^2 - \delta q\frac{(\alpha - \delta\gamma)^2}{1-\theta}},\\ \text{and } z^- &= \frac{\delta(\gamma q - \bar{r})}{2q} - \frac{1}{2q}\sqrt{\delta^2(\gamma q - \bar{r})^2 - \delta q\frac{(\alpha - \delta\gamma)^2}{1-\theta}}.\end{aligned}$$

Straightforward calculus shows that $z^- < \frac{\delta\gamma-\alpha}{2}$. Therefore, we only need to focus on z^+ (recall that, by assumption, $\frac{t}{n} > \frac{\delta\gamma-\alpha}{2}$). Hence, a deviation is not profitable as long as $\frac{t}{n} \leq z^+$. Hence, we need $\frac{t}{n} \leq \min\left\{\frac{2(\delta\gamma-1)}{3}, \delta\gamma - \bar{r}, z^+\right\}$ in order to support a CGE with deposit insurance δ. Long and tedious computations yield that

$$\frac{\partial z^+}{\partial \delta} > 0.$$

Hence, this threshold is increasing in δ.

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