Multiscale hedge ratio between the stock and futures markets: A new approach using wavelet analysis and high frequency data

Francis In[a *] and Sangbae Kim[b]

[a] *Department of Accounting and Finance, Monash University, Clayton Campus, Victoria, 3168, Australia*

[b] *School of Business Administration, Kyungpook National University, Sankyuk-dong, Puk-ku, Daegu, 702-701, Republic of Korea*

Abstract

In this paper, we investigate the multiscale relationship between the stock and futures markets over various time horizons. We propose a new approach – the wavelet multiscaling method – to undertake this investigation. This method enables us to make the *first* analysis of the multiscale hedge ratio using high frequency data (5 min). Wavelets are treated as a "lens" that enables the researcher to explore relationships that previously were unobservable. The approach focuses on the relationship in three ways: (1) the lead-lag causal relationship, (2) covariance/correlations, and (3) the hedge ratio and hedging effectiveness. Our empirical results show that the future market Granger causes the stock market. We find that the magnitude of the wavelet correlation between the two markets increases as the time scale increases, indicating that the stock and futures markets are not fundamentally different. We also find that the hedge ratio at the second scale has the lowest value and increases monotonically at a decreasing rate, converging toward the long horizon hedge ratio of one, which suggests that the shared permanent component ties the stock and futures series together, and the effect of the transitory components becomes negligible.

JEL classification: G1; G13; G15

Key words: Hedge ratio; Hedging effectiveness; Granger causality; Wavelet analysis; High frequency data.

---

* Corresponding Author, Francis In. Tel.: + 61 – 3 – 9905 – 1561; fax: +61 3 9905 5475

E-mail address: Francis.In@BusEco.monash.edu.au (Francis In)

1. Introduction

Many of the participants in futures markets are hedgers. Hedging is one of the important reasons for using derivative securities such as futures contracts. The literature shows that the study of the hedge ratio using futures contracts has been of interest to both academicians and practitioners and widely discussed. In addition to this aspect, understanding the long-run and short-run relationships between the stock and futures markets is important for portfolio management. For example, in financial risk management, risk is assessed at different time scales, which vary from intervals as small as a few minutes to longer time scales such as days or even months. A common practice in the risk management industry is the conversion of short-scale risk measures into longer horizons by taking the corresponding scaling quantity into account. Based on this practice, we would question whether the long-run covariance/correlation between the stock and futures markets is similar to that of the short-run. We can then question whether the long-run hedge ratio is similar to the short-run hedge ratio. As in the study of Lee (1999), given the time-varying nature of the covariance in many financial markets, the classical assumption of a time-invariant optimal hedge ratio[1] appears inappropriate.

To determine the optimal hedge ratio, some early studies assume that the hedge ratio is constant over time and estimate it using simple ordinary least square estimation (see Ederington, 1979; Hill and Schneeweis, 1982). However, given the time-varying nature of the covariance in many financial markets, the classical assumption of the time-invariant optimal

[1] Since risk in this context is usually measured as the volatility of portfolio returns, an intuitively plausible strategy might be to choose the hedge ratio that minimizes the variance of the returns of a portfolio containing the stock and futures position. This is known as the optimal hedge ratio (Brooks et al., 2002).

hedge ratio appears inappropriate. An improvement has been made by adopting a bivariate GARCH framework (see Kroner and Sultan, 1993; Choudhry, 2003; Wang and Low, 2003). Traders and investors working in the stock and futures markets have a different hedging horizon. Therefore, examining and applying the one-period hedge ratio could lead investors to invalid decision making. The previous literature has shown little attention to the multiscale hedge ratio, except for Howard and D'Antonio (1991), Lien and Luo (1993, 1994), Geppert (1995), and Lien and Wilson (2001). However, the models presented by these authors have at least three problems in estimating the multiscale hedge ratio. First, the ratio is an unreliable estimator due to a handful of independent observations generated from long-horizon return series (see Geppert, 1995). Second, computation is burdensome and difficult to calculate over longer investment horizons. Finally, it requires an assumption for the error term for GARCH/SV model estimation (see Lien and Wilson, 2001), which can cause inaccurate results.

To overcome these problems, In and Kim (2006) recently proposed a new approach for investigating the relationship between the stock and futures markets using wavelet analysis. The main advantage of using wavelet analysis is the ability to decompose the financial data into several time scales (investment horizons). All market participants such as regulators and speculative investors who trade in the stock and futures markets make decisions over different time scales. The time frame over which they operate differs immensely, ranging from seconds to months and beyond. In fact, due to the different decision-making time scales among traders, the true dynamic structure of the relationship between the stock and futures markets itself will vary over the different time scales associated with those different horizons. Although it is logical to assume that there are several time periods in decision making, economic and financial analyses have been restricted to at most two time scales (the short-run and the long-run), due to the lack of analytical tools to decompose data into more than two time scales. However, unlike previous studies, this paper uses wavelets to produce an orthogonal decomposition of correlation and the hedge ratio between the stock and futures indices over several different time scales (In and Kim, 2006).

Recently, several applications of wavelet analysis to economics and finance have been documented in the literature. To the best of our knowledge, applications in these fields include examination of foreign exchange data using waveform dictionaries (Ramsey and Zhang, 1997), decomposition of economic relationships of expenditure and income (Ramsey and Lampart, 1998a and 1998b), the multihorizon Sharpe ratio (Kim and In, 2005a), systematic risk in a capital asset pricing model (Gençay et al., 2003), and examination of the multiscale relationship between stock returns and inflation (Kim and In, 2005b).

This paper is different from the previous studies. We utilize intra-day data. Adopting intra-day data and wavelet analysis is very useful for examining the multiscale hedge ratio in that it allows us to investigate how the hedge ratio can be affected by hedge horizon from intra-day to longer horizons. More specifically, we can observe the difference between the 5-minute hedge ratio and the much longer horizon hedge ratio.

Our empirical results indicate that first, at 5-minute (original data) and approximately one-day dynamics (seventh scale), the futures market Granger causes the stock market. From this result, it is concluded that the futures market is more efficient. Examining the wavelet variance reveals an approximate decreasing linear relationship between the wavelet variance and the wavelet scale. This implies that an investor with a short investment horizon has to respond to every fluctuation in realized returns, while the long-run risk for an investor with a longer horizon is significantly less. We find that the magnitude of the wavelet correlation between the two markets increases as the time scale increases, indicating that the stock and futures markets are not fundamentally different. Finally, we also find that the hedge ratio at the second scale has the lowest value and increases monotonically at a decreasing rate, converging toward the long horizon hedge ratio of one, which suggests that the shared permanent component ties the stock and futures series together and the effect of the transitory components becomes negligible.

This paper is organized as follows. Section 2 derives the minimum variance hedge ratio. Section 3 describes the fundamental methods of wavelet analysis that we apply. Section 4 discusses the data and the empirical results. In section 5, concluding remarks are presented.

2. Minimum variance hedge

The most widely used static hedge ratio[2] is the Minimum Variance (MV) hedge ratio, which is derived Johnson (1960) by minimizing the portfolio risk. In this framework, the risk is given by the variance of changes in the value of the hedged portfolio. Assume that an individual has taken a fixed position in some asset and that this person is long one unit of the asset without loss of generality. Let h_t represent the short position taken in the futures market at time t under the adopted hedging strategy. Ignoring daily resettlement, the hedger's objective within this framework is to minimize the variance of the change in the value of the hedged portfolio:

$$\text{Min } Var(\Delta HP_t) = Var(\Delta S_t + h_t \Delta F_t)$$

$$= Var(\Delta S_t) + h_t^2 Var(\Delta F_t) + 2h_t Cov(\Delta S_t, \Delta F_t) \quad (1)$$

where ΔHP_t is the change in the value of the hedged portfolio during time *t*; ΔS_t and ΔF_t are the changes in the log of the stock and the futures prices at time *t*, respectively; and h_t is the optimal hedge ratio. Duffie (1989) shows that the optimal hedge ratio for a person with mean-variance utility can be decomposed into two portions: one reflecting speculative demand (which varies across individuals according to their risk aversion) and another reflecting a pure hedge (which is the same for all mean-variance utility hedgers). Because the pure hedge term is common to all hedgers, and the speculative demand term is both difficult to estimate and often close to zero, it is reasonable to focus attention on the pure hedge.

Suppose the hedger decides to pursue a hedging strategy. The optimal hedge is determined by solving equation (1).

$$\frac{\partial Var(\Delta HP_t)}{\partial h_t} = 2h_t Var(\Delta F_t) + 2Cov(\Delta S_t, \Delta F_t)$$

$$h_t^* = -\frac{Cov(\Delta S_t, \Delta F_t)}{Var(\Delta F_t)} = \rho_{sf} \frac{\sigma_s}{\sigma_f} \quad (2)$$

Where ρ_{sf} is the correlation coefficient between ΔS_t and ΔF_t, and σ_s and σ_f are standard deviations of ΔS_t and ΔF_t. This corresponds to the conventional hedge ratio, when changes

[2] We use the static MV hedge ratio, which changes with the holding period, not with time. Our purpose is not to examine the time-varying hedge ratio, but to investigate the horizon-varying hedge ratio.

in both stock and futures prices are homoskedastic. In the absence of conditional heteroskedasticity, both $Cov(\Delta S_t, \Delta F_t)$ and $Var(\Delta F_t)$ are independent of the information set. As a result, h_t^* is a constant term regardless of whatever information is available. Commonly, the value of the hedge ratio is less than unity, so that the hedge ratio that minimizes risk in the absence of basis risk turns out to be dominated by h_t^* when basis risk is taken into consideration (Choudhry, 2003). The attractive feature of the MV hedge ratio is that it is easy to understand and simple to compute. In general, it is considered that the MV hedge ratio is not consistent with the mean-variance framework, since it ignores the expected return on the hedged portfolio. However, for the MV hedge ratio to be consistent with the mean-variance framework, either the investors need to have an infinite risk aversion coefficient or the expected return on the futures contracts is zero, in other words, the futures price follows a pure martingale process.[3]

The degree of hedging effectiveness, proposed by Ederington (1979), is measured by the percentage reduction in the variance of the stock prices changes. Therefore, the degree of hedging effectiveness, denoted as *EH*, can be expressed as follows:

$$EH = \frac{Var(\Delta S_t) - Var(\Delta HP_t)}{Var(\Delta S_t)} = 1 - \frac{Var(\Delta HP_t)}{Var(\Delta S_t)} = \rho_{sf}^2 \tag{3}$$

where $\rho_{sf,t}^2$ is the square of the correlation between the change in the stock and futures prices.

[3] Other strategies that incorporate both the expected return and risk (variance) of the hedged portfolio have been recently proposed: Optimum mean-variance hedge ratio by Hsin et al. (1994), Sharpe hedge ratio by Howard and D'Antonio (1984), and minimum generalized semivariance (GSV) hedge ratio suggested by Chen et al. (2001). However, it is shown that if the futures price follows a pure martingale process or if the futures and spot returns are jointly normally distributed, the optimal mean-variance hedge ratio will be the same as the MV hedge ratio.

3. Wavelet analysis

The stock and futures markets are complex systems resulting from the action and reaction of many traders. Each trader has different motivations for buying and selling financial assets, and possibly different contractual or legal constraints, resulting in different trading time frames. The key fact is the existence of many traders working with very different time horizons when it comes to making an investment. Therefore, the relationship between the two markets should be examined at the different time scales. Wavelet analysis is a tool with a distinct advantage for investigating the relationship between the stock and futures markets in terms of correlations and the hedge ratio, as it enables us to decompose the data on a scale-by-scale basis. We first introduce general wavelet analysis and then present the method for calculating the wavelet variance and covariance from the data decomposed by wavelet analysis.

Throughout, we use multiresolution analysis (MRA), introduced in Mallat (1998). MRA is a mathematical method to decompose a square-integrable function or signal $f(\cdot)$ at different scales. The key of MRA is a mother wavelet function $\psi(\cdot)$. The mother wavelet function has three properties: sum to zero, unit energy and orthogonality. Orthogonality means that the shifted functions in the same scale are orthogonal and also that the functions at different scales are orthogonal. The orthonormality of $\psi(\cdot)$ implies that the doubly infinite sequence $\{\psi_{j,k}(\cdot)\}$ constitutes an orthonormal basis, where

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}}\psi(2^{-j}t - k) = 2^{-j/2}\psi\left(\frac{t - 2^j k}{2^j}\right) \tag{4}$$

This sequence is obtained from a single mother wavelet $\psi(\cdot)$ by dilations and translations. The integers j and k are called the dilation and translation parameters. In other words, to capture the high and low frequencies of the signal, the wavelet transformation utilizes a mother wavelet that is scaled and shifted.

Another basis function of wavelet analysis is the father wavelet as follows:

$$\phi_{j,k}(t) = 2^{-\frac{j}{2}}\phi(2^{-j}t - k) = 2^{-j/2}\phi\left(\frac{t - 2^{j}k}{2^{j}}\right) \tag{5}$$

where 2^{j} is a sequence of scales. The term $2^{-j/2}$ maintains the norm of the basis functions $\phi(t)$ at 1. Using these two wavelets, any time series can be expressed as follows:

$$x_t \approx \sum_{k} s_{J,k}\phi_{J,k}(t) + \sum_{k} d_{J,k}\psi_{J,k}(t) + \sum_{k} d_{J-1,k}\psi_{J-1,k}(t) + \Lambda + \sum_{k} d_{1,k}\psi_{1,k}(t) \tag{6}$$

where J is the number of scales, and k ranges from 1 to the number of coefficients in the specified component. The coefficients $s_{J,k}$, $d_{J,k}$, …, $d_{1,k}$ are the wavelet transform coefficients. J is the maximum integer such that 2^{j} is less than the number of data points. In this expression, the first expansion gives a function that is a low resolution or a coarse approximation of x_t. For each increasing index j in the second summation, a higher or finer resolution function is added. The coefficients in this wavelet expansion are called the discrete wavelet transform (DWT) of the signal x_t. More specifically, $s_{J,k}$ represents the smooth coefficients that capture the trend, while the detail coefficients $d_{j,k}$, …, $d_{1,k}$, which can capture the higher frequency oscillations, represent increasing finer scale deviations from the smooth trend.

In this study, we use the Maximal Overlap Discrete Wavelet Transform (MODWT). It is natural to ask why the MODWT is needed instead of the DWT. The motivation for formulating the MODWT is essentially to define a transform that acts as much as possible like the DWT, but does not suffer from the DWT's sensitivity[4] to the choice of a starting point for a time series (Percival and Walden, 2000). Importantly, the zero-phasing property of the MODWT permits meaningful interpretation of "timing" regarding the wavelet details. With this property, we can align perfectly the details from decomposition with the original time series. In comparison with the DWT, no phase shift will result in the MODWT.

Given the wavelet coefficients obtained from MODWT, the wavelet variance is estimated using the coefficients for scale $\lambda_j \equiv 2^{j\text{-}1}$ under the assumption that the dependence structure of our returns is independent of time through:

$$\sigma_l^2(\lambda_j) \equiv \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} [d_{j,t}^l]^2 \, , \; l = X, Y \tag{7}$$

where $d_{j,t}^l$ is the wavelet coefficient of variables l at scale λ_j.

[4] This sensitivity results from down-sampling the outputs from the wavelet and scaling filters at each stage of the pyramid algorithm (Percival and Walden, 2000).

The wavelet covariance can decompose the sample covariance into different time scales. The wavelet covariance at scale λ_j can be expressed as follows:

$$Cov_{XY}(\lambda_j) \equiv \frac{1}{\widetilde{N}_j} \sum_{t=L_j-1}^{N-1} d_{j,t}^{X} d_{j,t}^{Y} \tag{8}$$

Although the wavelet covariance decomposes the covariance between two stochastic processes on a scale-by-scale basis, in some situations it may be beneficial to normalize the wavelet covariance by the variability inherent in the observed wavelet coefficients. This leads us to calculate the wavelet correlation. The estimator of the wavelet correlation can be expressed as follows using equations (7) and (8):

$$\widetilde{\rho}_{XY}(\lambda_j) \equiv \frac{Cov_{XY}(\lambda_j)}{\widetilde{v}_X(\lambda_j)\widetilde{v}_Y(\lambda_j)} \tag{9}$$

As with the usual correlation coefficient between two random variables, $|\widetilde{\rho}_{XY}(\lambda_j)|<1$. Given the wavelet variance and covariance between two series, the hedge ratio at scale λ_j can be calculated using equations (7) and (8).

$$h_j^w = \frac{Cov_{sf}(\lambda_j)}{\widetilde{v}_f^2(\lambda_j)} \tag{10}$$

In this specification, h_j^w indicates the wavelet multiscale hedge ratio, which can vary depending on the wavelet scales (i.e., investment horizons). To understand how the wavelet scale represents the investment horizon, it is more informative to consider the spectral representation theorem. By this theorem, the spectrum of original 5-minute return series, R_t, contains all frequencies between zero and 1/2 cycles, equivalent to a 0 – 10 minute period in our data frequency. The wavelet coefficients at the first scale, d_1, are associated with the frequencies in the interval [1/4, 1/2], equivalent to a 10 – 20 minute period, while the wavelet coefficients at the second and third scales, d_2 and d_3, respectively, contain frequencies [1/8, 1/4] and [1/16, 1/8], respectively. Thus, the hedge ratios calculated at different scales represent the short position taken in the futures market at various frequencies (in other words, at various time scales).

4. Empirical results

In this paper, we use the S&P 500 Index and the S&P 500 Futures Index maturing in June 2004. Our data is composed of 5-minute prices for each index obtained from Tickdata (www.tickdata.com) for the period 22 March 2004 to 9 June 2004. The final three-months' trading data has been chosen for our study because most trading occurs in and close to the expiry month. The trading hours of the Chicago Mercantile Exchange (CME) are 8:30 to 15:15 local time, while the New York Stock Exchange (NYSE) opens from 9:30 to 16:00 local time. However, the local time of New York is one hour ahead of Chicago. Therefore, the opening hours of both markets are the same, while the CME closes 45 minutes earlier than the NYSE. Therefore, in our study we use the time period from 8:30 to 15:00, which generates 78 observations per day. In our sample we have 56 trading days, which generates a total of 4368

(56×78) observations. The 5-minute changes of both stock and futures indexes are calculated by $\log(P_t) - \log(P_{t-1})$.

Table 1 summarizes selected basic statistics. All sample means are positive in the sample period. Variances are 0.0016 for the futures index and 0.0014 for the stock index, showing that the futures market has a higher volatility than the stock market. The signs of skewness are all positive. The values of Ljung-Box up to 15 lags (hereafter LB(15)) for the return series are significant at the 5% level. The LB(15) for squared return series are highly significant for both markets, suggesting the possibility of the presence of autoregressive conditional heteroskedasticity. First-order autocorrelations of futures and stock returns (ρ) are –0.0374 and 0.0129, respectively. For the squared return data, the first-order serial correlations (ρ^2) for the futures and stock markets are 0.0075 and 0.0119, respectively, indicating that the stock returns are more persistent than the futures returns.

To analyze the relationship between the stock and the futures markets using wavelet analysis, requires choosing which wavelet filter to use from the various wavelet filters. In consideration of the balance between the sample size and the length of wavelet filter, we settle with the Daubechies least asymmetric wavelet filter of length 8 (LA(8))[5], while we decompose our data up to scale 9. To examine the lead-lag relationship in wavelet analysis, first, we test for Granger causality up to level 9, including the original data set.

The results of the Granger causality tests are reported in Table 2. As can be seen in Table 2, the futures market leads the stock market at the original data and the seventh scale, while at the other scales, there are feedback relationships. Note that lower scales correspond to higher

[5] In our study, to check the appropriateness of our choice of wavelet filter, we examined the same test using other wavelet filters, such as the Daubechies extremal phase wavelet filter of length 4, D(4) and the Daubechies

frequency bands. For example, the first scale is associated with 5-minute changes, the second scale is associated with $2 \times 5 = 10$-minute changes, and so on.

The first six scales capture the frequencies $1/64 \leq f \leq 1/2$; i.e., oscillations with a period length of 2 (10 minutes) to 64 (320minutes). Since there are $78 \times 5 = 390$ minutes in a day, the first six scales relate to the intra-day dynamics of our sample. This is of interest in that at 5-minute (original data) and approximately one-day dynamics (seventh scale), the futures market Granger causes the stock market. The cost-of-carry (COC) model states that as new information arrives simultaneously to the stock and futures markets and is reflected immediately in both the stock and futures prices, profitable arbitrage should therefore not exist, under the assumption that the two markets are perfectly efficient and frictionless and act as perfect substitutes. In other words, if both markets are efficient, the COC model indicates that the two markets have a feedback relationship in terms of Granger causality. Based on this finding of the Granger causality test, we can conclude that the futures market is more efficient. In other words, price discovery is greater in the futures market (Lin and Stevenson, 2001).

Turning to the second purpose of our paper (correlation in the various time scales), we first examine the variances of the futures and the stock markets' returns in various time scales. An important characteristic of the wavelet transform is its ability to decompose (analyze) the variance of the stochastic process. Figures 1a and b illustrate the MODWT-based wavelet variances of two series in log-log scales. The straight lines indicate the variance and the dotted lines indicate the 95% confidence interval.[6]

---

least asymmetric wavelet filter of length 16, D(16). The results are not quantitatively different from those of LA(8). To conserve space, the results are not reported but available from the authors on request.

[6] For a detailed explanation on how to construct the confidence interval of wavelet variance, see Gençay et al. (2002, p. 242).

There is an approximate linear relationship between the wavelet variance and the wavelet scale, indicating the potential for long memory in the volatility series. The variances of both the stock and futures markets decrease as the wavelet scale increases. Note that the variance-versus-wavelet scale curves show a broad peak at the first scale in both markets. More specifically, a wavelet variance in a particular time scale indicates the contribution to sample variance (Lindsay et al., 1996, p. 778). The sample variances of the stock and futures markets are 0.0014 and 0.0016, respectively, and 49%[7] and 51% of the total variances of the stock and futures markets, respectively, are accounted for by the first scale. This result implies that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk is significantly less. In addition, notice that the wavelet variances show that the futures market is more volatile than the stock market regardless of the time scale. This is consistent with the results of Lee (1999), who finds that the futures market has higher volatility than the stock market using a GARCH model.

Furthermore, the eighth scale, where we observe an apparent break in the variance for both series, is associated with $128 \times 5 = 640$-minute changes. Since there are 390 minute in one day, the eighth scale corresponds to 1.64 days. This implies that the stock and futures returns have a multiscaling behavior. There are two different scaling parameters corresponding to intra-day and higher scales, respectively. This apparent multiscaling behavior occurs approximately at the daily horizon where the intra-day seasonality is strongly present. The removal of the intra-day seasonality would not eliminate this multiscaling, but the transition between two

[7] This figure can be calculated by the normalization of the wavelet variance using the sample variance. For more detail, see Lindsay et al. (1996).

scaling regions would be more gradual by exhibiting a concave scaling behavior (Gençay et al., 2001).

In addition to examining the variances of two time series, a natural question is to consider how the two series are associated with one another. The wavelet correlation is constructed to examine how the two series are related over various time scales. Figure 2 illustrates the estimated wavelet correlation between the stock and futures returns against the various time scales. In this figure, the straight line indicates the wavelet correlations up to ninth scale, while the dotted lines represent the 95% confidence interval.

Two things are worth noting in Figure 2. First, the correlations up to the seventh scale are increasing, starting from the second scale. In the intra-day scales (up to seventh scale), the correlation coefficients are very different, depending on the specific time scale, while at one-day and higher dynamics, the correlation remains very high, close to one. This implies that in the intra-day scales, especially at very short scales, the primary purpose for traders' participation in the futures market is not hedging, but speculation. Second, the magnitude of the correlation increases as the time scale increases, indicating that the stock and futures markets are not fundamentally different (Lee, 1999).

The final purpose of this paper is to examine the multiperiod hedge ratio, based on the results of variance and covariance obtained from wavelet analysis. As indicated in Lien and Luo (1993, 1994), realism suggests that the hedger's planning horizon usually covers multiple periods. Therefore, examining the multiperiod hedge ratio is more appropriate than examining the one-period hedge ratio. Figure 3 shows the hedge ratio and hedging effectiveness using the various time scales up to scale 9. Note that these wavelet hedge ratios are estimated by a nonparametric method. Therefore, it is not necessary to assume a particular distribution of the error term as in the GARCH/SV model.

As can be seen in Figure 3, the hedge ratio at second scale has the lowest value, –0.06 and increases monotonically at a decreasing rate, converging toward the long horizon hedge ratio of one. As indicated in Figure 3, the degree of hedging effectiveness has the lowest value at the second scale, 0.004, and approaches one as the wavelet time scale increases. Intuitively, hedging effectiveness approaches one because, over long horizons, the shared permanent component ties the stock and futures series together, and the effect of the transitory components becomes negligible. In the long run, the stock and futures prices are perfectly correlated (Geppert, 1995). This result is consistent with the results of Low et al. (2002) and Geppert (1995), who compare the hedge ratios and hedging effectiveness obtained from various models.

5. Concluding remarks

This paper investigates the multiscale relationship between the stock and futures markets over various time horizons. Ours is the first analysis to undertake this investigation using wavelet analysis and high frequency intra-day data. The paper examines this relationship in three ways: (1) the lead-lag relationship, (2) covariance/correlations, and (3) the hedge ratio and hedging effectiveness. To examine the lead-lag relationship between the two markets, we employ the Granger causality test for various time scales utilizing the decomposed data. The wavelet correlation is estimated by testing the correlation between the two markets in the various time scales from the wavelet coefficients. The hedge ratio, defined by the covariance between the stock return and the futures return divided by the volatility of the futures return, is calculated from the wavelet covariance and variance. The main advantage of using wavelet analysis is the ability to decompose the data into various time scales. This advantage allows researchers to investigate the relationship between two variables in various time scales, while the traditional methodology allows examination of only two time scales – short- and long-run scales.

The wavelet analysis is undertaken using the LA(8) wavelet filter, and it supports four main conclusions. First, it is found that at 5-minute (original data) and approximately one-day dynamics (seventh scale), the futures market Granger causes the stock market. From this result, it is concluded that the futures market is more efficient. In other words, price discovery is greater in the futures market.

Second, it is found that there is an approximate decreasing linear relationship between the wavelet variance and the wavelet scale, indicating the potential for long memory in the volatility series. This result implies that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk is significantly less.

Third, at the eighth scale (associated with $128\times5 = 640$-minute changes), it is found that there is an apparent break in the variance for both series. Since there are 390 minutes in one trading day, the eighth scale corresponds to 1.64 days. This implies that the stock and futures returns have a multiscaling behavior. This apparent multiscaling behavior occurs approximately at the daily horizon, where the intra-day seasonality is strongly present.

Fourth, we also find that at the intra-day scales (up to seventh scale), the correlation coefficients between the two markets vary at different horizons, while at one-day and higher scales, the correlation remains very high and stabilizes close to one. In addition, the magnitude of the wavelet correlation between the two markets increases as the time scale increases, indicating that the stock and futures markets are not fundamentally different.

Fifth, the analysis of the wavelet hedge ratio and hedging effectiveness indicates that the hedge ratio at the second scale has the lowest value and increases monotonically at a decreasing rate, converging toward the long horizon hedge ratio of one. The degree of hedging effectiveness also has the lowest value at the second scale and approaches one as the wavelet time scale increases. Intuitively, hedging effectiveness approaches one because, over long horizons, the shared permanent component ties the stock and futures series together and the effect of the transitory components becomes negligible.

References

Brooks, C., Henry, O. T., Persand, G., 2002. The effect of asymmetries on optimal hedge ratios. Journal of Business 75, 333–352.

Chen, S. S., Lee, C. F., Shrestha, K., 2001. On a mean-generalized semivariance approach to determining the hedge ratio. Journal of Futures Markets 21, 581–598.

Choudhry, T., 2003. Short-run deviations and optimal hedge ratio: evidence from stock futures. Journal of Multinational Financial Management 13, 171–192.

Duffie, D., 1989. Futures Markets. Englewood Cliffs, NJ: Prentice-Hall.

Ederington, L., 1979. The hedging performance of the new futures markets. Journal of Finance 34, 15–7170.

Gençay, R., Selçuk F., Whitcher, B., 2001. Scaling properties of foreign exchange volatility. Physica A 289, 249–266.

Gençay, R., Selçuk F., Whitcher, B., 2002. An introduction to wavelets and other filtering methods in finance and economics. Academic Press, London.

Gençay, R., Selçuk, F., Whitcher, B., 2003. Systematic risk and time scales. Quantitative Finance 3, 108–116.

Geppert, J. M., 1995. A statistical model for the relationship between futures contract hedging effectiveness and investment horizon length. Journal of Futures Markets 15, 507–536.

Hill, J., Schneeweis, T., 1982. The hedging effectiveness of foreign currency futures. Journal of Financial Research 5, 95–104.

Howard, C. T., D'Antonio, L. J., 1984. A risk-return measure of hedging effectiveness. Journal of Financial and Quantitative Analysis 19, 101–112.

Howard, C. T., D'Antonio, L. J., 1991. Multiperiod hedging using futures: a risk minimization approach in the presence of autocorrelation. Journal of Futures Markets 11, 697–710.

Hsin, C., Kuo, W., Lee, C. F., 1994. A new measure to compare the hedging effectiveness of foreign currency futures versus options. Journal of Futures Markets 14, 685–707.

In, F., Kim, S., 2006. The hedge ratio and the empirical relationship between the stock and futures markets: a new approach using wavelet analysis. Journal of Business 79 (2), 799–820.

Johnson, L. L., 1960. The theory of hedging and speculation in commodity futures. Review of Economic Studies 27, 139–151.

Kim, S., In, F., 2005a. Multihorizon sharpe ratio. Journal of Portfolio Management 31, 105–111.

Kim, S., In, F., 2005b. Stock returns and inflation: new evidence from wavelet analysis. Journal of Empirical Finance 12, 435–444.

Kroner, K. F., Sultan, J., 1993. Time-varying distribution and dynamic hedging with foreign currency futures. Journal of Financial and Quantitative Analysis 28, 535–551.

Lee, G. G. J., 1999. Contemporary and long-run correlations: a covariance component model and studies on the S&P 500 cash and futures markets. Journal of Futures Markets 19, 877–894.

Lien, D., Luo, X., 1993. Estimating multiperiod hedge ratios in cointegrated markets. Journal of Futures Markets 13, 909–920.

Lien, D., Luo, X., 1994. Multiperiod hedging in the presence of conditional heteroskedasticity. Journal of Futures Markets 14, 927–955.

Lien, D., Wilson, B. K., 2001. Multiperiod hedging in the presence of stochastic volatility. International Review of Financial Analysis 10, 395–406.

Lin, S-J., Stevenson, M., 2001. Wavelet analysis of the Cost-of-Carry model. Studies in Nonlinear Dynamics and Econometrics, 5(1), art 7.

Lindsay, R. W., Percival, D. B., Rothrock, D. A., 1996. The discrete wavelet transform and the scale analysis of the surface properties of sea ice. IEEE Transactions on Geoscience and Remote Sensing 34, 771–787.

Low, A., Muthuswamy, J., Sakar, S., Terry, E., 2002. Multiperiod hedging with futures contracts. Journal of Futures Markets 22, 1179–1203.

Mallat, S., 1998. A wavelet tour of signal processing. Academic Press, San Diego.

Percival, D. B., Walden, A. T., 2000. Wavelet methods for time series analysis. Cambridge University Press, Cambridge, UK.

Ramsey, J. B., Lampart, C., 1998a. Decomposition of economic relationships by timescale using wavelets. Macroeconomic Dynamics 2, 49–71.

Ramsey, J. B., Lampart, C., 1998b. The decomposition of economic relationships by time scale using wavelets: expenditure and income. Studies in Nonlinear Dynamics and Economics 3, 23–42.

Ramsey, J. B., Zhang, Z., 1997. The analysis of foreign exchange data using waveform dictionaries. Journal of Empirical Finance 4, 341–372.

Wang, C., Low, S. S., 2003. Hedging with foreign currency denominated stock index futures: evidence from the MSCI Taiwan index futures market. Journal of Multinational Financial Management 13, 1–17.

Table 1
Basic statistics

	Futures market	Stock market
Mean	0.0003	0.0003
Variance	0.0016	0.0014
Skewness	0.2966	0.4281
Kurtosis	14.8162	13.5827
JB	39979.8616	33679.8516
	(0.0000)	(0.0000)
ρ	-0.0374	0.0129
LB(15) for R_t	22.2503	26.8247
	(0.0348)	(0.0082)
ρ^2	0.0075	0.0119
LB(15) for R_t^2	31.7695	55.2397
	(0.0015)	(0.0000)

Note: Sample period: 22 March 2004 – 9 June 2004. The means and variances are calculated by multiplying by 100 and 10,000, respectively. Significance levels are in parentheses. LB(n) is the Ljung-Box statistic for up to n lags, distributed as χ^2 with n degrees of freedom. ρ and ρ^2 indicate the first-order autocorrelations of returns and squared returns, respectively. Skewness and kurtosis are defined as $E[(R_t - \mu)]^3$ and $E[(R_t - \mu)]^4$, where μ is the sample mean.

Table 2
Granger causality test

	original	1	2	4	8	16	32	64	128	256
Stock	1.83	62.46*	108.30*	382.27*	38.34*	141.66*	21.10*	1.22	49.92*	77.29*
→Futures	(0.12)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.30)	(0.00)	(0.00)
Futures	818.80*	579.75*	431.55*	240.74*	435.70*	603.02*	631.01*	557.13*	514.61*	204.49*
→Stock	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: The original data has been transformed by the wavelet filter (LA(8)) up to time scale 8. * indicates significance at 5% level. The significance levels are in parentheses. The first detail (wavelet coefficient) d1 captures oscillations with a period length 5 to 10 min. The last wavelet scale captures oscillations with a period length of 1280 to 2560 min.

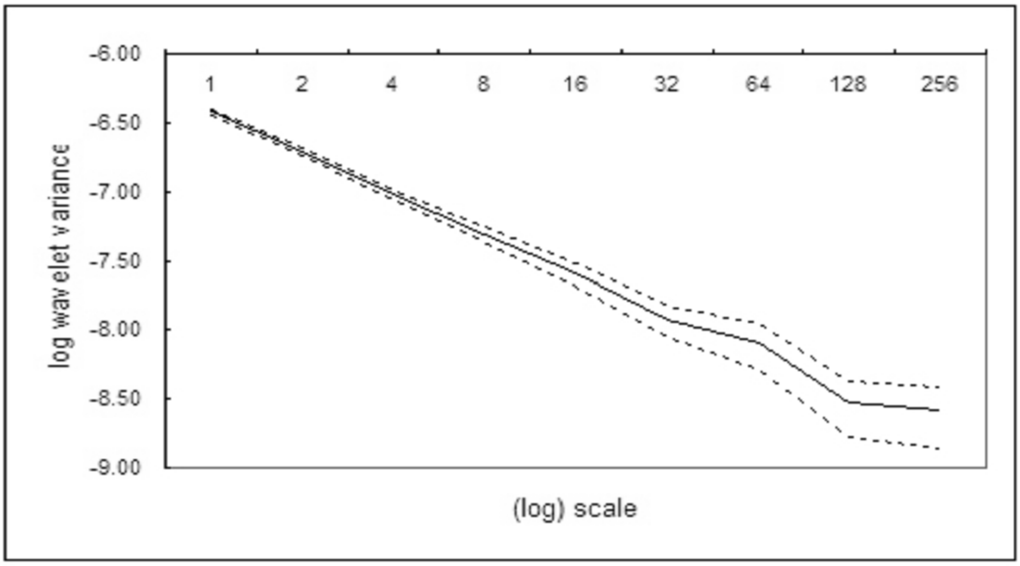

Fig. 1a. Estimated wavelet variance of stock returns. Wavelet variances for stock returns are plotted on different time scale x-axis. The dotted lines represent approximate 95% confidence interval. Each scale is associated with a particular time period. For example, the first scale is 5 min, the second scale is 2x5 = 10 min, the third scale is 4x5= 20 min and so on. The seventh scale is 64x5 = 320 min. Since there are 390 min per day, the seventh scale corresponds to approximately one day.

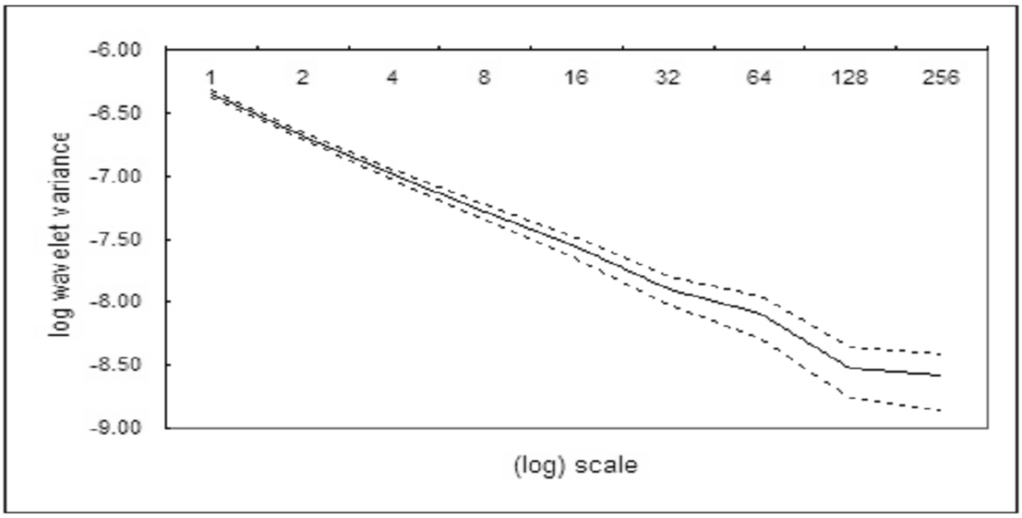

Fig. 1b. Estimated wavelet variance of futures returns. Wavelet variances for futures returns are plotted on different time scale x-axis. The dotted lines represent approximate 95% confidence interval. Each scale is associated with a particular time period. For example, the first scale is 5 min, the second scale is 2x5 = 10 min, the third scale is 4x5= 20 min and so on. The seventh scale is 64x5 = 320 min. Since there are 390 min per day, the seventh scale corresponds to approximately one day.

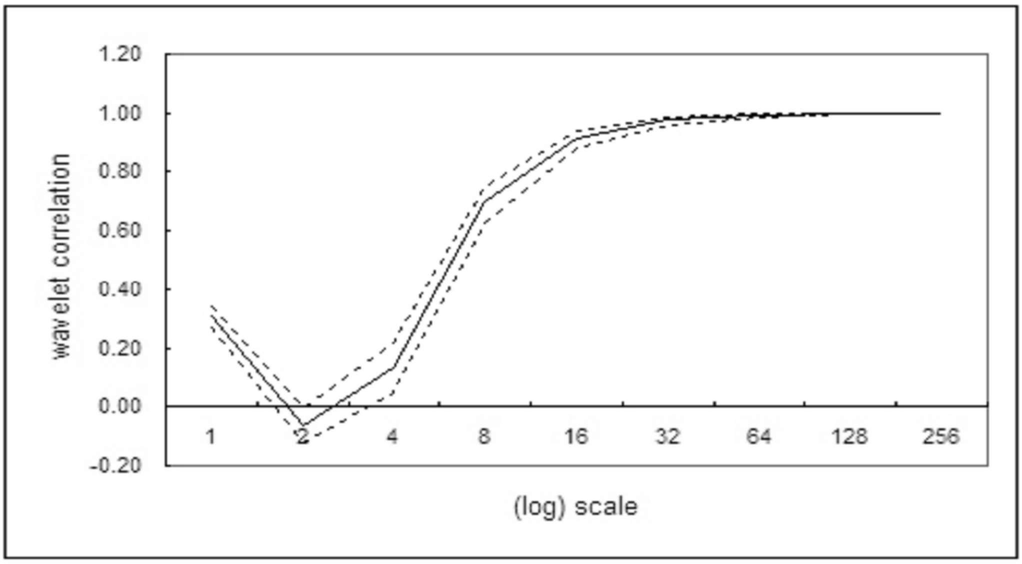

Fig. 2. Estimated wavelet correlation between stock and futures returns. The dotted lines denote approximate 95% confidence interval. At scales d1 – d3, correlation rapidly increases and at scales d4 – d8, gradual and persistent behavior becomes more visible. Each scale is associated with a particular time period. For example, the first scale is 5 min, the second scale is 2x5 = 10 min, the third scale is 4x5= 20 min and so on. The seventh scale is 64x5 = 320 min. Since there are 390 min per day, the seventh scale corresponds to approximately one day.

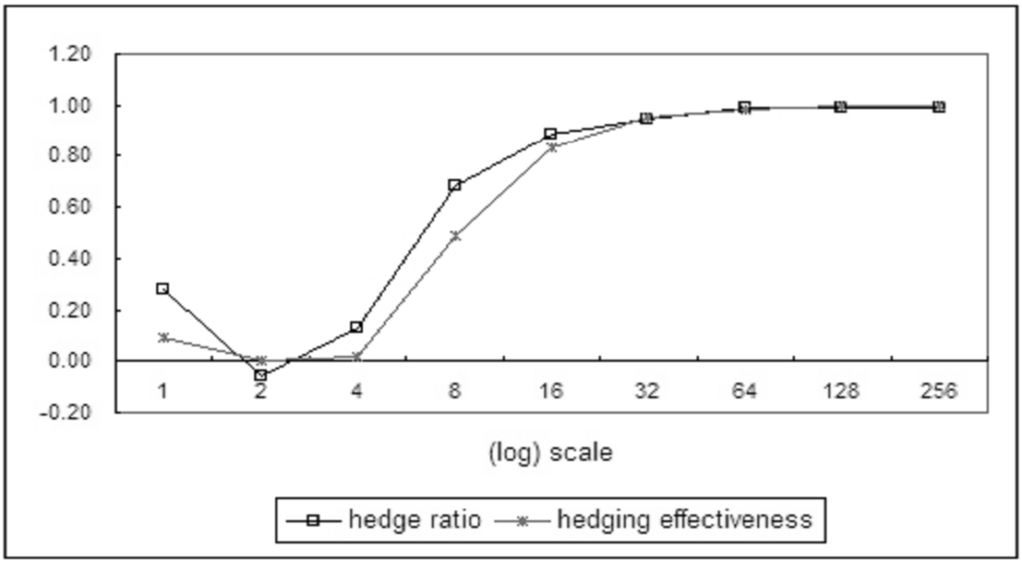

Fig. 3. Hedge ratio and hedging effectiveness against wavelet domains. Each scale is associated with a particular time period. For example, the first scale is 5 min, the second scale is 2x5 = 10 min, the third scale is 4x5= 20 min and so on. The seventh scale is 64x5 = 320 min. Since there are 390 min per day, the seventh scale corresponds to approximately one day.