Hedging Performance of Commodity Futures: Out-of-Sample Evaluation of Multivariate GARCH Models

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Abstract

This study examines the hedge performances of the daily, weekly and monthly commodities – crude oil and gold futures from March 31, 1983 through November 28, 2007. We employ the conventional hedging and various multivariate GARCH (1, 2) models during both in-sample and out-of-sample periods. We suggest six findings. First, a model that predicts well in sample does not necessarily predict well out-of-sample. Second, variance reduction from hedging varies across different periods. Third, by and large, we find that there is more variance reduction as the sample frequency declines, from daily to weekly to monthly. Fourth, in the short period of hedging performance, the conventional hedging model is not inferior to other popular and time-varying dynamic hedging models developed by multivariate GARCH methods. Fifth, in the longer period of hedging performance, multivariate GARCH models perform better than conventional hedge models. Sixth, BEKK GARCH and principal component GARCH models perform relatively better than other models in the out-of-sample period.

Keywords: Commodities futures; Crude oil; Gold; Hedge performance; Multivariate GARCH;

Out-of-sample forecast

1. Introduction

There are growing concerns of rising economic, business, and investment risks from the surging price of commodities markets, particularly crude oil and gold over the past several years. Commodity hedging is to reduce the risk and offset the loss generating from the spot position of commodities with the use of derivatives. Previous researches, such as Working (1953), Johnson (1960), Stein (1961), Ederington (1979), and Figlewski (1984), have investigated the conventional optimal hedging models, which restrict the hedge ratio to be constant over time. Recent studies, such as Baillie and Myers (1991), Myers (1991), Kroner and Sultan (1993), Ghosh (1993), Park and Switzer (1995), and Lee et al. (2006) present the improvement of hedging performance by time-varying minimum-variance hedge ratios based on either conditional second moments or time-varying coefficient of the model.

On the other hand, following the seminal work of multivariate generalized autoregressive conditional heteroskedasticity models (GARCH) by Bollerslev, Engle and Wooldridge (1988), a series of multivariate GARCH model extensions have been proposed. It is well known that GARCH models could well describe the in-sample distribution of stock and futures returns. However, few of these multivariate GARCH variants have been evaluated based on their out-of-sample forecasting performances.

This paper bridges the gap of the application and evaluation of various GARCH models in the in-sample and out-of-sample dynamic hedging. In particular, we estimate and forecast the variances and covariances of spots and futures of crude oil and gold markets based on several popular bivariate GARCH models. Furthermore, we evaluate the performance of variance reduction for the hedged portfolio according to predictions of these models.

The rest of this paper is organized as follows. Section 2 illustrates the models that we employ. Section 3 explains the data and shows the summary statistics. Section 4 presents the model estimation and evaluation of hedging performance. Section 5 concludes the paper.

2. Models and Methodology

This study employs seven models. The first one is the conventional ordinary least square (OLS),

$$r_{s,t} = \alpha + \beta r_{f,t} + e_t \tag{1}$$

where $r_{s,t}$ is the commodity spot return and $r_{f,t}$ is the commodity futures return. The OLS estimator

$$\beta^* = \frac{\sigma_{sf}}{\sigma_f^2} \tag{2}$$

where β^* is the optimal hedge ratio which will maximize utility function of an investor who faces the mean-variance expected utility function. This conventional hedging strategy assumes that the investor holds one unit in long position in the spot commodity market. To maximize his utility as well as minimize the variance of his long position, he holds the β^* unit of short position in the futures market. When β is one, it is called naïve hedge strategy.

Despite its simplicity and zero transaction cost, the conventional hedge model cannot incorporate and update the information from data, which could be a crucial problem when the out-of-sample is very volatile. Therefore, we propose a variant of OLS model, which is the rolling-window OLS model. We estimate OLS estimator β_t^* based on the in-sample data. When we estimate the out-of-sample hedge ratio, we use the latest available data instead of fixed in-sample data. For example, we use in-sample $r_1 : r_t$ to get OLS estimator β_t^*. β_t^* is the optimal hedge ratio and would be used for the first one-step-ahead out-of-sample data r_{t+1}. Then we use $r_2 : r_{t+1}$ to get OLS estimator β_{t+1}^*, the optimal hedge ratio, for the second one-step-ahead out-of-sample data r_{t+2}. In other words, the in-sample OLS estimation is rolling over as we hedge in the out-of-sample. This method could allow us to incorporate the latest information and discard the out-dated information as we go through the out-of-sample as well as the real-time hedging.

Since the joint distribution of commodity spots and futures market could be time-varying, we also consider the alternative models from the multivariate GARCH family. The third model is the simplified diagonal VECH GARCH (1,2) (DVEC GARCH) model, introduced by Bollerslev et al. (1988),

$$\begin{aligned} r_{s,t} &= \alpha_s + e_{s,t} \\ r_{f,t} &= \alpha_f + e_{f,t} \end{aligned} \tag{3}$$

$$\left. \begin{bmatrix} e_{s,t} \\ e_{f,t} \end{bmatrix} \right| \psi_{t-1} : \ N(0, H_t) \tag{4}$$

$$H_t = U + A \otimes e_{t-1} e_{t-1}' + B \otimes e_{t-2} e_{t-2}' + C \otimes H_{t-1} \tag{5}$$

$$\begin{aligned} H_t = \begin{bmatrix} h_{ss,t} & 0 \\ h_{fs,t} & h_{ff,t} \end{bmatrix} &= \begin{bmatrix} u_{ss} & 0 \\ u_{fs} & u_{ff} \end{bmatrix} + \begin{bmatrix} a_{ss} & 0 \\ a_{fs} & a_{ff} \end{bmatrix} \begin{bmatrix} e_{s,t-1} e_{s,t-1} & 0 \\ e_{f,t-1} e_{s,t-1} & e_{f,t-1} e_{f,t-1} \end{bmatrix} \\ &+ \begin{bmatrix} b_{ss} & 0 \\ b_{fs} & b_{ff} \end{bmatrix} \begin{bmatrix} e_{s,t-2} e_{s,t-2} & 0 \\ e_{f,t-2} e_{s,t-2} & e_{f,t-2} e_{f,t-2} \end{bmatrix} + \begin{bmatrix} c_{ss} & 0 \\ c_{fs} & c_{ff} \end{bmatrix} \begin{bmatrix} h_{ss,t-1} & 0 \\ h_{fs,t-1} & h_{ff,t-1} \end{bmatrix} \end{aligned} \tag{6}$$

where Equation (3) is the mean equation of the model; e_t is the innovation term, which follows a normal distribution with mean zero and conditional variance H_t; ψ_t is the information set at time *t*-1, and $\otimes$ is the Hadamard product. Equation (4) and (5) show that conditional variance follows an ARMA (1,2) process, which depends on its last-period variance and last-period squared residual. As shown in Equation (6), we only consider the lower triangular part of the symmetric metrics of *U*, *A*, and *B*. The covariance matrix must be positive semi-definite (PSD) but H_t in the DEVC model cannot be guaranteed to be PSD. Therefore, we adopt the fourth model – Matrix Diagonal GARCH (1,2) model, modified from Bollerslev et al. (1994),

$$H_t = UU^{'} + AA^{'} \otimes e_{t-1}e_{t-1}^{'} + BB^{'} \otimes e_{t-2}e_{t-2}^{'} + c \otimes H_{t-1} \quad (7)$$

where *b* is just a scalar. Equation (7) is a simple PSD version of the DEVC model. Although the Matrix Diagonal model has PSD covariance matrices, the dynamics in the covariance matrices is still restricted. Engel and Kroner (1995) propose the famous BEKK (Baba-Engle-Kraft-Kroner) GARCH (1,2) model, which would be our fifth model,

$$H_t = UU^{'} + A(e_{t-1}e_{t-1}^{'})A^{'} + B(e_{t-2}e_{t-2}^{'})B^{'} + CH_{t-1}C^{'} \quad (8)$$

where Equation (8) not only guarantees the PSD but also allows unrestricted matrices, where variances of the two variables have concurrent impact on each other by estimating two more parameters a_{sf} and b_{sf}. The sixth model is the constant conditional correlation - CCC GARCH (1,2) model, suggested by Bollerslev (1990), which assumes a time-invariant correlation, ρ, as shown in Equation (9).

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \quad (9)$$

The last model is the principal component GARCH (1,2) model,

$$H_t = \Lambda_t \Delta_t \Lambda_t^{'} \quad (10)$$

where Λ is an orthogonal matrix such that $\Lambda\Lambda^{'} = \mathrm{I}$ is an identity matrix; Δ is a diagonal matrix, of which the diagonal elements are the eigenvalues of *H* and the columns of Λ denote the eigenvectors of *H*.

It is worth noting that we do not incorporate the error correction terms into the mean equations of all the above models. It is because we fail to reject the null hypothesis of no cointegration for the commodities spot and futures returns, based on the two-step residual-based test from Engel and Granger (1988). In addition, we use the GARCH (1,2) instead of the simple GARCH (1,1) model specification because the former produces a more consistent and better in-sample fit than the latter.

3. Data

The article employs daily, weekly and monthly crude oil futures and gold futures traded at the New York Mercantile Exchange (NYMEX) from March 1983 to November 2007. These are among the most actively traded energy and metal futures commodities in terms of volume and open interest. The nearby futures prices of gold and crude oil in this paper are based on the prices of underlying assets of gold bullion traded in New York and West Texas intermediate oil respectively. These data are obtained from Global Financial Data. We construct the consistent time series of futures prices, in particular, daily time series, between contracts that occur on the last trading day of a month before the month of contract maturity. The returns are calculated in percentage terms as $[(\ln(P_t)-\ln(P_{t-1}))\times 100]$, where P_t is the spot and futures prices of commodities on day t. Figure 1 plots the spot and futures prices of crude oil and gold over the whole sample period. The surging prices of both commodities is evident since 2002.

There are 6,155 and 6,192 daily observations for oil and gold, respectively. There are 1,289 and 1,294 weekly observations for oil and gold, respectively. There are 296 observations for monthly oil and gold. Summary statistics of daily, weekly and monthly returns of the commodity spot and futures are reported in Table 1. In general, sample mean and standard deviation increase as the sample frequency increases. Daily and weekly series of both commodities spot and futures are non-normal because of excess kurtosis and skewness, especially in daily spot and futures returns.

4. Estimation Results and Hedging Performance

In this section, we present the estimations of the GARCH model family, its corresponding time-varying hedge ratios, and the hedge performance during both in-sample and out-of-sample periods. Because of different lengths of daily, weekly and monthly data, we choose different in-sample and out-of-sample sub-periods for two reasons. The first is to check the robustness of different samples. The second is to avoid possible parameter instability out of the long-range sample, particularly the daily series. For daily data, we have eight subsamples. Period 1 in sample is from March 31, 1983 to December 31, 1985; out-of-sample is from January 2, 1986 to December 31, 1986. Period 2 in sample is from January 2, 1987 to December 30, 1988; out-of-sample is from January 3, 1989 to December 29, 1989. Basically each period has a two-year in sample and one-year out of sample. For weekly data, we have two sub-periods with a nine-year in sample and a three-year out-of-sample. Period 1 in sample is from March 1983 through December 1992; out-of-sample is from January 1993 through December 1995. Period 2 in sample is from January 1996 through December 2004; out-of-sample is from January 2005 through November 2007. There is only one in sample (March 1983-December 2001) and one out-of-sample (January 2002-Novemebr 2007) from monthly data.

4.1. Estimation Results

To simplify the output, we only show, for example, daily in-sample parameter estimates in Period 8 (January 3, 2005-December 29, 2006) from various multivariate GARCH models in Table 2. The parameters are estimated by S-PLUS 7. In Table 2, most of the parameters are significant. As a preliminary model selection check, in crude oil returns, the matrix-diagonal GARCH model has the highest Akaike Information Criterion (AIC: 3374.05) and CCC GARCH has the highest Bayesian Information Criterion (BIC: 2215.80), which represent their better in-sample fit. On the contrary, the DVEC and principal component GARCH models have the lowest AIC (3244.93) and BIC (3301.50), respectively. In gold returns, the matrix-diagonal GARCH and DVEC GARCH have highest AIC (1827.04) and BIC (1879.18) respectively while the principal component GARCH has the lowest AIC (1674.79) and BIC (1725.41).

Figures 2 and 3 show in-sample Period 8 daily dynamic hedge ratios from Equation (2), which uses time-varying variances and covariances of five multivariate GARCH models during the in-sample period. The horizontal line is the constant hedge ratio (0.9423) based on the OLS estimation from Equation (1). The hedge ratios are time-varying as conditional covariances and variance are changing over time. In Figure 2, it is interesting to see that dynamic hedge ratios have diverse pattern from different models. For gold returns as shown in Figure 3, the dynamic hedge ratios of DEVC and matrix-diagonal have a similar pattern and are mean-reverting to the conventional constant hedge ratio. However, the BEKK, CCC and principal component models have quite different dynamic patterns.

4.2. Hedging Performance

In order to evaluate the performances of different hedge models, we construct the hedged portfolios based on the optimal hedge ratios for each trading day and then we compute the variances of the hedged portfolios,

$$\mathrm{Var}(r_{s,t} - \beta^{*} r_{f,t}) \tag{11}$$

where β^{*} is the optimal hedge ratio, calculated from the models mentioned in Section 2. We show the variance reduction, which is computed as the difference between the sample variance of the unhedged spot position and the estimated variance of the hedged portfolio of each model divided by the sample variance of the unhedged position. The hedge performances of models are divided into in-sample and out-of-sample periods.

Table 3 presents the results of 8 periods of in-sample daily data. In Table 3. A. of crude oil, principal component GARCH model with the lowest hedged variance (variance: 0.5101 and variance reduction: 52.18%) outperforms other models during Period 1 and 2. BEKK GARCH model with the lowest hedged variance (variance: 0.3556 and variance reduction: 88.01%) outperforms other models during Period 4. It is surprising to see that the naïve model outperforms other models in Period 3 (variance: 2.3604 and variance reduction: 84.31%). Moreover, OLS - conventional constant hedge model performs well and is superior to all other models during Period 5, 6, 7, and 8. In

Table 3.B. of gold, principal component GARCH model outperforms other models during Period 2, 3, 4, 6, 7, and 8 but only with small margin. CCC GARCH is superior to other models in Period 1 while BEKK GARCH model outperforms in Period 5.

It is more practical and informative to evaluate a model in terms of its out-of-sample performance. Table 4 reports the out-of-sample results of daily data. In Table 4.A. of crude oil, the OLS model outperforms other models in Period 1 (variance: 3.7122 and variance reduction: 77.62%), Period 6 (variance: 1.8353 and variance reduction: 79.30%), and Period 8 (variance: 0.2500 and variance reduction: 92.94%). BEKK GARCH model outperforms in Period 2 (variance: 1.8134 and variance reduction: 55.30%) and Period 4 (variance: 1.0545 and variance reduction: 52.28%). The rolling OLS (variance: 0.1619 and variance reduction: 90.22%) is superior in Period 3. The naïve model performs best (variance: 3.1138 and variance reduction: 72.19%) in Period 5 and principal component GARCH outperforms (variance: 0.4022 and variance reduction: 92.61%) in Period 7. In Table 4.B. of gold, no specific model dominantly outperforms other models. For example, the naïve model is the best in Period 1 and 3. BEKK GARCH model outperforms in Period 4, 6 and 7. OLS model is superior in Period 2 and 8. Principal component GARCH model is superior in Period 2 and 5. In summary, we find models perform differently in different out-of-sample periods. There is no difference of performance between conventional hedging models (naïve and OLS model) and dynamic hedging models based on multivariate GACH models. However, we do find that DVEC GARCH, diagonal matrix GARCH, and CCC GARCH are inferior to other models because none of them outperforms across all sample periods.

We also evaluate the weekly and monthly data for the in-sample and out-of-sample periods. The in-sample result is presented in Table 5. A. and out-of-sample result is shown in Table 5.B. In Table 5.A., we find principal component GARCH is superior in both weekly Period 1 (variance: 2.7222 and variance reduction: 89.50%) and Period 2 (variance: 2.5634 and variance reduction: 91.12%) for crude oil returns. For gold returns, principal component GARCH (variance: 0.0481 and variance reduction: 98.74%) is superior in Period 1 while BEKK GARCH (variance: 0.1191 and variance reduction: 96.81%) outperforms in Period 2. For monthly data, CCC GARCH model outperforms in crude oil market (variance: 0.5266 and variance reduction: 99.42%) while BEKK GARCH model is superior in gold market (variance: 0.4195 and variance reduction: 96.99%).

Table 5.B. reports the out-of-sample results of weekly and monthly data. Principal component GARCH model is superior in oil market in Period 1 (variance:

2.0436 and variance reduction: 85.26%) and BEKK GARCH model outperforms in oil market in Period 2 (variance: 1.9693 and variance reduction: 89.35%). In gold market, DVEC GARCH is superior in Period 1 (variance: 0.0584 and variance reduction: 97.13%) while OLS model is superior in Period 2. For monthly data, BEKK GARCH model outperforms in crude oil market (variance: 0.1669 and variance reduction: 99.76%) while OLS model is superior in gold market (variance: 0.0675 and variance reduction: 99.61%).

In summary, the study suggests six findings. First, a model that predicts well in sample does not necessarily predict out-of-sample well. Second, variance reduction from hedge varies across different periods. For example, in Table 4.A. of crude oil, the average variance reduction in Period 1 is about 75% while that in Period 2 is about 45%. Third, compared with Table 4 and 5, by and large, we find that there is more variance reduction as the sample frequency declines, from daily to weekly to monthly. Fourth, according to Table 4, in the short period of hedging performance the simple and conventional hedging model - naïve and OLS model is not inferior to other popular and time-varying dynamic hedging models by multivariate GARCH models. Fifth, according to Table 5, however, in the longer period of hedging performance like weekly and monthly frequency, multivariate GARCH models, in general, perform better than conventional hedge models. Sixth, BEKK GARCH and principal component GARCH models perform relatively better among other models in the out-of-sample period.

5. Conclusion

This study presents dynamic hedging models to calculate risk-minimizing hedge ratios in the daily, weekly and monthly commodities – crude oil and gold futures from March 31, 1983 through November 28, 2007. To compare the hedging performance of conditional hedging models with that of a conventional hedging method, we employ the rolling OLS and various multivariate GARCH (1,2) models during both in-sample and out-of-sample periods. We suggest six findings. First, a model that predict well in sample does not necessarily predict well out-of-sample. Second, variance reduction from hedge varies across different periods. Third, by and large, we find that there is more variance reduction as the sample frequency declines, from daily to weekly to monthly. Fourth, in the short period of hedging performance, the simple and conventional hedging model -naïve and OLS model is not inferior to other popular and time-varying dynamic hedging models by multivariate GARCH models. Fifth, in the longer period of hedging performance like weekly and monthly frequency, multivariate GARCH models perform better than conventional hedge models. Sixth, BEKK GARCH and principal component GARCH models perform relatively better than other models in the out-of-sample period.

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Table 1

Summary Statistics for Spot and Futures Returns of Commodities Contracts

	Crude Oil		Gold	
	spot	futures	spot	futures
		Daily		
Observations	6155		6192	
Mean	0.0182	0.0183	0.0097	0.0107
S.D.	2.4185	2.3456	0.9410	0.9657
Skewness	-1.0260	-1.1526	-0.1911	-0.0008
Kurtosis	18.8154	19.5017	6.8841	10.2791
		Weekly		
Observations	1289		1294	
Mean	0.0851	0.0860	0.0436	0.0488
S.D.	4.9651	4.8278	1.9813	2.0099
Skewness	-0.4481	-0.6181	-0.0110	0.0509
Kurtosis	4.7140	4.8679	2.4999	2.7209
		Monthly		
Observations	296		296	
Mean	0.3704	0.3746	0.1908	0.2134
S.D.	9.2551	9.3867	3.8991	3.9152
Skewness	-0.0058	0.0034	0.4102	0.4449
Kurtosis	2.4508	2.3848	0.9344	0.8534

Whole sample period: From March 31, 1983 through November 28, 2007

Table 2.A. Daily Parameter Estimates of Models for the In-Sample Period 8
Commodity: Crude Oil

	DVEC GARCH	Matrix-Diagonal GARCH	BEKK GARCH	CCC GARCH	Principal Component GARCH
α_s	**0.1936**(3.83***)	**0.0653**(0.76)	0.0785(0.76)	**0.2043**(**2.97****)	**-0.0794**(-0.65)
α_f	**0.2041**(3.56***)	**0.0674**(0.88)	**0.1109**(1.12)	**0.2064**(3.13***)	**-0.0155**(-1.70*)
u_{ss}	**2.9170**(18.34***)	**0.6230**(8.17***)	**1.0770**(3.25***)	**3.2658**(5.29***)	**0.5393**(0.20)
u_{fs}	**2.2529**(10.01***)	**0.5185**(7.49***)	**0.6100**(1.77**)	-	-
u_{ff}	**2.1978**(8.65***)	**0.2710**(10.58***)	**0.0022**(0.00)	**2.2577**(3.61***)	**0.2242**(9.80***)
a_{ss}	**0.5201**(8.92***)	0.3145(10.70***)	**0.5934**(2.83***)	**0.2375**(4.76***)	**-0.0011**(-0.06)
a_{fs}	**0.4009**(6.88***)	**0.3131**(7.80***)	**0.1727**(1.05)	-	-
a_{sf}	-	-	**-0.4601**(-2.57**)	-	-
a_{ff}	**0.3438**(5.99***)	0.0297(0.26)	**-0.0677**(-0.43)	**0.1134**(**3.64****)	**0.6200**(8.92***)
b_{ss}	**0.0930**(4.09***)	**-0.0140**(-0.04)	0.0631(0.21)	**0.1598**(3.39***)	**0.0030**(0.13)
b_{fs}	**-0.0166**(-0.36)	0.0209(0.01)	**0.1112**(0.78)	-	-
b_{sf}	-	-	0.0080(0.03)	-	-
b_{ff}	**0.0048**(0.11)	0.0007(0.00)	**0.0155**(0.10)	0.1557(4.18***)	**0.1511**(2.38**)
c_{ss}	**-0.0386**(-0.89)	-	0.5907(2.65***)	**-0.0995**(-0.57)	**0.9817**(48.92***)
c_{fs}	**0.1875**(2.40**)	-	**-0.1319**(-0.71)		-
c_{sf}	-	-	0.2546(1.51)	-	-
c_{ff}	**0.2281**(2.83***)	-	**1.0598**(8.50***)	**0.1593**(0.91)	**0.1903**(6.51***)
AIC	AIC(14): 3244.93	AIC(12): 3374.05	AIC(17): 3301.04	AIC(10): 3355.12	AIC(12): 3250.88
BIC	BIC(14): 3303.99	BIC(12): 3424.67	BIC(17): 3372.75	BIC(10): 3397.31	BIC(12): 3301.50

The daily in-sample Period 8 is from January 3, 2005 to December 29, 2006. DVEC-GARCH is diagonal VECH GARCH, CCC GARCH is the constant conditional correlation GARCH. ***, **,* indicate significance at a 1%, 5% and 10% levels, respectively.

Table 2.B. Parameter Estimates of Models for the In-Sample Period

Commodity: Gold

	DVEC GARCH	Matrix-Diagonal GARCH	BEKK GARCH	CCC GARCH	Principal Component GARCH
α_s	**0.1206**(2.64***)	**0.0767**(1.57)	0.05146(1.12)	**0.0252(0.64**)	**0.0761**(1.36)
α_f	**0.1202**(2.62***)	**0.0760**(1.53)	**0.0599**(1.35)	**0.0313**(0.81)	**-0.0070**(-1.24)
u_{ss}	**0.1735**(7.38***)	**0.3582**(9.40***)	**0.0947**(0.85)	**0.0016**(0.49)	**0.0099**(1.32)
u_{fs}	**0.1589**(7.60***)	**0.3409**(8.60***)	**0.2025**(2.54**)	-	-
u_{ff}	**0.1550**(7.38***)	**0.1064**(8.54***)	**0.0052**(0.00)	**0.0015**(0.52)	**0.0059**(1.67**)
a_{ss}	**0.1009**(4.17***)	0.3097(6.61***)	**0.2299**(1.60)	**0.0637**(3.84***)	**-0.0288**(-0.66)
a_{fs}	**0.0933**(3.69***)	**0.3080**(5.23***)	**-0.2413**(-1.94**)	-	-
a_{sf}	-	-	**-0.0830**(-0.47)	-	-
a_{ff}	**0.0925**(3.44***)	0.0417(1.01)	**0.3500**(2.12**)	**0.0967**(**3.47*****)	**0.4919**(6.81***)
b_{ss}	**0.0630**(2.11**)	**-0.0073**(-0.00)	0.4111(2.49**)	**0.0036**(0.18)	**0.0930**(1.87**)
b_{fs}	**0.0641**(2.20**)	0.0226(0.00)	**0.2695**(1.82)	-	-
b_{sf}	-	-	-0.2550(-1.54)	-	-
b_{ff}	**0.0599**(2.03**)	0.0026(0.29)	**-0.0241**(-0.14)	-0.0294(-0.96)	**-0.3240**(-5.36***)
c_{ss}	**0.7347**(42.31***)	-	0.7742(5.95***)	**0.9377**(91.11***)	**0.9429**(48.71***)
c_{fs}	**0.7441**(54.77***)	-	**-0.0187**(-0.16)		-
c_{sf}	-	-	0.2045(1.44)	-	-
c_{ff}	**0.7513**(54.77***)	-	**0.9604**(7.52***)	**0.9384**(94.22***)	**0.8410**(29.74***)
AIC	AIC(14): 1820.12	AIC(12): 1827.04	AIC(17): 1733.98	AIC(10): 1760.27	AIC(12): 1674.79
BIC	BIC(14): 1879.18	BIC(12): 1877.67	BIC(17): 1805.70	BIC(10): 1802.45	BIC(12): 1725.41

The daily in-sample Period 8 is from January 3, 2005 to December 29, 2006. DVEC-GARCH is diagonal VECH GARCH, CCC GARCH is the constant conditional correlation GARCH. ***, ** indicate significance at a 1% and 5% levels, respectively.

Table 3.A. In Sample Daily Hedging Performance of Models – Crude Oil

		Un-hedged	Naïve	OLS	DVEC GARCH	DMAT GARCH	BEKK GARCH	CCC GARCH	PC GARCH
Period 1	Variance	1.0669	0.5802	0.5327	0.5602	0.5391	0.5569	0.6433	0.5101
	Variance Reduction		45.62%	50.06%	47.49%	49.47%	47.80%	39.71%	52.18%
Period 2	Variance	3.9466	1.3064	1.2072	1.3277	1.2633	1.1568	1.2810	1.1234
	Variance Reduction		66.90%	69.41%	66.36%	67.99%	70.69%	67.54%	71.53%
Period 3	Variance	15.0441	2.3604	2.5678	2.4550	2.4204	2.5895	2.7596	2.4912
	Variance Reduction		84.31%	82.93%	83.68%	83.91%	82.79%	81.66%	83.44%
Period 4	Variance	2.9659	0.3665	0.3582	0.3726	0.3869	0.3556	0.3571	0.3769
	Variance Reduction		87.64%	87.92%	87.44%	86.96%	88.01%	87.96%	87.29%
Period 5	Variance	5.1842	2.2713	2.0869	2.1245	2.1883	2.1479	2.2964	2.1292
	Variance Reduction		56.19%	59.74%	59.02%	57.79%	58.57%	55.70%	58.93%
Period 6	Variance	6.8601	1.2106	1.1941	1.2735	1.2553	1.2095	1.2019	1.1978
	Variance Reduction		82.35%	82.59%	81.44%	81.70%	82.37%	82.48%	82.54%
Period 7	Variance	5.7734	1.3837	1.3259	1.3731	1.4056	1.3977	1.4509	1.3637
	Variance Reduction		76.03%	77.04%	76.22%	75.65%	75.79%	74.87%	76.38%
Period 8	Variance	4.0916	0.9263	0.9144	1.0024	0.9893	0.9537	1.0222	0.9585
	Variance Reduction		77.36%	77.65%	75.50%	75.82%	76.69%	75.02%	76.57%

Period 1: from March, 1983 to December, 1985. Period 2: from January, 1987 to December, 1988.

Period 3: from January, 1990 to December, 1991. Period 4: from January, 1993 to December, 1994.

Period 5: from January, 1996 to December, 1997. Period 6: from January, 1999 to December, 2000.

Period 7: from January, 2002 to December, 2003. Period 8: from January, 2005 to December, 2006.

Variance stands for the variance of the hedged portfolio calculation based on Equation (11). The percentage variance reductions are calculated as the differences of the variance of the unhedged position and the estimated variances of alternative models over the variance of the unhedged position multiplied by 100.

Table 3.B. In Sample Daily Hedging Performance of Models – Gold

Gold		Un-hedged	Naïve	OLS	DVEC GARCH	DMAT GARCH	BEKK GARCH	CCC GARCH	PC GARCH
Period 1	Variance	1.2617	0.2173	0.1807	0.1128	0.1294	0.1234	0.1064	0.1567
	Variance Reduction		82.78%	85.68%	91.06%	89.75%	90.22%	91.57%	87.58%
Period 2	Variance	0.9063	0.0085	0.0084	0.0086	0.0086	0.0087	0.0084	0.0084
	Variance Reduction		99.06%	99.07%	99.05%	99.05%	99.04%	99.07%	99.07%
Period 3	Variance	0.9693	0.0254	0.0254	0.0263	0.0262	0.0249	0.0264	0.0254
	Variance Reduction		97.38%	97.38%	97.29%	97.29%	97.43%	97.27%	97.38%
Period 4	Variance	0.6108	0.0078	0.0078	0.0089	0.0089	0.0085	0.0078	0.0078
	Variance Reduction		98.72%	98.72%	98.54%	98.55%	98.61%	98.72%	98.72%
Period 5	Variance	0.3833	0.0293	0.0286	0.0298	0.0297	0.0285	0.0297	0.0288
	Variance Reduction		92.35%	92.53%	92.23%	92.25%	92.56%	92.26%	92.48%
Period 6	Variance	0.9595	0.1042	0.0950	0.0885	0.0924	0.0942	0.1099	0.0767
	Variance Reduction		89.14%	90.10%	90.77%	90.37%	90.19%	88.55%	92.01%
Period 7	Variance	0.9220	0.0895	0.0872	0.0890	0.0884	0.0759	0.0894	0.0762
	Variance Reduction		90.30%	90.54%	90.35%	90.41%	91.77%	90.31%	91.73%
Period 8	Variance	1.4466	0.1385	0.1349	0.1412	0.1394	0.1392	0.1355	0.1304
	Variance Reduction		90.43%	90.67%	90.24%	90.36%	90.38%	90.64%	90.98%

See Table 3.A.

Table 4.A. Out-of-Sample Daily Hedging Performance of Models – Crude Oil

		Un-hedged	Naïve	OLS	Rolling OLS	DVEC GARCH	DMAT GARCH	BEKK GARCH	CCC GARCH	PC GARCH
Period 1	Variance	16.5840	4.0509	3.7122	3.7342	3.9557	4.1193	3.8887	4.1602	3.8529
	Variance Reduction		75.57%	77.62%	77.48%	76.15%	75.16%	76.55%	74.91%	76.77%
Period 2	Variance	4.0570	2.8174	2.3436	2.3447	2.2922	2.1796	1.8134	2.4524	2.1285
	Variance Reduction		30.56%	42.23%	42.21%	43.50%	46.28%	55.30%	39.55%	47.53%
Period 3	Variance	1.6554	0.1630	0.1905	0.1619	0.1709	0.1720	0.1678	0.1797	0.1729
	Variance Reduction		90.15%	88.49%	90.22%	89.67%	89.61%	89.87%	89.14%	89.56%
Period 4	Variance	2.2097	1.1000	1.0761	1.0750	1.0994	1.0831	1.0545	1.1572	1.0914
	Variance Reduction		50.22%	51.30%	51.35%	50.25%	50.99%	52.28%	47.63%	50.61%
Period 5	Variance	11.1955	3.1138	3.3242	3.1660	3.5755	3.3300	3.2371	4.2007	3.1831
	Variance Reduction		72.19%	70.31%	71.72%	68.06%	70.26%	71.09%	62.48%	71.57%
Period 6	Variance	8.8657	1.8574	1.8353	1.8481	1.8075	1.8793	1.9755	1.8818	2.0149
	Variance Reduction		79.05%	79.30%	79.16%	79.61%	78.80%	77.72%	78.77%	77.27%
Period 7	Variance	5.4458	0.4051	0.4308	0.4039	0.4049	0.4037	0.4166	0.4062	0.4022
	Variance Reduction		92.56%	92.09%	92.58%	92.56%	92.59%	92.35%	92.54%	92.61%
Period 8	Variance	3.5403	0.2716	0.2500	0.2539	0.3322	0.3054	0.2725	0.2927	0.3700
	Variance Reduction		92.33%	92.94%	92.83%	90.62%	91.37%	92.30%	91.73%	89.55%

Period 1: In-sample from March, 1983 to December, 1985. Out-of-sample from January, 1986 to December, 1986.

Period 2: In-sample from January, 1987 to December, 1988. Out-of-sample from January, 1989 to December, 1989.

Period 3: In-sample from January, 1990 to December, 1991. Out-of-sample from January, 1992 to December, 1992.

Period 4: In-sample from January, 1993 to December, 1994. Out-of-sample from January, 1995 to December, 1995.

Period 5: In-sample from January, 1996 to December, 1997. Out-of-sample from January, 1998 to December, 1998.

Period 6: In-sample from January, 1999 to December, 2000. Out-of-sample from January, 2001 to December, 2001.

Period 7: In-sample from January, 2002 to December, 2003. Out-of-sample from January, 2004 to December, 2004.

Period 8: In-sample from January, 2005 to December, 2006. Out-of-sample from January, 2007 to November, 2007.

Variance stands for the variance of the hedged portfolio calculation based on Equation (11). The percentage variance reductions are calculated as the differences of the variance of the unhedged position and the estimated variances of alternative models over the variance of the unhedged position multiplied by 100.

Table 4.B. Out-of-Sample Daily Hedging Performance of Models – Gold

		Un-hedged	Naïve	OLS	Rolling OLS	DVEC GARCH	DMAT GARCH	BEKK GARCH	CCC GARCH	PC GARCH
Period 1	Variance	1.5292	0.0079	0.0458	0.0534	0.0153	0.0268	0.0350	0.0255	0.0418
	Variance Reduction		99.49%	97.00%	96.51%	99.00%	98.25%	97.71%	98.33%	97.26%
Period 2	Variance	0.6755	0.0127	0.0126	0.0126	0.0130	0.0129	0.0128	0.0129	0.0126
	Variance Reduction		98.12%	98.14%	98.14%	98.08%	98.09%	98.11%	98.10%	98.14%
Period 3	Variance	0.2773	0.0020	0.0021	0.0021	0.0021	0.0021	0.0022	0.0023	0.0021
	Variance Reduction		99.26%	99.25%	99.26%	99.25%	99.24%	99.22%	99.18%	99.26%
Period 4	Variance	0.1763	0.0206	0.0207	0.0201	0.0186	0.0186	0.0185	0.0201	0.0187
	Variance Reduction		88.29%	88.27%	88.62%	89.45%	89.48%	89.51%	88.60%	89.42%
Period 5	Variance	0.6490	0.0114	0.0110	0.0110	0.0115	0.0114	0.0113	0.0112	0.0108
	Variance Reduction		98.24%	98.30%	98.31%	98.22%	98.24%	98.26%	98.27%	98.33%
Period 6	Variance	0.7338	0.1724	0.1504	0.1561	0.1542	0.1591	0.0699		0.0823
	Variance Reduction		76.50%	79.50%	78.72%	78.99%	78.32%	90.47%		88.78%
Period 7	Variance	0.9493	0.0805	0.0734	0.0728	0.0732	0.0753	0.0646	0.0751	0.0653
	Variance Reduction		91.52%	92.27%	92.33%	92.29%	92.07%	93.20%	92.09%	93.12%
Period 8	Variance	1.0755	0.1547	0.1487	0.1497	0.1574	0.1522	0.1541	0.1530	0.1473
	Variance Reduction		85.61%	86.17%	86.08%	85.36%	85.85%	85.67%	85.77%	86.30%

See Table 4. A.

Table 5.A. In-Sample Weekly and Monthly Hedging Performance of Models

		Un-hedged	Naïve	OLS	DVEC GARCH	DMAT GARCH	BEKK GARCH	CCC GARCH	PC GARCH
Weekly									
Oil	Variance	25.9358	2.9268	2.9126	2.9386	3.0261	3.2357	3.0495	2.7222
Period 1	Variance Reduction		88.72%	88.77%	88.67%	88.33%	87.52%	88.24%	89.50%
Oil	Variance	28.8513	2.6574	2.6630	2.7171	2.6983	2.6995	2.9419	2.5634
Period 2	Variance Reduction		90.79%	90.77%	90.58%	90.65%	90.64%	89.80%	91.12%
Gold	Variance	3.8285	0.0484	0.0481	0.0493	0.0492	0.0484	0.0483	0.0481
Period 1	Variance Reduction		98.74%	98.74%	98.71%	98.71%	98.74%	98.74%	98.74%
Gold	Variance	3.7352	0.1277	0.1201	0.1207	0.1207	0.1191	0.1195	0.1216
Period 2	Variance Reduction		96.58%	96.78%	96.77%	96.77%	96.81%	96.80%	96.74%
Monthly									
Oil	Variance	90.2222	0.5993	0.5659	0.5658	0.5580	0.5572	0.5266	0.5641
	Variance Reduction		99.34%	99.37%	99.37%	99.38%	99.38%	99.42%	99.37%
Gold	Variance	13.9356	0.4478	0.4442	0.4468	0.4477	0.4195	0.4344	0.4241
	Variance Reduction		96.79%	96.81%	96.79%	96.79%	96.99%	96.88%	96.96%

For weekly data, Period 1 in-sample is from March, 1983 to December, 1992 and out-of-sample is from January, 1993 to December, 1995. Period 2 in-sample is from January, 1996 to December, 2004 and out-of-sample is from January, 2005 to November, 2007. For monthly data, in-sample is from March, 1983 to December, 2001 and out-of-sample is from January, 2002 to November, 2007.

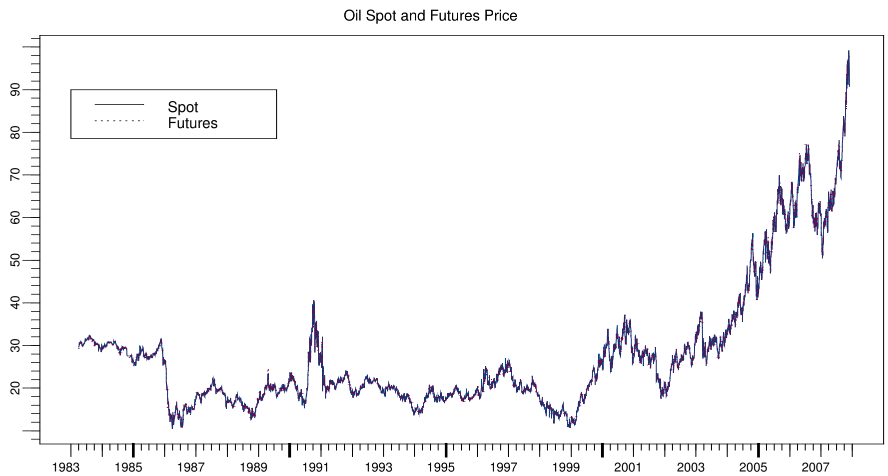
Variance stands for the variance of the hedged portfolio calculation based on Equation (11). The percentage variance reductions are calculated as the differences of the variance of the unhedged position and the estimated variances of alternative models over the variance of the unhedged position multiplied by 100.

Table 5.B. Out-of-Sample Weekly and Monthly Hedging Performance of Models

		Un-hedged	Naïve	OLS	Rolling OLS	DVEC GARCH	DMAT GARCH	BEKK GARCH	CCC GARCH	PC GARCH
Weekly										
Oil	Variance	13.8615	2.2219	2.1994	2.2132	2.2113	2.2835	2.1817	2.3770	2.0436
Period 1	Variance Reduction		83.97%	84.13%	84.03%	84.05%	83.53%	84.26%	82.85%	85.26%
Oil	Variance	18.4875	1.9973	1.9960	2.0026	2.0532	2.0578	1.9693	2.0460	2.0175
Period 2	Variance Reduction		89.20%	89.20%	89.17%	88.89%	88.87%	89.35%	88.93%	89.09%
Gold	Variance	2.0355	0.0599	0.0597	0.0596	0.0584	0.0585	0.0588	0.0597	0.0584
Period 1	Variance Reduction		97.06%	97.07%	97.07%	97.13%	97.13%	97.11%	97.07%	97.13%
Gold	Variance	6.7249	0.0900	0.0860	0.0883	0.0902	0.0909	0.0899	0.0935	0.0880
Period 2	Variance Reduction		98.66%	98.72%	98.69%	98.66%	98.65%	98.66%	98.61%	98.69%
Monthly										
Oil	Variance	68.2812	0.1734	0.1690	0.1698	0.1708	0.1748	0.1669	0.1744	0.1694
	Variance Reduction		99.75%	99.75%	99.75%	99.75%	99.74%	99.76%	99.74%	99.75%
Gold	Variance	17.3461	0.0736	0.0675	0.0687	0.0705	0.0715	0.0750	0.0724	0.0702
	Variance Reduction		99.58%	99.61%	99.60%	99.59%	99.59%	99.57%	99.58%	99.60%

See Table 5. A.

Figure 1. Spot and Futures Prices of Oil and Gold



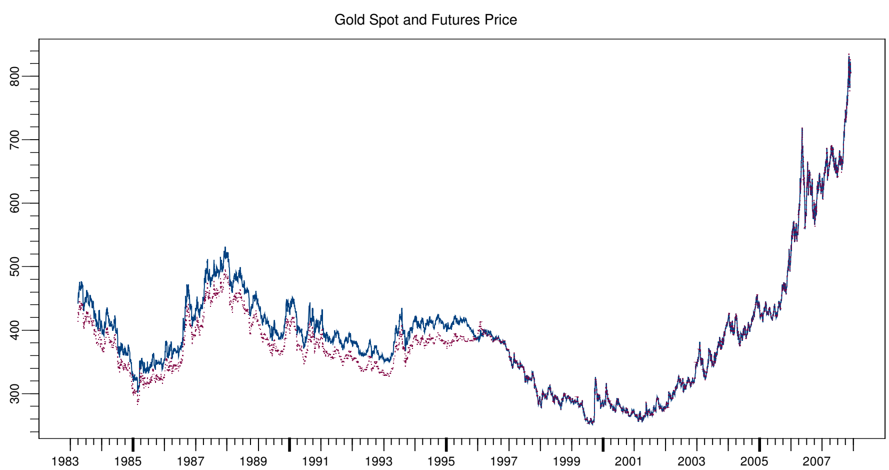
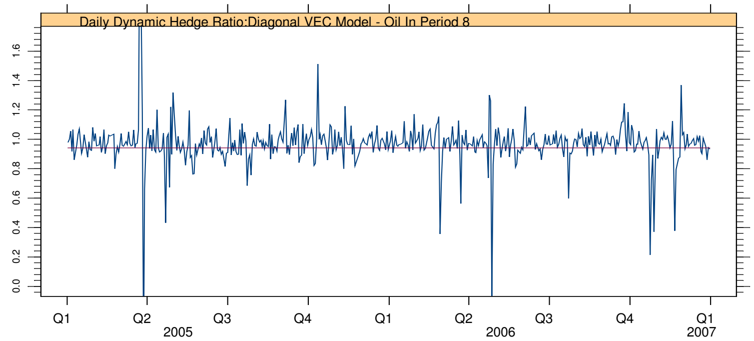
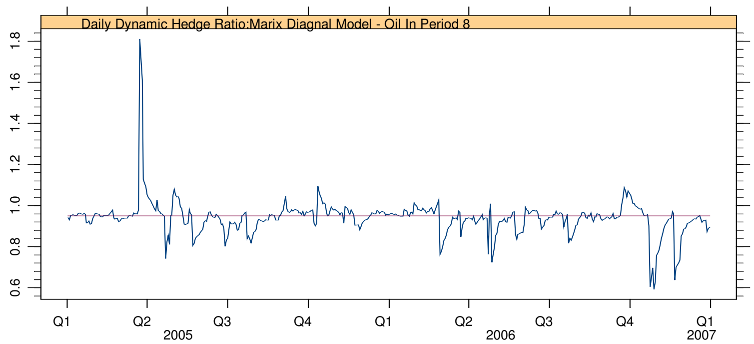


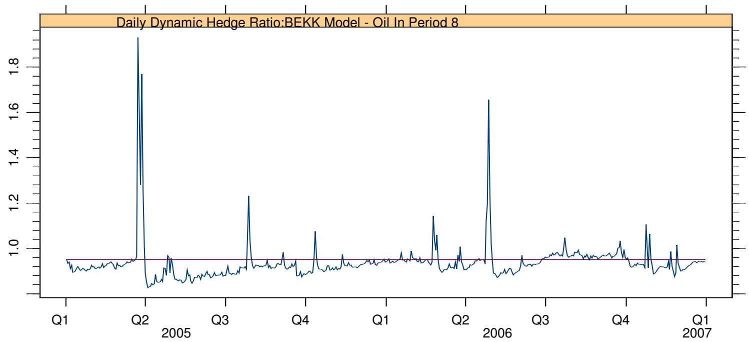
Figure 2. In Sample Daily Dynamic Hedge Ratio in Period 8 - Crude Oil

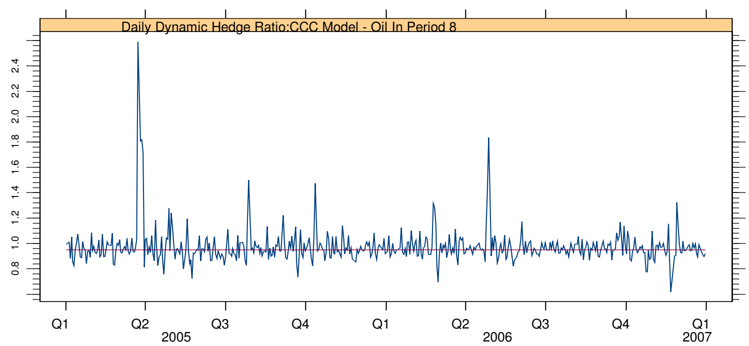




Note: The horizontal line is the constant hedge ratio (0.9423) computed by OLS.

In sample period: from January 3, 2005 to December 29, 2006


Daily Dynamic Hedge Ratio:BEKK Model - Oil In Period 8
1.0
1.2
1.4
1.6
1.8
Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
2005
2006
2007


Daily Dynamic Hedge Ratio:CCC Model - Oil In Period 8
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
2005
2006
2007

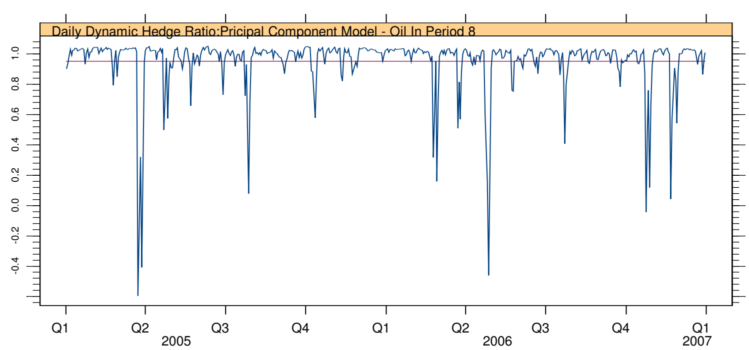
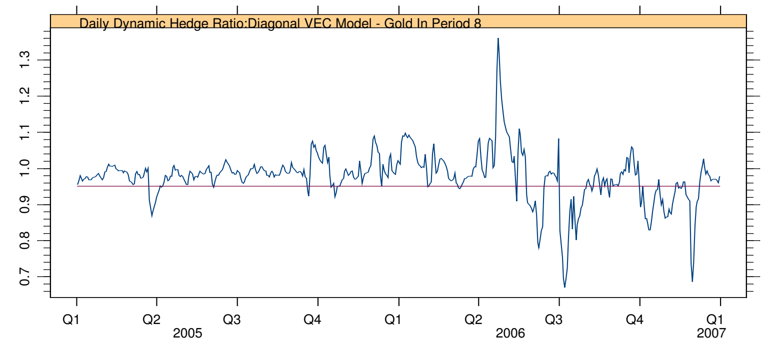
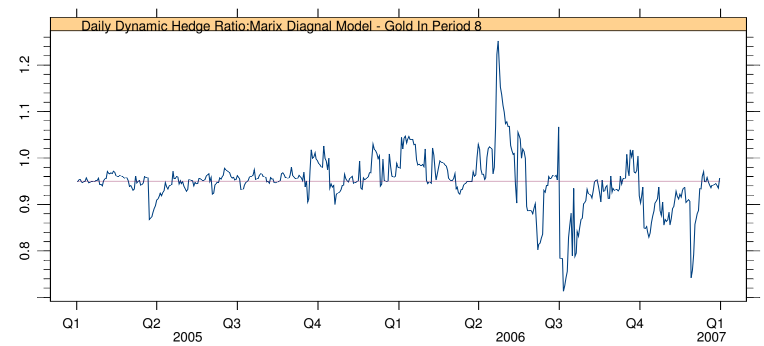
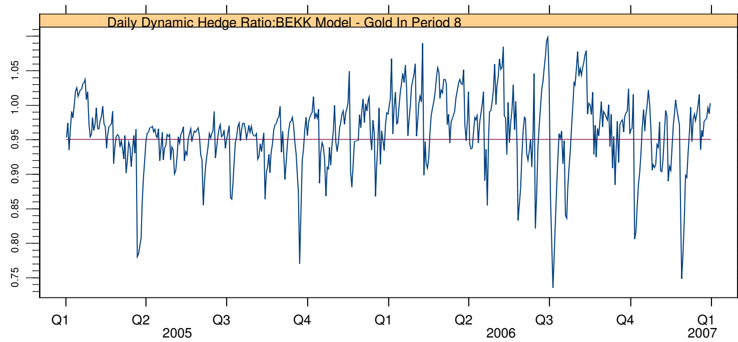
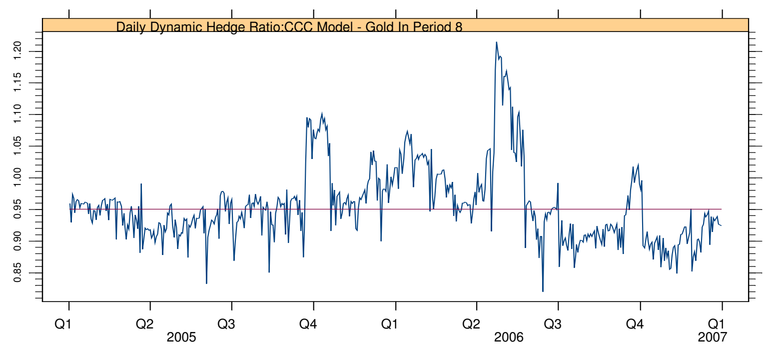

Daily Dynamic Hedge Ratio:Pricipal Component Model - Oil In Period 8
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
2005
2006
2007

Figure 3. In Sample Daily Dynamic Hedge Ratio in Period 8 - Gold






Daily Dynamic Hedge Ratio:BEKK Model - Gold In Period 8
0.75
0.80
0.85
0.90
0.95
1.00
1.05
Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
2005
2006
2007


Daily Dynamic Hedge Ratio:CCC Model - Gold In Period 8
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
2005
2006
2007

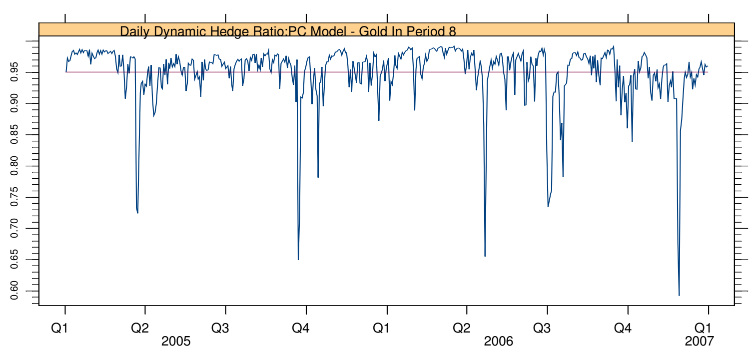
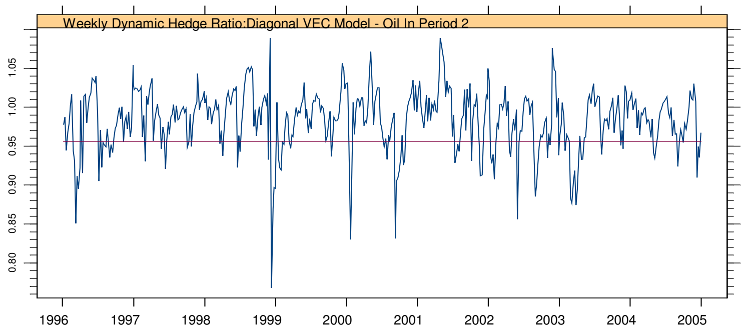
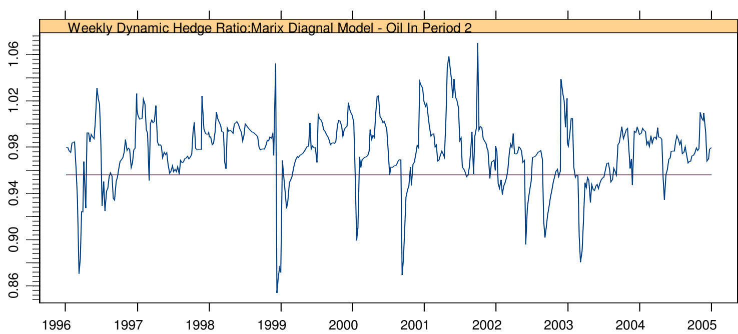
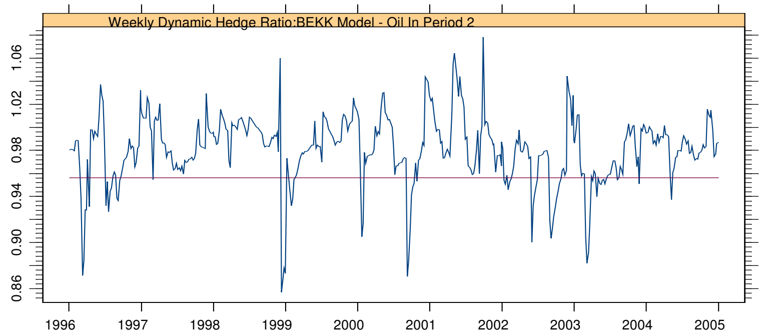
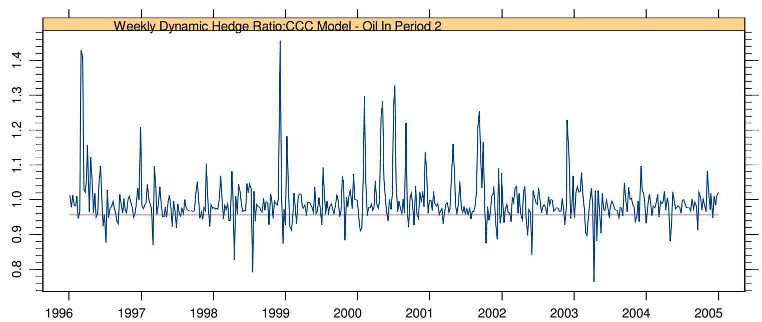

Daily Dynamic Hedge Ratio:PC Model - Gold In Period 8
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
Q1
Q2
Q3
Q4
Q1
Q2
Q3
Q4
Q1
2005
2006
2007

Figure 4. In Sample Weekly Dynamic Hedge Ratio in Period 2 - Crude Oil






Weekly Dynamic Hedge Ratio:BEKK Model - Oil In Period 2
0.86
0.90
0.94
0.98
1.02
1.06
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005


Weekly Dynamic Hedge Ratio:CCC Model - Oil In Period 2
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005

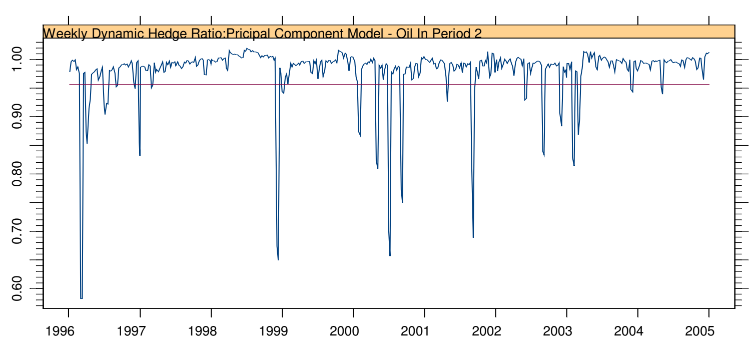
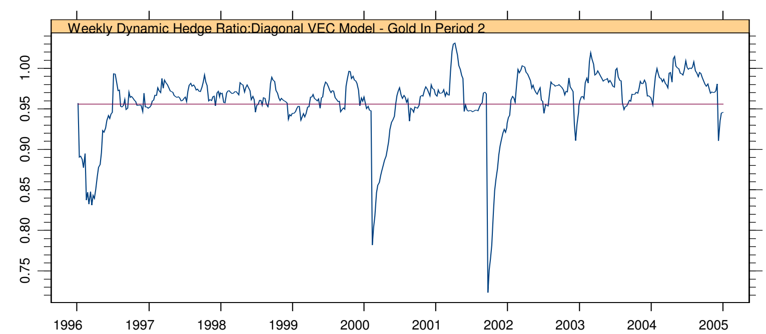
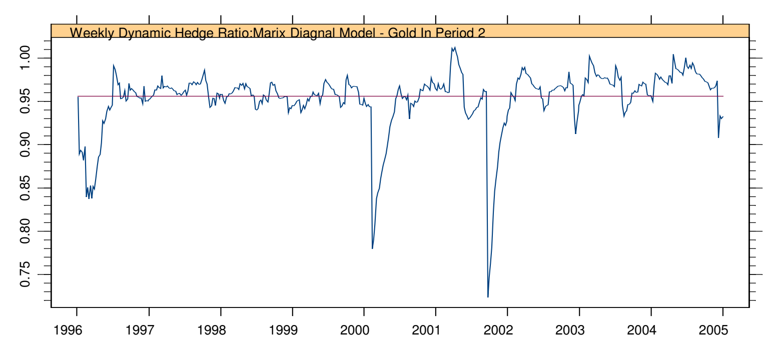
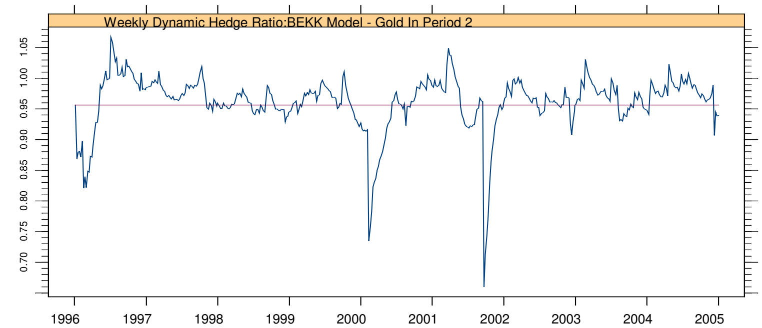
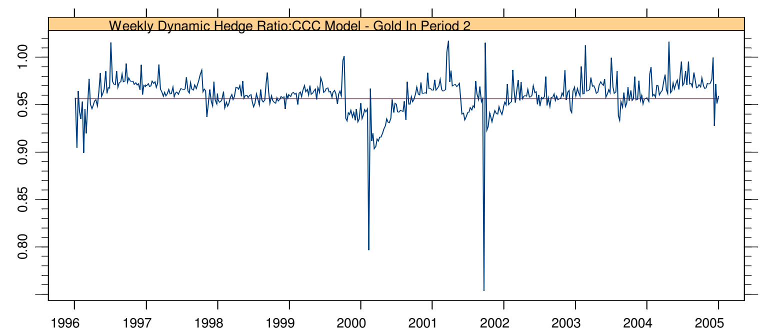

Weekly Dynamic Hedge Ratio:Pricipal Component Model - Oil In Period 2
0.60
0.70
0.80
0.90
1.00
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005

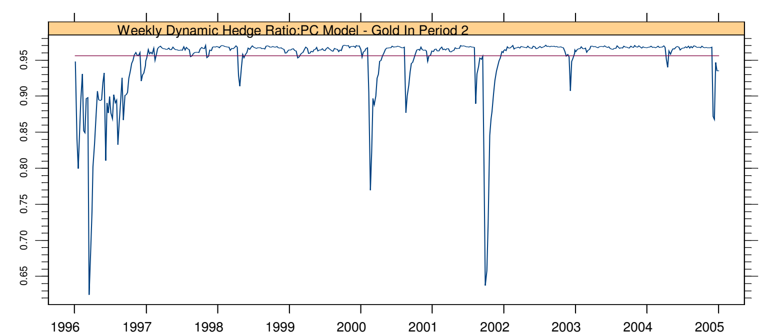
Figure 5. In Sample Weekly Dynamic Hedge Ratio in Period 2 - Gold






Weekly Dynamic Hedge Ratio:BEKK Model - Gold In Period 2
0.70
0.75
0.80
0.85
0.90
0.95
1.00
1.05
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005


Weekly Dynamic Hedge Ratio:CCC Model - Gold In Period 2
0.80
0.85
0.90
0.95
1.00
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005


Weekly Dynamic Hedge Ratio:PC Model - Gold In Period 2
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005