Herding, learning, and prices

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Abstract

We examine optimal investment strategy of informed traders in a futures' or equity market, where there are a more informed and a less informed trader, and many noise traders. The less informed trader has an incentive to follow the more informed because of informational advantage. The more informed can take advantage of that incentive by cheating the less informed and manipulating prices. Then prices are kept away from their fundamental values more often, and the profits of the more and the less informed trader are increased and decreased, respectively. Over time, both traders notice the other's strategy, and a long run equilibrium is attained, in which the more informed mixes his sincere and cheating strategy, and the less informed mixes his trust and distrust strategy. This model might be applicable to emerging markets, where foreign investors, local institutions, and local individuals are often regarded as more informed, less informed and noise traders, respectively.

Keywords : Information Asymmetry; Informed Trader; Noise Trader; Herd; Investor Performance;

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1. Introduction

Let us begin by recalling what happened to the Internet browser market in the late 1990s when Microsoft newly launched its Internet Explorer (IE). The market situation then was not very favorable to Microsoft since most customers had been using its powerful competitor, Netscape Navigator. In addition, not many technology experts saw in IE much better quality as an Internet browser. However, some market watchers like the *Economist*, an English weekly business magazine, predicted that it would be just a matter of time that IE would eventually dominate the browser market. This expectation was fulfilled afterwards as all of us know. How was IE able to defeat the Navigator to which most users had been used? One reason could be the well-known bundling strategy of Microsoft, which sold the combined package of the Windows and IE as a single product. Another reason might be the belief of Internet users that IE of Microsoft would eventually defeat the Navigator just as its Excel and Word had defeated their competitors before. Given the belief that most Internet users would use the Internet Explorer in the future anyway, the optimal decision of Internet users was to switch from Navigator to IE.

As this episode suggests, "belief" matters in markets, and market equilibriums could be affected partly by beliefs of market participants. Securities or commodities markets are no exception. For example, we saw the US stock markets plummeting when Alan Greenspan commented about "irrational exuberance" in 1996. Also the price of silver rose sharply soon after Warren Buffet betted a substantial amount of money on it in the late 1990's. These examples imply that people trusted by others affect market prices of financial securities or commodities. In an order-driven equity or futures market without market makers, investors are normally classified into informed and noise traders.[4] Informed traders usually have not only superior information but also a superior reputation or others' firm belief in them. Given this, they might be able to affect or even manipulate market prices to maximize their profits by taking advantage of others' belief in them. This possibility, however, has not been addressed in depth by existing finance literature; the behavior of informed or noise traders have been approached mainly in terms of (asymmetric) information only. In this paper, we classify informed traders into two groups, who are strongly informed and weakly informed ones, and examine how both groups optimally act to maximize their expected payoffs. It is possible that, by taking advantage of market's trusts in them, strongly informed traders make more profits than their superior information justifies.

The weakly informed trader has an incentive to follow the strongly informed one

[4] For the time being, we assume that arbitrageurs are a subset of informed traders.

because of the superior information of the latter. Also, the strongly informed trader has an incentive to cheat the weakly informed one if the former can enlarge his payoff by doing so. Over time both investors notice the other's incentive, and eventually a long run equilibrium is attained, where they both employ optimal mixed strategies. And, throughout this whole process, their payoffs and stock prices hinge on what they believe and how they acts. We also examine the case where the strongly informed trader has no more informational advantage. In addition, throughout the paper, we suggest interpretations and implications of many lemmas to be proven mathematically.

We organize the paper as follows. We present the motives and purposes of this study in Section 2, and explain the setup of the model in Section 3. Section 4 discusses informed traders' trading strategies, price movements, and the characteristics and market implications of the long run equilibrium. Section 5 concludes.

2. The Motivation

We have several motivations for this research. First, we want to explain how people's belief or reputation possibly influences market prices. For instance, suppose a very renowned (institutional) investor like Warren Buffet buys 1,000 shares of company A, and many investors, institutional or individual, follow suit. As a result, the share price rises from $100 to $150. Are the price increase of $50 and his investment profit of $50,000 purely attributable to his superior information on the intrinsic value of company A? Or are they a result from other investors' belief in him and following suit? Or are they a result from both? In this paper, we explore all of these possibilities; for example, the price increase and his profit might be attributable to people's belief in him as a super informed investor. This possibility has never been studied in depth, at least in the area of finance. As a matter of fact, we have hardly found any papers which take the issue of belief or reputation, and market prices as a main subject in finance.[5] In this sense, this paper might shed some light on the issue.

Second, we want to explain different investors' different behavior in a more realistic way. As in DeLong et al. (1990a, 1990b), most behavioral finance or market microstructure literature presumes that there are two kinds of investors in terms of information, i.e. informed and noise traders. But this dichotomy might be for the tractability of models, and in reality, there are investor groups in between, who are neither as irrational as noise traders nor as much informed as highly respected hedge funds. Also, in an emerging market like Korea, Taiwan or China, investors are often

[5] The only exception was Landon and Smith (1998), which found that the market price of Bordeaux wine was a function of wine maker's reputation.

classified into three categories: foreigners, local institutions, and local individuals. In light of this, we construct a model in such a way that there are three kinds of investors in a stock market, who are strongly informed, weakly informed and noise traders.

Third, we want to analyze investors' behavior better in an order-driven stock market such as many Asian and European stock exchanges. Apparently, many microstructure papers, including the seminal one by Kyle (1985), investigate market makers' optimal behavior in a quote-driven market. But, in order to analyze investors' behavior in many Asian and European order-driven markets without market makers, it would be more effective to specifically model an order-driven market as in this paper.

In order to build an acceptable model, we employ academic achievements of related fields, inside or outside the world of finance. For example, we solve for optimal strategies of both informed traders, using a game theoretic concept, Nash equilibrium or perfect Bayesian equilibrium because this concept could explain better two different informed investors' behavior in the presence of asymmetric information *and* asymmetric belief than does any other finance theory. We also use, explicitly or implicitly, concepts such as "information cascade" by Bikhchandani, Hirshleifer, and Welch (1992), Banerjee (1992) or Welch(1992), and "herding," which are (more) often found in economics literature (than in finance literature).[6] By doing these, we try to build a realistic model, which could serve the purposes of this paper aforementioned.

Last, as a matter of fact, a futures market might be more appropriate for this type of research than an equity market is. This is because a futures' "market equilibrium" price is always revealed at expiry. Trades are competing in terms of who can predict a futures' expiry price better, or in terms of information on a futures' expiry price. On the contrary, strictly speaking we do not know for sure when the fundamental value of a share of a stock is publicly revealed in the market. In this sense, this research covers not only spot markets but also futures markets with the latter being more eligible.

3. The Setup of the Model

3.1 Spot and futures markets

A futures' price is usually affected by the no arbitrage principle. In other words, a futures' price *usually* moves within a band where arbitrage using both futures and spot prices are impossible. This presumption, however, might become shaky when, in reality, carrying out arbitrage is not easy. First of all, in many equity markets, short sales are

[6] Information cascade indicates a phenomenon that people buy or sell securities following others since the former believe that the latter have superior information.

very limited or even impossible, so spot-futures arbitrage is seriously restricted. Second, for some futures contracts, conducting arbitrage is subject to risks. For example, replicating a stock index containing hundreds of stocks could be very risky due to tracking errors, market price impacts, liquidation risk, etc. In this circumstance, arbitrage might not be literally risk-free, and might not affect futures' prices greatly. Third, depending on how mature, developed or sophisticated futures markets or futures investors are, it is possible that not so many people are--or not so much money is--involved in arbitrage transactions, causing arbitrage to be less powerful in determining futures' prices. For instance, when a futures market is relatively recently introduced, when investors do not know arbitrage transactions much, when investors are mostly interested in speculative trading of futures, not in arbitrage transactions, then futures prices could be much more affected by speculative trading rather than by arbitrage transactions.

Another good reason for applying this research to a futures market is that a futures' "market equilibrium" price is realized with probability one at expiry. This is not always so when it comes to stock prices, although we assume in the paper that, at time T, the fundamental value becomes common knowledge. A futures' market could be even more eligible for this paper if we consider this point only. After all, for the reasons aforementioned, our model on equity markets could be extended to futures markets when arbitrage does not affect futures prices greatly. In this sense, although P_t in the paper denotes a stock price, it could mean a futures' price, too, as long as the aforementioned conditions are satisfied.

3.2 Information asymmetry

Following is the summary of the notations to be used. "Trader I" always means either "trader F" or "trader L."

① F, L; F and L mean a strongly and a weakly informed trader, respectively.
② P_t: share price of stock P at time t
③ θ: the true, liquidation value of a share of P
④ d: the size of price impact when trading one share of P
⑤ T: the final stage of stock investment (investment horizon)
⑥ s_F, s_I: signals about θ received by F and L, respectively
⑦ q_L: the precision of s_L
⑧ $x_{F,k}$, $x_{L,k}$: the trading position of F and L in his k'th trade, respectively
⑨ $\pi_{F,k}$, $\pi_{L,k}$: F's and L's (expected) payoff from his k'th trade, respectively

⑩ $P[\omega]$: the probability of even ω

In an order-driven stock market,[7] there are three types of risk neutral traders, who are a strongly informed trader (F), a weakly informed trader (L), and many noise traders.[8] We are interested in a particular stock named "P," and P_t denotes the share price of P at time t. The risk free rate is assumed to be zero for convenience.

Between time 0 and time 1, F and L receive s_F and s_L, respectively, which are their private signals about "θ," the true value of a share of P, and noise traders do not receive any signals. Either informed trader can receive his signal earlier than, later than, or at the same time as, the other does. The prior distribution of $\theta \in \{1, 2, 3, 4\} = \{\theta_1, \theta_2, \theta_3, \theta_4\}$ is common knowledge, and the nature chooses one (θ^*) from the set.[9] "$\theta = \theta^*$" becomes common knowledge at time T (>6) so $P_T = \theta^*$. At time 0, θ^* is expected to be equiprobably one of the four values, so the initial market price, P_0, is 2.5.

Assumption 1: The prior distribution of θ

① $\theta \in \{1, 2, 3, 4\} \equiv \{\theta_1, \theta_2, \theta_3, \theta_4\}$
② $P[\theta_i] = \frac{1}{4}$, where $i \in \{1, 2, 3, 4\}$
③ $P_0 = 2.5$
④ $P_T = \theta$, where $T > 6$.

The precisions of the two signals about θ^* are different. s_L correctly reveals whether θ^* is above or below P_0 (= 2.5), but reveals the exact value of θ^* with probability q_L ($>\frac{1}{2}$) only. For instance, given $\theta^* = 2$, s_L equals 2 with probability q_L and 1 with probability $1-q_L$ as in Figure 1.[10] On the contrary, s_F precisely reveals θ^*, so $s_F = \theta^*$. This assumption of θ^* as a perfect signal is made in an attempt to put an emphasis on F's informational advantage over L. The precisions of s_F and s_L are common knowledge between both traders, and Lemma 1 is derived in the Appendix (**Proof 1**).

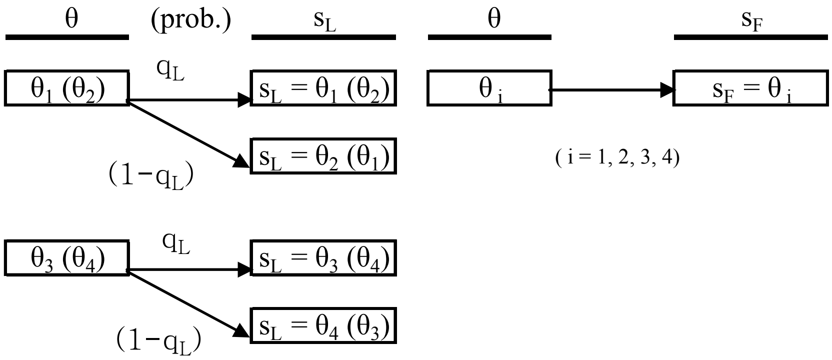
Assumption 2: The precisions of s_F and s_L

① $P[s_L = \theta_i | \theta_i] = q_L > \frac{1}{2}, \ \forall i$
② $P[s_L = \theta_2 | \theta_1] = P[s_L = \theta_1 | \theta_2] = P[s_L = \theta_4 | \theta_3] = P[s_L = \theta_3 | \theta_4] = 1-q_L$

[7] Again, although not explicitly expressed, a stock market in this paper could be replaced with a futures' market when the force of arbitrage is not strong in determining futures' prices.
[8] We assume that there are no market makers in this order-driven market.
[9] θ_k means $\theta = k$, where $k = \{1, 2, 3, 4\}$.
[10] This assumption of binary signals is popular as in Scharfstein and Stein (1990).

③ $P[s_F = \theta_i | \theta_i] = 1, \ \forall i$

Figure 1 States and signals



Lemma 1 Posterior probabilities and expected payoffs given s_L

① $P[\theta_i | s_L = \theta_i] \quad = q_L > \frac{1}{2}$

② $P[\theta_2 | s_L = \theta_1] \quad = P[\theta_1 | s_L = \theta_2] = P[\theta_4 | s_L = \theta_3] = P[\theta_3 | s_L = \theta_4] = 1 - q_L$

③ $E[P_T | s_L = \theta_1] = 2 - q_L$

④ $E[P_T | s_L = \theta_2] = 1 + q_L$

⑤ $E[P_T | s_L = \theta_3] = 4 - q_L$

⑥ $E[P_T | s_L = \theta_4] = 3 + q_L$

Whether F and L move sequentially or simultaneously, they can identify the other's trading position after the trading. By identifying the other's order flow, either trader can update his belief in the other's signal. Noise traders trade for idiosyncratic reasons, and are not affected by others' actions unless otherwise specified. Ex ante, their trades average out, and, as a whole, do not affect P_t unless otherwise specified.

Short sales are ruled out, and initially F and L have two shares of P.[11] At a trading round, F and L either trade or hold a share based s_F and s_L. They trade mostly at time 1, 2, 3, or 4. In our model, the exact value of T does not matter as long as T >6. $x_{F,k}$ ($x_{L,k}$) denotes the k'th trading position of F (L) so $x_{F,k}$ ($x_{L,k}$) $\in \{-1, 0, 1\}$, where −1, 1 and 0 mean selling, buying and holding a share, respectively. Exogenous sequencing models usually assume that the more informed trader, F here, moves first, which might be endogenously derived as in Zhang (1997). However, in reality, anyone could move earlier when he receives information earlier even if it is not very accurate. Therefore we let either F or L move earlier than, or at the same time as, the other does, based on

[11] Short sales are usually not permitted or very limited, if any, in real world.

how early he obtains his signal.

At any given price P_t, the excess demand for the shares by noise traders is expected to be 0 as assumed earlier. Accordingly, if an informed trader enters a buy (sell) order given P_t, the excess demand for the share is 1 (−1), and the market cannot clear at P_t; it can only clear at a higher (lower) price than P_t. We assume that he can buy (sell) a share at $P_t + d$ ($P_t - d$) or the price impact is d.[12] By the same token, if F and L simultaneously enter a buy (sell) order given P_t, the market-clearing share price will be $P_t + 2d$ ($P_t - 2d$). Meanwhile, the range of the two parameters, q_L and d, will be determined in such a way that, given $P_0 = 2.5$, either trader will optimally sell (buy) two shares when he thinks $\theta^* = 1$ (4), and one share when he thinks $\theta^* = 2$ (3). That is, optimally they sell one share (two shares) if they receive a weakly (strongly) bad signal, and buy one share (two shares) if a weakly (strongly) good signal. Finally, at T, θ^* becomes common knowledge and $P_T = \theta^*$.

3.3 Herding

Noise traders only know $\theta \in \{1, 2, 3, 4\}$, trade for personal, idiosyncratic reasons, and, as a whole, do not affect prices. But they do affect prices when they *herd*. Then when do they herd or when should we assume they herd? We set two conditions for herding. The first one is that they herd when P_t hits a critical threshold, which is to be announced. We set this condition as, in reality, stock prices do not move evenly when they plummet or skyrocket. For instance, there usually is a firing or burning point for stock prices to soar. A share price or a stock index hits up a critical point (or multiple points), and then most investors become belatedly over-confident and buy shares up and up. This phenomenon is probably more prominent for a stock than for a stock index like the Dow. Similarly, when a share price plummets, it usually does not fall proportionately but, while falling, it hits a critical point down and the falling process is accelerated. People might sell off shares for a loss-cut or out of panic when a share price hits a certain threshold down.

The second condition for herding is that the critical point should not be very close to the current price, $P_0 = 2.5$. If it were close to P_0, noise traders would herd too often, and the price would overshoot too frequently. Now, taking the two conditions into consideration, we set the critical threshold as $P_t = 2.5-4d$ ($2.5+4d$). In other words, when P_t hits $2.5-4d$ ($2.5+4d$), noise traders start selling off (buying up) shares out of panic (out of euphoria), and P_{t+1} will plummet to 1 (jump to 4), and stay at that level until

[12] Since a share in the model could amount to tens of thousands of shares in reality, d is not necessarily close to 0.

T. This is because, once all noise traders believe in $\theta^* = 1$ (4), each of them will be willing to sell (buy) a share at any price greater (less) than θ^*. As a result, the price impact from trading a share then will be negligible or just ε, the infinitesimal. This herding is common knowledge between both informed traders, but not for individual noise traders.

Assumption 3: The cause and effects of noise traders' herding

① The cause: $P_k = 2.5-4d$ $(2.5+4d)$.
② Effect 1: $P_{k+1} = 1$ $(P_{k+1} = 4)$.
③ Effect 2: $1 \le P_n \le 1+\varepsilon$ $(4-\varepsilon \le P_n \le 4)$, where $n = k+2, \ldots, T-1$

Another reason, not essential though, for picking 2.5-4d is that, noise traders often herd after they belatedly find that informed traders have kept selling or buying shares. In the model, 2.5-4d (2.5+4d) is the level of price to be realized only when F and L cumulatively net-sell (buy) fours shares, and is a good candidate as a critical threshold for the herding of noise traders.

3.4 Parameters

We like the model to have the feature that, when F or L receives a weakly (strongly) bad signal, he optimally sells one share (two shares), and when receiving a weakly (strongly) good signal, he optimally buys one share (two). And this should hold for F and L whether they move simultaneously or sequentially. To this end, we determine the ranges of their values as follows, which are common knowledge between F and L.

Assumption 4: The ranges of parameters

① $\frac{1}{6} < d < \frac{1}{4}$

② $1.5-3d < q_L < 1$

Suppose F receives a bad signal, i.e., $s_F = \theta^* = 1$ or 2. This automatically means that L also receives a bad signal, i.e., $s_L = 1$ or 2 by Assumption 2. In order for F to optimally sell one share given $\theta^* = 2$, it must hold that the least he gets from selling his first share is greater than what he gets by holding it till T. Since the former is 2.5-2d (assuming he sells *after* L) and the latter is $2 = E[P_T | \theta^*=2]$, we have $2<2.5-2d$. Furthermore, for him to sell only one share, not two, given $\theta^* = 2$, the most he gets from selling his second share must be less than what he gets by holding it till T. Since the former is 2.5−3d (assuming he sells *before* L) and the latter is 2, we have 2.5−3d<2. Similarly, for him to sell his second share, too, given $\theta^* = 1$, 1 should be less than the least he gets by selling it, which is 2.5−4d assuming he sells later than L. After all, we have

$$2.5-3d < 2 < 2.5-2d, \text{ and } 1 < 2.5-4d.$$

Solving these two simultaneous inequalities leads to ①. Similarly, for L, it must be true that $2.5-3d < E[P_T | s_L=2] = 1+q_L < 2.5-2d$ and $E[P_T | s_L=1] = 2-q_L < 2.5-4d$. This leads to ②, and ① and ② stay valid for good signals like s_F (s_L) = 3 or 4. The derivation of ① and ② are provided in the Appendix (**Proof 2**).

4. Trading in the presence of asymmetric information

In this section, we examine informed traders' trading when they trade either sequentially or simultaneously. We analyze this assuming $\theta^* \in \{1, 2\}$ since $\theta^* \in \{3, 4\}$ can be analyzed similarly and symmetrically. Particularly, we focus on the optimal strategy of either trader given the other's trading information, their learning process about the other's changing strategy, and the characteristics and market implications of the long run equilibrium.

4.1 Simultaneous trading

Suppose $\theta^* \in \{1, 2\}$ or equivalently s_F (s_L) $\in \{1, 2\}$ by Assumption 2, and F and L receive their bad signals at the same time. Neither F nor L will delay selling his first share because there is a waiting cost, the price impact. If the other sells earlier than he, he has to sell a share later at a more unfavorable price by "d." So F and L place a (limit) sell order as early as possible or simultaneously. Then the excess demand for the stock is two shares, and the market clearing price will be $P_1 = 2.5-2d$. Now further suppose $\{s_F, s_L\} = \{2, 2\}$. Then neither will place another sell order at 2.5−3d or 2.5−4d by Assumption 4. So the share price will stay at 2.5−2d till time T. What if $\{s_F, s_L\} = \{1, 2\}$? Then only F will place a sell order, which will be executed at 2.5−3d, and this price will last until T.

Now if $\{s_F, s_L\} = \{1, 1\}$, F and L will place a sell order simultaneously, and as a result, $P_2 = 2.5-4d$, the herding threshold. Then noise traders will herd and $P_3 = 1$. Also, if $\{s_F, s_L\} = \{2, 1\}$, F will hold but L will *wrongly* sell a share at 2.5−3d. The share price will stay at that level until T[13]. Lastly, if $\{s_F, s_L\} = \{1, 2\}$, L will hold and F will sell a share at 2.5−3d, which will stay at that level until T.[14]

Since either trader will always sell his first share simultaneously given $\theta^* \in \{1, 2\}$, the payoff from selling it for F or L is always 2.5−2d, i.e., $\pi_{F,1} = \pi_{L,1} = 2.5-2d$. Since this always holds true, their first trading will not be examined further in the paper. Now what about the expected payoff from trading the second shares ($x_{F,2}, x_{L,2} \in \{-1,0\}$)? It turns out that when F and L act on their signals only, the expected payoffs for them are as in Lemma 2 (**Proof 3**). As a result, the differential performance between F and L is $\frac{1}{2}(1-q_L)$, which comes from the informational advantage for F over L since $E[\pi_{F,2}] = E[\pi_{L,2}]$ if $q_L = 1 = q_F$.

Lemma 2 Investment Performances (when F and L act sincerely)

① $E[\pi_{F,2}] = \frac{1}{2}(4.5 - 3d - d \cdot q_L)$

② $E[\pi_{L,2}] = \frac{1}{2}\{3.5 - 3d + q_L(1-d)\}$

③ $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}(1-q_L) > 0$

Now will L be better off if he waits to see how F moves? Since L knows that F has a perfect signal, L has an incentive to follow F for an information externality. On the other hand, there will be the waiting cost, d, by waiting. Altogether, it turns out that he is better off by $\frac{1}{2}(1-q_L)(1-d)$ when following F as in Lemma 3. As for F, he is also better off by $\frac{1}{2}d \cdot q_L$ since he is not any more vulnerable to the disadvantageous price impact "d" when F and L sell shares simultaneously with probability $P[s_F = 1 \text{ and } s_L = 1] = P[\theta^* = 1 \text{ and } s_L = 1] = P[\theta^* = 1] \times P[s_L = 1 | \theta^* = 1] = \frac{1}{2}q_L$. In that both F and L can be better off when L follows F, Lemma 2 can be neither an (Pareto) optimal nor an efficient equilibrium. Finally, the differential performance now is larger ($\frac{1}{2}d$) than before ($\frac{1}{2}(1-q_L)$). The proof of Lemma 3 is in the Appendix (**Proof 4**).

Lemma 3 Investment Performances (when L follows F or $x_{L,2} = x_{F,2}$)

① $E[\pi_{F,2}] = \frac{1}{2}(4.5 - 3d)$

② $E[\pi_{L,2}] = \frac{1}{2}(4.5 - 4d)$

③ $\Delta E[\pi_{F,2}] = \frac{1}{2}d \cdot q_L$

④ $\Delta E[\pi_{L,2}] = \frac{1}{2}(1-q_L)(1-d)$

⑤ $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}d > \frac{1}{2}(1-q_L)$

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[13] F will not buy it at $2.5-3d+d = 2.5-2d > 2 = \theta^*$.

[14] L will not buy it at $2.5-3d+d = 2.5-2d > 1+q_L = E[P_T | s_L=2]$.

As a result, the time series of the share price given $\theta^* = 1$ is, $\{P_1, P_2, P_3, P_4\} = \{2.5-2d, 2.5-3d, 2.5-4d, 1 = \theta^*\}$.[15] That is, the price reflects its fundamental value as early as t = 4. If L does not follow F given $\theta^* = 1$, it will take at least as long and sometimes longer, depending on s_L; if $s_L = 2$ with probability $(1-q_L)$, the price will stay at 2.5−3d until T. It turns out that, when L follows F acting sincerely, L gets the most, and the price converges to its intrinsic value as early as possible.

Now, will L be even better off by *selectively* following F than by blindly following F as in Lemma 3? It turns out that selectively following F makes L worse off than does blindly following F (**Proof 5** in the Appendix).

4.2 Sequential trading

Suppose that F receives s_F earlier than L receives s_L, and places his first sell order earlier than L does. Then F and L will sell a share at $2.5-d = P_1$ and at $2.5-2d = P_2$, respectively. Similarly, if L receives his signal earlier, L and F will sell a share at $2.5-d = P_1$ and at $2.5-2d = P_2$, respectively. But, for t>2, i.e., from their second trading, how early F and L received their signals does not matter since, before time 2, both traders will have their signals anyway. What matters then is how accurate their signals are. L, the weakly informed trader, is better off by waiting to follow F, the strongly informed trader, as proven in the previous section. In this sense, it is of no use to blindly distinguish between simultaneous and sequential trading after time 2; F and L will trade earlier, later, or at the same time based on whether he will be better off by doing so. It turns out that F never delays his trading since he wants to sell at a higher price before his information is reflected on P_t with probability q_L (since $s_F = s_L$ with probability q_L). Meanwhile, L is always better off by waiting to see F's move. If L has a wrong signal, then L is always better off by doing so because of F's superior information. Even if L has a right signal (with probability q_L), he is not better off by acting (as) early (as F) since then both F and L will sell a share at 2.5−4d if $\theta^* = 1$ (and hold if $\theta^* = 2$). That is, given $s_L = s_F = 1$, L will never be able to sell a share at 2.5−3d even if he acts early. In short, by waiting to see F's move, L gains something with probability $1-q_L$ (when he is wrong) and loses nothing with probability q_L (right). Consequently L optimally waits to see F's move.

[15] When $\theta^* = 2$, P_t does not equal θ^* until T. This is because the price movement by d happens not to match 2 exactly.

4.3 Cheating

If L continues to copy F's trading for the payoff externality as in Lemma 3, F notices L copying his trading, and does not act always sincerely. Rather, he can cheat L if he can achieve an extra payoff from cheating. Given the assumptions and lemmas in the previous sections, the only way for F to get an extra payoff by cheating is to manipulate P_t, taking advantage of both L's trust in him and noise traders' herding behavior. More specifically, this means that, given $\theta^* = s_F = 2$, F takes $x_{F,2} = -1$ at 2.5−3d, pretending that he has "$s_F = 1$." When L follows by taking $x_{L,2} = -1$, P_t will reach 2.5−4d, and noise traders will herd to sell, causing P_{t+1} to plummet to 1. And, as all noise traders believe in $\theta^* = 1$, each of them will be willing to sell a share for any price greater than 1, such as $1+\varepsilon$. F enters here and buys back a share at $1+\varepsilon$ at, say, time T−1. And $P_T = \theta^* = 2$ will be realized at T, giving F a total payoff of $(2.5-3d)-(1+\varepsilon)+2 = 3.5-3d-\varepsilon$, which is higher than 2, the expected payoff he will get by acting on s_F to hold his second share till T.[16]

If F does this indeed, F and L's performances will be changed into Lemma 4. F's and L's expected payoffs get larger from $\frac{1}{2}(4.5–3d)$ to $\frac{1}{2}(6 – 6d -\varepsilon)$ and smaller from $\frac{1}{2}(4.5 – 4d)$ to $\frac{1}{2}(5 – 8d)$, respectively, than in Lemma 3, where L follows F (**Proof 6**).

Lemma 4 Investment Performances (when F cheats L)

$\pi_{F,2}$ here involves means F's selling and buying his second share.

① $E[\pi_{F,2}] = \frac{1}{2}(6 - 6d -\varepsilon) > \frac{1}{2}(4.5 - 3d)$
② $E[\pi_{L,2}] = 2.5-4d = \frac{1}{2}(5 - 8d) < \frac{1}{2}(4.5 - 4d)$
③ $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}(1+2d-\varepsilon) > \frac{1}{2}d$

4.4 Distrust

F can cheat L successfully once or so but not repeatedly because eventually L will notice F's cheating. Once L learns that F does not always act on s_F but cheat sometiems, L has no reason to blindly follow F. Eventually L can infer that F is not cheating when holding a share, i.e., when $x_{F,2} = 0$ (given $\theta^* = 1$). But when F sells a share ($x_{F,2} = -1$), he might or might not be cheating. Therefore, given $x_{F,2} = -1$, L has to decide whether to follow F blindly, to distrust him blindly, or to follow (distrust) him selectively, say,

[16] F has no incentives to cheat when $\theta^* = 1$. If he cheats by holding, the price will stay at 2.5−2d and, at T, will give him 1, which is less than 2.5−3d he will get by selling.

based on his own information, s_L. It turns out that L optimally relies back on s_L when he sees $x_{F,2} = -1$, and the resulting payoffs of F and L are in Lemma 5 (**Proof 7**).

Lemma 5 Investment Performances (when L distrusts F, who cheats)

① $E[\pi_{F,2}] = \frac{1}{2}\{6-6d-q_L-\varepsilon(1-q_L)\} < \frac{1}{2}(4.5 - 3d)$
② $E[\pi_{L,2}] = \frac{1}{2}(3.5-4d+q_L) > \frac{1}{2}(5 - 8d)$
③ $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}\{1+2d-q_L-\varepsilon(1-q_L)\} < \frac{1}{2}(1+2d-\varepsilon)$

4.5 The Long Run Equilibrium

F and L have different problems: given $\theta^* = 2 = \theta_2$, F has to decide whether to cheat L or not, and, given $x_{F,2} = -1$ and $s_L = 2$, L should decide whether to trust F or not. Their expected payoffs from different choices are provided in the cells in Tables 1 and 2 (**Proof 8**). The arrows show either trader's incentives to deviate given the other trader's strategy. For example, given that L trusts F and $\theta^* = 2$, F prefers to cheat since $3.5-3d-\varepsilon > 2$.

Table 1. Expected payoffs for F when $\theta^* = 2$: $E[\pi_{F,2} \mid \theta_2]$

The value in each cell shows F's expected payoff given $\theta^* = 2$, which depends on the strategy profile of F and L. L's trust (strategy) means $x_{L,2} = x_{F,2}$, and his distrust means $x_{L,2} = -1$ (0) if $s_L = 1$ (2). F's cheating strategy means $x_{F,2} = -1$ whether $s_F = 1$ or 2, while his sincere strategy means $x_{F,2} = -1$ (0) if $s_F = 1$ (2).

		L	
		Trust	Distrust
F	Act sincerely ($x_{F,2} = 0$)	2 ↓	2
	Cheat ($x_{F,2} = -1$)	$3.5 - 3d - \varepsilon$	$3.5-3d-q_L-\varepsilon(1-q_L)$ ↑

Suppose that initially F acts sincerely and L trusts him. Then if $s_F = 2$, both F and L will hold to earn 2 at T. But this is not optimal for F since he can deviate to the cheating strategy to earn $3.5-3d-\varepsilon>2$. If F repeatedly does this, L will notice it and deviate to his distrust strategy to earn $(1+q_L)$ ($>2.5-4d$) when he sees $s_L = 2$ and $x_{F,2} = -1$. Then F's cheating goes futile and his expected payoff will fall to $3.5-3d-q_L-\varepsilon(1-q_L)$, so he will return to his sincere strategy to earn 2 ($>3.5-3d-q_L-\varepsilon(1-q_L)$). Then $x_{F,2} = -1$ reveals truthfully $\theta^* = 1$ again, and L will follow F to earn $2.5-4d$, rather than follow $s_L = 2$ to earn 1. Then, again, F will have an incentive to cheat L and so on.

Table 2. Expected payoffs for L: $E[\pi_{L,2} \mid s_L = 2, x_{F,2} = -1]$

The value in each cell shows L's expected payoff given $s_L = 2$ and $x_{F,2} = -1$, which depends on the strategy profile of F and L.

		L	
		Trust ($x_{L,2} = -1$)	Distrust ($x_{L,2} = 0$)
F	Act sincerely	2.5−4d	← 1
	Cheat	2.5−4d →	1+ q_L

This process of selecting the best response to each other's strategy will eventually lead to a long-run equilibrium, where both traders have no further incentives to deviate. More specifically, in equilibrium, given $\theta^* = 2$, F cheats not always but with probability α, and, given $s_L = 2$ and $x_{F,3} = -1$, L trusts F not always but with probability β. Table 3 summarizes θ^*, α, β and the investment performances of F and L.[17]

Table 3. θ^*, α, β and investors' performances

"Prob." means probability, and Δ denotes $E[\pi_{F,2}] - E[\pi_{L,2}]$, the differential performance between F and L.

Prob.	θ^*	Prob.	$x_{F,2}$	Prob.	s_L	Prob.	$x_{L,2}$	$E[\pi_{F,2}]$	$E[\pi_{L,2}]$	Δ
0.5	1	1	−1	q_L	1	1	−1	2.5−3d	2.5−4d	d
				$1 - q_L$	2	β	−1	2.5−3d	2.5−4d	
						$1 - \beta$	0	2.5−3d	1	1.5−3d
0.5	2	α	−1	$1 - q_L$	1	1	−1	3.5 − 3d − ε	2.5−4d	1 + d − ε
				q_L	2	β	−1	3.5 − 3d − ε	2.5−4d	
						$1 - \beta$	0	2.5−3d	2	1.5−3d
		$1 - \alpha$	0	$1 - q_L$	1	1	0	2	2	0
				q_L	2					

The values of $\{\alpha, \beta\}$ will be determined in such a way that, at $\{\alpha, \beta\}$, neither trader has any incentives to deviate since, at $\{\alpha, \beta\}$, both are maximizing their expected payoffs, or they cannot increase them any more by switching to any other strategy. Lemma 6 shows the closed form solutions for α and β, and their ranges (Proof 9).

[17] Recall that $x_{F,2} = 0$ is a sufficient condition that $\theta^* = 2$, so L always holds to earn 2 at T.

Lemma 6 α^* and β^*

α^* (β^*) is the optimal level of probability, with which F cheats given $\theta^* = 2$ (L trusts F given $s_L = 2$ and $x_{F,2} = -1$).

$$\alpha^* = \frac{(3-8d)(1-q_L)}{(8d-1)q_L}, \quad \beta^* = 1 - \frac{3-6d}{2q_L}$$
$$where \ \ 0 < \alpha^* < \frac{1}{3}, \ \ 0 < \beta^* < \frac{1}{4}$$

The Proposition below summarizes F and L's optimal trading strategies in equilibrium given $\theta^* \in \{1, 2\}$.

The Proposition: The trading strategies of F and L in equilibrium

$P(x_{F,2} = -1 | \theta^* = 1) = 1$,
$P(x_{F,2} = -1 | \theta^* = 2) = \alpha^*$,
$P(x_{F,2} = 0 | \theta^* = 2) = 1 - \alpha^*$,
$P(x_{L,2} = -1 | s_L = 1, x_{F,2} = -1) = 1$,
$P(x_{L,2} = -1 | s_L = 2, x_{F,2} = -1) = \beta^*$,
$P(x_{L,2} = 0 | s_L = 2, x_{F,2} = -1) = 1 - \beta^*$, and
$P(x_{L,4} = 0 | s_L = 1, x_{F,2} = 0) = P(x_{L,4} = 0 | s_L = 2, x_{F,2} = 0) = 1$,

And the comparative statics of α^* and β^* show

$$\frac{\partial \alpha^*}{\partial q_L} = \frac{8d-3}{(8d-1)q_L^{\,2}} < 0 \qquad (1)$$

$$\frac{\partial \alpha^*}{\partial d} = \frac{-8(1-q_L)[q_L(8d-1)+3-8d]}{[(8d-1)q_L]^2} < 0 \qquad (2)$$

$$\frac{\partial \beta^*}{\partial q_L} = \frac{3-6d}{2q_L^{\,2}} > 0 \qquad (3)$$

$$\frac{\partial \beta^*}{\partial d} = \frac{3}{q_L} > 0 \,. \qquad (4)$$

All these results make sense intuitively. The higher the precision of L's information is, the more often F's cheating ends unsuccessfully. So he cheats less often ((1)) and L trusts him more often than otherwise ((3)). Similarly, the higher the price impact goes, the higher cost of cheating F has to incur (since he has to trade twice more). So he cheats less frequently ((2)) and L trusts him more frequently than elsewhere ((4)).

4.6 Investor Performances

Given q_L and d, F and L's investment performances are determined by their investment strategies (α and β). Table 4 shows their performances in each of various combinations of α and β, including {α^*, β^*}, where α = 0 (1) indicates F's sincere (cheating) strategy, and β = 0 (1) means L's distrust (trust) strategy. Initially, we assumed α = 0 and β = 1, where F and L's expected payoffs are $\frac{1}{2}$(4.5−3d) and $\frac{1}{2}$(4.5−4d), respectively (**Lemma 3**). In equilibrium, those are $\frac{1}{2}$(4.5−3d) and $\frac{1}{2}${5−8d + (1−α^*)(4d−0.5)}, respectively. Therefore, F's expected payoff has not changed but L has lost part of his payoff as $\frac{1}{2}${5−8d + (1−α^*)(4d−0.5)} < $\frac{1}{2}$(4.5−4d).[18] Table 4 summarizes F and L's expected payoffs in each of their strategy profiles, where the ranks mean the relatives sizes of the expected payoffs (with ① being the highest one) by Assumption 4 (**Proof 10**).

Table 4. Strategies and performances

This table shows various combinations of {α, β}, F and L's expected payoffs given a specific {α, β}, and the relative sizes of F's or L's payoffs in an descending order.

α =	β =	$E[\pi_{F,2} \| \alpha, \beta]$ =	rank	$E[\pi_{L,2} \| \alpha, \beta]$ =	rank
0	1	$\frac{1}{2}$(4.5−3d)	②	$\frac{1}{2}$(4.5−4d)	①
0	0	$\frac{1}{2}$(4.5−3d)	②	$\frac{1}{2}${3+ q_L(1.5−4d)}	②
1	1	$\frac{1}{2}$(6−6d)	①	$\frac{1}{2}$(5−8d)	④
1	0	$\frac{1}{2}$(6−q_L−6d)	④	$\frac{1}{2}$(3.5+ q_L−4d)	③
α^*	β^*	$\frac{1}{2}$(4.5−3d) = ②		③ < $\frac{1}{2}${5−8d + (1−α^*)(4d−0.5)} < ②	

4.7 Market Implications

What could be the implications of this investigation for stock markets in the presence of asymmetric information? First, more informed investors might take advantage of their

[18] $\frac{1}{2}${5−8d + (1−α^*)(4d−0.5)} is decreasing in α^*, and equals $\frac{1}{2}$(4.5−4d) when α^* = 0 or q_L = 1.

informational edge for an extra profit. This automatically inflicts damage on the investment performances of less informed investors as, in the model, L earns less in the long-run equilibrium than he does when F trades sincerely. This negative effect is the most pronounced when $\{\alpha, \beta\} = \{1,1\}$, where F and L achieve the maximum and minimum performance, respectively. Besides, in equilibrium, F gets as much as at $\{\alpha, \beta\} = \{0, 1\}$, which is effectively the starting point of the game, whereas L gets less than at {0, 1} as in Table 4.

Second, as more informed investors try to mislead less informed investors and noise traders, their private information is not revealed in the market as quickly as otherwise; rather the prices are often manipulated and kept away from their fundamental values for an extended period. When F acts sincerely (and L follows him) given θ_1, the path of P_t is such that $\{P_3, P_4, P_5\} = \{2.5-3d, 2.5-4d, 1\}$ with probability one. So the price converges to θ_1 by time 5. On the contrary, when, F cheats L, $\{P_3, P_4, P_5\} = \{2.5-3d, 2.5-4d, 1\}$ with probability β^*, and $\{P_3, P_4, P_5, P_6, \cdots, P_{T-1}, P_T\} = \{2.5-3d, 2.5-3d, 2.5-3d, 2.5-3d, \cdots, 2.5-3d, 1\}$ with probability $(1-\beta^*)$. In the latter's case, the price will never converge to θ_1 by the very last time T. In other words, stock markets will stay informationally inefficient for a longer period.

Third, in reality, α and β might be a little or far away from α^* and β^*. For example, if more informed investors do *not* cheat less informed investors *often*, the latter might not notice the cheating and β might be close to 1. For example, if $\alpha = 0.1$ and $\beta = 1$, the expected payoffs for F and L will be larger and smaller, respectively, than in equilibrium. Then the differential performance between the two will be expanded.

Our analysis could be applied to emerging markets like Korea, Taiwan, or China, where foreign investors are often regarded as more informed than local institutions or local individual since they have more experiences, more money, better know-how to value stocks, better global networks, etc. As a matter of fact, when they began to invest in these markets for the first time, they successfully established a good reputation as more informed investors because they showed fundamentals-oriented, scientific investment behavior and often outperformed markets, where local investors were less sophisticated than in developed markets. And then the Asian financial crisis in 1997 broke out *beyond* the expectations from Asians and *according to* the expectations from foreigners. As a result, foreigners' reputation and locals' belief in them soared. Given this, foreigners might have incentives to cheat locals or manipulate market prices to maximize their profits whenever they could. If they did it successfully indeed, it might take long before locals fully understand this sophisticated behavior of foreigners, or equivalently, locals might have to suffer unfairly bad performances for long before α and β converge to α^* and β^*.

4.8 Trading in the Absence of Information Asymmetry

Here, for reference, let us consider the case where F does not have any informational advantage over L. More specifically, let us assume that so far F has been trading sincerely based on his superior information and L has been following him. That is, we simply assume that **Lemma 3** holds. Now we additionally assume that F does not have his informational edge any more (for whatever reasons), and the accuracy of s_F now is exactly the same as that of s_L as in **Assumption 2-1**,[19] but L does not know this yet. Then how will F and L optimally trade?

Assumption 2-1: The quality of s_F and s_L

① $P[s_F = \theta_i | \theta_i] = P[s_L = \theta_i | \theta_i] = q_L$, $i \in \{1, 2, 3, 4\}$
② $P[s_L = \theta_2 | \theta_1] = P[s_L = \theta_1 | \theta_2] = P[s_L = \theta_4 | \theta_3] = P[s_L = \theta_3 | \theta_4] = 1-q_L$
③ $P[s_F = \theta_2 | \theta_1] = P[s_F = \theta_1 | \theta_2] = P[s_F = \theta_4 | \theta_3] = P[s_F = \theta_3 | \theta_4] = 1-q_L$

In Section 4.1, we learn that, given $\theta^* \in \{\theta_1, \theta_2\}$ or $s_L \in \{1, 2\}$, L will optimally sell his first share as soon as he receives s_L. Now, F's information is no better than L's, and F is exactly in the same situation as L is, although L does not know this. So F will optimally sell his first share immediately he receives s_F just as L does. Consequently, given $\theta^* \in \{\theta_1, \theta_2\}$ or s_F (s_L) $\in \{1, 2\}$, both traders will optimally sell their first shares as in Section 4.1.

Now what will they do to their second shares? Suppose F continues to trade sincerely. Then L will trade after F since L still believes that s_F is perfect. And their expected payoffs are as in **Lemma 7** (**Proof 11**), where the differential performance is $\frac{1}{2}d$ as in **Lemma 3**.

Lemma 7 Investment Performances (when L follows F or $x_{L,2} = x_{F,2}$)

① $E[\pi_{F,2}] = \frac{1}{2}(3.5+ q_L-3d)$
② $E[\pi_{L,2}] = \frac{1}{2}(3.5+ q_L-4d)$
③ $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}d$

In this circumstance, F has an incentive to cheat L as long as L believes that F still

[19] **Lemma 1** should be also changed accordingly; for example, $P[\theta_1 | s_F = \theta_1] = q_L$, $P[\theta_2 | s_F = \theta_1] = 1-q_L$ 이고, $E[P_T | s_F = \theta_1] = 2-q_L$, $E[P_T | s_F = \theta_2] = 1+ q_L$.

receives the perfect information. For example, given $s_F = 2$, he will get $E[P_T | s_F = 2] = 1+q_L$ if he acts on s_F. But if he sells his second share at 2.5−3d against s_F, L will sell his, too, at 2.5−4d, which will cause noise traders sell off theirs and the price will plummet to 1. If F buys back one share at $(1+\varepsilon)$ and holds till T, his total expected payoff will be $(2.5-3d)-(1+\varepsilon)+(1+q_L)$, and his extra payoff will be $(2.5-3d)-(1+\varepsilon) = 1.5-3d-\varepsilon>0$. Furthermore, even if $s_L = 1$, F can get an extra payoff by doing exactly the same thing. That is, F will sell his second share at 2.5−3d since $E[P_T | s_L = 1] = 2-q_L<2.5-3d$. Then L and noise traders will also sell their shares and the price will plunge to 1. If $s_F = 1 = \theta^*$ indeed, F would have no incentive to buy back a share at $(1+\varepsilon)$. But, now, his information, s_F, is imperfect and $E[P_T | s_F = 1] = 2-q_L>1+\varepsilon$. Consequently, he is better off by buying back a share at $(1+\varepsilon)$ now and earn the expected payoff of $2-q_L$ at T. Therefore he will optimally sell his second share even when he gets $s_F = 1$, and in this circumstance, their expected payoffs are as in Lemma 8 (Proof 12).[20]

Lemma 8 Investment Performances (when F cheats L)

① $E[\pi_{F,2}] = \frac{1}{2}(6-6d)$
② $E[\pi_{L,2}] = \frac{1}{2}(5-8d)$
③ $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}(1+2d)$

Over time, from repeated trading, L will realize that F neither has the perfect information nor acts sincerely, and eventually, he will be able to infer that the quality of s_F is as good as that of s_L. Given this, L has no reason to follow F, which will lead to Lemma 9 (Proof 13).

Lemma 9 Investment Performances (when L distrusts F, who cheats)

① $E[\pi_{F,2}] = \frac{1}{2}(6-q_L-6d)$
② $E[\pi_{L,2}] = \frac{1}{2}(3.5+q_L-4d)$
③ $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}(2.5-2q_L-2d)$

As a result, the expected payoffs for F and L are, respectively, less than and greater than when F cheats successfully (Proof 13). And L's expected payoff is as high as in Lemma 7, where it hinges entirely on the quality of s_F since L blindly follows F. Similarly, in Lemma 9, it depends critically on the quality of s_L since this time he acts on s_L. But the quality of s_L happens to equal that of s_F, so his expected payoff in either

[20] ε, the infinitesimal, is ignored since, effectively, it is like zero.

case is the same.

As for F' expected payoff, it could be greater or less than in **Lemma 7**. If q_L, the quality of s_L, is high enough for L not to fall prey to F's cheating or if d, a cost to F's cheating, is high enough, then F is worse off by cheating than by trading sincerely. Intuitively, this makes sense since q_L also indicates the quality of s_F since $q_L = q_F$. In other words, a high q_L implies a high q_F, so F is better off by acting honestly on s_F given a high q_L.[21]

Finally, in equilibrium, F and L trade neither later than, nor earlier than, the other but simultaneously since they are in exactly the same situations. If F is better off by trading earlier or later than L, so is L. Therefore in equilibrium, they will move simultaneously,[22] and their expected payoffs in equilibrium are as in **Lemma 10** (**Proof 14**).

Lemma 10 Investment Performances in Equilibrium

① $E[\pi_{F,2}] = \frac{1}{2}\{3.5+ q_L-4d+ 2q_L(1-q_L)d\}$
② $E[\pi_{L,2}] = \frac{1}{2}\{3.5+ q_L-4d+ 2q_L(1-q_L)d\}$
③ $E[\pi_{F,2}] - E[\pi_{L,2}] = 0$

In equilibrium, L's expected payoff is greater, by $\frac{1}{2}\times 2q_L(1-q_L)d = q_L(1-q_L)d$, than $\frac{1}{2}\{3.5+ q_L-4d\}$, the expected payoff when he acts on s_L or follows F who acts on s_F. The equilibrium expected payoffs for F and L match when the qualities of their signals equal, unlike those in the presence of information asymmetry.

5. Concluding Remarks

Securities investors with less information have an incentive to follow those with more information for information or payoff externality. Also noise traders, who trade securities for idiosyncratic reasons, tend to herd when they see stock prices soar or plunge quickly. Then more informed investors could take advantage of less informed investors' belief in them and noise traders' herd behavior to obtain an extra payoff. As a result, more and less informed investors achieve more and lee than they deserve,

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[21] The expected payoffs from the two strategies of F are the same when $6-q_L-6d = 3.5+ q_L-3d$, where the right hand side is the expected payoff when F trades sincerely. It will be easily shown that the higher q_L or d gets, the right hand side gets increasingly bigger than the left hand side. More specifically, the two sides of the equation equal only when $2.5 = 2q_L + 3d$. If $2.5 < 2q_L+ 3d$, then the right hand side is greater than the left hand side. For example, q_L and d are relatively high such as $q_L = 0.95$, and $d = 0.24$, $6-q_L-6d = 3.61 < 3.73 = 3.5+ q_L-3d$.

[22] Either F or L will end up selling at 2.5−3d if the other optimally holds.

respectively, and stock prices are kept away from their fundamental values for an extended time. Over time an equilibrium will be reached in which more informed investors mix their sincere and cheating strategy, and less informed ones mix their trust and distrust strategy. Furthermore, even if allegedly more informed investors do not actually have any informational edge, they might be able to obtain an extra payoff as long as less informed ones do not know of the fact.

One possible application of this model could be to emerging markets, where foreign investors mostly from developed markets like the US or the UK are considered more informed than local institutions. Particularly Asian emerging markets could be good examples, where foreigners have been enjoying markets' belief in them since some of them predicted successfully the Asian currency crisis in 1997. We think that this leaves a potential avenue for future research.

References

Avery, C., and P. Zemsky, 1998, "Multidimensional Uncertainty and Herd Behavior in Financial Markets," *American Economic Review*, 88(4), 724-748.

Banerjee, A., 1992, "A Simple Model of Herd Behavior," Quarterly Journal of Economics, 107, 797-817

Bikhchandani S., D. Hirshleifer, and J. Welch, 1992, "A Theory of Fad, Fashion, Custom and Cultural Change as Informational Cascades," *Journal of Political Economy*, 100, 992-1026

Brunnermeier, M., 2001, *Asset Pricing under Asymmetric Information*, Oxford, New York

DeLong, J..B., A. Shleifer, L. Summers, and R. Waldman, 1990a, "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, 98, 703-738

DeLong, J..B., A. Shleifer, L. Summers, and R. Waldman, 1990b, "Positive Feedback Investment Strategies and destabilizing Rational Speculation," *Journal of Finance*, 45, 375-395

Kreps, D., and R. Wilson, 1982, "Reputation and Imperfect Information," *Journal of Economic Theory*, 27, 253-279

Landon and Smith, 1998, "Quality Expectations, Reputation, and Price," *Southern Economic Journal*, 64(3), 628-647

Prendergast, C., and L. Stole, 1996, "Impetuous Youngsters and Jaded Old-Timers: Acquiring Reputation for Learning," *Journal of Political Economy*, 104, 1105-1134

Scharfstein, D., and J. Stein, 1990, "Herd Behavior and Investment," *American Economic Review*, 80, 465-479

Trueman, B., 1994, "Analyst Forecasts and Herding Behavior," *Review of Financial*

Studies, 7, 97-124
Wang, J., 1993, "A Model of Intertemporal Asset Prices under Asymmetric Information," *Review of Economics Studies*, 60, 249-282
Welch, I., 1992, "Sequential Sales, Learning and Cascades, *Journal of Finance*, 47, 695-732
Zhang, J., 1997, "Strategic Delay and the Onset of Investment Cascades," *Rand Journal of Economics*, 28, 188-205

Appendix

Proof 1: Lemma 1

Let us say $i = 1$. Then
$P[\theta_1 | s_L = \theta_1] = P[\theta_1 \text{ and } s_L = \theta_1]/P[s_L = \theta_1]$
$= \{P[\theta_1]\times P[s_L = \theta_1 | \theta_1]\}/\{P[\theta_1]\times P[s_L = \theta_1 | \theta_1] + P[\theta_2]\times P[s_L = \theta_1 | \theta_2]\}$
$= (\frac{1}{4}\times q_L)/\{\frac{1}{4}\times q_L + \frac{1}{4}\times(1-q_L)\}$ (by Assumption 2)
$= q_L$
This also holds for $i \in \{2, 3, 4\}$. Hence ①. Also, ② can be similarly derived by Assumption 2.

Now, $E[P_T | s_L=\theta_1]$
$= P[\theta_1 | s_L=\theta_1]\times E[P_T | \theta_1] + P[\theta_2 | s_L=\theta_1]\times E[P_T | \theta_2]$
$= q_L\times 1 + (1-q_L)\times 2 = 2-q_L$ by ① and ②. Hence ③. And ④, ⑤ and ⑥ can be similarly derived.

Proof 2: Assumption 4

Solving $2.5-3d< 2 <2.5-2d$ and $1 <2.5-4d$ leads to $\frac{1}{6} < d < \frac{1}{4}$ (①). Also, solving $2.5-3d< 1+q_L< 2.5-2d$ and $2-q_L< 2.5-4d$ leads to $1.5-3d<q_L<1.5-2d$ and $q_L>\frac{1}{2}-4d$, respectively. Since $q_L<1$ and ① implies that $1.5-2d>1$ and $\frac{1}{2}-4d<1.5-3d$, the range of q_L turns out to be $1.5-3d<q_L<1$.

Proof 3: Lemma 2

The current share price is $P_1 = 2.5-2d$. Now, in order to calculate the expected payoff from trading the second shares ($x_{F,2}$, $x_{L,2} \in \{-1,0\}$), we should recall that $x_{F,2} = -1$ (0) when $\theta^* = 1$ (2), and $x_{L,2} = -1$ (0) when $s_L = 1$ (2). For example, $E[\pi_{F,2} | \theta^*=1, s_L=1] = E[\pi_{L,2} | \theta^*=1, s_L=1] = 2.5-4d$ since both F and L sell a share simultaneously. Similarly, $E[\pi_{L,2} | \theta^*=2, s_L=1] = E[\pi_{F,2} | \theta^*=1, s_L=2] = 2.5-3d$ since only one trader sells a share. Then, L's expected payoff from $x_{L,2}$ is,

$$\begin{aligned} E[\pi_{L,2}] &= P[\theta_1]\times\{P[s_L=1 | \theta_1]\cdot E[\pi_{L,2} | \theta_1, s_L=1] + P[s_L=2 | \theta_1]\cdot E[\pi_{L,2} | \theta_1, s_L=2]\} \\ &\quad + P[\theta_2]\times\{P[s_L=1 | \theta_2]\cdot E[\pi_{L,2} | \theta_2, s_L=1] + P[s_L=2 | \theta_2]\cdot E[\pi_{L,2} | \theta_2, s_L=2]\} \\ &= \tfrac{1}{2}\{q_L(2.5-4d) + (1-q_L)(1)\} + \tfrac{1}{2}\{(1-q_L)(2.5-3d) + q_L(2)\} \\ &= \tfrac{1}{2}\{3.5-3d+ q_L(1-d)\} \end{aligned}$$

Similarly,

$$E[\pi_{F,2}] = P[\theta_1]\times\{P[s_L=1|\theta_1]\cdot E[\pi_{F,2}|\theta_1, s_L=1] + P[s_L=2|\theta_1]\cdot E[\pi_{F,2}|\theta_1, s_L=2]\}$$
$$+ P[\theta_2]\times\{P[s_L=1|\theta_2]\cdot E[\pi_{F,2}|\theta_2, s_L=1] + P[s_L=2|\theta_2]\cdot E[\pi_{F,2}|\theta_2, s_L=2]\}$$
$$= \tfrac{1}{2}\{q_L(2.5-4d) + (1-q_L)(2.5-3d)\} + \tfrac{1}{2}\{(1-q_L)\times 2 + q_L\times 2\}$$
$$= \tfrac{1}{2}\{4.5-3d-d\cdot q_L\}$$

Proof 4: Lemma 3

$$E[\pi_{F,2}] = P[\theta_1]\times E[\pi_{F,2}|\theta_1] + P[\theta_2]\times E[\pi_{F,2}|\theta_2]$$
$$= P[x_{F,2} = -1]\times E[\pi_{F,2}| x_{F,2} = -1] + P[x_{F,2} = 0]\times E[\pi_{F,2}| x_{F,2} = 0]$$
$$= \tfrac{1}{2}(2.5-3d) + \tfrac{1}{2}(2) = \tfrac{1}{2}(4.5-3d)$$

$$E[\pi_{L,2}] = P[\theta_1]\times E[\pi_{L,2}|\theta_1] + P[\theta_2]\times E[\pi_{L,2}|\theta_2]$$
$$= P[x_{F,2} = -1]\times E[\pi_{L,2}| x_{F,2} = -1] + P[x_{F,2} = 0]\times E[\pi_{L,2}| x_{F,2} = 0]$$
$$= P[x_{F,2} = -1]\times E[\pi_{L,2}| x_{L,2} = -1] + P[x_{F,2} = 0]\times E[\pi_{L,2}| x_{L,2} = 0]$$
$$= \tfrac{1}{2}(2.5-4d) + \tfrac{1}{2}(2) = \tfrac{1}{2}(4.5-4d)$$

$$\Delta E[\pi_{F,2}] = \tfrac{1}{2}(4.5-3d) - \tfrac{1}{2}(4.5-3d-d\cdot q_L) = \tfrac{1}{2}d\cdot q_L$$

$$\Delta E[\pi_{L,2}] = \tfrac{1}{2}(4.5-4d) - \tfrac{1}{2}\{3.5-3d+q_L(1-d)\}$$
$$= \tfrac{1}{2}\{1-d-q_L(1-d)\} = \tfrac{1}{2}\{1\times(1-q_L)-d(1-q_L)\}$$
$$= \tfrac{1}{2}(1-q_L)(1-d) > 0$$

$$E[\pi_{F,2}]-E[\pi_{L,2}] = \tfrac{1}{2}(4.5-3d) - \tfrac{1}{2}(4.5-4d) = \tfrac{1}{2}d > \tfrac{1}{2}(1-q_L)$$
since $d-(1-q_L) = d-1+q_L > d-1+(1.5-3d) = \tfrac{1}{2}-2d > 0$ by **Assumption 4.**

Proof 5: L is worse off by selectively following F than by blindly following F.

Basically there are two ways to selectively follow F. One way is to follow F with a certain probability; for example, follow F with probability f and do not follow F with probability (1-f). But this should not be optimal because the expected payoff from the former is always greater than that from the latter, i.e., $\tfrac{1}{2}(4.5-4d) > \tfrac{1}{2}\{3.5-3d+q_L(1-d)\}$. Therefore, if L is to follow F anyway, he should follow F *always* or *blindly*.

The other one is to follow F on the basis of his information s_L; for example, follow F if $s_L = 2$ and do not follow F otherwise. This also makes L worse off than does blindly following F. First, $P[s_L = 2|\theta^* \in \{\theta_1, \theta_2\}] = P[\theta_1]\times P[s_L = 2|\theta_1] + P[\theta_2]\times P[s_L = 2|\theta_2] =$

$\frac{1}{2}q_L + \frac{1}{2}(1-q_L) = \frac{1}{2}$. Now, given $s_L = 2$, F will hold to earn 2 with probability $P[\theta_2 | s_L = 2] = q_L$ and sell a share at 2.5-3d with probability $P[\theta_1 | s_L = 2] = 1-q_L$. If L follows F given $s_L = 2$, then L's expected payoff given $s_L = 2$ is $q_L \cdot 2 + (1-q_L)(2.5-4d)$. If $s_L = 1$, L will not follow F and sell a share. Since $P[s_F = \theta_1 | s_L=1] = q_L$ and $P[s_F = \theta_2 | s_L=1] = (1-q_L)$, L will sell a share at 2.5-4d with probability q_L and at 2.5-3d with probability $(1-q_L)$. So his expected payoff given $s_L = 1$ is $q_L(2.5-4d) + (1-q_L)(2.5-3d)$. Overall, the expected payoff for L is

$$
\begin{aligned}
&P[s_L = 2] \times E[\pi_{L,2} | s_L = 2] + P[s_L = 1] \times E[\pi_{L,2} | s_L = 1] \\
&= \tfrac{1}{2}\{q_L \cdot 2 + (1-q_L)(2.5-4d)\} + \tfrac{1}{2}\{q_L(2.5-4d) + (1-q_L)(2.5-3d)\} \\
&= \tfrac{1}{2}\{q_L(4.5-4d) + (1-q_L)(5-7d)\} \\
&= \tfrac{1}{2}\{5-7d + q_L(3d-0.5)\}
\end{aligned}
$$

Now, the incremental payoff of L's copying strategy over this strategy is positive since $\frac{1}{2}(4.5 - 4d) - \frac{1}{2}\{5-7d + q_L(3d-0.5)\} = \frac{1}{2}\{3d-0.5-q_L(3d-0.5)\} = \frac{1}{2}(3d-0.5)(1-q_L) > 0$ by Assumption 4 (①). Similarly it can be shown that the strategy of following F only if $s_L = 1$ is inferior to the strategy of blindly following F.

Proof 6: Lemma 4

When $s_F = \theta^* = 2$, F can cheat L to cause P_t to plummet to 1 and then buys back a share at the lowest price of $1+\varepsilon$. More specifically, given $\theta^* = 2$, he will sell a share at $2.5-3d = P_3$, and L will sell a share at $2.5-4d = P_4$. Then noise traders will herd to sell, which will bring about $P_5 = 1$. Once this happens, noise traders are willing to sell at any price greater than 1, and F can buy back a share at $1+\varepsilon$ at time, say, T-1. At T, $\theta^* = 2$ becomes common knowledge, and $P_T = \theta^* = 2$. Accordingly, his total payoff from this strategy given $\theta^* = 2$ is $(2.5-3d)-(1+\varepsilon)+2 = 3.5-3d-\varepsilon$. Also, his payoff given $\theta^* = 1$ is 2.5-3d. After all, the expected payoff of F when he cheats L given $\theta^* = 2$ is,

$$
\begin{aligned}
E[\pi_{F,2}] &= P[\theta_1] \times E[\pi_{F,2} | \theta_1] + P[\theta^*=2] \times E[\pi_{F,2} | \theta_2] \\
&= \tfrac{1}{2}(3.5-3d-\varepsilon) + \tfrac{1}{2}(2.5-3d) \\
&= \tfrac{1}{2}(6-6d-\varepsilon).
\end{aligned}
$$

This amount is greater than his expected payoff when he trades honestly and L follows F since $\frac{1}{2}(6-6d-\varepsilon) - \frac{1}{2}(4.5-3d) = \frac{1}{2}(1.5-3d-\varepsilon) > 0$. Meanwhile, L's payoff is always 2.5-4d, i.e., $E[\pi_{L,2}] = 2.5-4d = \frac{1}{2}(5-8d)$ as L will follow F, who sells a share at 2.5-3d whether $\theta^* = 1$ or 2. However, this amount is less than his expected payoff when L follows "always-honest" F as $\frac{1}{2}(5-8d) - \frac{1}{2}(4.5 - 4d) = \frac{1}{2}(0.5-4d) < 0$ where $\frac{1}{6} < d$. Last, the differential performance is $E[\pi_{F,2}] - E[\pi_{L,2}] = \frac{1}{2}(6 - 6d - \varepsilon) - \frac{1}{2}(5 - 8d) = \frac{1}{2}(1+2d-\varepsilon) > \frac{1}{2}d$.

Proof 7: Lemma 5

If L always sells a share following $x_{F,2} = -1$, his payoff is always 2.5−4d no matter what θ^* is. But L knows that F might be cheating when $x_{F,2} = -1$ ($\theta^* = \theta_1$ or θ_2), although F is not cheating when $x_{F,2} = 0$ (θ_1). What will be L's expected payoff if he bases his trading on s_L when he sees $x_{F,2} = -1$? Since F always takes $x_{F,2} = -1$ whether $\theta^* = \theta_1$ or θ_2, "$x_{F,2} = -1$" does not reveal any information on θ^*. In fact, given $\theta^* \in \{1, 2\}$, $P[\theta_1 | x_{F,2} = -1]$ =

$$\frac{P[x_{F,2} = -1, \theta^* = \theta_1]}{P[x_{F,2} = -1]} = \frac{P[x_{F,2} = -1, \theta^* = \theta_1]}{P[x_{F,2} = -1, \theta^* \in \{\theta_1, \theta_2\}]}$$

$$= \frac{P[\theta_1] \times P[x_{F,2} = -1 | \theta_1]}{P[\theta^* \in \{\theta_1, \theta_2\}] \times P[x_{F,2} = -1 | \theta^* \in \{\theta_1, \theta_2\}]} = \frac{0.5 \times 1}{1 \times 1} = 0.5,$$

and accordingly $P[\theta_2 | x_{F,2} = -1] = \frac{1}{2}$. Again, the event of $x_{F,2} = -1$ does not help L infer about θ^* at all. Therefore he will follow $x_{F,2} = -1$ only if s_L recommends him to sell, i.e., only if $s_L = 1$, and will not follow $x_{F,2} = -1$ if $s_L = 2$. Then L's expected payoff will depend on s_L. Suppose $s_L = 2$, then $P[\theta_1 | x_{F,2} = -1, s_L = 2] = P[\theta_1 | s_L = 2] = 1-q_L$, and $P[\theta_2 | x_{F,2} = -1, s_L = 1] = P[\theta_2 | s_L = 1] = q_L$. Therefore $E[\pi_{L,2} | x_{F,2} = -1, s_L = 2] = E[\pi_{L,2} | s_L = 2] = E[P_T | s_L = 2] = 1+q_L$, which is greater than 2.5−4d by Assumption 4. Therefore, given $x_{F,2} = -1$ and $s_L = 2$, L is better off by distrusting F and following s_L. Similarly it can be shown that, given $x_{F,2} = -1$ and $s_L = 1$, L is better off by following F (and following s_L), i.e., $2.5-4d > 2-q_L$ (by Assumption 4). After all L's expected payoff is,

$$\begin{aligned} E[\pi_{L,2}] &= P[s_L = 1 | \theta^* \in \{1, 2\}] \times E[\pi_{L,2} | s_L = 1] + P[s_L = 2 | \theta^* \in \{1, 2\}] \times E[\pi_{L,2} | s_L = 2] \\ &= \tfrac{1}{2} \times (2.5-4d) + \tfrac{1}{2} \times (1+q_L)^{23} \\ &= \tfrac{1}{2}(3.5-4d+q_L) > 2.5-4d \quad (\because q_L > 1.5-3d) \end{aligned}$$

Now, given θ_2, F cannot cheat L by taking $x_{F,2} = -1$ as easily as before; L will sell a share only if $s_L = 1$ with probability $1-q_L = P[s_L = 1 | \theta_2]$ and will not sell otherwise. F's payoff will be just 2.5−3d if L does not sell, and $(3.5-3d-\varepsilon)$ if he sells. Overall, F's expected payoff is,

[23] Refer to Proof 5 for $P[s_L = 1 | \theta^* = \{\theta_1, \theta_2\}] = P[s_L = 2 | \theta^* = \{\theta_1, \theta_2\}] = \frac{1}{2}$.

$$
\begin{aligned}
E[\pi_{F,2}] &= P[\theta_1]\times E[\pi_{F,2}|\theta_1] + P[\theta_2]\times E[\pi_{F,2}|\theta_2] \\
&= \tfrac{1}{2}\times(2.5-3d) + P[\theta_2]\times\{P[s_L=1|\theta_2]\times E[\pi_{F,2}|s_L=1] + P[s_L=2|\theta_2]\times E[\pi_{F,2}|s_L=2]\} \\
&= \tfrac{1}{2}\times(2.5-3d) + \tfrac{1}{2}\{(1-q_L)\times(3.5-3d-\varepsilon) + q_L\times(2.5-3d)\} \\
&= \tfrac{1}{2}\times(2.5-3d) + \tfrac{1}{2}\{3.5-3d-q-\varepsilon(1-q_L)\} \\
&= \tfrac{1}{2}\times\{6-6d-q_L-\varepsilon(1-q_L)\}
\end{aligned}
$$

This is even less than F's payoff when he acts sincerely, which is $\frac{1}{2}(4.5-3d)$ as $\frac{1}{2}\times\{6-6d-q_L-\varepsilon(1-q_L)\} - \frac{1}{2}(4.5-3d) = \frac{1}{2}\{1.5-3d-q-\varepsilon(1-q_L)\} < 0$ ($\because$ $q_L>1.5-3d$).

Proof 8: Expected payoffs in each of F and L's strategy profiles

Table 1. Expected payoffs for F when $\theta^* = 2$

$E[\pi_{F,2}\,|\theta_2]$		L	
		Trust	Distrust
F	Act sincerely ($x_{F,2} = 0$)	2 ↓	2
	Cheat ($x_{F,2} = -1$)	$3.5-3d-\varepsilon$	$3.5-3d-q_L-\varepsilon(1-q_L)$ ↑

When $\theta^* = \theta_2 = 2$, F's expected payoff from acting sincerely ($x_{F,2} = 0$) is 2, which is trivial. But that depends on L's response when he cheats. If F cheats L, who trusts him ($x_{L,2} = -1$), then F's expected payoff will be $3.5-3d-\varepsilon$ (Proof 6). If L distrusts F and relies on s_L, that will be

$$
\begin{aligned}
&P[s_L = 2|\theta_2]\times E[\pi_{F,2}|\theta_2, s_L = 2] + P[s_L = 1|\theta_2]\times E[\pi_{F,2}|\theta_2, s_L = 1] \\
&= q_L(2.5-3d)+(1-q_L)(3.5-3d-\varepsilon) = 3.5-3d-q_L-\varepsilon(1-q_L).
\end{aligned}
$$

$3.5-3d-\varepsilon$ is greater than 2 as $d<\frac{1}{4}$, and $3.5-3d-q_L-\varepsilon(1-q_L)$ is less than 2 as $q_L>1.5-3d$.

Table 2. Expected payoffs for L when $s_L = 2$ and $x_{F,2} = -1$

$E[\pi_{L,2}|s_L= 2, x_{F,2}= -1]$		L	
		Trust ($x_{L,2} = -1$)	Distrust ($x_{L,2} = 0$)
F	Act sincerely	$2.5-4d$	←1
	Cheat	$2.5-4d$ →	$1+q_L$

Given $x_{F,2} = -1$ and $s_L = 2$, L can follow F by selling a share and earn to $2.5-4d$ with probability one. If he distrusts F and holds his second share, his expected payoff is $E[P_T | s_L = 2] = 1+q_L$. And (2.5−4d) is greater than 1 and less than $(1+q_L)$ by Assumption 4. Notice that taking $x_{F,2} = -1$ while cheating does not always mean that F is cheating. Even if F employs the cheating strategy, $P[\theta_1 | x_{F,2} = -1] = P[\theta_2 | x_{F,2} = -1] = \frac{1}{2}$ (Proof 5). That is, $x_{F,2} = -1$ does not help L infer about θ^*. Only $s_L = 2$ helps him calculate the (ex post) probability that $\theta^* = 1$ or 2, which is $(1-q_L)$ or q_L.

Proof 9: Lemma 6

Given $\theta^* = 2 = \theta_2$, the expected payoff for F when he cheats ($x_{F,2} = -1$) is

$$
\begin{aligned}
&E[\pi_{F,2} | \theta_2, x_{F,2} = -1] \\
&= P[s_L = 1 | \theta_2]\times E[\pi_{F,2} | s_L = 1] + P[s_L = 2 | \theta_2]\times E[\pi_{F,2} | s_L = 2] \\
&= (1-q_L)\times E[\pi_{F,2} | x_{L,2} = -1] + q_L\times\{\beta\times E[\pi_{F,2} | x_{L,2} = -1] + (1-\beta)\times E[\pi_{F,2} | x_{L,4} = 0]\} \\
&= (1-q_L)\times(3.5-3d-\varepsilon) + q_L\times\{\beta\times(3.5-3d-\varepsilon) + (1-\beta)\times(2.5-3d)\} \\
&= (3.5-3d) - q_L\times(1-\beta) \qquad (5)
\end{aligned}
$$

Similarly, given $s_L = 2$ and $x_{F,2} = -1$, L's payoff is 2.5−4d by selling a share. If L does not trusts F and holds a share, his expected payoff will depend on $P[\theta^* = \theta_1 | x_{F,3} = -1, s_L = 2]$ such that

$$
\begin{aligned}
&E[\pi_{L,2} | x_{F,2} = -1, s_L = 2, x_{L,2} = 0] \\
&= P[\theta_1 | x_{F,2} = -1, s_L = 2]\times E[\pi_{L,2} | \theta_1, x_{L,2} = 0] + P[\theta_2 | x_{F,2} = -1, s_L = 2]\times E[\pi_{L2} | \theta_2, x_{L,2} = 0] \\
&= P[\theta_1 | x_{F,2} = -1, s_L = 2]\times 1 + P[\theta_2 | x_{F,2} = -1, s_L = 2]\times 2.
\end{aligned}
$$

Here, $P[\theta_1 | x_{F,2} = -1, s_L = 2] = \dfrac{P[\theta_1, x_{F,2} = -1, s_L = 2]}{P[x_{F,2} = -1, s_L = 2]}$

$$
\begin{aligned}
&= \frac{P[\theta_1]P[x_{F,2} = -1, s_L = 2 | \theta_1]}{P[\theta_1]P[x_{F,2} = -1, s_L = 2 | \theta_1] + P[\theta_2]P[x_{F,2} = -1, s_L = 2 | \theta_2]} \\
&= \frac{P[\theta_1]P[x_{F,2} = -1 | \theta_1]P[s_L = 2 | \theta_1]}{P[\theta_1]P[x_{F,2} = -1 | \theta_1]P[s_L = 2 | \theta_1] + P[\theta_2]P[x_{F,2} = -1 | \theta_2]P[s_L = 2 | \theta_2]} \\
&= \frac{0.5\times 1\times(1-q_L)}{0.5\times 1\times(1-q_L) + 0.5\times\alpha\times q_L} \\
&= \frac{1-q_L}{1-q_L+\alpha\cdot q_L}.
\end{aligned}
$$

Accordingly $P[\theta_2 | x_{F,2} = -1, s_L = 2] = \dfrac{\alpha \cdot q_L}{1 - q_L + \alpha \cdot q_L}$ and

$E[\pi_{L,2} | x_{F,2} = -1, s_L = 2, x_{L,2} = 0]$
$= P[\theta_1 | x_{F,2} = -1, s_L = 2] \times 1 + P[\theta_2 | x_{F,2} = -1, s_L = 2] \times 2.$

$$= \frac{1-q_L}{1-q_L+\alpha \cdot q_L} \times 1 + \frac{\alpha \cdot q_L}{1-q_L+\alpha \cdot q_L} \times 2 = \frac{1-q_L+2\alpha \cdot q_L}{1-q_L+\alpha \cdot q_L} \quad . \qquad (6)$$

In equilibrium, the expected payoff from F's cheating (=(5)) should be equal to 2, the payoff when he does not cheat ($x_{F,2}$ =0). This is because, if (5) is greater (less) than 2, he always (never) cheats. But a pure strategy such as cheating with probability 1 or 0 does not constitute a long-run equilibrium as shown in Tables 1 and 2. Therefore (5) should be equal to 2 and we have (ignoring ε)

$$\beta^* = 1 - \frac{3-6d}{2q_L} .$$

Similarly (6) should be equal to 2.5−4d. After a lengthy algebra, we have

$$\alpha^* = \frac{(3-8d)(1-q_L)}{(8d-1)q_L} .$$

And numerical analyses below show that the ranges of α^* and β^* are $0 < \alpha^* < \frac{1}{3}$, and $0 < \beta^* < \frac{1}{4}$, where $\alpha^* = \frac{1}{3}$ if $d = \frac{1}{4}$ and $q_L = 1.5 - 3d$, and $\beta^* = \frac{1}{4}$ if $d = \frac{1}{4}$ and $q_L = 1$. Notice that, while doing numerical analyses, the values of q_L and d are carefully picked to always satisfy the two inequalities of **Assumption 4** simultaneously.

< the range of α^* >

(q_L(min) is the minimum value of q_L to meet q_L>1.5-3d)

q_L =	d =																
	0.168	0.173	0.178	0.183	0.188	0.193	0.198	0.203	0.208	0.213	0.218	0.223	0.228	0.233	0.238	0.243	0.248
0.751																	
0.761																	0.33
0.771																	0.31
0.781																0.32	0.29
0.791															0.32	0.30	0.27
0.801															0.30	0.28	0.26
0.811														0.31	0.28	0.26	0.24
0.821													0.31	0.29	0.27	0.25	0.23
0.831													0.29	0.27	0.25	0.23	0.21
0.841												0.29	0.27	0.25	0.23	0.21	0.20
0.851											0.30	0.27	0.25	0.23	0.21	0.20	0.18
0.861											0.27	0.25	0.23	0.21	0.20	0.18	0.17
0.871										0.27	0.25	0.23	0.21	0.20	0.18	0.17	0.15
0.881									0.27	0.25	0.23	0.21	0.19	0.18	0.16	0.15	0.14
0.891								0.27	0.25	0.23	0.21	0.19	0.18	0.16	0.15	0.14	0.13
0.901								0.24	0.22	0.20	0.19	0.17	0.16	0.15	0.13	0.12	0.11
0.911							0.24	0.22	0.20	0.18	0.17	0.15	0.14	0.13	0.12	0.11	0.10
0.921							0.21	0.19	0.17	0.16	0.15	0.13	0.12	0.11	0.10	0.10	0.09
0.931						0.20	0.18	0.16	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.08	0.08
0.941					0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.08	0.08	0.07	0.07
0.951					0.15	0.14	0.13	0.11	0.10	0.10	0.09	0.08	0.07	0.07	0.06	0.06	0.05
0.961				0.14	0.12	0.11	0.10	0.09	0.08	0.08	0.07	0.06	0.06	0.05	0.05	0.05	0.04
0.971			0.11	0.10	0.09	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03	0.03
0.981			0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02	0.02	0.02
0.991		0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01
q_L(min)	0.997	0.982	0.967	0.952	0.937	0.922	0.907	0.892	0.877	0.862	0.847	0.832	0.817	0.802	0.787	0.772	0.757

< the range of β^* >

q_L =	d =																
	0.168	0.173	0.178	0.183	0.188	0.193	0.198	0.203	0.208	0.213	0.218	0.223	0.228	0.233	0.238	0.243	0.248
0.751																	
0.761																	0.01
0.771																	0.02
0.781																0.01	0.03
0.791															0.01	0.02	0.04
0.801															0.02	0.04	0.05
0.811														0.01	0.03	0.05	0.07
0.821													0.00	0.02	0.04	0.06	0.08
0.831													0.02	0.03	0.05	0.07	0.09
0.841												0.01	0.03	0.05	0.06	0.08	0.10
0.851											0.00	0.02	0.04	0.06	0.08	0.09	0.11
0.861											0.02	0.03	0.05	0.07	0.09	0.10	0.12
0.871										0.01	0.03	0.04	0.06	0.08	0.10	0.11	0.13
0.881									0.00	0.02	0.04	0.06	0.07	0.09	0.11	0.12	0.14
0.891									0.02	0.03	0.05	0.07	0.08	0.10	0.12	0.13	0.15
0.901								0.01	0.03	0.04	0.06	0.08	0.09	0.11	0.13	0.14	0.16
0.911							0.00	0.02	0.04	0.05	0.07	0.09	0.10	0.12	0.14	0.15	0.17
0.921							0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.13	0.15	0.16	0.18
0.931						0.01	0.03	0.04	0.06	0.07	0.09	0.11	0.12	0.14	0.15	0.17	0.19
0.941					0.00	0.02	0.04	0.05	0.07	0.08	0.10	0.12	0.13	0.15	0.16	0.18	0.20
0.951					0.01	0.03	0.05	0.06	0.08	0.09	0.11	0.13	0.14	0.16	0.17	0.19	0.20
0.961				0.01	0.02	0.04	0.06	0.07	0.09	0.10	0.12	0.13	0.15	0.17	0.18	0.20	0.21
0.971			0.00	0.02	0.04	0.05	0.07	0.08	0.10	0.11	0.13	0.14	0.16	0.17	0.19	0.20	0.22
0.981			0.01	0.03	0.04	0.06	0.08	0.09	0.11	0.12	0.14	0.15	0.17	0.18	0.20	0.21	0.23
0.991		0.01	0.02	0.04	0.05	0.07	0.08	0.10	0.12	0.13	0.15	0.16	0.18	0.19	0.21	0.22	0.24
q_L(min)	0.997	0.982	0.967	0.952	0.937	0.922	0.907	0.892	0.877	0.862	0.847	0.832	0.817	0.802	0.787	0.772	0.757

Proof 10:   Table 4

α	β	$E[\pi_{F,2} \mid \alpha, \beta]$	rank	$E[\pi_{L,2} \mid \alpha, \beta]$	rank
0	1	$\frac{1}{2}(4.5-3d)$	②	$\frac{1}{2}(4.5-4d)$	①
0	0	$\frac{1}{2}(4.5-3d)$	②	$\frac{1}{2}\{3+q_L(1.5-4d)\}$	②
1	1	$\frac{1}{2}(6-6d)$	①	$\frac{1}{2}(5-8d)$	④
1	0	$\frac{1}{2}(6-q_L-6d)$	④	$\frac{1}{2}(3.5+q_L-4d)$	③
α^*	β^*	$\frac{1}{2}(4.5-3d)$ = ②		③ < $\frac{1}{2}\{5-8d+(1-\alpha^*)(4d-0.5)\}$ < ②	

All the values of $E[\pi_{F,2} \mid \alpha, \beta]$ and $E[\pi_{L,2} \mid \alpha, \beta]$ but $E[\pi_{F,2} \mid \alpha = \beta = 0]$ and $E[\pi_{L,2} \mid \alpha = \beta = 0]$ have been already derived by Lemmas. And $E[\pi_{F,2} \mid \alpha = \beta = 0] = E[\pi_{F,2} \mid \alpha = 0, \beta = 1] = \frac{1}{2}(4.5-3d)$ since the expected payoff of F, who trades first based on s_F, is not affected by L's trading strategy. And, using Table 3,

$E[\pi_{L,2} \mid \alpha = \beta = 0]$
$= P[\theta_1]\times E[\pi_{L,2} \mid \theta_1, \alpha = \beta = 0] + P[\theta_2]\times E[\pi_{L,2} \mid \theta_2, \alpha = \beta = 0]$
$= \frac{1}{2}\times\{q_L(2.5-4d)+(1-q_L)\times 1\} + \frac{1}{2}\times 2$ ($\because$ $x_{F,2} = 0$ indicates θ_2)
$= \frac{1}{2}\{3+q_L(1.5-4d)\}$.

Now, in the ranks of F, ① – ② = (6−6d) – (4.5−3d) = 1.5−3d>0 as $d<\frac{1}{4}$. And ② – ④ = $(4.5-3d)-(6-q_L-6d) = -1.5+3d+q_L>0$ as $q_L>1.5-3d$. As for the ranks of L, ① – ② = $\frac{1}{2}\{4.5-4d-3-q_L(1.5-4d)\} = \frac{1}{2}(1.5-4d)(1-q_L) > 0$ ($\because 4d<1$). Also, ② – ③ = $\frac{1}{2}\{3+q_L(1.5-4d)-3.5-q_L+4d\} = \frac{1}{2}\{(4d-0.5)-q_L(4d-0.5)\} = \frac{1}{2}(4d-0.5)(1-q_L) > 0$. And ③ – ④ = $\frac{1}{2}(3.5+q_L-4d-5+8d) = \frac{1}{2}(4d+q_L-1.5) > 0$ ($\because q_L>1.5-4d$). Finally, numerical analyses below show that $E[\pi_{L,2} \mid \alpha = \alpha^*, \beta = \beta^*] = \frac{1}{2}\{5-8d+(1-\alpha^*)(4d-0.5)\}$ lies between ③ and ②.

< the range of $E[\pi_{L,2} | \alpha = \alpha^{*}, \beta = \beta^{*}]$ − ③ >[24]

q_L =	d =															
	0.168	0.173	0.178	0.183	0.188	0.193	0.198	0.203	0.208	0.213	0.218	0.223	0.228	0.233	0.238	0.243	0.248
0.751																	
0.761																	0.08
0.771																	0.08
0.781																0.07	0.08
0.791															0.06	0.07	0.07
0.801															0.06	0.07	0.07
0.811														0.06	0.06	0.07	0.07
0.821													0.05	0.05	0.06	0.06	0.07
0.831													0.05	0.05	0.06	0.06	0.07
0.841												0.04	0.05	0.05	0.06	0.06	0.06
0.851											0.04	0.04	0.05	0.05	0.05	0.06	0.06
0.861											0.04	0.04	0.04	0.05	0.05	0.05	0.06
0.871										0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05
0.881									0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05
0.891									0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05
0.901								0.02	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04
0.911							0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04
0.921							0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.04
0.931						0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03
0.941					0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03
0.951					0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
0.961				0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02
0.971			0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.981			0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.991		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
q_L(min)	0.997	0.982	0.967	0.952	0.937	0.922	0.907	0.892	0.877	0.862	0.847	0.832	0.817	0.802	0.787	0.772	0.757

< the range of $E[\pi_{L,2} | \alpha = \alpha^{*}, \beta = \beta^{*}]$ − ② >[25]

q_L =	d =															
	0.168	0.173	0.178	0.183	0.188	0.193	0.198	0.203	0.208	0.213	0.218	0.223	0.228	0.233	0.238	0.243	0.248
0.751																	
0.761																	−0.04
0.771																	−0.03
0.781																−0.03	−0.03
0.791															−0.03	−0.03	−0.03
0.801															−0.03	−0.03	−0.03
0.811														−0.03	−0.02	−0.02	−0.02
0.821													−0.02	−0.02	−0.02	−0.02	−0.02
0.831													−0.02	−0.02	−0.02	−0.02	−0.02
0.841												−0.02	−0.02	−0.02	−0.02	−0.02	−0.02
0.851											−0.02	−0.02	−0.02	−0.01	−0.01	−0.01	−0.01
0.861											−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
0.871										−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
0.881									−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
0.891									−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
0.901								−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
0.911							−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	0.00	0.00	0.00	0.00
0.921							0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.931						0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.941					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.951					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.961				0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.971			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.981			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.991		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
q_L(min)	0.997	0.982	0.967	0.952	0.937	0.922	0.907	0.892	0.877	0.862	0.847	0.832	0.817	0.802	0.787	0.772	0.757

[24] $\frac{1}{2}$, the common coefficients are omitted as in another table below.

[25] The 0.00's in this Excel spreadsheet are the numbers, which are less than, but very close to, zero.

Proof 11: Lemma 7

$$E[\pi_{F,2}] = P[\theta_1]\times\{P[s_F=1|\theta_1]\cdot E[\pi_{F,2}|\theta_1, s_F=1] + P[s_F=2|\theta_1]\cdot E[\pi_{F,2}|\theta_1, s_F=2]\}$$
$$+ P[\theta_2]\times\{P[s_F=1|\theta_2]\cdot E[\pi_{F,2}|\theta_2, s_F=1] + P[s_F=2|\theta_2]\cdot E[\pi_{F,2}|\theta_2, s_F=2]\}$$
$$= \tfrac{1}{2}\{q_L(2.5-3d) + (1-q_L)(1)\} + \tfrac{1}{2}\{(1-q_L)(2.5-3d) + q_L(2)\}$$
$$= \tfrac{1}{2}(3.5+q_L-3d).$$

Meanwhile,

$$P[s_F=1] = P[s_F=1, \theta_1] + P[s_F=1, \theta_2] = P[\theta_1]\times P[s_F=1|\theta_1] + P[\theta_2]\times P[s_F=1|\theta_2]$$
$$= \tfrac{1}{2}\times q_L + \tfrac{1}{2}\times(1-q_L) = \tfrac{1}{2}.$$

Therefore,

$$E[\pi_{L,2}] = P[s_F=1]\times E[\pi_{L,2}|s_F=1] + P[s_F=2]\times E[\pi_{L,2}|s_F=2]$$
$$= \tfrac{1}{2}\times E[\pi_{L,2}|x_{L,2} = -1] + \tfrac{1}{2}\times E[P_T|s_F=2]$$
$$= \tfrac{1}{2}(2.5-4d) + \tfrac{1}{2}(1+q_L) = \tfrac{1}{2}(3.5+q_L-4d).$$

Also $E[\pi_{F,2}] - E[\pi_{L,2}] = \tfrac{1}{2}(3.5+q_L-3d) - \tfrac{1}{2}(3.5+q_L-4d) = \tfrac{1}{2}d.$

Proof 12: Lemma 8

When F cheats given $\theta^* = \{\theta_1, \theta_2\}$, he always sells his second share and buy it back at $(1+\varepsilon)$. In this circumstance, the expected payoff for him is a function of θ^*, i.e., $E[\pi_{F,2}|\theta_1] = 2.5-3d-(1+\varepsilon)+1 = 2.5-3d-\varepsilon$, and $E[\pi_{F,2}|\theta_2] = 2.5-3d-(1+\varepsilon)+2 = 3.5-3d-\varepsilon$. His total expected payoff is, ignoring ε,

$$E[\pi_{F,2}] = P[\theta_1]\cdot E[\pi_{F,2}|\theta_1] + P[\theta_2]\cdot E[\pi_{F,2}|\theta_2]$$
$$= \tfrac{1}{2}(2.5-3d) + \tfrac{1}{2}(3.5-3d) = \tfrac{1}{2}(6-6d).$$

The expected payoff for L, who always sells following F, is

$E[\pi_{L,2}] = \tfrac{1}{2}(2.5-4d)+\tfrac{1}{2}(2.5-4d) = \tfrac{1}{2}(5-8d)$이다.

And the differential performance between them is

$$E[\pi_{F,2}] - E[\pi_{L,2}] = \tfrac{1}{2}(6-6d) - \tfrac{1}{2}(5-8d) = \tfrac{1}{2}(1+2d) > \tfrac{1}{2}d.$$

Therefore the performance gap is greater than when F sincerely trades.

Proof 13: Lemma 9

Let us say, {a, b} denotes the combination of "s_F = a" and "s_L = b." Then, both F and L sell their second shares (and F repurchase a share) only when {a, b} = {1,1} or {2,1} since L sells only when s_L= 1. Since F can achieve an extra payoff only when $\theta^* = \theta_2$, his expected payoff in each combination of {s_F, s_L} is as follows.

$E[\pi_{F,2}|\theta_1, \{1,1\}] = (2.5-3d)-\varepsilon$
$E[\pi_{F,2}|\theta_1, \{1,2\}] = (2.5-3d)$
$E[\pi_{F,2}|\theta_1, \{2,1\}] = (2.5-3d)-\varepsilon$
$E[\pi_{F,2}|\theta_1, \{2,2\}] = (2.5-3d)$

$E[\pi_{F,2}|\theta_2, \{1,1\}] = (3.5-3d)-\varepsilon$
$E[\pi_{F,2}|\theta_2, \{1,2\}] = (2.5-3d)$
$E[\pi_{F,2}|\theta_2, \{2,1\}] = (3.5-3d)-\varepsilon$
$E[\pi_{F,2}|\theta_2, \{2,2\}] = (2.5-3d)$

That is, (ignoring ε) $E[\pi_{F,2}] = (3.5-3d)$ when $\theta^* = \theta_2$ and {s_F, s_L} = {1,1} or {2,1}, and $E[\pi_{F,2}] = (2.5-3d)$ elsewhere. The probability that $\theta^* = \theta_2$ ***and*** {s_F, s_L} = {1,1} or {2,1} is nothing but $P[\theta_2, s_L = 1] = P[\theta_2]\times P[s_L = 1|\theta_2] = \frac{1}{2}(1-q_L)$, and the probability of its mutually exclusive events is $1-\frac{1}{2}(1-q_L) = \frac{1}{2}(1+q_L)$. Therefore his expected payoff is

$$\begin{aligned} E[\pi_{F,2}] &= P[\theta_2, s_L = 1]\times(3.5-3d) + \{1- P[\theta_2, s_L = 1]\}\times(2.5-3d) \\ &= \tfrac{1}{2}(1-q_L)(3.5-3d) + \tfrac{1}{2}(1+q_L)(2.5-3d) \\ &= \tfrac{1}{2}(6-q_L-6d), \end{aligned}$$

Which is less than $\frac{1}{2}(6-6d)$, the expected payoff when he successfully cheats L.

Meanwhile we know that $P[s_F=1] = \frac{1}{2}$ from Proof 11, and $P[s_L=1] = \frac{1}{2}$ can be proven similarly. We also know that, since F always sells his second share, L sells his at 2.5-4d and holds to get $E[P_T]$ when s_L = 1 and s_L = 2, respectively. Accordingly, his expected payoff is

$$\begin{aligned} E[\pi_{L,2}] &= P[s_L = 1]\times(2.5-4d) + P[s_L = 1]\times E[P_T|s_L = 2] \\ &= P[s_L = 1]\times(2.5-4d) + P[s_L = 1]\times(1+q_L) \\ &= \tfrac{1}{2}(2.5-4d) + \tfrac{1}{2}(1+q_L) \\ &= \tfrac{1}{2}(3.5+q_L-4d). \end{aligned}$$

Proof 14: Lemma 10

Here we only calculate L's expected payoff because F and L are in a perfectly symmetrical situation. L acts on s_L, so he sells if $s_L = 1$ and holds if $s_L = 2$. More specifically, he sells at 2.5-4d and 2.5-3d if $\{s_F, s_L\} = \{1, 1\}$ and $\{s_F, s_L\} = \{2, 1\}$, respectively, and holds to get $E[P_T | s_L= 2] = 1+q_L$. The probabilities of these events are as follows.

$P[s_L= 2] = \frac{1}{2}$ (Proof 13),

$P[s_L= s_F= 1] = P[\theta_1, s_L= s_F=1] + P[\theta_2, s_L= s_F=1] = \frac{1}{2}q_L^2 + \frac{1}{2}(1-q_L)^2 = \frac{1}{2}\{q_L^2 + (1-q_L)^2\}$

$P[s_L= 1, s_F= 2] = P[\theta_1, s_L= 1, s_F= 2] + P[\theta_2, s_L= 1, s_F= 2]$

$= \frac{1}{2}q_L(1-q_L) + \frac{1}{2}(1-q_L)q_L = \frac{1}{2}\{2q_L(1-q_L)\}$

Consequently, his eventual expected payoff is

$$
\begin{aligned}
E[\pi_{L,2}] &= P[s_L= 2]\times E[P_T | s_L= 2] + P[s_L= s_F= 1]\times(2.5-4d) + P[s_L= 1, s_F= 2]\times(2.5-3d) \\
&= \tfrac{1}{2}(1+q_L) + \tfrac{1}{2}\{q_L^2 + (1-q_L)^2\}(2.5-4d) + \tfrac{1}{2}\{2q_L(1-q_L)\}(2.5-3d) \\
&= \tfrac{1}{2}(1+q_L) + \tfrac{1}{2}\{q_L^2 + (1-q_L)^2\}(2.5-4d) + \tfrac{1}{2}2q_L(1-q_L)(2.5-4d+d) \\
&= \tfrac{1}{2}(1+q_L) + \tfrac{1}{2}\{q_L^2 + (1-q_L)^2 + 2q_L(1-q_L)\}(2.5-4d) + \tfrac{1}{2}2q_L(1-q_L)\times d \\
&= \tfrac{1}{2}(1+q_L) + \tfrac{1}{2}\{q_L + (1-q_L)\}^2\times(2.5-4d) + \tfrac{1}{2}\times 2q_L(1-q_L)d \\
&= \tfrac{1}{2}(1+q_L) + \tfrac{1}{2}\times 1\times(2.5-4d) + \tfrac{1}{2}2q_L(1-q_L)d \\
&= \tfrac{1}{2}\{3.5+q_L-4d+2q_L(1-q_L)d\},
\end{aligned}
$$

and so is $E[\pi_{F,2}]$. Meanwhile, $2q_L(1-q_L)$, as a quadratic function of q_L, reaches its maximum value at $q_L = \frac{1}{2}$. However, since $q_L>0.75$ by Assumption 4 or Proof 9, $2q_L(1-q_L)$ is less than $2q_L(1-q_L)$ at $q_L = 0.75$, which is $2\times0.75\times0.25 = 0.375 = \frac{3}{8}$. Hence

$$\tfrac{1}{2}\{3.5+q_L-4d\} < \tfrac{1}{2}\{3.5+q_L-4d+2q_L(1-q_L)d\} < \tfrac{1}{2}\{3.5+q_L-4d+\tfrac{3}{8}d\}.$$