Contemporaneous Spill-over among Equity, Gold, and Exchange Rate Implied Volatility Indices

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Abstract

This paper examines the contemporaneous spill-over effects among the CBOE implied volatility indices for stocks (VIX), gold (GVZ) and the exchange rate (EVZ). We use the "identification through heteroskedasticity" approach of Rigobon (2003) to decompose the contemporaneous relationship between these implied volatility indices into causal relationships. Our findings suggest that there is strong unidirectional, spill-over from VIX to GVZ and EVZ, where increases in stock market volatility lead to increases in gold and exchange rate volatility; and bi-directional spill-over between GVZ and EVZ. We emphasize the implications of our model by comparing the impulse-responses generated by our structural VAR with the impulse-responses of a traditional VAR. Our results show that the responses to shocks originating in GVZ and EVZ are seriously overestimated in the traditional VAR. These findings on the direction and magnitude of spill-over and the long-run impact on volatility have important implications for portfolio and risk management.

JEL Codes: C32; C58; G1.

Keywords: Spill-over; Implied Volatility Indices.

1. Introduction

The extraordinary concurrent upward trend in gold prices and downward trend in stock prices, especially after the onset of the global financial crisis, have triggered a renewed interest in investigating the information and trade flows between these assets. Traditionally, studies have investigated relationships between various properties of gold and stock markets, for example, gold has been considered as a safe haven for investors (in particular during periods of crisis), a natural hedge, and a potential diversifying asset.[1] In this paper, however, we examine the volatility transmission between gold and stock markets. Most studies that investigate the issue of volatility spill-over examine lead-lag relationships among markets and assets, and use correlations analysis to capture the contemporaneous relationship. But correlations do not indicate the direction of the spill-over. We explicitly model these contemporaneous volatility effects. Understanding these effects is paramount for portfolio and risk management purposes since spill-over may not always be bi-directional and of the same sign or magnitude.

An important issue that we need to deal with when studying volatility spill-over between gold and the stock market is the role of the exchange rate. The uncertainty in exchange rates may affect both gold and stock market volatility and vice versa. To obtain a complete picture of the direct transmission channels, as noted by Ehrmann et al. (2011), we also need to consider indirect transmission channels. Gold, for instance, has been considered as a hedge against currency risk particularly when investors are exposed to US dollar. Consequently, there is a

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[1] The literature is rather extensive to cite in full; for recent studies regarding safe haven and hedging properties see Baur and Lucey (2010), inflation hedging see Blose (2010), and diversifying benefits see Draper et al. (2006), and Bruno and Chicarini (2010).

direct relationship between the US dollar exchange rate and gold prices and any uncertainty around exchange rates could create uncertainty around the value of gold and vice versa.[2] We further expect a relationship between stock market volatility and exchange rate uncertainty, where exchange rate volatility may affect stock market volatility, especially when firms are not fully hedged. In similar vein, stock market volatility may affect exchange rate volatility if the former represents uncertainty about economic prospects leading investors to move out of domestic assets. We therefore need to investigate the triangular relationship of equity, gold and exchange rate volatility to find out the causal relationships among these assets.

This study contributes to the existing literature on volatility transmission in one important respect, namely the examination of the contemporaneous volatility spill-over among stocks, gold and the exchange rate. We estimate these spill-over effects using Rigobon's (2003) identification through heteroskedasticity methodology. Specifically, we use a multivariate GARCH model to explain the heteroskedasticity and use this to identify the causal spill-over effects between stock, gold and exchange rate volatility. Our paper, to the best of our knowledge, is the first empirical work examining the simultaneous volatility spill-over among gold, equity and foreign exchange markets.

Another contribution of this paper is the use of new implied volatility indices recently introduced by the Chicago Board Options Exchange (CBOE). Specifically, we examine the contemporaneous volatility spill-over effects between stocks, gold and the exchange rate by

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[2] Sari et al. (2010) argue that in periods of inflationary expectation, investors move away from dollar denominated soft assets, such as equities, to dollar denominated physical assets like gold.

using the VIX, GVZ and EVZ, respectively.[3] We use the VIX (the implied volatility index on stocks constructed using S&P 500 index options) as our measure for stock market volatility; the GVZ, also known as the Gold VIX (the implied volatility index on gold), constructed using options traded on the SPDR Gold Shares ETF as our measure for gold volatility; and the EVZ, also known as the Euro VIX (the implied volatility index on the euro-dollar exchange rate), constructed using the options traded on the CurrencyShares Euro Trust ETF as our measure for exchange rate volatility.[4] Given that the GVZ and the EVZ have only been introduced on August 1, 2008 (with calculations of the index going back to June 2008) we investigate the period starting June 3, 2008 to December 30, 2011.

Our results suggest that there are strong unidirectional causal spill-over effects from stock market volatility to gold and the euro-dollar exchange rate volatility, where higher stock market volatility leads to higher gold and exchange rate volatility. Gold and the exchange rate volatility, however, do not spill-over to the stock market. Moreover, we find bidirectional spill-over effects between gold and exchange rate volatility that are positive and of similar magnitude. We further observe that the volatility in the three volatility indices follow processes that can be captured by a multivariate GARCH (1, 1). Finally, we highlight the implications of our model by comparing the impulse-responses generated by our model with the impulse-responses generated by a traditional VAR. Our results clearly indicate that the responses to shocks originating in either gold or exchange rate volatility are seriously

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[3] The superiority of the information content of implied volatility over historical volatility measure in various markets has been extensively documented (see among others, Blair, Poon and Taylor, 2001; Poon and Granger, 2001, 2005; Christensen and Prabbala, 1998; Jorion, 1995).

[4] Lucey and Tully (2006) show that gold prices are especially sensitive to the dollar/euro exchange rate.

overestimated in traditional VARs. These findings on the direction of contemporaneous spill-over and the long-run impact on volatility have important implications for portfolio and risk management.

The rest of the paper is organized as follows. Section 2 briefly discusses the literature on identification through heteroskedasticity and its applications. Section 3 presents the model. Section 4 discusses the data and Section 5 presents the results. We conclude in Section 6.

2. Identification through Heteroskedasticity

The problem when investigating the spill-over effects among markets is essentially the same as that observed in simultaneous equation models, in that the contemporaneous causal effects cannot be identified due to endogeneity. As such, most of the literature to date has relied on lead-lag dynamics, be it in a VAR, GARCH or any other type of model, to identify spill-over effects between markets or different assets types. What is generally not captured in these models is the contemporaneous spill-over effect. This contemporaneous spill-over is generally measured by a correlation; however, correlations do not indicate the direction of the shock spill-over. As a solution to this problem many studies have relied on assumptions, such as orthogonal structures of residuals (Choleski factorization). However, orthogonalization is merely an assumption on the direction of causality.

Rigobon (2003) addresses this problem of simultaneity and proposes a technique that, under certain conditions, resolves the simultaneity issue. The technique relies on the heterogeneity in the data to identify the structural parameters in a simultaneous equation model. If, in a simultaneous equation model, there are non-proportional changes in volatility over time, then these changes in volatility affect the underlying relationship between the variables in the model. These changes in the underlying relationship can be used to identify the structural parameters of the model.

Applications of this "identification through heteroskedasticity" have mostly been applied to assessing the return spill-over between various markets and assets. Rigobon (2003), for instance, applies this technique to measure the contemporaneous relationships between Argentinean, Brazilian and Mexican sovereign bonds using a regime-switching model for the volatility process and finds that there are asymmetric reactions to shocks originating from the different markets. Rigobon and Sack (2003a) and Rigobon and Sack (2004) further use a regime-switching model for the volatility process to examine the impact of monetary policy on the stock market (Rigobon and Sack, 2003a) and the effect of the stock prices on monetary policy (Rigobon and Sack, 2004).

Ehrmann et al. (2011) apply the "identification through heteroskedasticity" technique, to examine international financial transmission between different asset classes (money, bond, foreign exchange and stock markets) and across different markets (US and the EURO area) and find that changes in the US market explain movements in the European markets by about 30%, whereas the European markets explain movements in the US markets by only 6%.

Rigobon and Sack (2003b) apply the same technique to assess the spill-over effects between US short- and long-term interest rates and stock prices, where they use a "structural"

GARCH model to identify the parameters in the structural VAR (SVAR). They find significant contemporaneous spill-over effects from the various markets, where stock market returns have a positive effect on short- and long-term interest rates, whereas both short- and long-term interest rates have a negative effect on stock market returns.

Andersen et al. (2007), extend this work to study real-time price discovery in money, bond and stock markets in the US, UK and Germany. They apply their model to intra-day data and use a structural GARCH model to identify the parameters in the structural VAR. Their results suggest that there are significant asymmetric spill-over effects in the three markets.

The studies discussed above all suggest that there are often asymmetric contemporaneous spill-over effects among markets and assets and that in some cases they even take on different signs. Hence relying on a simple correlation to describe the contemporaneous relationship between assets could lead to incorrect conclusions. The extant studies so far have employed the "identification through heteroskedasticity" to examine the contemporaneous spill-over effects at the return level. No study has yet investigated the contemporaneous spill-over effects at the volatility level. Given that volatilities are observed and traded, we extend this line of analysis and apply this methodology to assess the contemporaneous spill-over effects at the volatility level.

3. Model

To assess the volatility spill-over effects between the VIX, GVZ and EVZ, we follow the approach of Rigobon and Sack (2003b) and implemented by Andersen et al. (2007). Rigobon and Sack (2003b) investigate spill-over effects across US short- and long-term interest rates, and the stock market. In this paper, we apply a similar methodology to assess spill-over effects among the three implied volatility indices.

The issue that Rigobon and Sack (2003b) tackle is the fact that standard VAR analysis estimates the reduced-form VAR. These reduced-form VARs are not able to identify the contemporaneous causal effects of variables on each other. This is due to the presence of potential simultaneity issues, which essentially means that the structural VAR (SVAR) is underidentified.

Consider the following SVAR,

$$\mathbf{A}\Delta IV_t = c + \mathbf{\Phi(L)}\Delta IV_t + \varepsilon_t , \tag{1}$$

where ΔIV_t is a (3 × 1) vector of daily changes in VIX, GVZ and EVZ, i.e.,

$$\Delta IV_t \equiv \begin{pmatrix} \Delta VIX_t \\ \Delta GVZ_t \\ \Delta EVZ_t \end{pmatrix},$$

c is a (3 × 1) vector of constants and **Φ(L)** is a (3 × 3) matrix polynomial in the lag operator. The (3 × 3) matrix **A** contains the contemporaneous interactions between ΔVIX, ΔGVZ and ΔEVZ, i.e.

$$\mathbf{A} = \begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & 1 & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & 1 \end{pmatrix}, \tag{2}$$

where, e.g., α_{12} captures the contemporaneous spill-over from ΔGVZ_t to ΔVIX_t and α_{21} captures the contemporaneous spill-over from ΔVIX_t to ΔGVZ_t. The other parameters are defined likewise. The problem with Equation (1) is that the parameters in the contemporaneous matrix **A** are typically not identified, as they introduce simultaneity issues in the model. However, this issue can be resolved based on the heteroskedasticity in the data.

Following Rigobon and Sack (2003b), we make the following assumptions that allow for the parameters in Equation (2) to be identified. First, the residuals in Equation (1), ε_t, are treated as "structural shocks", which we assume have the standard zero-mean property, $E[\varepsilon_t] = 0$ and are contemporaneously and serially uncorrelated, i.e. $E[\varepsilon_{it}\varepsilon_{jt-k}] = 0$ for all $i \neq j$ and k. We can assume these innovation terms to be serially and contemporaneously uncorrelated if the SVAR is correctly specified and as the contemporaneous relations between the dependent variables come from the matrix **A**.

The second identifying assumption is that the innovation term in Equation (1) is conditionally heteroskedastic. Specifically, $\varepsilon_t \sim N(0, \mathbf{H_t})$, where $\mathbf{H_t}$ is diagonal (based on the first assumption) and where $h_t \equiv Diag(\mathbf{H_t})$ follows a GARCH(1, 1) process, i.e.,

$$h_t = \psi_h + \mathbf{\Gamma} h_{t-1} + \mathbf{\Lambda} \varepsilon_{t-1}^2, \tag{3}$$

with $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ being (3 × 3) matrices containing the persistence and shock-term parameters, respectively, and ψ_h being a (3 × 1) vector of constants.[5]

With these two assumptions, the parameters in **A** can be uniquely identified as follows. Equation (1) can be rewritten in its reduced form, i.e.

$$\Delta IV_t = c^* + \mathbf{\Phi}^*\mathbf{(L)} \Delta IV_t + \eta_t, \tag{4}$$

where $c^* = \mathbf{A}^{-1}c$, $\mathbf{\Phi}^*\mathbf{(L)} = \mathbf{A}^{-1}\mathbf{\Phi(L)}$ and $\eta_t = \mathbf{A}^{-1}\varepsilon_t$. The identification of the parameters in **A** comes from the volatility process for the residuals, $\eta_t \sim \mathrm{N}(0, \mathbf{\Omega_t})$, where $\mathrm{vech}(\mathbf{\Omega_t})$ follows the process

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[5] This second assumption is easily supported for return data, as volatility is shown to be heteroskedastic. In our case, we assume that the volatility of volatility is conditionally heteroskedastic.

$$\begin{pmatrix} \Omega_{VV,t} \\ \Omega_{VG,t} \\ \Omega_{VE,t} \\ \Omega_{GG,t} \\ \Omega_{GE,t} \\ \Omega_{EE,t} \end{pmatrix} = \mathbf{B}_1\psi_h + \mathbf{B}_1\boldsymbol{\Gamma}(\mathbf{B}^2)^{-1}\begin{pmatrix} \Omega_{VV,t} \\ \Omega_{GG,t} \\ \Omega_{EE,t} \end{pmatrix} + \mathbf{B}_1\boldsymbol{\Lambda}(\mathbf{B}^2)^{-1}\begin{pmatrix} \eta^2_{V,t-1} \\ \eta^2_{G,t-1} \\ \eta^2_{E,t-1} \end{pmatrix}, \tag{5}$$

with

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \mathbf{A}^{-1} \text{ and } \mathbf{B}_1 = \begin{pmatrix} b_{11}^2 & b_{12}^2 & b_{13}^2 \\ b_{11}b_{21} & b_{12}b_{22} & b_{13}b_{23} \\ b_{21}^2 & b_{22}^2 & b_{23}^2 \\ b_{11}b_{31} & b_{12}b_{32} & b_{13}b_{33} \\ b_{21}b_{31} & b_{22}b_{32} & b_{23}b_{33} \\ b_{31}^2 & b_{32}^2 & b_{33}^2 \end{pmatrix}.$$

This is essentially a Multivariate GARCH (1, 1) model with restrictions on parameters, and it is these restrictions that allow for the identification of the structural parameters. To be specific, an unrestricted version of the GARCH model in Equation (5) could have up to 42 parameters. However, our specification in Equation (5) contains only 27 parameters, 3 in ψ_h, 9 in each of **Γ** and **Λ** and 6 in **A**. We estimate the model using Quasi-Maximum Likelihood.

4. Data

We obtain daily data on the VIX, GVZ and EVZ from the CBOE website for the period June 3, 2008 to December 30, 2011. The VIX, which was introduced by the CBOE in September

2003, is based on the bid and ask prices of the cross-section of S&P 500 options.[6] The GVZ was introduced by the CBOE on 1 August 2008, and is based on the bid and ask prices of the cross-section of the SPDR Gold Shares ETF options.[7] Similarly, on 1 August 2008, the CBOE introduced the EVZ which is based on the bid and ask prices of the cross-section of the CurrencyShares Euro Trust ETF options.[8] These VIXs employ the model-free implied volatility methodology, which provides estimates of the expected future realized volatility (for the underlying) for 30 calendar days ahead. The VIX has probably been the greatest success story of these indices and now has become a traded product, with many other traded products that derive their value from the VIX. Likewise, the GVZ now has options and futures that derive their value from the GVZ.

In Figure 1, we provide a time series plot of the levels of the three implied volatility series. Overall, we observe that there is a commonality in the levels of these indices. Our sample period just includes the onset of the global financial crisis in October 2008, and we can see a sharp increase in the VIX, GVZ and EVZ during this period. Clearly, this crisis spilled over into various markets. We further note a decline in the volatility indices in the subsequent period, and observe two extra large increases in volatility in May 2010 and August 2011, which are related to the European Debt Crisis.

INSERT FIGURE 1 HERE

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[6] www.cboe.com/vix

[7] www.cboe.com/gvz

[8] www.cboe.com/evz

In Table 1, we present summary statistics for the VIX, GVZ, EVZ. Panel A contains summary statistics on the levels of the volatility indices. As can be seen, the VIX has the highest level on average followed by the GVZ. EVZ has about half the volatility of VIX. The indices are all positively skewed and show the presence of excess kurtosis. Based on Augmented Dickey Fuller (ADF) tests on the level data, we find that only in the case of the VIX there is some weak evidence of stationarity, producing an ADF test statistic that is significant at the 10% level. For the GVZ and EVZ, we cannot reject the null hypothesis of a unit root.

INSERT TABLE 1 HERE

In Panel B, we report the same summary statistics for the log changes in the implied volatility indices, ΔVIX, ΔGVZ and ΔEVZ, respectively. Overall, the mean values of these log changes are close to zero, although there is quite some variation on a daily basis as can be seen from the maximum, minimum and standard deviation. ΔVIX and ΔGVZ have positive skewness, however, ΔEVZ has a negative skewness. The positive skewness in ΔVIX and ΔGVZ imply that large positive changes in both VIX and GVZ occur more often than large negative changes. All series display excess kurtosis. The excess kurtosis in all three series imply that large changes occur more often than would be the case if these volatility changes series were normally distributed. Based on the unit root test on the log changes, we can strongly reject the presence of a unit root, which confirms that the log changes in the implied volatility indices are stationary.

5. Results

5.1 The Reduced Form VAR

Before estimating the SVAR described in Section 3, we first estimate the reduced form VAR. The VAR model was developed by Sims (1980) and treats each endogenous variable in the system as a function of lagged values of all endogenous variables. The reduced form VAR for volatility spill-over dynamics can be expressed by Equation (4). The lag length of the VAR is estimated using the Akaike information criterion (AIC), which suggests a lag length of 5 days. Hence, we conduct all our analysis with a 5-day lag length.

We start by following the standard approach of performing Granger (1969) causality tests, which establish the causal lead-lag relationships between the volatility indices. Granger causality establishes that volatility index i is Granger-caused by volatility index j if the information in the past values of volatility index j help to improve the forecasts of volatility index i. The results of the Granger causality tests for VIX, GVZ, and EVZ are reported in Table 2 with corresponding values of the F-tests. Unidirectional causality is observed between VIX and GVZ. VIX significantly Granger causes GVZ, but not vice versa. On the other hand, bidirectional causality is at work between VIX and EVZ. We observe strong causality running from VIX to EVZ. However, weak but statistically significant causality is also running from EVZ to VIX. Finally, there is a unidirectional causality observed between EVZ and GVZ. EVZ Granger causes GVZ, but not vice versa.

INSERT TABLE 2 HERE

In sum, these results suggest that stock market volatility significantly Granger causes the volatility in both gold and currency markets. Moreover, the uncertainty of the foreign exchange market also Granger causes both stock and gold markets. But there is no evidence that the uncertainty of the gold market Granger causes either of the other two markets.

Granger causality tests provide information about which variable impacts on the future values of each variable in the VAR. The F-test, however, does not provide an indication of the sign of the relationship, speed or persistence. This information can be gleaned from impulse response functions.

An impulse response function measures the responses of volatility indices in the VAR to a unit shock in each volatility index. We use the generalized impulse response function (GIRF) of Pesaran and Shin (1998), as these do not require orthogonalization of shocks and are invariant to the reordering of the volatility indices in the VAR.

Figure 2 provides the reduced-form accumulated generalized impulse responses of ΔVIX, ΔGVZ, and ΔEVZ for 30 days ahead. Panel A shows the accumulated impulse responses of ΔVIX, ΔGVZ, and ΔEVZ to a unit shock in ΔVIX. According to the GIRF, this leads to a contemporaneous increase of 0.43, and 0.37 units in GVZ, and EVZ, respectively. For the next few steps ahead the impact of a unit shock to ΔVIX decrease and the same pattern is observed for ΔGVZ and ΔEVZ. The impulse responses reach a steady state after about 10 days at about 0.7, 0.29 and 0.27 for ΔVIX, ΔGVZ, and ΔEVZ, respectively.

INSERT FIGURE 2 HERE

Panel B shows the accumulated impulse responses of ΔVIX, ΔGVZ, and ΔEVZ to a unit shock in the innovations of GVZ. The plot shows that a unit shock to ΔGVZ induces contemporaneous increases in VIX, and EVZ of about 0.43 and 0.36 units, respectively. For the next few days, we observe that the shock to ΔGVZ quickly reverts to its steady state and settles at a value of 0.71 for GVZ. For ΔVIX and ΔEVZ, we observe little dynamics after the initial reaction and these series settle at values of 0.47 and 0.33 units, respectively.

Panel C shows the accumulated impulse responses of ΔVIX, ΔGVZ, and ΔEVZ to a unit shock in the innovations of EVZ. A unit shock in ΔEVZ induces increases in VIX, and GVZ of about 0.37 and 0.36 units, respectively. Similar, to the shock in ΔVIX, it takes about 10 days for the impulse responses to reach their steady state and the responses settle at values of about 0.36, 0.28 and 0.72 units for ΔVIX, ΔGVZ, and ΔEVZ, respectively.

On the whole, these impulse responses suggest that there are bi-directional long-run effects for shocks to each market. Comparing the plots, we observe that the responses of the volatility indices are strongest when the shock is applied to the GVZ.

5.2 Structural Form Results

Our next step is to estimate the model described in Section 3, where we use the "identification through heteroskedasticity" approach to identify the structural parameters in the VAR. In Table 3, we report the results for the contemporaneous relations matrix **A** as given in Equation (2). We observe the causal contemporaneous effects of a change in the variable in the top row on the variable in the first column.

When we consider the contemporaneous causal effect of ΔVIX on the other two implied volatility indices, we observe a highly significant and positive causal effect, or spill-over, from VIX to GVZ, with the coefficient of approximately 0.3 (note that the coefficients in **A** have negative signs as **A** is on the left-hand side of Equation (1), when taken to the right-hand side the effects become positive), indicating that a 1% increase in VIX leads to a contemporaneous increase of 0.3% in the GVZ. Similarly, we observe a highly significant and contemporaneous causal effect of ΔVIX on ΔEVZ, with a value of about 0.23. These findings suggest that there is instantaneous spill-over from stock market volatility to exchange rate volatility.

The next column shows the contemporaneous causal effect of ΔGVZ on ΔVIX and ΔEVZ. We find that the instantaneous spill-over from GVZ to VIX is insignificant, suggesting that there is no contemporaneous causal effect of gold volatility on stock market volatility. Considering the causal effect of gold volatility on exchange rate volatility, we find a significant positive relationship of 0.13, suggesting that there is instantaneous spill-over from gold volatility to exchange rate volatility.

INSERT TABLE 3 HERE

The last column reports the instantaneous spill-over from ΔEVZ to ΔVIX and ΔGVZ. We observe no significant spill-over effect from exchange rate volatility to stock market volatility. There is, however, significant instantaneous spill-over from ΔEVZ to ΔGVZ of about 0.12.

Overall, the results of Table 3 suggest that there is instantaneous spill-over from ΔVIX to ΔGVZ and ΔEVZ, but the reverse is not the case. Further, there is bi-directional instantaneous spill-over between ΔGVZ and ΔEVZ.

If we compare the results reported in Table 3 with the Granger causality results reported in Table 2, we find strong contemporaneous volatility spill-over from ΔVIX to ΔGVZ and ΔEVZ, but the reverse is not the case. This is consistent with the Granger causality results which only consider lagged effects. We also find strong contemporaneous volatility spill-over from ΔGVZ to ΔEVZ and from ΔEVZ to ΔGVZ, which is not evident in the Granger causality test results. Hence, the Granger causality tests do not fully capture the contemporaneous relationships between ΔVIX, ΔGVZ and ΔEVZ.

In Table 4, we present the parameter estimates for the structural GARCH equation (Equation (3)), where, for brevity, we only report the coefficient on the diagonals of $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$. According to the parameters presented in Table 4, the volatility in the implied volatility indices follows a process that is similar to what one would expect of a GARCH (1, 1) model applied to return data. For the three series, we find that the GARCH coefficients on lagged volatility are quite high in magnitude and highly statistically significant, indicating that there is strong persistence in the volatility of the volatility indices; the exchange rate volatility having the highest volatility (0.838) and the gold volatility having the lowest (0.5032). The parameters on the "shock term" are also closely in line with what one would expect from a GARCH model on return data, with parameters ranging from 0.0835 to 0.1271 and are all significant.[9]

INSERT TABLE 4 HERE

The reduced-form impulse responses (generalized impulse response functions) do not capture the effects of the structural shocks on the system and therefore do not capture the contemporaneous spill-over effects accurately. The impulse responses computed from the SVAR however can overcome this shortcoming. These impulse responses can be obtained from the reduced form VAR, given that the contemporaneous effect is captured by **A**. This implies that contemporaneous reactions of shocks to ε_t, are determined by $\mathbf{A}^{-1}$. Hence, with $\mathbf{A}^{-1}$ given, we can compute the structural impulse responses.

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[9] The off-diagonal elements of the coefficient matrices are not reported here for the sake of brevity. To ensure non-negativity of the volatility processes, the elements of these coefficient matrices all need to be positive. In some cases, we find that these elements are not positive and in line with Rigobon and Sack (2003b), we impose non-negativity restrictions on the parameters. The non-negativity restriction is violated in 4 instances in the coefficients of $\mathbf{\Gamma}$ and never in the coefficients of $\mathbf{\Lambda}$.

In Figure 3, we plot the structural impulse responses for the three series. Panel A shows the accumulated impulse responses of ΔVIX, ΔGVZ, and ΔEVZ to a unit shock in the innovations of VIX. This unit shock induces contemporaneous increases in GVZ, and EVZ of about 0.34 and 0.28 units, respectively. The structural impulse responses converge in about 10 steps, where ΔVIX, ΔGVZ, and ΔEVZ settle at about 0.71, 0.38, and 0.27, respectively.

INSERT FIGURE 3 HERE

Panel B shows accumulated impulse responses of ΔVIX, ΔGVZ, and ΔEVZ to a unit structural shock in the innovations of GVZ. The unit shock in ΔGVZ induces instantaneous increases in VIX, and EVZ of about 0.05 and 0.14 units, respectively. Convergence again occurs fast, where ΔGVZ settles at about 0.62, and ΔVIX and ΔEVZ settle at 0.01 and 0.09, respectively.

Panel C shows the accumulated impulse responses of ΔVIX, ΔGVZ, and ΔEVZ to a unit structural shock in the structural innovations of EVZ. A unit shock in the EVZ induces an immediate decrease in VIX of -0.01 units, and increase in GVZ of 0.11 units. The impulse responses converge within 10 days, where ΔEVZ settles at a value of 0.67, and ΔVIX and ΔGVZ settle at values of about 0.01 and 0.12, respectively.

5.5 Reduced-Form versus Structural Impulse Response Functions

Table 5 provides a comparison of the long-run (30 days ahead) impact matrix of the reduced-form VAR and SVAR impulse responses. As can be seen, a unit shock in ΔVIX induces persistent increases in both GVZ and EVZ equal to 0.29 and 0.27 in the reduced-form, respectively, versus 0.38 and 0.27 in the structural form, respectively. This suggests that under both reduced-form and SVAR, we find evidence of strong volatility spill-over from the stock market to gold and the euro-dollar exchange rate.

INSERT TABLE 5 HERE

When we consider the long-run impact of a unit shock in ΔGVZ, we find that both VIX and EVZ increase to 0.47 and 0.33 units in the reduced-form, respectively, versus 0.01 and 0.09 in the structural model, respectively. In the structural impulse response function, the impact on ΔVIX is insignificant (the standard error is equal to 0.0574), while the impact on ΔEVZ is significant at the 5% level. This finding suggests that there is no long-run effect of volatility spill-over from gold to the stock market, and a much lesser effect of gold volatility on exchange rate volatility than suggested by the reduced-form impulse-responses. Finally, a unit shock in ΔEVZ induces long-run increases in VIX and GVZ of 0.36 and 0.28 units in the reduced-form, respectively, versus 0.01 and 0.12 units in the structural model, respectively. Again the long-run impact of a shock in ΔEVZ has no significant impact on ΔVIX (the standard error is 0.0689), while the impact on ΔGVZ is significant at the 5% level. Hence, the impulse-responses based on the reduced form VAR and the SVAR lead to very different conclusions, regarding 1) the direction of causality; and 2) the magnitude of the spill-over.

6. Conclusion

In this paper, we examine the contemporaneous implied volatility spill-over effects among stocks (VIX), gold (GVZ) and the exchange rate (EVZ). We employ the "identification through heteroskedasticity" approach of Rigobon (2003) which allows to decompose the contemporaneous relationship between these implied volatility indices into causal relationships. We observe strong unidirectional, spill-over effects from VIX to GVZ and EVZ, where increases in stock market volatility lead to increases in gold and exchange rate volatility; and bi-directional spill-over between GVZ and EVZ. We further show the implications of our model by comparing the impulse-responses generated by our structural VAR with the impulse-responses of a traditional VAR. Our results clearly indicate that the responses to shocks originating in GVZ and EVZ are overestimated in the traditional VAR. Our findings on the direction and magnitude of spill-over and the long-run impact on volatility have important implications for portfolio and risk management.

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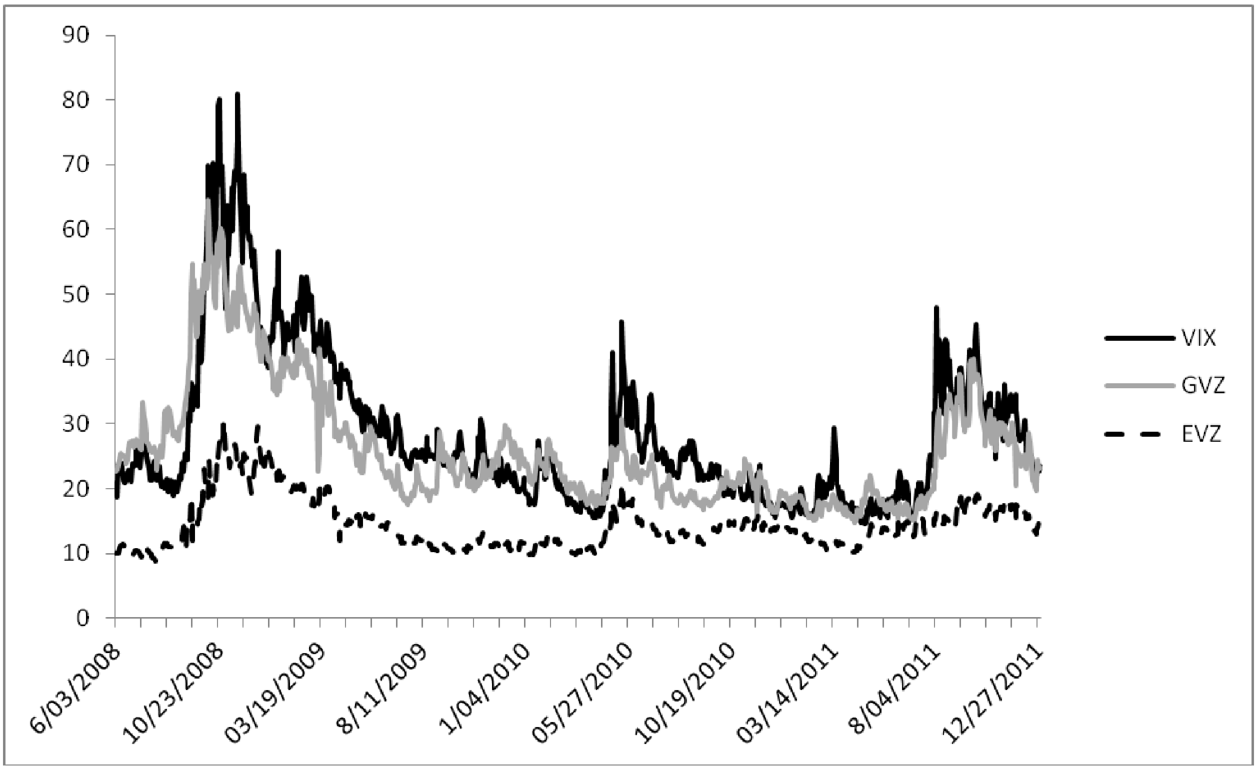
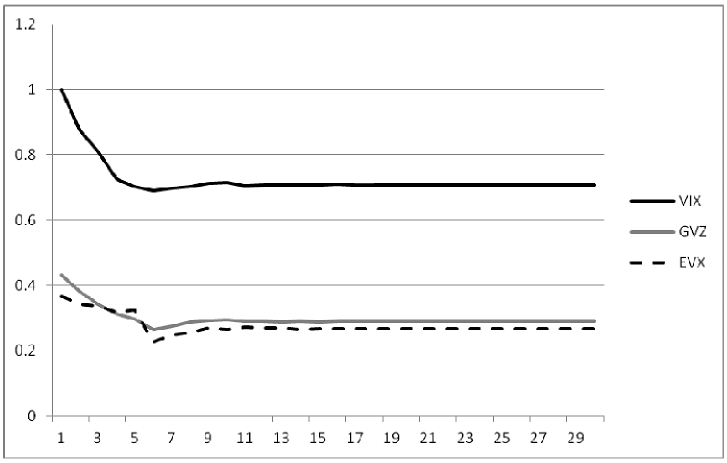
Figure 1. Implied Volatility Indexes



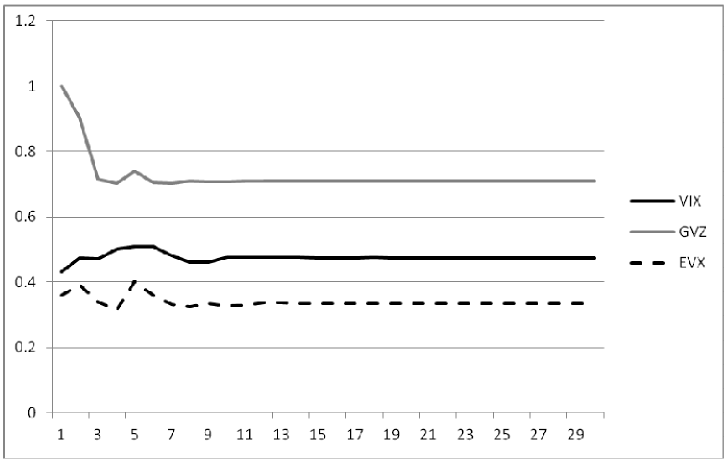
Note: This Figure shows a time series plot of the VIX, GVZ and EVZ over the sample period June 3, 2008 to December 30, 2011.

Figure 2. Reduced Form Generalized Impulse-Response Functions

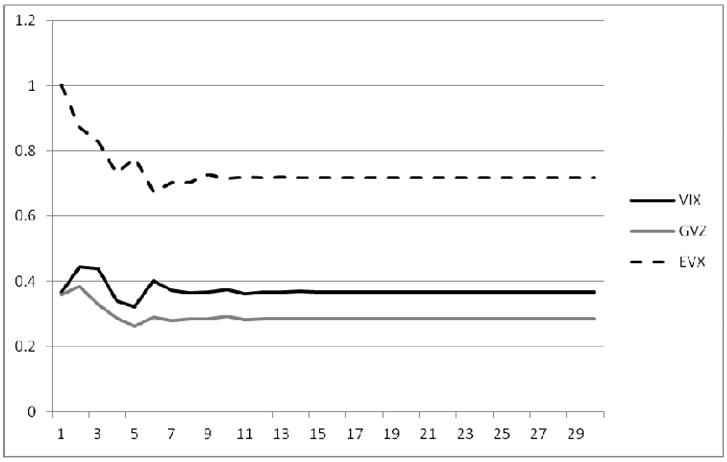
Panel A: Responses to a Unit Shock to ΔVIX



Panel B: Responses to a Unit Shock to ΔGVZ



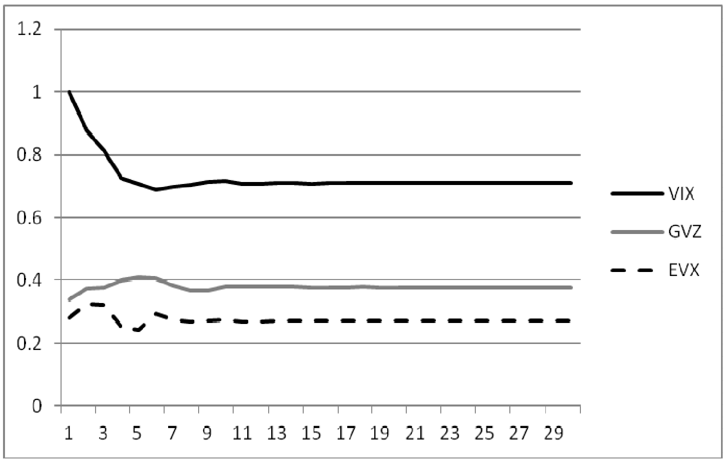
Panel C: Responses to a Unit Shock to ΔEVZ



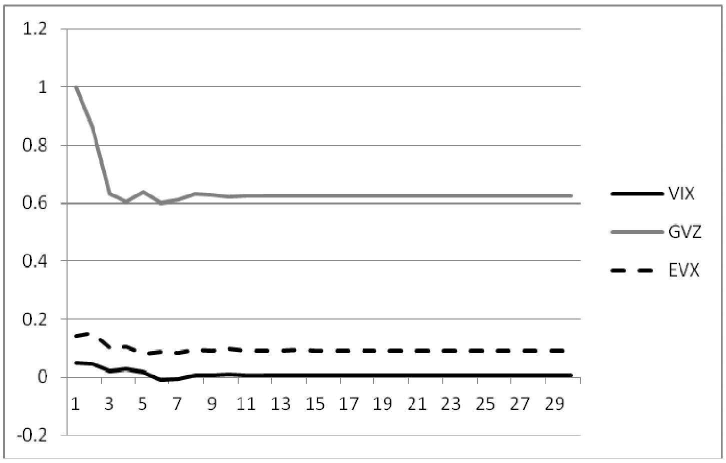
Note: This Figure shows the generalized impulse response functions of the reduced form VAR. Panel A, B, and C report the responses to a unit shock in ΔVIX, ΔGVZ and ΔEVZ, respectively. The x-axis is the number of days ahead and the y-axis is the accumulated response.

Figure 3. Structural Form Impulse-Responses

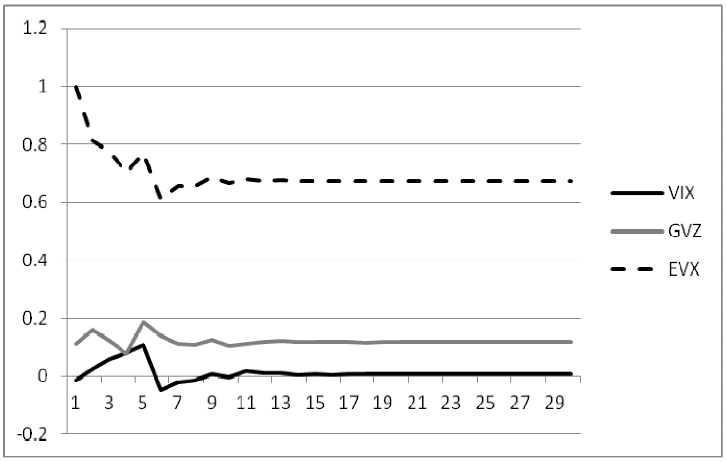
Panel A: Responses to a Unit Shock to ΔVIX



Panel B: Responses to a Unit Shock to ΔGVZ



Panel C: Responses to a Unit Shock to ΔEVZ



Note: This Figure shows the generalized impulse response functions of the structural VAR. Panel A, B, and C report the responses to a unit shock in ΔVIX, ΔGVZ and ΔEVZ, respectively. The x-axis is the number of days ahead and the y-axis is the accumulated response.

Table 1. Summary Statistics

Panel A: Summary Statistics for levels			
	VIX	GVZ	EVZ
Mean	28.22	25.92	14.45
Max	80.86	64.53	30.66
Min	14.62	14.72	8.81
Std. Dev.	11.91	9.35	3.92
Skewness	1.63	1.50	1.35
Kurtosis	5.71	4.92	4.62
ADF	-2.71*	-2.23	-1.98
Panel B: Summary Statistics for log volatility changes			
Mean	0.00018	0.000017	0.00042
Max	0.4055	0.4801	0.2815
Min	-0.3506	-0.4460	-0.4749
Std. Dev.	0.0747	0.0613	0.0524
Skewness	0.7624	0.6474	-0.2191
Kurtosis	6.4911	12.792	14.540
ADF	-33.56***	-26.37***	-20.82***

Note: This Table reports summary statistics for the three implied volatility series, VIX, GVZ, and EVZ. Panel A reports statistics for the levels, while Panel B reports results for log differences. ADF is the t-statistics for the Augmented Dickey-Fuller test. ***, ** and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 2. Granger causality for Implied Volatility Indexes.

Null Hypothesis	5 lags	
	F-Statistics	*P*-value
GVZ does not Granger Cause VIX	0.2277	0.951
VIX does not Granger Cause GVZ	4.3806***	0.001
EVZ does not Granger Cause VIX	2.0118*	0.075
VIX does not Granger Cause EVZ	6.5600***	0.000
EVZ does not Granger Cause GVZ	2.7474**	0.018
GVZ does not Granger Cause EVZ	1.7692	0.117

Note: This Table reports the results for the Granger causality tests on the reduced-form VAR. The reduced-form VAR is estimated using 5 lags. We report F-statistics and their associated P-values. ***, ** and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 3. Contemporaneous Relationship between VIX, GVX and EVX

	ΔVIX	ΔGVZ	ΔEVZ
ΔVIX	1	-0.0506 (0.0335)	0.0194 (0.0512)
ΔGVZ	-0.3054*** (0.0315)	1	-0.1152** (0.0481)
ΔEVZ	-0.2349*** (0.0279)	-0.1325*** (0.0282)	1

Note: This Table report the contemporaneous relationship matrix **A** as defined in Equation (2). We report coefficients and robust (QML) standard errors in parentheses. ***, ** and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 4. GARCH Parameters

	Coefficient	Standard Error
Γ_{11}	0.7818***	(0.0385)
Γ_{22}	0.5032***	(0.0899)
Γ_{33}	0.8380***	(0.0692)
Λ_{11}	0.1271***	(0.0416)
Λ_{22}	0.1029***	(0.0378)
Λ_{33}	0.0835**	(0.0343)

Note: This Table report the diagonal elements of the parameter matrices $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ as defined in Equation (3). We report coefficients and robust (QML) standard errors in parentheses. ***, ** and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 5. Long–Run Impact Matrix

		Reduced Form GIR			Structural IR		
		Shock			Shock		
		ΔVIX	ΔGVZ	ΔEVZ	ΔVIX	ΔGVZ	ΔEVZ
Response	ΔVIX	0.7088	0.4747	0.3639	0.7094	0.0082	0.0066
		(0.0483)	(0.0518)	(0.0516)	(0.0453)	(0.0574)	(0.0689)
	ΔGVZ	0.2892	0.7099	0.2849	0.3788	0.6248	0.1174
		(0.0504)	(0.0500)	(0.0516)	(0.0380)	(0.0482)	(0.0578)
	ΔEVZ	0.2676	0.3347	0.7177	0.2708	0.0921	0.6742
		(0.0500)	(0.0517)	(0.0488)	(0.0320)	(0.0405)	(0.0486)

Note: This Table report the long-run impact matrix of the reduced-form and structural VAR. Long-run impacts are computed at the 30-day ahead response to a unit shock. We also report standard errors in parentheses.