

A Joint Analysis of the Term Structure of Credit Default Swap Spreads and the Implied Volatility Surface

José Da Fonseca*

Katrin Gottschalk[†]

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Abstract

This paper presents a joint analysis of the term structure of credit default swap (CDS) spreads and the implied volatility surface. The rapid development of the CDS market has provided convenient products to extract credit risk, and its interaction with equity volatility has been analyzed in many studies. However, in most of them the 5-year credit default swap spread is used to measure credit risk, whilst the at-the-money 1-month implied volatility is used to measure equity volatility. Only very few studies analyze the entire smile and the term structure of CDS spreads.

The purpose of this paper is to study the co-movements of the term structure of credit default swap spreads and the implied volatility surface. We perform a factor decomposition for both the dynamics of the implied volatility surface and the CDS curve. Then we jointly analyze the factors. More precisely, we compute the information flow between the credit and volatility factors, scrutinize the contemporaneous interactions between the factors, and study the efficiency of cross-hedges between the credit and volatility markets.

Using time series of options and CDS curves for the U.S. and European markets, we find that the credit market is the main contributor to overall market innovations. Our methodology is parsimonious and allows to capture the intrinsic relationships between the two markets. The empirical study highlights the existing cross-market linkages during the Global Financial Crisis. It also underlines that factors with small associated eigenvalues can be of tremendous importance to perform efficient cross-hedges.

JEL Classification: C14, C58, G12, G13.

Keywords: Credit Default Swap, Term Structure, Implied Volatility Surface, Factor Decomposition, Information Flow, Market Linkages, Cross-Hedging.

*Corresponding Author: Auckland University of Technology, Business School, Department of Finance, Private Bag 92006, 1142 Auckland, New Zealand. Phone: ++64 9 9219999 extn 5063. Email: jose.dafonseca@aut.ac.nz.

[†]Auckland University of Technology, Business School, Department of Finance, Private Bag 92006, 1142 Auckland, New Zealand. Phone: ++64 9 9219999 extn 5707. Email: katrin.gottschalk@aut.ac.nz.

1 Introduction

The aim of this paper is to provide a joint analysis of the term structure of credit default swap (CDS) spreads and the implied volatility surface. The link between credit spreads and equity volatility is central to Merton's model and this relation has been studied extensively in the literature, see Campbell and Taksler (2003) and Collin-Dufresne et al. (2001), among many others. The rapid development of the credit default swap market has provided convenient products to extract credit risk, and its interaction with equity volatility has been studied in detail, see Benkert (2004), Forte and Pena (2009), and Zhang et al. (2009). In most of these studies the 5-year CDS spread is used to measure credit risk because it is the most liquid point of the curve, whilst the at-the-money (ATM) 1-month implied volatility is used to measure equity volatility.

However, the skewness of the smile and the slope of the credit default swap curve contain important information. For example, Cremers et al. (2008) analyze the impact of both implied volatility (ATM) and the implied volatility skew on corporate *bond* credit spreads (long and short maturities) and find that these variables have strong explanatory power. Carr and Wu (2010) find a significant correlation between the smile and the skew and the *average* (along the term structure axis) of the CDS spread on corporate data. Carr and Wu (2007) analyze the interaction between sovereign CDS spreads and currency options. The smile dynamics is synthesized through option strategies (straddles, risk reversals, butterfly spreads) that capture different aspects of the smile, such as the level or the slope, whilst each CDS maturity is studied individually. They find a strong relation between all these variables. Cao et al. (2010) analyze the 5-year CDS spread along with the at-the-money implied volatility and the implied volatility skew. Hui and Chung (2011) study the 10-delta dollar-euro implied volatility in relation to the *5-year* sovereign credit default swap spread. These authors conclude that both level and slope of the smile contain relevant information for the credit market.

The term structure of interest rates is known to convey relevant economic and financial information, see Viceira (2012) and António and Martins (2012), to name only a few studies. In comparison, the term structure of CDS spreads has attracted less attention, although some recent works underline its importance. For example, Zhang (2008) studies the default risk premium for Argentine sovereign debt jointly with some macroeconomic variables, whilst Pan and Singleton (2008) apply a similar framework to Mexico, Turkey, and Korea. Han and Zhou (2010) find that the term structure of CDS spreads explains log stock returns, hence the slope of the CDS curve contains relevant information for

the stock dynamics. To sum up, the slope of the CDS curve carries important financial information beyond the level of the CDS curve (usually given by the 5-year CDS spread).

The purpose of this work is to study the co-movements of the term structure of credit default swap spreads and the implied volatility surface. However, we will not restrict our analysis to a part of the CDS curve (such as the 5-year CDS spread) or the ATM skew for the implied volatility. Both of these objects will be analyzed globally. This aspect is essential if we want to manage portfolios of options and CDSs as inevitably they will contain different option maturities and strikes and different CDS maturities. Managing credit risk requires an understanding of the dynamics of the CDS curve, whilst for volatility risk knowing the dynamics of the implied volatility surface is crucial. Once this is achieved a joint analysis of these risks can be undertaken, one important application being the hedging of credit risk with volatility products,¹ see for example Carr and Wu (2011), and risk management at the portfolio level (i.e. at an aggregate level).

As the implied volatility surface and the term structure of CDS spreads are multidimensional, we first have to perform a principal component analysis in order to reduce their dynamics to a small number of factors. For the smile we follow the methodology proposed in Cont and Da Fonseca (2002) and build a factor decomposition for the dynamics of the implied volatility surface.² We find that the usual three factors (level, slope, and curvature) mainly explain the dynamics of the surface. As for the interest rate yield curve we can decompose the dynamics of the CDS curve with the usual three factors (level, slope, and curvature). Therefore, we can summarize the movements of the implied volatility surface as well as the CDS curve with few factors. Then, we can analyze these factors to understand the joint dynamics. We perform our analysis on a time series of implied volatility surfaces and credit default swap curves for the pairs S&P 500/CDX.NA.IG and Euro Stoxx 50/iTraxx Europe for the period 2007-2011. Our sample covers the Global Financial Crisis and, taking into account the very particular role played by the credit market during this period, our study allows to understand the cross-market linkages between these two derivatives markets.

We obtain the following results: first, using a similar methodology as in Hui and Chung (2011), we study the information flow between the credit and volatility markets and find that the former is the main contributor to overall market innovations. Second, by computing correlations between

¹We can also consider hedging volatility risk using credit products, but this seems of less interest.

²For related works on factor decomposition of the implied volatility surface see Skiadopoulos et al. (1999), Fengler et al. (2003), Fengler et al. (2007), Chalamandaris and Tsekrekos (2010) and Chalamandaris and Tsekrekos (2012).

contemporaneous factor changes and with the known relations between log stock returns and the credit/volatility market, we get a complete picture of the joint dynamics of these two derivatives markets. Therefore, our results underline intrinsic relations between the term structure of CDS spreads and the implied volatility surface. Third, we perform a regression analysis between contemporaneous factors that allows us to devise cross-hedging strategies between the credit and volatility markets. We find that the ability to achieve effective cross-hedging varies over time. When some market factors allow to perform effective cross-hedges, this does not necessarily imply effectiveness of the reciprocal cross-hedges. Lastly, factors with small eigenvalues can be very important from a cross-hedging point of view.

The structure of the paper is as follows. In the first part, we describe the data along with some descriptive statistics. In a second part, we present the factor decomposition for both the credit default swap spreads and the implied volatility surfaces. The third part contains the empirical results: the information flow analysis between the two markets, the contemporaneous analysis as well as the cross-hedging analysis. The last part concludes the paper. All tables and figures are gathered in the appendix.

2 Data Description

A credit default swap (CDS) is a credit derivative contract between two counterparties that essentially provides insurance against the default of an underlying entity. In a CDS, the protection buyer makes periodic payments to the protection seller until the occurrence of a credit event or the maturity date of the contract, whichever is first. The premium paid by the buyer is denoted as an annualized spread in basis points and referred to as CDS spread. If a credit event (default) occurs on the underlying financial instrument, the buyer is compensated for the loss incurred as a result of the credit event, i.e. the difference between the par value of the bond and its market value after default.

Our dataset comprises the evolution of the term structure of credit default swap spreads for both the U.S. market, given by the index CDX.NA.IG, and the European market, given by the index iTraxx Europe. We collect daily time series for both indices from Markit at maturities of 0.5, 1, 2, 3, 5, 7, and 10 years from January 24, 2007 to December 30, 2011. As the Global Financial Crisis is contained in our sample, we split the full 5-year period into two sub-samples for all our analyses. The

first sub-sample (January 24, 2007 - November 12, 2009) spans the turbulent crisis period, while the second sub-sample (November 13, 2009 - December 30, 2011) is more tranquil.

Figures 1 and 2 clearly reflect the turmoil of the Global Financial Crisis from mid-2007 onwards, with CDS levels peaking around the default of Lehman Brothers in September 2008. Maximum CDS spread levels reached in the U.S. (>500 basis points) are almost double the maximum levels reached in Europe (>300 basis points). While CDS spreads come down in mid-2009 and the term structure returns to a normal positively-sloped shape, the onset of the European debt crisis is visible in the iTraxx Europe index from mid-2010 onwards when CDS prices start to rise again.

[Insert Figure 1 here]

[Insert Figure 2 here]

Summary statistics for the CDS indices CDX.NA.IG and iTraxx Europe are reported in Table 1 for the full sample and two sub-samples. The mean CDS spread level for the CDX.NA.IG index ranges from 99 basis points for the shortest maturity (6 months) to 141 basis points for the longest maturity (10 years). The shorter maturities display significantly higher volatility than the longer maturities, with standard deviations between 105 basis points (6 months) and 60 basis points (10 years). European CDS spreads are lower across all maturities for the sample period 2007-2011, with mean CDS spreads ranging from 69 basis points (6 months) to 119 basis points and standard deviations between 72 basis points (6 months) and 47 basis points (10 years).

The first sub-sample (Jan 2007 - Nov 2009) displays significantly higher CDS prices and elevated volatility due to the Global Financial Crisis. In fact, the term structure of the CDX.NA.IG is now almost flat, with mean CDS spreads between 140 basis points (6 months) and 150 basis points (5 years). This stands in stark contrast to the second sub-sample (Nov 2009 - Dec 2011), when mean CDS spreads fall between 37 basis points (6 months) and 131 basis points (10 years). The steeper slope of the term structure is accompanied by drastically reduced volatility. The same observation applies to the iTraxx Europe index, although the differences between turbulent and tranquil time periods are somewhat less pronounced than for the CDX.NA.IG.

[Insert Table 1 here]

The implied volatility surface is constructed from European call and put options on the S&P 500 index for the United States and on the Euro Stoxx 50 index for Europe. Daily prices of all available options

were obtained from Datastream.

3 Factor Decompositions of CDS Spreads and the Implied Volatility Surface

Our purpose is to study the joint dynamics of the term structure of credit default swap spreads and the implied volatility surface. As both of them are multidimensional, we need to perform a factor decomposition in order to reduce the dimension. For the CDS curve we proceed as for the interest rate curve, see Litterman and Scheinkman (1991), whilst for the implied volatility surface we follow the approach proposed in Cont and Da Fonseca (2002), see also Skiadopoulos et al. (1999).

3.1 The Term Structure of CDS Spreads

The term structure of credit default swap spreads for the U.S. and European markets is given by the indices CDX.NA.IG and iTraxx Europe, respectively, as described in the Data section. Since the CDS curves have similar properties as the yield curve, we can apply a well-established factor decomposition. Denoting by $\{\ln \text{CDS}(t, \tau_i); i = 1 \dots N_1\}$ the time series of CDS spreads (logarithm) for the available maturities we can compute, using $\Delta x_t(\tau_i) = \ln \text{CDS}(t, \tau_i) - \ln \text{CDS}(t-1, \tau_i)$, a principal component analysis decomposition. Figures 3 and 4 show the eigenvectors for the U.S. and European markets, respectively. The corresponding eigenvalues are reported in Table 2. First, we note that both markets lead to the same decompositions, a result which is similar to what is obtained in yield curve studies. The first eigenvector is always positive and corresponds to a shift of the CDS spread curve. Its associated eigenvalue dominates as it represents a large fraction of the global variance (around 95% for both markets). The second eigenvector implies a change of the slope because the short-term part is positive whereas the long-term part is negative, and the second eigenvalue accounts for 2.5% of the global variance. The third factor has a U-shaped form and is related to a change of the convexity of the term structure. Similar to yield curve factor decompositions the third eigenvalue only represents a very small fraction of the global volatility. The overall results resemble what is obtained for yield curves in the sense that we get the usual level, slope and curvature factor decomposition, which is already fairly obvious from Figures 1 and 2. It is not necessary to go beyond the first three factors as their sum amounts to 98% of market volatility.

[Insert Figure 3 here]

[Insert Figure 4 here]

[Insert Table 2 here]

3.2 The Implied Volatility Surface

To build an implied volatility surface on which we can apply a factor decomposition we follow the approach developed in Cont and Da Fonseca (2002). If we denote by $c_{bs}(t, s_t, K, T, \sigma)$ the Black-Scholes formula for a European option (either call or put) at time t , with maturity T , strike K , when the stock price at time t is s_t and volatility σ , then the implied volatility for an option whose market price is $c(t, s_t, K, T)$ is the number $\sigma_t^{bs}(T, K)$ such that

$$c_{bs}(t, s_t, K, T, \sigma_t^{bs}(T, K)) = c(t, s_t, K, T) \quad (1)$$

As the Black-Scholes formula is monotonic with respect to volatility, this equation has a unique solution and the function

$$\sigma_t^{bs} : (K, T) \rightarrow \sigma_t^{bs}(K, T) \quad (2)$$

is called the implied volatility surface. We can parametrize this function in terms of time to maturity and moneyness ($m = \frac{K}{s_t}$), so we define the function: $I_t(m, \tau) = \sigma_t^{bs}(ms_t, t + \tau)$. As this surface is usually non-flat and exhibits a U-shaped form for all times to maturity with less convexity for long-term options, this surface is often referred to as the smile.³ Lastly, this smile fluctuates over time.

On the market for a given day t we observe a set of implied volatility values $\{I_t(m_i, \tau_i); i = 1 \dots N_t\}$ defined on a grid of pairs $\{(m_i, \tau_i); i = 1 \dots N_t\}$ that will change over time because as the underlying stock moves the available moneynesses will change because an option has a fixed strike. Similarly, as time passes the options get closer to their maturities so the available times to maturity will change over time. However, to perform a factor decomposition for the implied volatility surface we need to build a smile which is parametrized by a *fixed* grid of time to maturity and moneyness. To this end we interpolate by using a non-parametric Nadaraya-Watson estimator based on an independent bivariate Gaussian kernel as in Cont and Da Fonseca (2002), see also Carr and Wu (2010). This allows us to define a time series of implied volatility surface denoted $\{I_t(\bar{m}_j, \bar{\tau}_j); j = 1 \dots N\}$ on a fixed grid of

³More precisely, on the equity/index derivatives market we observe a smirk for each time to maturity.

points $\{(\bar{m}_j, \bar{\tau}_j); j = 1 \dots N\}$. To be more precise we compute the following quantities:⁴

$$I_t(\bar{m}_j, \bar{\tau}_j) = \frac{\sum_{l=1}^{N_t} I_t(m_l, \tau_l) g(\bar{m}_j - m_l, \bar{\tau}_j - \tau_l)}{\sum_{l=1}^{N_t} g(\bar{m}_j - m_l, \bar{\tau}_j - \tau_l)} \quad (3)$$

where $g(x, y) = e^{-x^2/(2h_1)} e^{-y^2/(2h_2)}$, with (h_1, h_2) being the bandwidth parameters of the kernel. For the optimal choice of these parameters we refer to the classical literature, see Härdle (1990). However, we found more convenient to set directly the values⁵, see table for precise values. Also, we only consider options with a time to maturity greater than one week to avoid the volatility spikes related to very short term options, see Cao et al. (2010) where a similar problem is underlined.

Having built a daily time series $\{I_t(\bar{m}_j, \bar{\tau}_j); j = 1 \dots N\}$, we can perform a factor decomposition for the CDS spread curves. Given the high autocorrelation, skewness, and positivity constraints on the implied volatility itself, we focus on daily variations of the logarithm of implied volatility. Thus, using $\Delta X_t(\bar{m}_j, \bar{\tau}_j) = \ln I_t(\bar{m}_j, \bar{\tau}_j) - \ln I_{t-1}(\bar{m}_j, \bar{\tau}_j)$ we can perform a factor decomposition and denote by $\{e_k(\bar{m}_j, \bar{\tau}_j); j = 1 \dots N\}$ and λ^k the k^{th} eigensurface and eigenvalue, respectively.⁶ Note that we have $\sum_{j=1}^N e_{k_1}(\bar{m}_j, \bar{\tau}_j) e_{k_2}(\bar{m}_j, \bar{\tau}_j) = \delta_{k_1 k_2}$, with $\delta_{k_1 k_2}$ the Kronecker function. Having these key quantities available, we can analyze the shape of the factors underlying the dynamics of the smile of our data.

The first three eigensurfaces are reported in Figures 5, 6, and 7 for the options on the S&P 500 and in Figures 8, 9, and 10 for the options on the Euro Stoxx 50. Table 3 contains the corresponding eigenvalues (expressed as a percentage of the global variance). Both options sets lead to same-shaped factors as well as the same eigenvalue decomposition. Since the first eigensurface is always positive, it is associated with a translation or shift of the smile. As the first eigenvalue accounts for 89% of the global variance, we conclude that a one-factor model, based on this eigensurface, provides a reasonably good model for the dynamics of the smile. If a more accurate model is required, then we need to go beyond this first factor. The second eigensurface is, for all times to maturity, positive for moneyness lower than one and negative otherwise. A shock along this mode implies that out-of-the-money (OTM) put options, whose volatility is given by the smile with moneyness lower than one, will become more expensive. OTM call options, whose volatility is given by the smile with moneyness

⁴For the CDS spreads this transformation is not needed as they are always quoted with the same time to maturity. A similar remark applies to FX options which are quoted in terms of fixed time to maturity and delta, see Chalamandaris and Tsekrekos (2010), and Chalamandaris and Tsekrekos (2012).

⁵For the S&P 500 we take $h_1 = 0.006$ and $h_2 = 0.14$, for the Euro Stoxx 50 $h_1 = 0.008$ and $h_2 = 0.09$.

⁶From a computational point of view we just need to stack column-wise all the columns of the matrix $\Delta X_t(\bar{m}_j, \bar{\tau}_j)$, compute the PCA decomposition, and rewrite the obtained eigenvectors in matrix form, by reversing the procedure, to get the eigensurfaces.

greater than one, will become less expensive. As a consequence, this eigensurface is associated with a bear market movement. The corresponding eigenvalue represents 7.5% of the total variance. Lastly, the third factor, given by Figure 7, is associated with a bull market movement. A shock along this eigensurface implies a decrease of long-term implied volatility for all times to maturity, a strong increase of short-term OTM call prices and a minor (negligible) increase of short-term OTM put prices. Its eigenvalue is equal to 2.8% of the total variance. As the first three eigenvalues account for 98% of the total variance, it is not necessary to go beyond these three factors.

[Insert Figure 5 here]

[Insert Figure 6 here]

[Insert Figure 7 here]

[Insert Figure 8 here]

[Insert Figure 9 here]

[Insert Figure 10 here]

[Insert Table 3 here]

Having built these factors, we can decompose the dynamics of the smile into these fundamental modes. We define the three scalar processes

$$\Delta \text{VOL}_{k,t} = \sum_{j=1}^N \Delta X_t(\bar{m}_j, \bar{\tau}_j) e^k(\bar{m}_j, \bar{\tau}_j) \quad k = 1, 2, 3 \quad (4)$$

which are the projection of the implied volatility change on the eigensurfaces, hence each one quantifies to which extent the smile "moves" along the direction given by the corresponding factor. Therefore, we will have $\Delta \text{VOL}_{1,t}$, which is associated with a shift of the smile, $\Delta \text{VOL}_{2,t}$, which is associated with a change of the skew (slope) of the smile, and $\Delta \text{VOL}_{3,t}$, which is associated with a change of the convexity of the smile. Note that we could have used other functions to decompose the dynamics of the implied volatility surface. The principal component analysis relates the functions used to the covariance structure of the process. The factor decomposition allows us to reduce the dynamics of the smile, which is a *surface*, into three scalar time series that encompass most of the statistical properties. In order to gain further understanding of the factors, it is fruitful to compute the correlation between $\Delta \text{VOL}_{k,t}$ and the log stock returns $\Delta \ln s_t = \ln s_t - \ln s_{t-1}$ for all factors. In Table 4 we report the results, which are consistent with intuition. The correlation between log stock returns and the first factor is negative because if the stock market goes down, the overall surface will go up due to the

leverage effect. The second correlation coefficient is negative because if the stock market goes down, it is a bear market configuration, which implies a steepening of the smile, hence an increase along the second factor. Lastly, if the market goes up, it is a bull market configuration, which implies an increase along the third factor, hence a positive correlation coefficient.

[Insert Table 4 here]

4 Cross-Market Linkages

The interaction between option implied volatility and credit default swap spreads has been studied in prior work. For the former either the at-the-money short-term volatility or the short-term slope of the smile is used, whilst for the latter the 5-year CDS spread is used. Our aim is to further investigate the interaction between these two markets, in order to reveal the existing cross-market linkages. Thanks to our factor decompositions we can analyze the interaction between the whole implied volatility surface and the whole CDS curve.

4.1 Information Flow between CDS and Volatility Markets

In order to understand the relation between implied volatility and CDS spreads we follow the methodology proposed by Acharya and Johnson (2007). It allows us to quantify to which extent market-specific innovations explain the dynamics of another market. For example, we can measure how the CDS innovation impacts the volatility market and, obviously, we can reverse the analysis and evaluate how the volatility innovation spreads into the CDS market. Hence, we can have a complete picture of the interaction between these two markets.

To implement this methodology we need to compute the lead-lag relationships between the CDS market and the volatility market through their respective factor decompositions. As we have three factors for each market, we first focus on the information flow between the factors of the same order. More precisely, we first compute

$$\Delta\text{CDS}_{1,t} = a + b\Delta\text{VOL}_{1,t} + \sum_{k=1}^N c_{1,k}\Delta\text{CDS}_{1,t-k} + \epsilon_{\text{CDS}_{1,t}}. \quad (5)$$

Hence, $\{\epsilon_{\text{CDS}_{1,t}}\}$ represents the information specific to the credit market, given by the first CDS factor, that is not explained by the first volatility factor (plus lagged values of the first CDS factor). To measure the impact of the CDS market on the volatility market we estimate

$$\Delta \text{VOL}_{1,t} = \alpha + \sum_{k=1}^N \beta_k \epsilon_{\text{CDS}_{1,t-k}} + \sum_{k=1}^N \nu_k \Delta \text{VOL}_{1,t-k} + \epsilon_t. \quad (6)$$

If $I = \sum_{k=1}^N \beta_k$ is found to be statistically significant, then we conclude that an information flow exists from the CDS market to the volatility market. Conversely, we can study the pair

$$\begin{aligned} \Delta \text{VOL}_{1,t} &= a + b \Delta \text{CDS}_{1,t} + \sum_{k=1}^N c_{1,k} \Delta \text{VOL}_{1,t-k} + \epsilon_{\text{VOL}_{1,t}} \\ \Delta \text{CDS}_{1,t} &= \alpha + \sum_{k=1}^N \tilde{\beta}_k \epsilon_{\text{VOL}_{1,t-k}} + \sum_{k=1}^N \nu_k \Delta \text{CDS}_{1,t-k} + \epsilon_t, \end{aligned}$$

with $\tilde{I} = \sum_{k=1}^N \tilde{\beta}_k$, if significant, suggesting an information flow from the volatility market to the credit market. This methodology to quantify information flow through innovations was used by Acharya and Johnson (2007) and Berndt and Ostrovnaya (2008), and after some modifications was applied by Hui and Chung (2011) to study the interaction between European sovereign 5-year CDS spreads and FX options (the 10-delta volatility point). We follow their approach to quantify information flow. However, for both markets we consider all the quotes and thanks to the factor decompositions the main features of the dynamics can be analyzed.

As we have three factors for each market, we can compute the information flow between the second (third) CDS factor and the second (third) volatility factor. This allows us to measure the cross-market interaction between the same factors. We can also analyze the interaction between different factors. It is natural to study the cross-market impact of a higher factor on a lower factor because in practice the question is whether we should increase the number of factors, hence to which degree an additional factor is appropriate. Therefore, we compute the information flow from the second volatility factor to the first CDS factor, and also the information flow from the third volatility factor to both the second and first CDS factors. The first specification leads to the following pair of equations:

$$\begin{aligned} \Delta \text{VOL}_{2,t} &= a + b \Delta \text{CDS}_{1,t} + \sum_{k=1}^N c_{1,k} \Delta \text{VOL}_{2,t-k} + \epsilon_{\text{IVOL}_{2,t}} \\ \Delta \text{CDS}_{1,t} &= \alpha + \sum_{k=1}^N \tilde{\beta}_k \epsilon_{\text{IVOL}_{2,t-k}} + \sum_{k=1}^N \nu_k \Delta \text{CDS}_{1,t-k} + \epsilon_t. \end{aligned}$$

We perform the reverse analysis and quantify the impact of CDS factors on volatility factors.

As described in the data section, we split our sample in two parts, a choice mainly motivated by the behavior of the U.S. CDS market, and report in Table 5 the information flow from the credit market to the volatility market for the pairs S&P 500/CDX.NA.IG and Euro Stoxx 50/iTraxx Europe. The reverse information flow from the volatility market to the credit market is presented in Table 6, although we only find one significant value for the European indices in the first sub-sample.

[Insert Table 5 here]

[Insert Table 6 here]

In the first sub-sample we observe only information flows from the credit market to the volatility market for the U.S. as all statistically significant values imply such a relation. This is consistent with the crisis having its roots in the credit market. The table also suggests that the third and second credit factors contain relevant information for the volatility dynamics, although the channel is through the first factor. For the second sub-sample the conclusions are quite similar. The information flows go from the credit market to the volatility market. However, the third credit factor now carries less information because it is involved only once and the corresponding coefficient is significant at the 10% level only.

For the European market we obtain qualitatively similar results. The information flow goes from the credit market to the volatility market in both sub-samples and all the credit factors seem to provide some information. Note that the third credit factor has a statistically significant coefficient at the 1% level in the second sub-sample, which is less turbulent than the first sub-sample, possibly because of first signs of the sovereign credit crisis around May 2010 and more turmoil by end-2011. Although the iTraxx Europe is a corporate CDS index, it is impacted by the sovereign CDS market.

In conclusion, the information flow goes from the credit market to the volatility market, a finding that is consistent with those obtained by Hui and Chung (2011) for the pair 5-year sovereign CDS spread/10-delta foreign exchange option. Also, even if the eigenvalue decomposition suggests a one-factor model, the second and third credit factors contain relevant information, thus emphasizing the interest of multi-factor models for the dynamics of the CDS curve and the implied volatility surface.

4.2 Contemporaneous Interactions

So far we have analyzed cross-market information flow based on innovation as described, for example, by (5) and (6). In equation (5) $\{\epsilon_{\text{CDS}_{1,t}}\}$ is the innovation specific to the credit market not explained

by the *contemporaneous* first volatility factor and the lags of the first CDS factor when the dynamics is described by a one-factor model. Equation (6) allows to test the existence of a flow from the credit market to the volatility market through a Wald test of the coefficients. We now turn our attention to contemporaneous effects by computing the correlations between the different variables and report the results in Table 7 whilst Table 8 gives the correlation between log stock returns and factor changes that will be useful to analyze the results.

[Insert Table 7 here]

[Insert Table 8 here]

For both pairs of indices the correlations are consistent across the samples. Their signs remain largely the same although we can observe some minor changes. For example, the correlation between ΔCDS_2 and ΔVOL_2 for the U.S. turns from negative to positive as the sub-sample changes. Similarly, the correlation for the U.S. between ΔCDS_3 and ΔVOL_2 becomes statistically insignificant in the second sub-sample, whilst it is negative in the first sub-sample. We observe a positive correlation between ΔCDS_1 and ΔVOL_1 , thereby implying that an increase of the smile is associated with an increase of the CDS spread. If we take into account the negative correlation between log returns and volatility as well as the negative correlation between log returns and the first CDS factor, we end up with a consistent dynamics of the stock, the level of volatility, and the level of the CDS spread.

The correlation between ΔCDS_1 and ΔVOL_2 is positive, whereas the correlation between ΔCDS_1 and ΔVOL_3 is negative for all pairs and sub-samples. An increase of the CDS level implies more default risk. This is associated with a bear stock market configuration, which in turn implies a steeper skew, hence a positive correlation sign for ΔVOL_2 because of the interpretation developed in the factor decomposition of the smile. The third factor is associated with a bull market configuration so that an increase of the level of the CDS curve should produce the opposite effect, hence a negative sign for the correlation.

At this stage it becomes complicated to provide an explanation valid for both markets. If we start with the European one, which contains more significant values, it might be useful to analyze the values in conjunction with the correlation of log stock returns with the derivatives markets reported in Table 8. In the European case the factor ΔVOL_1 is positively correlated with either ΔCDS_2 or ΔCDS_3 because to an increase of this factor corresponds a decrease of the stock price with two consequences: it will increase the likelihood of a default according to Merton (1974), and it will increase the second factor and third factor due to the negative correlation between the log returns and these

factors. As a consequence, we must have a positive correlation. For the correlation sign between ΔCDS_2 and ΔVOL_2 , the positiveness at least in the European case can also be understood through the stock market. An increase of ΔVOL_2 implies a decrease of the stock price, which in turn implies an increase along ΔCDS_2 . Lastly, an increase of ΔCDS_2 leads to a decrease of the stock price, which due to positive correlation with ΔVOL_3 implies a decrease of this volatility factor, hence a negative correlation between the second CDS factor and the third volatility factor. By using the correlation of spot log returns with the credit factors and volatility factors, the sign whenever statistically significant obtained between the factors can be understood. For the U.S. the results are less evident to analyze although to some extent they are consistent with the European ones. It is interesting to note that for the S&P500 the correlation between the log returns and the credit factors changes when the subsample changes and that the correlation between the S&P500 returns and the first volatility factor is lower (in absolute value terms) than what is obtained for the European market. This loose relation with respect to the stock might be the reason for the changing results observed in the U.S. market.

The contemporaneous correlations provide a complementary point of view to the information flow developed in the previous section. The important ingredient that allows to understand the relations between the credit factors and volatility factors is the correlation between these factors and the stock returns. Because of the leverage effect between stock price and volatility, as explained in Black (1976), and the tight relation between stock price and credit risk, as shown by Merton (1974), the interactions between credit and volatility factors can be analyzed through their relation with the stock.

4.3 Cross-Hedging Between Credit and Volatility Factors

Now that we have a better understanding of the relationships between the different factors we focus on a regression analysis of the first factor (i.e. the main factor). More precisely, we regress the first volatility factor on a set of explanatory variables chosen among the credit factors. As a given factor is worthy of consideration only if lower-order factors are taken into account, the sets of variables will be nested. Since we have three credit factors, we perform three regressions. Also, we reverse the analysis by regressing the first credit factor on a set of volatility factors. These regressions are of practical interest as they allow us to devise cross-hedging strategies.

[Insert Table 9 here]

[Insert Table 10 here]

We report in Tables 9 and 10 the results for the U.S. and European markets, respectively. For the U.S. market the results are consistent with those reported in Table 7, the signs of the regression coefficients are coherent with the correlation coefficients. However, for the first period (01/2007-11/2009) the (adjusted) R^2 is very small for all regressions, implying that a hedging strategy based on these results is likely to perform poorly. Whether we try to hedge credit risk (given by the first credit factor) using options (given by the volatility factors) or to hedge volatility risk (given by the first volatility factor) using credit default swaps (given by the credit factors), the results will be poor. During the second period (12/2009-12/2011) the results are different and interesting. Regressing the first volatility factor on credit factors, whatever the number of factors chosen is, leads to small R^2 . On the contrary, regressing the first credit factor on volatility factors gives reasonably good R^2 and, most surprisingly, it is the second and third factor that improve the results. We can draw several conclusions. First, the ability to perform cross-hedging changes over time. Second, the volatility market can be used to hedge the credit market during the second period, but we cannot hedge the volatility market using the credit market, hence the relation is not reciprocal. Third, even if a factor has a small eigenvalue it can be useful for hedging purposes.

For the European market the results are significantly different. During the first period all the regressions have a high R^2 , ranging from 14% to 19% when the dependent variable is the first volatility factor and from 14% to 33% when this variable is the first credit factor. The point of interest is the increase of the R^2 when higher factors are taken into account. For example, the second factor improves the R^2 by around 5% in both cases, whilst the third volatility factor increases the R^2 by an amount of 13%. For the second sub-sample the regressions also lead to interesting remarks. During this period, contrarily to what is obtained in the first sub-sample, the volatility factor can no longer be hedged using the credit factors as all the R^2 are small. This confirms the conjecture that cross-market correlations change over time. However, we can still hedge the first credit factor using the volatility factors as these variables appear to have strong explanatory power. What is more, the highest factors (the second and third factor) are responsible for this result. In fact, the second factor increases the R^2 from 6% to 21%, whereas the third factor nearly doubles the R^2 from 21% to 38%. Hence, a factor with a small eigenvalue can be of tremendous importance for an efficient hedging strategy.

5 Conclusion

In this work we propose a joint analysis of the term structure of credit default swap spreads and the implied volatility surface. To carry out this analysis we develop a factor decomposition for both markets which allows us to study them globally. We do not need to restrict our data to a part of the CDS curve, such as the 5-year swap spread as done in previous studies, and/or a part of the smile, such as the 1-month ATM implied volatility. We implement our methodology on a database of options and the term structure of CDS spreads for the U.S. and European markets covering the Global Financial Crisis. We quantify the information flow between the two markets and find the credit market to be the main contributor to global market innovations. A correlation analysis between contemporaneous factors along with stock returns confirms the ability of our methodology to perform viable factor decompositions for the implied volatility surface and the CDS curve. These allow to handle the joint statistical properties of the two markets. We also perform a regression analysis which underlines the cross-hedging opportunities between the two markets. They change over time and are not reciprocal. Furthermore, factors with small eigenvalues can be very important from a cross-hedging point of view; this has strong consequences from a risk management perspective as the number of factors chosen for a model should not depend only on the eigenvalue decomposition.

Our work suggests some extensions. We analyze the credit market through credit default swap contracts but collateralized debt obligations (CDOs) could be used instead, see Carverhill and Luo (2011). As these products are similar to options, they might lead to more effective cross-hedges. The correlations between the factors found in our study impose some strong constraints if we were to develop a model in the spirit of Carr and Wu (2010). We leave these open issues for future research.

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Appendix

Tables and Figures

Maturity		0.5	1	2	3	5	7	10
(a) Full sample: January 24, 2007 - December 30, 2011								
CDX.NA.IG	Mean	98.965	103.724	109.488	116.907	132.447	136.264	141.171
	Std. dev.	105.191	103.483	93.448	87.215	76.608	67.674	60.171
iTraxx Europe	Mean	69.399	74.988	85.353	94.933	108.838	113.653	118.766
	Std. dev.	72.187	71.534	66.962	62.965	55.810	51.460	47.177
(b) First sub-sample: January 24, 2007 - November 12, 2009								
CDX.NA.IG	Mean	140.156	142.933	142.298	144.731	150.115	147.943	147.917
	Std. dev.	118.553	117.601	108.086	102.727	93.700	83.832	75.035
iTraxx Europe	Mean	77.759	81.895	88.579	94.894	102.825	105.244	108.559
	Std. dev.	90.183	89.002	82.694	76.873	65.929	58.652	51.466
(c) Second sub-sample: November 13, 2009 - December 30, 2011								
CDX.NA.IG	Mean	37.135	44.870	60.239	75.141	105.926	118.733	131.045
	Std. dev.	14.019	15.519	15.898	16.371	17.876	19.660	20.689
iTraxx Europe	Mean	57.297	64.989	80.683	94.988	117.543	125.826	133.541
	Std. dev.	27.228	29.946	32.346	33.998	34.767	35.434	35.324

Table 1: Descriptive statistics for the CDS indices CDX.NA.IG and iTraxx Europe for the full sample and sub-samples. Maturities range from 0.5 years to 10 years. Means and standard deviations are based on daily data. CDS spreads are expressed in basis points.

Eigenvalue	CDX.NA.IG	iTraxx Europe
first	96.5	94.9
second	2.2	2.5
third	0.8	1.9

Table 2: Eigenvalues as a percentage of the total variance (daily data, same sample as the one used to compute the eigenvectors).

Eigenvalue	S&P 500	Euro Stoxx 50
first	88.8	88.3
second	7.3	7.6
third	2.8	2.7

Table 3: Eigenvalues as a percentage of the total variance (daily data, same sample as the one used to compute the eigensurfaces).

	ΔVOL_1	ΔVOL_2	ΔVOL_3
S&P 500	-0.163	-0.629	0.507
Euro Stoxx 50	-0.360	-0.606	0.737

Table 4: Correlation between log returns $\Delta \ln s_t = \ln s_t - \ln s_{t-1}$ and the factors $\Delta\text{VOL}_{k,t}$ for $k = 1, 2, 3$ (daily data, same sample as the one used to compute the eigensurfaces).

	01/2007-11/2009			12/2009-12/2011		
	ΔCDS_1	ΔCDS_2	ΔCDS_3	ΔCDS_1	ΔCDS_2	ΔCDS_3
Panel A: S&P 500 - CDX.NA.IG						
ΔVOL_1	1.482***	-7.196***	-9.128***	7.066***	16.998***	-11.672*
ΔVOL_2	—	-0.146	0.156	—	2.695***	-1.095
ΔVOL_3	—	—	2.055***	—	—	2.640
Panel B: Euro Stoxx 50 - iTraxx Europe						
ΔVOL_1	2.250***	2.403	1.195	4.523***	12.003***	23.996***
ΔVOL_2	—	1.555***	2.467***	—	-0.747	-1.766
ΔVOL_3	—	—	-0.042	—	—	-0.957

Table 5: Information flow from CDS market to volatility market for the pairs S&P 500/CDX.NA.IG and Euro Stoxx 50/iTraxx Europe for two subsamples. *** denotes statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

	01/2007-11/2009			12/2009-12/2011		
	ΔVOL_1	ΔVOL_2	ΔVOL_3	ΔVOL_1	ΔVOL_2	ΔVOL_3
Panel A: S&P 500 - CDX.NA.IG						
ΔCDS_1	-0.001	0.012	0.034	-0.006	-0.047	0.025
ΔCDS_2	—	-0.008	-0.005	—	0.008	-0.001
ΔCDS_3	—	—	0.004	—	—	-0.006
Panel B: Euro Stoxx 50 - iTraxx Europe						
ΔCDS_1	0.005	0.108**	0.049	0.019	0.054	-0.017
ΔCDS_2	—	0.017	0.018	—	0.010	-0.005
ΔCDS_3	—	—	0.007	—	—	-0.001

Table 6: Information flow from volatility market to CDS market for the pairs S&P 500/CDX.NA.IG and Euro Stoxx 50/iTraxx Europe for two subsamples. *** denotes statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

	01/2007-11/2009			12/2009-12/2011		
	ΔCDS_1	ΔCDS_2	ΔCDS_3	ΔCDS_1	ΔCDS_2	ΔCDS_3
Panel A: S&P 500 - CDX.NA.IG						
ΔVOL_1	0.072*	0.027	-0.024	0.060	-0.016	0.020
ΔVOL_2	0.166***	-0.139***	-0.151***	0.328***	0.090*	0.051
ΔVOL_3	-0.154***	0.137***	0.142***	-0.369***	-0.043	0.121***
Panel B: Euro Stoxx 50 - iTraxx Europe						
ΔVOL_1	0.373***	0.322***	0.245***	0.248***	0.216***	0.177***
ΔVOL_2	0.350***	0.091**	0.003	0.451***	0.315***	0.350***
ΔVOL_3	-0.532***	-0.315***	-0.307***	-0.501***	-0.397***	-0.400***

Table 7: Cross-market factor correlations for the pairs S&P 500/CDX.NA.IG and Euro Stoxx 50/iTraxx Europe for two subsamples. *** denotes statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

	ΔVOL_1	ΔVOL_2	ΔVOL_3	ΔCDS_1	ΔCDS_2	ΔCDS_3
01/2007-11/2009						
S&P 500	-0.163	-0.629	0.507	-0.257	0.190	0.221
Euro Stoxx 50	-0.360	-0.606	0.737	-0.609	-0.207	-0.165
12/2009-12/2011						
S&P 500	-0.102	-0.640	0.497	-0.619	-0.164	0.065
Euro Stoxx 50	-0.233	-0.544	0.605	-0.740	-0.492	-0.477

Table 8: Correlation between log returns $\Delta \ln s_t = \ln s_t - \ln s_{t-1}$ and the factors ΔVOL_k and ΔCDS_k for $k = 1, 2, 3$.

S&P 500 - CDX.NA.IG						
	01/2007-11/2009			12/2009-12/2011		
	Dependent variable: ΔCDS_1					
ΔVOL_1	0.014*	0.007	0.004	0.005	0.001	-0.005
ΔVOL_2		0.106***	0.093***		0.121***	0.104***
ΔVOL_3			-0.087**			-0.090***
Adj. R^2	0.003	0.030	0.034	0.000	0.123	0.156
Dependent variable: ΔVOL_1						
ΔCDS_1	0.336*	0.337*	0.332*	0.461	0.629	0.669
ΔCDS_2		0.824	0.837		-2.021	-2.090
ΔCDS_3			-0.104			2.495
Adj. R^2	0.003	0.003	0.001	0.000	0.000	0.000

Table 9: Cross-market factor regressions for the pair S&P 500/CDX.NA.IG for two subsamples. Regression intercepts have been suppressed in order to conserve space. *** denotes statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

Euro Stoxx 50 - iTraxx Europe						
	01/2007-11/2009			12/2009-12/2011		
Dependent variable: ΔCDS_1						
ΔVOL_1	0.066***	0.051***	0.026***	0.039***	0.019***	0.011*
ΔVOL_2		0.166***	0.106***		0.266***	0.219***
ΔVOL_3			-0.314***			-0.287***
Adj. R^2	0.139	0.195	0.331	0.060	0.213	0.377
Dependent variable: ΔVOL_1						
ΔCDS_1	2.128***	1.702***	1.706***	1.603***	1.312***	1.350***
ΔCDS_2		5.688***	4.328***		1.890	2.710
ΔCDS_3			4.554**			-2.847
Adj. R^2	0.139	0.181	0.185	0.060	0.059	0.057

Table 10: Cross-market factor regressions for the pair Euro Stoxx 50/iTraxx Europe for two subsamples. Regression intercepts have been suppressed in order to conserve space. *** denotes statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

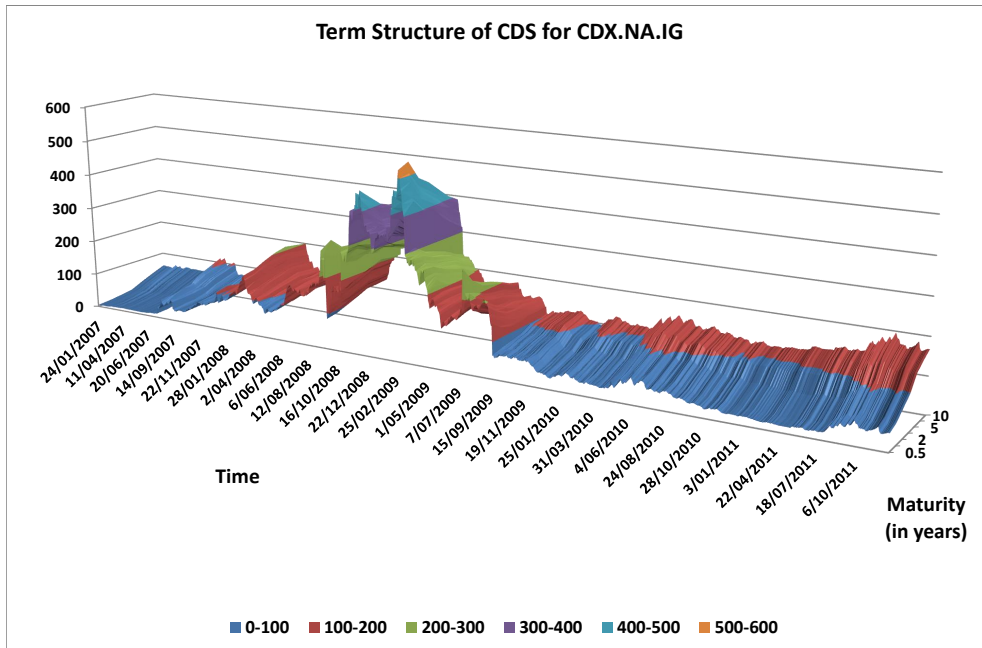


Figure 1: Term structure of CDS spreads for the index CDX.NA.IG. Daily observations from 24/1/2007 to 30/12/2011.

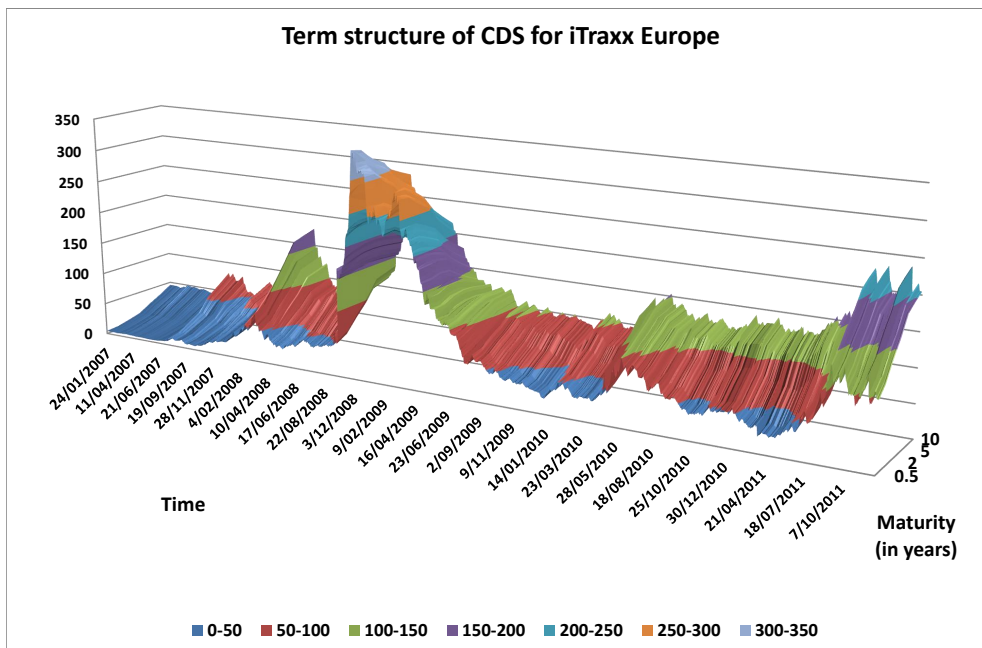


Figure 2: Term structure of CDS spreads for the index iTraxx Europe. Daily observations from 24/1/2007 to 30/12/2011.

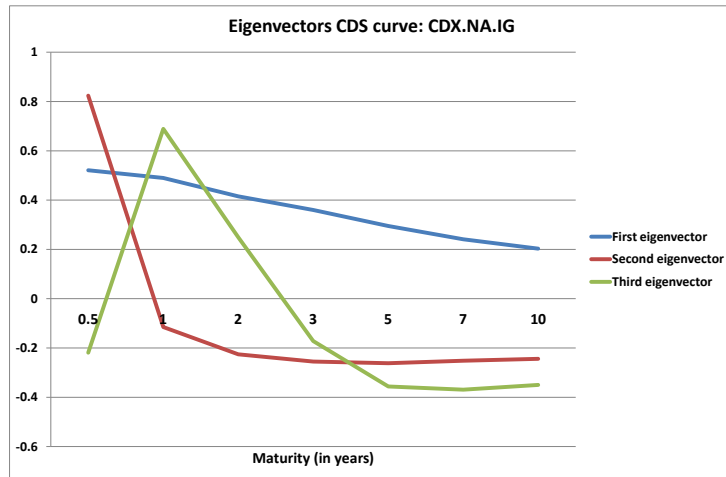


Figure 3: Eigenvectors for the CDS curve for the CDX.NA.IG, computed using a one-year daily sample (2007).

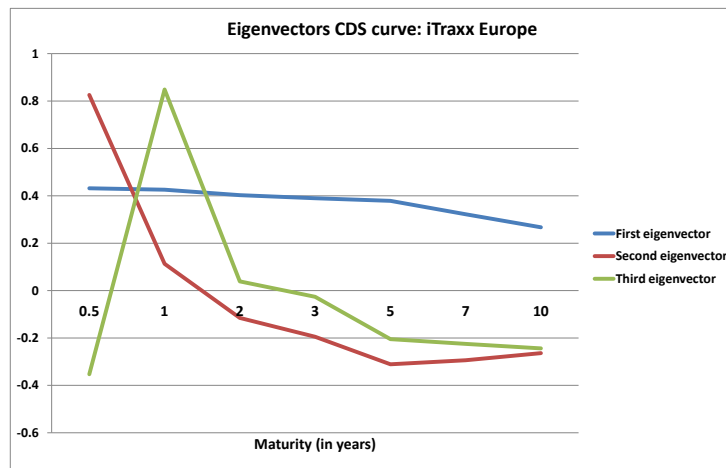


Figure 4: Eigenvectors for the CDS curve for the iTraxx Europe, computed using a one-year daily sample (2007).

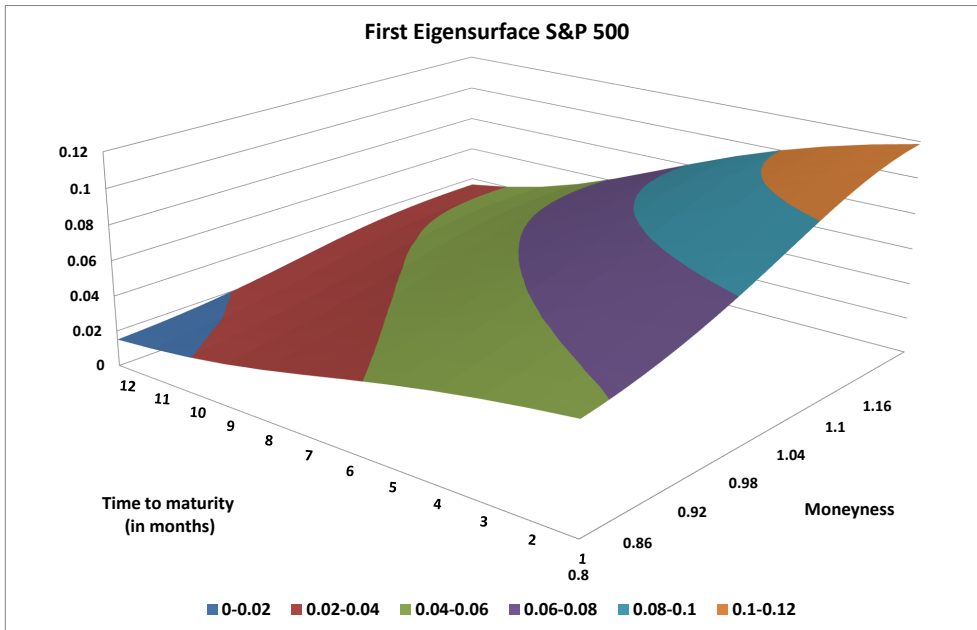


Figure 5: First eigensurface for the S&P 500, computed using a one-year daily sample (2007).

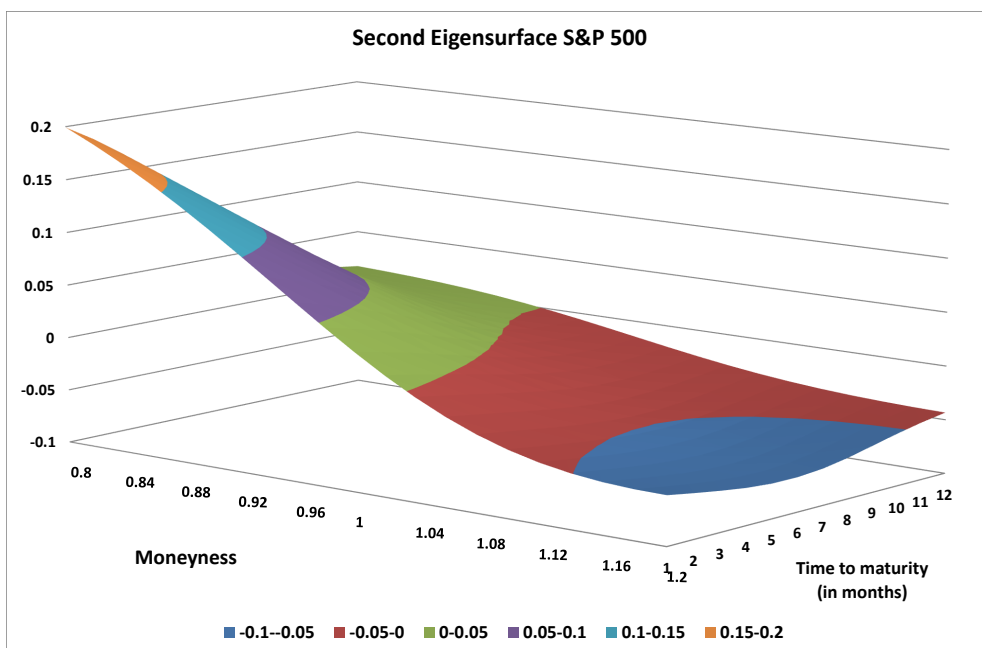


Figure 6: Second eigensurface for the S&P 500, computed using a one-year daily sample (2007).

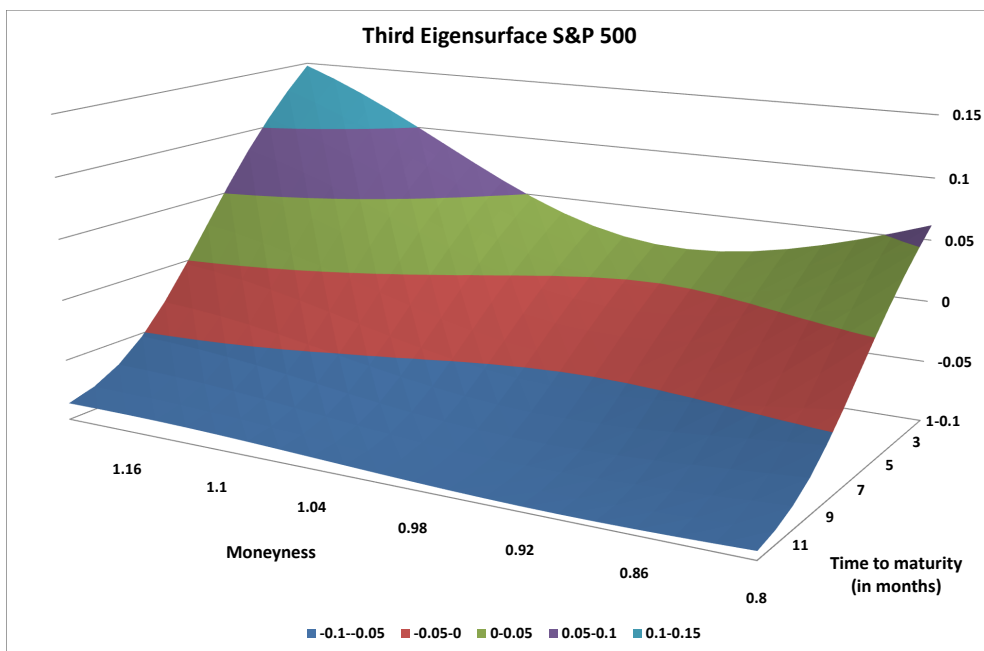


Figure 7: Third eigensurface for the S&P 500, computed using a one-year daily sample (2007).

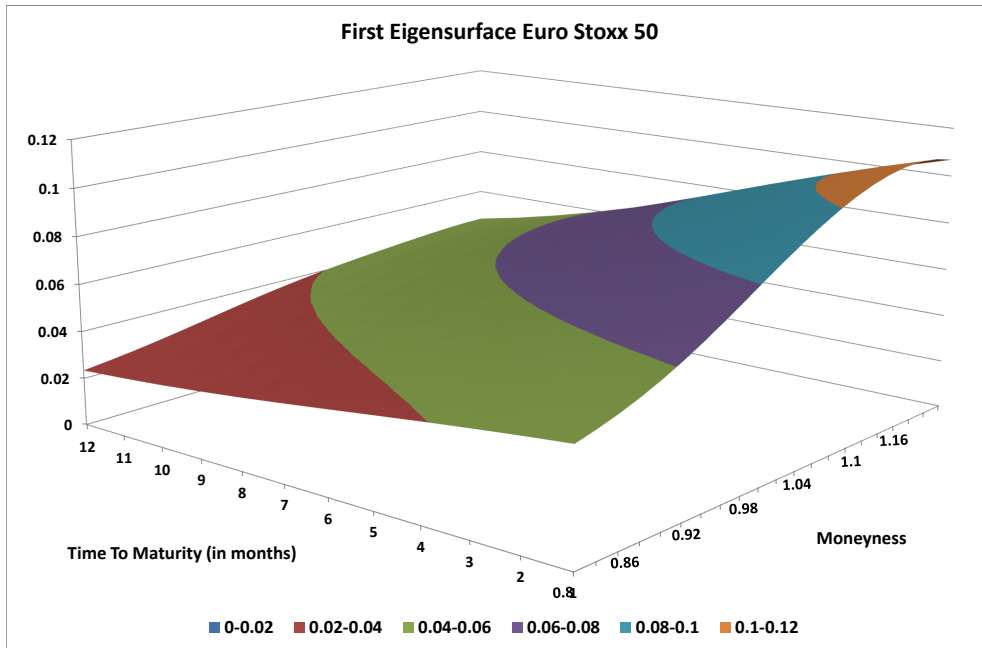


Figure 8: First eigensurface for the Euro Stoxx 50, computed using a one-year daily sample (2007).

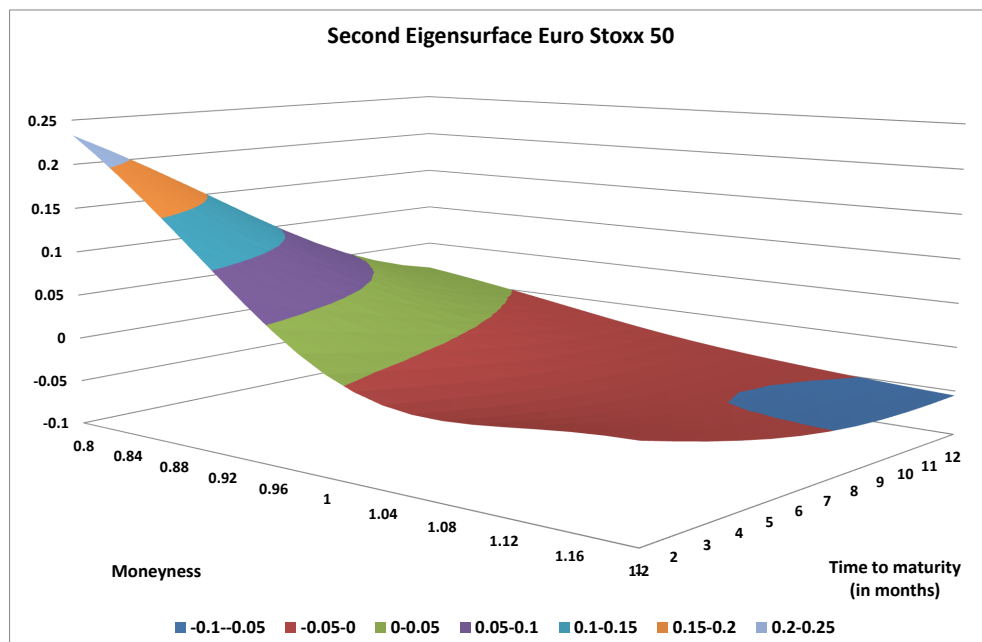


Figure 9: Second eigensurface for the Euro Stoxx 50, computed using a one-year daily sample (2007).

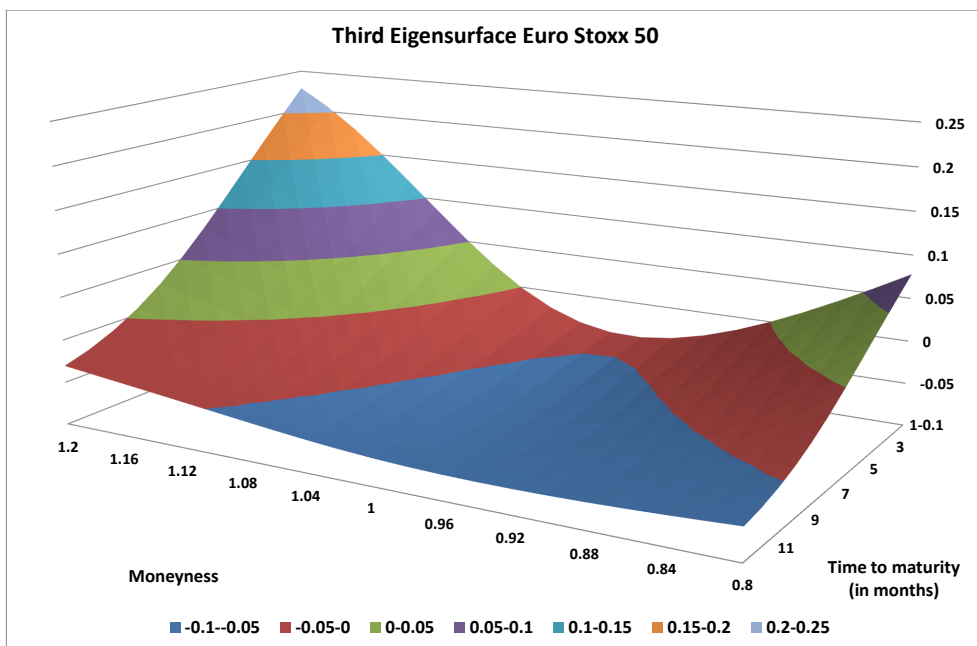


Figure 10: Third eigensurface for the Euro Stoxx 50, computed using a one-year daily sample (2007).