OPTION PRICING UNDER STOCK MARKET CYCLES WITH JUMP RISKS IN DOW JONES INDUSTRIAL AVERAGE INDEX AND S&P 500 INDEX

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Abstract

Hamilton (1989) proposed the regime-switching model to explain the different behaviors of mean and volatility for returns at different states of the stock market cycle. However, there are often abnormal jumps when the unanticipated information reaches the market such as during the periods of the internet bubble or the subprime crisis. Therefore, this study incorporates the regime-switching model with jump risks to model the behavior of stock returns in financial markets. We find that Dow Jones Industrial Average (DJIA) and S&P 500 suitable for the regime-switching model with jump risks based upon LR test from 1999 to 2010 using expectation-maximization (EM) algorithm and supplemented expectation-maximization (SEM) algorithm. In addition, both the regime-switching model and regime-switching model with jump risks can address the leptokurtic feature of the asset return distribution, volatility smile, and the volatility clustering phenomenon. Consequently, we develop the European option formula under stock market cycle with Poisson jump risks by capturing the stock market cycle and jump risks in the underlying assets. Finally, we offer some sensitivity analysis for the option pricing formula for a European call option.

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INTRODUCTION

The normality of economic behavior sometimes is disrupted by dramatic events. To capture the time-series behavior with business cycles, Hamilton (1989) pioneers in modeling changes in regime with a Markov chain process (also termed "Markov switching model"). Since the introduction of Markov switching models in the mainstream econometrics, it has received considerable attention from financial time-series analysis. There is a class of studies devoting to the forecasting of stock return, volatility, and the equity premium using Markov switching models.

Among these studies, Turner et al. (1989) is the earliest example of applying Markov- switching technique in describing the behavior of stock returns. They develop a two-regime Markov switching model whose transition probabilities remain constant. The main advantage of their model is to improve the accuracy of the stock return forecast under heteroscedasticity. Hamilton and Susmel (1994) distinguish a high-, median-, and low-volatility regime in stock return data, with the high-volatility regime being associated with economic recessions. A similar conclusion that volatilities are much higher in a bear market is also reached by Maheu and McCurdy (2000). Kim (2004) develops a stock return model with Markov switching volatility feedback effect to empirically test the positive relation between the equity premium and stock market volatilities. Extending the setting of regime-switching volatility, Kim et al. (2005) further examine the structural break in equity premium based on Bayesian margin likelihood analysis. More recently, Chen (2007) investigates the asymmetric effects of monetary policy on stock returns using Markov switching models. Note that above models all are based on an assumption that the dynamics of variables are continuous under a given regime. In brief, those studies ignore the discrete effects in describing the behavior of economic variables. To display the significance of such an effect, we take Dow Jones Industrial Average Index (hereafter DJIA) as an example. Figure 1 plots the index (Panel A) and daily returns (Panel B) of DJIA from Jan. 3, 1995 to Jan. 15, 2010.

[Insert Figure 1 here]

The data of sudden shocks in DJIA daily returns are given in Table I. Weak, median, and strong shocks are defined when the daily observation is over or below the level of single, double, and triple standard deviation of those computed by the full period. From Panel A of Figure 1, we observe that the index continued to grow during the period 1995-2000 due to the U.S. “new economy” effect, while in the second half of 2000, economy faced the burst of Internet bubbles. The expansion from 2003 to mid-2007 is attributed to the effect of oil-shock-based inflation. Global subprime mortgage crisis, however, appeared in 2008, and thus its aftermath prompts the depression. Such an undulating pattern for the index path is the so-called “stock market cycle” which can be captured well by existing models.

[Insert Table I here]

Compared to Panel B, it is notable that fluctuations in the daily returns are fiercer visibly, especially at the time where abnormal events occur (e.g., subprime mortgage crisis). The numbers shown in Table I give a clearer insight to the sudden shocks. For example, the ratio of strong-shock observation to total is 1.5048%, which approximates the likelihood of a strong sudden shock. Further, the means and variances of returns under shocks are also lower and higher respectively relative to those computed by full sample. These facts suggest that, as abnormal events strike the market, the index returns behave highly volatile within a short time period (e.g., 1 day). Such dynamic is obviously not in line with the assumption that time-series variables act continuously. Hence, the family of existing Markov switching models cannot explicitly capture the impacts of sudden shocks.

In 1976, Merton proposes the original type of jump-diffusion models.[1] He assumes that total changes in the assets prices can be divided into a normal variation part and an abnormal variation part. The former is modeled as a standard geometric Brownian motion with a fixed variance capturing continuous fluctuations in the prices because of strategic trading by informed or liquidity traders and the market microstructure effects; while the latter is modeled as a counting process that reflects discrete effects due to unanticipated information released to the public. To capture the sudden shock under switching regimes, this paper combines Markov regime-switching processes with jump risks, and jumps risk in the model are assumed to obey a Poisson process with a constant jump rate for the jump frequency and to follow a normal distribution for the jump sizes.

In this paper, we develop a estimating and testing methodology by the Expectation-Maximization (hereafter EM) algorithm (see Dempster et al., 1977), rather than the traditional maximum likelihood estimator for hidden states. The employment of the EM algorithm over MLE overcomes the problems of missing data and slow convergence (see Hamilton, 1990). Likelihood ratio test is also used in this study to compare the fitting performance of a standard-type regime-switching model with ours. The data we collected consists of Dow Jones Industrial Average Index and S&P 500

[1] There has been a vast amount of work on applying the jump-diffusion model in several dimensions. For example, option pricing (Kou, 2002; Kou and Wang, 2004; Hilliard and Schwartz, 2005; Duan et al., 2006; Ahn et al., 2007; and Feng and Linetsky, 2008;), financing structure (Dao and Jeanblanc, 2006; and Chen and Kou, 2009;), time-series analysis (Becker, 1981; Ball and Torous, 1985; Akgiray and Booth, 1988; Jorion, 1988; Bates, 1996; Pan, 2002; Eraker, 2004; Jiang and Oomen, 2007, and Lin, Wang and Tsai, 2009;), and term structure of interest rate and credit spreads (Ahn and Thompson, 1988; Duffie and Pan, 2001; Zhou, 2001; Das, 2002; Glasserman and Kou, 2003; Johannes, 2004; Guan et al., 2005;and Wang and Lin, 2010).

Index. Using the data, we test if the Markov switching model with jump risks fits better than regime-switching model. Based on the regime-switching model with jump risk, we derive the closed form formula for the European option and conduct sensitivity analysis.

The rest of this study is organized as follows. In the models of stock returns of Section 1, a regime-switching model with jump risks and a standard-type model are illustrated. Section 2 estimates the parameters and its variance using EM algorithm and SEM algorithm and give the empirical results in S&P 500 index and DJIA index. Section 3 evaluates European call options with the regime-switching model and the regime-switching model with jump risks based on the no-arbitrage theorem by he Esscher transform method. Numerical and empirical analyses are given in Sections 4. Section 5 draws the conclusion of this paper.

1. MODEL OF STOCK RETURNS

In the real world, the mean and volatility of a time series variable usually vary with market regimes. For instance, the mean of stock returns is positive in the bull market but negative in the bear market, while its volatility is significantly higher during a poor economic condition. Such structural changes in the economic series cannot be captured by traditional models, which assume that all the observations are drawn from a Gaussian distribution with fixed mean and variance throughout the sample period. Moreover, the Markov regime-switching models are especially useful for addressing financial phenomenon, such as leptokurtic feature of the asset return distribution, volatility smile, and the volatility clustering phenomenon.

1.1. Markov Switching Model

Consider a discontinuous trading economy. The uncertainty over this economy is defined on a finite space X consisting of states $1, 2, \ldots, I$. A standard-type Markov switching model governing the dynamics of stock returns with instantaneous mean u_{q_t} and constant volatility σ_{q_t} under the state $q_t \in X$ at time $t = 1, 2, \ldots, T$ is one that attaches a hidden Markov process following

$$R_t = u_{q_t} + \sigma_{q_t} Z_t \,, \tag{1.1}$$

where R_t denotes the stock return at time *t*; Z_t is a standard one-dimensional normal distribution at *t*; and q_t implicates the unobservable state of economy at *t* characterized by a hidden Markov Chain on X.

Consider Markov properties with one period which states that if the current state of the process is known, then the future behavior of the process is independent of its past. , we assume that the probability of Markov Chain and stock return satisfies

$$\Pr(q_t = i \mid q_{t-1}, \ldots, q_1) = \Pr(q_t = i \mid q_{t-1}) \,,$$

and

$$\Pr(R_t \mid R_{t-1}, q_{t-1}, \ldots, q_1) = \Pr(R_t \mid q_t) \,.$$

Now we are ready to form the migration of q_t as a transition probability matrix P with $p_{ij} = \Pr(q_{t+1} = j \mid q_t = i)$ for $\forall i, j \in X$ and $t = 1, 2, \ldots, T$:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1I} \\ p_{21} & p_{22} & \cdots & p_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ p_{I1} & p_{I2} & \cdots & p_{II} \end{pmatrix}.$$

Note that the sum of each entry in the same row must equal one; namely, $\sum_{j=1}^{I} p_{ij} = 1$ for $i \in S$. To define the initial state probability, let $\pi_i \equiv \Pr(q_1 = i)$ that satisfies $\sum_{i=1}^{I} \pi_i = 1$.

Since uncertainty associated with the stock market cycles is generally divided between expansion and recession, the standard Markov switching model can be reduced to a two-regime model (i.e., "regime-switching model"). A growing economy is classified as a recession ($q_t = 1$) one. In such a state $u_{q_t} = u_1$ and $\sigma_{q_t} = \sigma_1$. On the other hand, when the state is an expansion (i.e., $q_t = 2$), the value is taken as $u_{q_t} = u_2$ and $\sigma_{q_t} = \sigma_2$. Then we define a finite space containing all the relevant parameters as

$$\Theta_{RSM} \equiv \{\pi_i, p_{ij}, u_i, \sigma_i^2 \mid 0 < \pi_i, p_{ij} < 1, u_i \in R, \sigma_i^2 > 0, \forall i, j \in X, I = 2\}.$$

It has been mentioned that Markov switching–type models are successful in explaining the asymmetric behavior of time series caused by changes in the structural state. This is because this model can produce leptokurtosis (fat fail) and skewness (nonzero third-order cross-moments) in the return (see Krolzig, 1997; and Sola and Timmermann, 1998). Based upon our previous discussions, however, it is found that the arriving of unanticipated abnormal events delivers sudden shocks to the market states, hence the daily observations behave highly volatile within a short time period. Such dynamics cannot be captured using existing models that usually assume that variables act continuously under a given regime. To address this issue, in the following subsection a regime-switching model with jump risks will be proposed.

1.2. Regime-Switching Model with Jump Risks

The model we proposed combines a standard regime-switching process and a compound Poisson process N with a constant arrival rate of the abnormal events λ. First, let's consider an infinite sequence $\{Y_n\}_{n=1,2,\ldots,\infty}$ that consists of independent and identically distributed random variables representing the n-step jump size. A larger jump size implies a fiercer shock; and an upward jump indicates the release of unexpected good news. For tractability, assume that all the jumps follow a lognormal distribution with constant mean u_y and variance σ_y^2. Total number of jumps over the time period $(0, t)$ is counted using a Poisson process N_t. As usual, three sources of model's randomness $\{Y_n\}_{n=1,2,\ldots,\infty}$, $\{N_t\}_{t\geq 0}$, and $\{Z_t\}_{t\geq 0}$ are assumed to be mutually independent. The dynamic of stock returns governed by a switching regime with jump risks thus has the following explicit form

$$R_t = u_{q_t} + \sigma_{q_t} Z_t + \sum_{n=0}^{N_t} \log Y_n \ , \tag{1.2}$$

with a corresponding finite parameter space

$$\Theta_{RSMJ} \equiv \{\pi_i,\, p_{ij},\, u_i,\, \sigma_i^2,\, \lambda,\, u_y,\, \sigma_y^2 \mid 0 < \pi_i,\, p_{ij} < 1, u_i, u_y \in R, \sigma_i^2, \sigma_y^2 > 0, \forall i,\, j \in X, I = 2\}\,.$$

From Equation (1.2), the volatility of stock return is decomposed into two parts: continuous variation and discrete variation. The former is modeled as a product of standard normal distributed variable and regime-switching variance; while the latter is described using a jump process reflecting non-marginal effect of the information. Our idea serves as an extension of Merton's jump-diffusion model. Differing from Merton (1976), however, the key role in the model is the regime-switching process, instead of a pure standard Brownian motion. The estimation of relevant parameters in two spaces Θ_{RSM} and Θ_{RSMJ} will be discussed later.

We derived the descriptive statistics of stock returns under the regime-switching model with jump risks. The mean, variance, skewness and kurtosis of the returns are summarized in Table II.

[Insert Table II here]

Under the Black-Scholes model, the returns follow a normal distribution. Therefore, the skewness is 0 and kurtosis is 3. Under the regime-switching model, the formulae of skewness and kurtosis are derived by Timmermann (2000). In this paper, we derive the formulae of skewness and kurtosis under the regime-switching model with jump risks and summarize them in Table II. Furthermore, if jump term is ignored, that is $\lambda = 0$, the formulae of skewness and kurtosis will degenerate into those shown in the regime-switching model.

The regime-switching models with jump risks not only address the stock market cycles and the financial features, including the leptokurtic feature of the asset return distribution, volatility smile and the volatility clustering phenomenon, but also capture the effect of the abnormal events on the financial markets as described in Table I.

2. ESTIMATION AND TEST

In this section, we offer an estimate and test method of the regime-switching model with jump risks. Then, we use the DJIA and S&P 500 index to estimate the parameters of the Black-Scholes model, the regime-switching model and the regime-switching model with jump risks, and to test these models. Finally, we show the dynamics of the stock market cycles in the past ten years, and compute the daily jump probability dynamics.

2.1. Estimation and Test

Hamilton (1989) and Hardy (2001) employ the maximum likelihood estimation to compute the parameters of their models. However, in this paper, we use the EM algorithm to estimate the parameters. Suppose that the stock returns $\underset{\sim}{R} = \{R_1, R_2, \ldots, R_T\}$ are observable, the states of the financial market $\underset{\sim}{q} = \{q_1, q_2, \ldots, q_T\}$, the jump times $\underset{\sim}{N} = \{N_1, N_2, \ldots, N_T\}$ are unobservable, and the set of parameters under the regime-switching model with jump risks is Θ_{RSMJ}. Let $L_C^{RSMJ}(\Theta_{RSMJ} \mid \underset{\sim}{R}, \underset{\sim}{q}, \underset{\sim}{N})$ denote the complete-data likelihood function under the regime-switching model with jump risks, it can be shown as follows:

$$L_C^{RSMJ}(\Theta_{RSMJ} \mid \underset{\sim}{R}, \underset{\sim}{q}, \underset{\sim}{N}) = \pi_{q_1} \prod_{t=1}^{T} \Pr(R_t \mid q_t, N_t, \Theta_{RSMJ}) \prod_{t=1}^{T} \Pr(N_t \mid \Theta_{RSMJ}) \prod_{t=2}^{T} \Pr(q_t \mid q_{t-1}, \Theta_{RSMJ})$$

$$= \pi_{q_1} \prod_{t=1}^{T} p_{q_{t-1}q_t} \prod_{t=1}^{T} \Pr(R_t \mid q_t, N_t, \Theta_{RSMJ}) \prod_{t=1}^{T} \Pr(N_t \mid \Theta_{RSMJ}). \qquad (2.1)$$

Then the incomplete-data likelihood function can be written as follows:

$$L_{IC}^{RSMJ}(\Theta_{RSMJ} \mid \underset{\sim}{R}) = \sum_{q_1, \ldots, q_T = 1}^{2} \sum_{N_1, \ldots, N_T = 0}^{\infty} L_C^{RSMJ}(\Theta_{RSMJ} \mid \underset{\sim}{R}, \underset{\sim}{q}, \underset{\sim}{N}). \qquad (2.2)$$

However, too many observations will lead to numerous combinations of states and the number of shocks, causing computer unable to compute incomplete-data likelihood function. Therefore, in this study, we use Expectation-Maximization algorithm (EM algorithm) to find the maximum likelihood estimates of parameters. Under the regime-switching model with jump risks, the log-complete-data likelihood function is shown as follows:

$$\log L_C^{RSMJ}(\Theta_{RSMJ} \mid \underset{\sim}{R}, \underset{\sim}{q}, \underset{\sim}{N}) \qquad (2.3)$$

$$= \log \pi_{q_1} + \sum_{t=2}^{T} \log p_{q_{t-1}q_t} + \sum_{t=1}^{T} \left(-\lambda + n_t \log \lambda - \log(n_t!) - \frac{1}{2} \log[2\pi(\sigma_{q_t}^2 + n_t \sigma_y^2)] - \frac{(R_t - u_{q_t})^2}{2(\sigma_{q_t}^2 + n_t \sigma_y^2)} \right),$$

where π_{q_1} denotes the initial probability; the second term is the transition probabilities; the third term is the probabilities of the stock returns under the regime-switching model with jump risk; and the fourth term is the probabilities of occurring considerable events.

The EM algorithm is operated by E-step and M-step. In E-step, given the $(k-1)^{th}$ parameter, the conditional expectation of the log-complete-data likelihood function, $Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)})$ can be shown as follows:

$$Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)}) = E[\log L_C^{RSMJ}(\Theta_{RSMJ} \mid \underset{\sim}{R}, \underset{\sim}{q}, \underset{\sim}{N}) \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)}] \qquad (2.4)$$

$$= \sum_{i=1}^{2} \log \pi_{q_1} \Pr(q_1 = i \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)}) + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{t=2}^{T} \log p_{q_{t-1}q_t} \Pr(q_{t-1} = i, q_t = j \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)})$$

$$+ \sum_{i=1}^{2}\sum_{t=1}^{T}\sum_{n_t=0}^{\infty} \log \Pr(R_t \mid q_t = i, N_t = n_t) h(n_t, \lambda) \Pr(N_t = n_t \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)}) \Pr(q_t = i \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)}).$$

We could divide $Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)})$ into three components. The first part describes the initial probability; the second part is related to the transition probabilities; and the last part represents the behavior of the returns.

In the M-step of EM algorithm, we obtain the parameter estimates in the k^{th} iteration by maximizing $Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)})$ employing the estimated parameters from the (k-1)th iteration.

$$\hat{\pi}_i^{(k)} = \frac{\Pr(q_1 = i \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)})}{\sum_{i=1}^{2} \Pr(q_1 = i \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)})}, \quad \hat{p}_{ij}^{(k)} = \frac{\sum_{t=2}^{T} \Pr(q_{t-1} = i, q_t = j \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)})}{\sum_{j=1}^{2}\sum_{t=2}^{T} \Pr(q_{t-1} = i, q_t = j \mid \underset{\sim}{R}, \Theta_{RSMJ}^{(k-1)})}.$$

For the non-linear function of the third part, through Lagrange multiplier, we can finally get the estimates of $\hat{p}_{11}$, $\hat{p}_{22}$, $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_y$, $\hat{\sigma}_1$, $\hat{\sigma}_2$, $\hat{\sigma}_y$, and λ from EM gradient algorithm, which can be shown as follows:

$$\Theta_{RSMJ}^{(k)} = \Theta_{RSMJ}^{(k-1)} - a[d^{20} Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)})]^{-1} d^{10} Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)}), \tag{2.5}$$

where $\Theta_{RSMJ}^{(k)} = \arg\max_{\Theta} Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)})$, $a \in (0,1)$, d^{10} and d^{20} are the first order and second order condition of $Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)})$ with respect to Θ_{RSMJ}. Under the condition that $Q(\Theta_{RSMJ} \mid \Theta_{RSMJ}^{(k-1)})$ is monotonically increasing, we repeat the step E and the step M until the parameter estimates converge and the estimated parameters are the maximum likelihood estimators (Dempster et al., 1977). We then estimate the variance-covariance matrix of parameters via the supplemented EM algorithm (SEM) developed by Meng and Rubin (1991).

Based on the approximate theory of a large sample, the likelihood ratio test (LRT) Λ usually is employed for the goodness of fit tests of the model with the null hypothesis $H_0:\theta\in\Theta_0$. The alternative hypothesis $H_1:\theta\in\Theta_1\setminus\Theta_0$ where $\Theta_1\subset\Theta_0$, and the likelihood ratio test can be denoted as follows,

$$\Lambda=-2\ln\frac{L(R;\Theta_0)}{L(R;\Theta_1)},$$

where $L(R;\Theta_i)$ represents the maximum likelihood function under the hypothesis H_i, $i=0,1$. Under the null hypothesis H_0 with large sample, the likelihood test statistic Λ follows an asymptotically Chi-squared distribution $\chi^2_{1-\alpha}(\gamma)$ with a degree of freedom γ and significant level α. The test models include the Black Scholes model versus the regime-switching model, and the regime-switching model versus regime-switching model with jump risk.

2.2 Estimating the Parametric Variance

The asymptotic variance-covariance matrix of parametric estimators can be obtained by SEM algorithm (Meng and Rubin, 1991) in this subsection. An information matrix can be given by inverting the observed information matrix $I_o(\theta^*\,|\,Y_{obs})$, where Y_{obs} is observed data and θ^* is an maximum likelihood estimator (MLE) of θ, and the observed information matrix can be represented by the following:

$$I_o(\theta^*\,|\,Y_{obs})=-\frac{\partial^2 f(Y_{obs}\,|\,\theta)}{\partial\theta\cdot\partial\theta^T}.$$

However, according to the hidden variables of the states in models, it is difficult to calculate the observed information matrix $I_o(\theta^*\,|\,Y_{obs})$ directly. In contrast with the observed information matrix, it would be easy if we proceed to evaluate a complete data function $f(Y|\theta)$, where $Y=\{Y_{obs}, Y_{mis}\}$ is a complete data and Y_{mis} is a missing data. Therefore, the complete-data information matrix can be represented by the following

$$I_o(\theta^*|Y) = -\frac{\partial^2 f(Y|\theta)}{\partial\theta\cdot\partial\theta^T}.$$

The observed information matrix can be derived by complete-data information matrix and missing-data information matrix (Orchard and Woodbury, 1972), and expressed as the following formula:

$$I_o(\theta^*|Y_{obs}) = I_{oc} - I_{om},$$

where $I_{oc} = E[I_o(\theta|Y)|Y_{obs}, \theta=\theta^*]$ and $I_{om} = E\left(-\frac{\partial^2 f(Y_{mis}|Y_{obs},\theta)}{\partial\theta\cdot\partial\theta^T}|Y_{obs}, \theta=\theta^*\right)$.

Then, Louis (1982) proves that the missing information matrix can be rewritten as:

$$I_{om} = Cov\left(\frac{\partial f(Y|\theta)}{\partial\theta}\middle| Y_{obs}, \theta\right)$$

$$= E\left(\frac{\partial f(Y|\theta)}{\partial\theta}\cdot\frac{\partial f(Y|\theta)^T}{\partial\theta}\middle| Y_{obs}, \theta\right) - E\left(\frac{\partial f(Y|\theta)}{\partial\theta}\middle| Y_{obs}, \theta\right)\cdot E\left(\frac{\partial f(Y|\theta)}{\partial\theta}\middle| Y_{obs}, \theta\right)^T.$$

Moreover, we can obtain the complete information matrix as follows,

$$I_o(\theta|Y_{obs}) = I_{oc} - E\left(\frac{\partial f(Y|\theta)}{\partial\theta}\cdot\frac{\partial f(Y|\theta)^T}{\partial\theta}\middle| Y_{obs}, \theta\right) + E\left(\frac{\partial f(Y|\theta)}{\partial\theta}\middle| Y_{obs}, \theta\right)\cdot E\left(\frac{\partial f(Y|\theta)}{\partial\theta}\middle| Y_{obs}, \theta\right)^T.$$

Because the parameter θ^* is the MLE, the third part of the previous equation is zero. Therefore, the observed information matrix $I_o(\theta^*|Y_{obs})$ can be rewritten as:

$$I_o(\theta^*|Y_{obs}) = I_{oc} - E\left(\frac{\partial f(Y|\theta)}{\partial\theta}\cdot\frac{\partial f(Y|\theta)^T}{\partial\theta}\middle| Y_{obs}, \theta=\theta^*\right).$$

Dempster and Rubin (1977) develop the possibility of obtaining the asymptotic variance for the maximum likelihood estimator in single parameter case by using the rate of convergence of EM algorithm. We can thus give the relationship between the observed-data asymptotic variance and the complete-data asymptotic variance is as follows,

$$V = \frac{V_C}{1-r},$$

where V denotes the observed-data asymptotic variance, V_C represents the complete-data asymptotic variance, and r stands for the converge rate of the parameters in EM algorithm. Meng and Rubin (1991) propose a supplemented EM algorithm, where the algorithm is a method of calculating the asymptotic variance-covariance matrix based on EM algorithm. The process of the SEM algorithm requires only the calculations of complete data information matrix and the codes for the EM algorithm, and uses the rate of convergence of EM algorithm to obtain the asymptotic variance-covariance matrix.

Meng and Rubin (1991) show that the asymptotic variance-covariance matrix can be obtained by the following equation:

$$V = I_{oc}^{-1} + \Delta V, \tag{2.6}$$

where $\Delta V = I_{oc}^{-1} \cdot DM \cdot (I - DM)^{-1}$ and $DM = \left.\frac{\partial M_j(\theta)}{\partial \theta_i}\right|_{\theta=\theta^*}$ denotes the $d \times d$ convergence rate matrix of EM algorithm for parameters, $i, j = 1, 2, \ldots, d$. $M(\theta^{(k-1)}) = \theta^{(k)}$ represents a mapping for the EM algorithm. Therefore, the composition of the SEM algorithm is divided into three main components to obtain the estimation of the asymptotic variance-covariance matrix. The first component is calculating an inverse function of complete-data information matrix, the second component is to evaluate the convergence rate matrix DM, and the third component is to find the observed variance-covariance matrix V.

To obtain the observed variance-covariance matrix V, one needs to calculate the rate of convergence matrix DM . Meng and Rubin (1991) suggest the use of EM algorithms. Let r_{ij} be the $(i,\, j)^{th}$ element of the convergence rate matrix DM and define $\theta^{(t)}(i)$ $\theta^{(t)}(i)=(\theta_1^*,\, \ldots,\, \theta_{i-1}^*,\, \theta_i^{(t)},\, \theta_{i+1}^*,\, \ldots,\, \theta_d^*)$. That is, only the i^{th} component in $\theta^{(t)}(i)$ is in the sense and the other components are fixed at their MLE. By the definition of r_{ij} , we have

$$
\begin{aligned}
r_{ij} &= \frac{\partial M_j(\theta^*)}{\partial \theta_i} = \lim_{\theta_i \to \theta_i^*} \frac{\partial M_j(\theta_1^*, \ldots, \theta_{i-1}^*, \theta_i, \theta_{i+1}^*, \ldots, \theta_d^*) - M_j(\theta^*)}{\theta_i - \theta_i^*} \\
&= \lim_{t \to \infty} \frac{M_j[\theta^{(t)}(i)] - \theta_j^*}{\theta_i^{(t)} - \theta_i^*} \equiv \lim_{t \to \infty} r_{ij}^{(t)}
\end{aligned},
$$

where the convergence rate r_{ij} can be approximated by $r_{ij}^{(t)}$ when t is sufficiently large. $M(\theta^*)$ can be obtained by the EM algorithm program as $M(\theta)$ is the result of E-step and M-step in the EM algorithm.

Based on the same starting value of the SEM algorithm and EM algorithm, and setting the same value for θ^* and $\theta^{(t)}$, $r_{ij}^{(t)}$ can be obtained by the following procedure. First, we use the E and M step to obtain $\theta^{(t+1)}$, and repeat the steps for $i=1,2,\ldots,d$. Next, we calculate $\theta^{(t)}(i)$ and perform iteration it as the current estimate of θ, and give the iteration of EM to obtain $M_j[\theta^{(t)}(i)]$. We finally obtain the ratio

$$
r_{ij}^{(t)} = \frac{M_j[\theta^{(t)}(i)] - \theta_j^*}{\theta_i^{(t)} - \theta_i^*},
$$

where the output is $\theta^{(t+1)}$ and $\{r_{ij}^{(t)} : i,\, j=1,2,\ldots,d\}$. Let $r_{ij}^{(t+1)}$ substitute for r_{ij} when $|\, r_{ij}^{(t+1)} - r_{ij}^{(t)} \,| < 10^{-4}$, and then we have the DM matrix. Therefore, the asymptotic variance-covariance matrix can be obtained by equation (2.6).

2.3 Empirical Results

The data include the daily DJIA index and S&P 500 index from 1999/01/01 to 2009/12/31. The parameter estimates and tests for the Black-Scholes model (BSM), the regime-switching model (RSM) and the regime-switching model with jump risks (RSMJ) are presented in Table III based on the DJIA index and S&P 500 index. For the DJIA index, the return follows a normal distribution with a mean of 0.000046 and a standard deviation of 0.0129 under the Black-Scholes model. Under the regime-switching model, the transition probabilities, p_{11} and p_{22}, are 0.9798 and 0.9907, respectively. Both probabilities are close to one, implying that the probabilities of switching from expansion to recession and vice versa are very small. In recession, the mean stock return is a negative 0.008, with a standard deviation of 0.0196. In contrast, in expansion the mean stock return is a positive 0.0005, with a standard deviation of 0.0079. The volatility of stock returns in expansion is more stable than that in recession. Same results in the S&P 500 index using BSM and RSM are observed.

[Insert Table III here]

Using the regime-switching model with jump risk to fit DJIA index, the results of RSMJ are similar to those under the regime-switching model. The transition probabilities are still close to one. In recession, the mean return is negative and the volatility is high, while in expansion, the mean return is positive and the volatility is low. Moreover, comparing with the estimated results in the regime-switching model, the means are larger and the volatilities are smaller as part of them is explained by the jump term. The mean jump times is 0.2934, and the jump size follows a log-normal distribution with mean -0.0002 and standard deviation 0.0138, implying that the release of unanticipated information on average causes returns to go down. The same estimated results in S&P 500 for RSMJ are observed.

From LR test results, the null hypothesizes are rejected, meaning that at the 95% confidence level, the regime-switching model is better than the Black-Scholes model, and the regime-switching model with jump risks is better than the regime-switching model. The test results in S&P 500 index are the same.

Figure 2 shows the price, the return of DJIA index, the probability of recession under the regime-switching model with jump risks, and the probability of jumps. In panel A, the prices went upward in 1999 and 2003~2007 during the periods of economic expansion. The prices turned south during the periods of economic recession (from 2000 to 2003 and from 2008 to 2009) due to the dot-com bubble and the global financial crisis. Panel B shows that the volatilities from 1999 to 2002 are larger than the volatilities from 2003 to 2007, implying the returns become more volatile in the recession time. Moreover, we observe volatility clustering in panel B.

[Insert Figure 2 here]

Panel C indicates that the probability of recession from 1999 to 2002 is high because of the dotcom bubble, and the probability of recession from 2003 to 2007 is low. Hence, there is a transition of states in 2002 to 2003. In 2008, as the financial crisis progressed, the probability of recession in 2008 also became higher. There is also a switch of states in 2007 to 2008. Panel D shows the probability of jumps is large in 2000 to 2003, 2007 and 2008, consistent with the events of the dot-com bubble in 2000, the September 11 attacks in 2001, the end of Iraq war in 2003, the Yen carry trade in 2007, and the financial crisis in 2008. Based on Equation (1.8), the probability of jumps is one minus the probability of no jump given the daily returns and estimated parameters of the RSMJ. Figure 3 shows similar results for the S&P 500 index.

$$\Pr(N_t = n_t \mid \underset{\sim}{R}, \hat{\Theta}_{RSMJ}) = 1 - \sum_{i=1}^{2} \frac{\Pr(R_t, N_t = 0 \mid \hat{\Theta}_{RSMJ}, q_t = i)}{\sum_{t=1}^{T} \sum_{n_t=0}^{\infty} \Pr(R_t, N_t = n_t \mid \hat{\Theta}_{RSMJ}, q_t = i)} . \tag{1.8}$$

[Insert Figure 3 here]

3. OPTION PRICING

When the market is incomplete, the jump risks are considered as undiversifiable. The extended Girsanov's principle and the Esscher transform are often used to select the risk neutral pricing measure. The original Girsanov's theorem has been applied to change the measure of the Brownian motion under the continuous time. However, Elliott and Madan (1998) propose an extended Girsanov's principle which is then applied to change the measure of the Brownian motion under the discrete time. Liew and Siu (2010) further apply extended Girsanov's principle to change the measure of the regime-switch model. Another method of selecting the pricing measure under the incomplete market is the Esscher transform which is proposed by Gerber and Shiu (1994). When the stock price obeys a stochastic process in continuous time and the moment generating function of the stochastic process exists, the martingale measure can be found by the Esscher transform. In this section, we propose the Esscher transform to find the martingale measure for economic models, and we derive the European option pricing formula under regime switching with jump risks models.

3.1. Esscher Transform with No Arbitrage Condition

Merton (1976) assumes stock price follows the jump diffusion model and the jump risks are diversifiable. Therefore, jump risks can be avoided by the diversification of investments, and the investors will not demand the jump risk premium. However, Jarrow and Rosenfeld (1984) indicated that jump risks are the undiversifiable risks. When the stock price is expected to jump, the investors will demand the jump risk premium. When the jump risks are undiversifiable, the martingale measure is not unique. Therefore, we investigate the Esscher transform to derive the European option pricing formula. For such pricing problems, Elliott et al. (2005), Liew and Siu (2010) and others also show that the martingale measure can be found by the Esscher transform.

A. Esscher Transform under RSM

Under the true probability measure, the stock price dynamics of the regime switching model is

$$S(t) = S(0)\exp\left[\left(u_{q_t} - \frac{1}{2}\sigma_{q_t}^2\right)t + \sigma_{q_t}W(t)\right], \tag{3.1}$$

where $W(t)$ denotes the Brownian motion, $q_t \in \{1, 2\}$ address the state of stock market cycles, and u_{q_t} and σ_{q_t} present the mean and the volatility when the state is q_t.

Assume that the transition probability is known in the future, that is, there is no risk premium when the state changes into a different one. Let h_{Bq_t} be Esscher transform parameters of the Brownian motion with the state of stock market cycle, q_t. Suppose that $M_{R_t|F_{t-1}}(h_{Bq_t}) < \infty$ denotes the conditional moment generating function of returns R_t given the information set F_{t-1}. Thus, given the state is known in the future, the Randon-Nikodym derivative of the Brownian motion is as follows:

$$\eta_B(t) = \begin{bmatrix} \exp\left(-\frac{h_{B1}^2\sigma_1^2 t}{2} + h_{B1}\sigma_1 W(t)\right) & 0 \\ 0 & \exp\left(-\frac{h_{B2}^2\sigma_2^2 t}{2} + h_{B2}\sigma_2 W(t)\right) \end{bmatrix} \tag{3.2}$$

where h_{B1} and h_{B2} are the transformed parameters with the state 1 and 2, respectively. The Esscher transform from the physical measure to the risk neutral measure is denoted by

$$dQ(W^Q(t)) = dP(W(t))\eta_{Bq_t}(t) = \frac{1}{\sqrt{2\pi t}}\exp\left(-\frac{[W(t) - h_{Bq_t}\sigma_{q_t}t]^2}{2t}\right),$$

or, the new Brownian motion is presented as $W^Q(t) = W(t) - h_{Bq_t}\sigma_{q_t}t \sim N(0,t)$.

Since the expectations of the discounted stock price should equal to the present stock price in the risk neutral measure at any time *t* i.e.,

$$S(0) = E^Q[\exp(-rt)S(t) \mid F_0]. \tag{3.3}$$

No arbitrage condition is derived as $r = u_{q_t} + h_{Bq_t}\sigma_{q_t}^2$. A special case of Esscher parameters which satisfies the no arbitrage condition can be found as $h_{Bq_t} = (r - u_{q_t})/\sigma_{q_t}^2$. Under the risk neutral Esscher measure, the dynamics of the stock index can be denoted as follows:

$$S(t) = S(0)\exp\left[\left(r - \frac{1}{2}\sigma_{q_t}^2\right)t + \sigma_{q_t}W^Q(t)\right] \tag{3.4}$$

with the no-change transition probability P, which has two states, $I = 2$.

B. Esscher Transform under RSMJ

Under the physical probability measure, the stock price dynamics of the RSMJ is

$$S(t) = S(0)\exp\left[\left(u_{q_t} - \frac{1}{2}\sigma_{q_t}^2\right)t + \sigma_{q_t}W(t)\right]\prod_{m=1}^{N(t)} Y_m , \tag{3.5}$$

where $N(t)$ denotes the jump frequency and follows Poisson process with the arrival rate λt, and $\{Y_m\}$ presents the jump size that follows lognormal distribution with location parameter u_y and scale parameter σ_y^2. The transition probability is

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},$$

where $\sum_{j=1}^{2} p_{ij} = 1$ for $i = 1, 2$. We also assume that the transition probability is known in the future, i.e. the investors do not demand risk premium of changes in stock market states. h_{Bq_t} and h_J are respectively Esscher transform parameters of the Brownian motion and the systematic jump risk term. Suppose $M_{R_t|F_{t-1}}(h_{Bq_t}) < \infty$ denotes the conditional moment generating function of returns R_t given the information set F_{t-1}. we derive the Esscher transform $\eta_B(t)$ for the given states in Equation (3.6), because the Brownian motion term and the systematic jumps risk term are independent:

$$\eta_B(t) = \begin{bmatrix} \exp\left(-\frac{h_{B1}^2\sigma_1^2 t}{2} + h_{B1}\sigma_1 W(t)\right) & 0 \\ 0 & \exp\left(-\frac{h_{B2}^2\sigma_2^2 t}{2} + h_{B2}\sigma_2 W(t)\right) \end{bmatrix}, \tag{3.6}$$

where h_{B1} and h_{B2} are the transformed parameters with state 1 and 2, respectively. Similarly, the Brownian motion of a given state follows $W^Q(t) = W(t) - h_{Bq_t}\sigma_{q_t} t \sim N(0, t)$ under the risk neutral measure.

Because the jump risk is undiversifiable, the Esscher transform can be used to change the stock price dynamics from the true probability measure to the risk neutral probability measure. The Esscher transform of jump risks from the physical measure to the risk neutral measure is defined by

$$\eta_J(t) = \frac{dQ[N(t)=n, \log Y_1, \ldots, \log Y_n]}{dP[N(t)=n, \log Y_1, \ldots, \log Y_n]} = \prod_{m=1}^{n} Y_m^{h_J} e^{-\lambda \xi^{(h_J)} t} .$$

Based on the Esscher transform, we can obtain the new distribution of the jump sizes under risk neutral measure as $\log Y_m \sim N(u_y + h_J \sigma_y^2, \sigma_y^2)$. Suppose $M_{\log Y|F_{t-1}}(h_J) < \infty$ denote the conditional moment generating function of jump size $\log Y$ given the information set F_{t-1}, the derivation can be shown as:

$$dQ(\log Y_n) = dP(\log Y_n) \frac{\exp(h_J \log Y)}{E[\exp(h_J \log Y)]} = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{[\log Y - (u_y + h_J \sigma_y^2)]^2}{2\sigma_y^2} \right).$$

We also can derive the risk neutral distribution of the jump frequency $N(t) \sim Poi[\lambda(\xi^{(h_J)} + 1)t]$ by the following

$$dQ[N(t)=n] = dP[N(t)=n] \frac{\exp(\lambda \xi^{(h_J)} t)}{E[\exp(h_J \log Y)]^n} = \frac{e^{-\lambda(\xi^{(h_J)}+1)t} [\lambda(\xi^{(h_J)}+1)t]^n}{n!} .$$

Since the expectations of the discounted stock price should equal to the present stock price in the risk neutral measure at any time *t*, i.e.

$$S(0) = E^Q[\exp(-rt) S(t) \mid F_0]. \tag{3.7}$$

The no arbitrage condition can be derived as follows:

$$r = u_{q_t} + h_{Bq_t} \sigma_{q_t}^2 + \lambda(\xi^{(h_J+1)} - \xi^{(h_J)}) ,$$

And a special case of Esscher parameters which satisfies the no arbitrage condition can be found as $h_{Bq_t} = (r - u_{q_t}) / \sigma_{q_t}^2$ and $h_J = -(u_y / \sigma_y^2 + 0.5)$. Under this Esscher parameters special case, the jump size of the risk neutral measure is changed to $\log Y_m \sim N(-0.5\sigma_y^2, \sigma_y^2)$, and the arrival rate matrix of the risk neutral measure follows $\lambda t \exp[\sigma_y^2 / 8 - u_y^2 / 2\sigma_y^2]$.

Therefore, according to the no arbitrage condition, the dynamics of the stock index with RSMJ under the risk-neutral Esscher measure is stated as follows,

$$S(t) = S(0)\exp\left[\left(r - \frac{1}{2}\sigma_{q_t}^2\right)t + \sigma_{q_t} W^Q(t) + \sum_{m=1}^{N(t)} Y_m\right]. \tag{3.8}$$

with the no-changed transition probability P, which has two states, $I = 2$.

3.2. Valuation of Stock Index Options with Economic Models

Section 3.1 finds the Esscher transform parameters to satisfy the no arbitrage condition. Under the risk neutral measure, we obtain the stock index dynamics of the RSM and the RSMJ. Hence, in Section 3.2, the risk neutral stock price dynamics can be used to derive the European call stock index option formula and explain the association with economic models.

A. Valuation of European Stock Index Call Options with RSM

Under the risk-neutral world and the stock price dynamics in (3.4), we compute the European call option price with strike price *K*, risk-free interest rate *r*, and maturity date *T* of the RSM as follows:

$$C_{RSM} = \sum_{k=0}^{T}\sum_{i=1}^{2} \pi_i \cdot \gamma_{T,\,k|q_0=i} \cdot [S(0)N(d_{1k}) - Ke^{-rT}N(d_{2k})],$$

where π_i denotes the initial probability, and $\gamma_{T,\,k|q_0=i}$, as Duan et al. (2002), is the remaining probabilities that the amount, *k*, of the visits to state 1 in *T* years (days) given the initial state being *i*. Given that the initial state is state 1, for $k = 0$ and $t = 1, 2, \ldots, T$, the state immediately changes to 2 in the initial day and remains in state 2 later. The probability is denoted as $p_{12}p_{22}^{t-1}$. For $k = 1$ and $t = 1$, the probability of the state not changing in period $t = [0,1)$ is p_{11}. For $k = 1$ and $t = 2, 3, \ldots, T$, the probabilities of staying at state 1 in only one day and the others at state 2 is marked as $p_{11}p_{12}p_{22}^{t-2}+(t-2)\,p_{12}^{t-1}p_{21}p_{22}^{t-3}+p_{12}p_{21}p_{22}^{t-2}$. Finally, let $F(l\,|\,q_0=1)$ be the probability that state 1 occurs given initial state 1 after l periods. For $k=2,3,\ldots,T$ and $t=2,3,\ldots,T$, the probabilities are calculated by the previous daily probabilities and $F(l\,|\,q_0=1)$.

$$\gamma_{t,\,k|q_0=1}=\begin{cases} p_{12}p_{22}^{t-1} & ,\, k=0,\, t=1,2,\ldots,T \\ p_{11} & ,\, k=1,\, t=1 \\ p_{11}p_{12}p_{22}^{t-2}+(t-2)\,p_{12}^{t-1}p_{21}p_{22}^{t-3}+p_{12}p_{21}p_{22}^{t-2} & ,\, k=1,\, t=2,3,\ldots,T \\ \sum_{l=1}^{t-k+1}F(l\,|\,q_0=1)\cdot\gamma_{t-1,\,k-1|q_0=1} & ,\, k=2,3,\ldots,T,\, t=2,3,\ldots,T \end{cases}$$

$$F(l\,|\,q_0=1)=\begin{cases} p_{11} & ,\, l=1 \\ p_{12}p_{22}^{l-2}p_{21} & ,\, l=2,3,\ldots,T \end{cases}$$

Similarly, given that the initial state is state 2, the remaining probabilities are computed as below.

$$\gamma_{t,\,k|q_0=2}=\begin{cases} p_{22}^{t} & ,\, k=0,\, t=1,2,\ldots,T \\ p_{21} & ,\, k=1,\, t=1 \\ (t-1)\,p_{21}p_{12}p_{22}^{t-2}+p_{22}^{t-2}p_{21} & ,\, k=1,\, t=2,3,\ldots,T \\ \sum_{l=1}^{t-k+1}F(l\,|\,q_0=2)\cdot\gamma_{t-1,\,k-1|q_0=2} & ,\, k=2,3,\ldots,T,\, t=2,3,\ldots,T \end{cases}$$

$$F(l\,|\,q_0=2)=\begin{cases} p_{21} & ,\, l=1 \\ p_{22}^{l-1}p_{21} & ,\, l=2,3,\ldots,T \end{cases}$$

d_{1k} and d_{2k} are denoted as follows,

$$d_{1k}=\frac{\ln\dfrac{S(0)}{K}+rT+\dfrac{1}{2}\theta_k^2}{\theta_k}\,,\quad d_{2k}=d_{1k}-\theta_k\,,$$

where $\theta_k^2=k\sigma_1^2+(T-k)\sigma_2^2$, $k=0,1,\ldots,T$.

B. Valuation of European Stock Index Call Options with RSMJ

Under the risk-neutral world and the stock price dynamics in (3.8), we compute the European call option price with strike price *K*, risk-free interest rate *r*, and maturity date *t* of the RSMJ as follows:

$$C_{RSMJ} = \sum_{n=0}^{\infty}\sum_{k=0}^{T}\sum_{i=1}^{2}\pi_i \cdot \gamma_{T,\,k|q_0=i} \cdot \frac{e^{-\lambda^* T}(\lambda^* T)^n}{n!} \cdot [S(0)N(d_{1k}) - Ke^{-rT}N(d_{2k})],$$

where $\gamma_{T,\,k|q_0=i}$ presents the probability that state 1 will stay for *k* years (days) when the maturity is *T* years (days) and the initial state is $q_0 = i$. Such probability can be obtained from Duan et al. (2002) with hidden Markov Chain. And, λ^*, d_{1k} and d_{2k} are denoted as follows,

$$\lambda^* = \lambda \exp\left(\frac{\sigma_y^2}{8} - \frac{u_y^2}{2\sigma_y^2}\right),\ \ d_{1k} = \frac{\ln\frac{S(0)}{K} + rT + \frac{1}{2}(\theta_k^2 + n\sigma_y^2)}{\sqrt{\theta_k^2 + n\sigma_y^2}},\ \ d_{2k} = d_{1k} - \sqrt{\theta_k^2 + n\sigma_y^2}\ .$$

4. EMPIRICAL AND SENSITIVITY ANALYSIS

In this section, we first show the features of heavy tails and volatility clustering by the parameter estimates of the regime-switching model with jump risks. The volatility curve and volatility surface under the regime-switching model with jump risks are discussed next. Finally, sensitivity analysis for the European call option value under Black-Scholes model, the regime-switching model and the regime-switching model with jump risks is presented.

4.1. Financial Features

A. Volatility Clustering

The volatility clustering phenomenon was documented as early as Mandelbrot (1963) to describe that periods of high/low volatility are generally followed by periods of high/low volatility. Cont (2005) summarizes the characteristics of financial time series, and posits that volatility clustering does not imply the correlation of the returns, but the correlation of absolute returns or squared returns which displays a positive, significant and slowly decaying autocorrelation function (ACF).

Under the Black-Scholes model, since the returns are independent, the squared returns are also independent, implying that no significant correlation is found in the squared returns. As a result, the Black-Scholes model does not describe the characteristic of volatility clustering.

The regime-switching model and the regime-switching model with jump risks, on the other hand, can better describe the volatility clustering. Under the regime-switching model, the ACF of the squared returns is derived by Haldrup and Nielsen (2006). We extend their work to derive the ACF of the squared returns under the regime-switching model with jump risks, which can be shown in Table IV. As $\lambda = 0$, the ACF under the regime-switching model with jump risks reduces to the ACF under the regime-switching model.

[Insert Table IV here]

We consequently input the parameter estimates in Table IV to the ACF formulae and plot the ACF for different model settings in Figure 4. For comparison, panel A shows the returns of S&P 500 from 1999 to 2008. Panel B, C and D present the ACF of real data squared returns, the regime-switching model and the regime-switching model with jump risks, respectively. From panel B, we notice that the ACF of real data is positive, significant and slowly decaying, which means that the time series exhibits volatility clustering. Moreover, the ACFs of squared returns under the regime-switching model and the regime-switching model with jump risks conform to that of real data. In other words, the ACFs under both models decay fast if the ACF of real data decays fast. Conversely, the ACFs under both models decay slowly if the ACF of real data decays slowly. Similar results are found in DJIA index.

[Insert Figure 4 here]

B. Volatility Smile

Based on the information of option price, asset price, strike price, risk-free interest rate and maturity, the implied volatility can be computed under the Black-Scholes model. Harvey and Whaley (1992) indicate that implied volatility changes along with the expectation and the changes of the market. Therefore, implied volatility not only represents current price of the market but also reflects expectations of the market.

Volatility curve can be plotted under different ratios of strike to spot price. Since the implied volatility for at-the-money option is smaller than that for out-the-money or in-the-money options, the volatility curve is convex, hence named volatility smile. However, Schwert (1989), and Fleming et al. (1995) notice that the increasing speed of implied volatility for in-the-money options is faster than that for out-the-money options, making volatility smile like smirk. Harvey and Whaley (1992) verify the phenomenon of volatility smirk using S&P100 index.

We solve the implied volatilities, plot the volatility curve, and compute the one-year option prices for nine different strikes via the estimation under the regime-switching model with jump risks assuming the current time is September 30, 2008. A volatility surface is constructed with seven volatility curves by seven different maturities. Figure 5 shows the volatility curves and the volatility surface for the S&P500

index under the regime-switching model with jump risks. Panel A indicates that the curve has the feature of volatility smirk. The volatility surface for the S&P500 index in panel B shows that the implied volatility is also in smirk shape and is decaying along with maturities. These are the same results as in the DJIA index option.

[Insert Figure 5 here]

4.2. Sensitivity Analysis

Because the DJIA index and the S&P500 index exhibit stock market cycles, this section performs sensitivity analysis for the estimated parameters. Tables V and VI report sensitivity analysis of European call price assuming stock returns follow regime switching model with jump risks. The base volatility of state 1 (4%±2%) and the base volatility of state 2 (1%+1% and 1%-0.05%) show the effect of the volatility on European call option. Hence, the volatilities of daily stock returns are 2%, 4% and 6% for state 1；and 0.5%, 1% and 2% for state 2. According to the sensitivity analysis in Table V, there is a positive relationship between volatility and option value in state 1 and state 2, holding other parameters fixed, implying the larger the volatility, the higher the probability of increasing stock price, hence higher call price. In addition, there is a positive relationship between p_{11} and call value, other parameters held constant, because the volatility of the state 1 stays longer when p_{11} is closed to 1. The higher the p_{11}, the lower the probability that the economy will switch from state 1 to state 2.That is, in the long term, the longer the duration of state 1, which has higher volatility, the higher the call value will be. On the contrary, there is a negative relationship between p_{22} and call value. The higher the p_{22}, the lower the probability the economy will switch from state 2 to state 1. In the long term, the longer the duration of state 2, which has lower volatility, the smaller the call value.

[Insert Table V here]

This paper also discusses the influence of jump volatility on call price. Table VI illustrates the sensitivity analysis of the impact of jump size and jump frequency on call price. Other things held constant, there is a positive relationship between average jump size and call price. Since call price increases at expiration when stock price increases, the bigger jump size implies larger stock price upside volatility, hence higher call price.

[Insert Table VI here]

The relationship between the standard deviation of jump size and call price is concave, because the no arbitrage condition is satisfied. Finally, other parameters held constant, the higher jump frequency indicates more frequent jump volatility. Therefore, the option value is higher

5. CONCLUSION

This study proposes a regime-switching model with jump risks to price the European option. To capture the dynamics of stock returns over expansion-recession cycles and the occurrences of the abnormal events in financial markets, we assume the index return would follow the regime-switching model with jump risks.

In this study, we show that comparing to the Black-Scholes model and the regime-switching model, the regime-switching model with jump risks can better explain the dynamics of DJIA and the S&P 500 stock indices. In addition, both the regime-switching model and the regime-switching model with jump risks can address the leptokurtic feature of the asset return distribution, volatility smile, and the volatility clustering phenomenon.

We then examine the influence of parameters on European call value under the regime-switching model with jump risks, and find that option prices increase along with the probability of staying in the recession state, but decrease along with the probability of staying in the expansion state. Moreover, the increases of standard deviation (in either state), the mean of jump sizes, the standard deviation of jump sizes, and the mean of jump times, would all increase option prices. The differences among valuations under the Black-Scholes model, the regime-switching model and the regime-switching model with jump risks suggest that it is critical to value a European call option by an appropriate model.

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TABLE I

Sudden Shocks in the Daily Return of DJIA Index

Items	Shock Type			Total
	Weak ($\pm$ single std.)	Median ($\pm$ double std.)	Strong ($\pm$ triple std.)	
Obs.	824	183	57	3788
Percentage	21.7529%	4.8310%	1.5048%	100%
Avg. Daily Return	-4.9673×10^{-4}	-4.6166×10^{-3}	-7.2315×10^{-3}	3.2694×10^{-4}
Variance of Returns	5.2607×10^{-4}	7.4108×10^{-4}	8.2854×10^{-4}	1.3931×10^{-4}

Note. In this table, we use the DJIA index from Jan. 3rd, 1995 to Jan. 15th, 2010 as sample and present the weak, median, and strong shocks defined by the daily observation over or below the mean of single, double, and triple standard deviation, respectively.

TABLE II

The Moments in the Regime-Switching Model with Jump Risks

Mean (u)	$\pi_1 u_1 + \pi_2 u_2 + \lambda u_y$
Variance (σ_u^2)	$E\left(\sum_{n=1}^{N_t}\log Y_n\right)^2 + 2E\left(\sum_{n=1}^{N_t}\log Y_n\right)[\pi_1(u_1-u)+\pi_2(u_2-u)] + \{\pi_1[(u_1-u)^2+\sigma_1^2]+\pi_2[(u_2-u)^2+\sigma_2^2]\}$
Skewness	$\dfrac{E\left(\sum_{n=1}^{N_t}\log Y_n\right)^3 + 3E\left(\sum_{n=1}^{N_t}\log Y_n\right)^2[\pi_1(u_1-u)+\pi_2(u_2-u)] + 3E\left(\sum_{n=1}^{N_t}\log Y_n\right)\{\pi_1[(u_1-u)^2+\sigma_1^2]+\pi_2[(u_2-u)^2+\sigma_2^2]\}}{\sigma_u^3}$ $+\dfrac{\pi_1[(u_1-u)^3+3(u_1-u)\sigma_1^2]+\pi_2[(u_2-u)^3+3(u_2-u)\sigma_2^2]}{\sigma_u^3}$
Kurtosis	$\dfrac{E\left(\sum_{n=1}^{N_t}\log Y_n\right)^4 + 4E\left(\sum_{n=1}^{N_t}\log Y_n\right)^3[\pi_1(u_1-u)+\pi_2(u_2-u)] + 6E\left(\sum_{n=1}^{N_t}\log Y_n\right)^2\{\pi_1[(u_1-u)^2+\sigma_1^2]+\pi_2[(u_2-u)^2+\sigma_2^2]\}}{\sigma_u^4}$ $+\dfrac{4E\left(\sum_{n=1}^{N_t}\log Y_n\right)^2\{\pi_1[(u_1-u)^3+3(u_1-u)\sigma_1^2]+\pi_2[(u_2-u)^3+3(u_2-u)\sigma_2^2]\}+\{\pi_1[(u_1-u)^4+6(u_1-u)^2\sigma_1^2+3\sigma_1^4]+\pi_2[(u_2-u)^4+6(u_2-u)^2\sigma_2^2+3\sigma_2^4]\}}{\sigma_u^4}$

Note. $E(\sum_{n=1}^{N_t}\log Y_n)=\lambda u_y$, $E(\sum_{n=1}^{N_t}\log Y_n)^2 = u_y^2(\lambda^2+\lambda)+\sigma_y^2\lambda$,

$E(\sum_{n=1}^{N_t}\log Y_n)^3 = u_y^3(\lambda^3+3\lambda^2+\lambda)+3u_y\sigma_y^2(\lambda^2+\lambda)$, and $E(\sum_{n=1}^{N_t}\log Y_n)^4 = u_y^4(\lambda^4+6\lambda^3+7\lambda^2+\lambda)+6u_y^2\sigma_y^2(\lambda^3+3\lambda^2+\lambda)+3\sigma_y^4(\lambda^2+\lambda)$.

TABLE III

The Estimating and Testing of the BSM, RSM, RSMJ Models in DJIA Index and S&P 500 Index

Index	Model	$\hat{p}_{11}$	$\hat{p}_{22}$	$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_y$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\sigma}_y$	$\hat{\lambda}$	$-2\ln\Lambda$
	BSM			4.6E-5			0.0129				
				(0.0002)			(0.0002)				
DJIA	RSM	0.9798	0.9907	-0.0008	0.0005		0.0196	0.0079			919*
Index		(0.0069)	(0.0033)	(0.0007)	(0.0002)		(0.0007)	(0.0002)			
	RSMJ	0.9818	0.9936	-0.0009	0.0005	-0.0002	0.0196	0.0077	0.0138	0.2934	23*
		(0.0103)	(0.0082)	(0.0004)	(0.0044)	(0.0066)	(0.0009)	(0.0006)	(0.0044)	(0.0138)	
	BSM			-3.5E-5			0.0138				
				(0.0003)			(0.0002)				
S&P500	RSM	0.9783	0.9895	-0.0010	0.0004		0.0207	0.0082			945.4*
Index		(0.0057)	(0.0027)	(0.0007)	(0.0002)		(0.0006)	(0.0002)			
	RSMJ	0.9803	0.9925	-0.0010	0.0005	-0.0004	0.0207	0.0079	0.0144	0.3100	26.4*
		(0.0039)	(0.0013)	(0.0008)	(0.0016)	(0.0024)	(0.0002)	(0.0000)	(0.0025)	(0.0057)	

TABLE IV

The Autocorrelation Function of the Squared Return under RSMJ

Model	Autocorrelation Function
RSM	$\dfrac{[(u_1^2+\sigma_1^2)^2-(u_2^2+\sigma_2^2)^2]^2\dfrac{(-1+p_{11}+p_{22})^k(1-p_{11})(1-p_{22})}{(2-p_{11}-p_{22})^2}}{\pi_1(u_1^4+6u_1^2\sigma_1^2+3\sigma_1^4)+\pi_2(u_2^4+6u_2^2\sigma_2^2+3\sigma_2^4)-[\pi_1(u_1^2+\sigma_1^2)+\pi_2(u_2^2+\sigma_2^2)]^2}$
RSMJ	$\dfrac{[(u_y^2+\sigma_1^2)^2-(u_y^2+\sigma_2^2)^2]^2\dfrac{(-1+p_{11}+p_{22})^k(1-p_{11})(1-p_{22})}{(2-p_{11}-p_{22})^2}+4\lambda u_y[u_1(u_1^2+\sigma_1^2)+u_2(u_2^2+\sigma_2^2)]\dfrac{(-1+p_{11}+p_{22})^k(1-p_{11})(1-p_{22})}{(2-p_{11}-p_{22})^2}}{\pi_1(u_1^4+6u_1^2\sigma_1^2+3\sigma_1^4)+\pi_2(u_2^4+6u_2^2\sigma_2^2+3\sigma_2^4)-[\pi_1(u_1^2+\sigma_1^2)+\pi_2(u_2^2+\sigma_2^2)]^2+C(Jump)}$ $+\dfrac{2\lambda u_y\left\{\dfrac{1-(-1+p_{11}+p_{22})^k}{2-p_{11}-p_{22}}[u_1(1-p_{22})(u_2^2+\sigma_2^2)+u_2(1-p_{11})(u_1^2+\sigma_1^2)]\right\}+4\lambda^2u_y^2\dfrac{(-1+p_{11}+p_{22})^k(1-p_{11})(1-p_{22})}{(2-p_{11}-p_{22})^2}(u_1-u_2)^2}{\pi_1(u_1^4+6u_1^2\sigma_1^2+3\sigma_1^4)+\pi_2(u_2^4+6u_2^2\sigma_2^2+3\sigma_2^4)-[\pi_1(u_1^2+\sigma_1^2)+\pi_2(u_2^2+\sigma_2^2)]^2+C(Jump)}$

Note. $E(\sum_{n=1}^{N_t}\log Y_n)=\lambda u_y$, $E(\sum_{n=1}^{N_t}\log Y_n)^2=u_y^2(\lambda^2+\lambda)+\sigma_y^2\lambda$,

$E(\sum_{n=1}^{N_t}\log Y_n)^3=u_y^3(\lambda^3+3\lambda^2+\lambda)+3u_y\sigma_y^2(\lambda^2+\lambda)$, and $E(\sum_{n=1}^{N_t}\log Y_n)^4=u_y^4(\lambda^4+6\lambda^3+7\lambda^2+\lambda)+6u_y^2\sigma_y^2(\lambda^3+3\lambda^2+\lambda)+3\sigma_y^4(\lambda^2+\lambda)$.

$$C(Jump)=E(\sum_{n=1}^{N_t}\log Y_n)^4-[E(\sum_{n=1}^{N_t}\log Y_n)^2]^2+4\{[\pi_1(u_1^2+\sigma_1^2)+\pi_2(u_2^2+\sigma_2^2)][E(\sum_{n=1}^{N_t}\log Y_n)]^2\}$$
$$+4\{\pi_1(u_1^3+3u_1\sigma_1^2)+\pi_2(u_2^3+3u_2\sigma_2^2)-(\pi_1u_1+\pi_2u_2)[\pi_1(u_1^2+\sigma_1^2)+\pi_2(u_2^2+\sigma_2^2)]\}E(\sum_{n=1}^{N_t}\log Y_n)+4(\pi_1u_1+\pi_2u_2)[E(\sum_{n=1}^{N_t}\log Y_n)^3-E(\sum_{n=1}^{N_t}\log Y_n)^2E(\sum_{n=1}^{N_t}\log Y_n)]$$

Table V

The Impact of Pricing Cycles for Call Option under the RSMJ

p_{11}	p_{22}	σ_1	σ_2		
			0.005	0.01	0.02
0.90	0.90	0.02	5.0434	5.4076	6.6211
		0.04	8.9696	9.1878	9.9860
		0.06	13.0776	13.2298	13.8069
	0.95	0.02	4.3836	4.9355	6.6211
		0.04	7.3263	7.7030	8.9600
		0.06	10.4820	10.7665	11.7525
	0.99	0.02	3.2692	4.1854	6.6211
		0.04	4.2294	5.0511	7.2965
		0.06	5.3107	6.0709	8.1662
0.95	0.90	0.02	5.6055	5.8322	6.6211
		0.04	10.2699	10.4005	10.8905
		0.06	15.0730	15.1629	15.5091
	0.95	0.02	4.9970	5.3852	6.6211
		0.04	8.8035	9.0574	9.9286
		0.06	12.7836	12.9726	13.6388
	0.99	0.02	3.5854	4.4063	6.6211
		0.04	5.0518	5.7785	7.7803
		0.06	6.6377	7.3093	9.1635
0.99	0.90	0.02	6.3552	6.4123	6.6211
		0.04	11.9611	11.9927	12.1144
		0.06	17.6472	17.6687	17.7525
	0.95	0.02	6.1025	6.2215	6.6211
		0.04	11.3715	11.4457	11.7082
		0.06	16.7371	16.7915	16.9865
	0.99	0.02	4.8318	5.3086	6.6211
		0.04	8.1579	8.5744	9.7302
		0.06	11.5804	11.9657	13.0286

Note. $u_y = -0.0002$, $\sigma_y = 0.0138$, $\lambda = 0.2934$, $S = 100$, $K = 100$, $T = 60$, $r = 0.28\%$.

TABLE VI

The Impact of Jump Risk for Call Option under the RSMJ

λ	u_y	σ_y		
		0.005	0.01	0.02
0.1	-0.0003	3.5203	3.6330	4.0381
	0	3.5204	3.6331	4.0381
	0.0003	3.5203	3.6330	4.0381
0.5	-0.0003	3.6699	4.1683	5.6764
	0	3.6702	4.1686	5.6766
	0.0003	3.6699	4.1683	5.6764
1.0	-0.0003	3.8457	4.7359	7.1821
	0	3.8463	4.7364	7.1824
	0.0003	3.8457	4.7359	7.1821

Note. $p_{11} = 0.9818$, $p_{22} = 0.9936$, $\sigma_1 = 0.0196$, $\sigma_2 = 0.0077$, $S = 100$, $K = 100$, $T = 60$, $r = 0.28\%$.

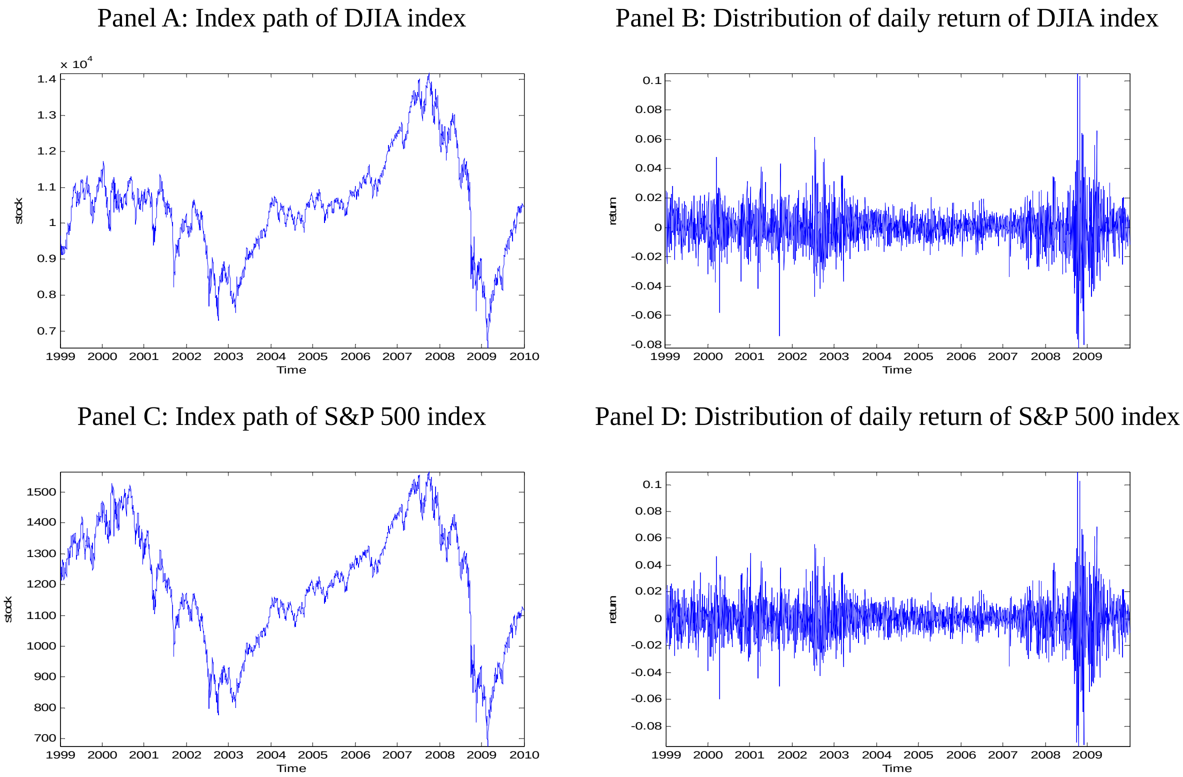
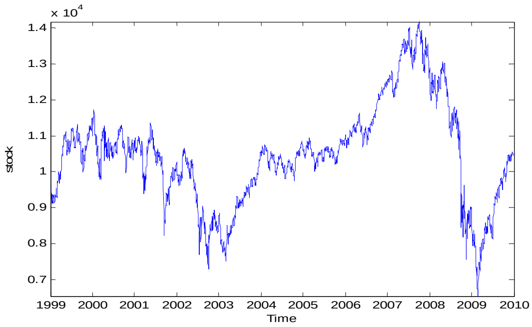


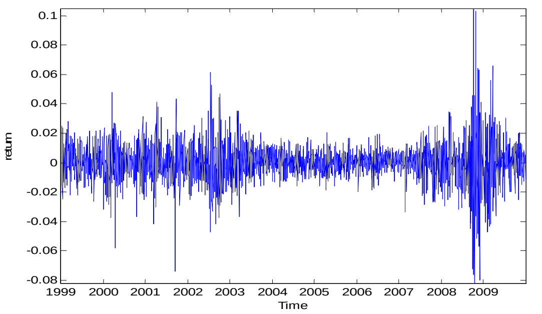
FIGURE 1

Index path and daily return of DJIA index and S&P 500 index from Jan. 4th, 1999 to Jan. 15th, 2010.

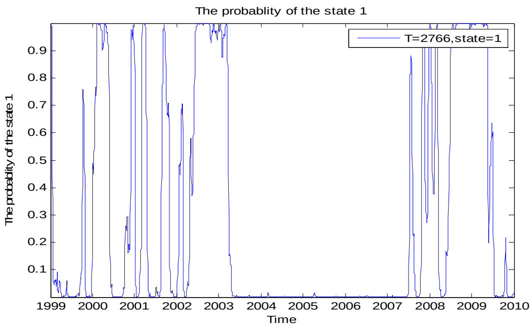
Panel A: The path index



Panel B: The returns of index



Panel C: The probabilities of recession



Panel D: The probabilities of jumps

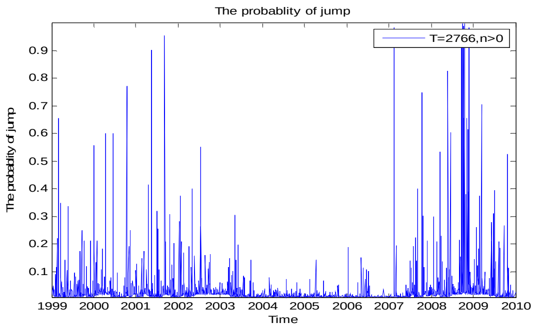
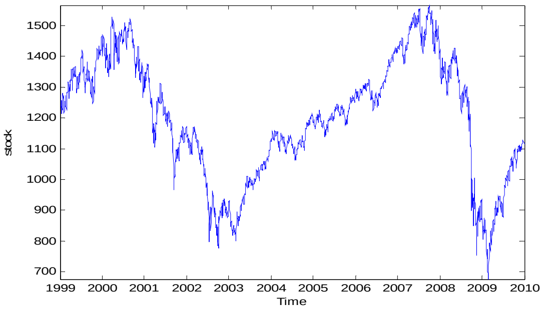


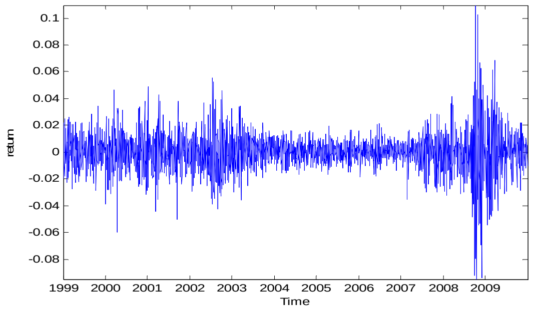
FIGURE 2

The index, index return, probabilities of the recession and probabilities of the jumps of the DJIA index from 1999 to 2010.

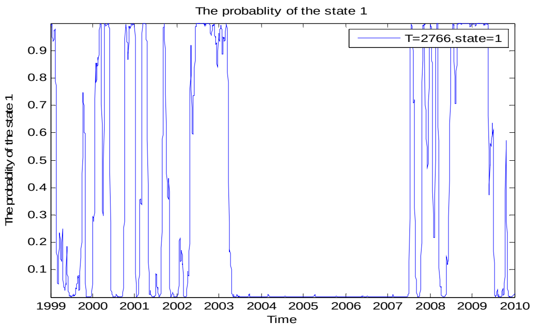
Panel A: The path of index



Panel B: The returns of index



Panel C: The probabilities of recession



Panel D: The probabilities of jumps

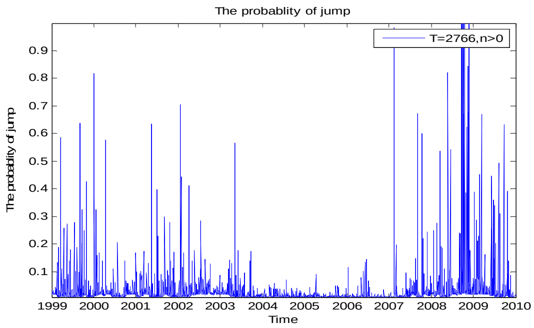
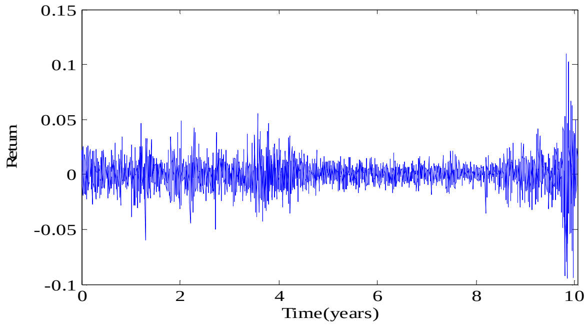


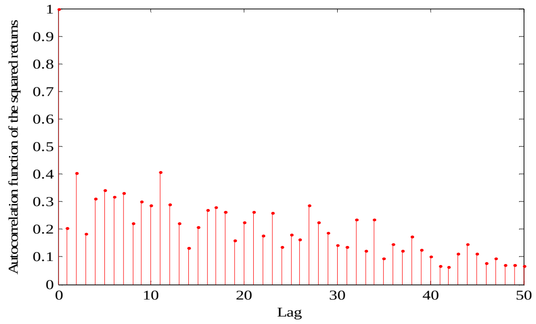
FIGURE 3

The index, index return, probabilities of the recession and probabilities of the jumps of the S&P 500 index from 1999 to 2010.

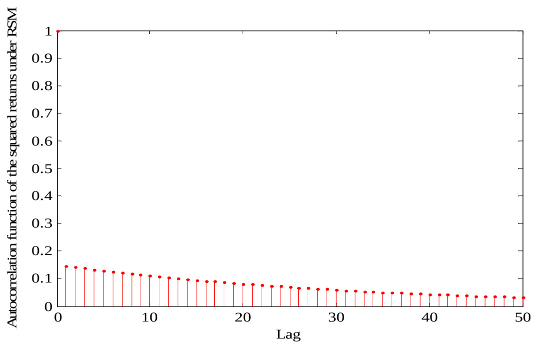
Panel A: The dynamic of the return in the S&P 500 index from 1999 to 2010



Panel B: The autocorrelation of the squared return in real data



Panel C: The autocorrelation of the squared return in regime- switching model



Panel D: The autocorrelation of the squared return in regime- switching model with jump risks

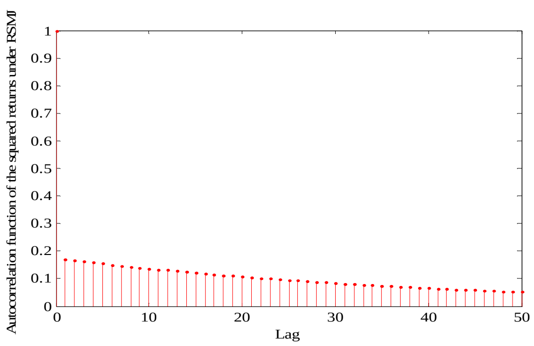
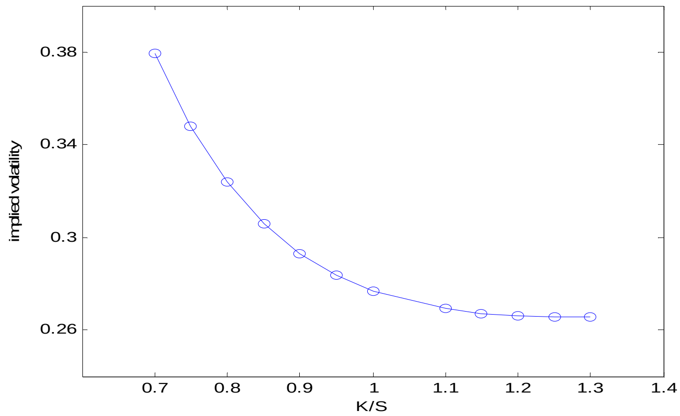


FIGURE 4

The plots of the returns and the autocorrelation of the squared returns.

Panel A: The Black-Scholes implied volatility curve of the S&P 500 index estimated under the regime-switching model with jump risks



Panel B: The Black-Scholes implied volatility surface of the S&P 500 index estimated under the regime-switching model with jump risks

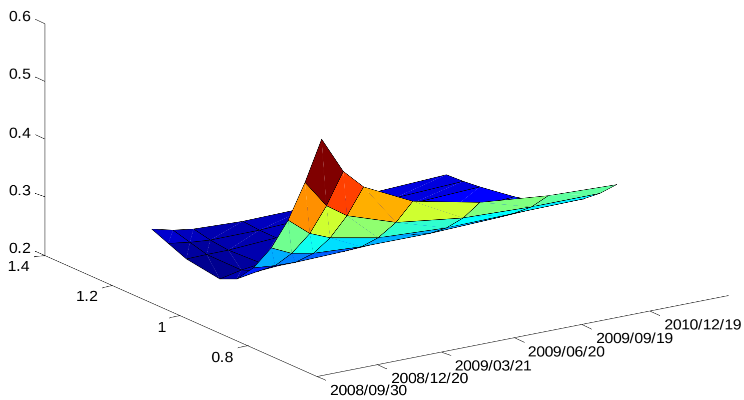


FIGURE 5

Black-Scholes implied volatility smirk and volatility surface estimated under the regime-switching model with jump risks.