## **Option Pricing Using Roll-Over Parameters**

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#### Abstract

For S&P 500 options, we examine the pricing and hedging performance of several options pricing models with respect to the roll-over strategies of parameters. The traditional roll-over strategy of the parameters, the nearest-to-next approach, and those using the new roll-over strategy, the next-to-next approach are compared. It is found that the next-to-next roll-over strategy can decrease pricing and hedging errors of all options pricing models and mitigate the over-fitting problems. The "absolute smile" traders' rule has the advantage of simplicity and is the best model for pricing and hedging options.

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### Abstract

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# 1. Introduction

Ever since Black and Scholes (1973) published their seminal article on option pricing in 1973, various theoretical and empirical researches have been conducted on option pricing. One important direction in which the Black and Scholes (1973) model can be modified is to generalize the geometric Brownian motion, which is used as a process for the dynamics of log stock prices. For example, Merton (1976) and Naik and Lee (1990) propose a jump-diffusion model. Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987) and Heston (1993) suggest a stochastic volatility model. Naik (1993) considers a regime-switching model that assumes jumps of the volatility. Duan (1995) and Heston and Nandi (2000) develop an option pricing framework based on the GARCH process. Madan, Carr and Chang (1998) use the Variance Gamma process as an alternative model for capturing the dynamics of log stock prices.

Bakshi, Cao and Chen (1997, 2000) and Kim (2005) have conducted a comprehensive empirical study on the relative merits of competing option pricing models. They have discovered that taking the stochastic volatility into account is of the first order in importance for improving the Black and Sholes (1973) model. However, among the striking empirical findings, Dumas, Fleming and Whaley (1998), Jackwerth and Rubinstein (2001) and Li and Pearson (2007) and Kim (2009) examine the performance of a number of these mathematically sophisticated models and find that they predict option prices less well than a pair of ad hoc approaches sometimes used by option traders. Ad hoc approaches can be an alternative to the complicated models for pricing options; they are known as ad hoc Black and Scholes models (henceforth AHBS).

There are two versions of the AHBS models. In the "relative smile" approach, the implied volatility skew is treated as a fixed function of moneyness, S/K, whereas the implied volatility for a fixed strike K varies as the stock index S varies. This is also known as the "sticky volatility"

method. In the "absolute smile" approach, the implied volatility is treated as a fixed function of the strike price K, and the implied volatility for a fixed strike does not vary with S. This is also known as the "sticky delta" method. Jackwerth and Rubinstein (2001), Li and Pearson (2007), Kim (2009), and Choi and Ok (2011) have found that the "absolute smile" approach shows better performance than the "relative smile" approach for pricing options. Further, a simpler model among the AHBS models shows better performance than the other models. That is, the presence of more parameters in AHBS models actually causes over-fitting.

When the options are priced and hedged, we need to estimate the parameters that are needed to plug into each model. For one day ahead pricing and hedging, the parameters are estimated using the previous days' options data. For one week ahead pricing and hedging, the options data before seven days are used. In general trading dates, there are no complicated problems. However, it is standard to eliminate the nearest option contracts with expiries less than 7 days, as well as to use the next-to-nearest option contracts with expiries less than 7 days plus 1 month for the empirical study. When forecasting the parameters for the next-to-nearest option contracts with expiry greater than 6 days (the nearest-to-next roll-over strategy) or the next-to-nearest contracts with expiry greater than 6 days plus 1 month (the next-to-next roll-over strategy). Recently, Choi and Ok (2011) have shown that the next-to-next roll-over strategy can mitigate the over-fitting problems of AHBS models and can make AHBS models with more parameters to become the better model than the AHBS model with less parameters. As a result, the next-to-next roll-over strategy of the parameters can be useful for the AHBS type models.

However, is the next-to-next roll-over strategy functioned only with the AHBS models? Or can this strategy also be functioned with mathematically complicated models, stochastic volatility (henceforth SV) and stochastic volatility with jump (henceforth SVJ) models? In this paper, we examine the empirical performance of several options pricing models with respect to the rollover strategies of parameters. Not only the traders' rules, that is, AHBS-type models, but also the SV model and the SVJ model are considered for a horse race competition. We compare the pricing and hedging performances of several option pricing models using the traditional rollover strategy of the parameters, which is the nearest-to-next approach, with those using the new roll-over strategy, the next-to-next approach. We examine whether the new roll-over strategy of the parameters can be functioned not only for the AHBS-type models but also for the mathematically complicated models, the SV and SVJ models. After considering the new rollover strategy of the parameters, we try to find out the best options pricing model.

We fill the gaps that have not been resolved in previous researches. First, when the roll-over strategies of the parameters are examined, Choi and Ok (2011) and Choi, Jordan, and Ok (2012) do not consider the mathematically complicated models that are shown to be competitive options pricing models. In this paper, we examine whether the new roll-over strategy of the parameters for the SV and the SVJ models is functioned. Second, in previous researches, the new roll-over strategy of the parameters, or the next-to-next strategy, is not considered for hedging performance. When we try to find out the best options pricing model, both pricing and hedging performance must be considered. Pricing performance of the options pricing models measures the ability to forecast the level of options price; however, hedging performance measures the ability to forecast the variability of options prices. If a specific model shows better performance than the other models for both performance measures, that model can truly be the best options pricing model. Third, Choi and Ok (2011) and Choi, Jordan and Ok (2012) consider the sample period with a span of just two years. For two years, the dates that require the rollover of the parameters are twenty four days when we examine the one day ahead out-of-sample pricing and hedging performance. The effect of the roll-over strategy can be exaggerated due to a small sample. In this paper, we examine the roll-over strategies using sample dates with a span of 13 years. If the new roll-over strategy works well, even for a long sample period, we can conjecture that there is a structural change of the parameters when the maturity of options is

rolled- over. Fourth, recent researches that examine the performance of AHBS-type models have considered KOSPI 200 options, which is one of the emerging markets. Although KOSPI 200 options are the biggest derivatives products in the world, in terms of trading volume, these products are traded in the emerging market. We use S&P 500 (SPX) option prices for our empirical work. S&P 500 options have been the focus of many existing investigations including, among others, Bakshi, Cao and Chan (1997), Bates (1996), Dumas, Fleming and Whaley (1995). Also, the roll-over strategies of the parameter are not compared in the SPX options market. If the new roll-over strategy is well functioned for SPX options, we can conjecture that it is not only fit to the emerging markets, but can also be generally applied to the advanced options markets.

It is found that the next-to-next strategy can decrease pricing and hedging errors of all options pricing models compared to the nearest-to-next approach. The AHBS-type models that more parameters show better performance than the ad-hoc approaches which have less parameters and the mathematically complicated models for both pricing and hedging options. That is, the next-to-next strategy can mitigate the over-fitting problem of AHBS-type models. The "absolute smile" traders' rule has the advantage of simplicity and is the best model for pricing and hedging options.

The outline of this paper is as follows. The AHBS-type models, the stochastic volatility with jumps model and the roll-over strategies of the parameters are reviewed in Section 2. The data used for the analysis are described in Section 3. Section 4 describes the parameter estimates of each model and evaluates pricing and hedging performances of alternative options pricing models. Section 5 concludes our study by summarizing the results.

## 2. Options Pricing Models

# 2.1 Ad Hoc Black-Scholes Model

Despite its significant pricing and hedging biases, the Black and Scholes (1973) model (henceforth the BS model) continues to be widely used by market practitioners. However, when practitioners apply the BS model, they commonly allow the volatility parameter to vary across strike prices of options as well as to fit the volatility to the observed smile pattern. As Dumas, Fleming and Whaley (1998) show, this procedure can avoid some of the biases associated with the BS model's constant volatility assumption.

We have to construct the AHBS model in which each option has its own implied volatility depending on a strike price (or moneyness) and the time to maturity. Specifically, the spot volatility of the asset that enters the BS model is a function of the strike price (or moneyness) and the time to maturity or a combination of both. However, we only consider the function of the strike price (or moneyness) because the liquidity of the index options market is concentrated in the nearest expiration contract. Dumas, Fleming and Whaley (1998) show that the specification that includes a time parameter performs worst of all, indicating that the time variable is an important cause of the over-fitting problem at the estimation stage.

There are two versions of the ad hoc approach. In the "relative smile" approach, the implied volatility skew is treated as a fixed function of moneyness, S/K, and the implied volatility for a fixed strike K varies as the stock index S varies. This is also known as the "sticky volatility" method. In the "absolute smile" approach, the implied volatility is treated as a fixed function of the strike price K, and the implied volatility for a fixed strike does not vary with S. This is also known as the "sticky delta" method. These models are so called the ad hoc Black-Scholes model (henceforth AHBS). Dumas, Fleming and Whaley (1998), Jackwerth and Rubinstein (2001) and Li and Pearson (2007), Kim (2009) and Choi and Ok (2011), Choi, Jordan and Ok (2012), who report that the AHBS model outperforms other options pricing models, adopt the "absolute smile" approach. On the other hand, Kirgiz (2001) and Kim and Kim (2004), who report the AHBS model does not outperform others, adopt the "relative smile" approach. That is, the specific type of the AHBS model seems to be important for pricing and hedging performances.

Specifically, we adopt the following six specifications for the BS implied volatilities:

R1: 
$$\sigma_i = \beta_1 + \beta_2 \cdot (S/K_i)$$
 (1)

R2: 
$$\sigma_i = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot (S/K_i)^2$$
 (2)

R3: 
$$\sigma_i = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot (S/K_i)^2 + \beta_4 \cdot (S/K_i)^3$$
 (3)

A1: 
$$\sigma_i = \beta_1 + \beta_2 \cdot K_i$$
 (4)

A2: 
$$\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot K_i^2$$
(5)

A3: 
$$\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot K_i^2 + \beta_4 \cdot K_i^3$$
(6)

where  $\sigma_i$  is the implied volatility for an i th option of strike  $K_i$  and spot price S.

R1, R2 and R3 models are the "relative smile" approaches using the "relative" moneyness as the independent variables. A1, A2 and A3 models are the "absolute smile" approaches using the "absolute" strike prices as the independent variables. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. Up to now, previous studies do not consider the third power of the moneyness and the strike price. In this paper, the performances of the AHBS models with higher degrees are examined.

For implementation, we follow a four-step procedure. First, we abstract the BS implied

volatilities from each option. Second, we set up the implied volatilities as the dependent variable and the moneyness or the strike price as the independent variables. We also estimate the  $\beta_i$  (i = 1,2,3,4) by ordinary least squares. Third, using the estimated parameters from the second step, we plug each option's moneyness or the strike price into the equation, and obtain the model-implied volatility for each option. Finally, we use volatility estimates computed in the third step in order to price options with the following BS formula.

$$C_{t,\tau;K} = S_t N(d_1) - K e^{-r\tau} N(d_2)$$
<sup>(7)</sup>

$$P_{t,\tau;K} = K e^{-r\tau} N(-d_2) - S_t N(-d_1)$$
(8)

$$d_1 = \frac{\ln[S_t/K] + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau}$$
(9)

where  $N(\cdot)$  is the cumulative standard normal density. The AHBS model, although theoretically inconsistent, can be a more challenging benchmark than the simple BS model for any competing options valuation model.

## 2.2 Stochastic Volatility with Jumps Model

Bakshi, Cao and Chan (1997) derived a closed-form option pricing model that incorporates both stochastic volatility and jumps. Under the risk neutral measure, the underlying nondividend-paying stock price  $S_t$  and its components for any time t are given by

$$\frac{dS_t}{S_t} = \left[r_t - \lambda \mu_J\right] dt + \sqrt{v_t} dz_{s,t} + J_t dq_t \tag{10}$$

$$dv_t = \left[\theta_v - \kappa_v v_t\right] dt + \sigma_v \sqrt{v_t} dz_{v,t}$$
(11)

$$\ln[1+J_{t}] \sim N(\ln[1+\mu_{J}]-1/2\sigma_{J}^{2},\sigma_{J}^{2})$$
(12)

where  $r_t$  is the instantaneous spot interest rate at time t,  $\lambda$  is the frequency of jumps per year and  $v_t$  is the diffusion component of return variance (conditional on no jump occurring).  $z_{s,t}$  and  $z_{v,t}$  are standard Brownian motions, with  $Cov_t [dz_{s,t}, dz_{v,t}] = \rho dt$ .  $J_t$  is the percentage jump size (conditional on a jump occurring) that is lognormally, identically and independently distributed over time, with unconditional mean,  $\mu_J$ . The standard deviation of  $\ln[1+J_t]$  is  $\sigma_J$ .  $q_t$  is a Poisson jump counter with intensity  $\lambda$ , that is,  $\Pr[dq_t = 1] = \lambda dt$ and  $\Pr[dq(t)=0]=1-\lambda dt$ .  $\kappa_v$ ,  $\theta_v/\kappa_v$  and  $\sigma_v$  are the speed of adjustment, long-run mean and variation coefficient of the diffusion volatility  $v_t$ , respectively.  $q_t$  and  $J_t$  are uncorrelated with each other or with  $z_{s,t}$  and  $z_{v,t}$ .

For a European call option written on the stock with strike price K and time to maturity  $\tau$ , the closed form formula for price  $C_{t,\tau}$  at time t is as follows.

$$C_{t,\tau} = S_t P_1(t,\tau; S_t, r_t, v_t) - K e^{-r_t \tau} P_2(t,\tau; S_t, r_t, v_t)$$
(13)

where the risk neutral probabilities,  $P_1$  and  $P_2$ , are computed from inverting the respective characteristic functions of the following:

$$P_{j}(t,\tau;S_{t},r_{t},v_{t})$$

$$=\frac{1}{2}+\frac{1}{\pi}\int_{0}^{\infty}\operatorname{Re}\left[\frac{\exp(-i\phi\ln K)f_{j}(t,\tau,S_{t},r_{t},v_{t};\phi)}{i\phi}\right] (j=1,2)$$
(14)

The characteristic functions,  $f_j$ , are given in equations (A-1) and (A-2) of the Appendix. The price of a European put on the same stock can be determined from the put-call parity.

The option valuation model in equation (13) and (14) contains the most existing models as special cases. For example, we obtain (i) the BS model by setting  $\lambda = \theta_v = \kappa_v = \sigma_v = 0$ ; we obtain (ii) the SV model by setting  $\lambda = 0$ , where in order to derive each special case from equation (14), one may need to apply L'Hopital's rule.

In applying the option pricing models, one always encounters the difficulty where spot volatilities and structural parameters are unobservable. As estimated in standard practice, we estimate the parameters of each model for every sample day. Since closed-form solutions are available for an option price, a natural candidate for the estimation of parameters in the formula is a non-linear least squares procedure, involving a minimization of the sum of squared errors between the model and the market prices.<sup>1</sup> Let  $O_{i,t}^*$  denote the model price of option i on day t, and  $O_{i,t}$  denote the market price of option i on day t. To estimate parameters for each model, we minimize the sum of percentage squared errors between the model and the market price of option i on day the market prices.

$$\min_{\phi_t} \sum_{i=1}^{N} \left[ O_{i,t} - O_{i,t}^* \right]^2 \quad (t = 1, ..., T)$$
(15)

where N denotes the number of options on day t, and T denotes the number of days in the sample.

Estimating the parameters from the asset returns can be an alternative method; however, historical data reflect only what happened in the past. Furthermore, the procedure using

<sup>&</sup>lt;sup>1</sup> The objective function to minimize the sum of percentage squared errors can be used. However, in this case, the AHBS models are calibrated by OLS on implied volatilities whereas the SV and SVJ models are calibrated on relative pricing errors. Christoffersen and Jacobs (2004) address such inconsistencies in the choice of objective function and argue that results from the mixture of two different objectives have little explanatory power. In particular, they argue that the in-sample objective function must be the same as the out-of-sample objective function. Also, in our sample, there is no large difference between the results using the sum of squared errors and those using the sum of percentage squared errors.

historical data is not capable of identifying risk premiums, which must be estimated from the options data conditional on the estimates of other parameters. The important advantage of using option prices to estimate parameters is to allow one to use the forward-looking information contained in the option prices.

### 2.3 Rollover Strategies

When options are priced and hedged, we need to estimate the parameters that are needed to plug into each model. As estimated in standard practice, we estimate the parameters of each model using the options data for every sample day. For one day ahead pricing and hedging performance, the parameters are estimated using the previous days' options data. For one week ahead pricing and hedging performance, the options data seven days ago are used. In general trading dates, there is no complicated problems to implement this methodology. However, it is standard to eliminate the nearest option contracts with expiries less than 7 days, and use the next-to-nearest option contracts with expiries less than 7 days plus 1 month for the empirical study due to the liquidity problems of options contract. When forecasting the parameters for the next-to-nearest option contracts with expiry less than 7 days plus 1 month, one can use either the nearest contracts with expiry greater than 7 days (the nearest-to-next roll-over strategy) or the next-to-nearest contracts with expiry greater than 7 days plus 1 month (the nextto-next roll-over strategy). The information content of these two contracts may differ and thus, the rollover procedure may be important to the accuracy of the parameter forecasting. Figure 1 shows the difference between the nearest-to-next strategy and the next-to-next strategy. Figure 1 represents the example for one day ahead pricing and hedging performance. The circle represents the options data from the nearest option contract. The diamond represents the options data from the next-to-nearest option contract. It is standard to eliminate the nearest option contracts with expiries less than 7 days, and use the next-to-nearest option contracts with expiries less than 7 days plus 1 month for the empirical study due to liquidity problems of the

options contract. When forecasting the parameters for the next-to-nearest option contracts with expiry less than 7 days plus 1 month, one can use either the nearest options contracts (circle) with expiry greater than 6 days or the next-to-nearest options contracts (diamond) with expiry greater than 6 days plus 1 month. When the nearest contract's expiry is less than 7 days, the next-to-next strategy uses the next-to-nearest contracts on the previous day(s), whereas the nearest-to-next one uses the nearest-term contracts. These two strategies are different only on the day(s) when the expiry of nearest-term option contracts is less than seven days.

In this paper, the performances of the nearest-to-next strategy and the next-to-next strategy are compared. If the next-to-next roll-over strategy shows better performance than the nearest-tonext one, we can conjecture that there is a structural change when the nearest-to-next contracts are changed into the nearest contract.

## 3. Data

The S&P 500 index option data used in this paper come from Option Metrics LLC. The data include the end-of day bid and ask quotes, implied volatilities, open interest and daily trading volume for the SPX options traded on the Chicago Board Options Exchange from January 4, 1996 through December 31, 2008. The data also include daily index values and estimates of dividend yields, as well as the term structures of zero-coupon interest rates constructed from LIBOR quotes and Eurodollar futures prices. We use the bid-ask average as our measure of the option price.

The following rules are applied in order to filter the data needed for the empirical test. We use out-of-the-money options for calls and puts. First of all, since there is only a very thin trading volume for the in-the-money (henceforth ITM) option, the credibility of price information is not entirely satisfactory. Therefore, we use the price data with regards to both put and call options that are near-the-money and out-of-the-money (henceforth OTM). Second,

if both call and put option prices are used, ITM calls and OTM puts, which are equivalent according to the put-call parity, are used to estimate the parameters. Third, as Huang and Wu (2004) mention, "the Black-Scholes model has been known to systematically misprice equity index options, especially those that are out-of-the-money (OTM)." We recognize the need for an alternative option pricing model in order to mitigate this effect.

As options with less than 7 days to expiration may induce biases due to low prices and bidask spreads, they are excluded from the sample. As we mentioned before, this is the reason why the roll-over strategies can be emerged. Because the liquidity is concentrated in the nearest expiration contract, we only consider options with the nearest maturity. To mitigate the impact of price discreteness on option valuation, prices lower than 0.4 are not included. Prices not satisfying the arbitrage restriction are excluded.

We divide the option data into several categories, according to the moneyness, S/K. Table 1 describes certain sample properties of the SPX option prices used in this study. Summary statistics are reported for the option price as well as for the total number of observations, according to each moneyness-option type category. Note that there are 42,396 call- and 64,316 put-option observations, with deep OTM<sup>2</sup> options, respectively, taking up 24% for calls and 49% for puts. Table 2 presents the "volatility smiles (or sneers)" effects for 26 consecutive sixmonth sub-periods. We employ six fixed intervals for the degree of moneyness, and compute the mean over alternative sub-periods of the implied volatility. SPX options market seems to be "sneer" independent of the sub-periods employed in the estimation. As the S/K increase, the implied volatilities decrease to near-the-money; however, after that, they increase to out-of-the-money put options. The implied volatility of deep out-of-the-money puts is larger than that of deep out-of-the-money calls. That is, a volatility snile is skewed towards one side. The skewed volatility smile is sometimes called a 'volatility sneer' because it looks more like a sardonic smile than a sincere smile. In the equity options market, the volatility sneer is often negatively skewed,

 $<sup>^2</sup>$  For the call option, deep OTM options are options in S/K < 0.94. For the put option, deep OTM options are options in S/K > 1.06.

where lower strike prices for out-of-the money puts have higher implied volatilities and, thus, higher valuations.<sup>3</sup> This is consistent with Rubinstein (1994), Derman (1999), Bakshi, Kapadia and Madan (2001), and Dennis and Mayhew (2002). As the smile evidence is indicative of negatively-skewed implicit return distribution with excess kurtosis, a better model must be based on a distributional assumption that allows for negative skewness and excess kurtosis.

## 4. Empirical Results

In this section, we examine the empirical performances of each model with respect to insample pricing, out-of-sample pricing and hedging performance. The analysis is based on two measures: mean absolute errors (henceforth MAE) and root mean squared errors, (henceforth RMSE) as follows.

$$MAE = \frac{1}{T} \sum_{t=1}^{T} \frac{1}{N} \sum_{i=1}^{N} \left| O_{i,t} - O_{i,t}^{*} \right|$$
(16)

RMSE = 
$$\sqrt{\frac{1}{T} \sum_{t=1}^{T} \frac{1}{N} \sum_{i=1}^{N} [O_{i,t} - O_{i,t}^{*}]^{2}}$$
 (17)

where  $O_{i,t}^*$  denotes the model price of option i on day t, and  $O_{i,t}$  denotes the market price of option i on day t. N denotes the number of options on day t, and T denotes the number of days in the sample. MAE measures the magnitude of pricing errors, while RMSE measures the volatility of errors. If some options pricing model has the lowest values for both MAE and RMSE, that model is the best.

## 4.1 In-sample Pricing Performance

<sup>&</sup>lt;sup>3</sup> See Rubinstein (1994) and Bakshi, Cao and Chen (1997).

Table 3 reports the mean and the standard error of the parameter estimates for each model. R<sup>2</sup> values for each AHBS-type model are also reported. For AHBS-type models, each parameter is estimated by the ordinary least squares every day. For the BS, SV and SVJ models, each parameter is estimated by minimizing the sum of the squared errors between the model and the market option prices every day. First, the daily estimates of each model's parameters have excessive standard errors. However, such estimation will be valuable for the following reasons. The estimated parameters can be generated by indicating market sentiment on a daily basis, and the estimated parameters may suggest future specification of more complicated dynamic models. Also, because the AHBS-type models are based on not theoretical backgrounds, but on the traders' rule, it is not a fatal problem. Second, as expected, the R3 and A3 models that have four independent variables show higher R<sup>2</sup> values compared to other models. Therefore, it is necessary to check for the over-fitting problem by examining the out-of-sample pricing performance. Third, the implied correlation between the index return and the level of the volatilities of the SV and SVJ models has negative values. The negative estimate indicates that the implied volatility and the index returns are negatively correlated and the implied distribution recognized by option traders is negatively skewed. Also, the mean of jumps of the SVJ model is negative. This is consistent with the volatility sneer pattern shown in table 2.

We evaluate the in-sample pricing performance of each model by comparing the market prices with the model's prices computed by using the parameter estimates from the current day. Table 4 reports in-sample valuation errors for the alternative models computed over the whole sample of options. The SVJ model shows the best performance, closely followed by the SV model for MAE; the SV model outperforms other models for RMSE. Roughly, the SV and SVJ, the mathematically complicated models, are better than the AHBS-type models for in-sample pricing. The in-sample pricing performance is simply contingent on the number of free parameters. Second, all models show moneyness-based valuation errors. The models exhibit the worst fit for the near-the-money options. The fit, as measured by MAE, steadily improves as we move from near-the-money to out-of-the-money options. Also, the SV and SVJ models do not show better performance than the AHBS-type models for OTM call options which S/K is less than one.

Overall, all AHBS-type models and mathematically complicated models demonstrate better performance than the BS model. Also, the traders' rule can explain the current market price in the options market although it is not rooted in rigorous theory.

## 4.2 Out-of-sample Pricing Performance

In-sample pricing performance can be perverted due to the dormant problem of over-fitting to the data. A good in-sample fit might be a result of having an increasingly larger number of parameters. To reduce the effect of this connection to inferences, we turn to examining the model's out-of-sample pricing performance. In the out-of-sample pricing, the presence of more parameters may actually cause over-fitting and thus, have the model penalized if the extra parameters do not improve its structural fitting. This analysis also has the purpose of evaluating the stability of each model's parameter over time. To control the parameters' stability over alternative time periods, we analyze out-of-sample valuation errors for one day or one week. We use the current day's estimated parameters in order to price options for the following day (or week).

Table 5 and table 6, respectively, report one-day and one-week ahead out-of-sample pricing errors for alternative models computed over the whole sample of options. First, we examine the out-of-sample pricing performance using the nearest-to-next roll-over strategy. Panel A of table 5 and table 6 represents the results using the nearest-to-next roll-over strategy. For one day ahead out-of-sample pricing, the A2 model shows the best performance, closely followed by the A1 model. The A1 model exhibits better fit for the one week ahead out-of-sample pricing. For the in-sample pricing performance, the mathematically complicated models are competitive. However, for the out-of-sample pricing performance, the AHBS-type models show better

performance than the SV and SVJ models. Also, for the in-sample pricing performance, the A3 and R3 models that have more parameters than other models show better performance. However, for the out-of-sample pricing performance, the simpler A1 or A2 model is the best. That is, the presence of more parameters actually causes the problem of over-fitting. Consistent with Jackwerth and Rubinstein (2001), Li and Pearson (2007) and Kim (2009), the traders' rules dominate more mathematically sophisticated models although the SV and SVJ models are not far behind. With respect to moneyness-based errors, similar to the results of the in-sample pricing performance, MAE steadily decreases as we move from near-the-money to deep out-of-the-money options for all models. Generally, simple absolute AHBS approaches, the A1 and A2 models, outperform all other models.

Pricing errors increase from in-sample to out-of-sample pricing. The average of MAE of all models is 0.5360 for in-sample pricing, and increases to 1.2184 for one-day ahead out-of-sample pricing. One-week ahead out-of-sample pricing errors grow to 1.9805, which is almost four times as much as the in-sample pricing errors. The relative margin of pricing performance is significantly changed when compared to that of the in-sample pricing results. The difference of the BS and the best model becomes smaller in the out-of-sample pricing. The ratio of the BS model to the SVJ model for MAE is 8.0882 for in-sample pricing errors. The ratio of the BS model to the A2 or A1 model decreases to 2.2312 and to 1.5415 for one-day ahead and one-week ahead out-of-sample errors, respectively. As the term of the out-of-sample pricing becomes longer, the difference between the BS model and the best model becomes smaller. The pricing performance of the SVJ model, which is the best model for in-sample pricing, is not maintained as the term of out-of-sample pricing gets longer, implying that the presence of more parameters actually cause over-fitting. For the AHBS-type models, A3 and R3, the best models among them for in-sample pricing do not remain their position for one day and one week ahead out-ofsample pricing. For out-of-sample pricing, the A3 and the R3 models are changed into the very last, implying that the presence of more parameters actually cause over-fitting. This result is

consistent with the result of Jackwerth and Rubinstein (2001), Li and Pearson (2007) and Kim (2009). As a result, both the mathematically complicated options pricing models and the AHBS-type models have over-fitting problems when the traditional nearest-to-next roll-over strategy is used. To mitigate these problems, we need to consider the new roll-over strategy, the next-to-next strategy.

Second, we examine the pricing performance for the next-to-next roll-over strategy, suggested by Choi and Ok (2011). Panel B of table 5 and table 6 represents the results using the next-to-next roll-over strategy. Above all, the next-to-next roll-over strategy decreases the pricing errors of all options pricing models. After using the next-to-next strategy, the averages of the MAEs of all options pricing models are decreased from 1.2184 (1.9805) to 1.0046 (1.4687) by 20% (35%) for one day (one week) ahead out-of-sample pricing. Panel A and panel B of Figure 1 represent the MAE of each options pricing model for both the nearest-to-next and the next-to-next roll-over strategies, respectively. The pricing errors of all models are decreased largely by the next-to-next strategy. Among them, the models with more parameters, R3, A3, and SVJ are favored the most. For both one day and one week ahead out-of-sample pricing, the A3 model generally shows the best performance. When the nearest-to-next roll-over strategy is applied, the simpler A1 and A2 models show better performance compared to other models. However, using the next-to-next strategy, the A3 model shows better performance than the complicated models. As a result, when the next-to-next roll-over strategy is applied, the over-fitting problems of the models are disappeared and the A3 model outperforms all other models.

Finally, we examine the relative strength of the absolute and relative smile approaches for pricing options. For the in-sample pricing performance, the averages of the MAEs of "relative smile" and "absolute smile" approaches are 0.4092 and 0.4129, respectively. Using the nearest-to-next strategy, for one day (one week) ahead out-of-sample pricing, the average MAEs of alternative "relative smile" and "absolute smile" approaches arile" approaches are 1.2186 (2.0395) and 0.9891 (1.6234), respectively. Using the next-to-next strategy, for one day (one week) ahead out-of-

sample pricing, the averages of the MAEs of the alternative "relative smile" and "absolute smile" approaches are 0.9704 (1.5087) and 0.7551 (1.0833), respectively. Irrespective of the type of the roll-over strategy, the effects of the reduction of pricing errors for the absolute smile approach are much better compared to those for the relative smile approach. This result is consistent with those of Jackwerth and Rubinstein (2001), Li and Pearson (2007), Kim (2009) and Choi and Ok (2011), who report that the "absolute smile" model beats the "relative smile" model for predicting prices. The result can be explained by the fact that the absolute smile model implicitly adjusts for the negative correlation between the index return and the level of the volatilities. Because the absolute model treats the skew as a fixed function of the strike instead of the moneyness S/K, it creates a smaller implied volatility than the relative smile model when there is an increase in the stock price.

### 4.3 Hedging Performance

Hedging performance is an important tool for gauging the forecasting power of the volatility of the underlying assets. In practice, option traders usually focus on the risk due to the underlying asset price volatility alone, and carry out a delta-neutral hedge, employing only the underlying asset as the hedging instrument. While this seems plausible for the BS and AHBStype models, the SV and SVJ models lead to incomplete markets. It is well known that a simple delta hedging strategy is sub-optimal in this setting. Additionally, Alexander and Nogueira (2007) show that the delta-hedge ratios of the SV and SVJ models should be theoretically identical (or only be driven by differences in the model fit) because of the homogeneity of the call option prices and the scale-invariance property of the SV and SVJ models.

Because there are several risk factors in the proposed SV and SVJ models, the need for a perfect hedge may arise in situations where not only is the underlying price risk present, but also is volatility, or jump risk present. To implement this hedging practice, we should recognize that a perfect hedge is not practically feasible in the presence of stochastic jump sizes. So, in line

with the measure of hedging performances in Dumas, Fleming, and Whaley (1998) and Gemmill and Saflekos (2000), we define the hedge portfolio error as follows.

$$\varepsilon_t = \Delta O - \Delta O^* \tag{18}$$

where  $\Delta O$  is the change in the reported market price from day t until day t+1 or t+7 and  $\Delta O^*$  is the change in the model's theoretical price.

Table 7 and table 8 present one day and one week hedging errors over alternative moneyness categories, respectively. First, using the nearest-to-next roll-over strategy, the A3 model has the best hedging performance for one day and one week. The SV or the SVJ model is the worst performer, and even the performances of those models are less than that of the BS model. For hedging performance, the AHBS-type models show better performance than the other models. The ratios of the BS model to the A3 model, which is the best performer, are 1.2915 and 1.3866 for one-day ahead and one-week ahead hedging errors, respectively. As the term of hedging becomes longer, the difference between the BS model and the best model becomes smaller. Second, we examine hedging performances using the next-to-next roll-over strategy. In Panel A and Panel B of Figure 2, the hedging errors of all models are decreased and the complicate models are favored the most, similar to the out-of-sample pricing results. The A3 model is the best performer for both one day and one week ahead hedging errors. After the next-to-next approach is applied into the models, the SV and SVI models become better than the BS model. Similar to the results of the out-of-sample pricing, the next-to-next strategy can mitigate the over-fitting problem of AHBS-type models. The AHBS-type models that have more parameters show better hedging performance compared to those with less parameters. However, the nextto-next strategy does not make extreme decreases of the hedging errors. The next-to-next rollover strategy can decrease 18% and 26% of one day and one week ahead out-of-sample pricing errors, but only 9% and 16% of one day and one week ahead hedging errors. As a result, the

next-to-next roll-over strategy can also decrease the hedging errors, but is not drastic.

## 5. Conclusion

For S&P 500 options, we implement a horse race competition among several options pricing models. We consider the traders' rules to predict future implied volatilities by applying simple ad hoc rules to the observed current implied volatility patterns as well as to the mathematically complicated options pricing models, the SV and SVJ models, for pricing and hedging options. The roll-over strategies of the parameters for each options pricing model are also compared. In the nearest-to-next strategy, the options data of the nearest term contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t, whereas in the next-to-next roll-over strategy, the option day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameter of the next-to-nearest contract on day t - k is used to estimate the parameter of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameter of the next-to-nearest contract on day t - k is used to estimate the parameter of the next-to-nearest contract on day t - k is used to estimate the parameter of the next-to-nearest contract on day t - k is used to estimate the parameter of the next-to-nearest contract on day t - k is used to estimate the parameter of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the parameters of the next-to-nearest contract on day t - k is used to estimate the par

When we use the traditional roll-over method, the nearest-to-next strategy, it is found that the SVJ, the mathematically complicated model, is the best models for in-sample pricing. However, for out-of-sample pricing, the A1 and A2 models show better performance than the SV and SVJ models. Also the absolute smile approaches show better performance than the relative smile approaches. Among AHBS-type models, a simpler model with less parameter shows better performance compared to other models. That is, the presence of more parameters actually causes the problem of over-fitting. For hedging performance, the AHBS-type models show better performance than the mathematically complicated models; yet, the differences among the models are not significant.

When we use the new roll-over method, the next-to-next strategy decreases the pricing and hedging errors of all options pricing models. The pricing errors of the AHBS-type models are decreased largely by the next-to-next strategy. Moreover, the A3 model is the best. The AHBStype models that more parameters show better performance than those which have less parameters for pricing options. That is, the next-to-next strategy can mitigate the over-fitting problem of AHBS-type models. For hedging performance, the next-to-next strategy also decreases the errors of all options pricing models; however, the difference between the results using the nearest-to-next strategy and those using the next-to-next strategy are not so large.

As a result, when the next-to-next strategy is considered, the "absolute smile" AHBS-type model that has more independent parameters can be a competitive model for pricing and hedging S&P 500 index options.

# Appendix

The characteristic functions  $\hat{f}_j$  for the SVJ model are respectively given by

$$\hat{f}_{1} = \exp\left[-i\phi \ln[B(t,\tau)] - \frac{\theta_{v}}{\sigma_{v}^{2}} \left[2\ln\left(1 - \frac{[\xi_{v} - \kappa_{v} + (1+i\phi)\rho\sigma_{v}](1-e^{-\xi_{v}\tau})}{2\xi_{v}}\right)\right]\right] \\ - \frac{\theta_{v}}{\sigma_{v}^{2}} [\xi_{v} - \kappa_{v} + (1+i\phi)\rho\sigma_{v}]\tau + i\phi \ln[S_{t}] \\ + \lambda(1+\mu_{J})\tau \left[(1+\mu_{J})^{i\phi}e^{(i\phi/2)(1+i\phi)\sigma_{J}^{2}} - 1\right] - \lambda i\phi\mu_{J}\tau \\ + \frac{i\phi(i\phi+1)(1-e^{-\xi_{v}\tau})}{2\xi_{v} - [\xi_{v} - \kappa_{v} + (1+i\phi)\rho\sigma_{v}](1-e^{-\xi_{v}\tau})}V_{t}\right],$$
(A-1)

and

$$\hat{f}_{2} = \exp\left[-i\phi \ln[B_{t,\tau}] - \frac{\theta_{v}}{\sigma_{v}^{2}} \left[2\ln\left(1 - \frac{\left[\xi_{v}^{*} - \kappa_{v} + i\phi\rho\sigma_{v}\right]\left(1 - e^{-\xi_{v}^{*}\tau}\right)}{2\xi_{v}}\right)\right] \right] - \frac{\theta_{v}}{\sigma_{v}^{2}} \left[\xi_{v}^{*} - \kappa_{v} + i\phi\rho\sigma_{v}\right]\tau + i\phi \ln[S_{t}] + \lambda(1 + \mu_{J})\tau\left[(1 + \mu_{J})^{i\phi}e^{(i\phi/2)(i\phi-1)\sigma_{J}^{2}} - 1\right] - \lambda i\phi\mu_{J}\tau + \frac{i\phi(i\phi-1)\left(1 - e^{-\xi_{v}^{*}\tau}\right)}{2\xi_{v}^{*} - \left[\xi_{v}^{*} - \kappa_{v} + i\phi\rho\sigma_{v}\right]\left(1 - e^{-\xi_{v}^{*}\tau}\right)}V_{t} \right].$$
(A-2)

where

$$\xi_{v} = \sqrt{\left[\kappa_{v} - (1 + i\phi)\rho\sigma_{v}\right]^{2} - i\phi(1 + i\phi)\sigma_{v}^{2}}$$
$$\xi_{v}^{*} = \sqrt{\left[\kappa_{v} - i\phi\rho\sigma_{v}\right]^{2} - i\phi(i\phi - 1)\sigma_{v}^{2}}$$

The characteristic functions for the SV model can be obtained by setting  $\lambda = 0$  in (A-1) and (A-2).

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# Table 1: S&P 500 Options Data

This table reports the average option price and the number of options, which are shown in parentheses, for each moneyness and options type (call or put) category. The sample period is from January 4, 1996 to December 31, 2008. The last bid-ask average of each option contract is used to obtain the summary statistics. Moneyness of an option is defined as S/K, where S denotes the spot price and K denotes the strike price.

	Call Options			Put Options	
Moneyness	Price	Number	Moneyness	Price	Number
S/K<0.94	2.7833	10,033	1.00 <s k<1.03<="" td=""><td>13.0104</td><td>17,966</td></s>	13.0104	17,966
0.94 <s k<0.96<="" td=""><td>4.8054</td><td>13,580</td><td>1.03<s k<1.06<="" td=""><td>6.5072</td><td>15,149</td></s></td></s>	4.8054	13,580	1.03 <s k<1.06<="" td=""><td>6.5072</td><td>15,149</td></s>	6.5072	15,149
0.96 <s k<1.00<="" td=""><td>12.2916</td><td>18,783</td><td>S/K&gt;1.06</td><td>3.0967</td><td>31,201</td></s>	12.2916	18,783	S/K>1.06	3.0967	31,201
Total	7.6435	42,396	Total	6.6693	64,316

## **Table 2: Implied Volatilities Sneer**

This table reports the implied volatilities calculated by inverting the Black-Scholes (1973) model separately for each moneyness category. The implied volatilities of individual options are then averaged within each moneyness category and across 26 consecutive six-month sub-periods. Moneyness is defined as S/K, where S denotes the spot price and K denotes the strike price. 1996 01-06 is the period from January, 1996 to June, 1996.

	<b>0</b> / <b>1</b>					
	S/K < 0.94	0.94 < S/K < 0.97	0.97 < S/K < 1.00	1.00 < S/K < 1.03	1.03 < S/K < 1.06	S/K > 1.06
1996 01-06	0.1308	0.1204	0.1207	0.1553	0.1841	0.2249
1996 07-12	0.1413	0.1243	0.1289	0.1606	0.1884	0.2332
1997 01-06	0.1626	0.1579	0.1636	0.1895	0.2108	0.2516
1997 07-12	0.1920	0.1867	0.1957	0.2223	0.2477	0.3092
1998 01-06	0.1517	0.1491	0.1588	0.1917	0.2226	0.2872
1998 07-12	0.2364	0.2037	0.2139	0.2406	0.2709	0.3492
1999 01-06	0.1831	0.1851	0.2011	0.2267	0.2511	0.3132
1999 07-12	0.1626	0.1658	0.1774	0.2035	0.2267	0.2887
2000 01-06	0.1922	0.1840	0.1969	0.2238	0.2417	0.3025
2000 07-12	0.2095	0.1844	0.1891	0.2070	0.2259	0.2841
2001 01-06	0.2120	0.1962	0.2040	0.2269	0.2391	0.2996
2001 07-12	0.2159	0.1977	0.2115	0.2401	0.2638	0.3525
2002 01-06	0.1781	0.1694	0.1740	0.1993	0.2256	0.2889
2002 07-12	0.2711	0.2683	0.2787	0.3070	0.3270	0.3850
2003 01-06	0.2427	0.2248	0.2208	0.2325	0.2504	0.2951
2003 07-12	0.1563	0.1459	0.1446	0.1689	0.1911	0.2444
2004 01-06	0.1464	0.1262	0.1243	0.1504	0.1768	0.2281
2004 07-12	0.1227	0.1127	0.1121	0.1354	0.1589	0.2036
2005 01-06	0.1197	0.1025	0.0986	0.1233	0.1510	0.1994
2005 07-12	0.1628	0.0902	0.0926	0.1205	0.1472	0.1958

2006 01-06	0.1205	0.0968	0.0981	0.1289	0.1562	0.2131
2006 07-12	0.2184	0.0875	0.0903	0.1201	0.1480	0.1954
2007 01-06	0.1666	0.0945	0.0955	0.1301	0.1618	0.2236
2007 07-12	0.1809	0.1548	0.1713	0.2124	0.2386	0.2917
2008 01-06	0.1902	0.1771	0.1942	0.2284	0.2471	0.2893
2008 07-12	0.3975	0.3296	0.3435	0.3754	0.4017	0.4722

### **Table 3: Parameters**

This table reports the mean and the standard error of the parameter estimates for each model. The mean and the standard deviation of R<sup>2</sup>s for each AHBS-type model are reported. For the AHBS-type models, each parameter is estimated by the ordinary least squares every day. For the BS, SV and SVJ models, each parameter is estimated by minimizing the sum of the squared errors between the model and the market option prices every day. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuoustime stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and the jumps.

			Panel A: A	AHBS-type	Models			
		$eta_0$	$eta_1$		$\beta_2$	$\beta_3$		$R^2$
Т	R1	-0.6097	0.800	6			(	).9326
1	<b>NI</b>	(0.0052)	(0.004	6)			(0	).1124)
т	R2	1.7984		74	2.3059		(	).9720
1	12	(0.0631)	(0.125	5)	(0.0621)		(0	).0427)
т	3	26.9431	-77.50	85	73.9906	-23.240	9 (	).9833
1	0	(1.0340)	(3.051	1)	(2.9996)	(0.9828	) (0	).0289)
	A1	1.0360	-0.000	)8			(	).9152
1	11	(0.0048)	(0.000	0)			(0	).1202)
	12	4.3900	-0.007	71	0.0000		(	).9761
1	12	(0.0656)	(0.0001)		(0.0000)		(0	).0377)
	43	-17.1892	0.055		-0.0001	0.0000		).9847
1	15	(1.0140)	(0.003	0)	(0.0000)	(0.0000	) (0	).0285)
			Panel I	3: Other M	odels			
	$\sigma$							
BS	0.1858							
D5	(0.0014)							
	λ	$\mu_{_J}$	$\sigma_{_J}$	$\kappa_v$	$ heta_v$	$\sigma_v$	ρ	$v_t$
SV				0.6431	0.3722	1.3843	-0.6305	0.0390
31				(0.0638)	(0.0417)	(0.0264)	(0.0046)	(0.0026)
сул	1.1187	-0.1777	0.2032	0.9792	0.6074	0.4631	-0.4239	0.0173
SVJ	(0.0358)	(0.0087)	(0.0172)	(0.0258)	(0.0212)	(0.0083)	(0.0201)	(0.0007)

### **Table 4: In-Sample Pricing Errors**

This table reports in-sample pricing errors with respect to moneyness. The in-sample pricing performance of each model is evaluated by comparing the market prices with the model's prices computed by using the parameter estimates from the current day. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. MAE denotes mean absolute errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	2.1424	0.4279	0.3100	0.1577	0.5455	0.2500	0.1381	0.1874	0.2493
	0.94 <s k<0.96<="" td=""><td>1.9385</td><td>0.5458</td><td>0.3979</td><td>0.2231</td><td>0.6187</td><td>0.3571</td><td>0.1891</td><td>0.1613</td><td>0.1842</td></s>	1.9385	0.5458	0.3979	0.2231	0.6187	0.3571	0.1891	0.1613	0.1842
	0.96 <s k<1.00<="" td=""><td>1.5185</td><td>0.9628</td><td>0.8138</td><td>0.5887</td><td>1.0738</td><td>0.7637</td><td>0.5516</td><td>0.4015</td><td>0.3704</td></s>	1.5185	0.9628	0.8138	0.5887	1.0738	0.7637	0.5516	0.4015	0.3704
MAE	1.00 <s k<1.03<="" td=""><td>1.5342</td><td>0.6935</td><td>0.5504</td><td>0.6171</td><td>0.7226</td><td>0.5696</td><td>0.5959</td><td>0.4527</td><td>0.3630</td></s>	1.5342	0.6935	0.5504	0.6171	0.7226	0.5696	0.5959	0.4527	0.3630
	1.03 <s k<1.06<="" td=""><td>2.2481</td><td>0.3666</td><td>0.2984</td><td>0.2129</td><td>0.3854</td><td>0.2981</td><td>0.1853</td><td>0.1840</td><td>0.1511</td></s>	2.2481	0.3666	0.2984	0.2129	0.3854	0.2981	0.1853	0.1840	0.1511
	S/K>1.06	1.9463	0.1896	0.1711	0.1197	0.2170	0.1563	0.1093	0.1866	0.1216
-	Total	1.8619	0.5034	0.4081	0.3160	0.5588	0.3873	0.2927	0.2657	0.2302
	S/K<0.94	2.7826	0.6683	0.4764	0.2383	2.8438	0.3796	0.2173	0.2936	0.7878
	0.94 <s k<0.96<="" td=""><td>2.3684</td><td>0.9581</td><td>0.6833</td><td>0.3747</td><td>1.0699</td><td>0.7589</td><td>0.3370</td><td>0.2696</td><td>0.6927</td></s>	2.3684	0.9581	0.6833	0.3747	1.0699	0.7589	0.3370	0.2696	0.6927
RMSE	0.96 <s k<1.00<="" td=""><td>1.8362</td><td>1.3847</td><td>1.1295</td><td>0.8592</td><td>1.5330</td><td>1.0672</td><td>0.8222</td><td>0.6835</td><td>0.8548</td></s>	1.8362	1.3847	1.1295	0.8592	1.5330	1.0672	0.8222	0.6835	0.8548
	1.00 <s k<1.03<="" td=""><td>1.9255</td><td>1.0109</td><td>0.8813</td><td>0.9327</td><td>1.0172</td><td>0.9080</td><td>0.8887</td><td>0.7399</td><td>0.7221</td></s>	1.9255	1.0109	0.8813	0.9327	1.0172	0.9080	0.8887	0.7399	0.7221
	1.03 <s k<1.06<="" td=""><td>2.6427</td><td>0.5752</td><td>0.5852</td><td>0.3648</td><td>0.5569</td><td>0.5734</td><td>0.3191</td><td>0.2791</td><td>0.3785</td></s>	2.6427	0.5752	0.5852	0.3648	0.5569	0.5734	0.3191	0.2791	0.3785

S/K>1.06	2.5675	0.3225	0.3329	0.2059	0.3643	0.3000	0.1828	0.2681	0.2456
Total	2.3618	0.8635	0.7191	0.5752	1.2557	0.7065	0.5432	0.4731	0.6114

## Table 5: One Day Ahead Out-of-Sample Pricing Errors

This table reports one day ahead out-of-sample pricing errors with respect to moneyness. Each model is estimated every day during the sample period; one day ahead out-of-sample pricing errors are computed using the estimated parameters from the previous trading day. Panel A reports one day ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. Panel B reports one day ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. Panel B reports one day ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. Panel B reports one day ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. MAE denotes mean absolute errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, and the square of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

				Panel A: N	earest-to-Nex	xt				
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	2.3820	1.7209	1.4492	1.7704	1.7932	1.2515	1.7001	1.0524	1.3395
	0.94 <s k<0.96<="" td=""><td>2.0893</td><td>1.2060</td><td>1.0784</td><td>1.0156</td><td>1.0759</td><td>0.8305</td><td>0.7952</td><td>1.0933</td><td>1.2787</td></s>	2.0893	1.2060	1.0784	1.0156	1.0759	0.8305	0.7952	1.0933	1.2787
	0.96 <s k<1.00<="" td=""><td>1.9595</td><td>1.6458</td><td>1.5069</td><td>1.3948</td><td>1.4724</td><td>1.2002</td><td>1.0755</td><td>1.5138</td><td>1.6761</td></s>	1.9595	1.6458	1.5069	1.3948	1.4724	1.2002	1.0755	1.5138	1.6761
MAE	1.00 <s k<1.03<="" td=""><td>1.8610</td><td>1.3506</td><td>1.2755</td><td>1.3002</td><td>1.1108</td><td>1.0091</td><td>1.0219</td><td>1.4552</td><td>1.5424</td></s>	1.8610	1.3506	1.2755	1.3002	1.1108	1.0091	1.0219	1.4552	1.5424
	1.03 <s k<1.06<="" td=""><td>2.2990</td><td>0.8960</td><td>0.8760</td><td>0.8786</td><td>0.7103</td><td>0.6666</td><td>0.6454</td><td>1.0387</td><td>1.0688</td></s>	2.2990	0.8960	0.8760	0.8786	0.7103	0.6666	0.6454	1.0387	1.0688
	S/K>1.06	1.9679	0.6636	0.9832	1.5465	0.5317	0.7510	1.1171	0.6946	0.6744
	Total	2.0498	1.1536	1.1653	1.3370	1.0080	0.9187	1.0406	1.1001	1.1923
	S/K<0.94	4.3852	9.8866	6.9483	11.2266	12.1547	7.4598	11.6676	2.4316	3.6415
DMCE	0.94 <s k<0.96<="" td=""><td>3.0969</td><td>2.5736</td><td>2.2606</td><td>2.2050</td><td>2.8509</td><td>1.9258</td><td>1.8610</td><td>2.3508</td><td>3.3254</td></s>	3.0969	2.5736	2.2606	2.2050	2.8509	1.9258	1.8610	2.3508	3.3254
RMSE	0.96 <s k<1.00<="" td=""><td>2.7602</td><td>2.6426</td><td>2.3948</td><td>2.3005</td><td>2.3494</td><td>1.9160</td><td>1.7958</td><td>2.8194</td><td>3.5162</td></s>	2.7602	2.6426	2.3948	2.3005	2.3494	1.9160	1.7958	2.8194	3.5162
	1.00 <s k<1.03<="" td=""><td>2.7072</td><td>2.2118</td><td>2.1536</td><td>2.1869</td><td>1.8059</td><td>1.7111</td><td>1.7057</td><td>2.7001</td><td>3.1183</td></s>	2.7072	2.2118	2.1536	2.1869	1.8059	1.7111	1.7057	2.7001	3.1183

	1.03 <s k<1.06<="" th=""><th>3.0361</th><th>1.7302</th><th>1.7318</th><th>1.8537</th><th>1.3526</th><th>1.3127</th><th>1.2769</th><th>2.1421</th><th>2.5299</th></s>	3.0361	1.7302	1.7318	1.8537	1.3526	1.3127	1.2769	2.1421	2.5299
	S/K>1.06	2.8041	1.6942	3.1487	12.5589	1.3521	2.3913	9.4232	1.6386	2.1283
-	Total	3.0350	3.6539	3.2101	7.7982	4.1522	2.9598	6.3635	2.3070	2.9496
				Panel B: I	Next-to-Next					
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	2.2890	1.2526	1.0825	1.3277	1.0851	0.7968	1.0984	0.9585	0.9453
	0.94 <s k<0.96<="" td=""><td>2.1098</td><td>1.0664</td><td>0.9990</td><td>0.9275</td><td>0.8735</td><td>0.7367</td><td>0.6670</td><td>0.9400</td><td>0.9276</td></s>	2.1098	1.0664	0.9990	0.9275	0.8735	0.7367	0.6670	0.9400	0.9276
	0.96 <s k<1.00<="" td=""><td>1.9423</td><td>1.5407</td><td>1.4337</td><td>1.3185</td><td>1.3561</td><td>1.1138</td><td>0.9807</td><td>1.3139</td><td>1.3267</td></s>	1.9423	1.5407	1.4337	1.3185	1.3561	1.1138	0.9807	1.3139	1.3267
MAE	1.00 <s k<1.03<="" td=""><td>1.7539</td><td>1.2706</td><td>1.1821</td><td>1.2049</td><td>1.0167</td><td>0.9067</td><td>0.9146</td><td>1.2125</td><td>1.2527</td></s>	1.7539	1.2706	1.1821	1.2049	1.0167	0.9067	0.9146	1.2125	1.2527
	1.03 <s k<1.06<="" td=""><td>2.2271</td><td>0.8463</td><td>0.8161</td><td>0.8077</td><td>0.6525</td><td>0.6072</td><td>0.5768</td><td>0.8584</td><td>0.8687</td></s>	2.2271	0.8463	0.8161	0.8077	0.6525	0.6072	0.5768	0.8584	0.8687
	S/K>1.06	1.9351	0.5325	0.5269	0.5276	0.4249	0.3838	0.3861	0.5694	0.5258
-	Total	2.0028	1.0144	0.9502	0.9467	0.8399	0.7158	0.7095	0.9335	0.9284
	S/K<0.94	3.4969	6.5118	4.4744	9.9655	7.7868	4.2148	10.1036	2.0523	2.0509
	0.94 <s k<0.96<="" td=""><td>2.9122</td><td>1.9145</td><td>1.8257</td><td>1.7539</td><td>1.5357</td><td>1.3176</td><td>1.2002</td><td>1.7983</td><td>1.8402</td></s>	2.9122	1.9145	1.8257	1.7539	1.5357	1.3176	1.2002	1.7983	1.8402
	0.96 <s k<1.00<="" td=""><td>2.6059</td><td>2.3118</td><td>2.1726</td><td>2.0654</td><td>1.9846</td><td>1.6376</td><td>1.4830</td><td>2.1155</td><td>2.1516</td></s>	2.6059	2.3118	2.1726	2.0654	1.9846	1.6376	1.4830	2.1155	2.1516
RMSE	1.00 <s k<1.03<="" td=""><td>2.4388</td><td>1.9392</td><td>1.8888</td><td>1.9098</td><td>1.4814</td><td>1.3808</td><td>1.3534</td><td>1.8983</td><td>1.9627</td></s>	2.4388	1.9392	1.8888	1.9098	1.4814	1.3808	1.3534	1.8983	1.9627
	1.03 <s k<1.06<="" td=""><td>2.8444</td><td>1.4731</td><td>1.4869</td><td>1.4545</td><td>1.0535</td><td>1.0306</td><td>0.9614</td><td>1.4924</td><td>1.5424</td></s>	2.8444	1.4731	1.4869	1.4545	1.0535	1.0306	0.9614	1.4924	1.5424
	S/K>1.06	2.6668	1.1429	1.1481	1.1399	0.8229	0.7548	0.7765	1.1794	1.1557
-	Total	2.7665	2.5918	2.1065	3.4315	2.7239	1.7325	3.2841	1.7102	1.7372

## Table 6: One Week Ahead Out-of-Sample Pricing Errors

This table reports one week ahead out-of-sample pricing with respect to moneyness. Each model is estimated every day during the sample period; one week ahead out-of-sample pricing errors are computed using estimated parameters from one week ago. Panel A reports one week ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. Panel B reports one week ahead out-of-sample pricing errors using the nearest-to-next roll over strategy. Panel B reports one week ahead out-of-sample pricing errors using the next-to-next roll over strategy. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. MAE denotes mean absolute errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept, the moneyness and the square of the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, and the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes the intercept, the strike price and the square of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

				Panel A: N	earest-to-Ne	xt				
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	2.6955	2.4740	2.5531	3.0536	2.4607	2.3104	3.1074	1.9402	2.3406
	0.94 <s k<0.96<="" td=""><td>2.3209</td><td>1.8113</td><td>1.6752</td><td>1.6174</td><td>1.5575</td><td>1.3106</td><td>1.3781</td><td>2.0172</td><td>2.6683</td></s>	2.3209	1.8113	1.6752	1.6174	1.5575	1.3106	1.3781	2.0172	2.6683
	0.96 <s k<1.00<="" td=""><td>2.4645</td><td>2.3196</td><td>2.1757</td><td>2.0906</td><td>1.9263</td><td>1.6263</td><td>1.5425</td><td>2.7361</td><td>3.3861</td></s>	2.4645	2.3196	2.1757	2.0906	1.9263	1.6263	1.5425	2.7361	3.3861
MAE	1.00 <s k<1.03<="" td=""><td>2.4457</td><td>2.1117</td><td>2.0762</td><td>2.0837</td><td>1.6234</td><td>1.5316</td><td>1.5427</td><td>2.8080</td><td>3.0853</td></s>	2.4457	2.1117	2.0762	2.0837	1.6234	1.5316	1.5427	2.8080	3.0853
	1.03 <s k<1.06<="" td=""><td>2.4741</td><td>1.5557</td><td>1.5498</td><td>1.5749</td><td>1.1563</td><td>1.1495</td><td>1.1733</td><td>2.0799</td><td>2.2210</td></s>	2.4741	1.5557	1.5498	1.5749	1.1563	1.1495	1.1733	2.0799	2.2210
	S/K>1.06	1.9914	1.3350	2.1132	2.8087	1.0396	1.7047	2.0681	1.4441	1.3936
-	Total	2.3278	1.8381	2.0237	2.2567	1.5101	1.5898	1.7702	2.1108	2.3977
	S/K<0.94	5.4083	11.7077	12.6689	15.3663	14.5579	11.5353	15.0424	4.4645	4.7711
DMCE	0.94 <s k<0.96<="" td=""><td>3.7497</td><td>3.4075</td><td>3.4598</td><td>3.3580</td><td>3.6598</td><td>2.6120</td><td>3.1233</td><td>4.1324</td><td>5.2583</td></s>	3.7497	3.4075	3.4598	3.3580	3.6598	2.6120	3.1233	4.1324	5.2583
RMSE	0.96 <s k<1.00<="" td=""><td>3.6732</td><td>3.6611</td><td>3.4406</td><td>3.3757</td><td>3.0477</td><td>2.5708</td><td>2.5055</td><td>4.7418</td><td>5.9154</td></s>	3.6732	3.6611	3.4406	3.3757	3.0477	2.5708	2.5055	4.7418	5.9154
	1.00 <s k<1.03<="" td=""><td>3.6445</td><td>3.3300</td><td>3.3049</td><td>3.3273</td><td>2.5613</td><td>2.4069</td><td>2.4213</td><td>4.7627</td><td>5.1920</td></s>	3.6445	3.3300	3.3049	3.3273	2.5613	2.4069	2.4213	4.7627	5.1920

	1.03 <s k<1.06<="" th=""><th>3.5529</th><th>2.7604</th><th>2.7448</th><th>2.8641</th><th>2.1222</th><th>2.0400</th><th>2.2054</th><th>3.9105</th><th>4.1467</th></s>	3.5529	2.7604	2.7448	2.8641	2.1222	2.0400	2.2054	3.9105	4.1467
	S/K>1.06	2.9730	2.5126	4.1344	15.3676	2.0307	3.4903	11.2947	3.0310	3.0275
-	Total	3.6779	4.6395	5.1587	9.8868	5.1212	4.4360	7.9132	4.0841	4.6342
				Damal P. I	Next-to-Next					
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	2.3866	1.6770	1.7050	1.5188	1.2440	1.2787	1.0834	1.5221	1.3450
	0.94 <s k<0.96<="" td=""><td>2.3951</td><td>1.5615</td><td>1.5110</td><td>1.4381</td><td>1.1851</td><td>1.0791</td><td>0.9971</td><td>1.5652</td><td>1.4809</td></s>	2.3951	1.5615	1.5110	1.4381	1.1851	1.0791	0.9971	1.5652	1.4809
	0.96 <s k<1.00<="" td=""><td>2.5028</td><td>2.2119</td><td>2.1317</td><td>2.0171</td><td>1.7648</td><td>1.541</td><td>1.4046</td><td>2.2169</td><td>2.1416</td></s>	2.5028	2.2119	2.1317	2.0171	1.7648	1.541	1.4046	2.2169	2.1416
MAE	1.00 <s k<1.03<="" td=""><td>2.1983</td><td>1.9819</td><td>1.9167</td><td>1.9159</td><td>1.4130</td><td>1.2908</td><td>1.2917</td><td>2.1895</td><td>2.1238</td></s>	2.1983	1.9819	1.9167	1.9159	1.4130	1.2908	1.2917	2.1895	2.1238
	1.03 <s k<1.06<="" td=""><td>2.2872</td><td>1.3932</td><td>1.3787</td><td>1.3991</td><td>0.9711</td><td>0.9368</td><td>0.9400</td><td>1.6557</td><td>1.6035</td></s>	2.2872	1.3932	1.3787	1.3991	0.9711	0.9368	0.9400	1.6557	1.6035
	S/K>1.06	1.9435	0.9062	0.9124	0.9214	0.6958	0.6473	0.6744	1.0499	0.9903
-	Total	2.2328	1.5420	1.5129	1.4713	1.1575	1.0683	1.0240	1.6430	1.5665
	S/K<0.94	3.8324	7.3373	8.4343	5.7274	7.4183	8.2250	3.0896	2.9123	2.6266
	0.94 <s k<0.96<="" td=""><td>3.5167</td><td>2.7285</td><td>2.6666</td><td>2.5978</td><td>2.0137</td><td>1.8453</td><td>1.7212</td><td>2.7959</td><td>2.6431</td></s>	3.5167	2.7285	2.6666	2.5978	2.0137	1.8453	1.7212	2.7959	2.6431
	0.96 <s k<1.00<="" td=""><td>3.5124</td><td>3.2995</td><td>3.2114</td><td>3.1088</td><td>2.5312</td><td>2.2361</td><td>2.0776</td><td>3.4452</td><td>3.2413</td></s>	3.5124	3.2995	3.2114	3.1088	2.5312	2.2361	2.0776	3.4452	3.2413
RMSE	1.00 <s k<1.03<="" td=""><td>3.2540</td><td>3.0122</td><td>2.9683</td><td>2.9811</td><td>2.0871</td><td>1.9427</td><td>1.9221</td><td>3.3412</td><td>3.1529</td></s>	3.2540	3.0122	2.9683	2.9811	2.0871	1.9427	1.9221	3.3412	3.1529
	1.03 <s k<1.06<="" td=""><td>3.2580</td><td>2.4232</td><td>2.4145</td><td>2.4438</td><td>1.6461</td><td>1.5466</td><td>1.5552</td><td>2.7657</td><td>2.6247</td></s>	3.2580	2.4232	2.4145	2.4438	1.6461	1.5466	1.5552	2.7657	2.6247
	S/K>1.06	2.8865	1.8045	1.8136	1.8225	1.3018	1.1693	1.2655	2.0088	1.9136
-	Total	3.2942	3.3527	3.5615	2.9933	2.9041	3.0086	1.8623	2.8330	2.6662

## **Table 7: One Day Ahead Hedging Errors**

This table reports one day ahead hedging error with respect to moneyness. For each option, its hedging error is the difference between the change in the reported market price and the change in the model's theoretical price from day t until day t+1. Panel A reports one day ahead hedging errors using the nearest-to-next roll over strategy. Panel B reports one day ahead hedging errors using the next-to-next roll over strategy. MAE denotes mean absolute errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the square of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

				Panel A: N	earest-to-Ne>	ĸt				
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	1.4032	1.1922	1.1215	1.0193	0.8805	0.8471	0.8117	0.9604	0.9999
	0.94 <s k<0.96<="" td=""><td>1.0095</td><td>1.1021</td><td>1.0441</td><td>0.9860</td><td>0.8010</td><td>0.7717</td><td>0.7490</td><td>0.9999</td><td>1.0270</td></s>	1.0095	1.1021	1.0441	0.9860	0.8010	0.7717	0.7490	0.9999	1.0270
	0.96 <s k<1.00<="" td=""><td>1.083</td><td>1.3724</td><td>1.3135</td><td>1.2989</td><td>1.0635</td><td>1.0418</td><td>1.0336</td><td>1.4638</td><td>1.5533</td></s>	1.083	1.3724	1.3135	1.2989	1.0635	1.0418	1.0336	1.4638	1.5533
MAE	1.00 <s k<1.03<="" td=""><td>1.0923</td><td>1.2066</td><td>1.2221</td><td>1.2702</td><td>0.9877</td><td>1.0004</td><td>1.0029</td><td>1.4359</td><td>1.4960</td></s>	1.0923	1.2066	1.2221	1.2702	0.9877	1.0004	1.0029	1.4359	1.4960
	1.03 <s k<1.06<="" td=""><td>0.8690</td><td>0.8436</td><td>0.8643</td><td>0.9074</td><td>0.6640</td><td>0.6573</td><td>0.6568</td><td>1.0009</td><td>0.9968</td></s>	0.8690	0.8436	0.8643	0.9074	0.6640	0.6573	0.6568	1.0009	0.9968
	S/K>1.06	0.6773	0.5671	0.6033	0.5743	0.4277	0.4380	0.4349	0.5749	0.5980
-	Total	0.8441	0.8649	0.8584	0.8466	0.6664	0.6608	0.6536	0.9048	0.9385
	S/K<0.94	2.8595	2.8751	2.7309	2.6181	2.3547	2.2827	2.2543	2.1086	2.6671
	0.94 <s k<0.96<="" td=""><td>1.9191</td><td>2.1929</td><td>2.0520</td><td>1.9817</td><td>1.7546</td><td>1.6135</td><td>1.5810</td><td>2.0084</td><td>2.8089</td></s>	1.9191	2.1929	2.0520	1.9817	1.7546	1.6135	1.5810	2.0084	2.8089
RMSE	0.96 <s k<1.00<="" td=""><td>1.8319</td><td>2.2460</td><td>2.1475</td><td>2.1491</td><td>1.8579</td><td>1.7735</td><td>1.7657</td><td>2.6404</td><td>3.2653</td></s>	1.8319	2.2460	2.1475	2.1491	1.8579	1.7735	1.7657	2.6404	3.2653
	1.00 <s k<1.03<="" td=""><td>1.7300</td><td>1.8846</td><td>1.9068</td><td>1.9861</td><td>1.5255</td><td>1.5485</td><td>1.5585</td><td>2.6153</td><td>2.8267</td></s>	1.7300	1.8846	1.9068	1.9861	1.5255	1.5485	1.5585	2.6153	2.8267

	1.03 <s k<1.06<="" th=""><th>1.5180</th><th>1.4926</th><th>1.5196</th><th>1.5993</th><th>1.1348</th><th>1.1344</th><th>1.1420</th><th>2.1019</th><th>2.3396</th></s>	1.5180	1.4926	1.5196	1.5993	1.1348	1.1344	1.1420	2.1019	2.3396
	S/K>1.06	1.3464	1.4179	1.5476	1.4854	1.0805	1.1690	1.1599	1.4923	2.0838
-	Total	1.6723	1.8186	1.7986	1.7842	1.4585	1.4362	1.4274	2.0078	2.4716
				Panel B: N	Next-to-Next					
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	1.3331	1.0988	1.0418	0.9487	0.7784	0.7592	0.7277	0.9243	0.9058
	0.94 <s k<0.96<="" td=""><td>0.9870</td><td>1.0404</td><td>0.9997</td><td>0.9520</td><td>0.7385</td><td>0.7296</td><td>0.7057</td><td>0.9171</td><td>0.8702</td></s>	0.9870	1.0404	0.9997	0.9520	0.7385	0.7296	0.7057	0.9171	0.8702
	0.96 <s k<1.00<="" td=""><td>1.0359</td><td>1.287</td><td>1.2412</td><td>1.2226</td><td>0.966</td><td>0.9556</td><td>0.9406</td><td>1.2862</td><td>1.2458</td></s>	1.0359	1.287	1.2412	1.2226	0.966	0.9556	0.9406	1.2862	1.2458
MAE	1.00 <s k<1.03<="" td=""><td>0.9944</td><td>1.1462</td><td>1.1411</td><td>1.1892</td><td>0.9183</td><td>0.9148</td><td>0.9151</td><td>1.2105</td><td>1.2548</td></s>	0.9944	1.1462	1.1411	1.1892	0.9183	0.9148	0.9151	1.2105	1.2548
	1.03 <s k<1.06<="" td=""><td>0.8044</td><td>0.8074</td><td>0.8222</td><td>0.8597</td><td>0.6214</td><td>0.6140</td><td>0.6063</td><td>0.8331</td><td>0.8277</td></s>	0.8044	0.8074	0.8222	0.8597	0.6214	0.6140	0.6063	0.8331	0.8277
	S/K>1.06	0.6561	0.5189	0.5412	0.5126	0.3814	0.3770	0.3736	0.5055	0.5134
-	Total	0.8004	0.8113	0.8024	0.7912	0.6086	0.6018	0.5924	0.7922	0.7869
	S/K<0.94	2.1383	2.0917	2.0195	1.9386	1.4771	1.3956	1.3575	1.9661	1.9233
	0.94 <s k<0.96<="" td=""><td>1.5449</td><td>1.7538</td><td>1.7008</td><td>1.6560</td><td>1.2528</td><td>1.2728</td><td>1.1720</td><td>1.6808</td><td>1.6468</td></s>	1.5449	1.7538	1.7008	1.6560	1.2528	1.2728	1.1720	1.6808	1.6468
	0.96 <s k<1.00<="" td=""><td>1.5557</td><td>1.9082</td><td>1.8753</td><td>1.8741</td><td>1.4536</td><td>1.4390</td><td>1.4225</td><td>2.0025</td><td>1.9958</td></s>	1.5557	1.9082	1.8753	1.8741	1.4536	1.4390	1.4225	2.0025	1.9958
RMSE	1.00 <s k<1.03<="" td=""><td>1.4955</td><td>1.7597</td><td>1.7601</td><td>1.8364</td><td>1.3619</td><td>1.3585</td><td>1.3565</td><td>1.8423</td><td>1.9266</td></s>	1.4955	1.7597	1.7601	1.8364	1.3619	1.3585	1.3565	1.8423	1.9266
	1.03 <s k<1.06<="" td=""><td>1.3219</td><td>1.4064</td><td>1.4371</td><td>1.5000</td><td>1.0126</td><td>1.0115</td><td>0.9981</td><td>1.4571</td><td>1.4507</td></s>	1.3219	1.4064	1.4371	1.5000	1.0126	1.0115	0.9981	1.4571	1.4507
	S/K>1.06	1.1857	1.1165	1.1536	1.0953	0.7589	0.7478	0.7451	1.1068	1.1882
-	Total	1.3894	1.5128	1.5033	1.4988	1.1077	1.0964	1.0737	1.5268	1.5469

## **Table 8: One Week Ahead Hedging Errors**

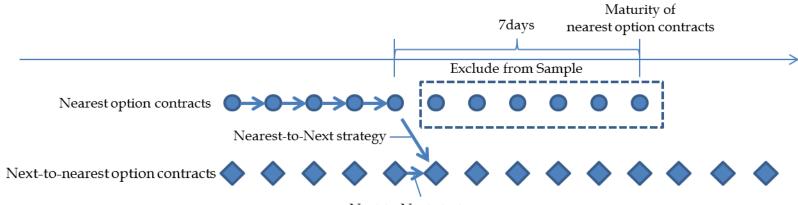
This table reports one week ahead hedging error with respect to moneyness. For each option, its hedging error is the difference between the change in the reported market price and the change in the model's theoretical price from day t until day t+7. Panel A reports one week ahead hedging errors using the nearest-to-next roll over strategy. Panel B reports one week ahead hedging errors using the next-to-next roll over strategy. MAE denotes mean absolute errors and RMSE denotes root mean squared errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A3 is the ad hoc Black-Scholes model that considers the intercept, the square of the strike price as the independent variables. BS is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and jumps.

Panel A: Nearest-to-Next										
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	2.6283	2.1145	2.0282	1.8813	1.6999	1.6574	1.5812	1.8439	1.6667
	0.94 <s k<0.96<="" td=""><td>1.5998</td><td>1.7879</td><td>1.6956</td><td>1.5704</td><td>1.2709</td><td>1.2070</td><td>1.1719</td><td>1.9364</td><td>2.0456</td></s>	1.5998	1.7879	1.6956	1.5704	1.2709	1.2070	1.1719	1.9364	2.0456
	0.96 <s k<1.00<="" td=""><td>1.5394</td><td>1.9869</td><td>1.873</td><td>1.8386</td><td>1.4784</td><td>1.4206</td><td>1.4123</td><td>2.5479</td><td>2.9488</td></s>	1.5394	1.9869	1.873	1.8386	1.4784	1.4206	1.4123	2.5479	2.9488
MAE	1.00 <s k<1.03<="" td=""><td>1.8656</td><td>1.8960</td><td>1.9405</td><td>2.0163</td><td>1.4660</td><td>1.4845</td><td>1.4931</td><td>2.6144</td><td>2.8695</td></s>	1.8656	1.8960	1.9405	2.0163	1.4660	1.4845	1.4931	2.6144	2.8695
	1.03 <s k<1.06<="" td=""><td>1.6018</td><td>1.4320</td><td>1.4528</td><td>1.5239</td><td>1.1520</td><td>1.1045</td><td>1.1097</td><td>1.9225</td><td>2.0450</td></s>	1.6018	1.4320	1.4528	1.5239	1.1520	1.1045	1.1097	1.9225	2.0450
	S/K>1.06	1.4885	1.1769	1.2948	1.2538	0.9082	0.9424	0.9383	1.3257	1.2967
-	Total	1.2291	1.1821	1.1886	1.1729	0.9061	0.8944	0.8864	1.4229	1.5082
	S/K<0.94	4.9952	4.5678	5.0059	4.8419	4.0583	4.2023	4.1496	4.1175	3.4577
	0.94 <s k<0.96<="" td=""><td>2.8507</td><td>3.2096</td><td>3.6100</td><td>3.4291</td><td>2.5558</td><td>2.3965</td><td>2.3661</td><td>3.9714</td><td>4.3152</td></s>	2.8507	3.2096	3.6100	3.4291	2.5558	2.3965	2.3661	3.9714	4.3152
RMSE	0.96 <s k<1.00<="" td=""><td>2.6294</td><td>3.2183</td><td>3.0550</td><td>3.0526</td><td>2.5423</td><td>2.4069</td><td>2.3966</td><td>4.4859</td><td>5.5497</td></s>	2.6294	3.2183	3.0550	3.0526	2.5423	2.4069	2.3966	4.4859	5.5497
	1.00 <s k<1.03<="" td=""><td>2.8621</td><td>2.9178</td><td>3.0120</td><td>3.1069</td><td>2.1942</td><td>2.2172</td><td>2.2360</td><td>4.4352</td><td>4.5892</td></s>	2.8621	2.9178	3.0120	3.1069	2.1942	2.2172	2.2360	4.4352	4.5892

	1.03 <s k<1.06<="" td=""><td>2.4656</td><td>2.3010</td><td>2.3629</td><td>2.4532</td><td>1.8311</td><td>1.7952</td><td>1.8093</td><td>3.6031</td><td>3.7004</td></s>	2.4656	2.3010	2.3629	2.4532	1.8311	1.7952	1.8093	3.6031	3.7004
	S/K>1.06	2.3053	2.3184	2.5515	2.5253	1.8371	1.9596	1.9377	2.9761	3.1366
-	Total	2.4342	2.4979	2.6397	2.6163	2.0158	2.0198	2.0078	3.2677	3.5177
				Panel B: N	Next-to-Next					
	Moneyness	BS	R1	R2	R3	A1	A2	A3	SV	SVJ
	S/K<0.94	2.2661	1.7700	1.6531	1.4952	1.3604	1.2076	1.1270	1.5597	1.4429
	0.94 <s k<0.96<="" td=""><td>1.5252</td><td>1.5835</td><td>1.5055</td><td>1.4096</td><td>1.1445</td><td>1.0808</td><td>1.0313</td><td>1.5388</td><td>1.3757</td></s>	1.5252	1.5835	1.5055	1.4096	1.1445	1.0808	1.0313	1.5388	1.3757
	0.96 <s k<1.00<="" td=""><td>1.5303</td><td>1.9347</td><td>1.8555</td><td>1.7919</td><td>1.4044</td><td>1.3465</td><td>1.2965</td><td>2.1387</td><td>1.8133</td></s>	1.5303	1.9347	1.8555	1.7919	1.4044	1.3465	1.2965	2.1387	1.8133
MAE	1.00 <s k<1.03<="" td=""><td>1.6108</td><td>1.7697</td><td>1.7585</td><td>1.8348</td><td>1.3191</td><td>1.3059</td><td>1.3062</td><td>1.9476</td><td>2.1042</td></s>	1.6108	1.7697	1.7585	1.8348	1.3191	1.3059	1.3062	1.9476	2.1042
	1.03 <s k<1.06<="" td=""><td>1.4347</td><td>1.2983</td><td>1.3056</td><td>1.3793</td><td>0.9914</td><td>0.9465</td><td>0.9438</td><td>1.5066</td><td>1.4817</td></s>	1.4347	1.2983	1.3056	1.3793	0.9914	0.9465	0.9438	1.5066	1.4817
	S/K>1.06	1.4236	0.9007	0.9327	0.9044	0.6875	0.6670	0.6662	1.0313	0.9922
-	Total	1.1352	1.0446	1.0278	1.0138	0.7799	0.7463	0.7305	1.1277	1.0730
	S/K<0.94	3.1783	2.9651	2.8073	2.6715	3.7514	2.0978	1.9467	2.9383	2.7330
	0.94 <s k<0.96<="" td=""><td>2.3139</td><td>2.5200</td><td>2.4310</td><td>2.3587</td><td>1.8485</td><td>1.7591</td><td>1.6915</td><td>2.6926</td><td>2.4580</td></s>	2.3139	2.5200	2.4310	2.3587	1.8485	1.7591	1.6915	2.6926	2.4580
RMSE -	0.96 <s k<1.00<="" td=""><td>2.2742</td><td>2.8370</td><td>2.7691</td><td>2.7306</td><td>2.0650</td><td>1.9893</td><td>1.9214</td><td>3.3470</td><td>2.8145</td></s>	2.2742	2.8370	2.7691	2.7306	2.0650	1.9893	1.9214	3.3470	2.8145
	1.00 <s k<1.03<="" td=""><td>2.5007</td><td>2.7508</td><td>2.7457</td><td>2.8430</td><td>1.9779</td><td>1.9697</td><td>1.9596</td><td>2.8990</td><td>3.1287</td></s>	2.5007	2.7508	2.7457	2.8430	1.9779	1.9697	1.9596	2.8990	3.1287
	1.03 <s k<1.06<="" td=""><td>2.2279</td><td>2.1269</td><td>2.1603</td><td>2.2536</td><td>1.5544</td><td>1.5348</td><td>1.5202</td><td>2.4300</td><td>2.3431</td></s>	2.2279	2.1269	2.1603	2.2536	1.5544	1.5348	1.5202	2.4300	2.3431
	S/K>1.06	2.1324	1.6784	1.7441	1.6765	1.2180	1.2029	1.1879	1.9382	1.8766
	Total	2.0121	2.0287	2.0101	2.0006	1.6541	1.4402	1.4024	2.2453	2.1268

## **Figure 1: Roll-over Strategies**

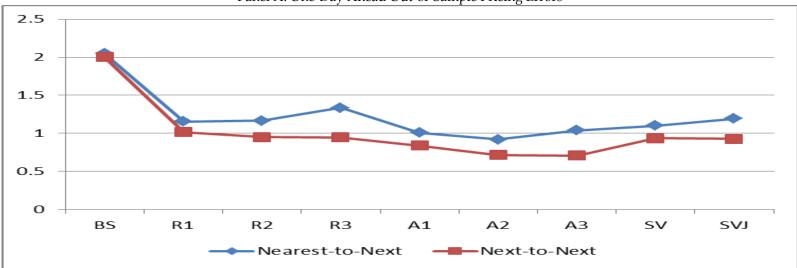
This figure shows the difference between the nearest-to-next roll-over strategy and the next-to-next roll-over strategy. This figure represents the example for one day ahead pricing and hedging performance. The circle represents the options data from the nearest option contract. The diamond represents the options data from the next-to-nearest option contract. It is standard to eliminate the nearest option contracts with expiries less than 7 days, and use the next-to-nearest option contracts with expiries less than 7 days plus 1 month for the empirical study due to liquidity problems of the options contract. When forecasting the parameters for the next-to-nearest option contracts with expiry greater than 6 days or the next-to-nearest options contracts (circle) with expiry greater than 6 days or the next-to-nearest options contracts (diamond) with expiry greater than 6 days plus 1 month. When the nearest contract's expiry is less than 7 days, the next-to-next strategy uses the next-to-nearest contracts on the previous day(s), whereas the nearest-to-next one uses the nearest-term contracts. These two strategies are different only on the day(s) when the expiry of nearest-term option contracts is less than seven days.



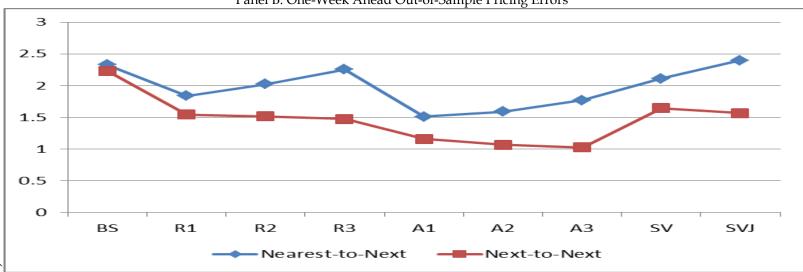
Next-to-Next strategy

### Figure 2: Out-of-Sample Pricing Errors

This figure shows the mean absolute errors (MAE) of out-of-sample pricing for each option pricing models with respect to the roll-over strategies. Panel A represents one-day ahead out-of-sample pricing errors and Panel B represents one-week ahead out-of-sample pricing errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variable. R2 is the ad hoc Black-Scholes model that considers the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept and the strike price as the independent variables. A2 is the ad hoc Black-Scholes model that considers the intercept, the strike price and the square of the strike price as the independent variables. B3 is the Black-Scholes model that considers the intercept, the strike price, and the square of the strike price as the independent variables. A3 is the Black-Scholes model that considers the intercept, the strike price, and the square and the third power of the strike price as the independent variables. B3 is the Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility and the jumps.



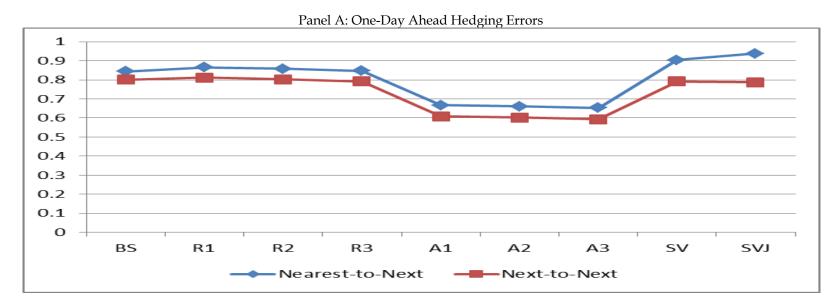


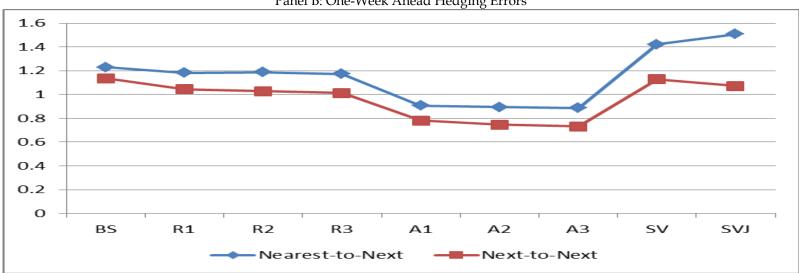


Panel B. One-Week Ahead Out-of-Sample Pricing Errors

## **Figure 3: Hedging Errors**

This figure shows the mean absolute errors (MAE) of hedging for each option pricing models with respect to the roll-over strategies. Panel A represents one-day ahead hedging errors and Panel B represents one-week ahead hedging errors. R1 is the ad hoc Black-Scholes model that considers the intercept and the moneyness as the independent variables. R2 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square of the moneyness as the independent variables. R3 is the ad hoc Black-Scholes model that considers the intercept, the moneyness, the square and the third power of the moneyness as the independent variables. A1 is the ad hoc Black-Scholes model that considers the intercept, the strike price as the independent variable. A2 is the ad hoc Black-Scholes model that considers the intercept, the square of the strike price as the independent variables. A3 is the ad hoc Black-Scholes (1973) option pricing model. SV is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility. SVJ is the option pricing model considering the continuous-time stochastic volatility.





Panel B: One-Week Ahead Hedging Errors