Modeling Infectious Mortality Risk and Its Application to the Valuation of Mortality-linked Securities

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Abstract

Due to the serious disease event such as the flu in 1918, the mortality rates in a group of nearby populations or worldwide may show a co-movement trend. In other word, a large numbers of deaths may occur at the same time caused by this catastrophic event, whereas we call it "infectious mortality risk" in this research. Nowadays, the trend of globalization and the progress of transportations may help the spread of infectious diseases across countries. Thus, we can't ignore the infectious risk, especially in pricing mortality-linked securities. This research attempts to model infectious mortality risk and investigate its effect in pricing mortality-linked securities. Using the Swiss Re mortality bond as an example, we derive a closed-form solution using Wang's transform (2000) and the corresponding fair spread of the Swiss Re bond is examined.

Keywords: Infectious Mortality Risk, Mortality Rates, Wang Transform (2000), Mortality Security, Jumps

1. Introduction

A large number of life insurance and pensions products have mortality as their primary source of risk. This means that products are exposed to unanticipated changes over time in the mortality rates of the underlying population. If future mortality improves relative to current expectations, life insurer liability decreases because death benefit payments are later than expected. However, annuity writers have a loss relative to current expectations because they have to pay annuity benefits longer than expected. If mortality deteriorates, the situation is reversed: life insurers have losses and annuity writers gain. Therefore, efficient mortality risk modeling for pricing mortality securities is an increasingly important concern for insurers.

In the past two decades, a wide range of mortality models have been proposed and discussed for modeling the dynamics of mortality over time. In 1992, Lee and Carter pioneered modeling the central mortality rates as log-linearly correlated with a time-dependent mortality factor, and adjusted for age-specific effects using two sets of age-dependent coefficients. Further, using USA data from 1933 to 1987 as a sample, they fit the model quite well. Tuljapurkar et al. (2000), Li et al. (2004), and Lundström and Qvist (2004) used the Lee-Carter (LC) (1992) model to forecast the mortality rates of G7 countries and Sweden. However, the Lee-Carter (LC) (1992) model cannot explicitly capture structural changes and short-term catastrophic shocks which may cause mortality jumps such as earthquakes and the tsunami in southern Asia and the east African killing of 182340 people in December 2004, or co-movement trends such as 1918 worldwide flu. Thus, the Lee-Carter approach with jump shocks has been presented by Cairns et al. (2006), Cox et al. (2006), Dahl and Møller (2006), Gründl et al. (2006), Lin and Cox (2008), Kogure and Kurachi (2010), Wills and Sherris (2010), Yang et al. (2010). Alternatively, Hardy(2001), Yuen and Yang (2010), Modisett and Maboudou-Tchao (2010) employed a regime-switching model to describe the phenomenon of structural changes in mortality rates. Above all, the literature ignores the impact of one country's mortality on another country's mortality. Cox, Lin, and Wang (2006) decomposed mortality shocks into two factors, a specific factor and a common factor, and applied multivariate exponential tilting to valuations of mortality-based securities written on the mortality indices of several countries. They regard the common factor as a substantial factor which causes the co-movement of the mortality indices of many countries. However, this phenomenon cannot be shown in Figure 1 which illustrates that in 2002 SARS killed 775 people in Europe, Asia, and

America, but deaths in France, England, Italy and Taiwan did not have a significant co-movement trend in 2002. Conversely, Figure 1 obviously demonstrates that there existed a co-movement phenomenon in France, England, Italy, Switzerland and the USA during the 1918 Spanish flu which killed at least 20 million people. Therefore, Figure 1 shows the fact that there is not a co-movement trend as a common factor does not cause substantially higher mortality rates, such as SARS in 2002. Mortality rates have significant co-movement phenomenon when the common factor leads to substantially higher deaths such as during the 1918 Spanish flu, namely infectious mortality.

Despite the fact that the phenomenon of infectious mortality exists in the world, modeling infectious mortality has not been proposed in the existing literature yet. This paper extends the Cox et al. (2006) model to present a multi-infectious mortality model and uses it to price Swiss Re bond, which is the first pure mortality security issued by the Swiss Reinsurance Company in late December 2003. The literature on pricing for the Swiss Re bond has a common conclusion that the fair spread of the bond is far less than the actual price, see Cox et al. (2006), Lin and Cox (2008), Chen and Cox (2009) and Liu and Yu (2010). Cox et al. (2006) showed that the actual par spread of the Swiss Re bond was three times bigger than the fair spread[4]. Considering infectious mortality effects, this paper aims to derive an analytical solution by means of the Wang transform (2000) to reexamine the par spread of the Swiss Re bond.

2. Model Formulation

Assuming there are m countries, and each country has n_i people for $i = 1, 2, 3, ..., m$. Let $\tau_{i,j}$ denote the death time of the j^{th} person of the i^{th} country for

[4] Cox et al. (2006) show the fair par spread of the Swiss Re bond is 0.45%. The actual par spread of the bond is 1.35%.

$j = 1, 2, 3, \ldots\ldots, n_i$; N_t stands for the total number of deaths in the world at time t, also denoted as

$$N_t = \sum_{i=1}^{m}\sum_{j=1}^{n_i} D_{i,j}(t) \ \text{ with } \ D_{i,j}(t) = \begin{bmatrix} 1, & \text{if } \tau_{i,j} \le t \\ 0, & \text{o.w} \end{bmatrix}, \ i = 1, 2, 3, \ldots, m;\ j=1, 2, 3, \ldots\ldots, n_i .$$

Suppose $N_t^{(i)}$ is the total number of deaths except for the i^{th} country, equally, $N_t^{(i)} \equiv \sum_{k \ne i=1}^{m}\sum_{j=1}^{n_i} D_{k,j}(t)$; $V_t^{(i)} = \dfrac{N_t^{(i)}}{N_t}$ is the ratio of the deaths except the i^{th} country relative to the total deaths in the world, and follows the geometric Brownian motion as follows.

$$\frac{dV_t^{(i)}}{V_t^{(i)}} = \mu_{v,i}dt+\sigma_{v,i}dW_{v,t}, \tag{1}$$

in which $\mu_{v,i}$ and $\sigma_{v,i}$ denote the drift term and volatility, respectively; $W_{v,t}$ is a one dimensional standard Brownian motion under the original probability measure, P. If $V_t^{(i)}$ is higher than the threshold a, then the death rate except for the i^{th} country can affect the death rates of other countries. Suppose the jump number of the impacts of the death rate except the i^{th} country on the death rates of the other countries follows Poisson distribution with a density of $\lambda_{i,t}$ at time t, also denoted as $I(t) \triangleq \int_0^t D_s^{(i)}\, ds \sim \text{Poisson}(\lambda_{i,t})$.

Let $q_{i,t}$ represent the population mortality index of the i^{th} country at time t and follow the dynamic process as below.

$$\frac{dq_{i,t}}{q_{i,t}} = \mu_i\, dt+\sigma_i\, dW_{i,t}+(\Lambda_i - 1)\, d\Gamma_{i,t}+(\pi_i - 1)\, dI_{i,t}, \tag{2}$$

where μ_i and σ_i are constants; $W_{i,t}$ is a one dimensional standard Brownian motion under the original probability measure, P. Also, the correlation coefficient between $W_{i,t}$ and $W_{v,t}$ is denoted as $\mathrm{corr}(dW_{v,t}, dW_{i,t}) = \rho_{v,i}$. $\pi_i - 1$ is the random variable percentage in the mortality index of the i^{th} country resulting from common jumps of deaths in other countries. Assume that the natural logarithm of π_i follows normal distributions with the mean of u_{π_i} and variance of $\sigma^2_{\pi_i}$, also denoted as $\ln \pi_i \sim N(u_{\pi_i}, \sigma^2_{\pi_i})$, $\pi_i > 0, i = 1, 2, 3, ..., m$. On the other hand, $\Lambda_i - 1$ denotes the percentage in the mortality index of the i^{th} country resulting from specific jumps in deaths of the i^{th} county, and $\ln \Lambda_i \sim N(u_{\Lambda_i}, \sigma^2_{\Lambda_i})$, $\Lambda_i > 0$, $i = 1, 2, 3, ..., m$. π_i is independent of Λ_i. $I_{i,t}$ and $\Gamma_{i,t}$ are independent Poisson processes with the intensity $\lambda_{I_{i,t}}$ and λ_{Γ_i} at time t, respectively. $dI_{i,t}$ is independent of $d\Gamma_{i,t}$.

In the same vein, when $V^{(k)}_t$ is higher than the threshold (a), the death rate except in the k^{th} country can affect the death rates of other countries. Suppose the jump number of the impacts of the death rate except in the k^{th} country on the death rates of the other countries follows Poisson distribution with a density of $\lambda_{k,t}$ at time t, also denoted as $I_{k,t} \triangleq \int_0^t D^{(k)}_s ds \sim \mathrm{Poisson}(\lambda_{k,t})$, $i \neq k$.

Let $q_{k,t}$ represent the population mortality index of the k^{th} country at time t and follows the dynamic process as below.

$$\frac{dq_{k,t}}{q_{k,t}} = \mu_k dt + \sigma_k dW_{k,t} + (\Lambda_k - 1) d\Gamma_{k,t} + (\pi_k - 1) dI_{k,t}, \tag{3}$$

where μ_k and σ_k are constants; $W_{k,t}$ is a one dimensional standard Brownian motion under the original probability measure. Also, the correlation coefficients are denoted as $\mathrm{corr}(dW_{k,t}, dW_{i,t}) = \rho_{v,k}$ and $\mathrm{corr}(dW_{v,t}, dW_{k,t}) = \rho_{v,k}$ for $i, k = 1, 2, 3, \ldots, m$, $i \neq k$. $\pi_k - 1$ is the random variable percentage in the mortality index of the k^{th} country resulting from common jumps of deaths in the other countries. Assume that the natural logarithm of π_k follows normal distributions with the mean of u_{π_k} and variance of $\sigma^2_{\pi_k}$. $\pi_k > 0, k = 1, 2, 3, \ldots, m$. On the other hand, $\Lambda_k - 1$ denotes the percentage in the mortality index of the k^{th} country resulting from specific jumps in deaths of the k^{th} country, and $\ln\Lambda_k \sim N(u_{\Lambda_k}, \sigma^2_{\Lambda_k})$, $\Lambda_k > 0$, $k = 1, 2, 3, \ldots, m$. π_k is independent of Λ_k. $I_{k,t}$ and $\Gamma_{k,t}$ are independent Poisson processes with the intensity $\lambda_{I_{k,t}}$ and λ_{Γ_k} at time t, respectively. $dI_{k,t}$ is independent of $d\Gamma_{k,t}$.

From equations (2) and (3), $\ln\pi_i$ and $\ln\pi_k$ are also called the effects of infectious mortality. When the threshold (a) is infinite, the model can be reduced to Lin and Cox (2008).

3. Description and Valuation of Swiss Re Bond

3.1 Description of Swiss Re Bond

Swiss Re bond was a mortality security which can transfer mortality risk into investors in the capital market. The bond was issued by Swiss Reinsurance Company in 2003 and matured on January 1, 2007. It was a three-year deal. The principal was exposed to mortality risk. The mortality risk was defined in terms of an index based on the average annual population death rates in the US, UK, France, Italy, and Switzerland. If the index exceeded 130% of the actual 2002 level, then investors had have a reduced principal payment at maturity (T) as follows.

$$B_T = Max(1-\sum_{i=1}^{3} L_{t_i},\ 0), \tag{4}$$

with $\sum_{i=1}^{3} L_{t_i} = \dfrac{Max(Y_{\max} - 1.3Y_{t_0^-},\ 0) - Max(Y_{\max} - 1.5Y_{t_0^-},\ 0)}{0.2Y_{t_0^-}}$,

$Y_{\max} = Max(Y_{t_1}, Y_{t_2}, Y_{t_3})$ and $Y_t = (q_{1,t}^{a_1} q_{2,t}^{a_2} \ldots\ldots q_{5,t}^{a_5})^{\frac{1}{a_1+a_2+\ldots+a_5}}$.

$Y_{t_0^-}$, Y_{t_0}, Y_{t_1}, Y_{t_2} and Y_{t_3} stand for the geometric average population death rates in the US, UK, France, Italy, and Switzerland in 2002, 2003, 2004, 2005, and 2006, respectively. $a_1, a_2, \ldots, a_5$ denote the weights of population mortality indices for the US, UK, France, Italy, and Switzerland, respectively. The fair price of the bond is shown in equation (5).

$$B_0 = 400000000 \times e^{-rT} E^Q[B_T], \tag{5}$$

in which r is the riskless rate; $E_t^Q(.)$ denotes the expectation value under the risk neutral probability measure, Q, at time t. Let $K_1 = 1.3Y_{t_0^-}$ and $K_2 = 1.5Y_{t_0^-}$. Thus, L_{t_i} can be written as equation (6).

$$\sum_{i=1}^{3} L_{t_i} = \frac{Max(Y_{\max} - K_1,\ 0) - Max(Y_{\max} - K_2,\ 0)}{K_2 - K_1}. \tag{6}$$

Further, substituting equations (4) and (6) into (5) becomes equation (7).

$$B_0 = 400000000 \times e^{-rT} E^Q\left[Max\left(1 - \frac{Max(Y_{\max} - K_1, 0) - Max(Y_{\max} - K_2, 0)}{K_2 - K_1}, 0\right)\right]. \tag{7}$$

3.2 Valuation of Swiss Re Bond

Pricing derivative securities in the complete market involves replicating portfolios. This also means if there is a traded bond and stock index, then options on the stock index can be replicated by holding bonds and the index, which are priced. Swiss Re bond was a mortality derivative, but there is no efficiently traded mortality index with which to create a replicating hedge. These situations are called incomplete markets. In incomplete markets, Wang transform (2000) is a popular pricing methodology based on the following transformation: For a risk with CDF F(x) under the original probability measure, P, the risk-adjusted CDF $F^*(x)$ under the risk-neutral probability measure, Q for the pricing of the risk is given by

$$F^*(x) = \Phi(\Phi^{-1}(F(x)) + \theta), \tag{8}$$

where θ is a constant risk premium. In this section, this paper proposes a methodology to solve equation (7).

Let $X_{i,t} = \Gamma_{i,t} + I_{i,t}$, which follows the Poisson distribution with the intensity $\lambda_{X_{i,t}}$.

Suppose $\ln x_i$ follows normal distributions with the mean of u_x and variance of σ_x^2; $(x_i - 1)dX_{i,t} \overset{d}{=} (\Lambda_i - 1)\, d\Gamma_{i,t} + (\pi_i - 1)\, dI_{i,t}$. Therefore,

$$E\left[(x_i - 1)dX_{i,t}\right] = E\left[(\Lambda_i - 1)\, d\Gamma_{i,t} + (\pi_i - 1)\, dI_{i,t}\right], \tag{9}$$

$$Var\left[(x_i - 1)dX_{i,t}\right] = Var\left[(\Lambda_i - 1)\, d\Gamma_{i,t} + (\pi_i - 1)\, dI_{i,t}\right]. \tag{10}$$

From equations (9) and (10), one can obtain

$$E\left[x_i - 1\right] = \frac{(e^{u_{\pi_i} + \frac{1}{2}\sigma_{\pi_i}^2} - 1)\lambda_{I_{i,t}} + (e^{u_{\Lambda_i} + \frac{1}{2}\sigma_{\Lambda_i}^2} - 1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i} + \lambda_{I_{i,t}}},$$

$$Var(x_i - 1) = \frac{A + \left[(e^{u_{\pi_i} + \frac{1}{2}\sigma_{\pi_i}^2} - 1)\lambda_{I_{i,t}} + (e^{u_{\Lambda_i} + \frac{1}{2}\sigma_{\Lambda_i}^2} - 1)\lambda_{\Gamma_i}\right]^2}{(\lambda_{\Gamma_i} + \lambda_{I_{i,t}} + 1)(\lambda_{\Gamma_i} + \lambda_{I_{i,t}})} - \left[\frac{(e^{u_{\pi_i} + \frac{1}{2}\sigma_{\pi_i}^2} - 1)\lambda_{I_{i,t}} + (e^{u_{\Lambda_i} + \frac{1}{2}\sigma_{\Lambda_i}^2} - 1)\lambda_{\Gamma_i}}{\lambda_{\Gamma_i} + \lambda_{I_{i,t}}}\right]^2,$$

$$A=\left[e^{2u_{\Lambda_i}+\sigma_{\Lambda_i}^2}(e^{\sigma_{\Lambda_i}}-2e^{-u_{\Lambda_i}-\frac{1}{2}\sigma_{\Lambda_i}^2}+2)+1\right](\lambda_{\mathrm{T}_i}+\lambda_{\mathrm{T}_i}^2)-\lambda_{\mathrm{T}_i}^2\left(e^{u_{\Lambda_i}+\frac{1}{2}\sigma_{\Lambda_i}^2}-1\right)^2$$
$$+\left[e^{u_{\pi_i}+\sigma_{\pi_i}^2}(e^{\sigma_{\pi_i}}+1)+\left(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1\right)^2\right]\left[\lambda_{\mathrm{I}_{i,,t}}+\lambda_{\mathrm{I}_{i,,t}}^2\right]-\left[(e^{u_{\pi_i}+\frac{1}{2}\sigma_{\pi_i}^2}-1)\lambda_{\mathrm{I}_{i,,t}}\right]^2.$$

Under probability measure, P, equation (2) can be rewritten as

$$\mathrm{q}_{\mathrm{i,T}}=\mathrm{q}_{\mathrm{i,t_0}}\,\mathrm{e}^{(\mu_\mathrm{i}-\frac{1}{2}\sigma_\mathrm{i}^2)(\mathrm{T-t_0})+\sigma_\mathrm{i}\mathrm{W}_{\mathrm{i,T-t0}}}\prod_{\mathrm{l=1}}^{\mathrm{X_{i,T}}}\mathrm{x}_{\mathrm{i,l}}\ ,\quad \text{i=US, UK, Swiss, Italy, France,}$$

$$\text{or}\quad \ln q_{i,T}=\ln q_{i,t_0}+(\mu_i-\frac{1}{2}\sigma_i^2)(T-t_0)+\sigma_i W_{i,T-t_0}+\sum_{l=1}^{\mathrm{X}_{i,T}}x_{i,l}\quad . \tag{11}$$

Furthermore, one can obtain

$$\ln\mathrm{Y}_T=\ln\mathrm{Y}_{t_0}+\mu_y(T-t_0)+\sigma_y W_{T-t_0}+\frac{1}{a_1+a_2+...+a_5}\left[a_1\sum_{l=1}^{\mathrm{X}_{1,T}}\ln\mathrm{Z}_{1,l}+a_2\sum_{l=1}^{\mathrm{X}_{2,T}}\ln\mathrm{Z}_{2,l}+...+a_5\sum_{l=1}^{\mathrm{X}_{5,T}}\ln\mathrm{Z}_{5,l}\right], \tag{12}$$

with $\mu_y=\frac{1}{a_1+a_2+...+a_5}\sum_{i=1}^{5}(\mu_i-\frac{1}{2}\sigma_i^2)$, $\sigma_y W_t=\frac{1}{5}\sum_{i=1}^{5}\sigma_i W_{i,t}$, and

$$\sigma_y=\frac{1}{a_1+a_2+...+a_5}\sqrt{\left[a_1\sigma_1\ \ a_2\sigma_2\ \ a_3\sigma_3\ \ a_4\sigma_4\ \ a_5\sigma_5\right]\begin{pmatrix}1 & \cdots & \rho_{15}\\ \vdots & \ddots & \vdots\\ \rho_{51} & \cdots & 1\end{pmatrix}\left[a_1\sigma_1\ \ a_2\sigma_2\ \ a_3\sigma_3\ \ a_4\sigma_4\ \ a_5\sigma_5\right]'}.$$

Suppose $\mathrm{X}_t=\mathrm{X}_{1,t}+\mathrm{X}_{2,t}+....+\mathrm{X}_{5,t}$. X_t follows the Poisson distribution with the intensity λ_t , $\lambda_t=\sum_{i=1}^{5}(\lambda_{\mathrm{T}_i}+\lambda_{\mathrm{I}_{i,,t}})$.

Let $\sum_{l=1}^{\mathrm{X}_t}\ln\mathrm{Z}_l=a_1\sum_{l=1}^{\mathrm{X}_{1,t}}\ln x_{1,l}+a_2\sum_{l=1}^{\mathrm{X}_{2,t}}\ln x_{2,l}+...+a_5\sum_{l=1}^{\mathrm{X}_{5,t}}\ln x_{5,l}$, and $\ln\mathrm{Z}\sim\mathrm{N}(u_z,\sigma_z^2)$.

Given $\mathrm{X}_t=s$, $\ln\mathrm{Z}_t\,|\,\mathrm{X}_t=s$ has a normal distribution with mean u_z and variance σ_z^2, in which there is $u_z=\frac{s_1}{s}u_{\pi_1}+\frac{s_2}{s}u_{\pi_2}+...+\frac{s_5}{s}u_{\pi_5}$ and $\sigma_z^2=\frac{s_1}{s}\sigma_{\pi_1}^2+\frac{s_2}{s}\sigma_{\pi_2}^2+...+\frac{s_5}{s}\sigma_{\pi_5}^2$.

In addition,

$$
\begin{aligned}
\lambda_{1_{i,,t}} &= E\left[I_{i,t}\right] \\
&= \int_0^t E\left[D_s^{(i)}\right] d\delta \\
&= \int_0^t P_r\ (V_\delta^{(i)} \geq a) d\delta \\
&= \sum_{\delta=0}^{t} \Phi\left(\frac{\ln\left(\frac{\mathrm{V}_0^{(i)}}{a}\right) + (\mu_{v,i} - \frac{1}{2}\sigma_{v,i}^2)\delta}{\sigma_{v,i}\sqrt{\delta}} \right)
\end{aligned}.
$$

Equation (12) can be written as

$$
\mathrm{Y}_T = \mathrm{Y}_{t_0}\, e^{\mu_y (T-t_0)+\sigma_y W_T + \sum_{l=1}^{\mathrm{X}_T} \ln \mathrm{Z}_l} . \tag{13}
$$

Let $S_T = \dfrac{Max(Y_{\max} - K_1, 0) - Max(Y_{\max} - K_2, 0)}{K_2 - K_1}$. Equation (7) becomes

$$
B_0 = 400000000 \times e^{-rT}\ E^Q\left[(1 - S_T) 1_{\{S_T < 1\}}\right]. \tag{14}
$$

If $Y_{\max} = Y_{t_1}$, then $S_T = \dfrac{(Y_{t_1} - K_1) 1_{\{Y_{t_1} > K_1\}} - (Y_{t_1} - K_2) 1_{\{Y_{t_1} > K_2\}}}{K_2 - K_1}$.

If $Y_{\max} = Y_{t_2}$, then $S_T = \dfrac{(Y_{t_2} - K_1) 1_{\{Y_{t_2} > K_1\}} - (Y_{t_2} - K_2) 1_{\{Y_{t_2} > K_2\}}}{K_2 - K_1}$.

If $Y_{\max} = Y_{t_3}$, then $S_T = \dfrac{(Y_{t_3} - K_1) 1_{\{Y_{t_3} > K_1\}} - (Y_{t_3} - K_2) 1_{\{Y_{t_3} > K_2\}}}{K_2 - K_1}$.

Therefore, equation (7) becomes

$$
B_0 = e^{-rT} \left\{ \begin{array}{l} E^Q\left[(1 - S_T) 1_{\{S_T < 1\}} \middle| Y_{\max} = Y_{t_1}\right] P_r^Q(Y_{\max} = Y_{t_1}) + E^Q\left[(1 - S_T) 1_{\{S_T < 1\}} \middle| Y_{\max} = Y_{t_2}\right] P_r^Q(Y_{\max} = Y_{t_2}) \\ + E^Q\left[(1 - S_T) 1_{\{S_T < 1\}} \middle| Y_{\max} = Y_{t_3}\right] P_r^Q(Y_{\max} = Y_{t_3}) \end{array} \right\}. \tag{15}
$$

Using Wang transform (2000), one can obtain

$$
P_r^Q(Y_{\max} = Y_{t_1}) = \Phi(\Phi^{-1}(P_r^p(Y_{\max} = Y_{t_1})) + \theta_1) = \Phi(\Phi^{-1}(\Phi(d_1, d_2, \rho_{1,2})) + \theta_1),
$$

$$
P_r^Q(Y_{\max} = Y_{t_2}) = \Phi(\Phi^{-1}(P_r^p(Y_{\max} = Y_{t_2})) + \theta_2) = \Phi(\Phi^{-1}(\Phi(d_3, d_4, \rho_{3,4})) + \theta_2),
$$

$$P_r^Q(Y_{\max}=Y_{t_3})=\Phi(\Phi^{-1}(P_r^p(Y_{\max}=Y_{t_3}))+\theta_3)=\Phi(\Phi^{-1}(\Phi(d_5,d_6,\rho_{5,6}))+\theta_3),$$

in which θ_1,θ_2 , and θ_3 are the risk premiums of Y_{t_1},Y_{t_2}, and Y_{t_3};

$$P_r^p(Y_{\max}=Y_{t_1})=P_r^p(Y_{t_1}>Y_{t_2},Y_{t_1}>Y_{t_3})=\Phi(d_1,d_2,\rho_{1,2}) ,$$

$$P_r^p(Y_{\max}=Y_{t_2})=P_r^p(Y_{t_2}>Y_{t_1},Y_{t_2}>Y_{t_3})=\Phi(d_3,d_4,\rho_{3,4}) ,$$

$$P_r^p(Y_{\max}=Y_{t_3})=P_r^p(Y_{t_3}>Y_{t_1},Y_{t_3}>Y_{t_2})=\Phi(d_5,d_6,\rho_{5,6}) \quad , \quad \text{with} \quad d_1=\frac{-\mu_y(t_2-t_1)}{\sigma_y\sqrt{|t_2-t_1|}} ,$$

$$d_2=\frac{-\mu_y(t_3-t_1)}{\sigma_y\sqrt{|t_3-t_1|}} \quad , \quad d_3=\frac{\mu_y(t_2-t_1)}{\sigma_y\sqrt{|t_1-t_2|}} \quad , \quad d_4=\frac{-\mu_y(t_3-t_2)}{\sigma_y\sqrt{|t_3-t_2|}} \quad , \quad d_5=\frac{\mu_y(t_3-t_1)}{\sigma_y\sqrt{|t_3-t_1|}} ,$$

$$d_6=\frac{\mu_y(t_3-t_2)}{\sigma_y\sqrt{|t_3-t_2|}} \ , \ \rho_{1,2}=corr(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}}) \ , \ \rho_{3,4}=corr(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}}) ,$$

$$\rho_{5,6}=corr(\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}}) .$$

Alternatively,

$$\begin{aligned}E^Q\left[1_{\{S_T<1\}}\middle|Y_{\max}=Y_{t_1}\right]&=P_r^Q(\frac{(Y_{t_1}-K_1)1_{\{Y_{t_1}>K_1\}}-(Y_{t_1}-K_2)1_{\{Y_{t_1}>K_2\}}}{K_2-K_1}<1)\\&=1-P_r^Q(Y_{t_1}<K_1)=1-\Phi(\Phi^{-1}(F_{Y_{t_1}}(K_1))+\theta_1)\end{aligned},$$

$$\begin{aligned}E^Q\left[1_{\{S_T<1\}}\middle|Y_{\max}=Y_{t_2}\right]&=P_r^Q(\frac{(Y_{t_3}-K_1)1_{\{Y_{t_2}>K_1\}}-(Y_{t_2}-K_2)1_{\{Y_{t_2}>K_2\}}}{K_2-K_1}<1)\\&=1-P_r^Q(Y_{t_2}<K_1)=1-\Phi(\Phi^{-1}(F_{Y_{t_2}}(K_1))+\theta_2)\end{aligned},$$

$$\begin{aligned}E^Q\left[1_{\{S_T<1\}}\middle|Y_{\max}=Y_{t_3}\right]&=P_r^Q(\frac{(Y_{t_3}-K_1)1_{\{Y_{t_3}>K_1\}}-(Y_{t_3}-K_2)1_{\{Y_{t_3}>K_2\}}}{K_2-K_1}<1)\\&=1-P_r^Q(Y_{t_3}<K_1)=1-\Phi(\Phi^{-1}(F_{Y_{t_3}}(K_1))+\theta_3)\end{aligned}$$

with

$$F_{Y_{t_1}}(K_2) = P_r^P(Y_{t_1} \le K_2) = \sum_{s=0}^{\infty} \frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!} \Phi(\frac{\ln \frac{K_2}{Y_{t_0}} - \mu_y(t_1 - t_0) - su_z}{\sqrt{\sigma_y^2(t_1 - t_0) + s\sigma_z^2}}),$$

$$F_{Y_{t_1}}(K_1) = P_r^P(Y_{t_1} \le K_1) = \sum_{s=0}^{\infty} \frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!} \Phi(\frac{\ln \frac{K_1}{Y_{t_0}} - \mu_y(t_1 - t_0) - su_z}{\sqrt{\sigma_y^2(t_1 - t_0) + s\sigma_z^2}}),$$

$$F_{Y_{t_2}}(K_2) = P_r^P(Y_{t_2} \le K_2) = \sum_{s=0}^{\infty} \frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!} \Phi(\frac{\ln \frac{K_2}{Y_{t_0}} - \mu_y(t_2 - t_0) - su_z}{\sqrt{\sigma_y^2(t_2 - t_0) + s\sigma_z^2}}),$$

$$F_{Y_{t_2}}(K_1) = P_r^P(Y_{t_2} \le K_1) = \sum_{s=0}^{\infty} \frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!} \Phi(\frac{\ln \frac{K_1}{Y_{t_0}} - \mu_y(t_2 - t_0) - su_z}{\sqrt{\sigma_y^2(t_2 - t_0) + s\sigma_z^2}}),$$

$$F_{Y_{t_3}}(K_2) = P_r^P(Y_{t_3} \le K_2) = \sum_{s=0}^{\infty} \frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!} \Phi(\frac{\ln \frac{K_2}{Y_{t_0}} - \mu_y(t_3 - t_0) - su_z}{\sqrt{\sigma_y^2(t_3 - t_0) + s\sigma_z^2}}) \qquad ,$$

$$F_{Y_{t_3}}(K_1) = P_r^P(Y_{t_3} \le K_1) = \sum_{s=0}^{\infty} \frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!} \Phi(\frac{\ln \frac{K_1}{Y_{t_0}} - \mu_y(t_3 - t_0) - su_z}{\sqrt{\sigma_y^2(t_3 - t_0) + s\sigma_z^2}}).$$

In addition,

$$E^Q\left[S_T 1_{\{S_T < 1\}} \middle| Y_{\max} = Y_{t_1} \right] = E^Q\left[\frac{(Y_{t_1} - K_1)1_{\{Y_{t_1} > K_1\}} - (Y_{t_1} - K_2)1_{\{Y_{t_1} > K_2\}}}{K_2 - K_1} 1_{\left\{ \frac{(Y_{t_1} - K_1)1_{\{Y_{t_1} > K_1\}} - (Y_{t_1} - K_2)1_{\{Y_{t_1} > K_2\}}}{K_2 - K_1} < 1 \right\}} \right],$$

$$= P_r^Q(Y_{t_1} > K_2) + E^Q\left[\frac{Y_{t_1} - K_1}{K_2 - K_1} \right] 1_{\{Y_{t_1} < K_2\}} P_r^Q(K_1 < Y_{t_1} \le K_2)$$

$$E^Q\left[S_T 1_{\{S_T < 1\}} \middle| Y_{\max} = Y_{t_2} \right] = P_r^Q(Y_{t_2} > K_2) + E^Q\left[\frac{Y_{t_2} - K_1}{K_2 - K_1} \right] 1_{\{Y_{t_2} < K_2\}} P_r^Q(K_1 < Y_{t_2} \le K_2),$$

$$E^Q\left[S_T 1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_3}\right.\right]=P_r^Q(Y_{t_3}>K_2)+E^Q\left[\frac{Y_{t_3}-K_1}{K_2-K_1}\right]1_{\{Y_{t_3}<K_2\}}P_r^Q(K_1<Y_{t_3}\le K_2)\,.$$

Using Wang transform (2000), one can obtain

$$\begin{aligned}&P_r^Q(Y_{t_i}>K_2)=\Phi(\Phi^{-1}(P_r^P(Y_{t_i}>K_2))+\theta_i)\\&=\Phi(\Phi^{-1}(\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!}\Phi(\frac{\ln(\frac{Y_{t_0}}{K_2})+\mu_y(t_i-t_0)-su_z}{\sqrt{\sigma_y^2(t_1-t_0)+s\sigma_z^2}}))+\theta_i)\end{aligned}\quad ,i=1,2,3\,,$$

$$\begin{aligned}&P_r^Q(K_1<Y_{t_i}\le K_2)=P_r^Q(Y_{t_i}\le K_2)-P_r^Q(Y_{t_i}\le K_1)\\&=\Phi(\Phi^{-1}(\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!}\Phi(\frac{\ln(\frac{K_2}{Y_{t_0}})-\mu_y(t_i-t_0)-su_z}{\sqrt{\sigma_y^2(t_1-t_0)+s\sigma_z^2}}))+\theta_i)\\&\quad-\Phi(\Phi^{-1}(\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!}\Phi(\frac{\ln(\frac{K_1}{Y_{t_0}})-\mu_y(t_i-t_0)-su_z}{\sqrt{\sigma_y^2(t_1-t_0)+s\sigma_z^2}}))+\theta_i)\end{aligned}\quad ,$$

and

$$\begin{aligned}&E^P\left[Y_{t_1}\right]=\sum_{s=0}^{\infty}E^P\left[Y_{t_1}\left|I_t=s\right.\right]P_r^P(I_t=s)\\&=Y_{t_0}\sum_{s=0}^{\infty}\frac{e^{-\lambda_t(t_1-t_0)}(\lambda_t(t_1-t_0))^s}{s\,!}e^{\mu_y(t_1-t_0)+s\,(u_z+\sigma_z^2)+\frac{1}{2}\sigma_y^2(t_1-t_0)}\end{aligned}\,.$$

Thus, the formula can be written as follows.

$$\begin{aligned}&E^Q\left[S_T 1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_i}\right.\right]=1-\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i)\\&+\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i)\times\left\{\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i)-\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_1))+\theta_i)\right\}\\&\times\left\{\frac{1}{K_2-K_1}\left[\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!}Y_{t_0}e^{\mu_y(t_i-t_0)+s\,(u_z+\sigma_z^2)+\frac{1}{2}\sigma_y^2(t_i-t_0)}+\theta_i\sqrt{Var^P(Y_{t_i})}\right]-\frac{K_1}{K_2-K_1}\right\}\end{aligned}\quad ,$$

$i=1\,,2\,,3.$

Consequently, the fair par spread of Swiss Re bond is shown in equation (16).

$$B_0=400000000\,e^{-rT}\left\{\begin{aligned}&E^Q\left[(1-S_T)1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_1}\right.\right]P_r^Q(Y_{\max}=Y_{t_1})\\&+E^Q\left[(1-S_T)1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_2}\right.\right]P_r^Q(Y_{\max}=Y_{t_2})\\&+E^Q\left[(1-S_T)1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_3}\right.\right]P_r^Q(Y_{\max}=Y_{t_3})\end{aligned}\right\},\qquad(16)$$

with $E^Q\left[1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_1}\right.\right]=1-\Phi(\Phi^{-1}(F_{Y_{t_1}}(K_1))+\theta_1)$,

$E^Q\left[1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_2}\right.\right]=1-\Phi(\Phi^{-1}(F_{Y_{t_2}}(K_1))+\theta_2)$,

$E^Q\left[1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_3}\right.\right]=1-\Phi(\Phi^{-1}(F_{Y_{t_3}}(K_1))+\theta_3)$,

$$\begin{aligned}
&E^Q\left[S_T 1_{\{S_T<1\}}\left|Y_{\max}=Y_{t_i}\right.\right]=1-\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i)\\
&+\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i)\times\left\{\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_2))+\theta_i)-\Phi(\Phi^{-1}(F_{Y_{t_i}}(K_1))+\theta_i)\right\}\\
&\times\left\{\frac{1}{K_2-K_1}\left[\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}Y_{t_0}e^{\mu_y(t_i-t_0)+s(u_z+\sigma_z^2)+\frac{1}{2}\sigma_y^2(t_i-t_0)}+\theta_i\sqrt{Var^P(Y_{t_i})}\right]-\frac{K_1}{K_2-K_1}\right\}
\end{aligned},$$

$i=1,2,3$,

$P_r^P(Y_{\max}=Y_{t_1})=\Phi(d_1,d_2,\rho_{1,2})$, $P_r^P(Y_{\max}=Y_{t_2})=\Phi(d_3,d_4,\rho_{3,4})$,

$P_r^P(Y_{\max}=Y_{t_3})=\Phi(d_5,d_6,\rho_{5,6})$,

$$d_1=\frac{-\mu_y(t_2-t_1)}{\sigma_y\sqrt{|t_2-t_1|}},\ d_2=\frac{-\mu_y(t_3-t_1)}{\sigma_y\sqrt{|t_3-t_1|}},\ \rho_{1,2}=corr\left(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}}\right),$$

$$d_3=\frac{\mu_y(t_2-t_1)}{\sigma_y\sqrt{|t_1-t_2|}},\ d_4=\frac{-\mu_y(t_3-t_2)}{\sigma_y\sqrt{|t_3-t_2|}},\ \rho_{3,4}=corr\left(\frac{W_{t_2-t_1}}{\sqrt{|t_2-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}}\right),$$

$$d_5=\frac{\mu_y(t_3-t_1)}{\sigma_y\sqrt{|t_3-t_1|}},\ d_6=\frac{\mu_y(t_3-t_2)}{\sigma_y\sqrt{|t_3-t_2|}},\ \rho_{5,6}=corr\left(\frac{W_{t_3-t_1}}{\sqrt{|t_3-t_1|}},\frac{W_{t_3-t_2}}{\sqrt{|t_3-t_2|}}\right),$$

$$E^P(Y_{t_i})=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}Y_{t_0}e^{\mu_y(t_i-t_0)+s(u_z+\sigma_z^2)+\frac{1}{2}\sigma_y^2(t_i-t_0)},$$

$$Var^P(Y_{t_i})=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}Y_{t_0}^2e^{2\mu_y(t_i-t_0)+2su_z+2\sigma_y^2(t_i-t_0)}-\left\{\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}Y_{t_0}e^{2\mu_y(t_i-t_0)+2s(u_z+\sigma_z^2)+\frac{1}{2}\sigma_y^2(t_i-t_0)}\right\}^2,$$

$$E^Q\left[Y_{t_i}\right]=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}Y_{t_0}e^{\mu_y(t_i-t_0)+s(u_z+\sigma_z^2)+\frac{1}{2}\sigma_y^2(t_i-t_0)}+\theta_i\sqrt{Var^P(Y_{t_i})},\ i=1,2,3,$$

$$F_{Y_{t_i}}(K_1)=\sum_{s=0}^{\infty}\frac{e^{-\lambda_t}(\lambda_t)^s}{s!}\Phi\left(\frac{\ln\frac{K_1}{Y_{t_0}}-\mu_y(t_i-t_0)-su_z}{\sqrt{\sigma_y^2(t_i-t_0)+s\sigma_z^2}}\right),$$

$$F_{Y_{t_i}}(K_2) = P_r^P(Y_{t_i} \le K_2) = \sum_{s=0}^{\infty} \frac{e^{-\lambda_t}(\lambda_t)^s}{s\,!} \Phi\left(\frac{\ln\frac{K_2}{Y_{t_0}} - \mu_y(t_i - t_0) - su_z}{\sqrt{\sigma_y^2(t_i - t_0) + s\sigma_z^2}}\right),\ i = 1\,,2\,,3\,.$$

4. Empirical Results

In this section, HMD (Human Mortality Data) is used to estimate the parameters $(\mu_i\,,\ \sigma_i\,,\ u_{\pi_i}\,, \sigma_{\pi_i}\,, \mu_{v,i}\,,\ \sigma_{v,i})$ for the US, UK, France, Italy, and Switzerland. The time window is the period from 1933 to 2007. Further, the fair par spread of Swiss Re bond was obtained through equation (16), and comparative statics were performed.

4.1 Estimation of Model Parameters

A calibration approach was adopted to estimate the variables $(\mu_i\,,\ \sigma_i\,,\ u_{\pi_i}\,, \sigma_{\pi_i}\,, \mu_{v,i}\,,\ \sigma_{v,i})$ for the US, UK, France, Italy, and Switzerland. The term calibration indicates the task of estimating best-fitted parameters in a parametric model in comparison with a chosen observable quantity. The source of information consists typically of historical data for liquid instruments. The procedure of fitting prices was based on the assumption that a trader agreed with the view that the historical data were fully consistent with a true process. Different jump-diffusion processes were calibrated using actual log returns of the population mortality index for each country. The detailed procedure is described in the following steps.

(1) Collect the actual log returns of the population mortality indexes of the US, UK, France, Italy, and Switzerland. $d(\ln \hat{q}_{i,t})$ was the model log returns of the five countries from equation (11), $d(\ln q_{i,t})$ the observed log returns of the population mortality index of each country. The differences of $d(\ln q_{i,t}) - d(\ln \hat{q}_{i,t})$ was a function of the values taken by $\Theta = (\mu_i\,,\ \sigma_i\,,\ u_{\pi_i}\,, \sigma_{\pi_i}\,, \mu_{v,i}\,,\ \sigma_{v,i})$.

(2) Given the initial values of $(\mu_i\,,\ \sigma_i\,,\ u_{\pi_i}\,, \sigma_{\pi_i}\,, \mu_{v,i}\,,\ \sigma_{v,i})$, the parameter vector Θ

was found to solve the nonlinear the sum of the squared errors during Period I and Period II as follows:

$$SSE = \min_{\Theta} \sum_{j=1}^{n} \left| \varepsilon_j [\Theta] \right|^2$$

Through the above procedure, the estimated parameters are disclosed in Tables 1 and 2. Based on equation (16), the fair par spread of Swiss Re bond was found to be 1.0958973% which is higher than that of Cox et al. (2006), and closes to the actual par spread of 1.35%. This shows that considering infectious effects of mortality rates enables the par spread Swiss Re bond to fit the real world.

4.2 Analysis of Comparative Statics

Given that the risk premiums for Swiss Re bond presented by Cox et al. (2006), Lin and Cox (2008), Chen and Cox (2009) and Lin, Liu and Yu (2010) were 0.83, 0.8657, 1.5, and 1.21, respectively, the impacts of mean and volatility on the magnitudes of infectious mortality, the threshold values (a), and jump intensities on the par spread of Swiss Re bond are demonstrated in Table 3.

Table 3 shows a common phenomenon, that the fair spread of the Swiss Re bond decrease as mortality increased. In Panel A, the impacts of mean of the magnitudes of infectious mortality on the par spread of the Swiss Re bond are uncertain. However, the par spreads of the Swiss Re bond decrease as volatilities of the magnitudes of infectious mortality increased on account of the higher mortality rates as demonstrated in Panel B.

Panel C illustrates that the relationship of the threshold values and the par spreads of the Swiss Re bond is positive. This reason is that the higher the threshold values are, the lower infectious mortality is. Conversely, Panel D explains that when the jump intensities increase, mortality rates increase so that the par spread of the bond decline.

Also, the sensitivity of volatilities of magnitudes of infectious mortality is the largest among model parameters, whereas that of the threshold values is the smallest.

5. Conclusion

Previously much literature studied mortality rates with jumps such as Cairns et al. (2006), Cox et al. (2006), Dahl and Møller (2006), Gründl et al. (2006), Lin and Cox (2008), Kogure and Kurachi (2010), Wills and Sherris (2010), Yang et al. (2010). General speaking, this literature explains the co-movement of mortality rates using common jumps in the world. However, actual data report from when an event occurs show that mortality rates do not significantly co-move in the world until large deaths occur. There is no existing literature to model the real phenomenon of mortality rates. This paper fills the gap by offering a fresh look at the infectious effects of mortality rates on the valuation of mortality securities.

From empirical results, the fair par spread of the Swiss Re bond in the model was found to be far higher than that of Cox et al. (2006), and closer to the actual par spread. This shows that considering infectious effects of mortality rates enables the par spread Swiss Re bond to fit into the real world. This is helpful to price mortality securities for insurance issuers.

Table 1 Parameter Estimation of Dynamic Processes of the Mortality Index of the 5 Countries

	US	UK	France	Italy	Switzerland
μ_i	-0.007635698	-0.0019212853	-0.0023350499	-0.0022268845	-0.0017903048
σ_i	0.0353039713	0.0303390187	0.0208596021	0.0346345882	0.0041059500
u_{π_i}	-0.4080070179	-0.0685499403	-0.0480721838	-0.0797427948	-0.0545940218
σ_{π_i}	0.1727495194	0.0287313469	0.0201543124	0.00328499675	0.0227485367

Note that i=US, UK, France, Italy, and Switzerland, $a_1 = 0.7$, $a_2 = 0.15$, $a_3 = 0.025$,

$a_4 = 0.05$, $a_5 = 0.075$.

Table 2 Parameter Estimation of Dynamic Processes of the Ratio of the Deaths Except in the i^{th} country

	US	UK	France	Italy	Switzerland
$\mu_{v,i}$	-0.0002826201	-0.0018850476	-0.0021297657	-0.0018799084	-0.0004465767
$\sigma_{v,i}$	0.0184113032	0.0126177338	0.0093617168	0.0124682592	0.0122047626

Note that i=US, UK, France, Italy, and Switzerland, $a_1 = 0.7$, $a_2 = 0.15$, $a_3 = 0.025$,

$a_4 = 0.05$, $a_5 = 0.075$.

Table 3 Impacts of Various Important Model Parameters on Swiss Re Bond

Parameter	$\theta = 0.83$	$\theta = 0.8657$	$\theta = 1.21$	$\theta = 1.5$
u_z	Panel A: u_z changes			
-0.001	0.5163	0.5196	0.5897	0.6125
-0.003	0.4987	0.5011	0.5734	0.5813
-0.005	0.4593	0.4972	0.5539	0.5712
-0.007	0.4886	0.5313	0.5618	0.5896
-0.009	0.5098	0.5478	0.5715	0.5947
σ_z	Panel B: σ_z changes			
0.1	0.9125	0.9237	0.9358	0.9399
0.2	0.8143	0.8168	0.8915	0.9141

0.3	0.7759	0.7825	0.8598	0.8611
0.4	0.5647	0.5998	0.6315	0.7014
0.5	0.3325	0.3985	0.4918	0.5481
a	Panel C: a changes			
0.01	0.8169	0.8198	0.8245	0.8266
0.02	0.8256	0.8267	0.8309	0.8351
0.03	0.8321	0.8357	0.8401	0.8416
0.04	0.8395	0.8400	0.8415	0.8423
0.05	0.8411	0.8425	0.8438	0.8509
λ_t	Panel D: λ_t changes			
0.01	0.6458	0.6511	0.6715	0.6798
0.02	0.6135	0.6212	0.6598	0.6613
0.03	0.5123	0.5237	0.5997	0.6011
0.04	0.4978	0.5198	0.5498	0.5599
0.05	0.4569	0.4986	0.5058	0.5149

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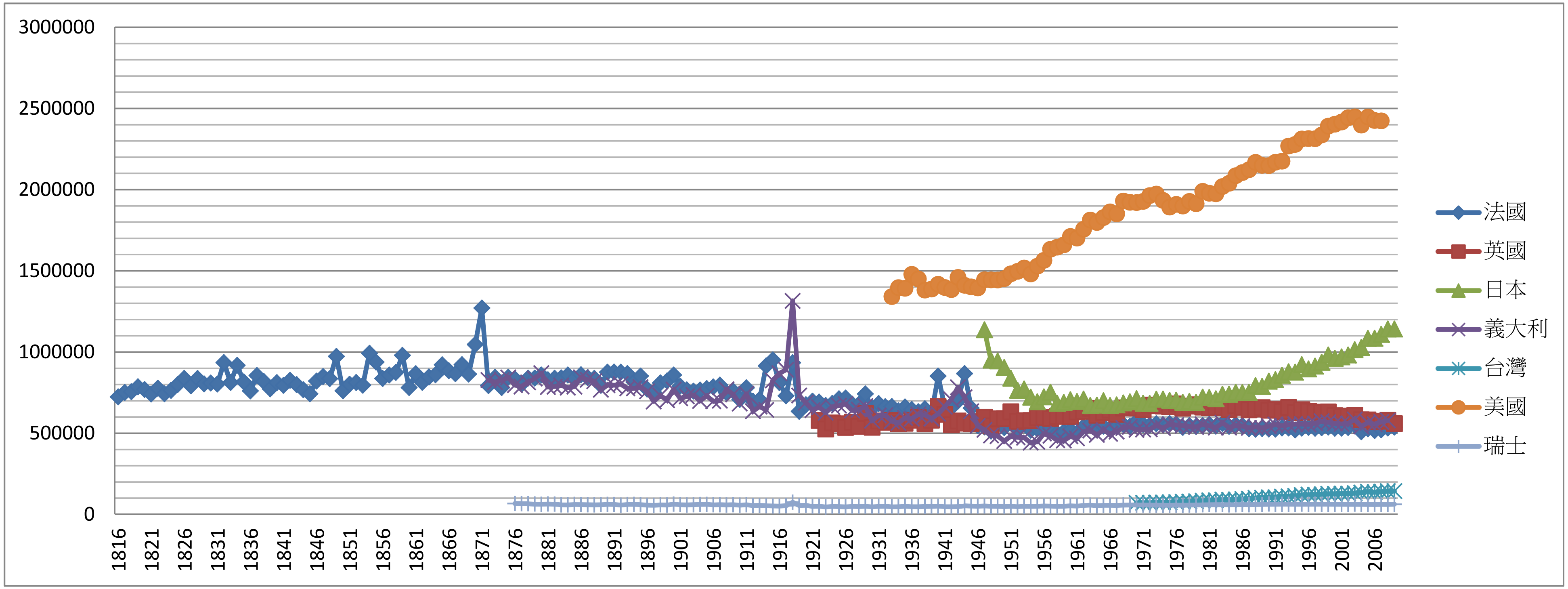

Figure 1 The Deaths in the World from 1816 to 2006